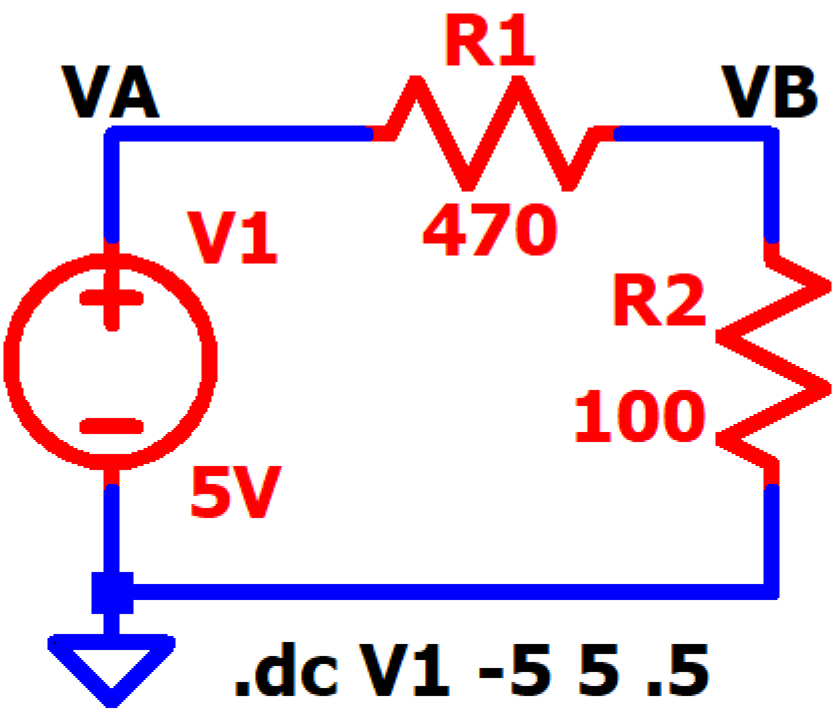
1. Slope of an I-V curve and Ohm's Law

Building Block



Ohm's Law: $V = IR$

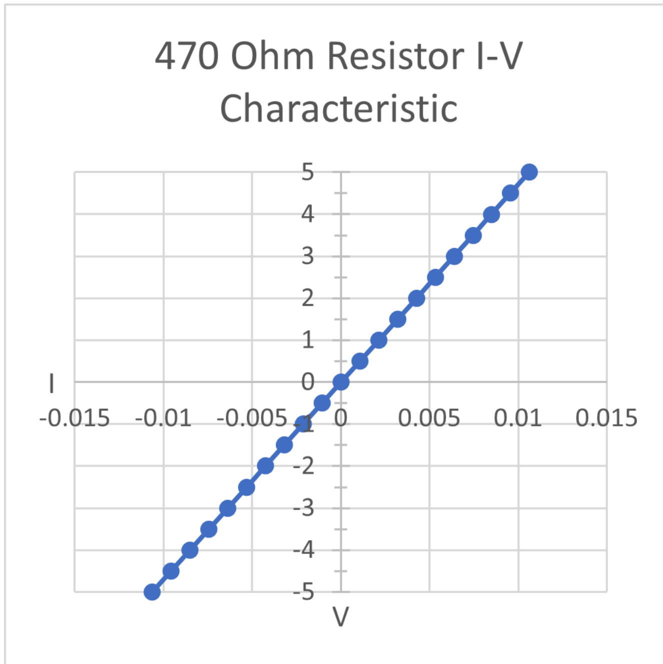
I-V Characteristic: A graph that shows the relationship between voltage and current for a circuit element. For a resistor, this relationship is linear.

Mathematical Analysis

Using Ohm's Law, we can determine the current flowing through a resistor given the voltage, or the voltage given the current. Knowing both of these allows us to create an I-V characteristic graph.

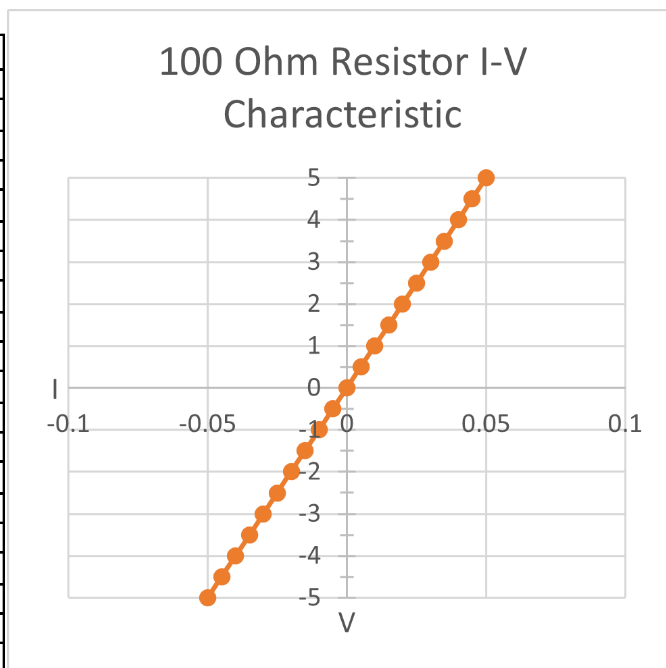
470Ω Resistor (R1)

Current (A)	Voltage (V)
-0.01064	-5
-0.00957	-4.5
-0.00851	-4
-0.00745	-3.5
-0.00638	-3
-0.00532	-2.5
-0.00426	-2
-0.00319	-1.5
-0.00213	-1
-0.00106	-0.5
0	0
0.001064	0.5
0.002128	1
0.003191	1.5
0.004255	2
0.005319	2.5
0.006383	3
0.007447	3.5
0.008511	4
0.009574	4.5
0.010638	5

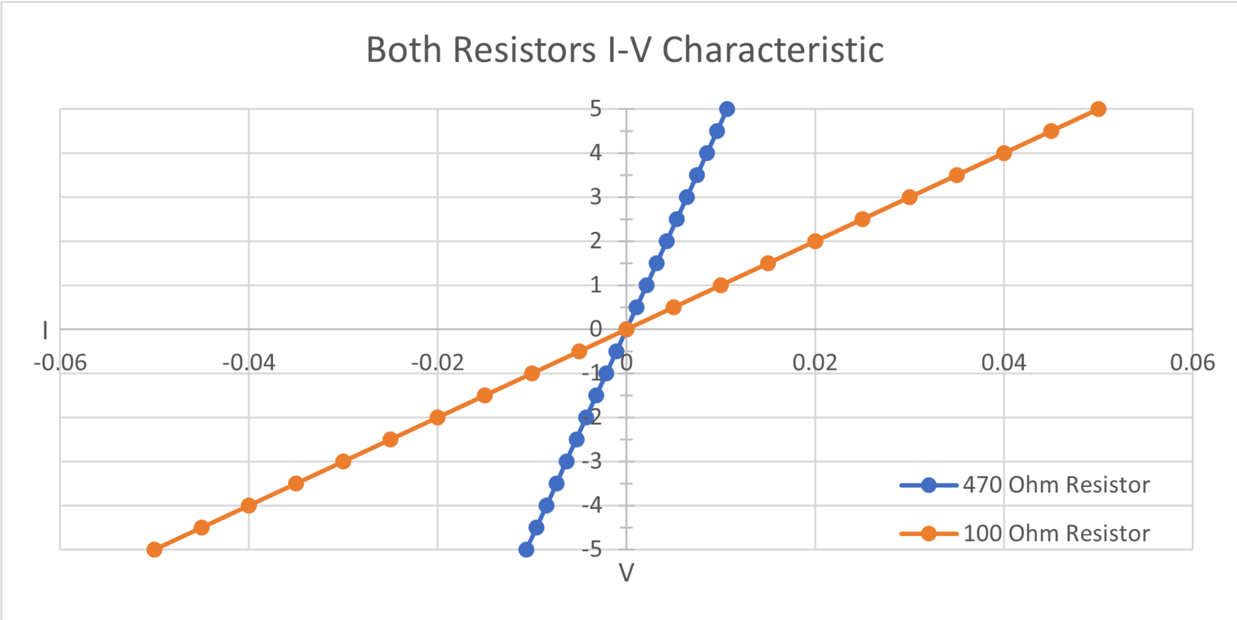


100Ω Resistor (R2)

Current (A)	Voltage (V)
-0.05	-5
-0.045	-4.5
-0.04	-4
-0.035	-3.5
-0.03	-3
-0.025	-2.5
-0.02	-2
-0.015	-1.5
-0.01	-1
-0.005	-0.5
0	0
0.005	0.5
0.01	1
0.015	1.5
0.02	2
0.025	2.5
0.03	3
0.035	3.5
0.04	4
0.045	4.5
0.05	5



Both Resistors



With I-V characteristics for multiple resistors plotted together, it is apparent that the slope of each I-V characteristic shows the resistance. This can also be used to demonstrate Ohm's Law:

