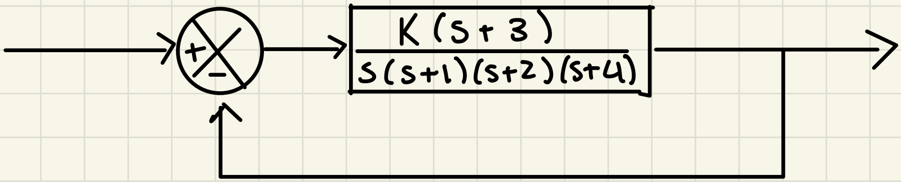
Let's do examples!

Sketch the root locus



1) Poles and zeros of OLTF

$z_1 = -3$

$p_1 = 0 \qquad p_3 = -2$

$p_2 = -1 \qquad p_4 = -4$

$\Rightarrow$ We will have 3 lines to ∞

2) Real axis segments b/w 0 & 1, 2 & 3, after 4

3) Asymptotes

$$\sigma = \frac{(-1-2-4)-(-3)}{4-1} = \frac{-4}{3}$$

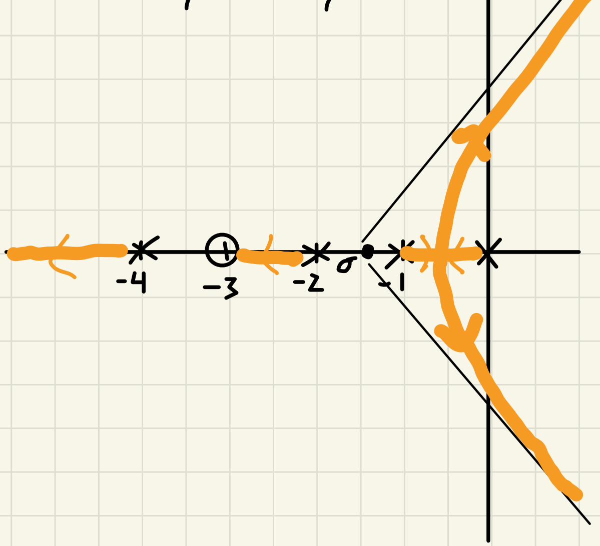
$$\theta_A = \frac{(2K+1)\cdot 180^\circ}{4-1}$$

for

$K=0 \Rightarrow 180^\circ/3 = 60^\circ$

$K=1 \Rightarrow 180^\circ \cdot 3/3 = 180^\circ$

$K=2 \Rightarrow 180^\circ \cdot 5/3 = 300^\circ = -60^\circ$

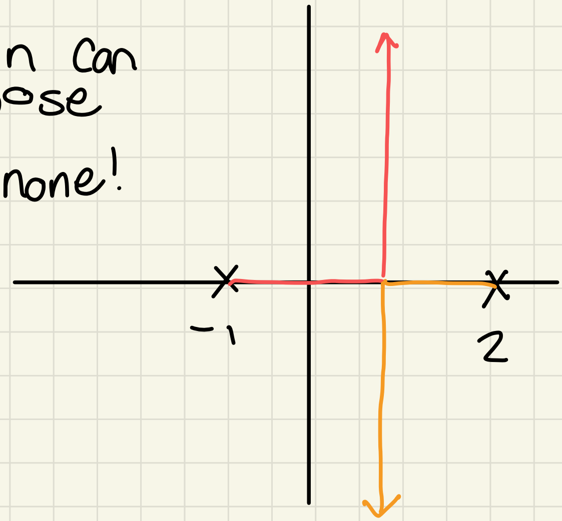


Let's look at a quick example of using a by-hand sketch to evaluate a controller

Let's say you want to stabilize this system.

What gain can you choose

→ none!

