Homework 6

Aleck Zhao

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1. Recall that by strong duality there are exactly four possibilities for any linear program LP and its dual DP. Namely,

   i) LP and DP are both feasible and have equal optimal objective function values
   ii) LP is unbounded and DP is infeasible
   iii) LP is infeasible and DP is unbounded
   iv) LP and DP are both infeasible

   Consider the linear program (LP) min $c^T x$ such that $Ax = b$ and $x \geq 0$ where A happens to be a matrix of all zeros. Determine and show which of the four above scenarios occurs in (each possible) different choices of b and c.

2. Consider the following LP:

$$\begin{aligned} \max \quad & 4x_1 - 6x_2 \\ \text{s.t.} \quad & x_1 + 3x_2 \geq 3 \\ & 5x_1 - x_2 \leq 7 \\ & -2x_1 + 3x_2 = 9 \\ & x_1 \geq 0 \quad x_2 \text{ unrestricted} \end{aligned}$$

   a) Write LP in standard form, then write its standard form dual.
   b) Write LP in canonical form, then write its canonical form dual.
   c) Show how the dual programs from part a, b, are equivalent.

3. Consider the linear program (LP) min $c^T x$ such that $Ax = b, x \geq 0$ where

$$A = \begin{bmatrix} -6 & -5 & 25 & 3 & -85 & 4 & 30 \\ 24 & -2 & 28 & 6 & -55 & 1 & -9 \\ 9 & -5 & 11 & 2 & -55 & -1 & 10 \end{bmatrix} \quad b = \begin{bmatrix} 62 \\ 62 \\ 3 \end{bmatrix} \quad c = \begin{bmatrix} 23 & 1 & -17 & -1 & 52 & -6 & -12 \end{bmatrix}^T$$

   Write the dual program (DP) and then solve the dual problem.

4. Concerning the specific LP discussed in the previous problem:

   a) Suppose you may change the value of b_1 (currently 62) to anything you want. To what value should you set b_1 in order to have the adjusted LP have optimal objective function value -70? Compute the optimal solution for the adjusted LP.
   b) Suppose you may change the value of b_2 (currently 62) to anything you want. To what value should you set b_2 in order to have the adjusted LP have optimal objective function value -68.5? Compute the optimal solution for the adjusted LP.

5. Consider the linear program (LP) min $c^T x$ such that $Ax = b, x \geq 0$ where

$$A = \begin{bmatrix} 7 & 7 & 45 & -1 & 3 & -53 & -68 \\ 9 & -5 & 27 & -115 & 7 & -129 & 42 \\ 5 & -3 & 63 & -96 & 10 & -109 & 86 \end{bmatrix} \quad b = \begin{bmatrix} 26 \\ 18 \\ 34 \end{bmatrix} \quad c = \begin{bmatrix} 1 & 7 & -37 & 94 & -9 & 76 & -146 \end{bmatrix}^T$$

   Write the dual program (DP) and then solve the dual problem. Compute the optimal dual variables using one of the optimal bases for (LP), and then repeat this for the other optimal basis.