Homework 9

Aleck Zhao

November 9, 2017

Chapter 9: Category

5. If A is a subset of $\mathbb{R}$ and if x is in the interior of A, show that x is a point of continuity for χ_A (the characteristic function of A). Are there any other points of continuity?

9. If E is a closed set in $\mathbb{R}$, show that $E = D(f)$ for some bounded function f. (Hint: A sum of two characteristic functions will do the trick.)

12. More generally, in any metric space, show that every open set is an F_σ and that every close set is a G_δ.

14. Prove that A has an empty interior in M if and only if A^c is dense in M.

28. In a metric space M, show that any subset of a first category set is still first category, and that a countable union of first category sets is again first category.

30. Show that $\mathbb{N}$ is first category in $\mathbb{R}$ but second category in itself.

32. In $\mathbb{R}$, show that any open interval (and hence any nonempty, open set) is a second category set.

47. Let $\mathcal{P}$ be the vector space of all polynomials supplied with the norm $\|p\| = \max_{0\le i\le n} |a_i|$, where $p(x) = a_0 + a_1 x + \cdots + a_n x^n \in \mathcal{P}$. Show that P is not complete.