

Homework 9

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Chapter 7: Renewal Theory

1. Is it true that
 - (a) $N(t) < n$ if and only if $S_n > t$?
 - (b) $N(t) \leq n$ if and only if $S_n \geq t$?
 - (c) $N(t) > 0$ if and only if $S_n < t$?
2. Suppose that the inter-arrival distribution for a renewal process is Poisson distributed with mean μ . That is, suppose

$$P[X_n = k] = e^{-\mu} \frac{\mu^k}{k!}, \quad k = 0, 1, \dots$$

- (a) Find the distribution of S_n .
 - (b) Calculate $P[N(t) = n]$.
4. Let $\{N_1(t), t \geq 0\}$ and $\{N_2(t), t \geq 0\}$ be independent renewal processes. Let $N(t) = N_1(t) + N_2(t)$.
 - (a) Are the inter-arrival times of $\{N(t), t \geq 0\}$ independent?
 - (b) Are they identically distributed?
 - (c) Is $\{N(t), t \geq 0\}$ a renewal process?
5. Let U_1, U_2, \dots be independent uniform $(0, 1)$ random variables, and define N by

$$N = \min\{n \mid U_1 + U_2 + \dots + U_n > 1\}$$

What is $E[N]$?

7. Mr. Smith works on a temporary basis. The mean length of each job he gets is three months. If the amount of time he spends between jobs is exponentially distributed with mean 2, then at what rate does Mr. Smith get new jobs?
9. A worker sequentially works on jobs. Each time a job is completed, a new one is begun. Each job, independently, takes a random amount of time having distribution F to complete. However, independently of this, shocks occur according to a Poisson process with rate λ . Whenever a shock occurs, the worker discontinues working on the present job and starts a new one. In the long run, at what rate are jobs completed?
12. Events occur according to a Poisson process with rate λ . Any event that occurs within a time d of the event that immediately preceded it is called a d -event.
 - (a) At what rate do d -events occur?
 - (b) What proportion of all events are d -events?