Week 4: Value Function Approximation and Deep Q-Learning

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Announcement

1. Assignment 1 was due by the end of last week
2. Assignment 2 is released at `https://github.com/ucla-rlcourse/cs260r-assignment-2026winter/`
    1. Please start early, due by the end of next week.
3. The instruction for redeeming the Google Cloud credit is sent out
    1. TAs will give a tutorial on how to properly use the credit in this week's discussion

This Week's Plan

1. Introduction to function approximation
2. Value function approximation for prediction
3. Value function approximation for control
4. Batch RL and least square prediction and control
5. Deep Q-Learning: Entering the modern RL world

Introduction: Scaling up RL from Tabular Setting

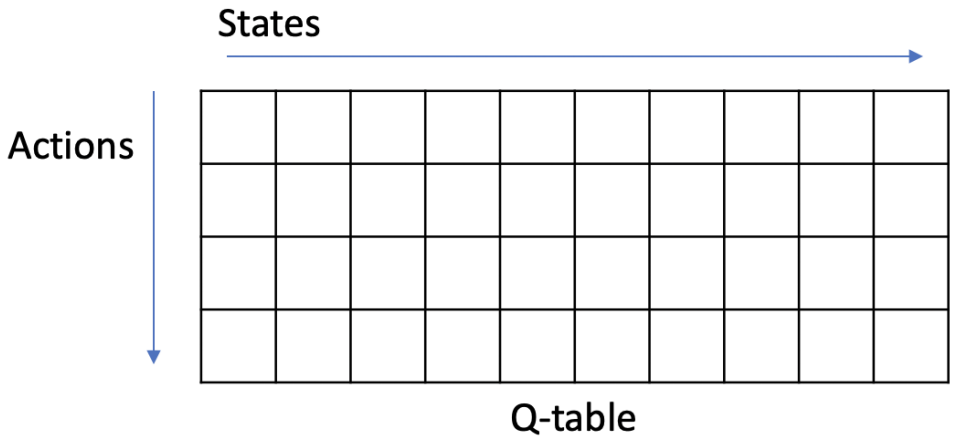
1. Previous lectures on small (tabular) RL problems:
   1. Cliff walk: 4×16 states
   2. Mountain car: 1600 states
   3. Tic-Tac-Toe: 10^3 states
2. Large-scale problems:
   1. Backgammon: 10^{20} states
   2. Chess: 10^{47} states
   3. Game of Go: 10^{170} states
   4. Robot Arms and Helicopters have continuous state space
   5. Number of atomics in universe: 10^{80}

hard to explore state space with decision tree

3. Challenge: How can we scale up the model-free methods for prediction and control to large-scale problems?

Introduction: Scaling up RL from Tabular Setting

1. In tabular methods we represent value function by a lookup table:
   1. Every state s has an entry $V(s)$
   2. Every state-action pair s, a has an entry $Q(s, a)$



2. Challenges with large MDPs:
   1. too many states or actions to store in memory
   2. too slow to learn the value of each state individually

Scaling up RL with Function Approximation

1. How to avoid explicitly learning or storing for every single state:
   1. Dynamics or reward model
   2. Value function, state-action function
   3. Policy
2. Solution: Estimate with function approximation

if we don't have all state action pairs, a function approximator can estimate some of the missing values using the ones we know

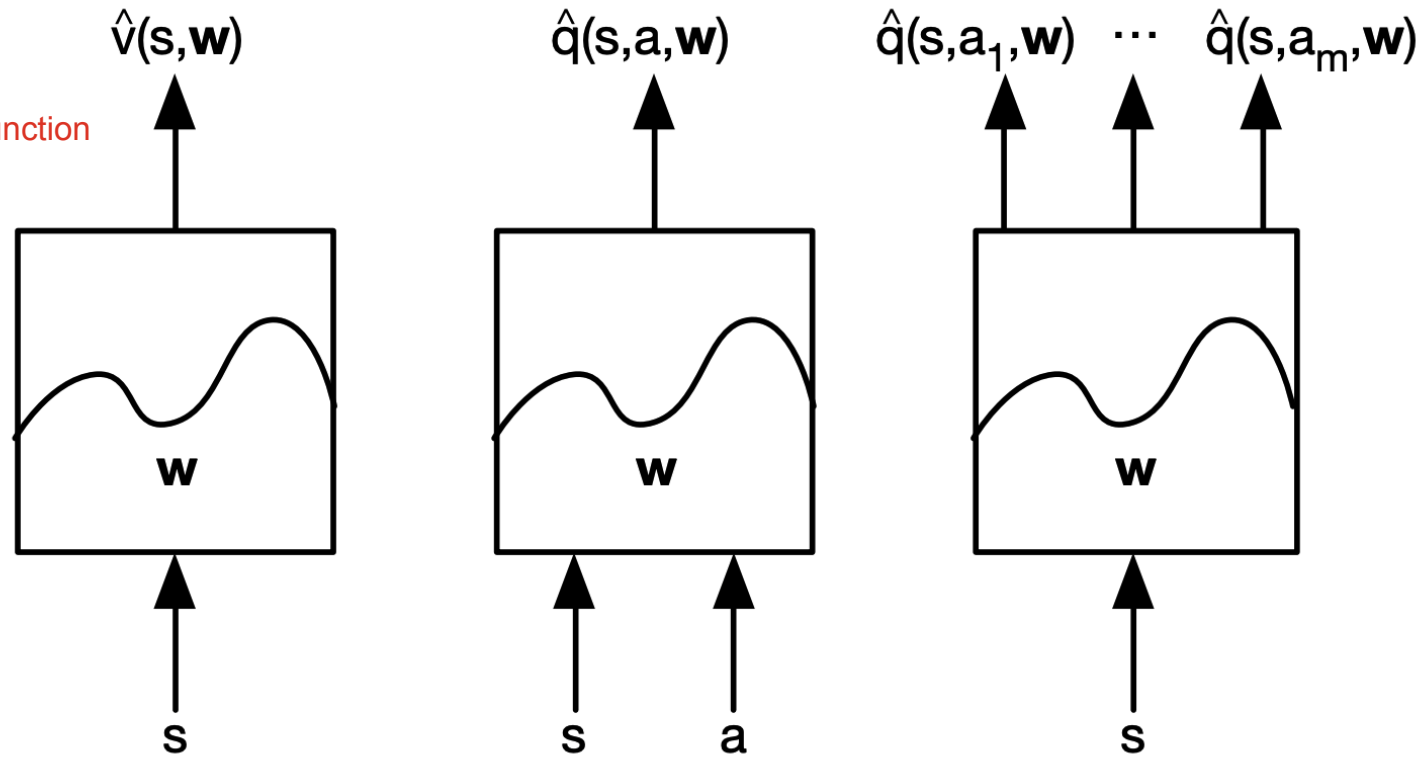
$$\hat{v}(s, \mathbf{w}) \approx v^{\pi}(s)$$
$$\hat{q}(s, a, \mathbf{w}) \approx q^{\pi}(s, a)$$
$$\hat{\pi}(a, s, \mathbf{w}) \approx \pi(a|s)$$

1. Generalize from seen states to unseen states
2. Update the parameter $\mathbf{w}$ using MC or TD learning

Types of value function approximation

Several function designs:

input some states
regress to value function



inefficient because we have to loop thru each possible action

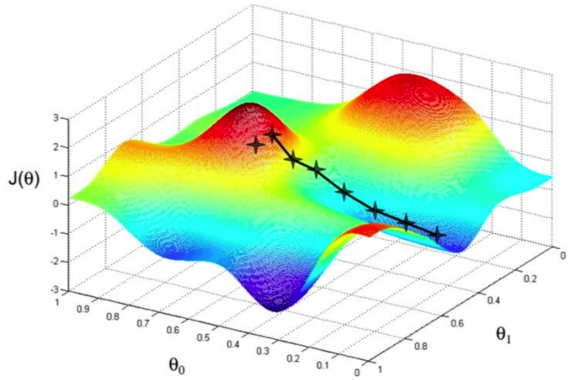
just input state and get the q-values of the possible actions we want to take

Function Approximators

where f(s) = [w_1, w_2, ...] or [f_1(s), f_2(s), f_k(s)]

1. Many possible function approximators:
   1. Linear combinations of features v(s) = \sum_k f_k(s)
   2. Neural networks does feature learning on its own
   3. Decision trees
   4. Nearest neighbors
2. We will focus on function approximators that are differentiable
   1. Linear feature representations
   2. Neural networks

Review on Gradient Descend



1. Consider a function $J(\mathbf{w})$ that is a differentiable function of a parameter vector $\mathbf{w}$
2. Goal is to find parameter $\mathbf{w}^*$ that minimizes J
3. Define the gradient of $J(\mathbf{w})$ to be

$$\nabla_{\mathbf{w}} J(\mathbf{w}) = \left(\frac{\partial J(\mathbf{w})}{\partial w_1}, \frac{\partial J(\mathbf{w})}{\partial w_2}, ..., \frac{\partial J(\mathbf{w})}{\partial w_n}\right)^T$$

