Problem 1: (a) $\dot{x} = v, \dot{v} = -\omega^2 x$.

Dividing the two, we get

$$\frac{dx}{dt} \cdot \frac{dt}{dv} = \frac{dx}{dv} = \frac{v}{-\omega^2 x}$$

Doing some rearranging and integration we get

$$-\frac{\omega^2}{2}x^2 = \frac{1}{2}v^2 + c$$

So, $\omega^2 x^2 + v^2 C$.

(b) We know that $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$. Let $k = \omega^2$. Then, $\frac{1}{2}mv^2 + \frac{1}{2}\omega^2 x^2 = E$. Mulitplying by $2m$ which should be constant, $C = 2mE = v^2 + \omega^2 x^2$.

Problem 2: We get that $A = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$. Solving for the eigenvalues of the vector, we find that

$$\lambda = \frac{1 \pm \sqrt{1 - 4(1-2)}}{2} = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$$

Since the eigenvalue is a complex number, and the real part is negative, we find that the system is a unstable spiral. To interpret this in terms of Romeo and Juliets love, we can say that Romeo and Juliets feelings towards each other get more and more intense as time progresses.

Problem 3: Given $\ddot{\theta} + \sin\theta = \gamma$. Let $v = \dot{\theta}$. So $\dot{v} + \sin\theta = \gamma$.

We know that $\dot{(\frac{1}{2}v^2)} = v\dot{v}$.

Let's multiply the original equation by v.

$$v\dot{v} + v\sin\theta - \gamma v = 0$$

Integrating, we get

$$\frac{1}{2}v^2 + \sin\theta - \gamma\theta = C$$

Since this only depends on θ and v, we find that it is a conservative system.

If we map $\theta \to \theta$ and $v \to -v$, we find that $\frac{d(-v)}{d(-t)} + \sin\theta = \gamma$. This simplifies to $\frac{dv}{dt} + \sin\theta = \gamma$. Hence, the system is reversible on this map.