Homework 4

Aleck Zhao

October 13, 2016

1: Consider the linear program (LP) min $c^T x$ such that $Ax = b, x \geq 0$ where

$$A = \begin{bmatrix} -6 & -5 & 25 & 3 & -85 & 4 & 30 \\ 24 & -2 & 28 & 6 & -55 & 1 & -9 \\ 9 & -5 & 11 & 2 & -55 & -1 & 19 \end{bmatrix}, \quad b = \begin{bmatrix} 62 \\ 62 \\ 3 \end{bmatrix}, \quad c = \begin{bmatrix} 23 & 1 & -16 & -1 & 52 & -6 & -12 \end{bmatrix}^T$$

Solve this problem using the Simplex Method, starting from the basis consisting of A's columns 1, 3, 6.

```
M = [1 -23 -1 16 1 -52 6 12 0;
0 -6 -5 25 3 -85 4 30 62;
0 24 -2 28 6 -55 1 -9 62;
0 9 -5 11 2 -55 -1 19 3];
% swap columns 3 and 7 in M (correspond to columns 2 and 6 in A)
M(:,[3, 7])=M(:,[7, 3]);
A=rref(M)

A =

    1.0000         0         0         0    0.4444   -1.4444    0.7778  -14.4444
        -63.2222
         0    1.0000         0         0    0.1111    3.8889    0.4444   -3.1111
             1.4444
         0         0    1.0000         0    0.2222   22.7778    2.8889  -15.2222
             14.8889
         0         0         0    1.0000    0.1111   -6.1111   -0.5556    2.8889
             0.4444

% use column 7 since it is largest positive number
% pivot on row 2 since ratio (13/9)/(4/9) is smallest, positive
% swap columns 2 and 7
A(:,[2, 7])=A(:,[7, 2]);
B=rref(A)

B =

    1.0000         0         0         0    0.2500   -8.2500   -1.7500   -9.0000
        -65.7500
         0    1.0000         0         0    0.2500    8.7500    2.2500   -7.0000
             3.2500
         0         0    1.0000         0   -0.5000   -2.5000   -6.5000    5.0000
             5.5000
         0         0         0    1.0000    0.2500   -1.2500    1.2500   -1.0000
             2.2500

% use column 5 since it is largest positive number
% pivot on row 4 since ratio (9/4)/(1/4) is smallest, positive
% swap columns 4 and 5
B(:,[4, 5])=B(:,[5, 4]);
C=rref(B)

C =

     1     0     0     0    -1    -7    -3    -8   -68
     0     1     0     0    -1    10     1    -6     1
     0     0     1     0     2    -5    -4     3    10
```

```
     0     0     0     1     4    -5     5    -4     9

% minimum is -68.
diary off
```

2: Consider the linear program (LP) min $c^T x$ such that $Ax = b, x \geq 0$ where

$$A = \begin{bmatrix} 8 & -226 & -33 & 10 & 9 & 49 & -1 \\ 9 & -199 & -51 & 10 & 3 & 25 & -25 \\ 2 & 24 & 45 & -6 & 3 & -45 & -15 \end{bmatrix}, \quad b = \begin{bmatrix} 107 \\ 55 \\ 25 \end{bmatrix}, \quad c = \begin{bmatrix} -4 & 63 & 7 & -2 & -2 & 0 & 21 \end{bmatrix}^T$$

Solve this problem using the Simplex Method, starting from the basis consisting of A's columns 1, 3, 4.

```
M=[1 4 -63 -7 2 2 0 -21 0;
0 8 -226 -33 10 9 49 -1 107;
0 9 -199 -51 10 3 25 -25 55;
0 2 24 45 -6 3 -45 -15 25];
% swap rows 3 and 5
M(:,[3, 5])=M(:,[5, 3]);
A=rref(M)

A =

    1.0000         0         0         0    3.0000    0.3333    5.0000   -6.0000
        -25.0000
         0    1.0000         0         0   -9.0000         0   -6.0000   -6.0000
             2.0000
         0         0    1.0000         0  -22.0000    2.0000   13.0000    8.0000
             19.0000
         0         0         0    1.0000   -2.0000    0.3333    1.0000    1.0000
             3.0000

% use column 7 since first row value is greatest
% pivot on row 4 since 19/13 is smallest
% swap columns 3 and 7
A(:, [3, 7])=A(:, [7, 3]);
B=rref(A)

B =

    1.0000         0         0         0   11.4615   -0.4359   -0.3846   -9.0769
        -32.3077
         0    1.0000         0         0  -19.1538    0.9231    0.4615   -2.3077
             10.7692
         0         0    1.0000         0   -1.6923    0.1538    0.0769    0.6154
             1.4615
         0         0         0    1.0000   -0.3077    0.1795   -0.0769    0.3846
             1.5385

% use column 5 since first row value is greatest
% all entries in column 5 are negative, so function is unbounded.
diary off
```

3: Consider the linear program (LP) min $c^T x$ such that $Ax = b, x \geq 0$ where

$$A = \begin{bmatrix} 7 & 7 & 45 & -1 & 3 & -53 & -68 \\ 9 & -5 & 27 & -115 & 7 & -129 & 42 \\ 5 & -3 & 63 & -96 & 10 & -109 & 86 \end{bmatrix}, \quad b = \begin{bmatrix} 26 \\ 18 \\ 34 \end{bmatrix}, \quad c = \begin{bmatrix} 1 & 7 & -37 & 94 & -9 & 76 & -146 \end{bmatrix}^T$$

a) Solve this problem using the Simplex Method, starting from the basis consisting of A's columns 1, 2, 5.

b) Solve this problem using the Simplex Method, starting from the basis consisting of A's columns 1, 2, 7. Comment on the difference in outcome between this part b and the previous part a.

c) Solve this problem using the Simplex Method, starting from the basis consisting of A's columns 1, 3, 6. Observe how the objective function changes through this particular Simplex Method implementation, and comment on an anomaly.