

# Graphing Lines

## PhET Sim Design Document

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### Public URL:

[https://docs.google.com/document/d/1551QbIT5yKjlrkB9vQ-GFwXP-qby3Fo\\_yO1ksZFTNS4/edit#](https://docs.google.com/document/d/1551QbIT5yKjlrkB9vQ-GFwXP-qby3Fo_yO1ksZFTNS4/edit#)

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## Audience

This simulation is targeted at upper middle school and lower high school students who are working on understanding linear equations and how the equations relate to their graphical representations.

## Learning Goals

The basic learning goals focus on understanding how each variable in the linear equation affects the graphical representation of the line. Specifically:

- Interpret “b” in the equation  $y = mx + b$  as the “y intercept” and recognize the link between changing b and vertically translating the line.
- Interpret “m” in the equation  $y = mx + b$  as the “slope” of the line, and predict how changing the slope will change the orientation of the line
- Understand qualitatively how a negative slope differs from a positive slope
- Recognize a line in both “slope-intercept form” and “point-slope form”, and be able to interpret the connection between each variable and the graph of the line in these forms
- Predict if a specific point lies on a line of a given equation
- Given a particular line be able to construct the equation of the line
- Understand how the equation of a line can be manipulated such that a given line can be transformed to a line with a different slope and position.

TL: Ariel, I wanted to make some suggested changes to the vocabulary of the goals so that they are “measurable” , in smaller chunks, and also more grade level language appropriate, but I didn’t want to remove your ideas and my word choice may still be too high level. I also split the list into 3 sections of goals. I also recommend that we place the greater than, less, etc as an advanced goal that might be even a separate sim, but have the plan in place so that Chris can plan the programming and Ariel can consider the feature in the design.

### **Students will use a line graph or equation in the form $y = mx + b$ to:**

- Sketch and describe what the “m” means.
- Define “slope” in own words and tell how to find slope
- Sketch and describe how a line with a negative slope differs from one with a positive slope
- Explain how knowing the slope can help graph a line.
- Identify the points where the line crosses the y and x axes
- Define “y- intercept” and “x-intercept” using words, sketches, and the form (x,y).
- Explain how the equation can be used to find the “y- intercept”
- Explain how knowing y-intercept would help graph a line.

- Write the equation for a line from the graph
- Graph a line from the equation.
- Given a point, identify if it is on the line [ie Is it a solution to the equation?]
- If  $m$  and/or  $b$  is changed, predict what you would change on your graph. Use your predictions to graph the line.

**Students will use a line graph or equation in the form  $y = m(x - x_1) + y_1$  to:**

- Sketch and describe what the “ $m$ ” means.
- Define “slope” in own words and tell how to find slope
- Sketch and describe how a line with a negative slope differs from one with a positive slope
- Explain how knowing the slope can help graph a line.
- Identify what  $x_1$  and  $y_1$  mean (or  $h$  and  $k$  if we chose that notation)
- Explain how knowing  $x_1$  and  $y_1$  would help graph a line.
- Write the equation for a line from the graph
- Graph a line from the equation.
- Given a point, identify if it is on the line [ie Is it a solution to the equation?]
- If  $m$ ,  $x_1$ , and/or  $y_1$  is changed, predict what you would change on your graph. Use your predictions to graph the line.

TL: [I am still thinking about the material below. I think Ariel and I need to discuss them.

- Recognize a line in both “slope-intercept form” and “point-slope form”, I see this need, but I wonder how this can be accomplished with a sim that only shows lines. KP: does “point-slope form” mean that if you know any point on the line and the slope, you can draw the line? If this is the case, then on the “game” tab could have a challenge around this... give a point not at the intercept and state the slope and require them to draw the line as the answer.  
TL: Sounds like a good plan

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## Standards

• **Math TEKS 6.5** Patterns, relationships, and algebraic thinking. The student uses letters to represent an unknown in an equation. The student is expected to formulate equations from problem situations described by linear relationships.

**Math TEKS 8.3** Patterns, relationships, and algebraic thinking. The student uses letters to represent an unknown in an equation. The student is expected to formulate equations from problem situations described by linear relationships.

· **CCSS 8.EE Understand the connections between proportional relationships, lines, and linear equations.** 5. *Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.*

**CCSS 8.F Define, evaluate, and compare functions.** 3. Interpret the equation  $y = mx + b$  as defining a linear function, whose graph is a straight line.