Slope for 470Ω resistor = $\frac{4-3.5}{0.008511-0.007447} = 470 = R_1$

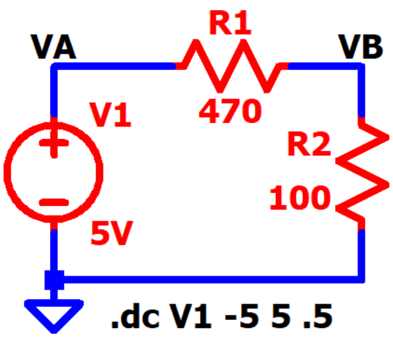
Slope for 100Ω resistor = $\frac{4-3.5}{0.04-0.035} = 100 = R_2$

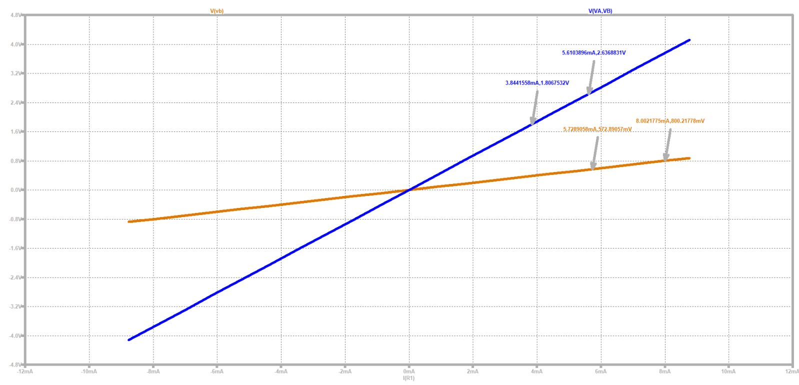
Generally:

$$\text{Slope} = \frac{\Delta V}{\Delta I} = R \rightarrow \Delta V = \Delta I(R) \rightarrow V = IR$$

The slope of the I-V characteristic is the resistance.

Simulation





The blue line shows the I-V characteristic of R1, and the orange line shows the I-V characteristic of R2 (same colors as mathematical analysis).

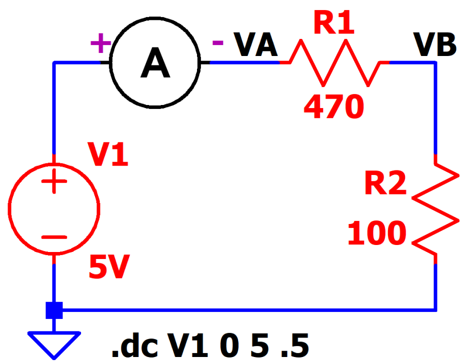
The same strategy of using the slope of the I-V characteristic to find resistance applies:

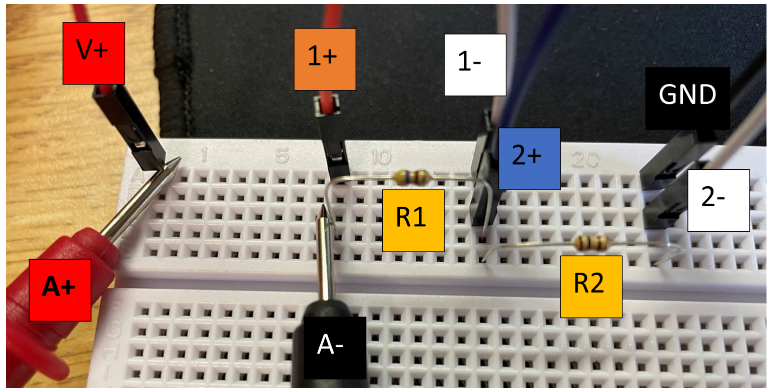
Slope for R1 = $\frac{2.6368831 - 1.8067532V}{5.6103896 - 3.8441558mA} = 470\Omega = R_1$

Slope for R2 = $\frac{800.21778 - 572.89057mV}{8.0021775 - 5.7289058mA} = 100\Omega = R_2$

Resistor	**Calculated Resistance (Analysis)**	**Calculated Resistance (Simulation)**
R1	470Ω	470Ω
R2	100Ω	100Ω

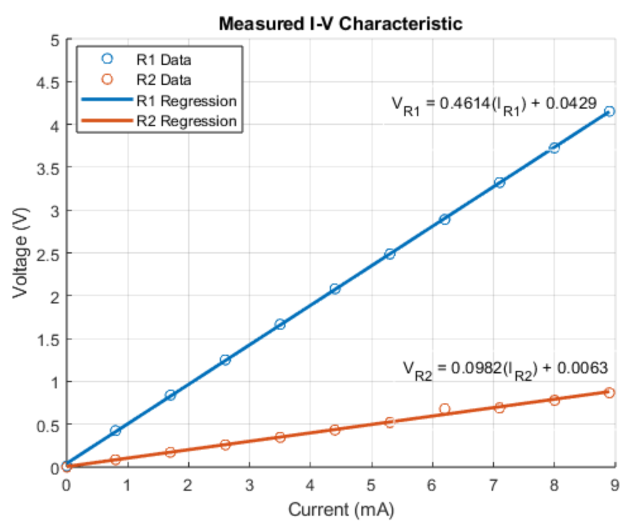
Measurement





To construct an I-V characteristic, current through and voltage across each resistor must be measured. To measure current, an ammeter was connected in series with R1 and R2 (represented by A above). Because this is a series circuit, current through the ammeter, R1 and R2 will be equal. To measure voltage, the two M2K channels were connected across resistors R1 and R2.

V+ Voltage (Set)	R1 Voltage (Measured)	R2 Voltage (Measured)	Current (mA) (Measured)
0	0.011	0	0
0.5	0.425	0.086	0.8
1	0.84	0.173	1.7
1.5	1.251	0.259	2.6
2	1.666	0.346	3.5
2.5	2.081	0.432	4.4
3	2.487	0.519	5.3
3.5	2.89	0.68	6.2
4	3.321	0.691	7.1
4.5	3.725	0.779	8.0
5	4.153	0.866	8.9



The data and regression lines for R1 (Blue) and R2 (Orange) are plotted in the graph on the right.

From the slope of the regression lines:

$R1 = 0.4614m\Omega \rightarrow 461.4\Omega$

$R2 = 0.0982m\Omega \rightarrow 98.2\Omega$

This is very close to the measured actual resistances of R1 and R2, which were 464Ω and 97Ω respectively.

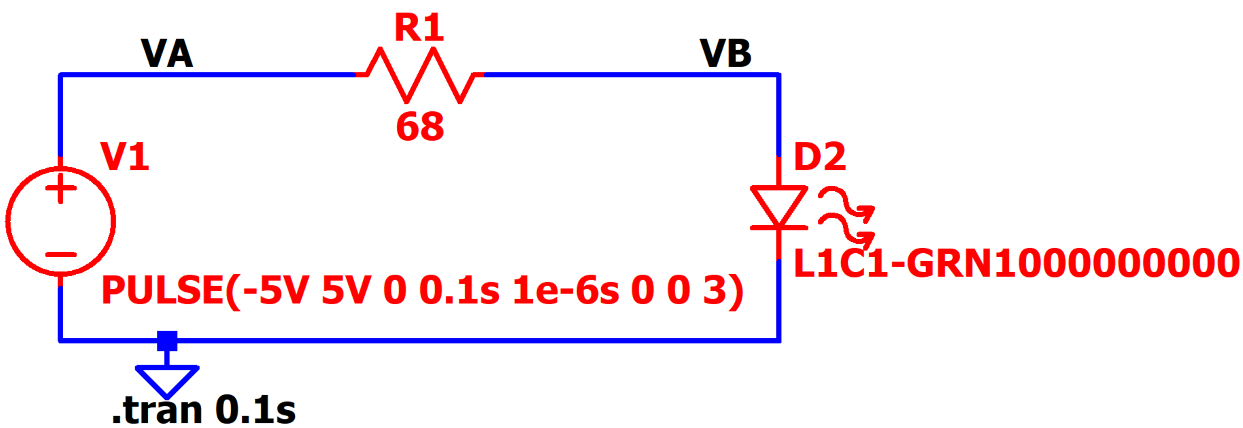
Resistor	Calculated Resistance (Analysis)	Calculated Resistance (Simulation)	Calculated Resistance (Measurement)	Actual Resistance (Measurement)
R1	470Ω	470Ω	461.4Ω	464Ω
R2	100Ω	100Ω	98.2Ω	97Ω

Discussion

The calculated resistances are exactly the same as the simulated values, and the measured values are only slightly lower. This variation may have been caused by the wires having some resistance that was not accounted for in the mathematical analysis or simulations, and the M2k's actual output voltage being slightly higher than the set 5 volts. Overall, these results prove that the slope of a resistor's I-V curve is the resistance, matching Ohm's Law.

