Static Error Constants from Bode Diagrams

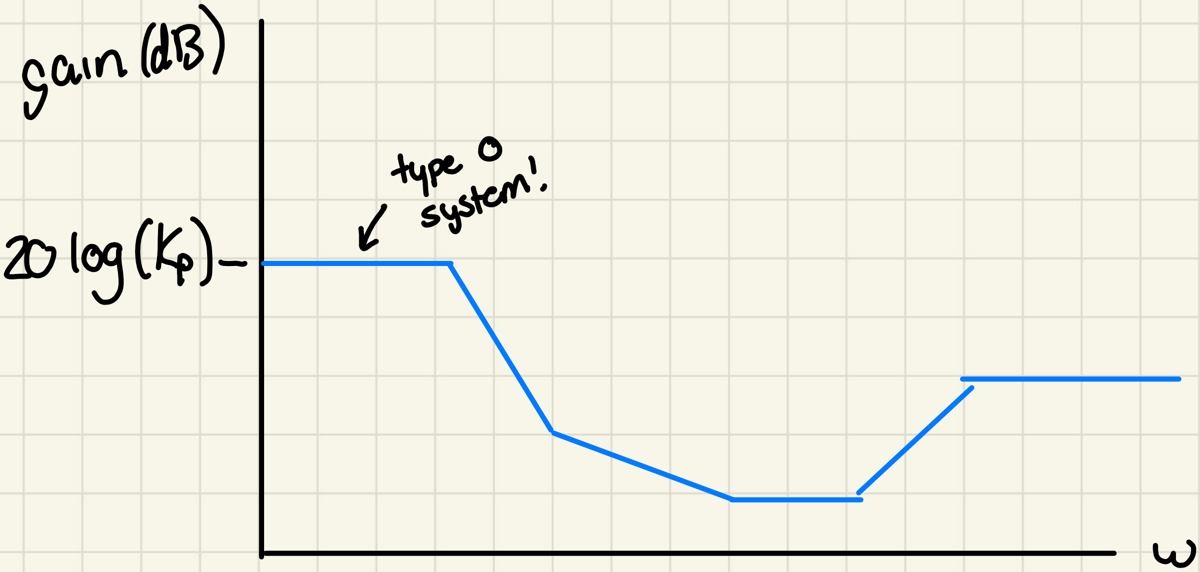
K_p: Type 0 system $G(s) = K\dfrac{\prod(s+z_i)}{\prod(s+p_i)}$

if $s = j\omega$, and ω is small (ω_0) $\Rightarrow 20\log M = 20\log\left(\underbrace{\dfrac{K\prod z_i}{\prod p_i}}\right)$

↑ low freq. value

the 0 frequency gain $= K_p$

A system of type 0 will have a Bode plot that begins with a constant value (think back to our pole & zero rules) and this constant value is $20\log(K_p)$



K_v: Type 1 System $\qquad G(s) = \dfrac{K\prod(s+z_i)}{s\prod(s+p_i)}$

the low frequency value is

$$20\log M = 20\log \frac{K\prod(z_i)}{\omega_0\prod(p_i)}$$

this is a line with slope -20 dB/dec from:

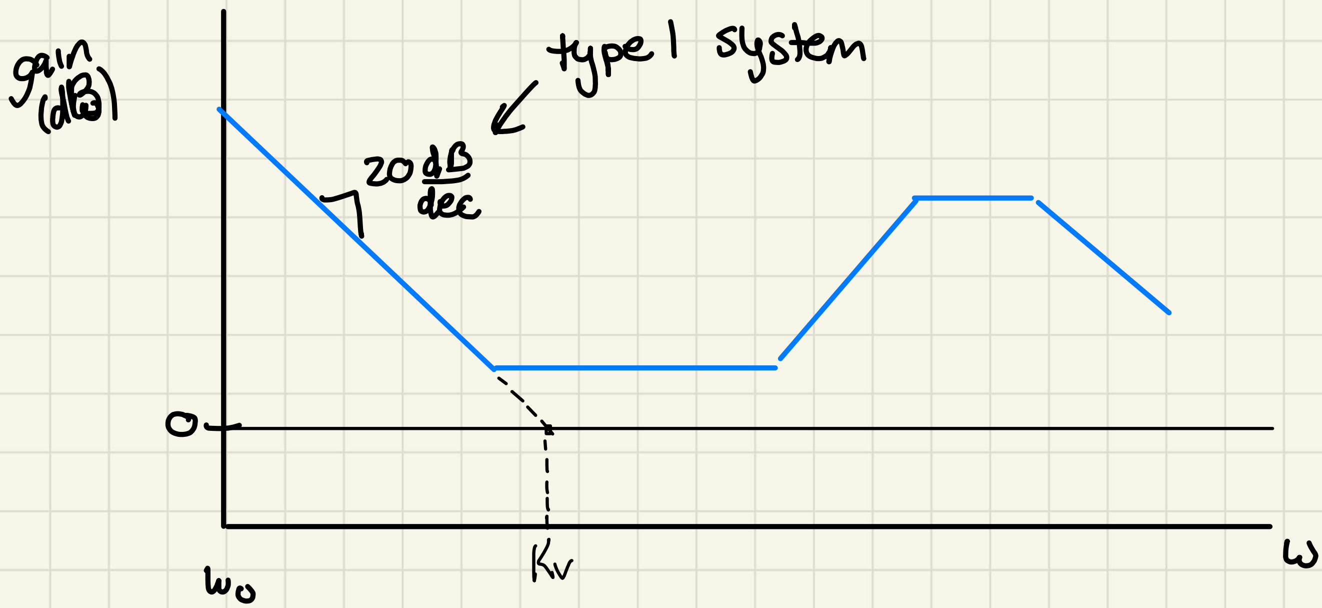
$$G'(s) = \frac{K}{s}\,\frac{\prod(z_i)}{\prod(p_i)}$$

Type 1 systems start w/ a -20 dB/dec slope!

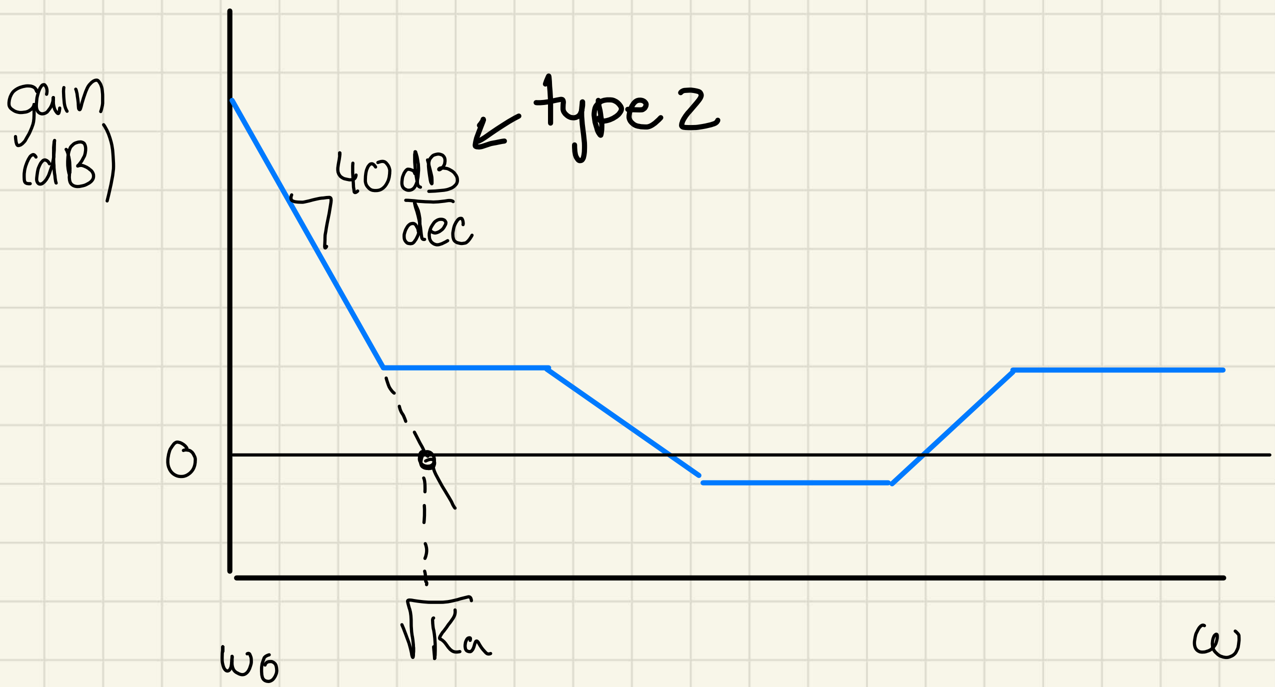
so this intersects the frequency axis at

