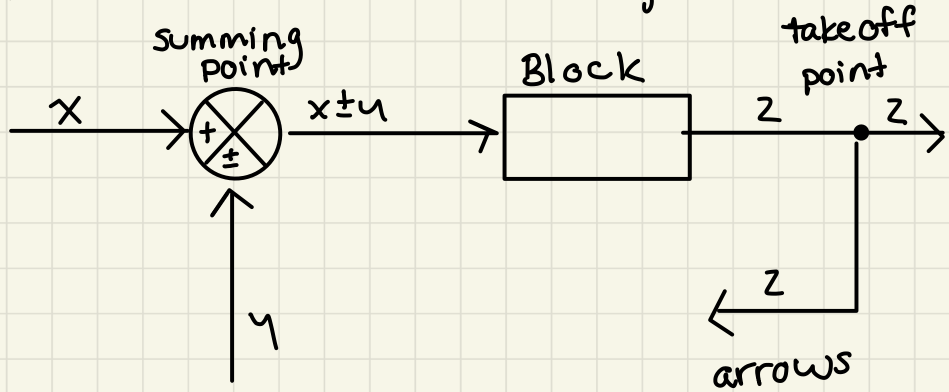
Block Diagrams

Block diagrams are a simple and useful way to graphically represent control systems

These provide a way to map and analyze the flow of information in a control s stem

Types of elements in a block diagram



In general, capital letters represent LaPlace transforms and lowercase letters are used to represent time-domain quantities

Block: A block represents a mathematical operation that transforms an input into an output (transfer function)

Arrows: Indicate the direction and flow of signals. Signals can pass only in the direction of the arrow

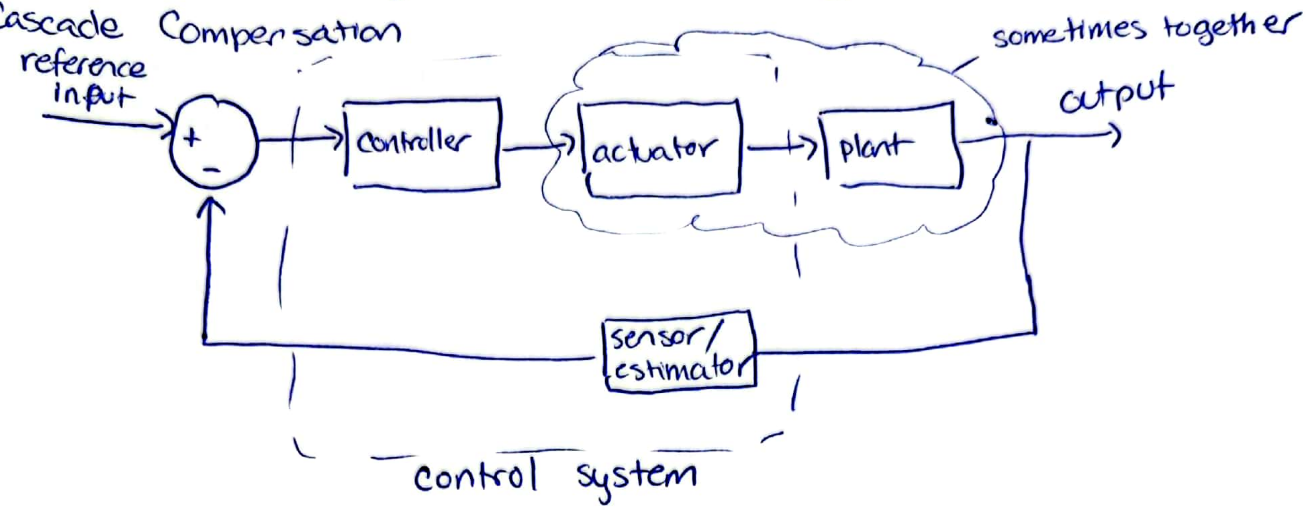
Summing Point: Summing points represent plus or minus operations. The plus or minus sign at the arrowhead indicates if the signal should be added or subtracted