2. Non-linear I-V curve for a diode

Building Block



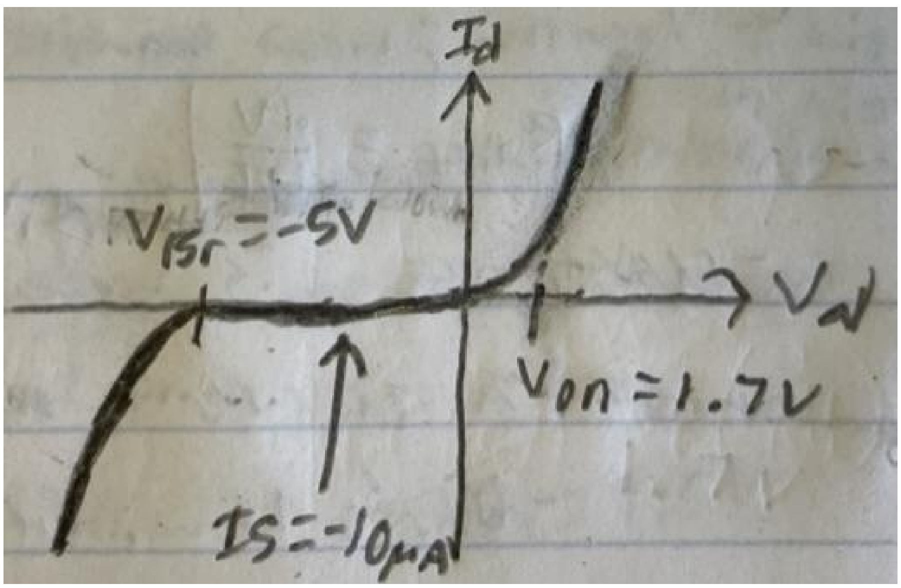
Ohm's Law: $V = IR$
I-V Characteristic: A graph that shows the relationship between voltage and current for a circuit element. For a diode, this relationship is not linear.

Mathematical Analysis

Diode Information (from data sheet)

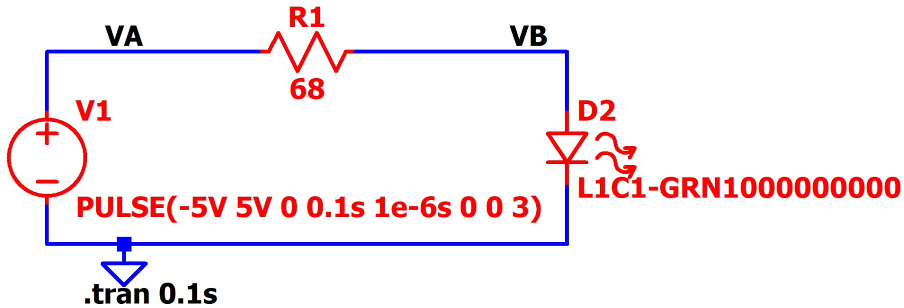
Type	Green LED
Turn on voltage	1.7 V
Breakdown voltage	-5 V
Saturation current	-10 µA

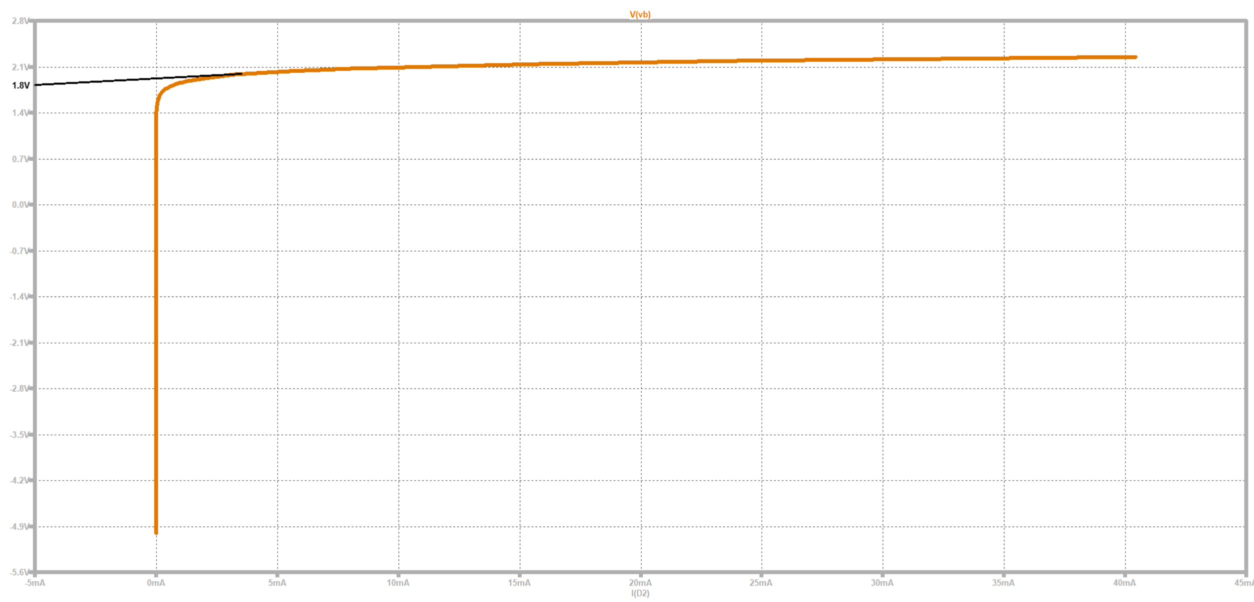
I-V Characteristic



An ideal diode only allows current to flow in one direction, but requires a certain voltage, the turn on voltage, to allow the current flow. Above this turn on voltage, significant current is allowed to flow. A diode has very high resistance before this point, and when the voltage is reached, the resistance lowers significantly, resulting in an exponential increase in current flow. These properties of diodes result in a non-linear I-V characteristic.

Simulation



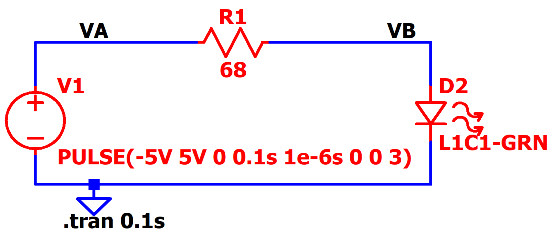


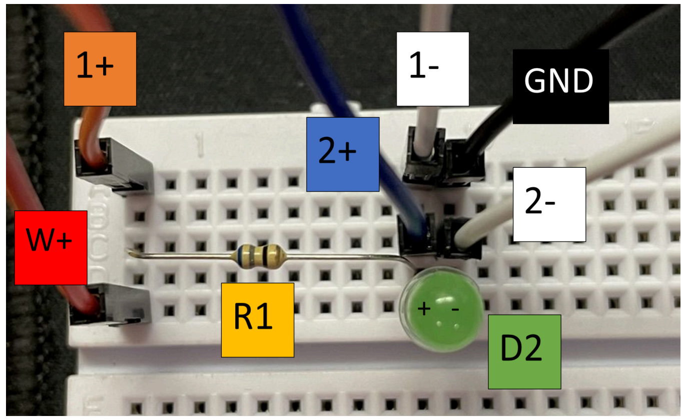
X-axis is current through the LED (D2), y-axis is voltage across the LED.

Turn on voltage is about 1.8 V
Reverse breakdown voltage is not modeled by this simulation
Saturation current is a very small negative value, consistent with mathematical analysis.

These values are not the same, but the turn on voltage is very similar, breakdown voltage is at the end of the simulated voltage range, and the saturation current being a small negative value is still true. These small differences could be based on the simulated LED being modeled on a different real LED. The exponential shape of the graph is still the same as the mathematical analysis graph in the simulated range.

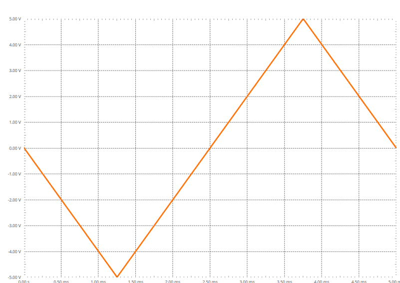
Measurement

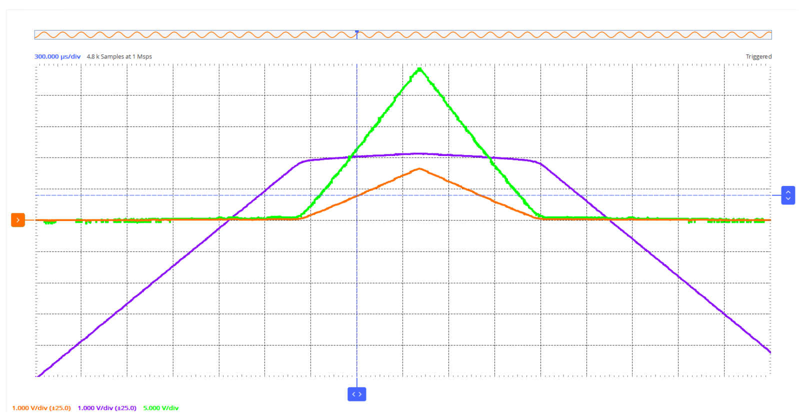




To construct an I-V characteristic for a diode, current through and voltage across that diode must be measured. To measure current, we measured the voltage across R1 and used Ohm's Law to calculate the current. Because this is a series circuit, current through R1 and D2 will be equal. To measure voltage across the diode, channel 2 was connected across D2.

