Problem 1: 2.2.10

(a) $\dot{x} = f(x) = 0$. For all $x \in \mathbb{R}$, $\dot{x} = 0$, so every real number is a fixed point.

(b) If there are no fixed points, for all $x \in \mathbb{R}$, $\dot{x} = f(x) \neq 0$. One function that satisfies $\dot{x} = 1$.

Problem 2: 2.4.1

$\frac{dx}{dt} = x(1-x)$. The fixed points are $x = 0, 1$.

$$\frac{df}{dt} = \frac{df}{dx} \cdot \frac{dx}{dt} = \frac{d}{dx}(x - x^2) \cdot 1 = 1 - 2x$$

Evaluated at $x = 0$, $\frac{df}{dt} = 1 > 0$. Hence, $x = 0$ is a unstable fixed point. Evaluated at $x = 1$, $\frac{df}{dx} = 1 - 2(1) = -1$. So, $x = 1$ is a stable fixed point.

Problem 3: 2.4.2

$\dot{x} = x(1-x)(2-x)$. Fixed points: $x = 0, 1, 2$.

$$\frac{df}{dt} = 3x^2 - 6x + 2$$

Evaluated at $x = 0$, $\frac{df}{dt} = 2 > 0$. So, $x = 0$ is a unstable fixed point. Evaluated at $x = 1$, $\frac{df}{dt} = -1 < 0$. So $x = 1$ is a stable fixed point. Evaluated at $x = 2$, $\frac{df}{dt} = -2 < 0$. So $x = 2$ is a stable fixed point.

Problem 4: 2.4.4

$\dot{x} = x^2(6-x)$. Fixed points: $x = 0, 6$.

$$\frac{df}{dt} = 12x - 3x^2$$

Evaluated at $x = 0$, $\frac{df}{dt} = 0$. Linear Stability Analysis fails at $x = 0$. Evaluated at $x = 6$, $\frac{df}{dt} = -36 < 0$. So $x = 6$ is a stable fixed point.

Problem 5: 2.7.2

$$-\frac{dV}{dx} = 3$$
$$V = -3x + c$$

Choose $c = 0$. $V(x) = 0$ is not a local minimum and maximum, so equilibrium does not occur. This is obvious since $\dot{x} = 3 \neq 0$.