## Mockup

### Intro tab

The intro tab will have a simple user interface that allows the user to interact with both the equation of a line, and the line itself to see how the two are related.

The values that will be varied by spinners (or sliders or buttons?) will be

- 1) “+/-”,
- 2) “numerator” , and 3) “denominator” will be able to be changed from 0 to +10 in integer increments. the slope will be shown as an improper nonreduced fraction (reduced?) . For instance, “2” will be shown as “2/1” and “18/4” might be shown as “9/2” if there is a “reduce” option.
- 4) “b” will also be variant but only integer values (-10 to 10 if feasible; need to see scenarios)

The student can also grab the line and make changes, but the line will snap to the grid so that the values are all integers.

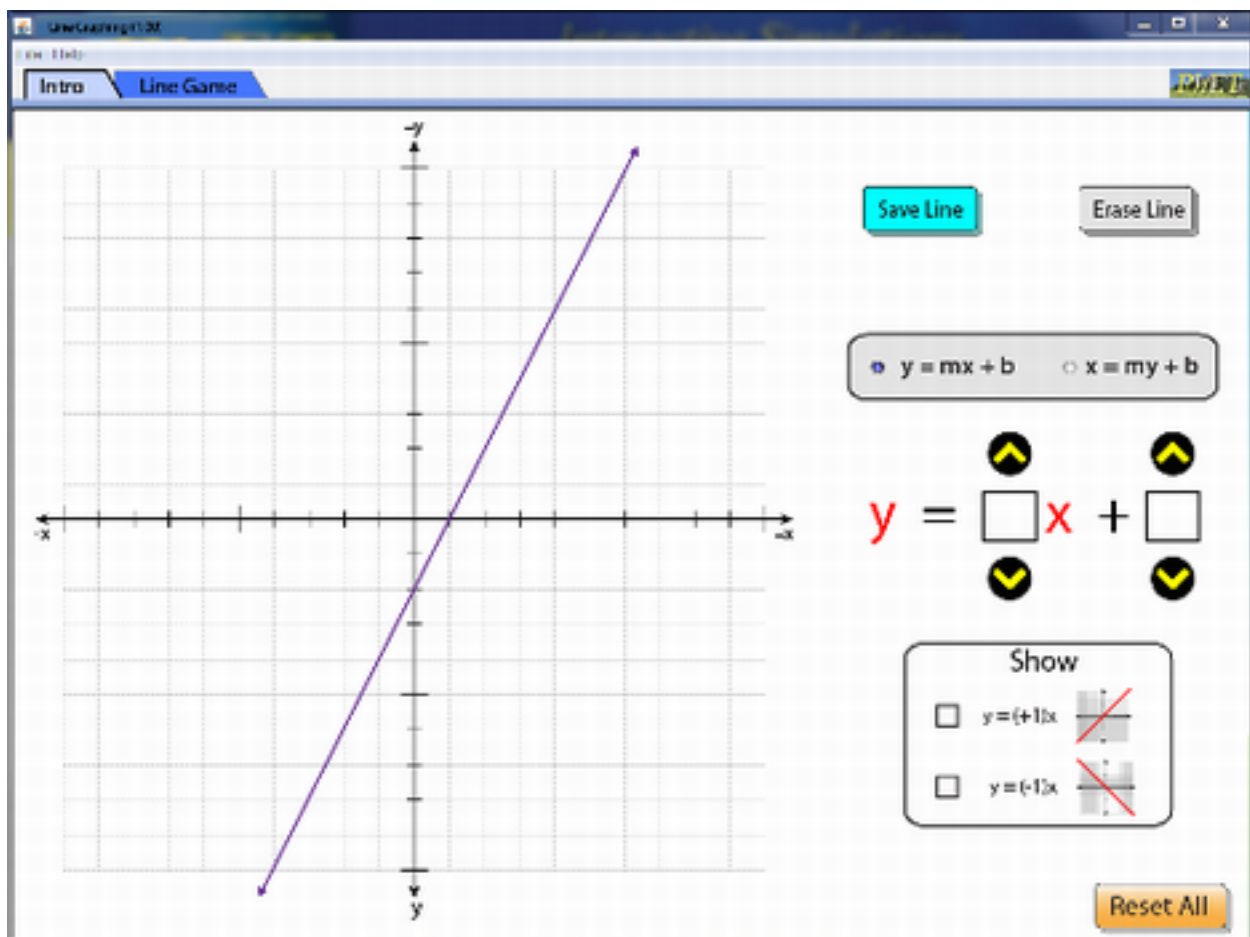
The user will be able to choose a line of “ $y = mx + b$ ” or a line of the form “ $x = my + b$ ” which will accommodate the possibility of vertical lines (we have so far not included recognizing “undefined” slope as a learning goal.

