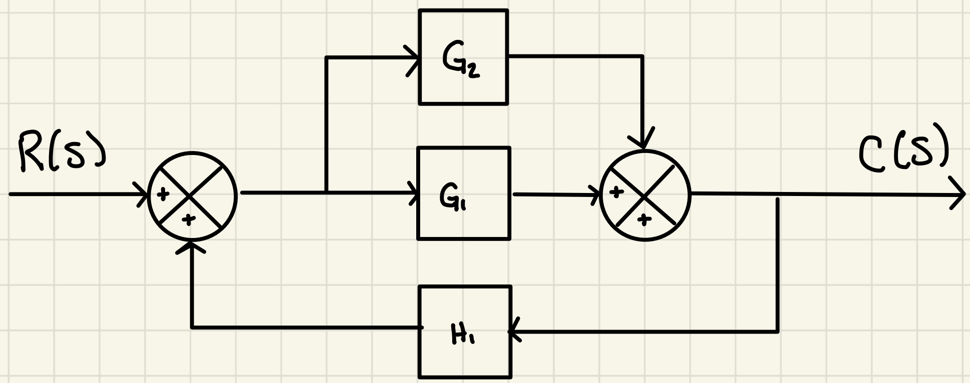
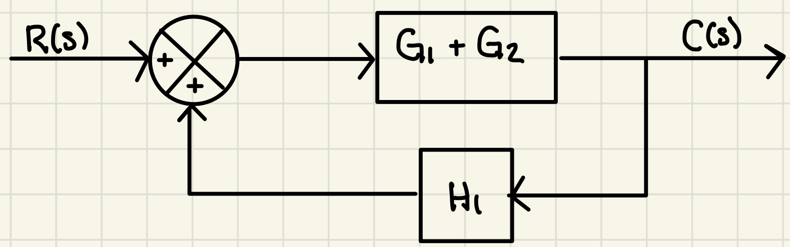
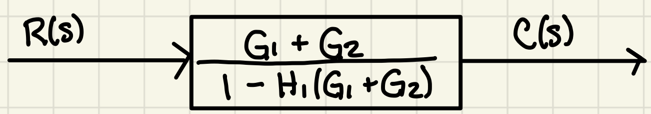
Example 2 Determine $C(s)/R(s)$ for the system
(Try first on your own)



1) Combine the feed forward path

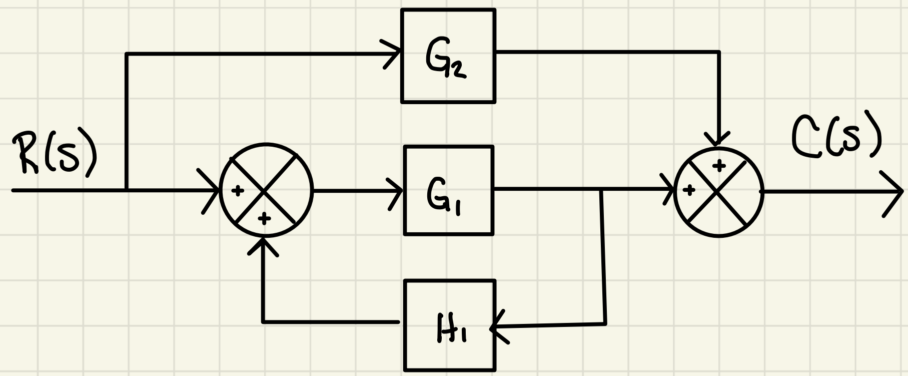


2) Remove the feed back loop

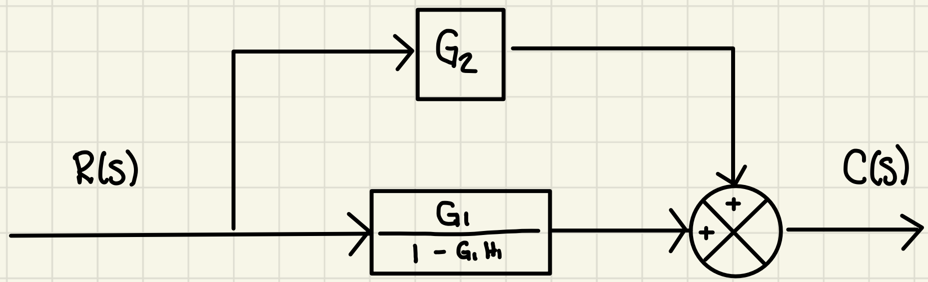


$$\boxed{\frac{C(s)}{R(s)} = \frac{G_1 + G_2}{1 - G_1H_1 - G_2H_1}}$$

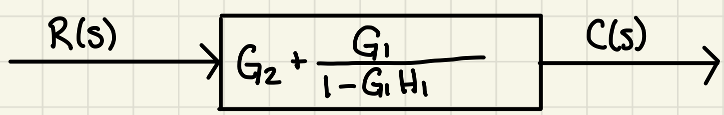
Example 3) Find $C(s)/R(s)$
(Try on your own)



