Homework 4

Aleck Zhao

October 13, 2016

1:

$$A = \begin{bmatrix} -6 & -5 & 25 & 3 & -85 & 4 & 30 \\ 24 & -2 & 28 & 6 & -55 & 1 & -9 \\ 9 & -5 & 11 & 2 & -55 & -1 & 19 \end{bmatrix}, \quad b = \begin{bmatrix} 62 \\ 62 \\ 3 \end{bmatrix}, \quad c = \begin{bmatrix} 23 & 1 & -16 & -1 & 52 & -6 & -12 \end{bmatrix}^T$$

```
M = [1 -23 -1 16 1 -52 6 12 0;
0 -6 -5 25 3 -85 4 30 62;
0 24 -2 28 6 -55 1 -9 62;
0 9 -5 11 2 -55 -1 19 3];
% swap columns 3 and 7 in M (correspond to columns 2 and 6 in A)
M(:,[3, 7])=M(:,[7, 3]);
A=rref(M)

A =

    1.0000         0         0         0    0.4444   -1.4444    0.7778  -14.4444
        -63.2222
         0    1.0000         0         0    0.1111    3.8889    0.4444   -3.1111
             1.4444
         0         0    1.0000         0    0.2222   22.7778    2.8889  -15.2222
             14.8889
         0         0         0    1.0000    0.1111   -6.1111   -0.5556    2.8889
             0.4444

% use column 7 since it is largest positive number
% pivot on row 2 since ratio (13/9)/(4/9) is smallest, positive
% swap columns 2 and 7
A(:,[2, 7])=A(:,[7, 2]);
B=rref(A)

B =

    1.0000         0         0         0    0.2500   -8.2500   -1.7500   -9.0000
        -65.7500
         0    1.0000         0         0    0.2500    8.7500    2.2500   -7.0000
             3.2500
         0         0    1.0000         0   -0.5000   -2.5000   -6.5000    5.0000
             5.5000
         0         0         0    1.0000    0.2500   -1.2500    1.2500   -1.0000
             2.2500

% use column 5 since it is largest positive number
% pivot on row 4 since ratio (9/4)/(1/4) is smallest, positive
% swap columns 4 and 5
B(:,[4, 5])=B(:,[5, 4]);
C=rref(B)

C =

     1     0     0     0    -1    -7    -3    -8   -68
     0     1     0     0    -1    10     1    -6     1
     0     0     1     0     2    -5    -4     3    10
     0     0     0     1     4    -5     5    -4     9

% minimum is -68.
diary off
```

2:

$$A = \begin{bmatrix} 8 & -226 & -33 & 10 & 9 & 49 & -1 \\ 9 & -199 & -51 & 10 & 3 & 25 & -25 \\ 2 & 24 & 45 & -6 & 3 & -45 & -15 \end{bmatrix}, \quad b = \begin{bmatrix} 107 \\ 55 \\ 25 \end{bmatrix}, \quad c = \begin{bmatrix} -4 & 63 & 7 & -2 & -2 & 0 & 21 \end{bmatrix}^T$$

```
M=[1 4 -63 -7 2 2 0 -21 0;
0 8 -226 -33 10 9 49 -1 107;
0 9 -199 -51 10 3 25 -25 55;
0 2 24 45 -6 3 -45 -15 25];
% swap rows 3 and 5
M(:,[3, 5])=M(:,[5, 3]);
A=rref(M)

A =

    1.0000         0         0         0    3.0000    0.3333    5.0000   -6.0000
        -25.0000
         0    1.0000         0         0   -9.0000         0   -6.0000   -6.0000
             2.0000
         0         0    1.0000         0  -22.0000    2.0000   13.0000    8.0000
             19.0000
         0         0         0    1.0000   -2.0000    0.3333    1.0000    1.0000
             3.0000

% use column 7 since first row value is greatest
% pivot on row 4 since 19/13 is smallest
% swap columns 3 and 7
A(:, [3, 7])=A(:, [7, 3]);
B=rref(A)

B =

    1.0000         0         0         0   11.4615   -0.4359   -0.3846   -9.0769
        -32.3077
         0    1.0000         0         0  -19.1538    0.9231    0.4615   -2.3077
             10.7692
         0         0    1.0000         0   -1.6923    0.1538    0.0769    0.6154
             1.4615
         0         0         0    1.0000   -0.3077    0.1795   -0.0769    0.3846
             1.5385

% use column 5 since first row value is greatest
% all entries in column 5 are negative, so function is unbounded.
diary off
```

