3.3.2) Nyquist

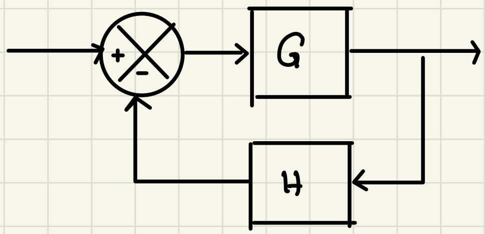
Let's look at one more tool in the frequency space. This one is primarily used for determing the stability of a closed loop system from its open loop frequency response.

Nyquist plots are sort of like root locus for the frequency space, in that we use the OL info to plot contours that can give us information about the closed loop stability

And actually, the Nyquist criterion fundamentally come from something we already know, but mapped into the Nyquist Space.

Remember when we learned about RL?

If we have



$$G = \frac{N_G}{D_G}, \quad H = \frac{N_H}{D_H}$$

OLTF: $GH = \dfrac{N_G N_H}{D_G D_H}$ ← poles of OLTF

CLTF: $\dfrac{G}{1+GH} = \dfrac{N_G D_H}{D_G D_H + N_G N_H}$

char. eqn: $1 + GH = 1 + \dfrac{N_G N_H}{D_G D_H} = \dfrac{D_G D_H + N_G N_H}{D_G D_H}$

From those, we can see that:

1) Poles of the char. eqn. $1+GH$ are the poles of the OLTF

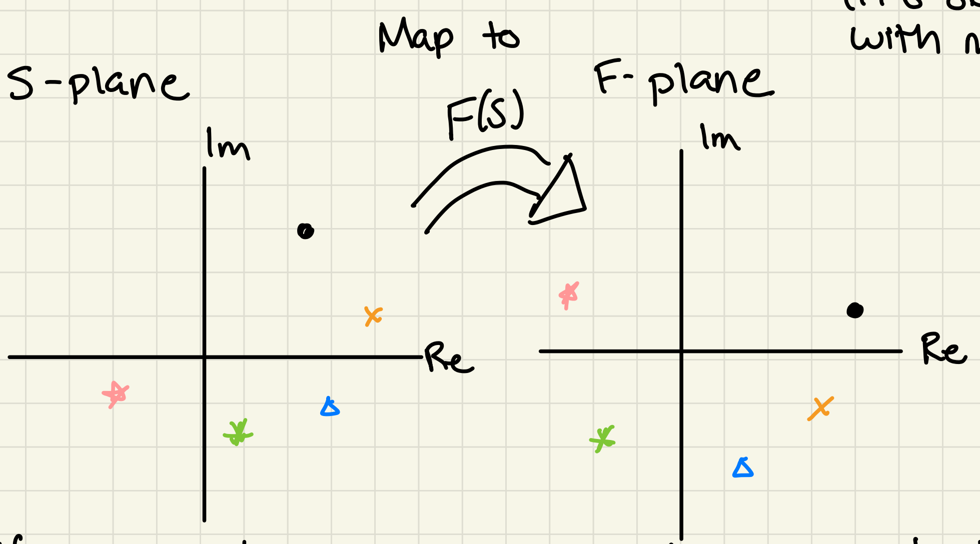
2) Zeros of the char. eqn $1+GH$ are the poles of the CLTF

Let's hold that thought, and talk about another concept: Mapping from s to F plane

Let's say we have some function

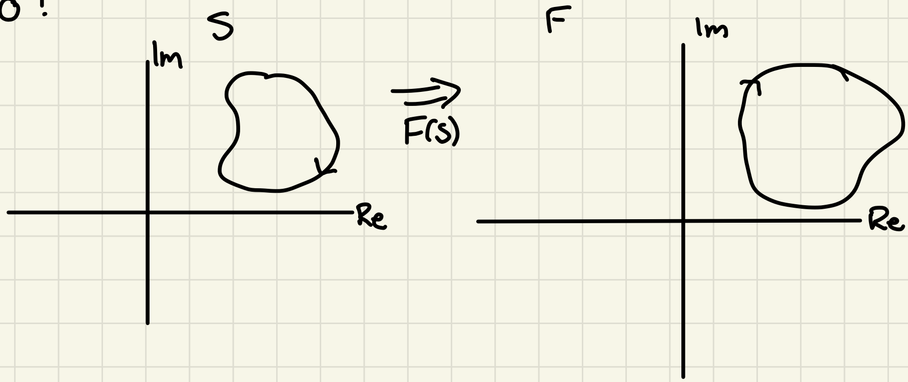
$$F(s) = \frac{(s-z_1)(s-z_2)\ldots}{(s-p_1)(s-p_2)\ldots}$$

zeros & poles in RHP! AHHH!!! (it's okay, bear with me!)