Takeoff (Branch) Point: A point from which the signal goes concurrently to other points

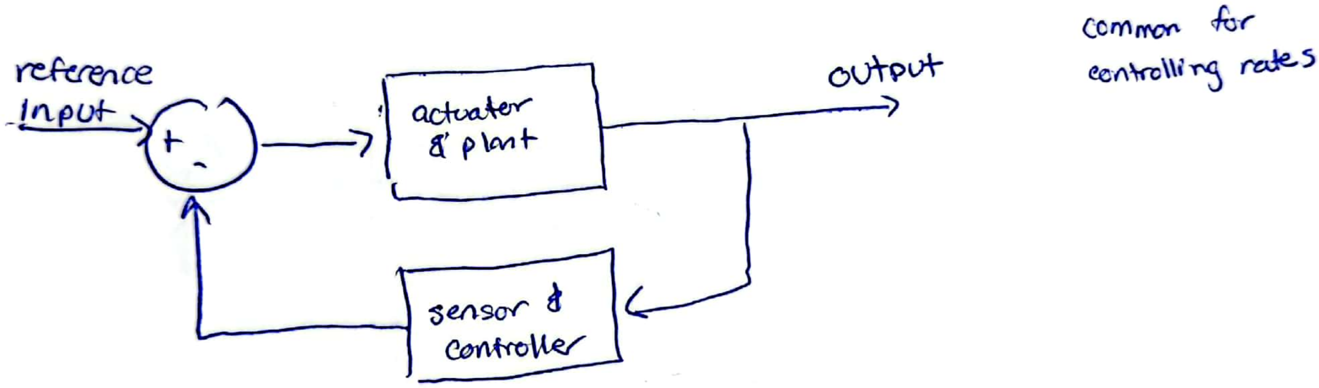
Block Diagrams & Control Systems

Most real (simple) control systems look something like one of these:

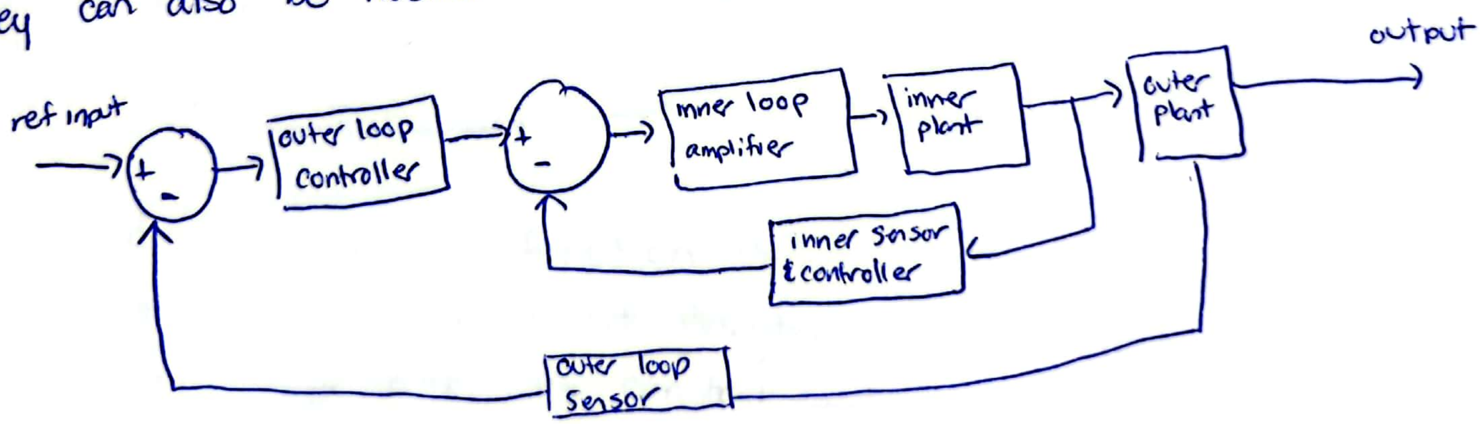
a) Cascade Compensation



b) Feedback Compensation

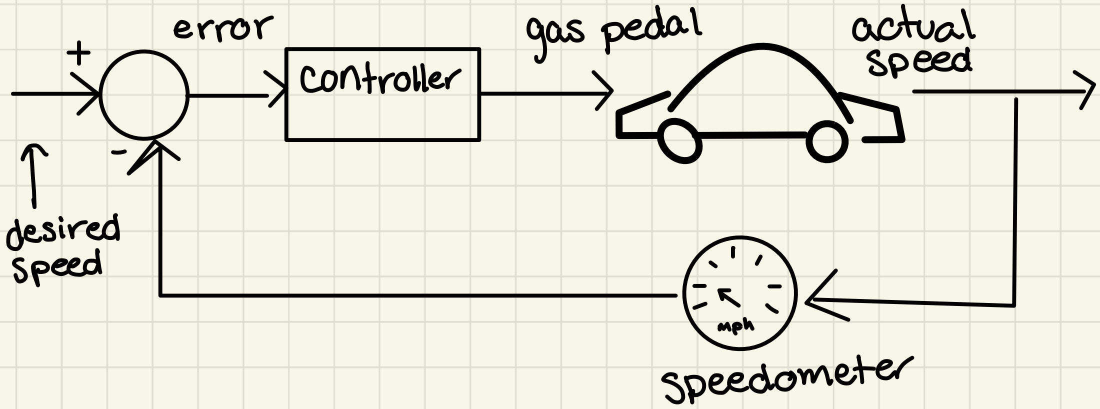


They can also be nested combinations of these (like this)



For example: inner loop controls aielorons & outer loop controls rudder - combination controls aircraft turn rate

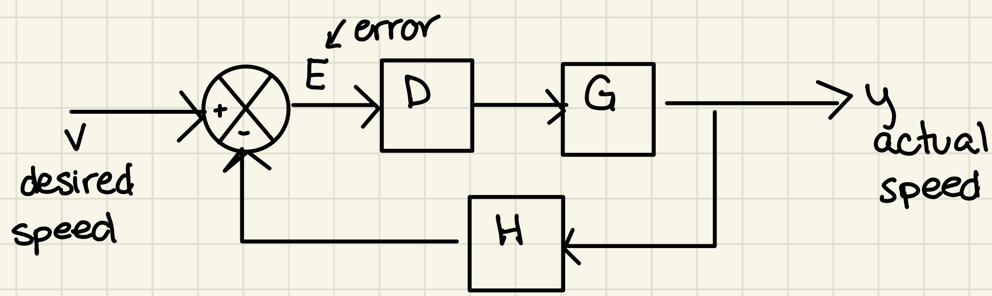
Let's take an example we are familiar with, and represent it as a block diagram: a car's cruise control



goal: desired speed = actual speed

error = desired speed - actual speed

so let's represent this as a block diagram

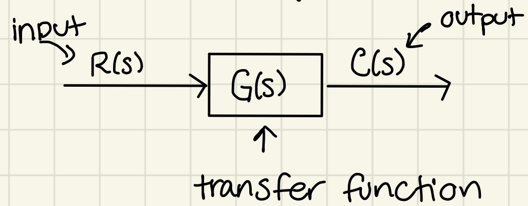


D, G, H are functions (La Place transforms) representing the behavior of the controller, car, and speedometer respectively

More on these later... we'll keep them abstract for now

⇒ takeaway: block diagrams give us a graphical bridge between real systems and mathematical models of these systems

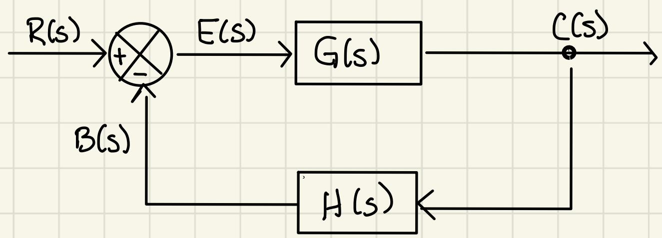
Let's consider a simple example of an open loop system:



