Homework 11

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Chapter 11: The Space of Continuous Functions

1. For each n, let Q_n be the set of all polygonal functions that have nodes at $k/n, k = 0, \cdots, n.$, and that take on only rational values at those points. Check that Q_n is a countable set, and hence tha the union of the Q_n's is a countable dense set in $C[0, 1]$.

7. If p is a polynomial and $\varepsilon > 0$, prove that there is a polynomial q with rational coefficients such that $\|p - q\|_\infty < \varepsilon$ on $[0, 1]$.

9. Let $\mathcal{P}_n$ denote the set of polynomials of degree at most n, considered as a subset of $C[a, b]$). Clearly $\mathcal{P}_n$ is a subspace of $C[a, b]$ of dimension $n + 1$. Also, $\mathcal{P}_n$ is closed in $C[a, b]$. How do you know that $\mathcal{P}$, the union of all of the P_n, is not all of $C[a, b]$? That is, why are there necessarily non-polynomial elements in $C[a, b]$?

12. Let p_n be a polynomial of degree m_n, and suppose that $p_n \Rightarrow f$ on $[a, b]$, where f is not a polynomial. Show that $m_n \to \infty$.

14. Let $f \in C[a, b]$ be continuously differentiable, and let $\varepsilon > 0$. Show that there is a polynomial p such that $\|f - p\|_\infty < \varepsilon$ and $\|f' - p'\|_\infty < \varepsilon$. Conclude that $C^{(1)}[a, b]$ is separable.

27. Let T be a trig polynomial. Prove:

    (a) If T is an odd function, then T can be written using only cosines.

    (b) If T is an even function, then T can be written using only sines.