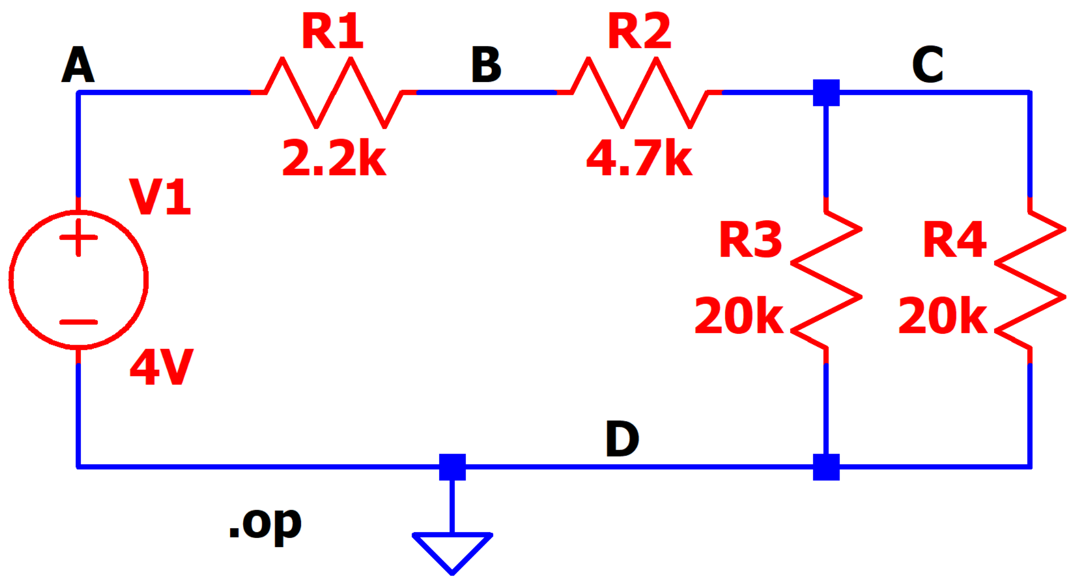
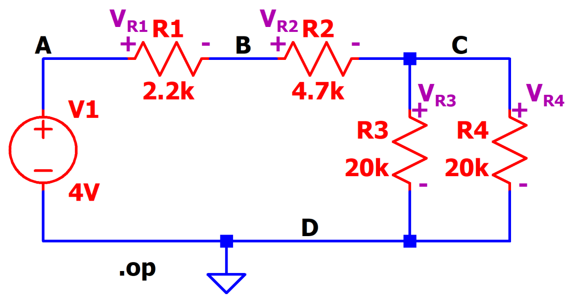
1. Ohm’s Law, KCL, and KVL in a circuit

Building Block



Ohm’s Law: $V = IR$
Kirchhoff’s Current Law: The sum of currents entering and leaving a node is zero
Kirchhoff’s Voltage Law: The sum of all voltages around a loop in a circuit is zero

Mathematical Analysis

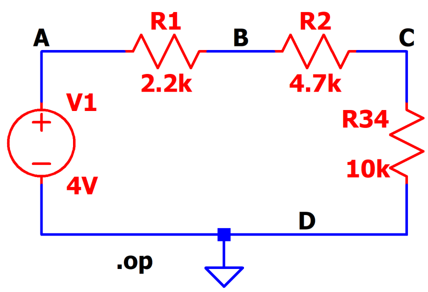


Using circuit reduction, we can find voltages across all resistors in this circuit.

Combined resistance of R3 and R4 (parallel resistors)

R3 and R4 are in parallel, so we can simplify them into a single resistor value.

$R_{34} = (\frac{1}{R_3} + \frac{1}{R_4})^{-1} = (\frac{1}{20000} + \frac{1}{20000})^{-1} = 10k\Omega$



Voltages across resistors (Voltage divider equation)

The circuit is now simplified into a series circuit (R1, R2, R34), and the voltage divider equation can be used.

