Homework 7

Aleck Zhao

October 29, 2017

Chapter 11: Properties of Stock Options

7. The price of a non-dividend-paying stock is \$19 and the price of a 3-month European call option on the stock with a strike price of \$20 is \$1. The risk-free rate is 4% per annum. What is the price of a 3-month European put option with a strike price of \$20?

   Solution. By put-call parity, we have

$$\begin{aligned} c + Ke^{-rT} &= p + S_0 \\ \implies p &= 1 + 20e^{-0.04\cdot\frac{1}{4}} - 19 \\ &= \boxed{\$1.801} \end{aligned}$$

□

14. The price of a European call that expires in 6 months and has a strike price of \$30 is \$2. The underlying stock price is \$29, and a dividend of \$0.50 is expected in 2 months and again in 5 months. Interest rates (all maturities) are 10%. What is the price of a European put option that expires in 6 months and has a strike price of \$30?

   Solution. By put-call parity, we have

$$\begin{aligned} c + D + Ke^{-rT} &= p + S_0 \\ \implies p &= 2 + \left(0.5e^{-0.10\cdot\frac{1}{6}} + 0.5e^{-0.10\cdot\frac{5}{12}}\right) + 30e^{-0.10\cdot\frac{1}{2}} - 29 \\ &= \boxed{\$2.508} \end{aligned}$$

□

15. Explain the arbitrage opportunities in problem 11.14 if the European put price is \$3.

   Solution. We short the put and the stock for $3 + 29 = \$32$, and buy the call for \$2, for a net of \$30, which matures to $30e^{0.10\cdot\frac{1}{2}} = \$31.538$ after 6 months.

   If the price at maturity is less than 30, the counterparty will exercise the put, and we buy the stock at \$30 to close out the short position for a riskless profit of $31.538 - 30 = \$1.538$.

   If the price at maturity is greater than 30, we exercise the call to buy the stock at \$30 to close out the short position for a riskless profit of $31.538 - 30 = \$1.538$. □

18. Prove the result in equation (11.7). (Hint: for the first part of the relationship, consider (a) a portfolio consisting of a European call plus an amount of cash equal to K, and (b) a portfolio consisting of an American put option plus one share.)

Proof. From put-call parity of European options, and the fact that American options cost at least as much as European options, we have

$$\begin{aligned} p + S_0 &= c + Ke^{-rT} \\ \implies P + S_0 &\geq p + S_0 = c + Ke^{-rT} \\ \implies c - P &\leq C - P \leq S_0 - Ke^{-rT} \end{aligned}$$

Next, consider portfolio A: a European call plus an amount of cash equal to K, and portfolio B: an American put plus one share. At expiration, we have

A	$S_T > K$	$S_T \leq K$
call	$S_T - K$	0
cash	Ke^{rT}	Ke^{rT}
total	$S_T + (Ke^{rT} - K)$	Ke^{rT}

B	$S_T > K$	$S_T \leq K$
put	0	$K - S_T$
share	S_T	S_T
total	S_T	K

Thus, since $Ke^{rT} \geq K$, at expiration, portfolio A is worth at least as much as portfolio B, so it is worth at least as much at any point in time, and thus

$$\begin{aligned} c + K &\geq P + S_0 \\ \implies S_0 - K &\leq c - P \leq C - P \end{aligned}$$

as desired. □

19. Prove the result in equation (11.11). (Hint: for .the first part of the relationship, consider (a) a portfolio consisting of a European call plus an amount of cash equal to $D + K$, and (b) a portfolio consisting of an American put option plus one share.)

Proof. From put-call parity of European options, and the fact that American options cost at least as much as European options, we have

$$\begin{aligned} p + S_0 &= c + D + Ke^{-rT} \\ \implies P + S_0 &\geq p + S_0 \geq c + D + Ke^{-rT} \\ \implies c - P &\leq C - P \leq S_0 - D - Ke^{-rT} \leq S_0 - Ke^{-rT} \end{aligned}$$

Next, consider portfolio A: a European call plus an amount of cash equal to $D + K$, and portfolio B: an American put plus one share. At expiration, we have

A	$S_T > K$	$S_T \leq K$
call	$S_T - K$	0
cash	$(D + K)e^{rT}$	$(D + K)e^{rT}$
total	$S_T + (Ke^{rT} - K) + De^{rT}$	$(D + K)e^{rT}$

B	$S_T > K$	$S_T \leq K$
put	0	$K - S_T$
share	$S_T + De^{rT}$	$S_T + De^{rT}$
total	$S_T + De^{rT}$	$K + De^{rT}$

At expiration, portfolio A is worth at least as much as portfolio B, so it is worth as least as much at any point in time, and thus

$$\begin{aligned} c + D + K &\geq P + S_0 \\ \implies S_0 - D - K &\leq c - P \leq C - P \end{aligned}$$

as desired. □

25. Suppose that c_1, c_2, and c_3 are the prices of European call options with strike prices K_1, K_2, and K_3, respectively, where $K_3 > K_2 > K_1$ and $K_3 - K_2 = K_2 - K_1$. All options have the same maturity. Show that

$$c_2 \leq 0.5(c_1 + c_3)$$

(Hint: Consider a portfolio that is long one option with strike price K_1, long one option with strike price K_3, and short two options with strike price K_2.)

Proof. At maturity, if $S_T > K_3$, then all 3 options are exercised. If $K_2 < S_T \leq K_3$, then options 1 and 2 are exercised. If $K_1 < S_T \leq K_2$ then option 1 is exercised, and if $K_1 \leq S_T$, then none are exercised. Thus, we have

	$S_T > K_3$	$K_2 < S_T \leq K_3$	$K_1 < S_T \leq K_2$	$K_1 \leq S_T$
call 1	$S_T - K_1$	$S_T - K_1$	$S_T - K_1$	0
-2 × call 2:	$2(K_2 - S_T)$	$2(K_2 - S_T)$	0	0
call 3	$S_T - K_3$	0	0	0
total	$-K_1 + 2K_2 - K_3$	$2K_2 - K_1 - S_T$	$S_T - K_1$	0

In the first case, we have

$$K_3 - K_2 = K_2 - K_1 \implies -K_3 + 2K_2 - K_1 = 0$$

In the second case, we have

$$\begin{aligned} 2K_2 - K_1 - S_T &= (K_2 - K_1) + K_2 - S_T \\ &= (K_3 - K_2) + K_2 - S_T = K_3 - S_T \\ &\geq 0 \end{aligned}$$

In the third case, $S_T - K_1 \geq 0$. Thus, in any scenario, the portfolio is worth at least 0, so at the outset,

$$c_1 - 2c_2 + c_3 \geq 0 \implies c_2 \leq 0.5(c_1 + c_3)$$

as desired. □