1) Remove the feedback loop



2) Remove the feedforward loop

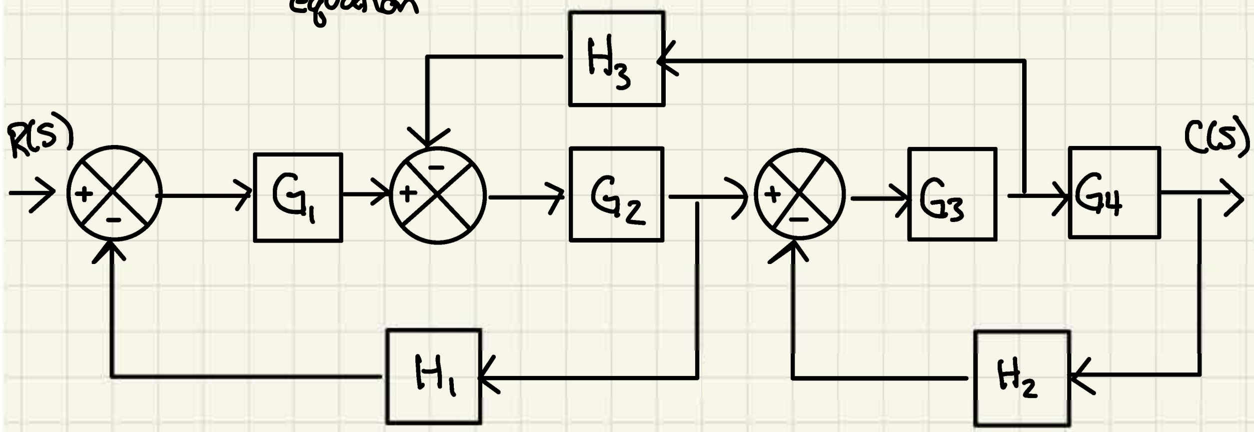


3) Create a common denominator

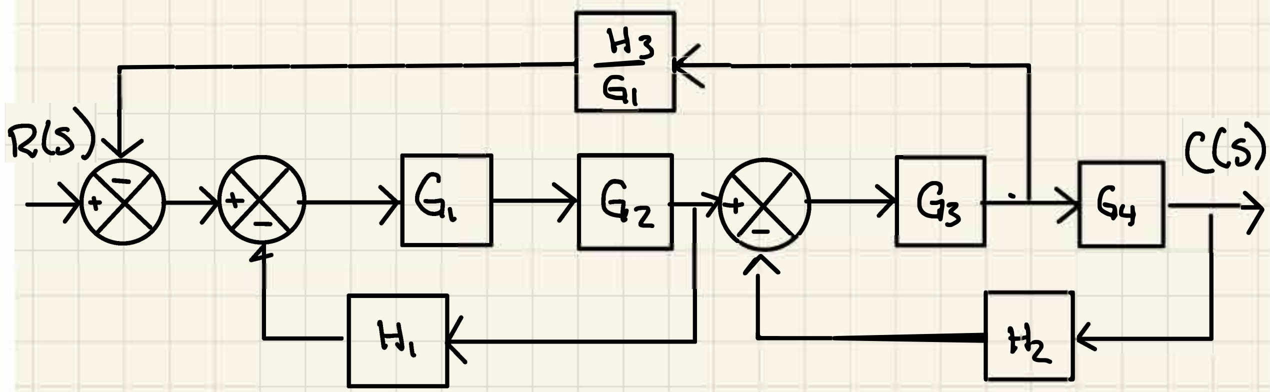
$$\frac{C(s)}{R(s)} = \frac{G_1 + G_2(1-G_1H_1)}{1-G_1H_1}$$

★ Compare the result of this example to the previous one. How do the structural differences in the block diagram affect the final transfer function?