$V_{R1} = V1\frac{R1}{R1+R2+(R34)} = 4\frac{2200}{2200+4700+10000} = 0.521V$

$V_{R2} = V1\frac{R2}{R1+R2+(R34)} = 4\frac{4700}{2200+4700+10000} = 1.112V$

$V_{R34} = V1\frac{R34}{R1+R2+(R34)} = 4\frac{10000}{2200+4700+10000} = 2.367V$

$V_{R3} = V_{R4} = 2.367V$

Current through each resistor (Ohm's Law)

Using Ohm's Law, we can solve for the current through each resistor.

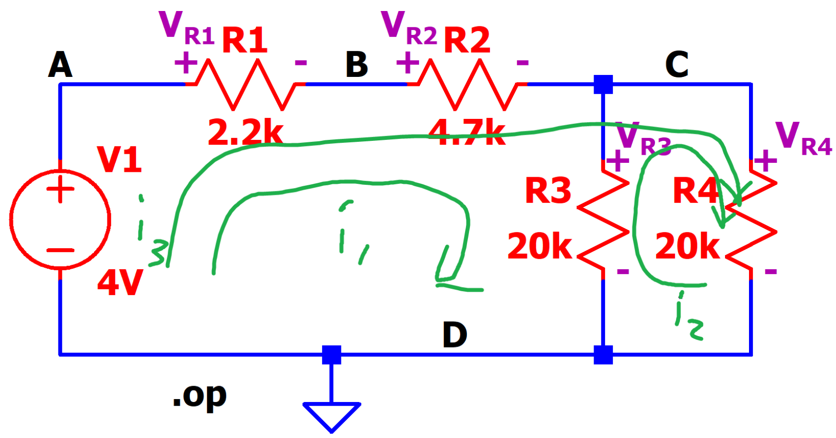
$V = IR \rightarrow I = \frac{V}{R}$

$I_{R1} = \frac{V_{R1}}{R_1} = \frac{0.521}{2200} = 237\mu A$

$I_{R2} = \frac{V_{R2}}{R_2} = \frac{1.112}{4700} = 237\mu A$

$I_{R3} = \frac{V_{R3}}{R_3} = \frac{2.367}{20000} = 118\mu A$

$I_{R4} = \frac{V_{R4}}{R_4} = \frac{2.367}{20000} = 118\mu A$



Proving KCL at each node

Sign convention: positive current flows into the node, negative current flows out of the node

Node B: $I_{R1} - I_{R2} = 0 \rightarrow 237\mu A - 237\mu A = 0\checkmark$

Node C: $I_{R2} - I_{R3} - I_{R4} = 0 \rightarrow 237\mu A - 118\mu A - 118\mu A = 0\checkmark$

Proving KVL in each loop

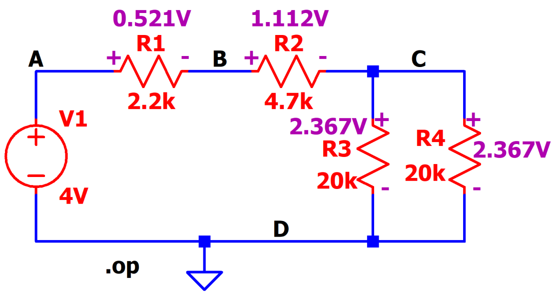
Sign convention: first sign encountered when going around the loop (clockwise)

Loop 1: $-V_{V1} + V_{R1} + V_{R2} + V_{R3} = 0 \rightarrow -4V + 0.521V + 1.112V + 2.367V = 0\checkmark$

Loop 2: $-V_{R3} + V_{R4} = 0 \rightarrow -2.367 + 2.367 = 0\checkmark$

Loop 3: $-V_{V1} + V_{R1} + V_{R2} + V_{R4} = 0 \rightarrow -4V + 0.521V + 1.112V + 2.367V = 0\checkmark$

Simulation




```
--- Operating Point ---

V(a):        4              voltage
V(b):        3.47929        voltage
V(c):        2.36686        voltage
I(R1):       -0.000236686   device_current
I(R2):       -0.000236686   device_current
I(R3):       -0.000118343   device_current
I(R4):       -0.000118343   device_current
I(V1):       -0.000236686   device_current
```

Calculating Resistor Voltages

$V_{R1} = V_A - V_B = 4 - 3.47929 = 0.521V$
$V_{R2} = V_B - V_C = 3.47929 - 2.36686 = 1.112V$
$V_{R3} = V_{R4} = V_C - V_D = 2.36686 - 0 = 2.367V$
These values match the mathematical analysis results.