4. Adjust $\mathbf{w}$ in the direction of the negative gradient, where α is step-size

$$\Delta \mathbf{w} = -\frac{1}{2}\alpha \nabla_{\mathbf{w}} J(\mathbf{w})$$

Value Function Approximation with an Oracle

assume we know true values for v^{\pi}

1. We assume that we have the oracle for knowing the true value for $v^{\pi}(s)$ for any given state s
2. Then the objective is to find the best approximate representation of $v^{\pi}(s)$
3. Thus use the mean squared error and define the loss function as

original value function

$$J(\mathbf{w}) = \mathbb{E}_{\pi}\left[(v^{\pi}(s) - \hat{v}(s, \mathbf{w}))^2\right]$$

use linear/neural function approximator

4. Follow the gradient descend to find a local minimum

$$\Delta \mathbf{w} = -\frac{1}{2}\alpha \nabla_{\mathbf{w}} J(\mathbf{w})$$

$$\mathbf{w}_{t+1} = \mathbf{w}_t + \Delta \mathbf{w}$$

Representing State with Feature Vectors

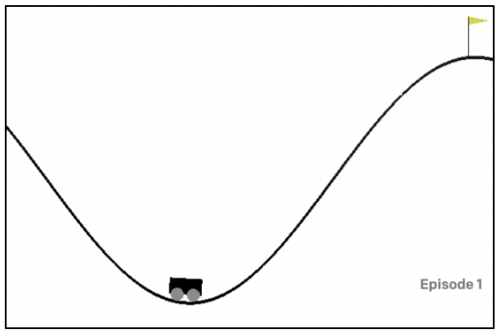
1. Represent state using a feature vector

$$\mathbf{x}(s) = \left(x_1(s), ..., x_n(s)\right)^T$$

in many environments, state is already represented as a vector

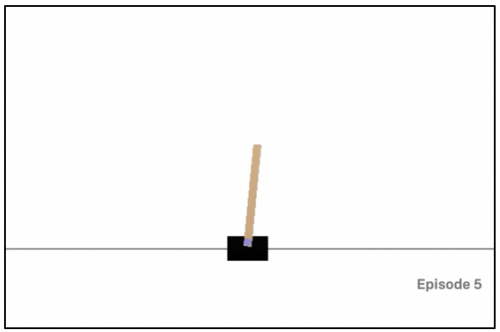
2. For example:

Mountain Car



Position of car, velocity of car

Cart Pole



Position of cart, velocity of cart, angle of pole, rotation rate of pole

Game of Go in AlphaGo

Extended Data Table 2 | Input features for neural networks

Feature	# of planes	Description
Stone colour	3	Player stone / opponent stone / empty
Ones	1	A constant plane filled with 1
Turns since	8	How many turns since a move was played
Liberties	8	Number of liberties (empty adjacent points)
Capture size	8	How many opponent stones would be captured
Self-atari size	8	How many of own stones would be captured
Liberties after move	8	Number of liberties after this move is played
Ladder capture	1	Whether a move at this point is a successful ladder capture
Ladder escape	1	Whether a move at this point is a successful ladder escape
Sensibleness	1	Whether a move is legal and does not fill its own eyes
Zeros	1	A constant plane filled with 0
Player color	1	Whether current player is black

48 places of 19x19 feature maps

Linear Value Function Approximation

1. Represent value function by a linear combination of features

$$\hat{v}(s, \mathbf{w}) = \mathbf{x}(s)^T \mathbf{w} = \sum_{j=1}^{n} x_j(s) w_j$$

2. The objective function is quadratic in parameter $\mathbf{w}$

$$J(\mathbf{w}) = \mathbb{E}_\pi \left[(v^\pi(s) - \mathbf{x}(s)^T \mathbf{w})^2 \right]$$

3. Thus the update rule is as simple as

step size, prediction error, feature value

$$\Delta \mathbf{w} = \alpha (v^\pi(s) - \hat{v}(s, \mathbf{w})) \mathbf{x}(s)$$

Update = *StepSize* × *PredictionError* × *FeatureValue*

4. Stochastic gradient descent converges to global optimum. Because in the linear case, there is only one optimum, thus local optimum is automatically converge to or near the global optimum.

Linear Value Function Approximation with Table Lookup Feature

1. Table lookup is a special case of linear value function approximation
2. Table lookup feature is one-hot vector as follows

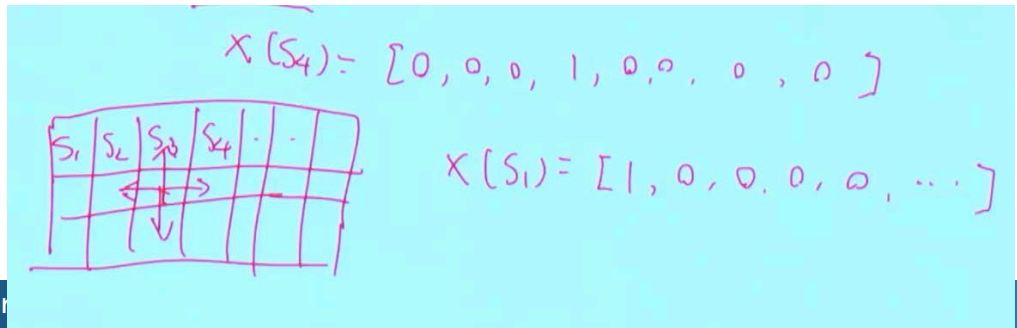
$$\mathbf{x}^{table}(S) = \big(\mathbf{1}(S = s_1), ..., \mathbf{1}(S = s_n)\big)^T$$

one hot vector will indicate where the location of the agent is?

3. Then we can see that each element on the parameter vector $\mathbf{w}$ indicates the value of each individual state

$$\hat{v}(S, \mathbf{w}) = \big(\mathbf{1}(S = s_1), ..., \mathbf{1}(S = s_n)\big)\big(w_1, ..., w_n\big)^T$$

4. Thus we have $\hat{v}(s_k, \mathbf{w}) = w_k$



Value Function Approximation for Model-free Prediction

1. In practice, no access to oracle of the true value $v^{\pi}(s)$ for any state s
2. Recall model-free prediction estimate v^\pi using some fixed policy \pi
   1. Goal is to evaluate v^{π} following a fixed policy π
   2. A lookup table is maintained to store estimates v^{π} or q^{π}
   3. Estimates are updated after each episode (MC method) or after each step (TD method)
3. Thus what we can do is to **include the function approximation step in the loop**

Incremental VFA Prediction Algorithms

1. We assumed that true value function $v^\pi(s)$ given by supervisor/oracle

$$\Delta \mathbf{w} = \alpha \Big(v^\pi(S) - \hat{v}(S, \mathbf{w}) \Big) \nabla_{\mathbf{w}} \hat{v}(S_t, \mathbf{w})$$

2. But in RL there is no supervisor, only rewards
3. In practice, we substitute the target for $v^\pi(s)$
   1. For MC, the target is the actual return G_t

$$\Delta \mathbf{w} = \alpha \Big(G_t - \hat{v}(S_t, \mathbf{w}) \Big) \nabla_{\mathbf{w}} \hat{v}(S_t, \mathbf{w})$$

   2. For TD(0), the target is the TD target $R_{t+1} + \gamma \hat{v}(S_{t+1}, \mathbf{w})$

$$\Delta \mathbf{w} = \alpha \Big(R_{t+1} + \gamma \hat{v}(S_{t+1}, \mathbf{w}) - \hat{v}(S_t, \mathbf{w}) \Big) \nabla_{\mathbf{w}} \hat{v}(S_t, \mathbf{w})$$

this works because G_t/TD Target is a noisy estimate for v^\pi

Monte-Carlo Prediction with VFA

1. Return G_t is an unbiased, but noisy sample of true value $v^\pi(S_t)$
2. Why unbiased? $\mathbb{E}[G_t] = v^\pi(S_t)$
3. So we have the training data that can be used for supervised learning in VFA:

$$< S_1, G_1 >, < S_2, G_2 >, ..., < S_T, G_T >$$

4. Using linear Monte-Carlo policy evaluation

error between current return and actual observed return

$$\begin{aligned}\Delta \mathbf{w} =& \alpha \Big(G_t - \hat{v}(S_t, \mathbf{w}) \Big) \nabla_{\mathbf{w}} \hat{v}(S_t, \mathbf{w}) \\ =& \alpha \Big(G_t - \hat{v}(S_t, \mathbf{w}) \Big) \mathbf{x}(S_t)\end{aligned}$$

5. Monte-Carlo prediction converges, in both linear and non-linear value function approximation.

TD Prediction with VFA

1. TD target $R_{t+1} + \gamma \hat{v}(S_{t+1}, \mathbf{w})$ is a biased estimate of true value $v^{\pi}(S_t)$
   current estimate is not the true value, biased because we are using our own weights to generate value
2. Why biased? It is drawn from our previous estimate, rather than the true value: $\mathbb{E}[R_{t+1} + \gamma \hat{v}(S_{t+1}, \mathbf{w})] \neq v^{\pi}(S_t)$
3. We have the training data used for supervised learning in VFA:

$$< S_1, R_2 + \gamma \hat{v}(S_2, \mathbf{w}) >, < S_2, R_3 + \gamma \hat{v}(S_3, \mathbf{w}) >, ..., < S_{T-1}, R_T >$$

4. Using linear TD(0), the stochastic gradient descend update is

   gradient = how to adjust weights to reduce err

   gradient contains w

   (target - curr prediction)

$$\begin{aligned}\Delta \mathbf{w} &= \alpha \Big(R + \gamma \hat{v}(S', \mathbf{w}) - \hat{v}(S, \mathbf{w}) \Big) \nabla_{\mathbf{w}} \hat{v}(S, \mathbf{w}) \\ &= \alpha \Big(R + \gamma \hat{v}(S', \mathbf{w}) - \hat{v}(S, \mathbf{w}) \Big) \mathbf{x}(S)\end{aligned}$$