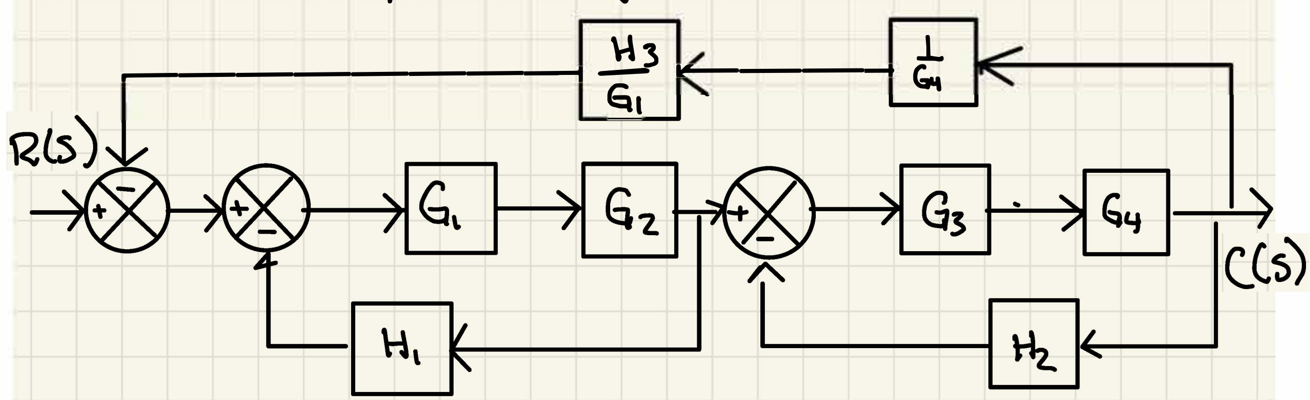
Example 4 Determine $C(s)/R(s)$ and write the characteristic equation



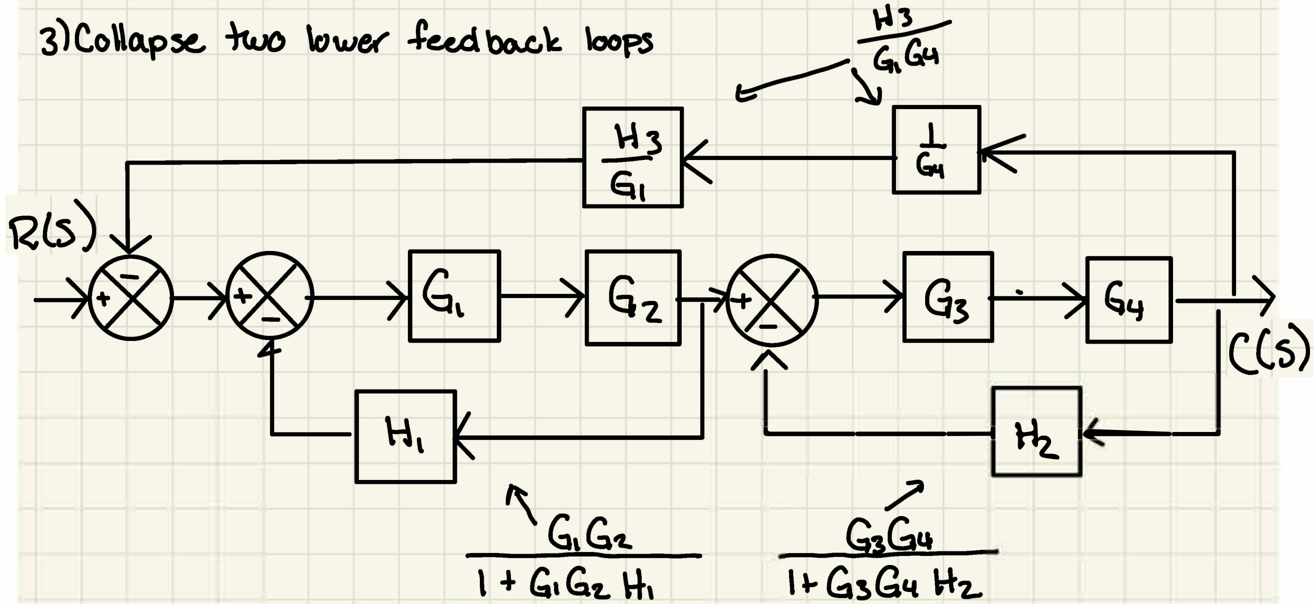
1) Move the summing point from H_3 ahead of G_1



