AE 3531 Notes

Introduction to Control Systems

What is controls engineering?

- Discipline concerned with modeling the dynamic behavior of systems and designing systems to monitor and adjust the behavior of the system to produce a desired response
  - → Understand the system
  - → Model the system
  - → Monitor the system
  - → Influence the system ✱

  We need to

Can you think of some examples of control systems you encounter in daily life?

- Thermostat to home HVAC
- Cruise control on car
- Temperature control on oven
- Charging module on your phone
- ....

Before we go further, let's define some terms:

Control Variable: The variable representing the quantity or condition that is measured or controlled

Control Signal / Manipulated Variable: The quanity or condition that is varied by the controller so as to affect the control variable

Example: In a car's cruise control system, we control the speed (control variable) by adjusting the fuel flow rate (control signal / manipulated variable) to the engine

> **Control** means measuring the value of the control variable and adjusting the control signal to achive a target value of the control variable and limit its deviation

In general, we are concerned with behavior over time ⇒ Differential Equations

$$\xrightarrow{\text{input}} \boxed{\text{system}} \xrightarrow{\text{output}}$$

Control : how do I change this (vary) ... to get what I want here (consistently)?

This is the big question we will spend our term answering

Some more important terms:

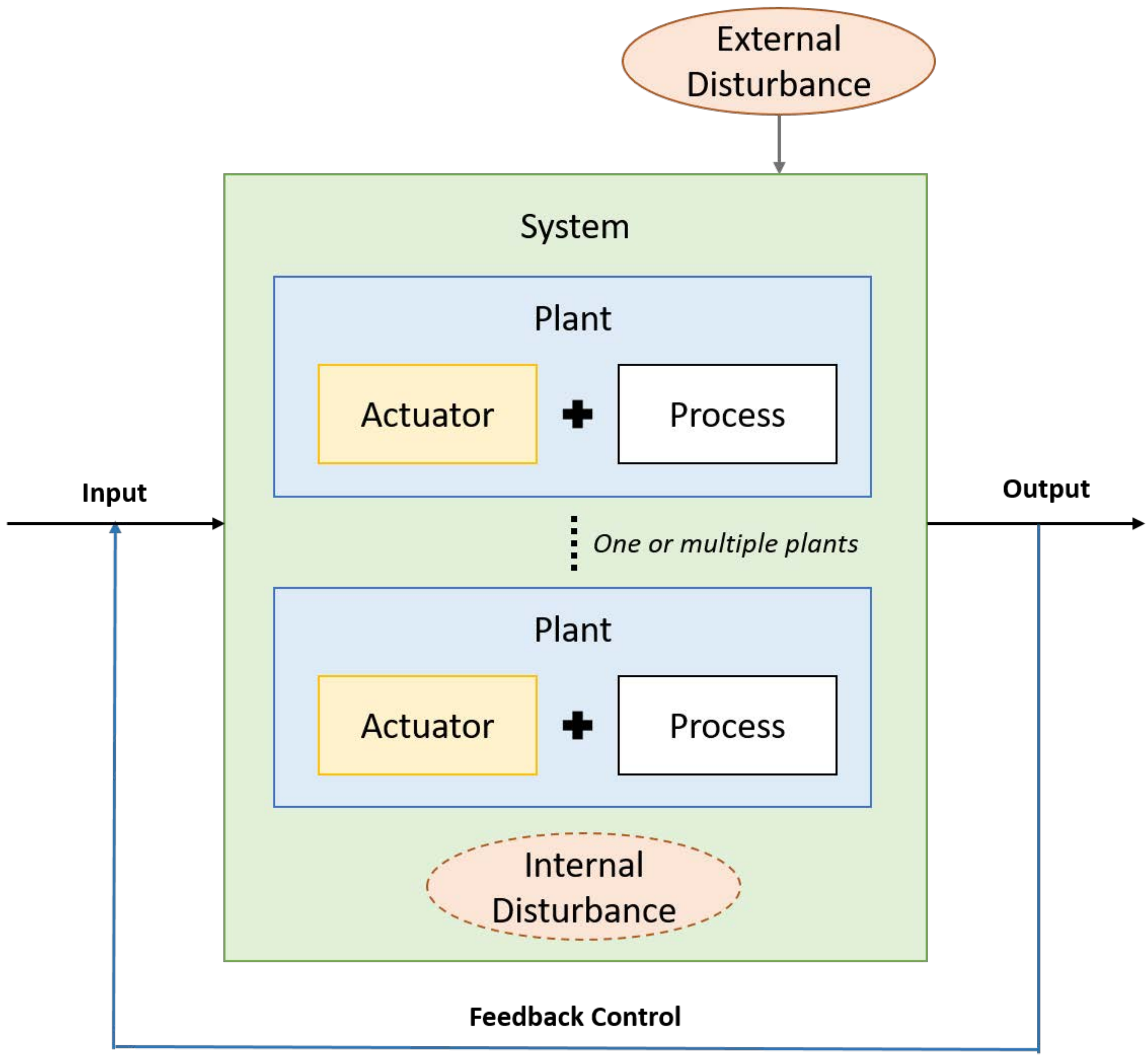
Plant: A piece of equipment or subsystem that performs a particular operation. For this class, the plant is the physical object we want to control
⇒ ex: car, spacecraft, aircraft