$$\omega = K\frac{\prod(z_i)}{\prod(p_i)} \Rightarrow \text{the eqn for } K_v$$

K_v is found by extending the inital -20 dB/dec line to the frequency axis and finding the intercept



K_a: By a similar derivation, a type 2 system is identified by an inital slope of 40 dB per decade, with $\sqrt{K_a}$ found at the value this low frequency asymtote intersects the frequency axis



So now that we understand how frequency response relates to performance, let's look at how it can be used for controller design

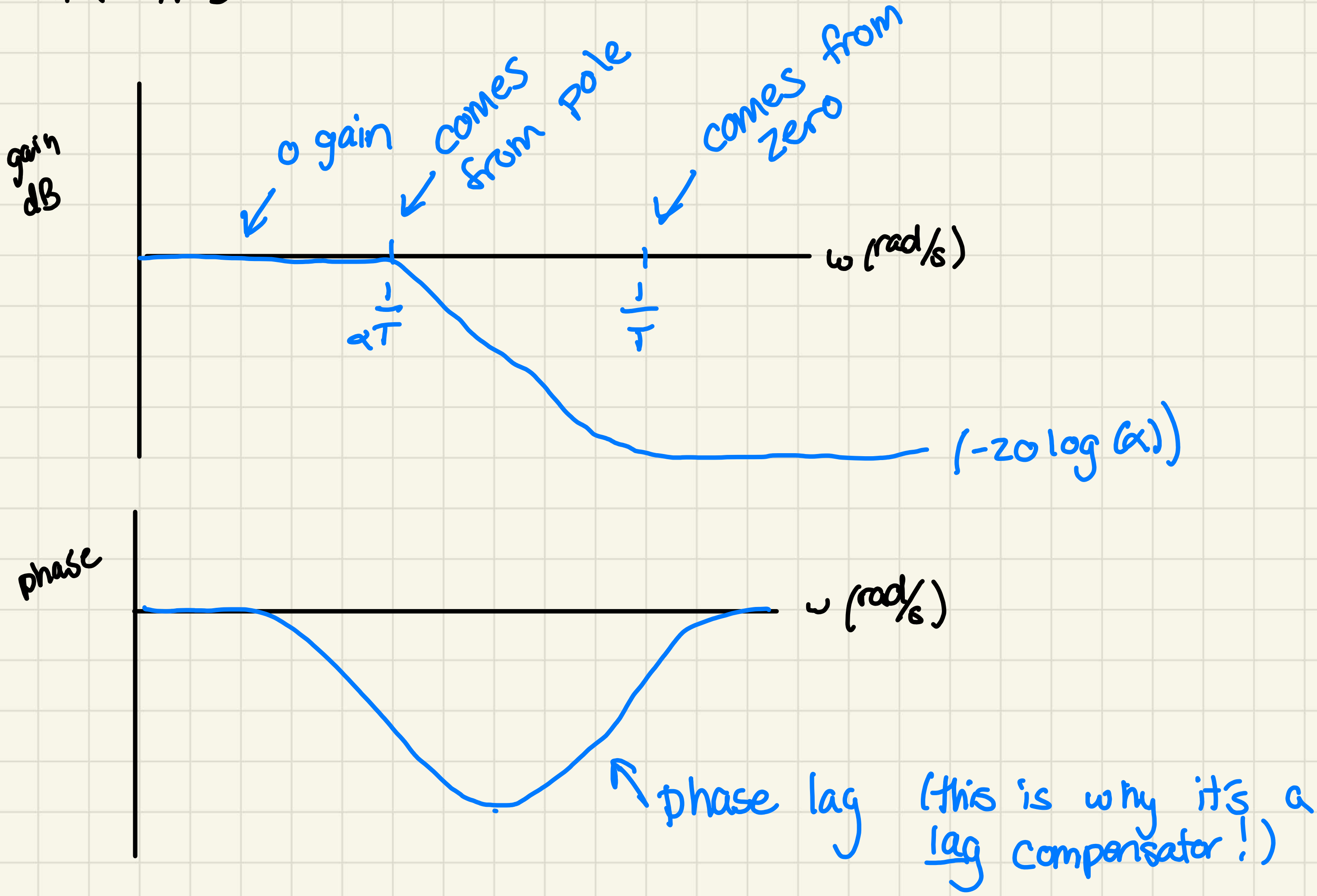
Let's look at a lag compensator design

First, consider that a lag compensator is

$$G_c(s) = \frac{Ts+1}{\alpha Ts+1}$$

$T, \alpha > 1$

So the Bode plot contribution from the compensator looks like this:



and a lead is just upside down!

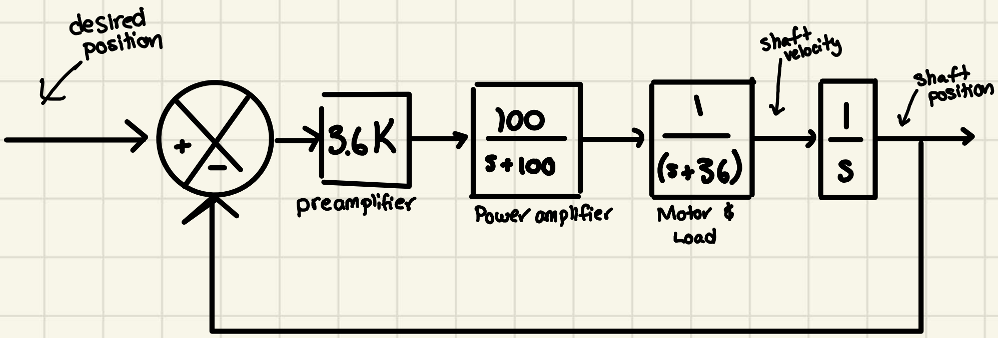
So, we know we use lag compensators to help improve steady state error, but it looks like it has no affect on K_p! So how does it work?

→ We'll rely on adjusting the overall system gain to meet the SSE

→ The lag compensator will move the crossover frequency, thus affecting the phase margin, giving more room to modify the transient response without going unstable

→ we'll choose our lag compensator to achieve the crossover frequency needed for the desired phase margin.

Let's do an example



First, we are going to look at gain adjustment. Let's adjust our K to get a desired overshoot of 9.5%. Then, we'd like to improve K_v by a factor of 10.

run bodedesignexample.m part1&2 find $K = 162.2$

With this, the OLTF is

$$G_p(s) = \frac{3.6K(100)}{s(s+36)(s+100)}$$

so we can calculate the current $K_v = 16.22$

a tenfold improvement sets our goal at $K_v = 162.2$

to achieve this, we need to ↑ K by 10x, or K = 5839

so our new OLTF is

$$G_p(s) = \frac{583900}{s(s+36)(s+100)}$$

The problem is, we've lost our 9.5% OS!

