Homework 1

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Chapter 2: The Basic Theory of Interest

2. The number of years n required for an investment at interest rate r to double in value must satisfy $(1+r)^n = 2$. Using $\ln 2 = 0.69$ and the approximation $\ln(1+r) \approx r$ valid for small r, show that $n \approx 69/i$, where i is the interest rate percentage. Using the better approximation $\ln(1+r) \approx r - \frac{1}{2}r^2$, show that for $r \approx 0.08$ there holds $n \approx 72/i$.

   Proof. Solving $(1+r)^n = 2$, we have

$$(1+r)^n = 2 \implies \ln(1+r)^n = n\ln(1+r) = \ln 2$$
$$\implies nr \approx 0.69 \implies n \approx \frac{0.69}{r} = \frac{69}{i}$$

   Using a better approximation $\ln(1+r) \approx r - \frac{1}{2}r^2$, we have

$$n\ln(1+r) = \ln 2 \implies n\left(r - \frac{1}{2}r^2\right) \approx 0.69$$
$$\implies n\left(0.08 - \frac{1}{2}\cdot 0.08^2\right) = 0.0768n = 0.69 \implies n \approx 8.984 \approx \frac{72}{8} = \frac{72}{i}$$

   $\square$

3. Find the corresponding effective rates for:

   (a) 3% compounded monthly

   Answer.

$$r = \left(1 + \frac{0.03}{12}\right)^{12} - 1 = 3.042\%$$

   (b) 18% compounded monthly

   Answer.

$$r = \left(1 + \frac{0.18}{12}\right)^{12} - 1 = 19.562\%$$

   (c) 18% compounded quarterly

   Answer.

$$r = \left(1 + \frac{0.18}{4}\right)^{4} - 1 = 19.252\%$$

6. A young couple has made a nonrefundable deposit of the first month's rent (equal to \$1000) on a 6-month apartment lease. The next day they find a different apartment that they like just as well, but its monthly rent is only \$900. They plan to be in the apartment only 6 months. Should they switch to the new apartment? What if they plan to stay 1 year? Assume an interest rate of 12%.

   Solution. The monthly interest rate is 12%/12 = 1%. By switching, the couple will have to pay \$1900 the first month, then \$900 for 5 months, whereas by keeping, the couple will be paying \$1000 for 6 months. The present values are

$$NPV_k = \sum_{i=0}^{5} \frac{-1000}{1.01^i} = -5853.43$$
$$NPV_s = -1900 - \sum_{i=1}^{5} \frac{-900}{1.01^i} = -6268.09$$

   Thus, the couple should keep the current apartment since the NPV is greater.

   If they plan to stay for 1 year, then the net present values are

$$NPV_k = \sum_{i=0}^{11} \frac{-1000}{1.01^i} = -11367.6$$
$$NPV_s = -1900 - \sum_{i=1}^{11} \frac{-900}{1.01^i} = -11230.9$$

   In this case, it makes sense to switch because the NPV of the alternative is greater. □

9. You are considering the purchase of a nice home. It is in every way perfect for you and in excellent condition, except for the roof. The roof has only 5 years of life remaining. A new roof would last 20 years, but would cost \$20,000. The house is expected to last forever. Assuming that costs will remain constant and that the future interest rate is 5%, what value would you assign to the existing roof?

   Solution. Since a roof lasts for 20 years and costs \$20,000 up front, this is equivalent to costing C each year, where C is the value such that

$$20,000 = \sum_{i=0}^{19} \frac{C}{(1+0.05)^i} \implies C = 1528.4$$

   Now, if the existing roof will last 5 more years, its present value should be

$$\sum_{i=0}^{4} \frac{1528.4}{(1+0.05)^i} = 6948$$

□

11. Consider the two projects whose cash flows are shown in Table 2.8. Find the IRRs of the two projects and the NPVs at 5%. Show that the IRR and NPV figures yield different recommendations. Can you explain this?

	Years					
	0	1	2	3	4	5
Project 1	-100	30	30	30	30	30
Project 2	-150	42	42	42	42	42

Solution. Project 1: For IRR, we must find the rate r_1

$$0 = -100 + \frac{30}{1+r_1} + \frac{30}{(1+r_1)^2} + \frac{30}{(1+r_1)^3} + \frac{30}{(1+r_1)^4} + \frac{30}{(1+r_1)^5}$$
$$\implies r_1 = 15.2\%$$

The NPV is calculated using $d = \frac{1}{1+0.05}$:

$$NPV_1 = -100 + \frac{30}{1+0.05} + \frac{30}{(1+0.05)^2} + \frac{30}{(1+0.05)^3} + \frac{30}{(1+0.05)^4} + \frac{30}{(1+0.05)^5} = 29.88$$

Project 2: For IRR, we must find the rate r_2

$$0 = -150 + \frac{42}{1+r_2} + \frac{42}{(1+r_2)^2} + \frac{42}{(1+r_2)^3} + \frac{42}{(1+r_2)^4} + \frac{42}{(1+r_2)^5}$$
$$\implies r_2 = 12.4\%$$

The NPV is calculated using $d = \frac{1}{1+0.05}$:

$$NPV_2 = -150 + \frac{42}{1+0.05} + \frac{42}{(1+0.05)^2} + \frac{42}{(1+0.05)^3} + \frac{42}{(1+0.05)^4} + \frac{42}{(1+0.05)^5} = 31.84$$

IRR recommends taking project 1 because $r_1 > r_2$, but NPV recommends taking project 2 because $NPV_2 > NPV_1$. These criteria disagree because the sizes of the projects are different. □