Homework 2

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Section 1.6

(a) $|z - 1 + i| \leq 3$

(b) $|\arg z| < \pi/4$

(c) $0 < |z - 2| < 3$

(d) $-1 < \operatorname{Im} z \leq 1$

(e) $|z| \geq 2$

(f) $(\operatorname{Re} z)^2 > 1$

2. Sketch each of the given sets.
3. Which of the given sets are open?
4. Which of the given sets are domains?
6. Describe the boundary of each of the given sets.

extra. Which of the sets are closed?

Section 2.1

3. Describe the range of each of the following functions.
   (a) $f(z) = z + 5$ for $\operatorname{Re} z > 0$
   (b) $g(z) = z^2$ for z in the first quadrant, $\operatorname{Re} z \geq 0, \operatorname{Im} z \geq 0$.
   (c) $h(z) = \frac{1}{z}$ for $0 < |z| \leq 1$
   (d) $p(z) = -2z^3$ for z in the quarter-disk $|z| < 1, 0 < \arg z < \pi/2$.
5. (e) For the complex exponential function $f(z) = e^z$ defined in Sec 1.4, describe the image of the infinite strip $0 \leq \operatorname{Im} z \leq \pi/4$.
6. The Joukowski mapping is defined by

$$w = J(z) = \frac{1}{2}\left(z + \frac{1}{z}\right)$$

   Show that
   (a) $J(z) = J(1/z)$
   (b) J maps the unit circle $|z| = 1$ onto the real interval $[-1, 1]$.
   (c) J maps the circle $|z| = r \quad (r > 0, r \neq 1)$ onto the ellipse

$$\frac{u^2}{\left[\frac{1}{2}\left(r + \frac{1}{r}\right)\right]^2} + \frac{v^2}{\left[\frac{1}{2}\left(r - \frac{1}{r}\right)\right]^2} = 1$$

   which has foci at ± 1.

Section 2.2

2. Sketch the first five terms of the sequence $(2i)^n, n = 1, 2, 3, \cdots$ and then describe the divergence of this sequence.

7. Decide whether each of the following sequences converges, and if so, find its limit.

   (a) $z_n = \frac{i}{n}$

   (b) $z_n = i(-1)^n$

   (c) $z_n = \arg\left(-1 + \frac{i}{n}\right)$

   (d) $z_n = \frac{n(2+i)}{n+i}$

   (e) $z_n = \left(\frac{1-i}{4}\right)^n$

   (f) $z_n = \exp\left(\frac{2n\pi i}{5}\right)$

21. (d) Find the limit

$$\lim_{z \to -\pi i} \exp\left(\frac{z^2 + \pi^2}{z + \pi i}\right)$$