Voltage	Math. Analysis	Simulation
V_{R1}	0.521V	0.521V
V_{R2}	1.112V	1.112V
V_{R3}	2.367V	2.367V
V_{R4}	2.367V	2.367V

Proving Ohm's Law for each resistor

$V = IR$
R1: $V_{R1} = I_{R1}(R1) \rightarrow 0.521 = 0.000237(2200)\checkmark$
R2: $V_{R2} = I_{R2}(R2) \rightarrow 1.112 = 0.000237(4700)\checkmark$
R3: $V_{R3} = I_{R3}(R3) \rightarrow 2.367 = 0.000118(20000)\checkmark$
R4: $V_{R4} = I_{R4}(R4) \rightarrow 2.367 = 0.000118(20000)\checkmark$
The voltages, currents, and resistances follow the relationship described by Ohm's Law.

Proving KCL at each node

Sign convention: positive current flows into the node, negative current flows out of the node
Node B: $I_{R1} - I_{R2} = 0 \rightarrow 237\mu A - 237\mu A = 0\checkmark$
Node C: $I_{R2} - I_{R3} - I_{R4} = 0 \rightarrow 237\mu A - 118\mu A - 118\mu A = 0\checkmark$
The current through resistors in the simulation adds correctly to follow KCL.

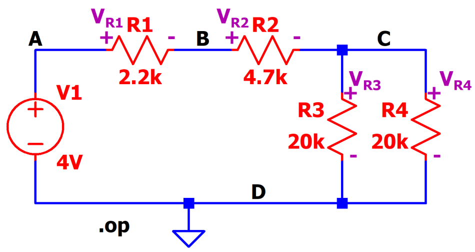
Proving KVL in each loop

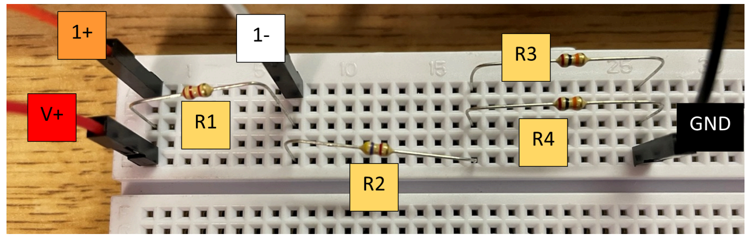
Sign convention: first sign encountered when going around the loop (clockwise)
Loop 1: $-V_{V1} + V_{R1} + V_{R2} + V_{R3} = 0 \rightarrow -4V + 0.521V + 1.112V + 2.367V = 0\checkmark$
Loop 2: $-V_{R3} + V_{R4} = 0 \rightarrow -2.367 + 2.367 = 0\checkmark$
Loop 3: $-V_{V1} + V_{R1} + V_{R2} + V_{R4} = 0 \rightarrow -4V + 0.521V + 1.112V + 2.367V = 0\checkmark$
The voltages in each loop in the circuit add correctly to follow KVL.