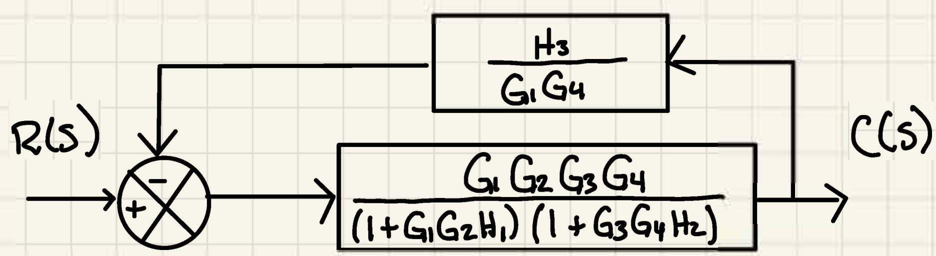
2) Move the takeoff point of H_3 beyond G_4



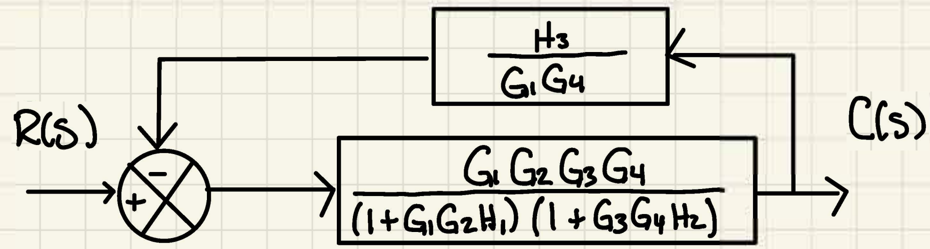
3) Collapse two lower feedback loops



These can be multiplied



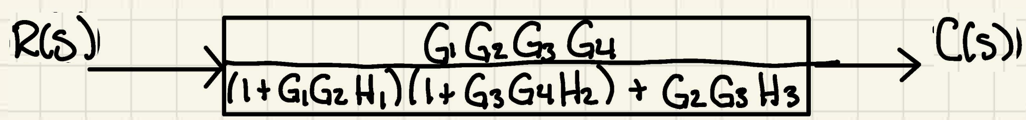
4) Collapse the feedback loop



$$\frac{\dfrac{G_1G_2G_3G_4}{(1+G_1G_2H_1)(1+G_3G_4H_2)}}{1+\dfrac{\cancel{G_1}G_2G_3\cancel{G_4}}{(1+G_1G_2H_1)(1+G_3G_4H_2)}\cdot\dfrac{H_3}{\cancel{G_1G_4}}}$$

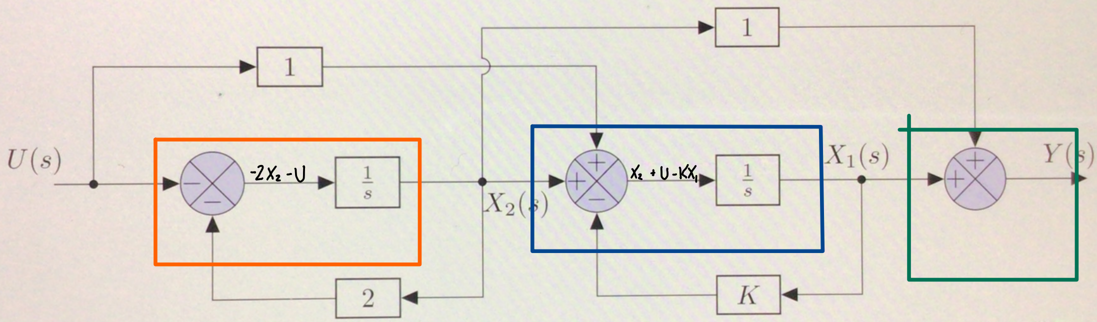
multiply through by $(1+G_1G_2H_1)(1+G_3G_4H_2)$

$$\frac{G_1G_2G_3G_4}{(1+G_1G_2H_1)(1+G_3G_4H_2)+G_2G_3H_3}$$



$$\boxed{\frac{C(s)}{R(s)}=\frac{G_1G_2G_3G_4}{(1+G_1G_2H_1)(1+G_3G_4H_2)+G_2G_3H_3}}$$

4. (25 points) The block diagram of a linear time invariant system is



Find the state space model (not the overall x-fer function)