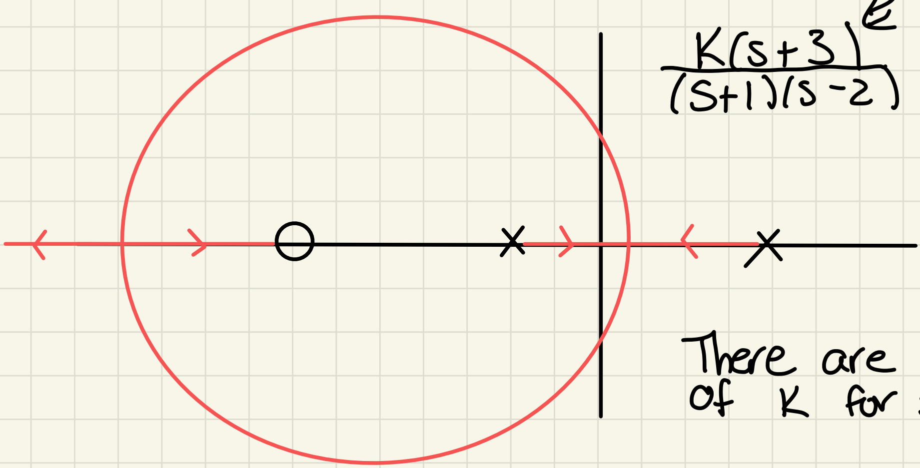


$$\frac{K}{(s+1)(s-2)}$$

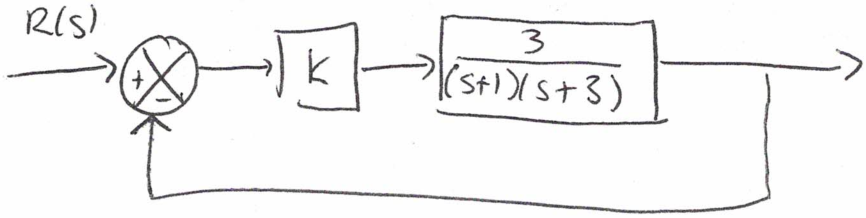
So you know your controller has to alter the pole-zero configuration

So let's add a zero and redraw

$$\frac{K(s+3)}{(s+1)(s-2)} \Leftarrow \text{PD controller}$$



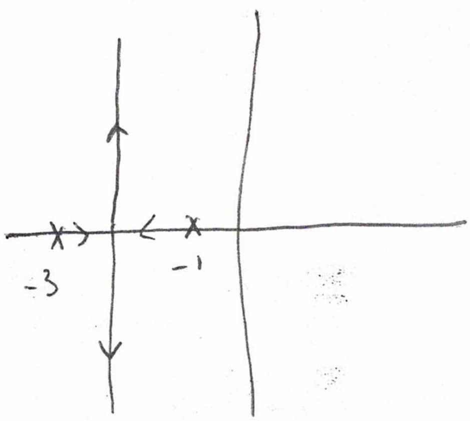
There are values of K for stability



Let's use the RL to find the desired value K if we are targeting a 8% OS.

① Draw the RL

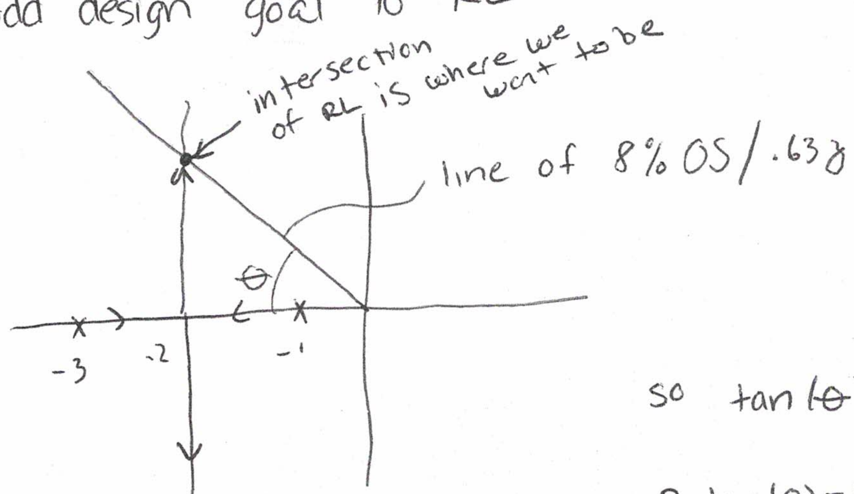
Breakaway is at -2



② Identify design constraint (ζ for 8% OS)

