Homework 5

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October 6, 2017

Chapter 6: Interest Rate Futures

4. A Eurodollar futures price changes from 96.76 to 96.82. What is the gain or loss to an investor who is long two contracts?

   Solution. The price increases by 6 bps, so the investor who is long 2 contracts gains $6 \cdot 2 \cdot \$25 = \$300$. □

6. The 350-day LIBOR rate is 3% with continuous compounding and the forward rate calculated from a Eurodollar futures contract that matures in 350 days is 3.2% with continuous compounding. Estimate the 440-day zero rate.

   Solution. Using $F_0 = 3.2\%$ and $R_0 = 3\%$, we have

$$\begin{aligned} R_1 &= \frac{F_0(T_1 - T_0) + R_0 T_0}{T_1} = \frac{3.2\% \left(\frac{440}{365} - \frac{350}{365}\right) + 3\% \cdot \frac{350}{365}}{\frac{440}{365}} \\ &= 3.0409\% \end{aligned}$$

   □

9. It is May 5, 2014. The quoted price of a government bond with a 12% coupon that matures on July 27, 2024, is 110-17. What is the cash price?

   Solution. The current coupon period ends on July 27, 2014 and started on Jan 27, 2014, which is 181 days. 98 days have elapsed since Jan 27, each coupon is $\frac{12\%}{2} \cdot 100 = \$6$, so the cash price is

$$110\frac{17}{32} + \frac{98}{181} \cdot 6 = \$113.78$$

   □

11. It is July 30, 2015. The cheapest-to-deliver bond in a September 2015 Treasury bond futures contract is a 13% coupon bond, and delivery is expected to be on September 30, 2015. Coupon payments on the bond are made on February 4 and August 4 each year. The term structure is flat, and the rate of interest with semiannual compounding is 12% per annum. The conversion factor for the bond is 1.5. The current quoted bond price is \$110. Calculate the quoted futures price for the contract.

   Solution. The coupon period started on Feb 4, so 176 days have elapsed, and the coupon period is 181 days. Thus the cash price is

$$110 + \frac{176}{181} \cdot \frac{13}{2} = 116.3204$$

The semi-annual rate is 12% per annum which is equal to $\ln(1.06^2) = 11.65\%$ with continuous compounding. The coupon of \$6.50 will be received in 5 days, so it has present value $6.5e^{-0.1165\cdot\frac{5}{365}} = 6.4896$. The futures contract lasts for 62 days, so the cash futures price would be

$$(116.3204 - 6.4896)e^{0.1165\cdot\frac{62}{365}} = 112.0259$$

At delivery, there are $62 - 5 = 57$ days of accrued interest, out of a 184 day period, so the quoted futures price on a 13% bond should be

$$112.0259 - \frac{57}{184}\cdot\frac{13}{2} = 110.0123$$

Finally, the quoted futures price of the contract should be $\frac{110.0123}{1.5} = \$73.34$. □

14. Suppose that the 300-day LIBOR zero rate is 4% and the Eurodollar quotes for contracts maturing in 300, 398, and 489 days are 95.83, 95.62, and 95.48. Calculate 398-day and 489-day LIBOR zero rates. Assume no difference between forward and futures rates for the purposes of your calculation.

Solution. The forward rate for the 300-day ED contract is $100 - 95.83 = 4.17\%$, compounded quarterly under ACT/360, which is equivalent to $F_0 = \frac{365}{90}\ln\left(1 + \frac{4.17\%}{4}\right) = 4.2060\%$ compounded continuously under ACT/365. Using $R_0 = 4\%$, we have

$$\begin{aligned} R_1 &= \frac{F_0(T_1 - T_0) + R_0T_0}{T_1} = \frac{4.206\%(398 - 300) + 4\%\cdot 300}{398} \\ &= 4.0507\% \end{aligned}$$

is the 398-day LIBOR zero rate. Similarly, the forward rate for the 398-day ED contract is $100 - 95.62 = 4.38\%$, compounded quarterly under ACT/360, which is equivalent to $F_1 = \frac{365}{90}\ln\left(1 + \frac{4.38\%}{4}\right) = 4.4167\%$ compounded continuously under ACT/365. Thus, we have

$$\begin{aligned} R_2 &= \frac{F_1(T_2 - T_1) + R_1T_1}{T_2} = \frac{4.4167\%(489 - 398) + 4.0507\%\cdot 398}{489} \\ &= 4.1188\% \end{aligned}$$

is the 489-day LIBOR zero rate. □

21. The 3-month Eurodollar futures price for a contract maturing in 6 years is quoted as 95.20. The standard deviation of the change in the short term interest rate in 1 year is 1.1%. Estimate the forward LIBOR interest rate for the period between 6.00 and 6.25 years in the future.

Solution. The futures rate is $100 - 95.20 = 4.80\%$ with quarterly compounding under ACT/360, which is equivalent to $\frac{365}{90}\ln\left(1 + \frac{4.80\%}{4}\right) = 4.8377\%$ with continuous compounding under ACT/365. We have $\sigma = 0.011, T_1 = 6, T_2 = 6.25$, so using the convexity adjustment, we have the forward rate is

$$4.8377\% - \frac{1}{2}\sigma^2T_1T_2 = 4.8377\% - \frac{1}{2}(0.011)^2\cdot 6\cdot 6.25 = 4.6108\%$$

□

26. A Eurodollar futures quote for the period between 5.1 and 5.35 years in the future is 97.1. The standard deviation of the change in the short-term interest rate in one year is 1.4%. Estimate the forward interest rate in an FRA.

Solution. The futures rate is $100 - 97.1 = 2.9\%$ with quarterly compounding under ACT/360, which is equivalent to $\frac{365}{90} \ln\left(1 + \frac{2.9\%}{4}\right) = 2.9297\%$ with continuous compounding under ACT/365. We have $\sigma = 0.014, T_1 = 5.1, T_2 = 5.35$, so using the convexity adjustment, we have the forward rate is

$$2.9297\% - \frac{1}{2}\sigma^2 T_1 T_2 = 2.9297\% - \frac{1}{2}(0.014)^2 \cdot 5.1 \cdot 5.35 = 2.6623\%$$

□

27. It is March 10, 2014. The cheapest-to-deliver bond in a December 2014 Treasury bond futures contract is an 8% coupon bond, and the delivery is expected to be made on December 31, 2014. Coupon payments on the bond are made on March 1 and September 1 each year. The rate of interest with continuous compounding is 5% per annum for all maturities. The conversion factor for the bond is 1.2191. The current quoted bond price is \$137. Calculate the quoted futures price for the contract.

Solution. The coupon period started on March 1, so 9 days have elapsed, and the coupon period is 184 days. Thus the cash price is

$$137 + \frac{9}{184} \cdot \frac{8}{2} = 137.1957$$

The coupon of \$4 will be received in 175 days, so it has present value $4e^{-0.05 \cdot \frac{175}{365}} = 3.9052$. The futures contract lasts for 296 days, so the cash futures price would be

$$(137.1957 - 3.9052)e^{0.05 \cdot \frac{296}{365}} = 138.8062$$

At delivery, there are $296 - 175 = 121$ days of accrued interest, out of a 181 day period, so the quoted futures price on an 8% bond should be

$$138.8062 - \frac{121}{181} \cdot \frac{8}{2} = 136.1322$$

Finally, the quoted futures price of the contract should be $\frac{136.1322}{1.2191} = \$111.67$. □