① Let's understand what is happening at the summing points

$X_2 = (-2X_2 - U)\frac{1}{s} \Rightarrow sX_2 = -2X_2 - U$

In other words, to get X_2, we integrate the signal coming into the block → $1/s$ is an integrator

we can $\mathcal{L}^{-1}$ to get back to time domain

$\dot{x}_2 = -2x_2 - u$

$X_1 = (X_2 + U - KX_1)\frac{1}{s} \Rightarrow X_1 s = X_2 + U - KX_1$

similarly to above, $\mathcal{L}^{-1}$

$\dot{x}_1 = x_2 + u - kx_1$

finally $Y = X_1 + X_2 \Rightarrow y = x_1 + x_2$

② Now we can write in SS

$\dot{x}_1 = x_2 + u - kx_1$

$\dot{x}_2 = -2x_2 - u$

$y = x_1 + x_2$

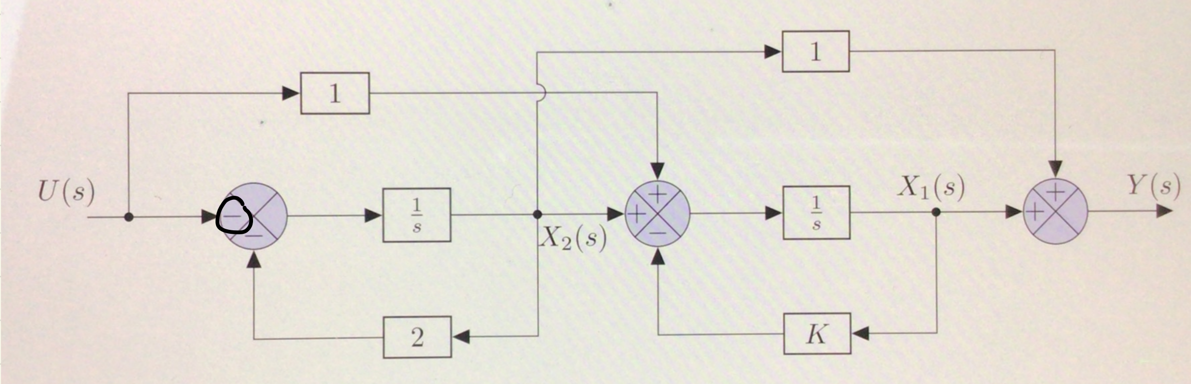
$$\Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -K & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

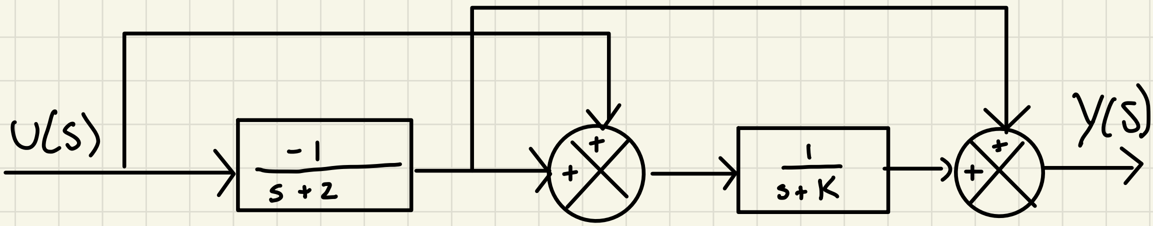
This is a different use of the block diagram than we looked at last class. We have been working to find the CL transfer function for the system, which we can put into the tools we will learn soon.

Let's find the CLTF for this system

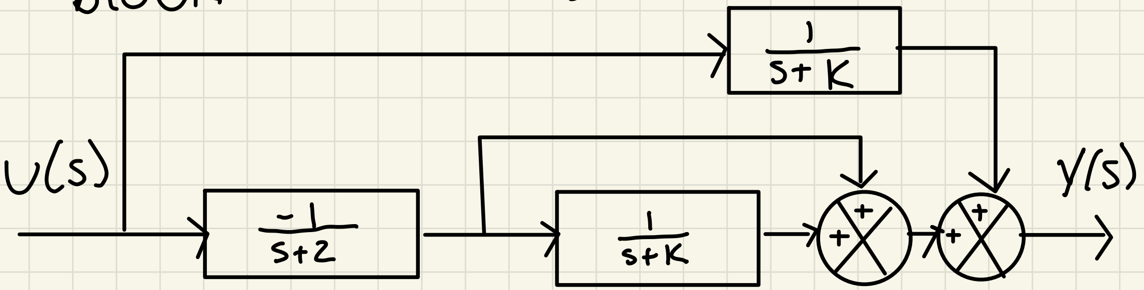
4. (25 points) The block diagram of a linear time invariant system is