The transfer function relates the output to the input, such that:

$$G(s) = \frac{C(s)}{R(s)} \begin{array}{l} \leftarrow \text{output} \\ \leftarrow \text{input} \end{array}$$

Now, let's consider a simple closed loop system



$B(s)$ is the feedback signal $\quad B(s) = H(s)C(s)$

There are a few quantities to be familiar with:

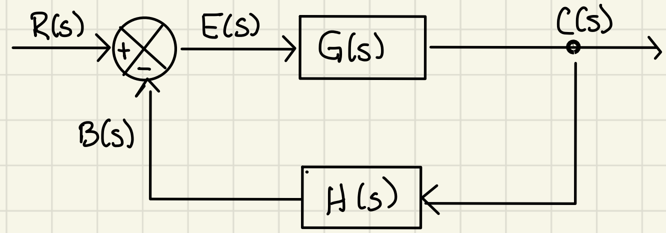
$\frac{B(s)}{E(s)} = G(s)H(s) \Rightarrow$ Open loop transfer function

$\frac{C(s)}{E(s)} = G(s) \Rightarrow$ Feedforward Transfer Function

$\frac{E(s)}{R(s)} \Rightarrow$ Error ratio

$\frac{B(s)}{R(s)} \Rightarrow$ Primary Feedback Ratio

Now, let's consider a simple closed loop system



For the system shown, we can describe the input output relationship:

$$C(s) = G(s)E(s)$$

$$E(s) = R(s) - B(s) = R(s) - H(s)C(s)$$

$$C(s) = G(s)\left[R(s) - H(s)C(s)\right]$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$ ⇐ closed loop transfer function

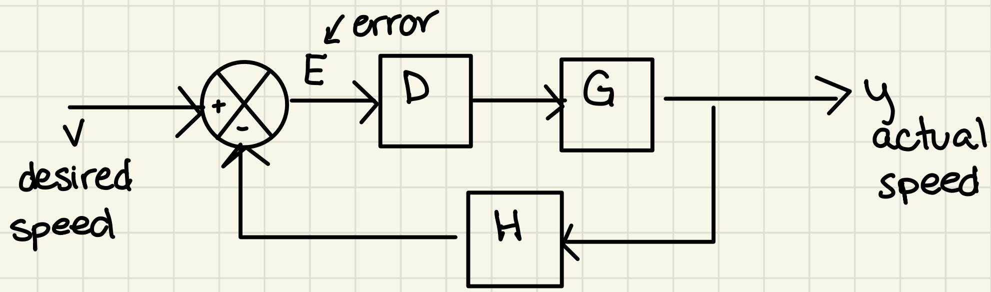
The denominator of the closed loop transfer function is the characteristic equation of the system

More complex systems can be represented using more complex block diagrams, where different blocks are used to represent different subsystems, components, and control elements

We are often interested in reducing these block diagrams to a simplified form represented by a single block containing the closed loop transfer function for analysis

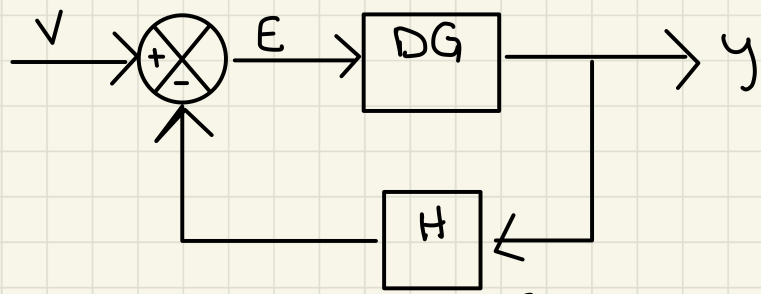
Think of this as the process of taking a system from it's physical representation to the mathematical formulation useful for analysis

Let's look at our cruise controller → can we simplify?



remember that the block diagram is a graphical representation of the equations of the system

blocks in series represent multiplication, so



but can we go even further?