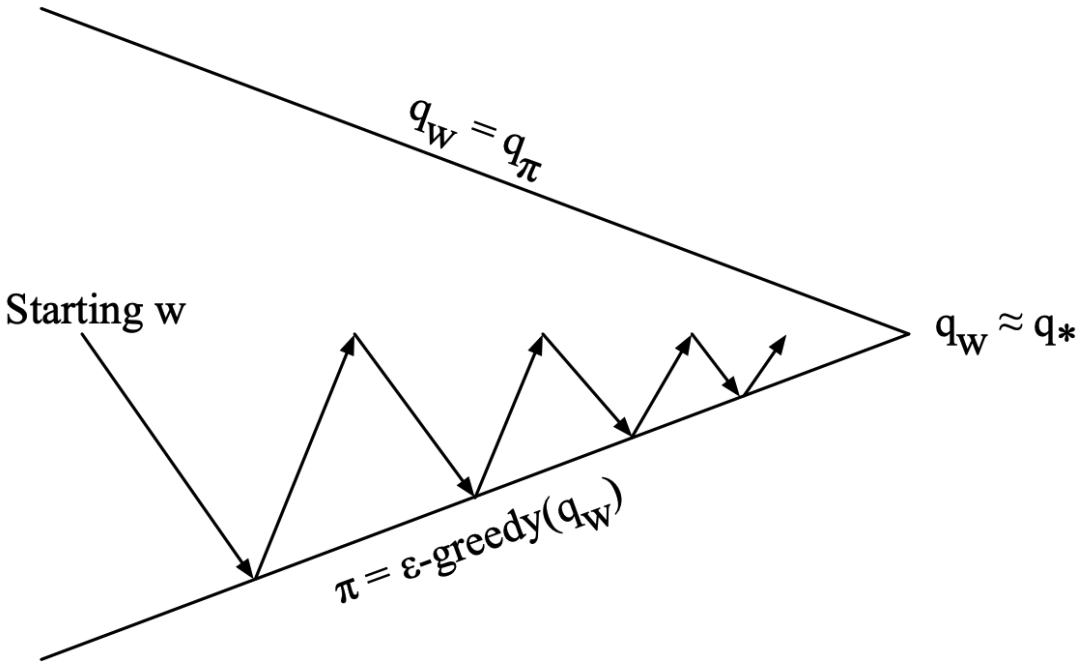
   simplifies to just the feature vector if using lin. approx

   need to call value function approximator twice

   This is also called as semi-gradient, as we ignore the effect of changing the weight vector $\mathbf{w}$ on the target (treat R + \gamma vhat(S', w)) as fixed
5. Linear TD(0) converges(close) to global optimum

Control with Value Function Approximation

Generalized policy iteration



1. Policy evaluation: approximate policy evaluation, $\hat{q}(.,.,\mathbf{w}) \approx q^\pi$
2. Policy improvement: ϵ-greedy policy improvement

Action-Value Function Approximation

1. Approximate the action-value function

$$\hat{q}(s, a, \mathbf{w}) \approx q^{\pi}(s, a)$$

2. Minimize the MSE (mean-square error) between approximate action-value and true action-value (assume oracle)

$$J(\mathbf{w}) = \mathbb{E}_{\pi}[(q^{\pi}(s, a) - \hat{q}(s, a, \mathbf{w}))^2]$$

3. Stochastic gradient descend to find a local minimum

$$\Delta \mathbf{w} = \alpha(q^{\pi}(s, a) - \hat{q}(s, a, \mathbf{w}))\nabla_{\mathbf{w}}\hat{q}(s, a, \mathbf{w})$$

linear, x(s, a)

Linear Action-Value Function Approximation

1. Represent state and action using a feature vector

$$\mathbf{x}(s,a) = \left(x_1(s,a), ..., x_n(s,a)\right)^T$$

2. Represent action-value function by a linear combination of features

$$\hat{q}(s,a,\mathbf{w}) = \mathbf{x}(s,a)^T\mathbf{w} = \sum_{j=1}^{n} x_j(s,a)w_j$$

3. Thus the stochastic gradient descend update

$$\Delta\mathbf{w} = \alpha(q^\pi(s,a) - \hat{q}(s,a,\mathbf{w}))\mathbf{x}(s,a)$$

Incremental Control Algorithm

Same to the prediction, there is no oracle for the true value $q^\pi(s, a)$, so we substitute a target

replace true value with the estimated value in each of the methods

1. For MC, the target is the return G_t

$$\Delta \mathbf{w} = \alpha\big(G_t - \hat{q}(S_t, A_t, \mathbf{w})\big)\nabla_{\mathbf{w}}\hat{q}(S_t, A_t, \mathbf{w})$$

2. For Sarsa, the target is the TD target $R_{t+1} + \gamma\hat{q}(S_{t+1}, A_{t+1}, \mathbf{w})$

$$\Delta \mathbf{w} = \alpha\big(R_{t+1} + \gamma\hat{q}(S_{t+1}, A_{t+1}, \mathbf{w}) - \hat{q}(S_t, A_t, \mathbf{w})\big)\nabla_{\mathbf{w}}\hat{q}(S_t, A_t, \mathbf{w})$$

3. For Q-learning, the target is the TD target $R_{t+1} + \gamma \max_a \hat{q}(S_{t+1}, a, \mathbf{w})$

$$\Delta \mathbf{w} = \alpha\big(R_{t+1} + \gamma \max_a \hat{q}(S_{t+1}, a, \mathbf{w}) - \hat{q}(S_t, A_t, \mathbf{w})\big)\nabla_{\mathbf{w}}\hat{q}(S_t, A_t, \mathbf{w})$$

Semi-gradient Sarsa for VFA Control

Episodic Semi-gradient Sarsa for Estimating $\hat{q} \approx q_*$

Input: a differentiable function $\hat{q} : \mathcal{S} \times \mathcal{A} \times \mathbb{R}^d \to \mathbb{R}$

differentiable to calculate the gradient

Initialize value-function weights $\mathbf{w} \in \mathbb{R}^d$ arbitrarily (e.g., $\mathbf{w} = \mathbf{0}$)

Repeat (for each episode):

$S, A \leftarrow$ initial state and action of episode (e.g., ε-greedy)

Repeat (for each step of episode):

Take action A, observe R, S'

If S' is terminal:

$\mathbf{w} \leftarrow \mathbf{w} + \alpha\big[R - \hat{q}(S, A, \mathbf{w})\big]\nabla\hat{q}(S, A, \mathbf{w})$

Go to next episode

Choose A' as a function of $\hat{q}(S', \cdot, \mathbf{w})$ (e.g., ε-greedy)

$\mathbf{w} \leftarrow \mathbf{w} + \alpha\big[R + \gamma\hat{q}(S', A', \mathbf{w}) - \hat{q}(S, A, \mathbf{w})\big]\nabla\hat{q}(S, A, \mathbf{w})$

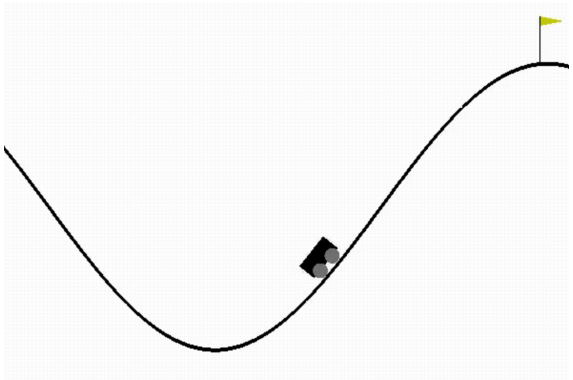
target

prev estimate

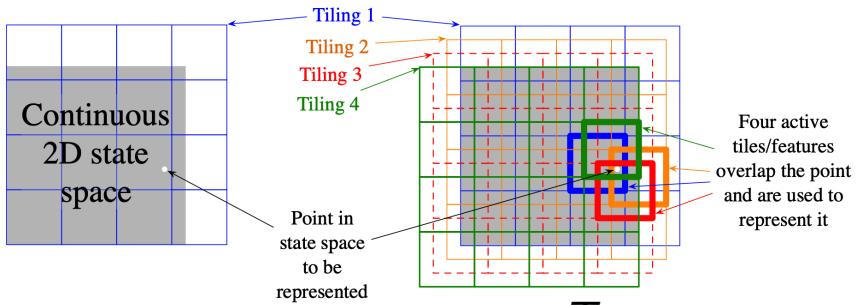
$S \leftarrow S'$

$A \leftarrow A'$

Mountain Car Example



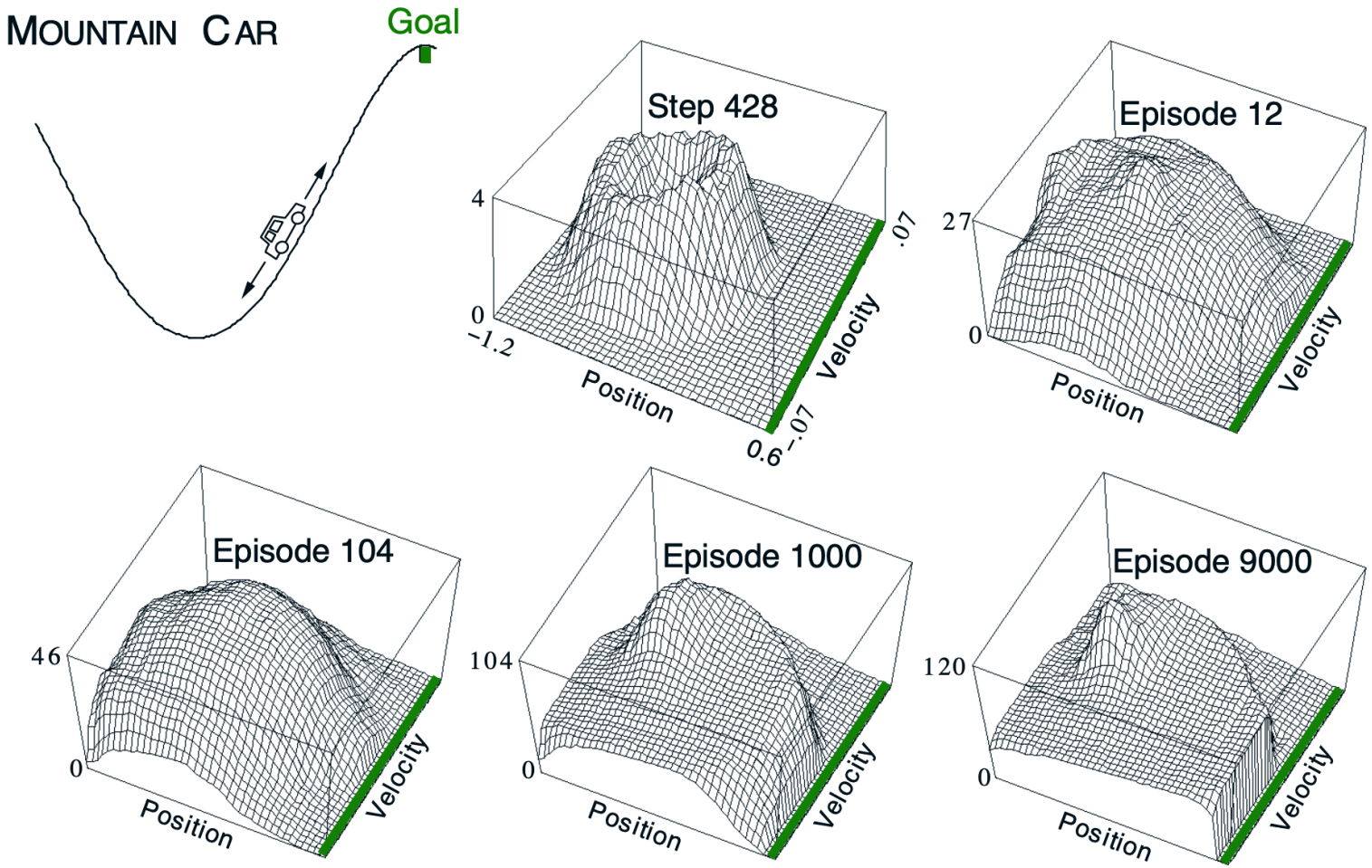
1. Control task: move the car up the hill with forward and backward acceleration
   1. Continuous state: [position of the car, velocity of the car]
   2. Action: [full throttle forward, full throttle backward, zero throttle]
2. Grid-tiling coding of the continuous 2D state space with 4 tilings



