Trajectories for Rutherford Scattering

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Here is the algorithm for computing the alpha particle trajectories for the Plum Pudding, Bohr, deBroglie, and Schrodinger models in the "Models of the Hydrogen Atom" simulation. The only difference between the models is the value of the constant D, discussed below.

Variables and Constants:

x,y = coordinates in Cartesian coordinates
r,ϕ = coordinates in Polar coordinates
t = time
v = particle speed
v_o = initial particle speed
Δt = size of time step
L = width of box.
b = initial distance between alpha particle and y-axis.
R = Radius of red blob in Plum Pudding model (can be approximate). $D = 2ke^2/(mv_0^2/2)$ = a constant with units of length.
For Bohr/deBroglie/Schrodinger, $D = D_B = L/4$ (We'll probably need to tweak this number a bit.)
For Plum Pudding, $D = D_P = \begin{cases} D_B b^2/R^2 & \text{if } b \leq R, \\ D_B & \text{if } b \geq R. \end{cases}$
Note that in Bohr/deBroglie/Schrodinger, D is the same for all particles, but in Plum Pudding, D depends on the initial position of the particle in the box.

Conversion from Cartesian coordinates to Polar coordinates:

$r = \sqrt{x^2 + y^2}$ $\qquad\qquad$ $\phi = arctan(-x/y)$

Conversion from Polar coordinates to Cartesian coordinates:

$x = r\sin(\phi)$ $\qquad\qquad$ $y = -r\cos(\phi)$

Initial Conditions:

$x = b$ $\qquad$ $y = -L/2$ $\qquad$ $v = v_o$
Convert x and y to r and ϕ to input into loop:

Loop:

1. $\phi_{new} = \phi + \frac{bv_\Delta t}{r\sqrt{b + r^2(b\cos\phi - \frac{D}{2}\sin\phi)^2}}$
2. $r_{new} = \frac{b^2}{b\sin(\phi_{new}) + \frac{D}{2}(\cos(\phi_{new}) - 1)}$
3. $v_{new} = v_o\sqrt{1 - D/r_{new}}$
4. Convert r_{new} and ϕ_{new} to x_{new} and y_{new} and move alpha particle to new coordinates.
5. r_{new}, ϕ_{new}, $v_{new} \rightarrow r$, ϕ, v for next iteration of loop.

Note: In the plum pudding model, you will run into divide by zero problems when b=0, because this makes D=0. In this case, the correct behavior is that the alpha particle should just continue to move straight up at the same speed it started with.