$E = V - yH$ and $y = EDG$ so $E = \frac{y}{DG}$

$\therefore \; V - yH = \frac{y}{DG}$ and we can find y as a function of V

$$y = \frac{DG}{1 + DGH} V \quad \text{or} \quad \frac{y}{V} = \frac{DG}{1 + DGH}$$ ⇐ transfer function

now, we can represent this as an open loop system!

$$V \longrightarrow \boxed{\frac{DG}{1 + DGH}} \longrightarrow y$$

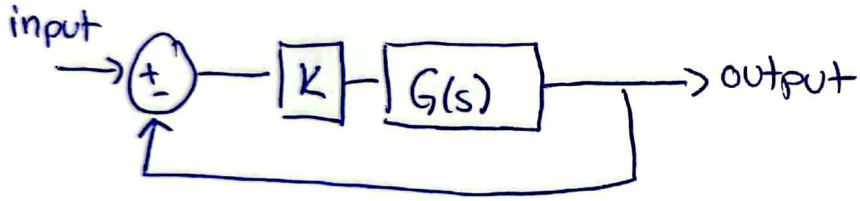
and $1 + DGH$ is the characteristic equation of the system

We often "reduce" our block diagrams to one of a few standard forms that are useful viewpoints for the various control system design and analysis methods

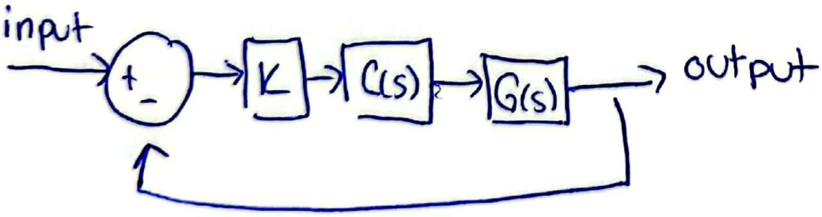
① Reduce the block diagram to find a single overall transfer function for the system

- Can be analyzed as an open-loop, feed forward system
- Useful for stability analysis methods

② Reduce the block diagram to a single "unity" feedback



OR



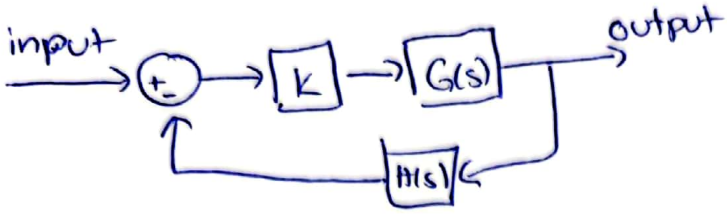
- useful in root locus, bode, frequency domain
- most common design architecture for simple control systems
- corresponds to direct observation of feedbac variable (control variable)
- Use to Identify "Closed Loop" and "Open Loop" Transfer functions for a system

"Open loop" transfer function is simply the product of everything in the loop except the constant gain term

$$\text{OLTF} = \cancel{K}G(s) \quad \text{or} \quad \cancel{K}C(s)G(s)$$

often not included

③ Reduce the block diagram to a single non-unity feedback loop



- also useful for methods utilizing open loop - RL, Bode, Nyquist...
- OLTF = $G(s)H(s)$

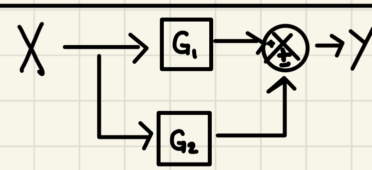
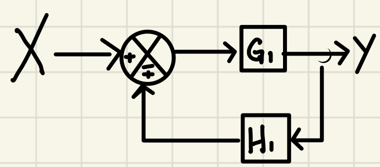
We use the rules of block diagram algebra to transform our block diagram to the desired form

Block Diagram Reduction (Algebra)

It can be useful to reduce more complex block diagrams into simpler ones. This can help find the characteristic eqn.