Voltage and current plotted against time





To measure different voltages and currents, the voltage of V1 is being varied over time (function shown on left graph)

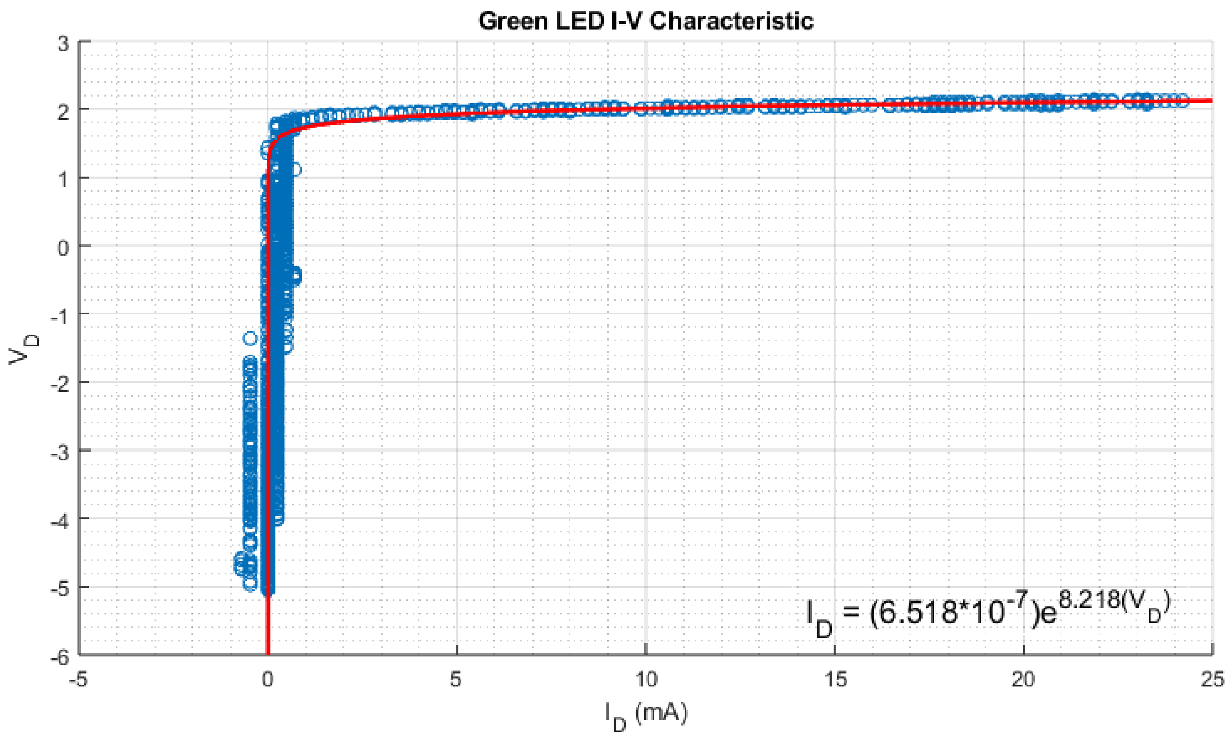
The oscilloscope output (right graph) shows the voltage across and current through each of the circuit components over time. The orange line is the voltage across R1, and the green line is the current through the series circuit, calculated using Ohm's Law and the known value of R1, scaled to be measured in mA:

$$V = IR \rightarrow I = \frac{V}{R} \rightarrow I_D = \frac{V_{R1}}{68\Omega}$$

The purple line shows the voltage across the diode.

Constructing an I-V characteristic

To create an I-V characteristic, these values of current and voltage can be plotted against each other rather than against time.



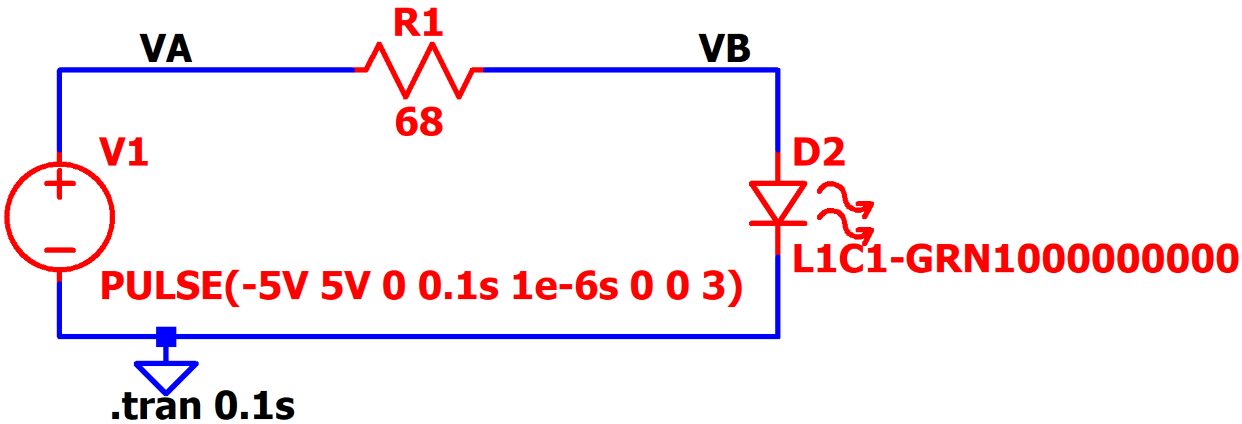
The diode's measured I-V characteristic follows an exponential shape, matching the shape of the mathematical analysis and simulation

Discussion

The graphs for simulation and measurement had the same exponential shape, and the graph for analysis had the same shape in the regions covered by simulation and measurement. The analysis graph has the axes opposite the simulation and measurement to better show the reverse breakdown voltage, causing the slope to be 1/R and not R. This still is able to demonstrate the varying slope in the different regions of the I-V characteristic. These graphs are not the same, but this is caused by the hand drawing not being based on data, but rather the ideal diode model, and the simulation and real LED are not based on the same model of LED. Even with these differences, the similar exponential shape in the modeled regions proves the common nonlinear nature of the I-V curve for diodes.

3. Differential resistance in a diode's I-V curve

Building Block



Ohm's Law: $V = IR$

I-V Characteristic: A graph that shows the relationship between voltage and current for a circuit element. For a diode, this relationship is not linear.

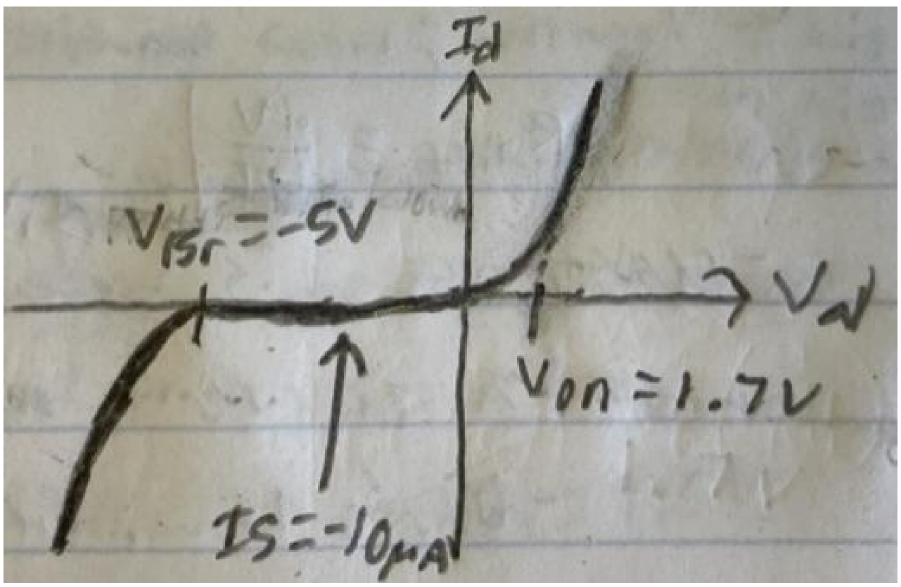
Differential Resistance: For non-linear circuit components, resistance is not constant, so the resistance must be found by taking the derivative at a particular point.

