Homework 10

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Section 3.4: Homomorphisms

3. Show that a general ring homomorphism $\theta : \mathbb{Z} \to \mathbb{Z}$ is either a ring isomorphism or $\theta(k) = 0$ for all $k \in \mathbb{Z}$.
4. Determine all onto ring homomorphisms $\mathbb{Z}_{12} \to \mathbb{Z}_6$.
20. If $n > 0$ in $\mathbb{Z}$, describe all the ideals of $\mathbb{Z}$ that contain $n\mathbb{Z}$.

Section 4.1: Polynomials

2. (c) Compute $(1 + x)^5$ in $\mathbb{Z}_5[x]$.
4. (a) Find all roots of $(x - 4)(x - 5)$ in $\mathbb{Z}_6$; in $\mathbb{Z}_7$.
13. Divide $x^3 - 4x + 5$ by $2x + 1$ in $\mathbb{Q}[x]$. Why is it impossible in $\mathbb{Z}[x]$?
24. If R is a commutative ring, a polynomial f in $R[x]$ is said to **annihilate** R if $f(a) = 0$ for every $a \in R$.
    (a) Show that $x^p - x$ annihilates $\mathbb{Z}_p$.

Section 4.2: Factorization of Polynomials over a Field

5. (a) Determine whether the polynomial $x^2 - 3$ is irreducible over each of the fields $\mathbb{Q}, \mathbb{R}, \mathbb{C}, \mathbb{Z}_2, \mathbb{Z}_3, \mathbb{Z}_5, \mathbb{Z}_7$.
9. Show that an odd degree polynomial has a real root.
10. Find all monic irreducible cubics in $\mathbb{Z}_2[x]$.