Homework 2

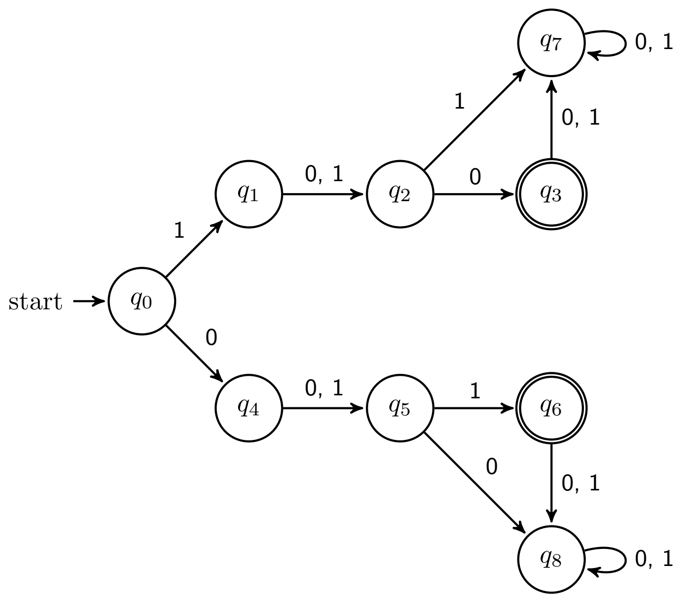
Aleck Zhao

February 14, 2018

1. Give the state diagram of a DFA recognizing the following language. The alphabet is $\{0, 1\}$.

$$\{w : w \text{ has length exactly 3 and its last symbol is different from its first symbol}\}$$

Solution. Let q_0 be the start state.



$\square$

2. Give a DFA (both a state diagram and a formal description) recognizing the following language. The alphabet is $\{0,1\}$.

$$\{w : w \text{ has odd length or contains an even number of 0s}\}$$

Solution. Let q_0 be the start state. Then let

$$\begin{aligned} q_1 &:= \text{ odd length, even number of 0s} \\ q_2 &:= \text{ odd length, odd number of 0s} \\ q_3 &:= \text{ even length, even number of 0s} \\ q_4 &:= \text{ even length, odd number of 0s} \end{aligned}$$

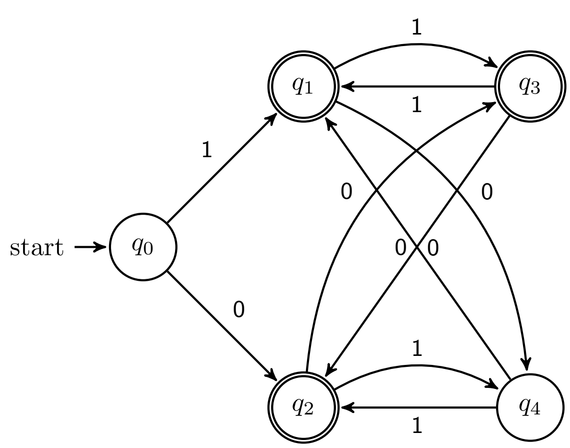
Thus, states q_1, q_2, q_3 are accepting states. Then $M = (Q, \Sigma, \delta, q_0, F)$ where

$$\begin{aligned} Q &= \{q_0, q_1, q_2, q_3, q_4\} \\ F &= \{q_1, q_2, q_3\} \end{aligned}$$

and the transition function is described as

δ	0	1
q_0	q_2	q_1
q_1	q_4	q_3
q_2	q_3	q_4
q_3	q_2	q_1
q_4	q_1	q_2

Thus, the state diagram is given by

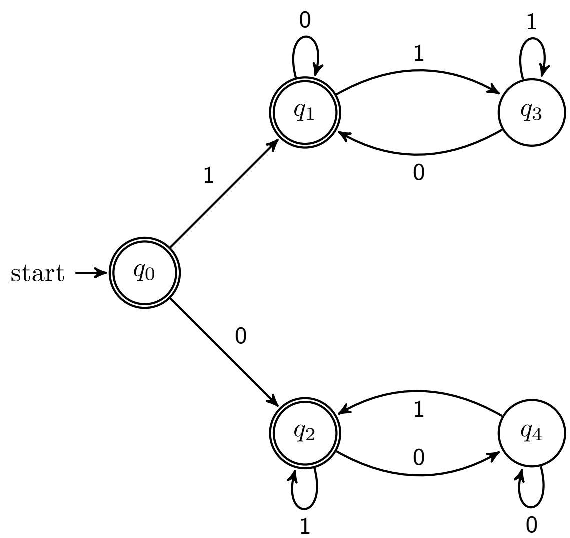


$\square$

3. Show that the following language is regular, where the alphabet is $\{0, 1\}$.

$$\{w : w \text{ contains an equal number of occurrences of the substrings 01 and 10}\}$$

Solution. Let q_0 be the start state. The DFA represented by the following state diagram accepts this language, so it is regular, as desired.



□

4. For any string $w = w_1 w_2 \cdots w_n$, the reverse of w, written as $w^{\mathcal{R}}$, is the string w in reverse order $w_n \cdots w_2 w_1$. For any language A, let $A^{\mathcal{R}} = \left\{ w^{\mathcal{R}} : w \in A \right\}$. Show that if A is regular, so is $A^{\mathcal{R}}$.

   Proof. Since A is regular, it is accepted by some DFA $M = (Q, \Sigma, \delta, q_0, F)$. Construct the following NFA $N = (Q', \Sigma', \delta', q_0', F')$ where

$$\begin{aligned} Q' &= Q \cup \{q\} \\ \Sigma' &= \Sigma \\ \delta'(q_i, w_j) &= \{q : \delta(q, w_j) = q_i\}, \forall q_i \in Q \\ \delta'(q, \varepsilon) &= F \\ q_0' &= q \\ F' &= q_0 \end{aligned}$$

   In this NFA N, we have reversed the direction of every transition in M. We created a new dummy start state that transitions to each of the original accept states under ε, and the old start state became the new accept state.

   By construction, N accepts $A^{\mathcal{R}}$ because if M accepts $w_1 w_2 \cdots w_n \in A$, then the series of transitions from q_0 ends up in F. Then in N, starting at q, we can go to any of the original accept states under ε, then all the transitions are done in reverse order, so we will end up at q_0, which is the accept state in N. Since every NFA is equivalent to some DFA, it follows that a DFA accepts $A^{\mathcal{R}}$, so it is regular, as desired. □