Problem 1: (2.4.7) $\dot{x} = ax - x^3$. Note that $\frac{df}{dx} = a - 3x^2$.

Case: $a = 0$. Then, $\dot{x} = x^3 = 0$. So the system has one fixed point at $x = 0$. $\frac{df}{dx} = 0 - 0 = 0$. So linear stability analysis fails. On a graphical inspection, $x = 0$ is a stable fixed point.

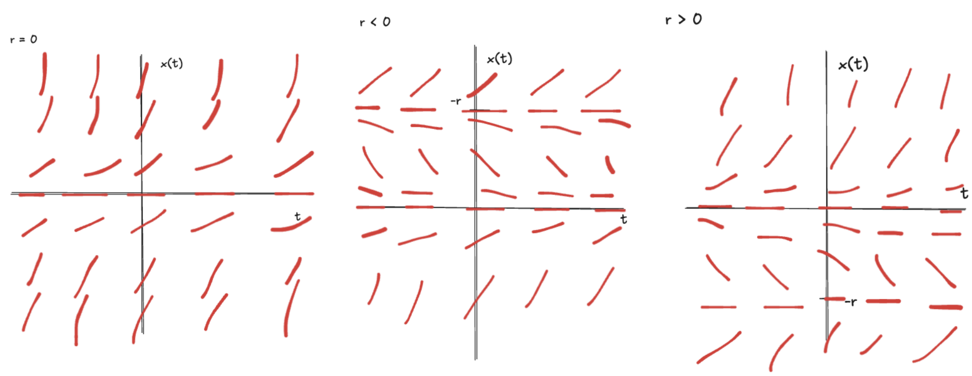
Case: $a < 0$. The system has one fixed point at $x = 0$. At $x = 0$, $\frac{df}{dx} = a < 0$, which shows that it is stable.

Case: $a > 0$. The system has three fixed points at $x = 0, \sqrt{a}, -\sqrt{a}$. At $x = 0$, $\frac{df}{dx} = a > 0$, so it is unstable.

At $x = \sqrt{a}$, $\frac{df}{dx} = a - 3(a) = -2a < 0$. So it is stable.

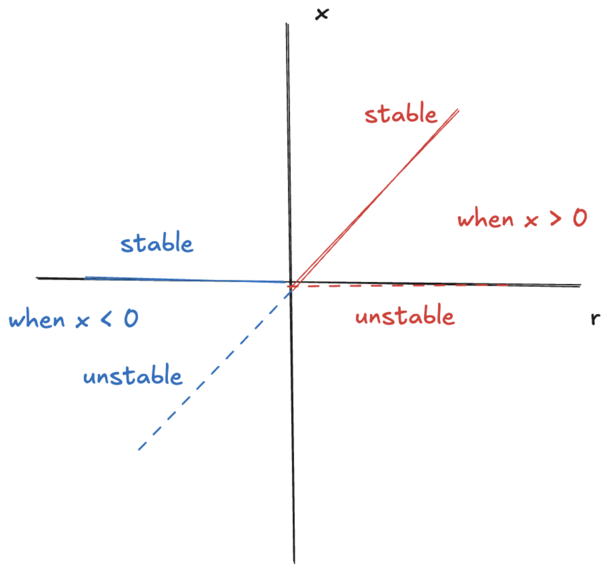
At $x = -\sqrt{a}$, $\frac{df}{dx} = a - 3(a) = -2a < 0$. So it is stable as well.

Problem 2: (3.2.1) $\dot{x} = rx + x^2$



$$\frac{df}{dx} = r + 2x = 0$$
$$r = -2x$$
$$\frac{dx}{dt} = rx + x^2 = x(r + x) = 0$$

So, $x = -r$ and $x = 0$ are solutions of $\frac{dx}{dt} = 0$. Plugging in both, results in $r_c = 0$.



Problem 3: (3.4.5) $\dot{x} = r - 3x^2$.

Using a change of variables of $x = -\frac{1}{3}u$, we get $\frac{du}{dt} = -3r + u^2$, so this is a saddle node bifurcation.

$$\begin{aligned} \frac{dx}{dt} &= r - 3x^2 = 0 \\ r &= 3x^2 \\ \frac{df}{dx} &= 6x = 0 \\ r_c &= 0 \end{aligned}$$