The “Save Line” feature will allow a student to save the current line in a displayed form in the graph (perhaps grayed out) and then manipulate the equation or the active line to see how it compares with the saved line. When a line is saved, the “erase” button will become active, and allow the user to erase the saved line. The saved line equation will be labelled on the line

So, many lines could be displayed at once. The two standard lines, saved line,s and an active line.

The grid will be in one unit increments from -10 to 10 with (0,0) at the center of the play area.

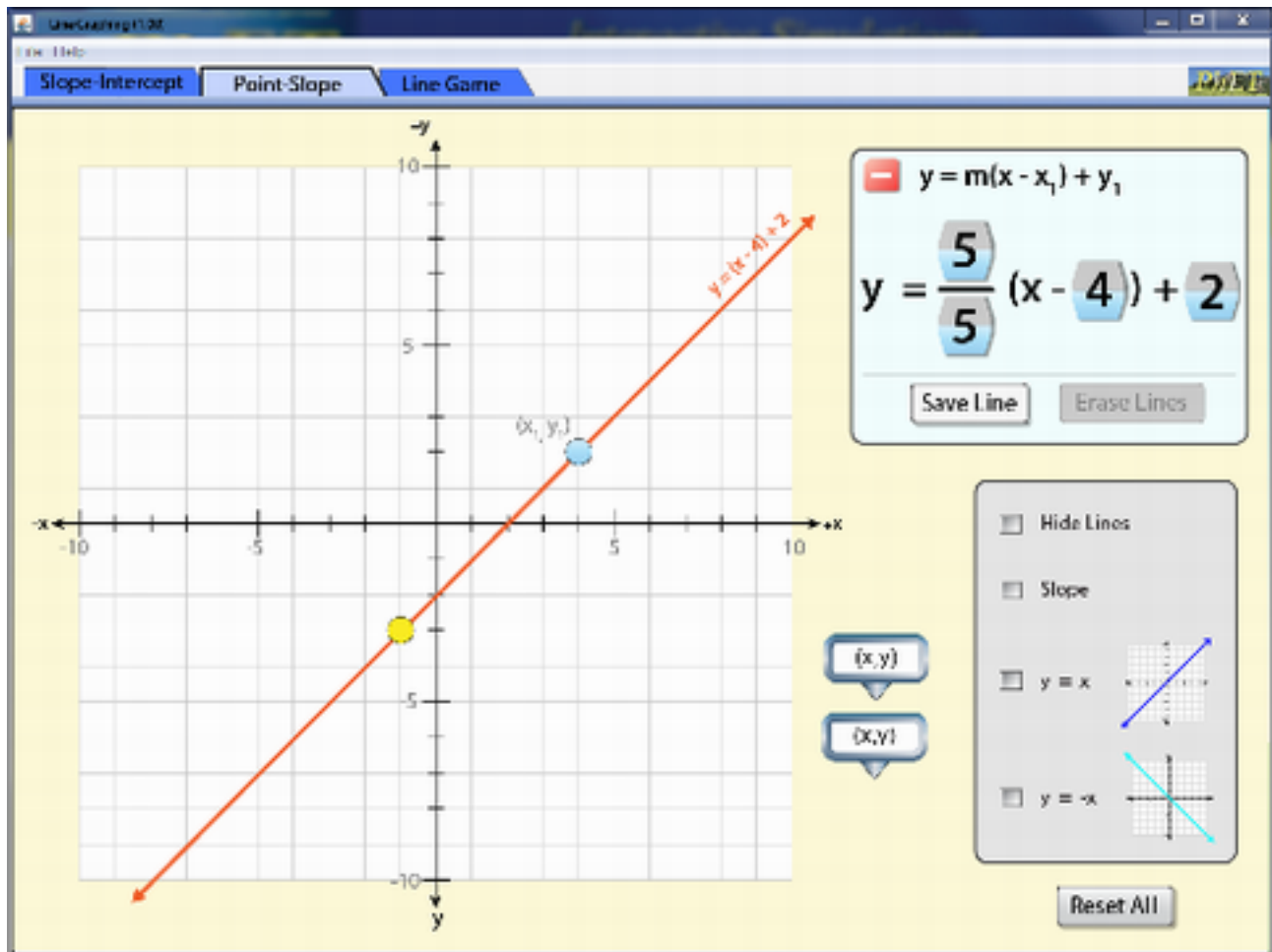
A “point tool” will allow students to see points, but will snap to the grid (ie show only integer values) . It would operate something like the speed tool in Bending light. Chris is going to try some ideas.