3:

$$A = \begin{bmatrix} 7 & 7 & 45 & -1 & 3 & -53 & -68 \\ 9 & -5 & 27 & -115 & 7 & -129 & 42 \\ 5 & -3 & 63 & -96 & 10 & -109 & 86 \end{bmatrix}, \quad b = \begin{bmatrix} 26 \\ 18 \\ 34 \end{bmatrix}, \quad c = \begin{bmatrix} 1 & 7 & -37 & 94 & -9 & 76 & -146 \end{bmatrix}^T$$

a) Solve this problem using the Simplex Method, starting from the basis consisting of A's columns 1, 2, 5.

```
M=[1 -1 -7 37 -94 9 -76 146 0;
0 7 7 45 -1 3 -53 -68 26;
0 9 -5 27 -115 7 -129 42 18;
0 5 -3 63 -96 10 -109 86 34];
% swap columns 4 and 6
M(:,[4, 6])=M(:,[6, 4]);
A=rref(M)

A =

     1     0     0     0    -5    -8    -9     0   -22
     0     1     0     0    -5    -1    -8    -7     0
     0     0     1     0     7     4     3    -7     2
     0     0     0     1    -5     8    -6    10     4

% top row is non-positive, so mininum objective function value is -22.
diary off
```

b) Solve this problem using the Simplex Method, starting from the basis consisting of A's columns 1, 2, 7. Comment on the difference in outcome between this part b and the previous part a.

```
M=[1 -1 -7 37 -94 9 -76 146 0;
0 7 7 45 -1 3 -53 -68 26;
0 9 -5 27 -115 7 -129 42 18;
0 5 -3 63 -96 10 -109 86 34];
% swap columns 4 and 8
M(:, [4, 8])=M(:, [8, 4]);
A=rref(M)

A =

    1.0000         0         0         0   -5.0000         0   -9.0000   -8.0000
          -22.0000
         0    1.0000         0         0   -8.5000    0.7000  -12.2000    4.6000
                2.8000
         0         0    1.0000         0    3.5000    0.7000   -1.2000    9.6000
                4.8000
         0         0         0    1.0000   -0.5000    0.1000   -0.6000    0.8000
                0.4000

% top row is non-positive, so minimum objective function value is -22.
% the entries in columns 5 through 8 have different values (aside from top row)
diary off
```

c) Solve this problem using the Simplex Method, starting from the basis consisting of A's columns 1, 3, 6. Observe how the objective function changes through this particular Simplex Method implementation, and comment on an anomaly.

```
M=[1 -1 -7 37 -94 9 -76 146 0;
0 7 7 45 -1 3 -53 -68 26;
0 9 -5 27 -115 7 -129 42 18;
0 5 -3 63 -96 10 -109 86 34];
% swap columns 3 and 7
M(:, [3, 7])=M(:, [7, 3]);
A=rref(M)

A =

    1.0000         0         0         0   13.7500   -0.2500    2.5000  -20.0000
        -18.0000
         0    1.0000         0         0    8.2292   -0.6042    1.4583  -23.2500
                0.5000
         0         0    1.0000         0    1.5833   -0.0833    0.1667   -2.0000
                   0
         0         0         0    1.0000    0.5625    0.0625    0.1250   -0.2500
                0.5000

% objective function value is -18 right now.
% use column 5 since first row value is greatest
% pivot on row 3 since ratio is 0
% swap columns 3 and 5
A(:,[3, 5])=A(:,[5, 3]);
B=rref(A)

B =

    1.0000         0         0         0   -8.6842    0.4737    1.0526   -2.6316
        -18.0000
         0    1.0000         0         0   -5.1974   -0.1711    0.5921  -12.8553
                0.5000
         0         0    1.0000         0    0.6316   -0.0526    0.1053   -1.2632
                   0
         0         0         0    1.0000   -0.3553    0.0921    0.0658    0.4605
                0.5000

% objective function value is still -18.
% use column 7 since first row value is greatest
% pivot on row 3 since ratio of 0 is smallest
% swap columns 3 and 7
B(:,[3, 7])=B(:,[7, 3]);
C=rref(B)

C =

    1.0000         0         0         0  -15.0000    1.0000  -10.0000   10.0000
        -18.0000
         0    1.0000         0         0   -8.7500    0.1250   -5.6250   -5.7500
                0.5000
         0         0    1.0000         0    6.0000   -0.5000    9.5000  -12.0000
                   0
         0         0         0    1.0000   -0.7500    0.1250   -0.6250    1.2500
                0.5000

% objective function value is still -18.
% use column 8 since first row value is greatest
% pivot on row 4 since ratio is smallest non-negative (row 3 ratio is "negative"
    0)
% swap columns 4 and 8
C(:,[4, 8])=C(:,[8,4]);
D=rref(C)
```

```
50
51 D =
52
53     1.0000         0         0         0   -9.0000         0   -5.0000   -8.0000
           -22.0000
54          0    1.0000         0         0  -12.2000    0.7000   -8.5000    4.6000
                 2.8000
55          0         0    1.0000         0   -1.2000    0.7000    3.5000    9.6000
                 4.8000
56          0         0         0    1.0000   -0.6000    0.1000   -0.5000    0.8000
                 0.4000
57
58 % top row is non-positive. minimum objective function value is -22.
59 % multiple steps of the algorithm did not change the objective function value
       since the ratio was 0 each time.
60 diary off
```