In fact, it is possible to reduce the reprensentation all the way to a single block system

Block Diagram Simplification Rules:

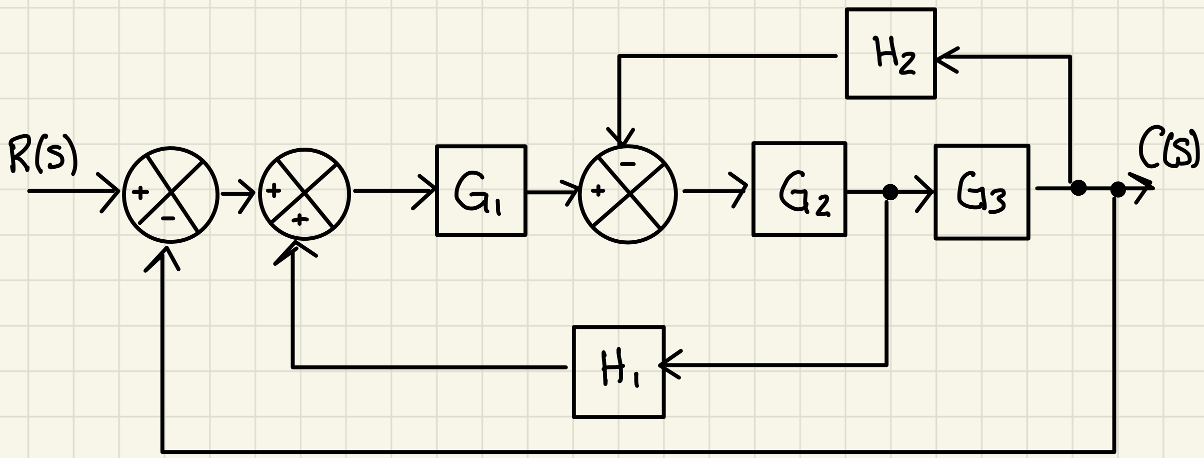
Reduction	Original Diagram	New Diagram	Eqn
Combining blocks in series (cascade)	$X \rightarrow \boxed{G_1} \rightarrow \boxed{G_2} \rightarrow Y$	$X \rightarrow \boxed{G_1 G_2} \rightarrow Y$	$Y = (G_1 G_2) X$
Combining blocks in parallel or eliminating foreward loop		$X \rightarrow \boxed{G_1 \pm G_2} \rightarrow Y$	$Y = (G_1 \pm G_2) X$
Eliminating a feed back loop		$X \rightarrow \boxed{\dfrac{G_1}{1 \pm G_1 H_1}} \rightarrow Y$	$Y = G_1 (X \mp H_1 Y)$