3. Q function approximator: $\hat{q}(s, a, \mathbf{w}) = \mathbf{w}^T \mathbf{x}(s, a) = \sum_{i=1}^{4} w_i x_i(s, a)$

Mountain Car Example

1. Visualization of cost-to-go function ($-\max_a \hat{q}(s, a, \mathbf{w})$) learned over episodes



Mountain Car Example

1. Code example: `https://github.com/ucla-rlcourse/RLexample/blob/master/modelfree/q_learning_mountaincar.py`

Convergence of Control Methods with VFA

1. For Sarsa,

$$\Delta \mathbf{w} = \alpha\big(R_{t+1} + \gamma \hat{q}(s_{t+1}, a_{t+1}, \mathbf{w}) - \hat{q}(s_t, a_t, \mathbf{w})\big)\nabla_{\mathbf{w}} \hat{q}(s_t, a_t, \mathbf{w})$$

2. For Q-learning,

$$\Delta \mathbf{w} = \alpha\big(R_{t+1} + \gamma \max_a \hat{q}(s_{t+1}, a, \mathbf{w}) - \hat{q}(s_t, a_t, \mathbf{w})\big)\nabla_{\mathbf{w}} \hat{q}(s_t, a_t, \mathbf{w})$$

"chasing our tail, because td target is using our past w"

3. TD with VFA doesn't follow the gradient of any objective function
4. The updates involve doing an approximate Bellman backup followed by fitting the underlying value function
5. That is why TD can diverge when off-policy or using non-linear function approximation
6. Challenge for off-policy control: behavior policy and target policy are not identical, thus value function approximation can diverge

The Deadly Triad for the Danger of Instability and Divergence

Potential problematic issues:

1. **Function approximation**: A scalable way of generalizing from a state space much larger than the memory and computational resources
2. **Bootstrapping**: Update targets that include existing estimates (as in dynamic programming or TD methods) rather than relying exclusively on actual rewards and complete returns (as in MC methods)
3. **Off-policy training**: training on a distribution of transitions other than that produced by the target policy
4. See details at Textbook Chapter 11.3

Convergence of Control Methods

Algorithm	Table Lookup	Linear	Non-Linear
Monte-Carlo Control	$\sqrt{}$	$(\sqrt{})$	X
Sarsa	$\sqrt{}$	$(\sqrt{})$	X
Q-Learning	$\sqrt{}$	X	X

$(\sqrt{})$ moves around the near-optimal value function

Batch Reinforcement Learning

1. Incremental gradient descend update is simple
2. But it is not sample efficient
3. Batch-based methods seek to find the best fitting value function for a batch of the agent's experience

Least Square Prediction

1. Given the value function approximation $\hat{v}(s, \mathbf{w}) \approx v^{\pi}(s)$
2. The experience $\mathcal{D}$ consisting of $< state, value >$ pairs (may from one episode or many previous episodes)

$$\mathcal{D} = \{ < s_1, v_1^{\pi} >, ..., < s_T, v_T^{\pi} > \}$$

3. Objective: To optimize the parameter $\mathbf{w}$ that best fit all the experience $\mathcal{D}$
4. Least squares algorithms are used to minimize the sum-squared error between $\hat{v}(s_t, \mathbf{w})$ and the target values v_t^{π}

$$\begin{aligned} \mathbf{w}^* &= \arg\min_{\mathbf{w}} \mathbb{E}_{\mathcal{D}}[(v^{\pi} - \hat{v}(s, \mathbf{w})^2] \\ &= \arg\min_{\mathbf{w}} \sum_{t=1}^{T} (v_t^{\pi} - \hat{v}(s_t, \mathbf{w}))^2 \end{aligned}$$

Stochastic Gradient Descent with Experience Replay

1. Given the experience consisting of $< state, value >$ pairs

$$\mathcal{D} = \{ < s_1, v_1^\pi >, ..., < s_T, v_T^\pi > \}$$

2. Iterative solution could be to repeat the following two steps
   1. Randomly sample one pair $< state, value >$ from the experience

take somesubset of <state, value> from experiences

$$< s, v^\pi > \sim \mathcal{D}$$

   2. Apply the stochastic gradient descent update

update so we can match the <s, v\pi> data points

$$\Delta \mathbf{w} = \alpha(v^\pi - \hat{v}(s, \mathbf{w}))\nabla_{\mathbf{w}} \hat{v}(s, \mathbf{w})$$

3. The solution from the gradient descend method converges to the least squares solution

$$\mathbf{w}^{LS} = \arg\min_{\mathbf{w}} \sum_{t=1}^{T} (v_t^\pi - \hat{v}(s_t, \mathbf{w}))^2$$

Solving Linear Least Squares Prediction

1. Experience replay finds least squares solution, but it may take many iterations
2. Using linear value function approximation $\hat{v}(s, \mathbf{w}) = \mathbf{x}(s)^T \mathbf{w}$, we can solve the least squares solution directly

Solving Linear Least Squares Prediction 2

1. At minimum of LS($\mathbf{w}$), the expected update must be zero

no gradient means updates will stop which means we reached optimal solution

$$\mathbb{E}_{\mathcal{D}}[\Delta \mathbf{w}] = 0$$

2. thus

$$\Delta \mathbf{w} = \alpha \sum_{t=1}^{T} \mathbf{x}(s_t)(v_t^{\pi} - \mathbf{x}(x_t)^T \mathbf{w}) = 0$$

$$\sum_{t=1}^{T} \mathbf{x}(s_t) v_t^{\pi} = \sum_{t=1}^{T} \mathbf{x}(s_t)\mathbf{x}(s_t)^T \mathbf{w}$$

$$\mathbf{w} = \left(\sum_{t=1}^{T} \mathbf{x}(s_t)\mathbf{x}(s_t)^T \right)^{-1} \sum_{t=1}^{T} \mathbf{x}(s_t) v_t^{\pi}$$

matrix inversion is slow!! so we can use the iterative form described in slide 31

3. For N features, the matrix inversion complexity is $O(N^3)$

Linear Least Squares Prediction Algorithms

1. Again, we do not know the true values v_t^π
2. In practice, the training data must use noisy or biased samples of v_t^π
   1. LSMC: Least squares Monte-Carlo uses return
      $v_t^\pi \approx G_t$
   2. LSTD: Least squares TD uses TD target
      $v_t^\pi \approx R_{t+1} + \gamma \hat{v}(S_{t+1}, \mathbf{w})$

Linear Least Squares Prediction Algorithms

analytical solutions

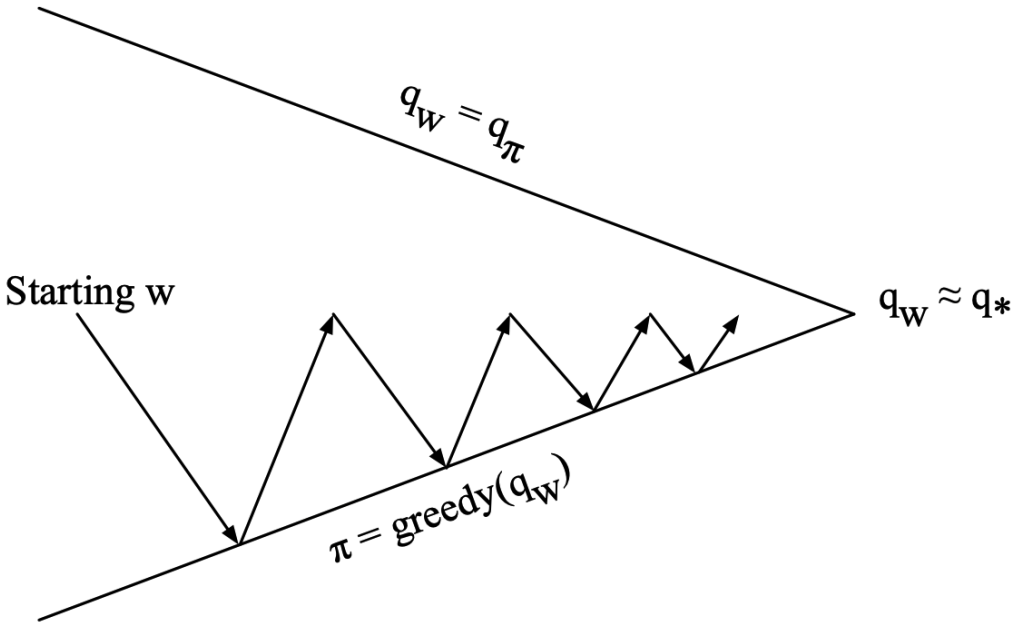
q-learning on slide 39

LSMC
$$0 = \sum_{t=1}^{T} \alpha(G_t - \hat{v}(S_t, \mathbf{w}))\mathbf{x}(S_t)$$
$$\mathbf{w} = \Big(\sum_{t=1}^{T} \mathbf{x}(S_t)\mathbf{x}(S_t)^T \Big)^{-1} \sum_{t=1}^{T} \mathbf{x}(S_t) G_t$$