## Second Tab

The second tab will have an interface similar to the first but will also allow students to explore the line in point-slope form using the equation:

$$(y - y_1) = m(x - x_1)$$



## Game Tab

### General:

- same framework as RPAL, ABS, Balancing Act
- game settings with on/off controls for timer and sound
- check answer, smiley/frowny face, try again, next, show correct answer
- scoring: +2 first try, +1 second try, 0 points when shown correct answer
- before "try again" is pressed, the "incorrect answer" should remain displayed
- "show answer" should show both the incorrect guess and correct answer
- reward for perfect score
- Default line generally displayed will be  $y=x$
- $y=x$  will be excluded as a challenge, but something like  $y=2x$  will still be possible
- Both slope-intercept and point-slope form will appear on all levels
- Slope will not be labeled in the graph

### Basic Layout:

- Given on left, "create/find/make" on right
- Some sort of Challenge Label, such as "Make the Line", likely centered
- Initial feeling would be to give the two point tools, placed below equation on the corner of the graph
- Manipulated line is black, displayed correct answer is red

Challenges: 6 total on each level

Given an Equation, manipulate graph:

1. Graph, S-I, manipulate slope
2. Graph, S-I, manipulate intercept
3. Graph, S-I, manipulate slope and intercept
4. Graph, S-I, free manipulation
5. Graph, P-S, manipulate slope
6. Graph, P-S, manipulate point
7. Graph, P-S, free manipulate
8. Graph, P-S, place 3 points
9. Graph, S-I, place 3 points

Given a Graph, manipulate equation

10. Eqn, S-I, manipulate slope
11. Eqn, S-I, manipulate intercept
12. Eqn, S-I, manipulate slope and intercept

13. Eqn, P-S, manipulate slope
14. Eqn, P-S, manipulate point
15. Eqn, P-S, manipulate point and slope

Slopes: Positive, Negative,  $2/1$  vs  $2/3$  vs  $7/4$ , Fractions (greater than 1, less than 1), Steep, Shallow, Unity, must move 2nd point “backward” -- must place in negative-negative space, (Slope always simplified)

CM: Last sentence above can be rephrased as: Placement of point on the graph, and the limits of the graph range, forces the user to invert the slope.

### Difficulty:

determined by:

- # variables in equations, or number of points to position
- sign of numbers
- whether form of equation matches information/manipulation available
- whether can graph slope in the “straightforward” direction (e.g. positive slope is up and to right) versus “negative-negative” directions (e.g. positive slope is down and to the left)

### Level 1:

**summary:** 1 variable, easy positive slopes, same sign for  $x_1$  and  $y_1$

(randomize order)

- 1 variable (either slope, intercept, or point on line)
- 3 interactive equations (S-I form: 1 slope, 1 intercept, P-S form: either slope or point)
- 3 interactive graphs (S-I form: 1 slope, 1 intercept, P-S form: opposite of other P-S form, so if slope chosen for interactive equation challenge, use point, and vice versa).
- available positive number slopes  $\{1/2, 1/3, 2/3, 3/4, 3/2, 4/3, 1/4, 5/2, 5/3, 1/5, 2/5, 3/5\}$

On slope questions:

At least one slope question from  $\{3/2, 4/3, 5/2, 5/3\}$

At least one slope question from  $\{1/2, 1/3, 1/4, 1/5\}$

Last slope question from  $\{2/3, 3/4, 3/5, 2/5\}$

-Only show one manipulator. When intercept/point is fixed, plot it.

CM: As it's currently implemented.

- negative and positive intercept, range= $(-6,4)$



On intercept questions:

At least 1 positive intercept

At least 1 negative intercept

- point in point-slope y range is  $(-9, +4)$ , x range is  $(-9, +4)$ , 0's are OK, but not  $(0,0)$ .

On point in point-slope questions: restrict to (pos,pos) or (neg,neg) points

-forms of equation always match graph (in terms of manipulators, constraints, etc.)