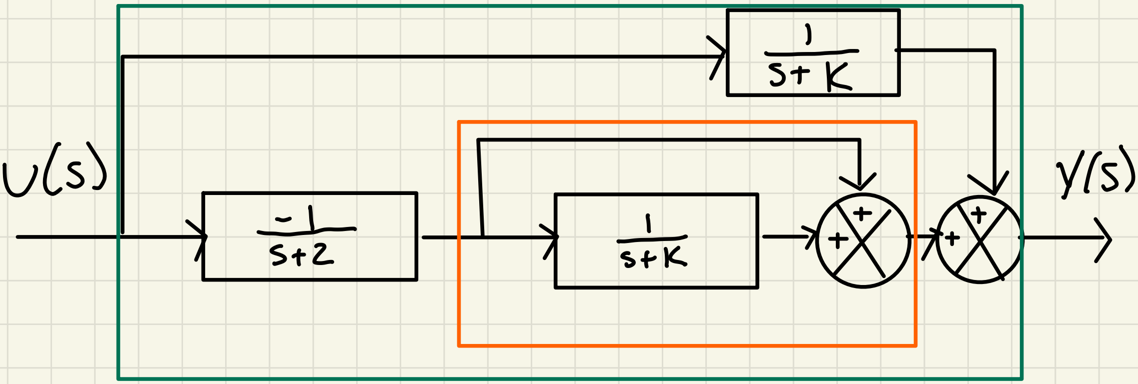
① Let's collapse the two feedback loops



② Move the summing point after the block



3) Collapse the loops



Inner loop: $\left(1 + \frac{1}{s+K}\right)$

Outer: $\frac{-1}{s+2}\left(1 + \frac{1}{s+K}\right) + \frac{1}{s+K}$

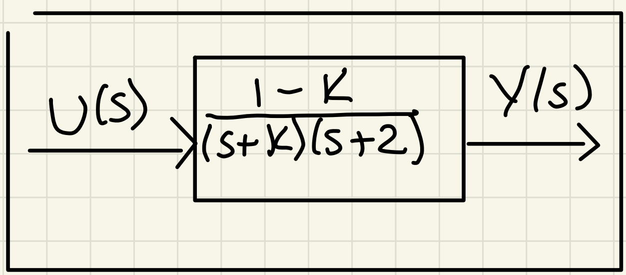
Reduce algebraically

$$= \frac{-1}{s+2}\left(\frac{s+K}{s+K} + \frac{1}{s+K}\right) + \frac{1}{s+K}$$

$$= -\frac{(s+K+1)}{(s+K)(s+2)} + \frac{(s+2)}{(s+K)(s+2)}$$

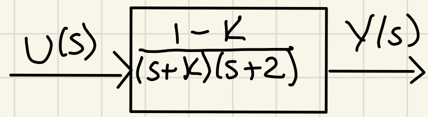
$$= \frac{-s-K-1 + s + 2}{(s+K)(s+2)}$$

$$= \frac{1-K}{(s+K)(s+2)} \implies$$



Is this equivalent to the state space model? Let's check!

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -K & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u \quad \overset{?}{\equiv}$$



$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

We'll convert SS to TF!

$$G(s) = C(sI - A)^{-1} B + D$$

$$(sI - A) = \begin{bmatrix} s+K & -1 \\ 0 & s+2 \end{bmatrix}$$

$$(sI - A)^{-1} = \frac{1}{(s+2)(s+K)} \begin{bmatrix} s+2 & 1 \\ 0 & s+K \end{bmatrix}$$

$$G(s) = \underbrace{\begin{bmatrix} 1 & 1 \end{bmatrix}}_{C} \underbrace{\frac{1}{(s+2)(s+K)} \begin{bmatrix} s+2 & 1 \\ 0 & s+K \end{bmatrix}}_{(sI-A)^{-1}} \underbrace{\begin{bmatrix} 1 \\ -1 \end{bmatrix}}_{B}$$

$$= \frac{1}{(s+2)(s+K)} \left((s+1) - (s+K) \right)$$

$$= \frac{1 - K}{(s+2)(s+K)}$$

Yes, these are equivalent!

Aside: We have poles at $s = -2$ and $s = -K$ $\Rightarrow$ for what values of K will the system be stable?

$\Rightarrow$ Positive ones! (more on this soon!)