LSTD
$$0 = \sum_{t=1}^{T} \alpha(R_{t+1} + \gamma \hat{v}(S_{t+1}, \mathbf{w}) - \hat{v}(S_t, \mathbf{w}))\mathbf{x}(S_t)$$
$$\mathbf{w} = \Big(\sum_{t=1}^{T} \mathbf{x}(S_t)(\mathbf{x}(S_t) - \gamma \mathbf{x}(S_{t+1}))^T \Big)^{-1} \sum_{t=1}^{T} \mathbf{x}(S_t) R_{t+1}$$

Least Square Control with Value Function Approximation

Generalized policy iteration



1. Policy evaluation: Policy evaluation by least squares Q-learning
2. Policy improvement: greedy policy improvement

Least Squares Action-Value Function Approximation

1. Approximate action-value function $q^{\pi}(s, a)$
2. using linear combination of features $\mathbf{x}(s, a)$

$$\hat{q}(s, a, \mathbf{w}) = \mathbf{x}(s, a)^T \mathbf{w} \approx q^{\pi}(s, a)$$

3. minimize least squares error between $\hat{q}(s, a, \mathbf{w})$ and $q^{\pi}(s, a)$
4. The experience is generate from using policy π, consisting of $< state, action, action - value >$

Least Squares Control

1. For policy evaluation we want to efficiently use all the experience
2. For control we want to improve the policy
3. The experience is generated from many policies (previous old policies)
4. So to evaluate $q^{\pi}(s, a)$ we must learn off-policy
5. We follow the same idea of Q-Learning:
   1. Use the experience generated by old policy:

$$S_t, A_t, R_{t+1}, S_{t+1} \sim \pi_{old}$$

   2. Consider the alternative successor action $A' = \pi_{new}(S_{t+1})$ using greedy
   3. Update $\hat{q}(S_t, A_t, \mathbf{w})$ towards value of alternative action $R_{t+1} + \gamma \hat{q}(S_{t+1}, A', \mathbf{w})$

Least Squares Q-Learning

1. Consider the following linear Q-learning update

$$\begin{aligned}\delta =& R_{t+1} + \gamma \hat{q}(S_{t+1}, \pi(S_{t+1}), \mathbf{w}) - \hat{q}(S_t, A_t, \mathbf{w}) \\ \Delta \mathbf{w} =& \alpha \delta \mathbf{x}(S_t, A_t)\end{aligned}$$

2. LSDQ algorithm: solve the total sum of the gradient = zero:

$$0 = \sum_{t=1}^{T} \alpha(R_{t+1} + \gamma \hat{q}(S_{t+1}, \pi(S_{t+1}), \mathbf{w}) - \hat{q}(S_t, A_t, \mathbf{w}))\mathbf{x}(S_t, A_t)$$

$$\mathbf{w} = \Big(\sum_{t=1}^{T} \mathbf{x}(S_t, A_t)(\mathbf{x}(S_t, A_t) - \gamma \mathbf{x}(S_{t+1}, \pi(S_{t+1})))^T \Big)^{-1} \sum_{t=1}^{T} \mathbf{x}(S_t, A_t) R_{t+1}$$

Least Squares Policy Iteration Algorithm

1. Use LSDQ for policy evaluation
2. Repeatedly re-evaluate experience $\mathcal{D}$ with different policies

function LSPI-TD$(\mathcal{D}, \pi_0)$
  $\pi' \leftarrow \pi_0$
  repeat
    $\pi \leftarrow \pi'$
    $Q \leftarrow$ **LSTDQ**$(\pi, \mathcal{D})$
    for all $s \in \mathcal{S}$ **do**
      $\pi'(s) \leftarrow \underset{a \in \mathcal{A}}{\text{argmax}}\ Q(s, a)$
    end for
  until $(\pi \approx \pi')$
  return π
end function

Convergence of Control Methods

Algorithm	Table Lookup	Linear	Non-Linear
Monte-Carlo Control	$\surd$	$(\surd)$	X
Sarsa	$\surd$	$(\surd)$	X
Q-Learning	$\surd$	X	X
LSPI	$\surd$	$(\surd)$	-

$(\surd)$ moves around the near-optimal value function

Deep Q-Learning

1. DeepMind's **Nature** paper: Mnih, Volodymyr; et al. (2015). **Human-level control through deep reinforcement learning**
    1. Entering the period of DRL: deep learning + reinforcement learning

Review on Stochastic Gradient Descend for Function Approximation

1. Goal: Find the parameter vector $\mathbf{w}$ that minimizes the loss between a true value function $v_\pi(s)$ and its approximation $\hat{v}_\pi(s, \mathbf{w})$ as represented with a particular function approximator parameterized by $\mathbf{w}$
2. The mean square error loss function is as

$$J(\mathbf{w}) = \mathbb{E}_\pi \left[(v_\pi(S) - \hat{v}(s, \mathbf{w}))^2 \right]$$

3. Follow the gradient descend to find a local minimum

$$\Delta \mathbf{w} = -\frac{1}{2} \alpha \nabla_{\mathbf{w}} J(\mathbf{w})$$
$$\mathbf{w}_{t+1} = \mathbf{w}_t + \Delta \mathbf{w}$$

Linear Function Approximation

1. Represent value function by a linear combination of features

linear regression over features

w \in R^n

$$\hat{v}(S, \mathbf{w}) = \mathbf{x}(S)^T \mathbf{w} = \sum_{j=1}^{n} x_j(S) w_j$$

2. Objective function is $J(\mathbf{w}) = \mathbb{E}_\pi \left[(v_\pi(S) - \mathbf{x}(S)^T \mathbf{w})^2 \right]$
3. Update is as simple as $\Delta \mathbf{w} = \alpha (v_\pi(S) - \hat{v}(S, \mathbf{w})) \mathbf{x}(S)$
4. But there is no oracle for the true value $v_\pi(S)$, we substitute with the target from MC or TD
   1. for MC policy evaluation,

$$\Delta \mathbf{w} = \alpha \Big(G_t - \hat{v}(S_t, \mathbf{w}) \Big) \mathbf{x}(S_t)$$

   2. for TD policy evaluation,

$$\Delta \mathbf{w} = \alpha \Big(R_{t+1} + \gamma \hat{v}(S_{t+1}, \mathbf{w}) - \hat{v}(S_t, \mathbf{w}) \Big) \mathbf{x}(S_t)$$

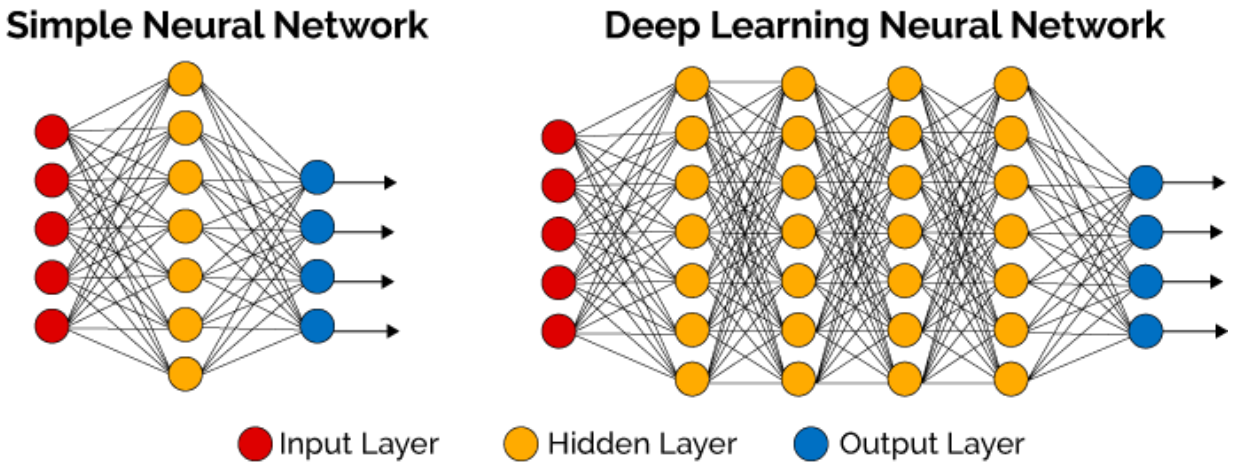
Linear vs Nonlinear Value Function Approximation