-Label challenges with vocab, example: Set the slope, Set the intercept

## Level 2:

**summary:** 1 variable, positive and negative slopes, some more difficult slopes

(randomize order)

- 1 variable (either slope, intercept, or point on line)

- 3 interactive equations (S-I form: 1 slope, 1 intercept, P-S form: either slope or point)

- 3 interactive graphs (S-I form: 1 slope, 1 intercept, P-S form: opposite of other P-S form, so if slope chosen for interactive equation challenge, use point, and vice versa).

- positive and negative slopes

- available positive number slopes harder  $\{\frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{2}{5}, \frac{3}{5}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{2}, \frac{5}{3}, \frac{7}{2}, \frac{7}{3}, \frac{7}{4}, 1, 2, 3, 4, 5\}$

- available **negative** number slopes easier  $\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{2}{3}, \frac{3}{4}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{3}{2}, \frac{4}{3}, \frac{5}{2}, \frac{5}{3}, \frac{5}{4}, 1, 2, 3, 4, 5\}$

On slope questions:

One integer slope (positive or negative number)

One draw from negative slope (fractions)

One draw from positive slope (fractions)

-Only show one manipulator. When intercept/point is fixed, plot it.

Run must always be positive

$x_1$  and  $y_1$  range constrained by slope, sign of  $x_1$  and  $y_1$  can differ

On intercept questions: 1 positive intercept, 1 negative intercept

-forms of equation always match graph (in terms of manipulators, constraints, etc.)

-Label challenges with vocab, example: Set the slope, Set the intercept

### **Level 3:**

**summary:** 2 variables, introduce concept of graphing a line using 2 points.

Same as Level 2, with the following differences:

- 2 variables
- 1 "Graph the Line" challenge S-I equation with S-I manipulators
- 1 "Graph the Line" challenge P-S equation with P-S manipulators
- 1 "Graph the Line" challenge with 2 point manipulators (SI or PS equation form chosen randomly)
- 1 "Make the Equation" challenge S-I
- 1 "Make the Equation" challenge P-S
- 1 "Make the Equation" randomly choose S-I or P-S

-No vocab labeling

### **Level 4**

**summary:** 2 variables, force inversion of slope, 2 free ranging points

- 3 "Make the Equation" challenges from Level 3 (possibly with more randomly-chosen slopes)
- 1 "Graph the Line" challenge with 2 point manipulators, randomly choose equation form either P-S or S-I
- 2 "Graph the Line" challenges, one P-S, one S-I, both force "invert" the given slope, due to where the point is placed within the limits of the graph

### **Level 5**

**summary:** 2 variables, Introduce "Place the Points", introduce horizontal line "Make the Equation"

- 2 "Place the Points" challenges (1 P-S, 1 S-I), ranges of  $x_1, y_1, \text{rise}, \text{run}$  limited to  $[-5, 5]$
- 1 "Make the Equation" challenges (either P-S or S-I) Randomly generate a slope using  $[-10, 10]$  axes ranges, no slopes from this set:  $\{1, 2, 1/2, 1/3, 1/4, 2/3, -1, -2, -1/2, -1/3, -1/4, -2/3\}$

- 1 “Make the Equation” with horizontal line given, S-I form
- 1 “Graph the Line” challenge with 2 point manipulators, (randomly choose form),  $(x_1, y_1)$  and  $(x_2, y_2)$  coordinates constrained to  $[-5, 5]$ .
- 1 “Graph the Line” challenge (S-I manipulators) with horizontal line (zero slope)

## Level 6

**summary:** 2 variables, require mapping between different line forms, “Place the Points”

- 3 “Graph the Line” challenge where line forms are different (eg, P-S equation with S-I manipulators), limit point range to  $[-7, 7]$ , keep slope manip on graph
- 3 “Place the Points” challenges (1 P-S, 1 S-I, 1 horizontal [eg  $y=3$ ]), ranges of  $x_1, y_1, \text{rise}, \text{run}$  limited to  $[-5, 5]$

## Online Resources

<http://www.mathopenref.com/graphfunctions.html>

<http://phet.colorado.edu/en/simulation/equation-grapher>

<http://www.shodor.org/interactivate/activities/Graphit/>

<http://rentcalculators.org/stheli.html>

## References

## Interview and Classroom Feedback

## Discussion

Meeting 01-03-2013

1. Slope tool might go off the screen in some cases in game challenges when the answer is wrong and the line is manipulated, these cases are rare and won't be considered an issue.