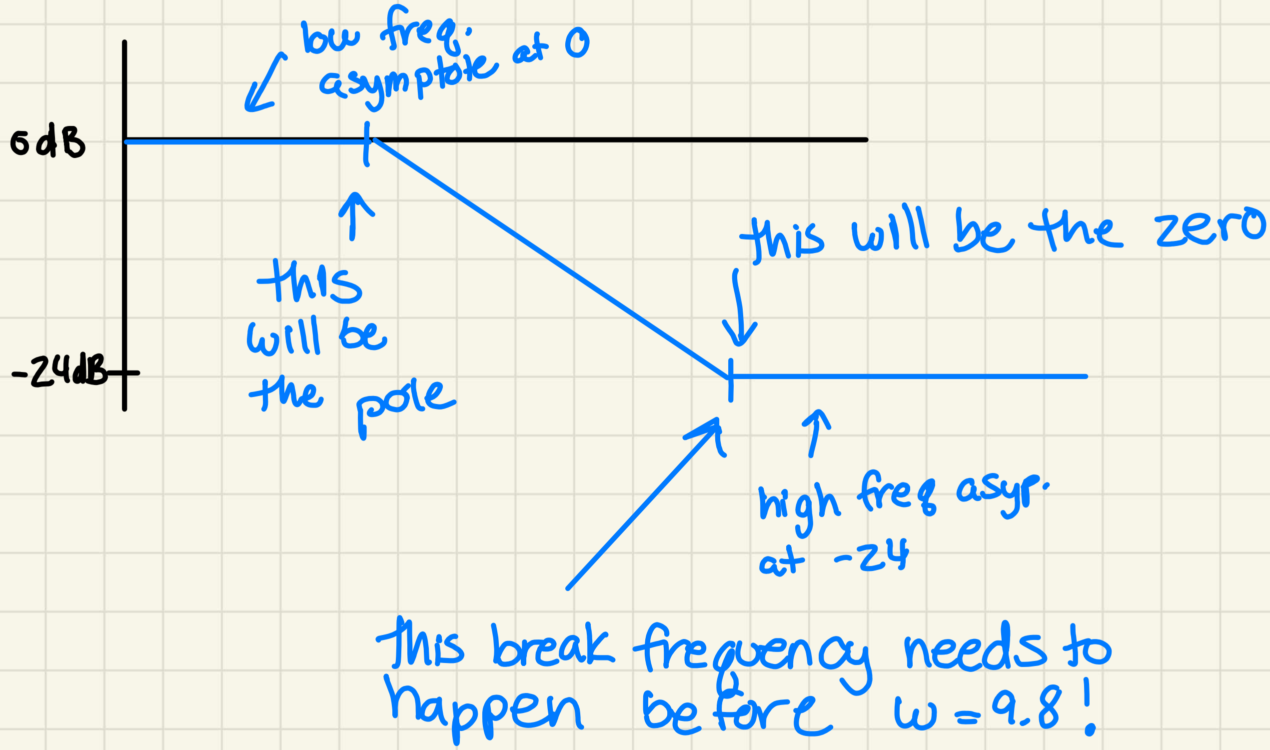
We need our phase margin to be 59.2 and it is now 4.37!

But, we will add phase lag with our compensator so we'll set our target at 69.2°

So $-180 + 69.2 = -110.8°$, and we can find the frequency at this value to be 9.8° and the gain is 24 dB. In order to get the desired phase margin, we need -24 dB. So this will be our high frequency asymptote.

bodedesignexample.m parts 3 and on

Let's sketch our controller gain plot to help us.



So from this, we can design our controller.

Let's pick our high frequency break a decade before 9.8, so .98. So our zero will be at $(s + .98)$

Now we need to travel backwards along a line of -20 dB/dec slope to get to the low frequency asymptote at 0 and find the low frequency break. And, we need to find the Kc value to get the asymptote at -24

$$G_c(s) = K_c \frac{(s + \omega_{high})}{(s + \omega_{low})}$$

at ω_{high}, gain $= -20 \log(K) = -20 \log(1/M) = -24$

so $K_c = \frac{1}{M} = \frac{1}{15.87} = .063$

not in dB!

at ω_{low}, $K_c \cdot \frac{\omega_{high}}{\omega_{low}} = 1 \Rightarrow \frac{.063(.98)}{\omega_{low}} = 1 \Rightarrow \omega_{low} = .062$

$$\text{So} \quad G_c = \frac{.063\,(s+.98)}{(s+.062)}$$

now we can check our response for our controlled system

our final OLTF is:

$$G_{final} = G_c G_p = \frac{36786\,(s+.98)}{s(s+.062)(s+36)(s+100)}$$

our target PM was 59.2, we achived 63.7

$$K_v = \frac{36786\,(.98)}{(.062)(36)(100)} = 161.5$$

very close to our goal of 162.2

If we look at our step response, we see an acceptable behavior

Bode Plots & System Id

Every system will have a unique frequency response

One way to obtain G(s) for a system is to develop the differential equations

Another way is to do it experimentally - one strategy is Bode

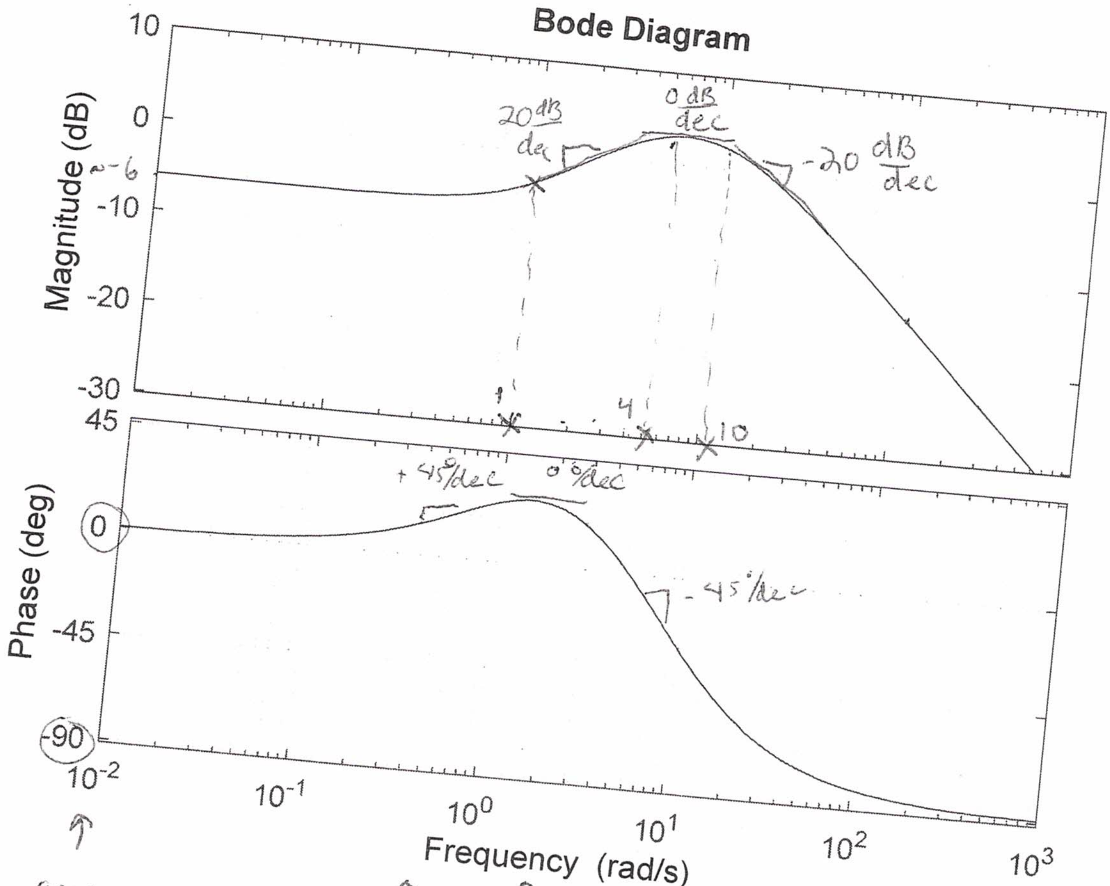
We can input frequency inputs across a range of frequencie and record mag/phase of response

We can build the bode plot from these responses, and use the Bode plot to develop the transfer function

Let's do a quick example

Estimate the transfer function from the bode plot

$$G(s) = \frac{20(s+1)}{(s+4)(s+10)}$$



↑ gain

↑ Zero indicated ~ -1

↑ pole indicated ~ -4

↑ pole indicated ~ -10

$$G(s) = \frac{K(s+1)}{(s+4)(s+10)}$$

low freq gain is ~6 ⇒ s = 0

$$20 \log\left(\frac{K(1)}{(4)(10)}\right) = \sim -6$$

what is K?

$$10^{-6/20} \approx .5 = \frac{K(1)}{(4)(10)}$$

K = 20

$$G(s) = \frac{20(s+1)}{(s+4)(s+10)}$$