Mathematical Analysis

Diode Information (from data sheet)

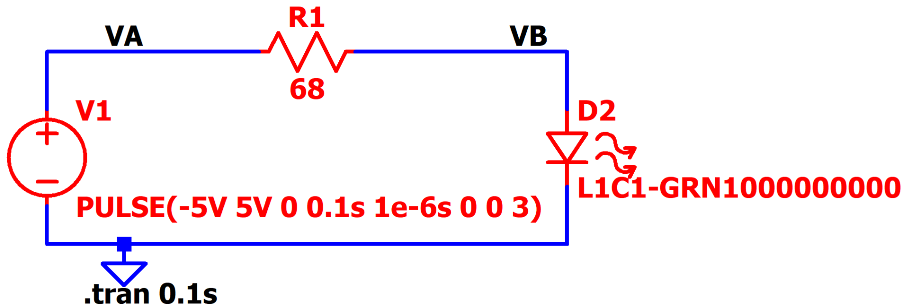
Type	Green LED
Turn on voltage	1.7 V
Reverse breakdown voltage	5 V
Saturation current	10 μA

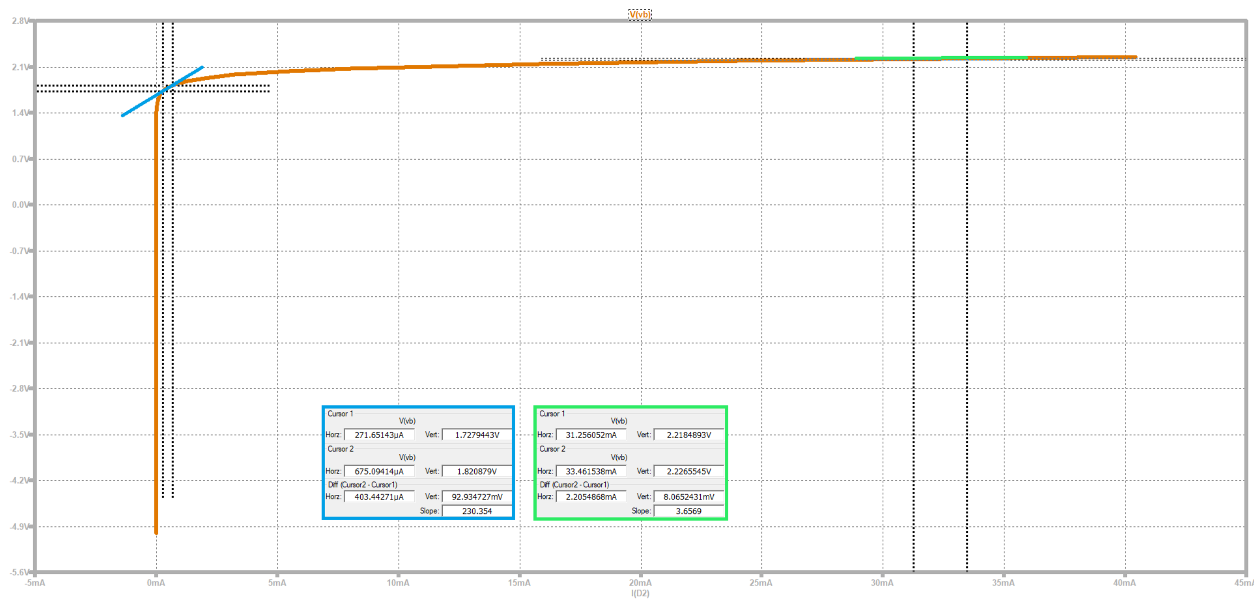
I-V Characteristic



Because the I-V characteristic for a diode is not linear, the standard Ohm's Law approach of calculating resistance will not apply for the entire graph. Instead, the resistance must be calculated using the slope at a specific point, known as the *differential resistance*. The slope of this graph is 1/R, so horizontal sections (saturation current) represent very high resistance and vertical sections (above turn on voltage, below breakdown voltage) represent very low resistance.

Simulation





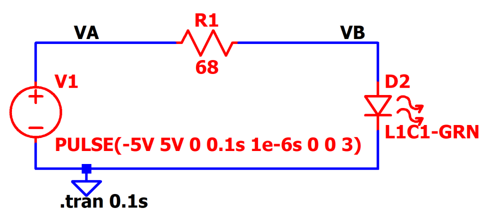
X-axis is current through the LED (D2), y-axis is voltage across the LED (D2).

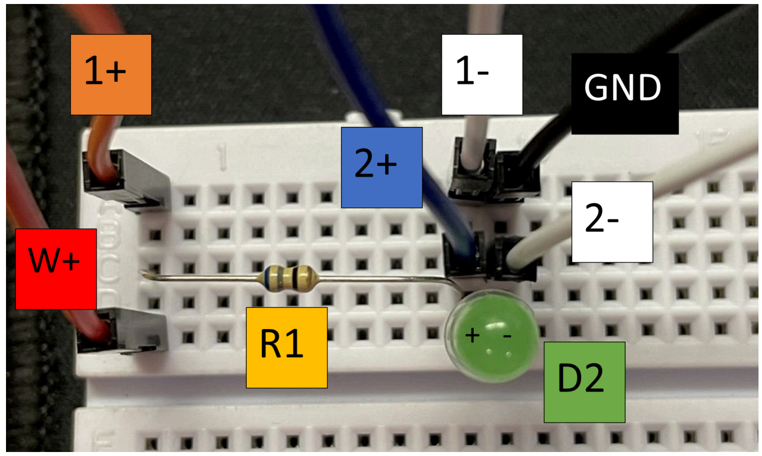
The differential resistance (slope at a particular point) can be calculated by taking the derivative (slope) at that point. The slope of this graph is the resistance, and the cursor values in the simulation do this calculation automatically.
The resistance at the blue line, right at the turn on voltage, is 230.354Ω, while the resistance above the turn on voltage (green line) is substantially lower, at only 3.6569Ω.

Value	Math. Analysis	Simulation
R(below Von)	Large	230.354Ω
R(above Von)	Small	3.6569Ω

The resistance below the turn on voltage is very high, and the resistance above the turn on voltage is very low, matching the observations from mathematical analysis.

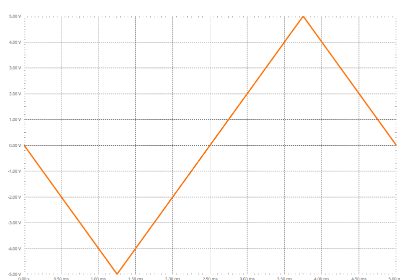
Measurement

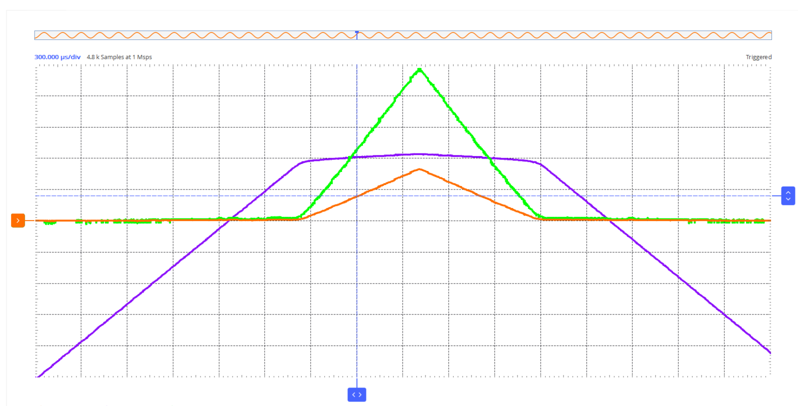




To construct an I-V characteristic for a diode, current through and voltage across that diode must be measured. To measure current, we measured the voltage across R1 and used Ohm's Law to calculate the current. Because this is a series circuit, current through R1 and D2 will be equal. To measure voltage across the diode, channel 2 was connected across D2.

Voltage and current plotted against time





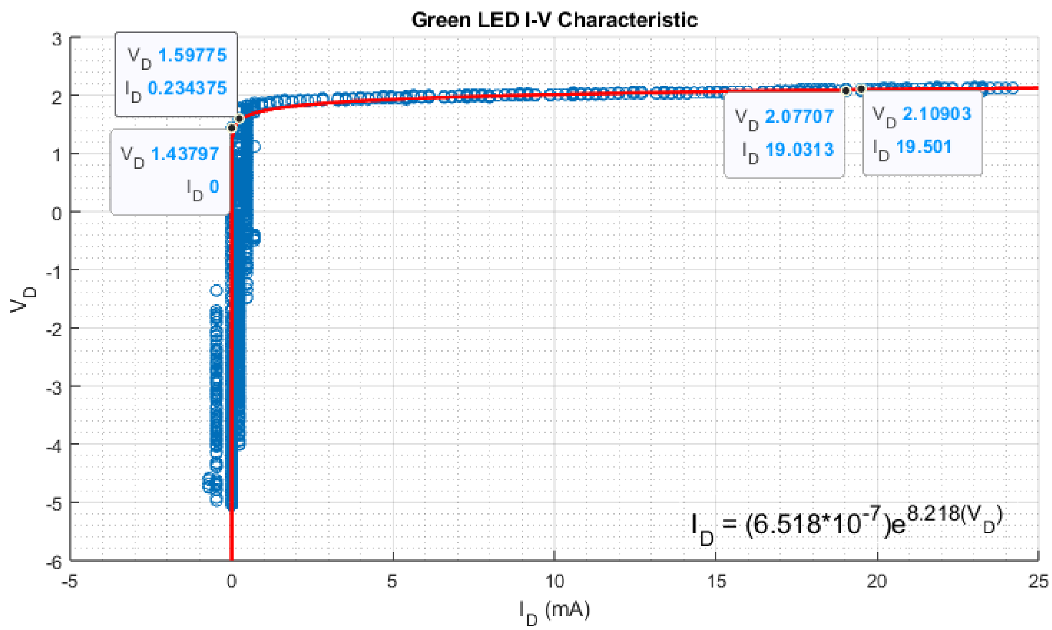
To measure different voltages and currents, the voltage of V1 is being varied over time (function shown on left graph)
The oscilloscope output (right graph) shows the voltage across and current through each of the circuit components over time. The orange line is the voltage across R1, and the green line is the current through the series circuit, calculated using Ohm's Law and the known value of R1, scaled to be measured in mA:

$$V = IR \rightarrow I = \frac{V}{R} \rightarrow I_D = \frac{V_{R1}}{68\Omega}$$

The purple line shows the voltage across the diode.