#### Meeting 11-8-2012

1. Watch for "multiple saving lines", consider limiting number of saved lines to 5
2. Watch for confusions about multiple equal signs
3. Maybe more negative space between control box and grid (sim interface is compressed towards center on PC)
4. Make blue slope color coordination a single rectangular box around fractional representation of slope
5. Watch in interviews for use of "save line" and "reset all"
6. On PS tab,  $x_1$  and  $y_1$  in the interactive equation can drive the manipulators off the grid.

#### Meeting 11-1-2012

1. "Slope" tab as first tab
2. Label on line "m is (simplified number)"
3. Remove "standard lines" from Slope tab
4. Think about distributing levels

#### Meeting 10-25-2012

1. Navigation challenge: work out carefully before deciding whether same code base.
2. Possible navigation challenge: Make these objects crash, where do they crash, two racers going parallel
3. Slope tool should appear after students get a challenge wrong on a make the line challenge
4. Symmetrize point tools on first 2 tabs
- 5.

#### Meeting 10-16-2012

1. Keep current slope tool but include negatives (watch in interviews if causing confusion)
  2. Definitely have a challenge of two manipulatable points but given slope-intercept form
  3. Navigation style challenge - not in first published version of this sim, but decide whether it would make a 4th tab. Should we develop such a challenge, and if so, should it be in the same code base?
  4. Have vocab on early level challenges "Set the intercept" and "Set the Slope"
  5. Green light on "Place N' Points" challenge
- .

## Meeting with Kathy 10-9-2012

- Up for discussion:

1. Should slope tool include negatives?
2. Should there be a challenge with two manipulatable points but in slope-intercept (probably yes)
3. Multiple lines that go through the same intercept (or point)? (probably not)
4. Consider parallel and perpendicular in another sim.

## Physics Meeting 9-20-2012

- Change slope brackets to arrows

## Physics Meeting 9-13-2012

- After “show answer” in the game, the line challenges will still be interactive, and also the equation will be interactive for equation challenges

- For incorrect answers in the game at “show answer”, show users incorrect equation and the target equation, as well as incorrect line and correct line

- Slope tool will not be displayed in game at any point

- For interactive equation in the game: equation in “standard” form except for where interactive, ie slope is simplified

- For point-slope interactive equation easiest levels will just have one point interactive equation

- For point-slope if “0” is in the equation for instance  $(y-0) = \frac{3}{4}(x-0)$  we will write the equation as  $y = \frac{3}{4}x$

- For noninteractive slope try to put the minus sign more centered on the division bar

- Horizontal lines will appear in the game, but vertical lines will not

## Physics Meeting 9-6-2012

-  $x=c$  label for vertical lines

- transparent red x on slope-intercept equation in the entire sim when slope undefined

## Physics Meeting 7/26/2012

- Variations on manipulation in game for the graph. Control just slope (intercept show fixed intercept), control just intercept, slope and intercept adjust, and 2 freely ranging points.
- Variations for equations in game...slope only interactive, intercept only, points only, everything interactive
- For the moment start interactive numbers default to "0" in game
- Keep moving point tools available after correct answer displayed
- For inequalities, no save line, no erase line. Hide "plot" still available. no standard lines. Slope tool still maybe still available. Only use slope-intercept form. Possibility for using "tool tip" when hovering over. Color coding of manipulators, perhaps different shapes. Think about making it clear which line (on second tab with two lines) is being controlled.

#### General Update Notes 7/19/2012

- Discreet jumping on the grid seems to work well
- The hybrid spinner/picker design has been finalized and appears to work well in the interactive equation
- only one form of "point slope form" is being used (only one form of the equation can truly be called "point slope")
- RTL languages appear to write math formulas LTR, so the equations and such are all left in LTR format
- Everyone seems happiest with "dynamic constraints" on the interactive line for both tabs
- In "point slope" form the numbers for the "point" change to negative numbers with a fixed minus sign preceding them. Even though this makes a "negative minus a negative" in the equation it keeps a straightforward connection to the interaction of the equation to the translation of the interactive line.

#### Physics Meeting 5/31/2012

- Decide on option for slope-intercept display on point-slope tab after equation is pretty up and such
- See how discreet jumping to grid points looks
- try hybrid spinner/picker

#### Physics Meeting 5/23/2012

Traditional game:

- given equation, two points in bucket to be placed to form correct line. Check button appears when both points are placed. As soon as second point is in the grid, the line will show.

- given a line, write in the equation....starting from  $y=1/1*x$   
partially filled in equation

- Is this point on the line?