Measurement

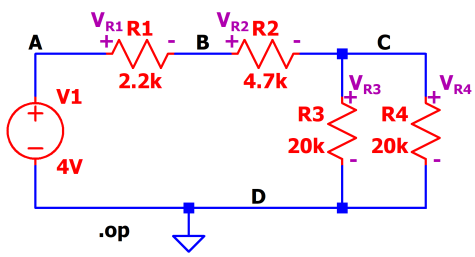
Measuring voltage across each resistor

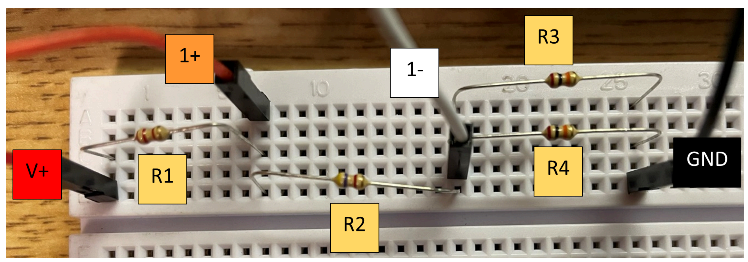
R1

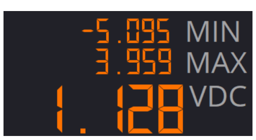




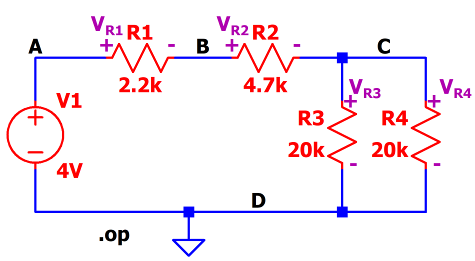
R2

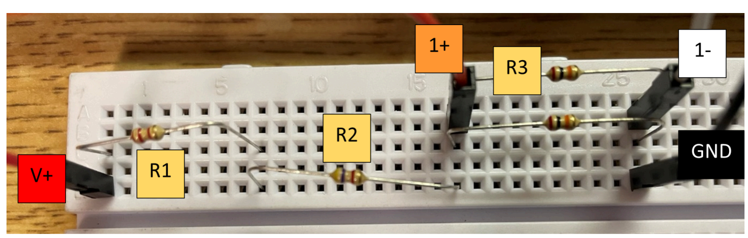






R3 and R4





Supply Voltage

Comparison to mathematical analysis and simulation

Voltage	Math. Analysis	Simulation	Measurement
V_{in}	4.000V	4.000V	3.959V
V_{R1}	0.521V	0.521V	0.551V
V_{R2}	1.112V	1.112V	1.128V
V_{R3}	2.367V	2.367V	2.384V
V_{R4}	2.367V	2.367V	2.384V

Calculating current through each resistor (Ohm's Law)

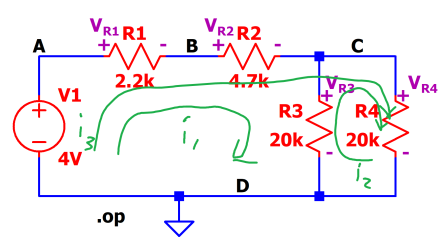
I_R1 = 0.551 / 2.2 = 0.250 mA
I_R2 = 1.128 / 4.7 = 0.240 mA
I_R3 = 2.384 / 20 = 0.1192 mA
I_R4 = 2.384 / 20 = 0.1192 mA

KCL at each node

Node B: I1 = I2 (0.10mA error)
Node C: I2 = I3 + I4 (0.116 mA error)

KVL in each loop

Req(12) = 1.657 V
Req(34) = 2.366 V
Loop 1 and 3: Vin = Req(12) + Req(34) = 4.023V
(4.023 - 3.959) / 4.023 * 100% = 1.59% error
Loop 2: V3 = V4   2.384 V = 2.384 V   0% error
Vin = V_R1 + V_R2 + V_R3
Vin = 0.551 V + 1.128 V + 2.384 V = 4.063 V
(4.063 - 3.959) / 4.063 * 100% = 2.56% error



Discussion

The calculated and simulated values exactly match each other, and the measured values are very close to the calculated and simulated values. This variation may have been caused by resistance of the wires that was not accounted for in calculations, or resistor values and source voltages that do not exactly match the nominal values. Overall, these results prove Ohm’s Law, KCL, and KVL in a circuit.