Homework 7

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Chapter 6: Continuous-Time Markov Chains

2. Suppose that a one-celled organism can be in one of two states - either A or B. An individual in state A will change to state B at an exponential rate α; an individual in state B divides into two new individuals of type A at an exponential rate β. Define an appropriate continuous-tine Markov chain for a population of such organisms and determine the appropriate parameters for this model.

5. There are N individuals in a population, some of whom have a certain infection that spreads as follows. Contacts between two members of this population occur in accordance with a Poisson process having rate λ. When a contact occurs, it is equally likely to involve any of the $\binom{N}{2}$ pairs of individuals in the population. If a contact involves an infected and non-infected individual, then with probability p the non-infected individual becomes infected. Once infected, an individual remains infected throughout. Let $X(t)$ denote the number of infected members of the population at time t.

   (a) Is $\{X(t), t \geq 0\}$ a continuous-time Markov chain?

   (b) Specify its type.

   (c) Starting with a single infected individual, what is the expected time until all members are infected?

6. Consider a birth and death process with birth rates $\lambda_i = (i+1)\lambda, i \geq 0$, and death rates $\mu_i = i\mu, i \geq 0$.

   (a) Determine the expected time to go from state 0 to state 4.

   (b) Determine the expected time to go from state 2 to state 5.

   (c) Determine the variances in parts (a) and (b).

9. The birth and death process with parameters $\lambda_n = 0$ and $\mu_n = \mu, n < 0$ is called a pure death process. Find $P_{ij}(t)$.