Constructing an I-V characteristic

To create an I-V characteristic, these values of current and voltage can be plotted against each other rather than against time.



This can then be used to calculate differential resistance, as the slope of the I-V characteristic is proportional to resistance.

Left points (below turn on voltage): $R = \frac{1.59775 - 1.82143V}{0.234375 - 0mA} = 681.7\Omega$

Right points (above turn on voltage): $R = \frac{2.10903 - 2.07707V}{19.501 - 19.0313mA} = 68.04\Omega$

While these values are not the same as the simulation, they are taken from the same areas of the graph, and the difference in resistances at these points is similar.

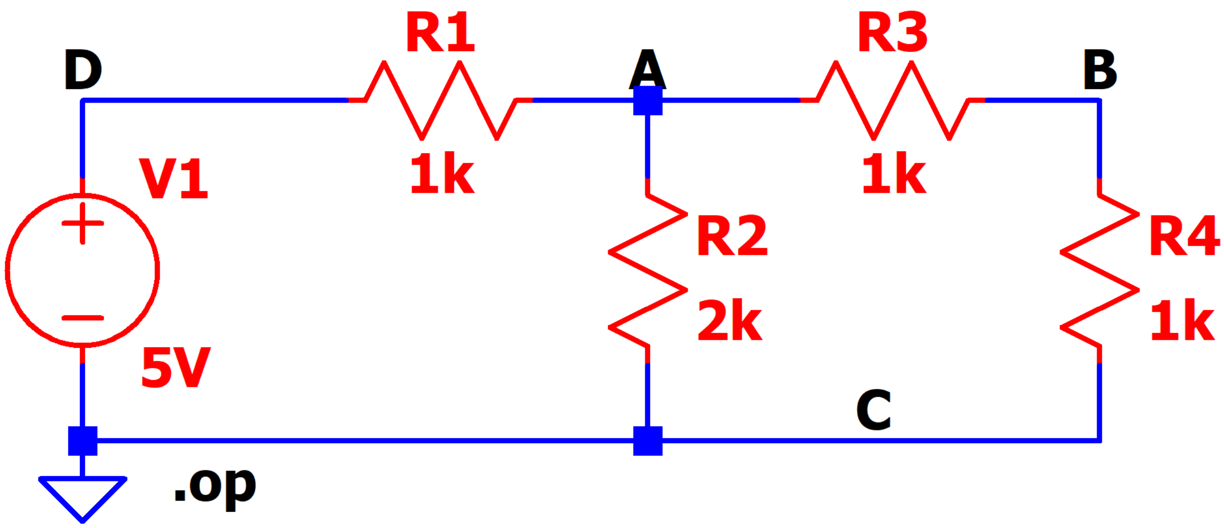
Value	Math. Analysis	Simulation	Measurement
R(below Von)	Large	230.354Ω	681.7Ω
R(above Von)	Small	3.6569Ω	68.04Ω

Discussion

The resistance values for different regions of the diode’s I-V characteristic are based on the slope at that point, and the slopes are in similar ranges for the different areas of the graph. The analysis graph has the axes opposite the simulation and measurement to better show the reverse breakdown voltage, causing the slope to be 1/R and not R, but this still shows the difference in resistance in different regions. Below the turn on voltage, resistance was very high, and above the turn on voltage, the resistance dropped dramatically. While the simulation and analysis resistances are based on a continuous curve, the measurement data was not. A major source of error in the measurement is that the M2k is only able to measure the voltage (and the calculated current) in discrete steps, rather than as a continuous scale. While this did not affect the overall resistance trend, it did cause significant error in the calculated resistance. At the points chosen above Von, the slope between them is much larger than the overall trend in that area, causing the calculated resistance to be much higher. While this causes a problem for specific measurements, the sample size is large enough that the overall trend of changing resistance in different regions holds, proving the concept of varying differential resistance in a diode’s I-V characteristic.

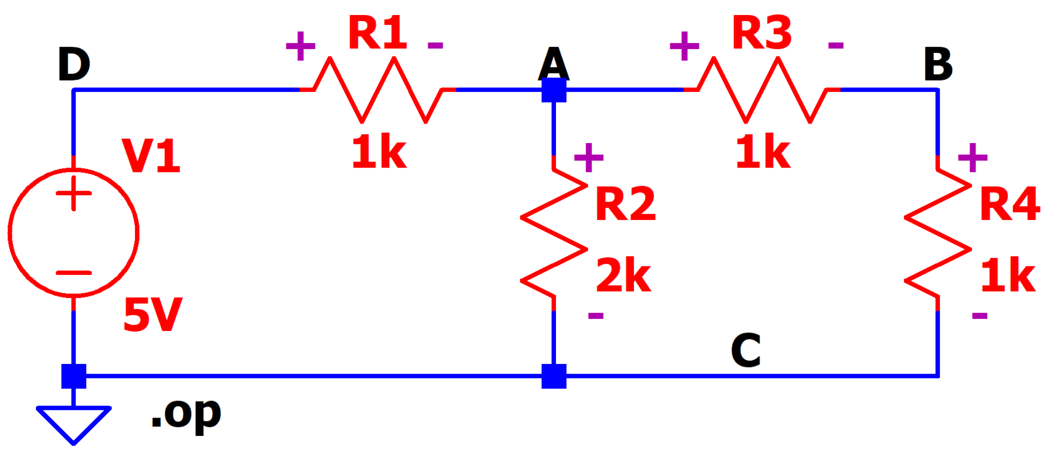
4. Nodal Analysis of a circuit

Building Block



Ohm's Law: $V = IR$
Kirchoff's Current Law: The sum of currents entering and leaving a node is zero
Nodal Analysis: A circuit analysis strategy that solves for nodal voltages

Mathematical Analysis



Knowns:	Unknowns:
V(C) = 0V (ground), V(D) = 5V (voltage source)	V(A), V(B)

KCL for unknown nodes

Sign convention: current flow into a node is positive, current flow out of a node is negative (based on reference marks)

Node A: $I_{R1} - I_{R2} - I_{R3} = 0$

Node B: $I_{R3} - I_{R4} = 0$

KCL in terms of nodal voltages

Current can be converted into voltage using Ohm's Law: $V = IR \rightarrow I = \frac{V}{R}$

Voltage across a circuit element can be written in terms of nodal voltages: $V_{R1} = V_D - V_A$

These can be combined to write KCL equations in terms of voltages and known resistances

Node A: $\frac{V_{R1}}{R1} - \frac{V_{R2}}{R2} - \frac{V_{R3}}{R3} = 0 \rightarrow \frac{V_D - V_A}{R1} - \frac{V_A - V_C}{R2} - \frac{V_A - V_B}{R3} = 0$

Node B: $\frac{V_{R3}}{R3} - \frac{V_{R4}}{R4} = 0 \rightarrow \frac{V_A - V_B}{R3} - \frac{V_B - V_C}{R4} = 0$

KCL equations into standard form

The unknown nodal voltages can be factored out, to put the equations into matrix form:

Node A: $V_A(-\frac{1}{R1} - \frac{1}{R2} - \frac{1}{R3}) + V_B(\frac{1}{R3}) + V_C(-\frac{1}{R2}) + V_D(\frac{1}{R1}) = 0$

Node B: $V_A(\frac{1}{R3}) + V_B(-\frac{1}{R3} - \frac{1}{R4}) + V_C(\frac{1}{R4}) = 0$

These equations can then be simplified using known nodal voltages ($V_C = 0V$ and $V_D = 5V$), and constants can be moved to the right side:

Node A: $V_A(-\frac{1}{R1} - \frac{1}{R2} - \frac{1}{R3}) + V_B(\frac{1}{R3}) = -\frac{V_D}{R1}$

