Homework 8

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1. (a) Let $z = a + bi \in \mathbb{C}$ with $a, b \in \mathbb{R}$. Explain why the quantities

$$\frac{a + \sqrt{a^2 + b^2}}{2} \quad \text{and} \quad \frac{-a + \sqrt{a^2 + b^2}}{2}$$

are non-negative, and hence have real square roots. Then use these square roots to produce a square root of z in $\mathbb{C}$, i.e. a $w \in \mathbb{C}$ such that $w^2 = z$. (Be careful about signs)

(b) Let $f(x) = x^2 + \alpha x + \beta \in \mathbb{C}[x]$, with $\alpha, \beta \in \mathbb{C}$. Use the quadratic formula to show directly that f splits into linear factors over $\mathbb{C}$, and hence the roots of f lie in $\mathbb{C}$.

Section 4.5: Symmetric Polynomials

6. Show that $f(x_1, \cdots, x_n)$ is homogeneous of degree m in $R[x_1, \cdots, x_n]$ if an only if $f(tx_1, \cdots, tx_n) = t^m \cdot f(x_1, \cdots, x_n)$ in $R[t, x_1, \cdots, x_n]$, t another indeterminate.

9. Show that the number of terms in $s_k(x_1, \cdots, x_n)$ is nk.

10. Show that the number of monomials of degree m in $R[x_1, \cdots, x_n]$ is $\binom{m+n-1}{m}$.