Physics Meeting 5/10/2012

Consider how to make “negative over negative” more visible

Second tab, try to implement both forms of point-slope form

See cost for implementing RTL language

Considering two “game tabs”....one more explicit, one more implicit (possibly a navigation challenge). Almost definitely a “pick two points to make a line” type challenge. Also maybe try to scaffold, so perhaps first just finding slope of a line.

Physics Meeting 4/19/2012

Everyone seems happy to try refining “pickers”

Dynamic constraints seem worth interviewing on

Make decisions on Line Game next week

Ping Karina/Ian for feedback on point-slope form

Physics Meeting 4/12/2012

Everyone happy with equation not showing while line is dragged

Everyone seems happier with dynamic constraints

For 1st tab, constrain so slope and intercept manipulators cannot be overlapped.

For 2nd tab,  $x_1$  and  $y_1$  will translate the line horizontally and vertically, but keep the slope fixed. The line will be “anchored” at the “ $x_1, y_1$ ” and rotate about this point when the slope manipulator is changed, keeping  $x_1$  and  $y_1$  unchanged as the slope changes.

Going to try “number on spinner” idea.

Physics Meeting 4/5/2012

Design issues:

- Is integer-based manipulation of the line acceptable? Or do we need to implement something more sophisticated?

To be answered with interviews

We will put representation of equation for lines with fractional slope (not / representation)

- Keeping the slope manipulator inside the grid can only be accomplished by using a grid size that is 2x the range of rise, run, and intercept.

Try dynamically changing constraints to keep manipulators in grid

- Should we constrain run to positive values (quadrants 1 & 4) so that minus sign is in the numerator?

Unconstrained

- Currently using a single color for all saved lines, with highlighting on mouse over. is that OK? Having separate colors for each line is problematic.

OK

- Layout problem with interactive equation, generally looks too spaced out, but needs to accommodate “-10”

Going to work on

- When you set run=0 in the interactive equation, it feels odd/confusing to replace the equation with “ $x=0$ ”

Equation stays up, line labelled sloped undefined

- What should point tool look like?

Looks fine

- What should point tool display while it's being dragged? Nothing? Closest grid point?

Closest grid point



- When should point tool "light up"? When it's on the interactive line? When it's on any line?  
What does "light up" look like?

On any line....

- When you press "Save Line", you can't see your saved line until you move the interactive line.  
This seems confusing.

Saved line shows up on top

- When "Lines" is unchecked, should controls related to graph lines be disabled? (rise/run,  $y=x$ ,  $y=-x$ , Save Line, Erase Lines)

Change to "hide lines",

- Look and formatting of reduced equations on the graphed line.

- If interactive line is the same as one of the standard lines, you won't see those standard lines when they are turned on.

- Should the graph's origin be labeled? If so, using what convention?

No need

- Does shape and color coding of manipulators and spinner buttons help make a connection between them?

Yes

- Overall color scheme (lines, buttons, manipulators, backgrounds, etc.)

Decent

- Does everything we're doing in the first tab work for inequality and absolute value?

**EMAIL NOTES:** On 3/20/2012 11:58 PM, Ariel J Paul wrote:

I will try to address your questions now. Chris, I put in a few comments directly for you.

1. KP: I thought this simulation was going to address absolute value graphs...  $y = m|x| + b$

No, this sim will not address this goal. Trish never brought up absolute value you with me, and it is beyond the scope of this sim. However, we are thinking of doing a "Line Graphing: "XXXX" series. This first one being "Line Graphing" The next being "Line Graphing: Inequalities" Certainly Graphing absolute value could be another or incorporated to a related sim.

KP: I ask because that was the original sim proposed by Trish --- that is sketched on my whiteboard. I'm OK with focusing on regular lines - and putting this off.

2. KP: does “point-slope form” mean that if you know any point on the line and the slope, you can draw the line? If this is the case, then on the “game” tab could have a challenge around this... give a point not at the intercept and state the slope and require them to draw the line as the answer.

This is the point-slope form we will use:  $y = m(x - x_1) + y_1$   
sometimes also written as:  $(y - y_1) = m(x - x_1)$

And yes, the game tab could have such a challenge

KP: I think we should make the decision of which form of point-slope to use informed by which is most popular form ... can get this by looking across texts or writing to one of the math ed list serves?

3KP: I don't see this (numerator/denominator) behavior reflected in the mock-up? Is idea to mirror slope as (change in y over change in x) with these?

KP: Can we put some numbers on the axes? Maybe just at 5 and 10 points?

