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Homework 4

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1: Consider the linear program (LP) min $c^T x$ such that $Ax = b, x \ge 0$ where

$$A = \begin{bmatrix} -6 & -5 & 25 & 3 & -85 & 4 & 30 \\ 24 & -2 & 28 & 6 & -55 & 1 & -9 \\ 9 & -5 & 11 & 2 & -55 & -1 & 19 \end{bmatrix}, \quad b = \begin{bmatrix} 62 \\ 62 \\ 3 \end{bmatrix}, \quad c = \begin{bmatrix} 23 & 1 & -16 & -1 & 52 & -6 & -12 \end{bmatrix}^T$$

Solve this problem using the Simplex Method, starting from the basis consisting of A's columns 1, 3, 6.

```
_{1} M = [1 -23 -1 16 1 -52 6 12 0;
 2 0 -6 -5 25 3 -85 4 30 62;
3 0 24 -2 28 6 -55 1 -9 62;
 4 0 9 -5 11 2 -55 -1 19 3];
5 % swap columns 3 and 7 in M (correspond to columns 2 and 6 in A)
6 M(:,[3, 7]) = M(:,[7, 3]);
7 \text{ A=rref}(M)
9 A =
10
11 <<<<<  HEAD
          1
                           0
                                            0
                                                             0
                                                                              4/9
                                                                                             -13/9
12
                           7/9
                                         -130/9
                                                          -569/9
          0
                           1
                                            0
                                                             0
                                                                              1/9
                                                                                              35/9
                           4/9
                                          -28/9
                                                            13/9
                           0
                                                             0
                                                                              2/9
                                                                                             205/9
14
                                            1
                          26/9
                                         -137/9
                                                           134/9
                           0
                                            0
                                                             1
                                                                              1/9
                                                                                             -55/9
15
                          -5/9
                                           26/9
                                                             4/9
16
       1.0000
                                              0
                                                    0.4444
                                                               -1.4444
                                                                           0.7778
                                                                                    -14.4444
17
            -63.2222
             0
                  1.0000
                                   0
                                              0
                                                    0.1111
                                                               3.8889
                                                                           0.4444
                                                                                     -3.1111
18
                 1.4444
                                                    0.2222
                              1.0000
                                                               22.7778
                                                                           2.8889
                                                                                    -15.2222
19
                 14.8889
             0
                        0
                                   0
                                         1.0000
                                                               -6.1111
                                                                          -0.5556
                                                                                      2.8889
                                                    0.1111
20
                 0.4444
21 >>>>> f5d31343c2616764d3e6e7032cdf1a02353f09fc
22
^{23} % use column 7 since it is largest positive number
_{24} % pivot on row 2 since ratio (13/9)/(4/9) is smallest, positive
25 % swap columns 2 and 7
26 A(:,[2, 7]) = A(:,[7, 2]);
27 B=rref(A)
28
29 B
30
       1.0000
                                                                                     -9.0000
                                   0
                                              0
                                                    0.2500
                                                               -8.2500
                                                                          -1.7500
31
            -65.7500
                 1.0000
32
            0
                                   0
                                              0
                                                    0.2500
                                                               8.7500
                                                                           2.2500
                                                                                     -7.0000
                 3.2500
                              1.0000
             0
                       0
                                              0
                                                   -0.5000
                                                               -2.5000
                                                                          -6.5000
                                                                                      5.0000
                 5.5000
                                   0
                                         1.0000
                                                    0.2500
                                                               -1.2500
                                                                           1.2500
                                                                                     -1.0000
34
                 2.2500
```

```
36 % use column 5 since it is largest positive number
37 % pivot on row 4 since ratio (9/4)/(1/4) is smallest, positive
_{\rm 38} % swap columns 4 and 5
39 B(:,[4, 5])=B(:,[5, 4]);
40 C=rref(B)
41
42 C =
43
               44
45
46
47
48
49 % minimum is -68.
50 diary off
```

2: Consider the linear program (LP) min $c^T x$ such that $Ax = b, x \ge 0$ where

$$A = \begin{bmatrix} 8 & -226 & -33 & 10 & 9 & 49 & -1 \\ 9 & -199 & -51 & 10 & 3 & 25 & -25 \\ 2 & 24 & 45 & -6 & 3 & -45 & -15 \end{bmatrix}, \quad b = \begin{bmatrix} 107 \\ 55 \\ 25 \end{bmatrix}, \quad c = \begin{bmatrix} -4 & 63 & 7 & -2 & -2 & 0 & 21 \end{bmatrix}^T$$

Solve this problem using the Simplex Method, starting from the basis consisting of A's columns 1, 3, 4.

3: Consider the linear program (LP) min $c^T x$ such that $Ax = b, x \ge 0$ where

$$A = \begin{bmatrix} 7 & 7 & 45 & -1 & 3 & -53 & -68 \\ 9 & -5 & 27 & -115 & 7 & -129 & 42 \\ 5 & -3 & 63 & -96 & 10 & -109 & 86 \end{bmatrix}, \quad b = \begin{bmatrix} 26 \\ 18 \\ 34 \end{bmatrix}, \quad c = \begin{bmatrix} 1 & 7 & -37 & 94 & -9 & 76 & -146 \end{bmatrix}^T$$

- a) Solve this problem using the Simplex Method, starting from the basis consisting of A's columns 1, 2, 5.
- b) Solve this problem using the Simplex Method, starting from the basis consisting of A's columns 1, 2, 7. Comment on the difference in outcome between this part b and the previous part a.
- c) Solve this problem using the Simplex Method, starting from the basis consisting of A's columns 1, 3, 6. Observe how the objective function changes through this particular Simplex Method implementation, and comment on an anomaly.