Homework 1

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Section 1.1

8. Write the number in the form $a + bi$.

$$\frac{(8+2i)-(1-i)}{(2+i)^2}$$

10. Write the number in the form $a + bi$.

$$\left[\frac{2+i}{6i-(1-2i)}\right]^2$$

Section 1.2

7. (e) Describe the set of points z in the complex plane that satisfies $z = \operatorname{Re} z + 2$.

16. Prove that if $|z| = 1$ $(z \neq 1)$, then $\operatorname{Re}[1/(1-z)] = \frac{1}{2}$.

Section 1.3

5. (d) Find the value of

$$\left|\frac{(\pi+i)^{100}}{(\pi-i)^{100}}\right|$$

7. (h) Find the argument of this complex number and write it in polar form.

$$\frac{-\sqrt{7}(1+i)}{\sqrt{3}+i}$$

28. Let the crankshaft pivot O lie at the right of the origin of the coordinate system, and let z be the complex number giving the location of the base of the piston rod, as depicted in Fig 1.14,

$$z = \ell + id$$

where ℓ gives the piston's linear excursion and d is a fixed offset. The crank arm is described by $A = a(\cos\theta_1 + i\sin\theta_1)$ the connecting arm by $B = b(\cos\theta_2 + i\sin\theta_2)$ (θ_2 is negative in Fig 1.14). Exploit the obvious identity $A + B = z = \ell + id$ to derive the expression relating the piston position to the crankshaft angle:

$$\ell = \cos\theta_1 + b\cos\left[\sin^{-1}\left(\frac{d - a\sin\theta_1}{b}\right)\right]$$

Solution. Because of the identity

$$\begin{aligned} A + B &= a(\cos\theta_1 + i\sin\theta_1) + b(\cos\theta_2 + i\sin\theta_2) \\ &= (a\cos\theta_1 + b\cos\theta_2) + i(a\sin\theta_1 + b\sin\theta_2) \\ &= \ell + id \end{aligned}$$

we must have

$$\begin{aligned} &a\cos\theta_1 + b\cos\theta_2 = \ell \\ &a\sin\theta_1 + b\sin\theta_2 = d \implies \theta_2 = \sin^{-1}\left(\frac{d - a\sin\theta_1}{b}\right) \\ &\implies \ell = a\cos\theta_1 + b\cos\left[\sin^{-1}\left(\frac{d - a\sin\theta_1}{b}\right)\right] \end{aligned}$$

as desired. □

Section 1.4

11. Determine which of the following properties of the real exponential function remain true for the complex exponential function

    (a) e^x is never zero.

    (b) e^x is a one-to-one function.

    (c) e^x is defined for all x.

    (d) $e^{-x} = 1/e^x$.

18. Sketch the curves that are given for $0 \le t \le 2\pi$ by

    (a) $z(t) = e^{(1+i)t}$

    (b) $z(t) = e^{(1-i)t}$

    (c) $z(t) = e^{(-1+i)t}$

    (d) $z(t) = e^{-1-i)t}$

22. Show that if n is an integer then

$$\int_0^{2\pi} e^{in\theta}\,d\theta = \int_0^{2\pi} \cos(n\theta)\,d\theta + i\int_0^{2\pi} \sin(n\theta)\,d\theta = \begin{cases} 2\pi & \text{if } n = 0 \\ 0 & \text{if } n \neq 0 \end{cases}$$

Section 1.5

4. Use the identity (1) to show that

    (a) $(\sqrt{3} - i)^7 = -64\sqrt{3} + 64i$

    (b) $(1 + i)^{95} = 2^{47}(1 - i)$

5. (f) Find the value of $\left(\frac{2i}{1+i}\right)^{1/6}$