

Homework 4

ALECK ZHAO

October 13, 2016

1: Consider the linear program (LP) $\min c^T x$ such that $Ax = b, x \geq 0$ where

$$A = \begin{bmatrix} -6 & -5 & 25 & 3 & -85 & 4 & 30 \\ 24 & -2 & 28 & 6 & -55 & 1 & -9 \\ 9 & -5 & 11 & 2 & -55 & -1 & 19 \end{bmatrix}, \quad b = \begin{bmatrix} 62 \\ 62 \\ 3 \end{bmatrix}, \quad c = [23 \quad 1 \quad -16 \quad -1 \quad 52 \quad -6 \quad -12]^T$$

Solve this problem using the Simplex Method, starting from the basis consisting of A 's columns 1, 3, 6.

```

1 M = [1 -23 -1 16 1 -52 6 12 0;
2 0 -6 -5 25 3 -85 4 30 62;
3 0 24 -2 28 6 -55 1 -9 62;
4 0 9 -5 11 2 -55 -1 19 3];
5 % swap columns 3 and 7 in M (correspond to columns 2 and 6 in A)
6 M(:,[3, 7])=M(:,[7, 3]);
7 A=rref(M)
8
9 A =
10
11      1          0          0          0          4/9          -13/9
12          0          1          0          0          1/9          35/9
13          0          4/9         -28/9         13/9          2/9         205/9
14          0         26/9        -137/9        134/9          1/9         -55/9
15          0          0          0          1          1/9         -55/9
16          0         -5/9         26/9         4/9
17
18 % use column 7 since it is largest positive number
19 % pivot on row 2 since ratio (13/9)/(4/9) is smallest, positive
20 % swap columns 2 and 7
21 A(:,[2, 7])=A(:,[7, 2]);
22 B=rref(A)
23
24 B =
25      1          0          0          0          1/4          -33/4
26          0         -7/4         -9        -263/4          1/4          35/4
27          0          1          0          0          1/4          35/4
28          0          9/4         -7         13/4          1/4          35/4
29          0          0          1          0         -1/2         -5/2
30          0        -13/2          5         11/2          1/4         -5/4
31          0          0          0          1          1/4         -5/4
32          0          5/4         -1          9/4
33
34 % use column 5 since it is largest positive number
35 % pivot on row 4 since ratio (9/4)/(1/4) is smallest, positive
36 % swap columns 4 and 5
37 B(:,[4, 5])=B(:,[5, 4]);
38 C=rref(B)
39
40 C =
41      1          0          0          0          0         -1         -7
42          0         -3         -8        -68

```

```

38      0      1      0      0      -1      10
39      0      0      1      0      2      -5
40      0      0      0      1      4      -5
41      5      -4      9
42 % minimum is -68.
43 diary off

```

2: Consider the linear program (LP) $\min c^T x$ such that $Ax = b, x \geq 0$ where

$$A = \begin{bmatrix} 8 & -226 & -33 & 10 & 9 & 49 & -1 \\ 9 & -199 & -51 & 10 & 3 & 25 & -25 \\ 2 & 24 & 45 & -6 & 3 & -45 & -15 \end{bmatrix}, \quad b = \begin{bmatrix} 107 \\ 55 \\ 25 \end{bmatrix}, \quad c = [-4 \quad 63 \quad 7 \quad -2 \quad -2 \quad 0 \quad 21]^T$$

Solve this problem using the Simplex Method, starting from the basis consisting of A 's columns 1, 3, 4.

3: Consider the linear program (LP) $\min c^T x$ such that $Ax = b, x \geq 0$ where

$$A = \begin{bmatrix} 7 & 7 & 45 & -1 & 3 & -53 & -68 \\ 9 & -5 & 27 & -115 & 7 & -129 & 42 \\ 5 & -3 & 63 & -96 & 10 & -109 & 86 \end{bmatrix}, \quad b = \begin{bmatrix} 26 \\ 18 \\ 34 \end{bmatrix}, \quad c = [1 \quad 7 \quad -37 \quad 94 \quad -9 \quad 76 \quad -146]^T$$

- Solve this problem using the Simplex Method, starting from the basis consisting of A 's columns 1, 2, 5.
- Solve this problem using the Simplex Method, starting from the basis consisting of A 's columns 1, 2, 7. Comment on the difference in outcome between this part b and the previous part a.
- Solve this problem using the Simplex Method, starting from the basis consisting of A 's columns 1, 3, 6. Observe how the objective function changes through this particular Simplex Method implementation, and comment on an anomaly.