1. Linear VFA often works well given the right set of features
2. But it requires manual designing of the feature set <------- MANUAL DESIGN IS UNCHILL (Domain Expertise)
3. Alternative is to use a much richer function approximator that is able to directly learn from states without requiring the feature design
4. Nonlinear function approximator: Deep neural networks

we can also optimize the features

Deep Neural Networks

state is inputted into the neural net



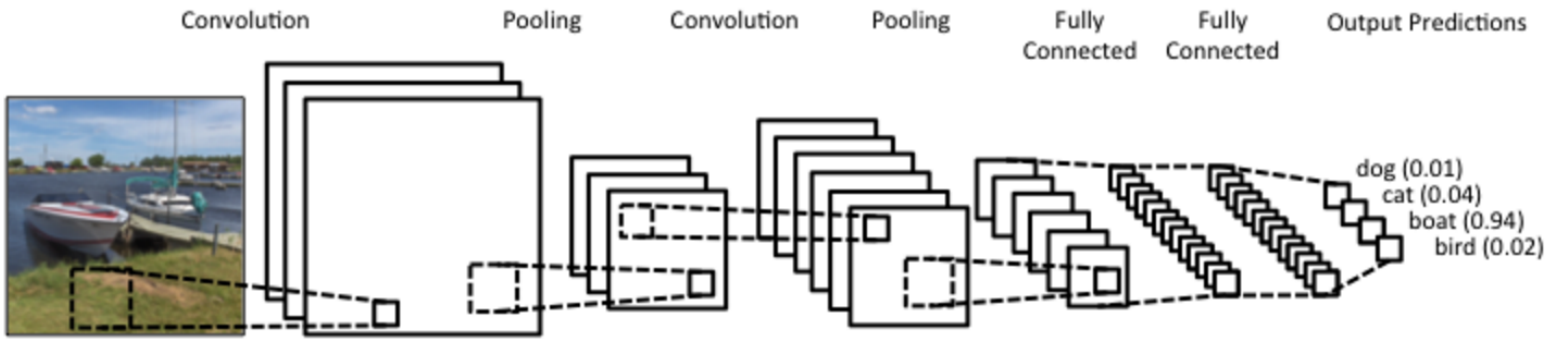
regress to the value/q function

1. Multiple layers of linear functions, with non-linear operators between layers

$$f(\mathbf{x};\theta) = \mathbf{W}_{L+1}^T \sigma(\mathbf{W}_L^T \sigma(...\sigma(\mathbf{W}_1^T \mathbf{x} + \mathbf{b}_1) + ... + \mathbf{b}_{L-1}) + \mathbf{b}_L) + \mathbf{b}_{L+1}$$

2. The chain rule to backpropagate the gradient to update the weights using the loss function $L(\theta) = \frac{1}{n} \sum_{i=1}^{n} \left(y^{(i)} - f(\mathbf{x};\theta) \right)^2$

Convolutional Neural Networks



1. Convolution encodes the local information in 2D feature map
2. Layers of convolution, reLU, batch normalization, etc.
3. CNNs are widely used in computer vision (more than 70% top conference papers using CNNs)
4. A detailed introduction on CNNs: `http://cs231n.github.io/convolutional-networks/`

Deep Reinforcement Learning

1. Frontier in machine learning and artificial intelligence
2. Deep neural networks can be used to parameterize
    1. Value function
    2. Policy function (policy gradient methods to be introduced) learn policy function directly
    3. World model predict next state (modeling transitions)
3. Loss function is optimized by stochastic gradient descent (SGD)
4. Challenges long convex optimizaiton => we are only getting the local min
    1. Efficiency: too many model parameters to optimize
    2. The Deadly Triad for the danger of instability and divergence in training
        1. Nonlinear function approximation
        2. Bootstrapping
        3. Off-policy training

Deep Q-Networks (DQN)

1. DeepMind's **Nature** paper: Mnih, Volodymyr; et al. (2015). **Human-level control through deep reinforcement learning**
2. DQN represents the action value function with neural network approximator
3. DQN reaches a professional human gaming level across many Atari games using the same network and hyperparameters

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1. 4 Atari Games: Breakout, Pong, Montezuma's Revenge, Private Eye
2. OpenAI gym env: `https://www.gymlibrary.dev/environments/atari/complete_list/`

Recall: Action-Value Function Approximation

1. Approximate the action-value function

$$\hat{q}(S, A, \mathbf{w}) \approx q_\pi(S, A)$$

2. Minimize the MSE (mean-square error) between approximate action-value and true action-value (assume oracle)

$$J(\mathbf{w}) = \mathbb{E}_\pi[(q_\pi(S, A) - \hat{q}(S, A, \mathbf{w}))^2]$$

this is if we know the real q

3. Stochastic gradient descend to find a local minimum

$$\Delta \mathbf{w} = \alpha(q_\pi(S, A) - \hat{q}(S, A, \mathbf{w}))\nabla_{\mathbf{w}} \hat{q}(S, A, \mathbf{w})$$

Recall: Incremental Control Algorithm

Same to the prediction, there is no oracle for the true value $q_\pi(S, A)$, so we substitute a target

1. For MC, the target is the return G_t

$$\Delta \mathbf{w} = \alpha\big(G_t - \hat{q}(S_t, A_t, \mathbf{w})\big)\nabla_{\mathbf{w}}\hat{q}(S_t, A_t, \mathbf{w})$$

2. For Sarsa, the target is the TD target $R_{t+1} + \gamma\hat{q}(S_{t+1}, A_{t+1}, \mathbf{w})$

$$\Delta \mathbf{w} = \alpha\big(R_{t+1} + \gamma\hat{q}(S_{t+1}, A_{t+1}, \mathbf{w}) - \hat{q}(S_t, A_t, \mathbf{w})\big)\nabla_{\mathbf{w}}\hat{q}(S_t, A_t, \mathbf{w})$$

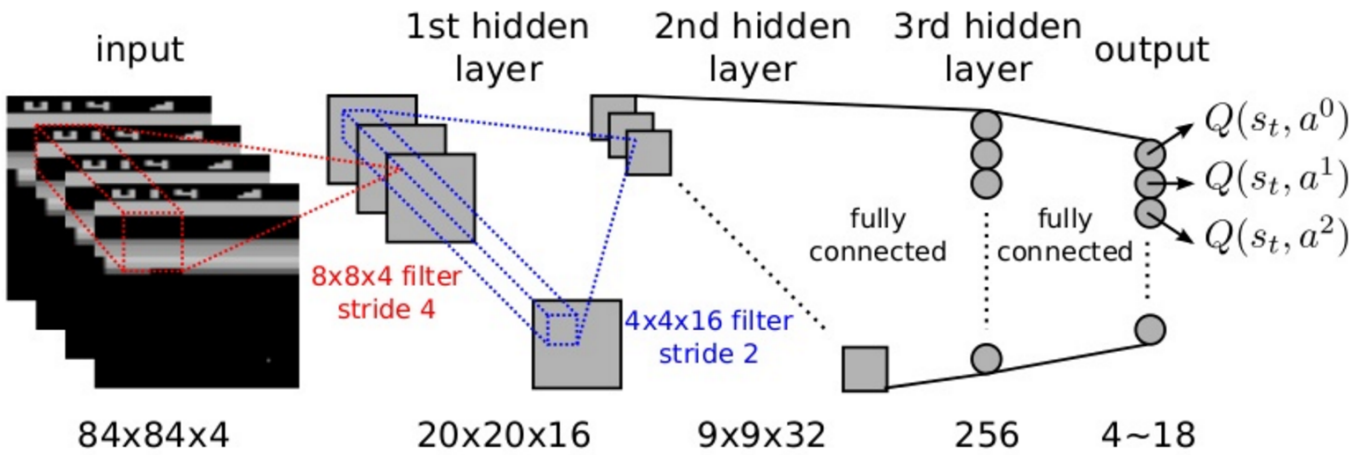
3. For Q-learning, the target is the TD target $R_{t+1} + \gamma \max_a \hat{q}(S_{t+1}, a, \mathbf{w})$

$$\Delta \mathbf{w} = \alpha\big(R_{t+1} + \gamma \max_a \hat{q}(S_{t+1}, a, \mathbf{w}) - \hat{q}(S_t, A_t, \mathbf{w})\big)\nabla_{\mathbf{w}}\hat{q}(S_t, A_t, \mathbf{w})$$

DQN for Playing Atari Games

using directly from the video frames
wanna capture dynamics, so they have a stack of frames

1. End-to-end learning of values $Q(s, a)$ from input pixel frame
2. Input state s is a stack of raw pixels from latest 4 frames
3. Output of $Q(s, a)$ is 18 joystick/button positions
4. Reward is the change in score for that step
5. Network architecture and hyperparameters fixed across all games



generate feature maps using first set of kernels

delayed reward for this game (breaking brick and kicking the ball occur at different times)

Q-Learning with Value Function Approximation

1. Two of the issues causing problems:
   1. Correlations between samples
   2. Non-stationary targets
2. Deep Q-learning (DQN) addresses both of these challenges by
   1. Experience replay
   2. Fixed Q targets

small pixel differences

try to reuse previous trajectories

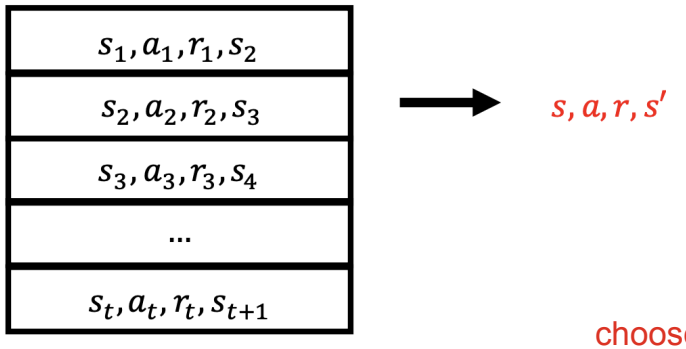
create time differences between target and predictions

this is so we don't have subgradient (veresioned w's)

