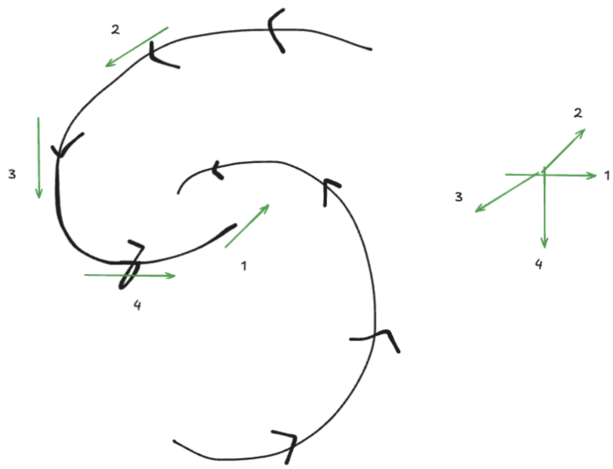
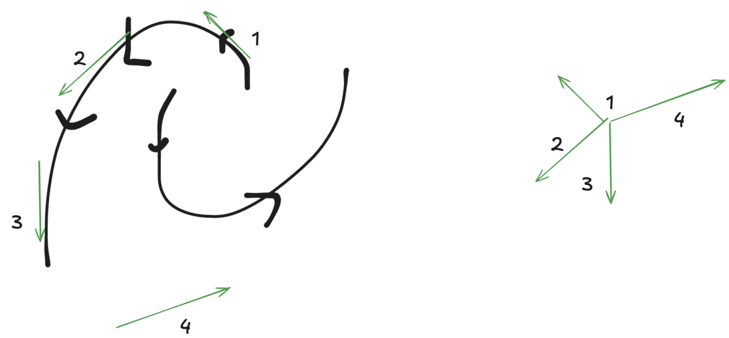
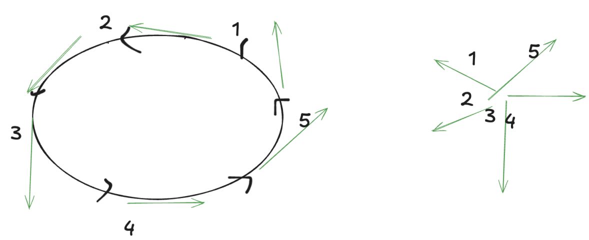
Problem 1: (6.8.1) stable spiral = 1



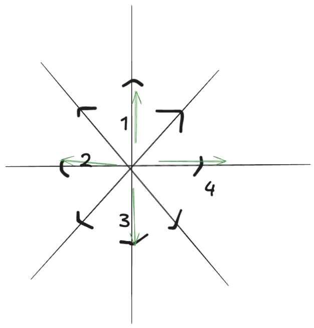
unstable spiral = 1

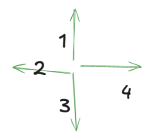


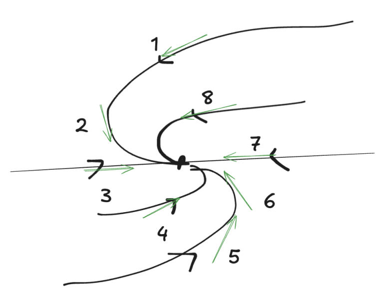
center = 1

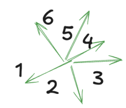


star = 1



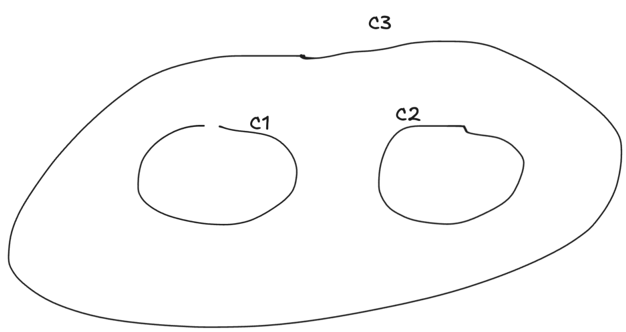






degenerate node = 1

Problem 2: (6.8.6) In a closed orbit, $I_\Gamma = 1$. We also know that $I_\Gamma = I_1 + I_2 + \cdots + I_n$ Each saddle node contributes -1 to the index, while degenerate nodes, spirals, and centers all contribute $+1$ per. Hence, we get $I_\Gamma = 1 = N + F + C - S$.



Problem 3: (6.8.8)

We Know that $I_{C_3} = 1 = I_{C_1} + I_{C_2} +$ index of any fixed points. Since, $I_{C_1} = I_{C_2} = 1$ because they are closed orbits, there must be at least one fixed point that contributes -1 to the index of I_{C_3} to fufill the first equality.

Problem 4: (6.8.13)

(a) Let $u = \frac{g(x,y)}{f(x,y)}$. So, $du = \frac{(f)dg-(g)df}{f^2}$

So, $\phi = \arctan(u)$ and $d\phi = \frac{du}{1+u^2}$. Substituting du, we get

$$d\phi = \frac{(f)dg - (g)df}{f^2(1+\frac{g^2}{f^2})}$$

Simplifying some things,

$$d\phi = \frac{(f)dg - (g)df}{f^2(1+\frac{g^2}{f^2})} = \frac{fdg - gdf}{f^2+g^2}$$

(b) So, $I_C = \frac{1}{2\pi}\oint d\phi = \frac{1}{2\pi}\oint \frac{fdg-gdf}{f^2+g^2}$

Problem 5: $(6.8.14bc)$ (b) Let $x = r\cos\theta$ and $y = r\sin\theta$. Then

$$f(x,y) = f(r,\theta) = r\cos\theta\cos\alpha - r\sin\theta\sin\alpha = r\cos(\theta+\alpha)$$
$$g(x,y) = g(r,\theta) = r\cos\theta\sin\alpha + r\sin\theta\cos\alpha = r\sin(\theta+\alpha)$$

So, $\phi = \arctan(\frac{r\sin(\theta+\alpha)}{r\cos(\theta+\alpha)}) = \arctan(\tan(\theta+\alpha)) = \theta+\alpha$. Thus, $d\phi = d\theta$. Therefore, $I_C = \frac{1}{2\pi}\oint d\phi = \frac{1}{2\pi}\oint d\theta$. The integral is independent of α.
(c)

$$I_C = \frac{1}{2\pi}\oint d\theta = \frac{1}{2\pi}2\pi = 1$$