Homework 8

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Section 8.2: Cauchy's Theorem

2. Partition D_n into conjugacy classes where n is odd.

Section 8.3: Group Actions

2. If $|G| = 24$ and G has a subgroup of order 8, show that G is not simple.
4. Show that every group of order 15 is cyclic.
14. Let $X = \mathbb{R}[x_1, \cdots, x_n]$, the polynomial ring in the indeterminates $x_1, \cdots, x_n$. Given $\sigma \in S_n$ and $f = f(x_1, \cdots, x_n) \in X$, define $\sigma \cdot f = f(x_{\sigma 1}, x_{\sigma 2}, \cdots, x_{\sigma n})$. Show that this is an action and describe the fixer. If $n = 3$, give three polynomials in the fixer and compute $S_3 \cdot g$ and $S(g)$, where $g(x_1, x_2, x_3) = x_1 + x_2$.
26. Let G be a finite p-group. If $\{1\} \neq H \trianglelefteq G$, show that $H \cap Z(G) \neq \{1\}$.

Section 8.4: The Sylow Theorems

1. Find all Sylow 3-subgroups of S_4, and show explicitly that all are conjugate.
2. Find all Sylow 2-subgroups of D_n, where n is odd, and show explicitly that all are conjugate.
10. Show that G has a cyclic normal subgroup of index 5 if
    (a) $|G| = 385$
    (b) $|G| = 455$
12. If $|G| = pq$ where $p < q$ are primes and p does not divide $q - 1$, show that G is cyclic.
16. Let $P \trianglelefteq H$ and $H \trianglelefteq P$. If P is a Sylow subgroup of G, show that $P \trianglelefteq G$.