Node B: $V_A(\frac{1}{R3}) + V_B(-\frac{1}{R3} - \frac{1}{R4}) = 0$

Matrix form

These equations can be written as a matrix equation of the form $Ax = B$

$$\begin{bmatrix} \left(-\frac{1}{R1}-\frac{1}{R2}-\frac{1}{R3}\right) & \frac{1}{R3} \\ \frac{1}{R3} & \left(-\frac{1}{R3}-\frac{1}{R4}\right) \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} -\frac{V_D}{R1} \\ 0 \end{bmatrix}$$

Resistance values can then be substituted in to solve the matrix:

$$\begin{bmatrix} \left(-\frac{1}{1000}-\frac{1}{2000}-\frac{1}{1000}\right) & \frac{1}{1000} \\ \frac{1}{1000} & \left(-\frac{1}{1000}-\frac{1}{1000}\right) \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} -\frac{5}{1000} \\ 0 \end{bmatrix}$$

$$x = A^{-1}B = \begin{bmatrix} 2.5 \\ 1.25 \end{bmatrix}$$

V(A) = 2.5V, V(B) = 1.25V

Matlab verification of matrix math

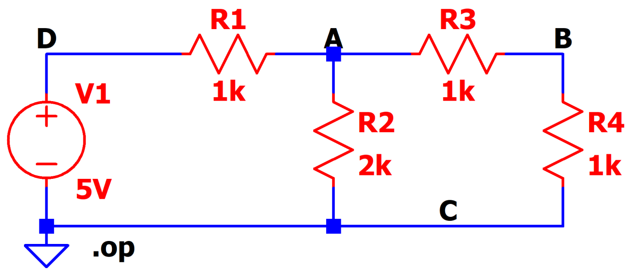
```
>> R1=1000; R2=2000; R3=1000; R4=1000;
>> VC=0; VD=5;
>> A = [(-1/R1)-(1/R2)-(1/R3), 1/R3; (1/R3), (-1/R3)-(1/R4)]
A =
   -0.0025    0.0010
    0.0010   -0.0020
>> B = [-VD/R1; 0]
B =
   -0.0050
         0
>> x = inv(A)*B
x =
    2.5000
    1.2500
```

V(A) = 2.5V, V(B) = 1.25V

Value	**Math. Analysis (TI-84)**	**Math. Analysis (Matlab)**
V(A)	2.5V	2.5V
V(B)	1.25V	1.25V

The voltages are the same whether the TI-84 or Matlab is used to solve the matrix.

Simulation

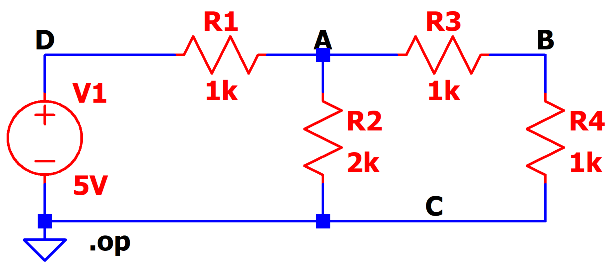


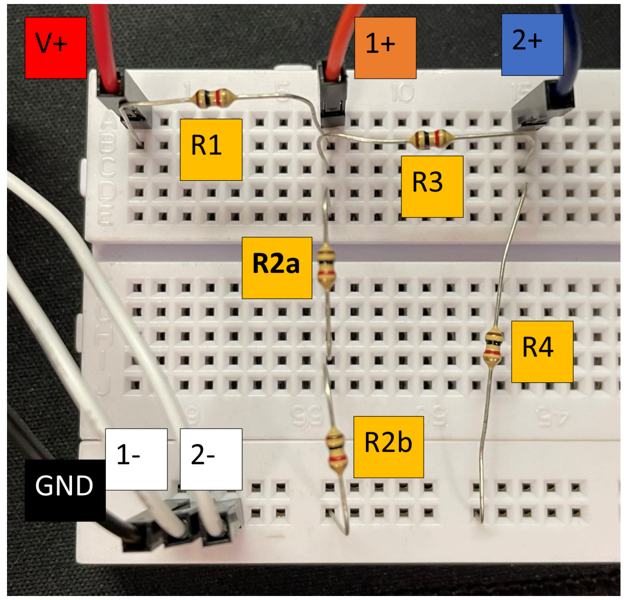

```
--- Operating Point ---

V(d):        5            voltage
V(a):        2.5          voltage
V(b):        1.25         voltage
I(R1):       -0.0025      device_current
I(R2):       0.00125      device_current
I(R3):       -0.00125     device_current
I(R4):       0.00125      device_current
I(V1):       -0.0025      device_current
```

Value	**Math. Analysis (TI-84)**	**Math. Analysis (Matlab)**	**Simulation (LTspice)**
V(A)	2.5V	2.5V	2.5V
V(B)	1.25V	1.25V	1.25V

The simulated nodal voltages match the values calculated in mathematical analysis.

Measurement





Because we did not have a 2kΩ resistor, R2 has been replaced with two 1kΩ resistors in series (labeled R2a and R2b), to create the equivalent of 2kΩ of resistance.

Voltage source V1	V(A)	V(B)
5.000 VDC Set 5.061 VDC Measure	2.506 VDC	1.263 VDC

Value	Math. Analysis (TI-84)	Math. Analysis (Matlab)	Simulation (LTspice)	Measurement
V(A)	2.5V	2.5V	2.5V	2.506V
V(B)	1.25V	1.25V	1.25V	1.263V

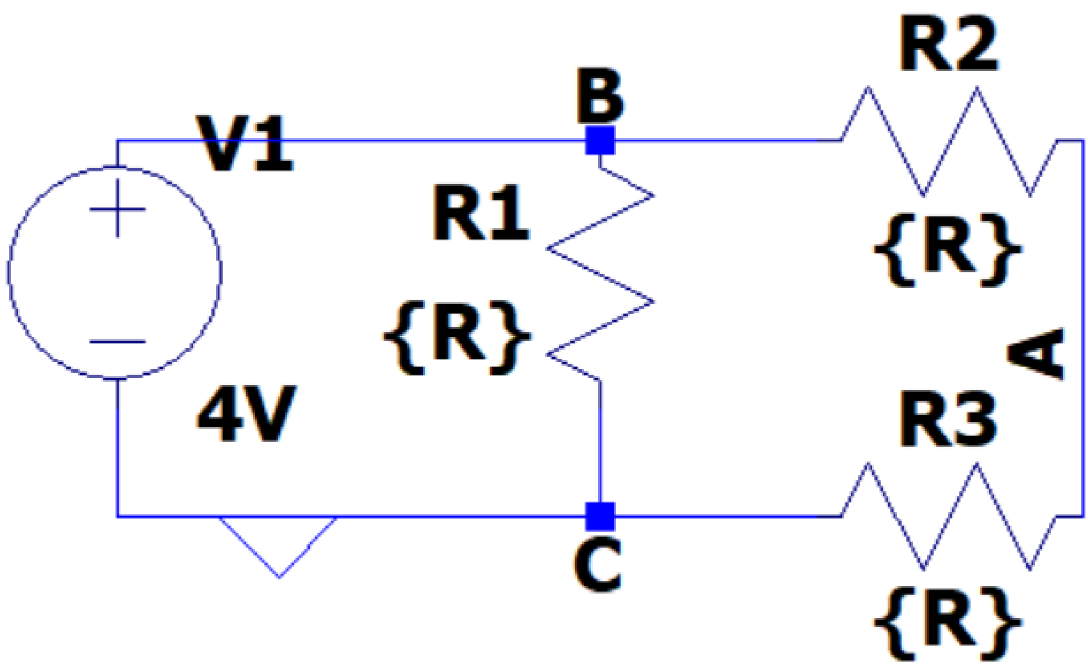
The measurement values are within 1.1% of the mathematical analysis and simulation values, showing that nodal analysis accurately reflects real circuit behavior.

Discussion

The calculated nodal voltages are exactly the same as the simulated values, regardless of whether Matlab or the TI-84 was used to calculate them, and the measured values are only slightly higher. This increase may have been caused by the M2k's actual output voltage being slightly higher than the set 5 volts, causing the nodal voltages to be slightly higher as well. Overall, these results prove that nodal analysis is an effective method to calculate nodal voltages in a circuit.

5. Approaching circuit design using Nodal Analysis

Building Block



Ohm's Law: $V = IR$
Kirchoff's Current Law: The sum of currents entering and leaving a node is zero
Nodal Analysis: A circuit analysis strategy that solves for nodal voltages
Design challenge 1: Design a circuit with a node other than the source node at 2V, with one voltage source and three resistors. The resistors cannot be in series, and two resistors can be in parallel.