These are just a subset of the key rules. See the handout for the full set

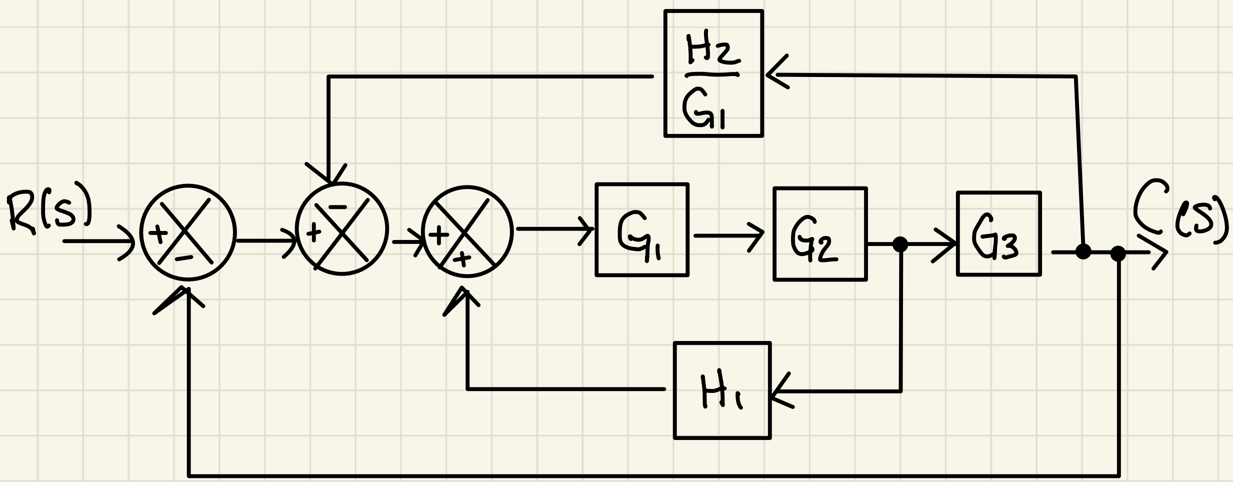
⇒ GIVE HANDOUT OF BLOCK RULES

Lets do some examples

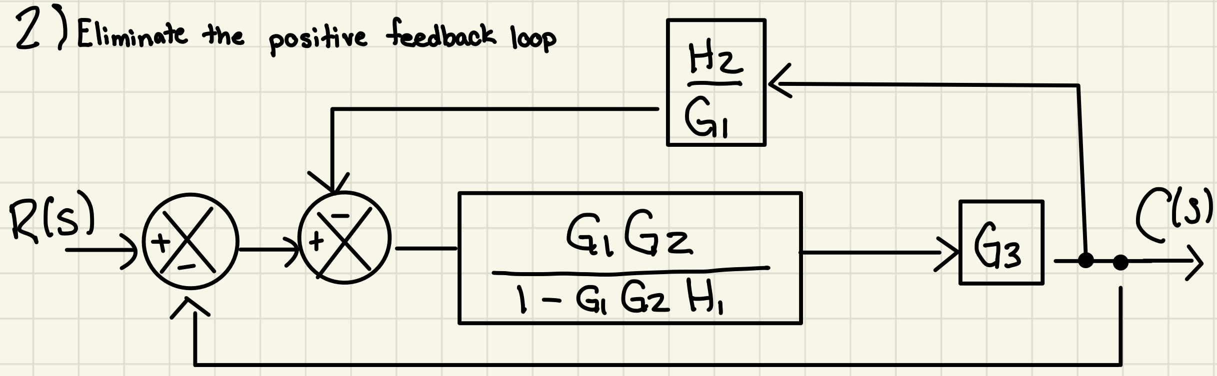
Example 1 : Simplify the diagram



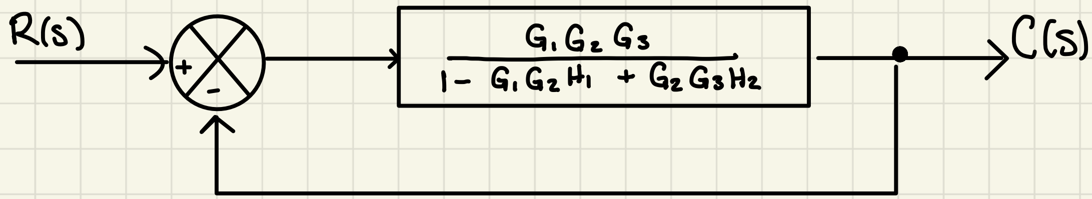
1) Move the summing point of the negative feedback loop H_2 to before G_1



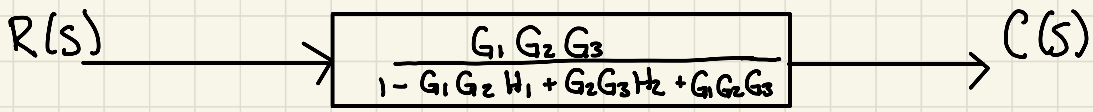
2) Eliminate the positive feedback loop



3) Combine the center blocks and remove upper feedback loop



4) Eliminate the feedback loop



- Note that the numerator is the product of the transfer functions of the feedforward path

- The denominator is

$$1 + \sum \text{(product of the transfer functions around each loop)}$$

feedforward functions

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3}{1 - \underset{\uparrow}{G_1 G_2 H_1} + \underset{\uparrow}{G_2 G_3 H_2} + \underset{\uparrow}{G_1 G_2 G_3}}$$

⊕ inner loop ⊖ top loop ⊖ outer loop