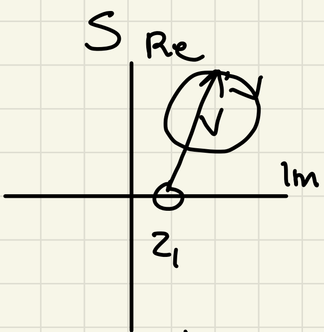
If we pick any complex number s, and plug it into F(s), we get a new complex number out that we can map on our F graph, and we can do that for any point in s

Any collection of points that forms a closed loop is called a contour, and we can map those too!

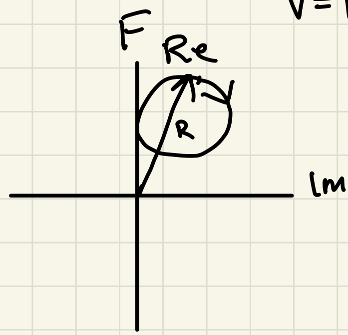


So lets look at some simple ones

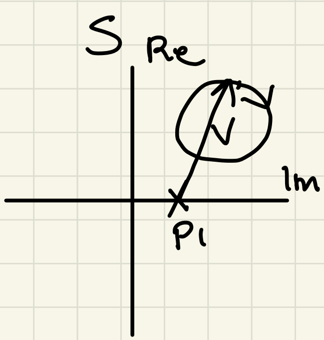
$V = R$



$F = (s - z_1)$

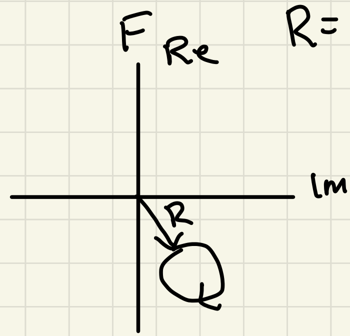


contour does not encircle zero

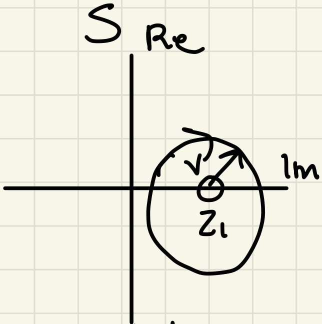


$F = \frac{1}{(s - p_1)}$

$R = \frac{1}{V}$



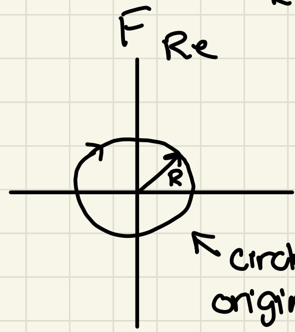
contour does not encircle pole



$\Longrightarrow$

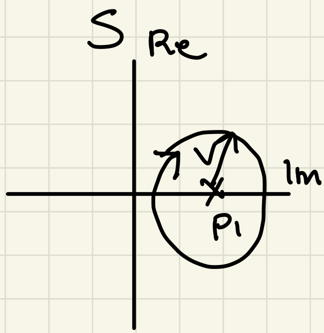
$F = (s - z_1)$

$R = V$



circles origin

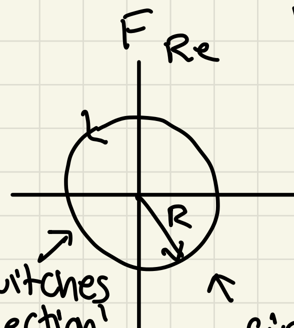
contour encircles zero



$\Longrightarrow$

$F = \dfrac{1}{s - p_1}$

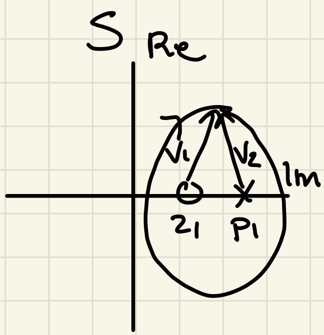
$R = \dfrac{1}{V}$



switches direction!