DQNs: Experience Replay

this is a solution to the method of nearby samples being highly correlated

1. To reduce the correlations among samples, store transition (s_t, a_t, r_t, s_{t+1}) in replay memory $\mathcal{D}$



learn from the past

choose a random sample, this is so the samples in a batch are NOT correlated

2. To perform experience replay, repeat the following
   1. sample an experience tuple from the dataset: $(s, a, r, s') \sim \mathcal{D}$
   2. compute the target value for the sampled tuple: $r + \gamma \max_{a'} \hat{Q}(s', a', \mathbf{w})$ (best action in next state)
   3. use stochastic gradient descent to update the network weights

$$\Delta \mathbf{w} = \alpha \Big(r + \gamma \max_{a'} \hat{Q}(s', a', \mathbf{w}) - Q(s, a, \mathbf{w}) \Big) \nabla_{\mathbf{w}} \hat{Q}(s, a, \mathbf{w})$$

target — prediction — gradient

DQNs: Fixed Targets

1. To help improve stability, fix the target weights used in the target calculation for multiple updates
2. Let a different set of parameter $\mathbf{w}^-$ be the set of weights used in the target, and $\mathbf{w}$ be the weights that are being updated
3. To perform experience replay with fixed target, repeat the following
   1. sample an experience tuple from the dataset: $(s, a, r, s') \sim \mathcal{D}$
   2. compute the target value for the sampled tuple: $r + \gamma \max_{a'} \hat{Q}(s', a', \mathbf{w}^-)$ use old parameters for the TD target
   3. use stochastic gradient descent to update the network weights

$$\Delta \mathbf{w} = \alpha\Big(r + \gamma \max_{a'} \hat{Q}(s', a', \mathbf{w}^-) - Q(s, a, \mathbf{w})\Big)\nabla_{\mathbf{w}} \hat{Q}(s, a, \mathbf{w})$$

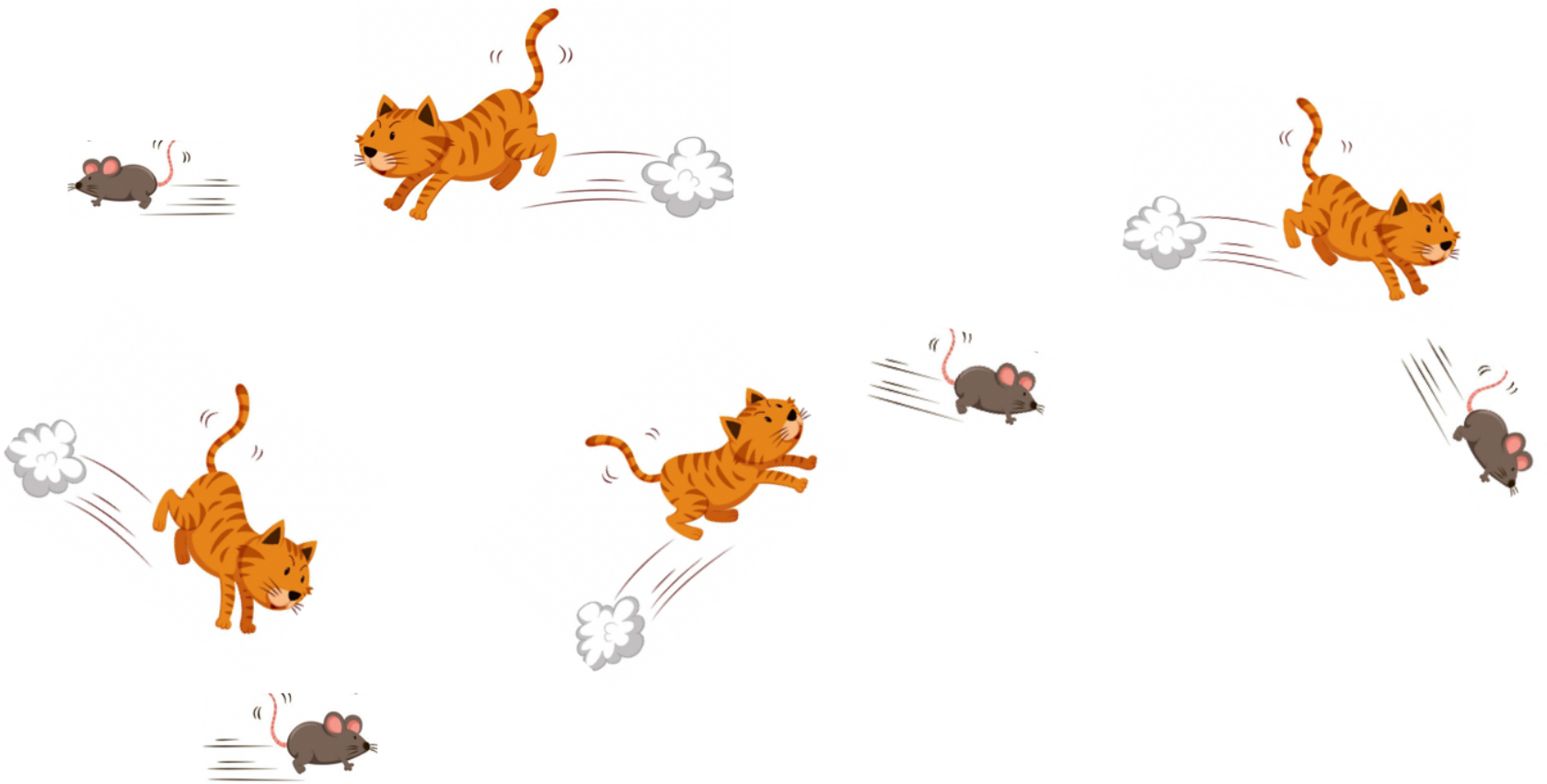
every C steps we copy the new weights into w^-

Why fixed target

1. In the original update, both Q estimation and Q target shifts at each time step
2. Imagine a cat (Q estimation) is chasing after a mouse (Q target)
3. The cat must reduce the distance to the mouse

Why fixed target

1. Both the cat and mouse are moving,

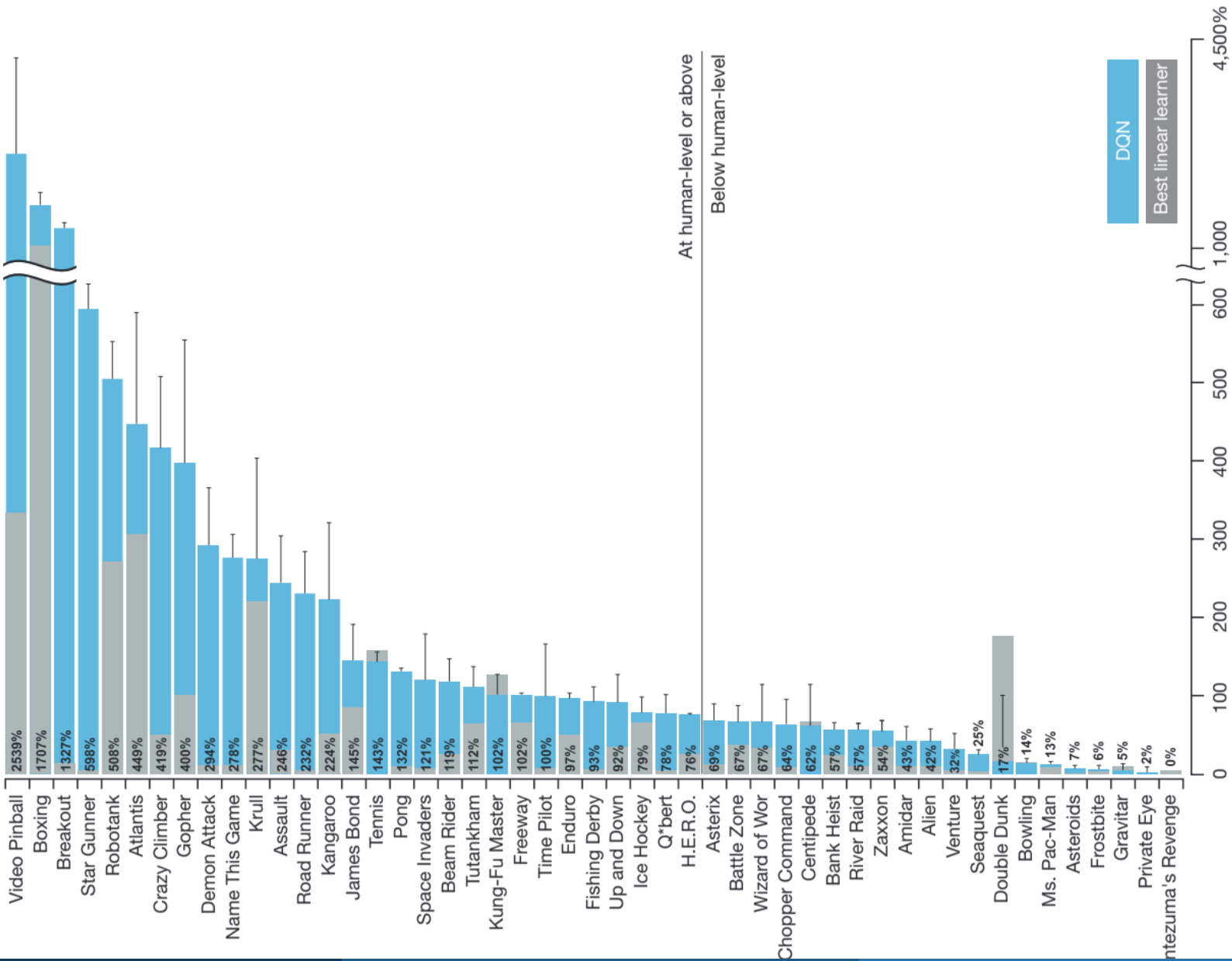


Why fixed target

1. This could lead to a strange path of chasing (an oscillated training history)

2. Solution: fix the target for a period of time during the training

Performance of DQNs on Atari



Ablation Study on DQNs

1. Game score under difference conditions

	Replay Fixed-Q	Replay Q-learning	No replay Fixed-Q	No replay Q-learning
Breakout	316.81	240.73	10.16	3.17
Enduro	1006.3	831.25	141.89	29.1
River Raid	7446.62	4102.81	2867.66	1453.02
Seaquest	2894.4	822.55	1003	275.81
Space Invaders	1088.94	826.33	373.22	301.99

