Homework 4

ALECK ZHAO

October 13, 2016

1: Consider the linear program (LP) min $c^T x$ such that $Ax = b, x \geq 0$ where

$$A = \begin{bmatrix} -6 & -5 & 25 & 3 & -85 & 4 & 30 \\ 24 & -2 & 28 & 6 & -55 & 1 & -9 \\ 9 & -5 & 11 & 2 & -55 & -1 & 19 \end{bmatrix}, \quad b = \begin{bmatrix} 62 \\ 62 \\ 3 \end{bmatrix}, \quad c = \begin{bmatrix} 23 & 1 & -16 & -1 & 52 & -6 & -12 \end{bmatrix}^T$$

Solve this problem using the Simplex Method, starting from the basis consisting of A's columns 1, 3, 6.

```
1  M = [1 -23 -1 16 1 -52 6 12 0;
2  0 -6 -5 25 3 -85 4 30 62;
3  0 24 -2 28 6 -55 1 -9 62;
4  0 9 -5 11 2 -55 -1 19 3];
5  % swap columns 3 and 7 in M (correspond to columns 2 and 6 in A)
6  M(:,[3, 7])=M(:,[7, 3]);
7  A=rref(M)
8
9  A =
10
11       1.0000         0         0         0    0.4444   -1.4444    0.7778  -14.4444
                -63.2222
12            0    1.0000         0         0    0.1111    3.8889    0.4444   -3.1111
                 1.4444
13            0         0    1.0000         0    0.2222   22.7778    2.8889  -15.2222
                14.8889
14            0         0         0    1.0000    0.1111   -6.1111   -0.5556    2.8889
                 0.4444
15
16  % use column 7 since it is largest positive number
17  % pivot on row 2 since ratio (13/9)/(4/9) is smallest, positive
18  % swap columns 2 and 7
19  A(:,[2, 7])=A(:,[7, 2]);
20  B=rref(A)
21
22  B =
23
24       1.0000         0         0         0    0.2500   -8.2500   -1.7500   -9.0000
                -65.7500
25            0    1.0000         0         0    0.2500    8.7500    2.2500   -7.0000
                 3.2500
26            0         0    1.0000         0   -0.5000   -2.5000   -6.5000    5.0000
                 5.5000
27            0         0         0    1.0000    0.2500   -1.2500    1.2500   -1.0000
                 2.2500
28
29  % use column 5 since it is largest positive number
30  % pivot on row 4 since ratio (9/4)/(1/4) is smallest, positive
31  % swap columns 4 and 5
32  B(:,[4, 5])=B(:,[5, 4]);
33  C=rref(B)
34
35  C =
36
37       1         0         0         0        -1        -7        -3        -8       -68
38       0         1         0         0        -1        10         1        -6         1
39       0         0         1         0         2        -5        -4         3        10
```

```
40        0       0       0      1      4     -5     5     -4     9
41
42 % minimum is -68.
43 diary off
```

2: Consider the linear program (LP) min $c^T x$ such that $Ax = b, x \geq 0$ where

$$A = \begin{bmatrix} 8 & -226 & -33 & 10 & 9 & 49 & -1 \\ 9 & -199 & -51 & 10 & 3 & 25 & -25 \\ 2 & 24 & 45 & -6 & 3 & -45 & -15 \end{bmatrix}, \quad b = \begin{bmatrix} 107 \\ 55 \\ 25 \end{bmatrix}, \quad c = \begin{bmatrix} -4 & 63 & 7 & -2 & -2 & 0 & 21 \end{bmatrix}^T$$

Solve this problem using the Simplex Method, starting from the basis consisting of A's columns 1, 3, 4.

3: Consider the linear program (LP) min $c^T x$ such that $Ax = b, x \geq 0$ where

$$A = \begin{bmatrix} 7 & 7 & 45 & -1 & 3 & -53 & -68 \\ 9 & -5 & 27 & -115 & 7 & -129 & 42 \\ 5 & -3 & 63 & -96 & 10 & -109 & 86 \end{bmatrix}, \quad b = \begin{bmatrix} 26 \\ 18 \\ 34 \end{bmatrix}, \quad c = \begin{bmatrix} 1 & 7 & -37 & 94 & -9 & 76 & -146 \end{bmatrix}^T$$

a) Solve this problem using the Simplex Method, starting from the basis consisting of A's columns 1, 2, 5.

b) Solve this problem using the Simplex Method, starting from the basis consisting of A's columns 1, 2, 7. Comment on the difference in outcome between this part b and the previous part a.

c) Solve this problem using the Simplex Method, starting from the basis consisting of A's columns 1, 3, 6. Observe how the objective function changes through this particular Simplex Method implementation, and comment on an anomaly.