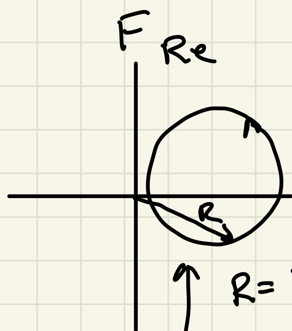
circles origin

contour encircles pole



$\Longrightarrow$

$F = \dfrac{(s - z_1)}{(s - p_1)}$



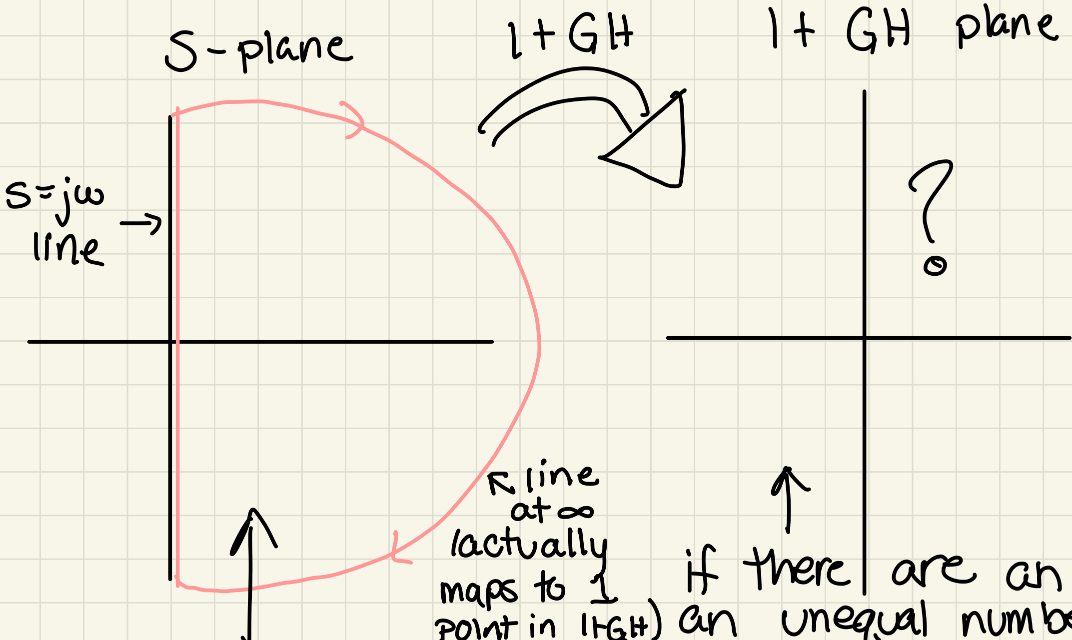
$R = \dfrac{V_1}{V_2}$

does not circle origin

contour encircles both

So that was a fun exercise in drawing circles! How is it useful?

Let's take a very specific mapping function, and apply a very specific contour



contour encircles entire unstable region (ie the whole RHP)

if there are an unequal number of poles & zeros in the RHP, we'll see circles around the origin. The direction, CW or CCW, will tell us if there are more poles (CCW) or more zeros (CW)

if N = # of circles about the origin

$N = P - Z$ ← zeros of 1+GH in RHP ← poles of CLTF

P: poles of 1+GH in RHP ← poles of OLTF

Let's think about what this is telling us.

$$N = P - Z$$ ← zeros of 1+GH in RHP ← poles of CLTF in RHP

P: poles of 1+GH in RHP ← poles of OLTF in RHP

Let's rewrite as:

NYQUIST CRITERION

$Z = P - N$ = # CLTF poles inside the contour (ie the RHP)

Z: CLTF poles in RHP

P: OLTF poles in RHP ← known from TF

N: # of origin circles ← CW circles are ⊖, CCW circles are ⊕ ← easy to count from Nyquist

if $Z = 0$, the system is stable

and we didn't actually need the CLTF to figure that out.