Demo of DQNs

1. Demo of deep Q-learning for Breakout: `https://www.youtube.com/watch?v=V1eYniJ0Rnk`
2. Demo of Flappy Bird by DQN: `https://www.youtube.com/watch?v=xM62SpKAZHU`
3. Code of DQN in PyTorch: `https://github.com/ucla-rlcourse/DeepRL-Tutorials/blob/master/01.DQN.ipynb`
4. Code of Flappy Bird: `https://github.com/xmfbit/DQN-FlappyBird`

Summary of DQNs

1. DQN uses experience replay and fixed Q-targets
2. Store transition $(s_t, a_t, r_{t+1}, s_{t+1})$ in replay memory $\mathcal{D}$
3. Sample random mini-batch of transitions (s, a, r, s') from $\mathcal{D}$
4. Compute Q-learning targets w.r.t. old, fixed parameters $\mathbf{w}^-$
5. Optimizes MSE between Q-network and Q-learning targets using stochastic gradient descent

Improving DQN

1. Success in Atari has led to a huge excitement of using deep neural networks for value function approximation in RL
2. Many follow-up works on improving DQNs
    1. **Double DQN**: Deep Reinforcement Learning with Double Q-Learning. Van Hasselt et al, AAAI 2016
    2. **Dueling DQN**: Dueling Network Architectures for Deep Reinforcement Learning. Wang et al, best paper ICML 2016
    3. **Prioritized Replay**: Prioritized Experience Replay. Schaul et al, ICLR 2016
3. A nice tutorial on the relevant algorithms: `https://github.com/ucla-rlcourse/DeepRL-Tutorials`

Improving DQN: Double DQN

tends to think states/actions are more valuable then they actually are because we are taking the max q-value at each next state

1. Handles the problem of the overestimation of Q-values
2. Idea: use the two networks to decouple the action selection from the target Q value generation

selection: deciding which action is the best
evaluation: calculating the value of the chosen action

3. Vanilla DQN:

$$\Delta \mathbf{w} = \alpha \Big(r + \gamma \max_{a'} \hat{Q}(s', a', \mathbf{w}^-) - Q(s, a, \mathbf{w}) \Big) \nabla_{\mathbf{w}} \hat{Q}(s, a, \mathbf{w})$$

4. Double DQN:

use the current/online network to decide the action (NOT the frozen one)

$$\Delta \mathbf{w} = \alpha \Big(r + \gamma \hat{Q}(s', \arg\max_{a'} Q(s', a', \mathbf{w}), \mathbf{w}^-) - Q(s, a, \mathbf{w}) \Big) \nabla_{\mathbf{w}} \hat{Q}(s, a, \mathbf{w})$$

this makes sure we don't reinforce the out-of-date q-values

recall Q hat is the target network and Q is the online network

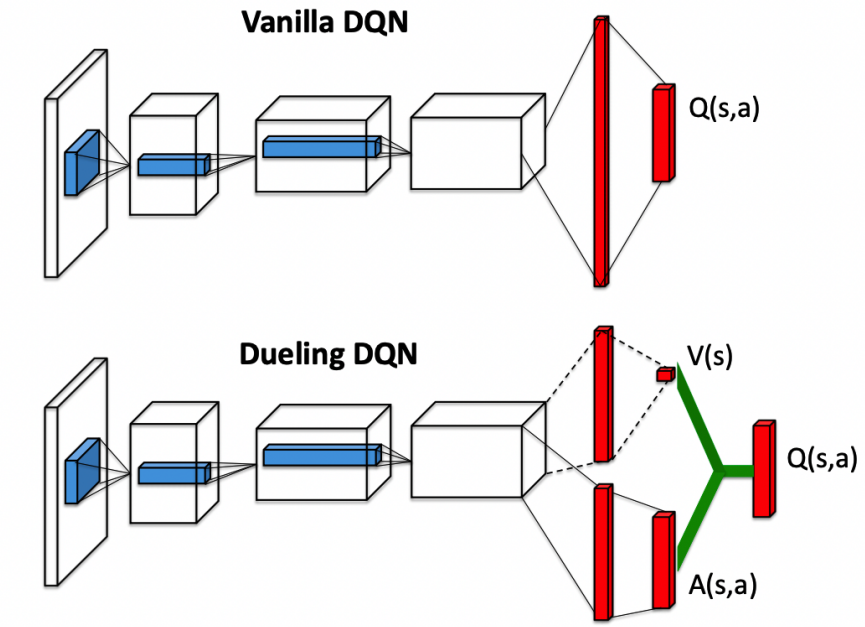
5. Code: `https://github.com/ucla-rlcourse/DeepRL-Tutorials/blob/master/03.Double_DQN.ipynb`

Improving DQN: Dueling DQN

1. One branch estimates $V(s)$, other branch estimates the advantage for each action $A(s, a)$. Then $Q(s, a) = A(s, a) + V(s)$

advantage function A

recall V(s) = \sum_a Q(s,a)
A(s, a) = Q(s, a) - V(s)
so A is kind of like how much above the average an action is for a state

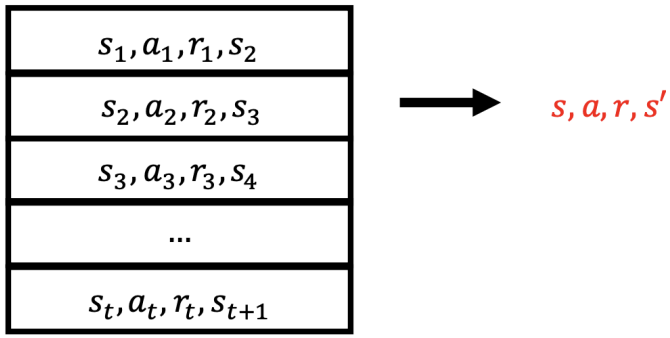


2. By decoupling the estimation, intuitively the DuelingDQN can learn which states are (or are not) valuable without having to learn the effect of each action at each state
3. Code: `https://github.com/ucla-rlcourse/DeepRL-Tutorials/blob/master/04.Dueling_DQN.ipynb`

Improving DQN: Prioritized Experience Replay

1. Transition $(s_t, a_t, r_{t+1}, s_{t+1})$ is stored in and sampled from the replay memory $\mathcal{D}$

priority scores to weigh how often we select a sample



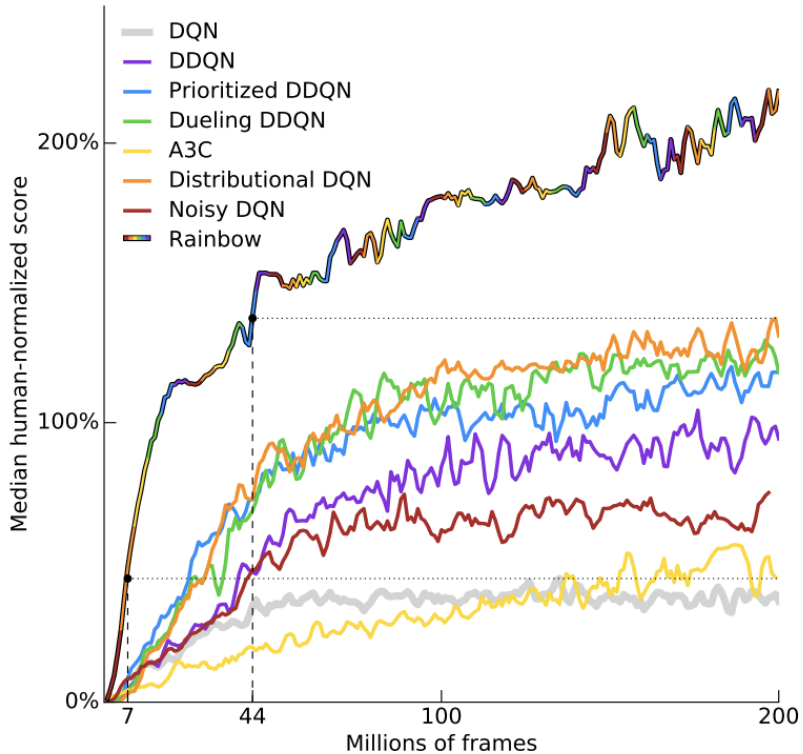
2. Priority is on the experience where there is a big difference between our prediction and the TD target, since it means that we have a lot to learn about it.
3. Define a a priority score for each tuple i

$$p_i = |r + \gamma \max_{a'} Q(s_{i+1}, a', \mathbf{w}^-) - Q(s_i, a_i, \mathbf{w})|$$

4. Code: `https://github.com/ucla-rlcourse/DeepRL-Tutorials/blob/master/06.DQN_PriorityReplay.ipynb`

Improving over the DQN

1. Rainbow: Combining Improvements in Deep Reinforcement Learning. Matteo Hessel et al. AAAI 2018. `https://arxiv.org/pdf/1710.02298.pdf`
2. It examines six extensions to the DQN algorithm and empirically studies their combination



Summary

1. Introduction to function approximation
2. Value function approximation for prediction
3. Value function approximation for control
4. Batch RL and least square prediction and control
5. Deep Q-Learning
6. Improvements over deep q-learning

End

1. Go through the tutorial and train your own gaming agent: `https://github.com/ucla-rlcourse/DeepRL-Tutorials`
2. Good resource for training your driving agent in mini-project or your future research project
    1. Make sure it works for simple environments such as Pong and Breakout before adapting it to your own environment
3. Next week: Policy-based RL
    1. Read Textbook Chapter 13 (yes, we will finish the >300 pages of the textbook next week)