$$\zeta = -\frac{\ln(.08)}{\sqrt{\pi^2 + \ln^2(.08)}} = .63$$

③ Add design goal to RL



Recall that $\theta = \cos^{-1}(\zeta)$

$\theta = .889$ rad

$= 50.95°$

so $\tan(\theta) = \frac{Im}{Re}$ at desired location and we know Re = -2

$Re\tan(\theta) = Im = (-2)\tan(50.95) = -2.5$

desired pole loc is $-2 \pm 2.5j$

4) What value of K will get us there?

$$\text{OLTF} = \frac{3K}{(s+1)(s+3)}$$

$$\text{CLTF} = \frac{3K}{s^2+4s+3+3K}$$

find poles of char. eqn

$$\frac{-4 \pm \sqrt{4^2-4(1)(3+3K)}}{2(1)} = -\frac{4}{2} \pm \frac{\sqrt{16-12-12K}}{2}$$

$$= -2 \pm \frac{\sqrt{4-12K}}{2}$$

set equal to desired poles & solve for K

$$-2 \pm 2.5j = -2 \pm \frac{1}{2}\sqrt{4-12K}$$

focus on Im part

$$2.5j = \frac{1}{2}\sqrt{4-12K}$$

$$5j = \sqrt{4-12K}$$

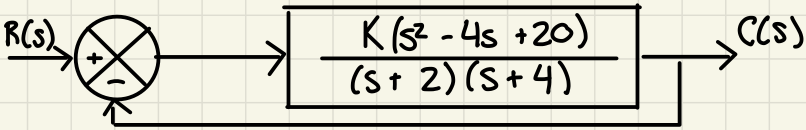
$$4-12K = -25$$

$$-12K = -29 \rightarrow K = 2.4$$

We should set our K to 2.4 to achive 8% OS on this system

Now let's talk about the power of making the root locus in matlab and how to use these tools to design a desired transient response

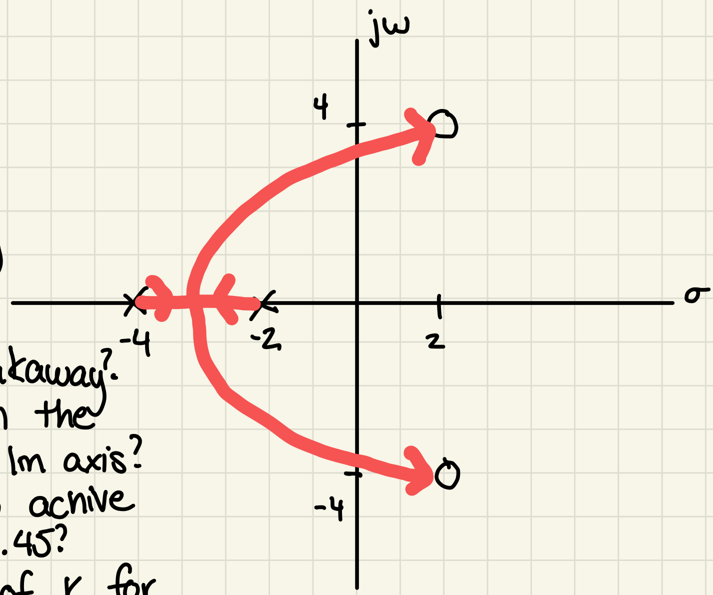
Let's sketch the root locus, then use Matlab to generate it. In Matlab we'll find some critical points:



First, let's practice by making a sketch of what the root locus will look like:

OLP: $s_{1,2} = -2, -4$

OLZ: $s_{1,2} = 2 \pm 4i$



Some points we may wish to know:

1) Where is the breakaway?
2) What is the gain when the locus crosses the Im axis?
3) What is the gain to achive a damping ratio of .45?
4) What is the range of K for which the system is stable?

use rootlocus_example1.m