Cool! But's let's make one small change to make this even easier!

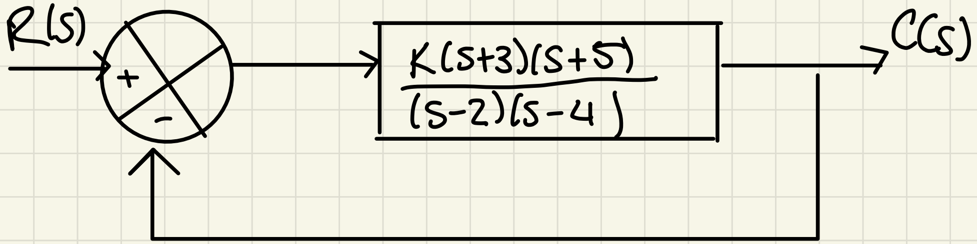
What if we map through the OLTF instead of the characteristic eqn?

i.e. we use GH instead of 1 + GH

All this will do is move the whole mapping left by 1.

So instead of origin circles, we'll count −1 circles

Let's do an example!



Nyquistex1.m

The OLTF has 2 poles in the RHP

if $K=1 \Rightarrow$ 2 CCW about -1

$Z = P - N = 2 - 2 = 0$ stable!

(we can see that on the root locus)

if $K = .1 \Rightarrow$ 0 CCW about -1

$Z = P - N = 2 - 0 = 2$ unstable! (2 CLTF poles in RHP)

(we can see that on the root locus)

Now, let's incrament K by .05 and watch what happens → somewhere around $K = .75$, the CLTF poles pass into LHP, and our circles encompass the -1, and Z becomes 0

So the system is stable for $K > .75$ (or so)

An easy way to count encirclements

① Draw a line out from -1 point in any direction

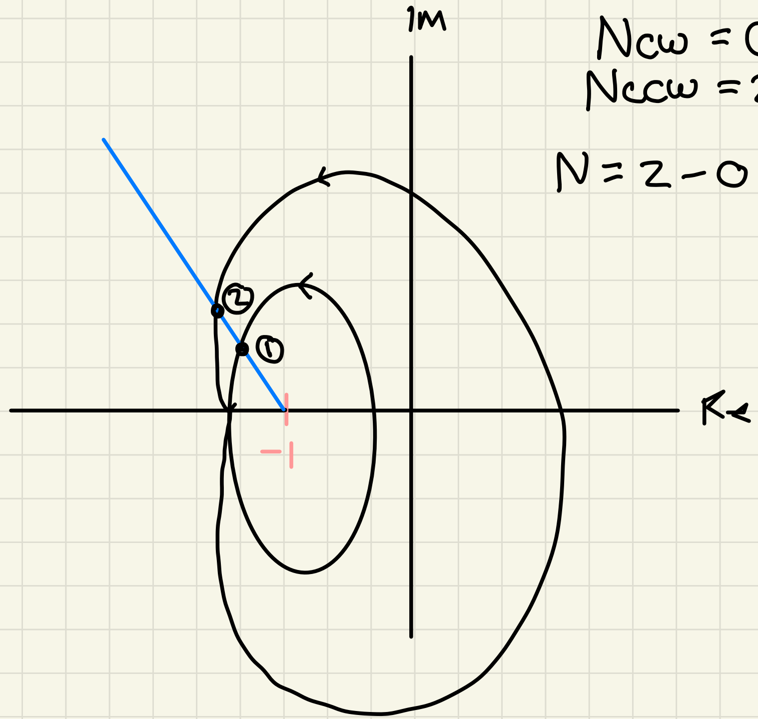
② Count the number of times it crosses the nyquist path with the arrow:

- pointing clockwise ⇒ ⊖ N_{cw}
- pointing counterclockwise ⇒ ⊕ N_{ccw}

③ $N_{ccw} - N_{cw} = N$

④ Plug into $Z = P - N$ to asses stability

Lets do this for our last example



$N_{cw} = 0$

$N_{ccw} = 2$

$N = 2 - 0$