Yes, you are correct, this is not currently reflected in the mockup, and came from a discussion with Chris today. I can tweak the mockup if you prefer. And yes, it is to emphasize the "rise over run" idea

Yes, number can be on the axes, if it is not too cluttered with them. 5 and 10 seems like a good place to start

KP: I'm trying to visualize this behavior --- it feels like it might be hard for the student to make sense of because the value will be changing pretty randomly if they are just controlling the line? Switching from say  $2/1$  to  $3/2$  --- both numerator and denominator switching. It requires pretty darn good understanding of fractions to get the overall trend of  $m$  is smaller or bigger, it seems. Perhaps we need to be able to switch between a fractions display for  $m$  and a decimal display for  $m$ ?

Decimal display was ruled out by the team in several discussions. Currently we are thinking of one tool for changing the slope and one tool for changing the  $y$  intercept, so I think the issue you are concerned with will not be a major issue.

KP: I have a suggestion for the current mock-up regarding the fraction implementation --- easier to do in the drawing, so I'll send the suggestion out in a minute.

KP: Another thought related to this that might help a bit ... if in "fractions" mode for m, the line had 2 points plotted on it that were interactive, and showed the run and rise relationship. These points would always snap to the grid and the fraction displayed would always be the rise/run of those points. Like this. See top image

TL: Explicitly showing the delta's seems very helpful . I wonder if we could show this with a slope tool sort of like the point tool. See second image with my poorly drawn "slope tool" Also, Ariel and I have talked about a slope sim that would use real world examples like house roofs, mountains, playground equipment, etc.

@Chris, please see in the Google doc, this seems like a reasonable function if it can be incorporated into the sim.

KP: Again, I think having the slope displayed as a decimal will help make the connection between standard line and slope greater or less than 1.

TL: we discussed the option of using a radio button to switch between fraction vs decimal and began to worry about clutter. If both can be shown, I can see where decimal might be helpful, but usually, decimal slopes are part of more advanced graphing.

Unfortunately decimal does not get at the "rise over run" behavior well, and also Trish specifically said it would not be good to have in this introductory sim. However, it would be appropriate for a higher level sim.

KP: Maybe another tab has continuous lines.

KP: If use the interact with line using 2-points as above, will have to decide what to do when they do the vertical line. What word you call that line? Undefined? 'Not a function'?

TL: "not a function" is an advanced goal, but being able to graph and write equations for vertical lines is fundamental. In texts, vertical lines are usually just written as  $x =$  number since there is no slope or y intercept.

KP: How common is it to teach  $x = my + b$  --- I don't recall that? I would have thought the other line equation option would have been  $(y - y_0) = m(x - x_0)$ . I see that in the learning goals, but I don't see that form addressed in the sim?

TL: we discussed both having  $(y - y_0) = m(x - x_0)$  vs  $y = m(x - x_1) + y_1$ , and thought sticking to  $y =$  might help students since it aligns with their calculator and  $f(x)$  notation. Ariel has noted that we right now thought it might be best introduced in a second tab and haven't really flushed out the details. I think we recognise that we might want to enable students to compare representations.

The vertical line will only be "allowed" in the  $x = my + b$  mode. It is beyond the scope of the learning goals of this sim to deal with "undefined" slopes.

@Chris, I do not think I made this constraint explicit, though Trish and I had talked about it.  
KP: Suggested switching line reading to read " $x=0$ " at vertical line.

KP: I think the equation of the saved line also needs to be displayed somewhere for this comparison feature to be more helpful.

TL: yes!

Right now there is room in the interface for this possibility. Seems worthwhile to me.

KP: Why not let them measure any point anywhere on the whole coordinate system?

Will they feel like non-integers cannot be displayed as points in the (x,y) format?

Perhaps the

point changes color -- from black or grey to the "color" of the line when the point is actually on the line --- and "snaps to be on the line" if they are within some reasonable amount?

Trish and I decided specifically today that only displaying integers was definitely a "pedagogically appropriate simplification" since students are specifically taught to find the integer locations on a graph when looking for slope as they are introduced to line graphing. Again, we do not want decimal display in the sim at this level.

KP: Other questions ...

I thought that when Trish first explained this simulation to me, it was going to address the absolute value line graphing issues. I don't see that here, or discussed in this document at all. Did this idea get dropped? If so, what is the plan for that?

TL: The idea is that we would have a series of graphing line sims: equality, inequality, and absolute value. Whether these are all separate or one big one is still up for discussion