Homework 8

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Problems on Expected Time Until Hitting a State

1. Two possible infinitesimal generators for a 4-state Markov Process are given below. For each generator, find the expected time until the process hits state 4 if it starts in state 1.

(a) $\begin{bmatrix} -2 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 1 & 1 & -3 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

(b) $\begin{bmatrix} -3 & 1 & 1 & 1 \\ 0 & -3 & 2 & 1 \\ 1 & 2 & -4 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

Chapter 6: Continuous-Time Markov Chains

8. Consider two machines, both of which have an exponential lifetime with mean $1/\lambda$. There is a single repairman that can service machines at an exponential rate μ. Set up the Kolmogorov backward equations; you do not need to solve them.

12. Each individual in a biological population is assume to give birth at an exponential rate λ, and to die at an exponential rate μ. In addition, there is an exponential rate of increase θ due to immigration. However, immigration is not allowed when the population size is N or larger.

    (a) Set this up as a birth and death model.

    (b) If $N = 3, \lambda = \theta = 1, \mu = 2$, determine the proportion of time that immigration is restricted.

13. A small barbershop, operated by a single barber, has room for at most two customers. Potential customers arrive at a Poisson rate of three per hour, and the successive service times are independent exponential random variable with mean 1/4 hour.

    (a) What is the average number of customers in the shop?

    (b) What is the proportion of customers that enter the shop?

    (c) If the barber could work twice as fast, how much more business would he do?

14. Potential customers arrive at a full-service, one-pump gas station at a Poisson rate of 20 cars per hour. However, customers will only enter the station for gas if there are no more than 2 cars (including the one currently being attended to) at the pump. Suppose the amount of time required to service a car is exponentially distributed with a mean of five minutes.

    (a) What fraction of the attendant's time will be spent servicing cars?

    (b) What fraction of potential customers are lost?

22. Customers arrive at a single-server queue in accordance with a Poisson process having rate λ. However, an arrival that finds n customers already in the system will only join the system with probability $1/(n+1)$. Show that the limiting distribution of the number of customers in the system is Poisson with mean λ/μ.