Mathematical Analysis

Using Nodal Analysis to find V_A

Since Node B is connected to source voltage and Node C is connected to ground, their KCL equations can be factored out.
#Unknowns = #Nodes - #Source Voltage - 1
#Unknowns = $3 - 1 - 1$ = 1 unknown
Using KCL at node A, we get the equation

$\frac{V_A - V_B}{R_1} + \frac{V_A - V_C}{R_3} = 0A$ but since $R_1 = R_2 = R_3$ and $V_B = V_1$

V_C is connected to ground so we can set $V_C = 0V$

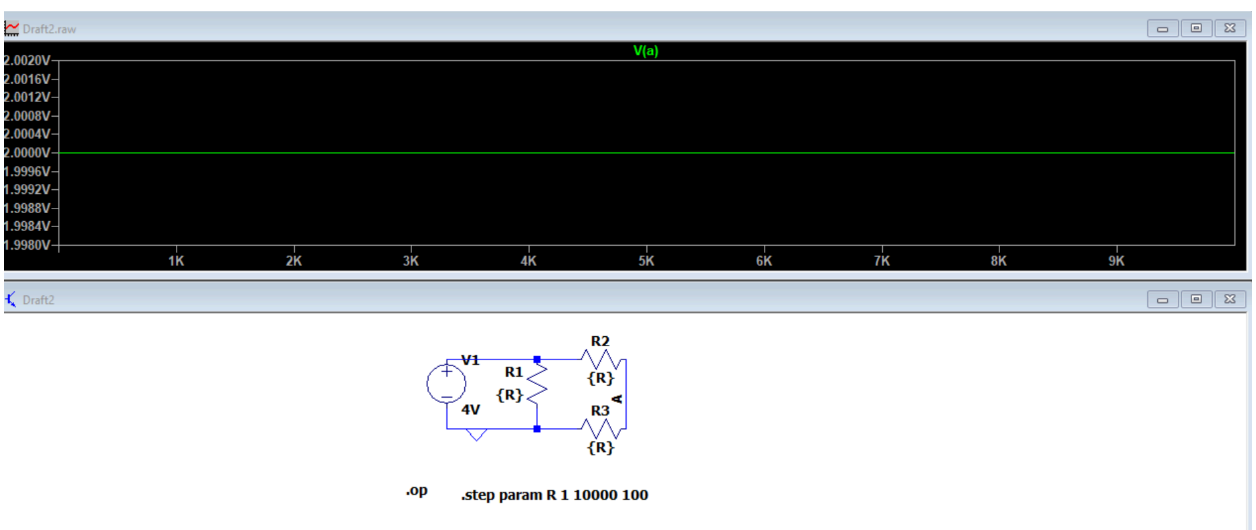
$\frac{V_A - V_1}{R} + \frac{V_A}{R} = 0V$ gives us the equation $\frac{2V_A}{R} = \frac{V_1}{R}$

Simplified: $V_1 = 2V_A$

Since we want $V_A = 2V$, we can set $V_A = 2V$

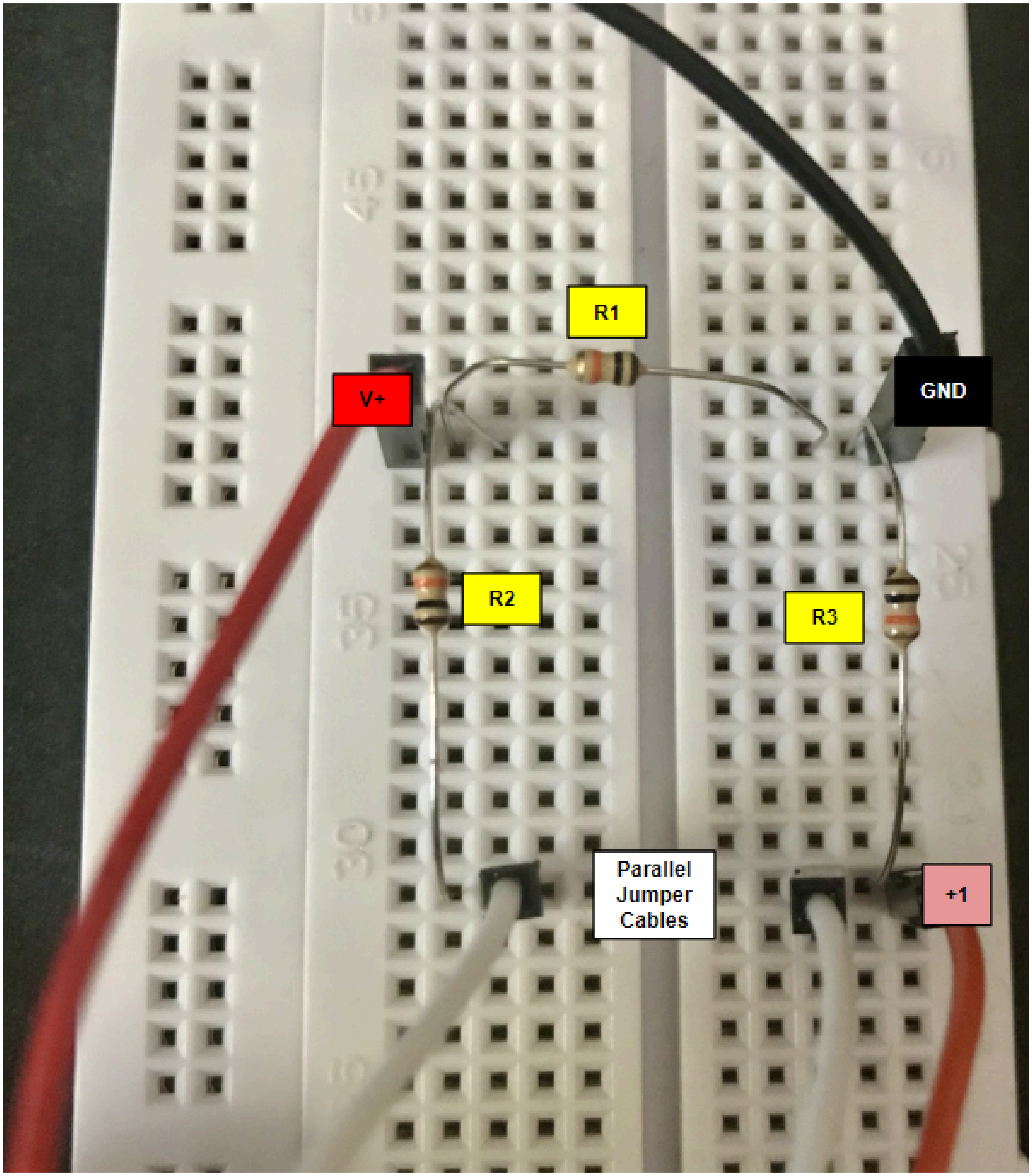
$V_1 = 4V$

Simulation



Here, we can see that for all resistor values, $V_A = 2V$ when $V_1 = 4V$

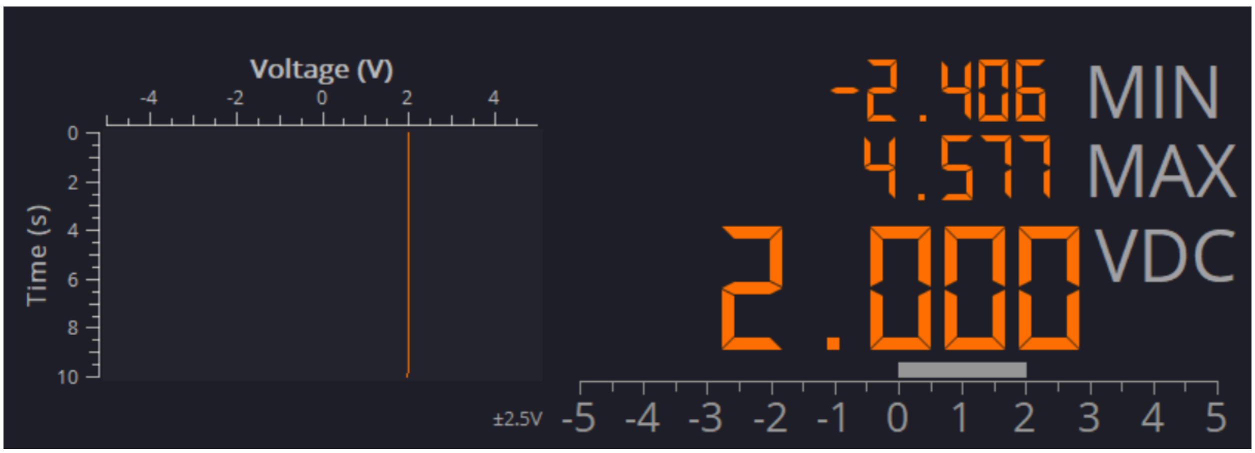
Measurement



- All resistors are 1k ohms
- -1 Connected to ground

Vin:

Voltage at Node A (to ground):



Discussion

The mathematical analysis and simulation results are exactly the same, and the measured results are very similar. The simulated nodal voltage is exactly 2.000V, but the voltage source is outputting less than 4.000V. This difference may have been caused by the wires having some resistance in reality, which was not accounted for in the mathematical analysis or simulation. Overall, these results show that this circuit accomplishes the goal of design challenge 1, creating a non-source node with a voltage of 2V, demonstrating the role of nodal analysis in circuit design.