Process: A progressively continuing operation marked by a series of changes that suceed one another to achieve a particular result. In this class, any operation we wish to control is a process.

System: A combination of components that act together and perform a certain objective. These may or may not be physical, and may include more than one plant/process

Disturbance: A disturbance is a signal that tends to adversely affect the output of a system. These can be either internal (generated within the system) or external (generated outside the system)

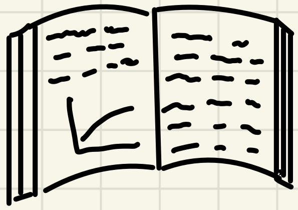
Feedback Control: Feedback control is an operation that, in the presence of a disturbance, tends to minimize the effect of the disturbance by reducing the difference between the output and some reference value based on the size of the difference.



In control theory, a plant is the combination of process and actuator. An actuator is a component of a machine that helps convert a supplied input signal into the required form of mechanical energy.

Note that depending on the complexity of the system, the system/plant/process may collapse into one unit. While there are subtle differences, these terms are often used interchangeably.

Let's do a thought experiment before diving into feedback control a bit more



When you study for an exam, do you use an open-loop or closed-loop approach?

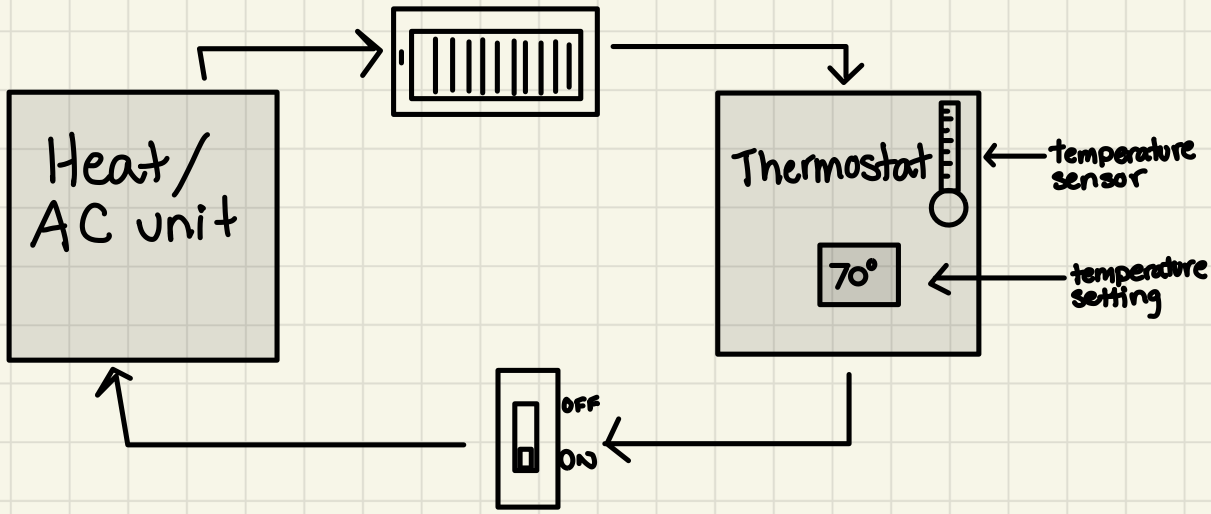
a) Open loop - I have 4 hours until the exam, so I will study for 4 hours
b) Open loop - The exam covers 4 topics, so I will do 1 hr of practice problems on each
c) Closed loop - I will study until I am able to finish the practice exam easily
d) Closed loop - I will study until I am able to correctly complete 2 new practice problems on each topic without looking at my notes.

Which strategy do you think is more likely to yield a higher grade?

What do the two strategies labeled "open-loop" have in common when compared to the two labeled "closed-loop"

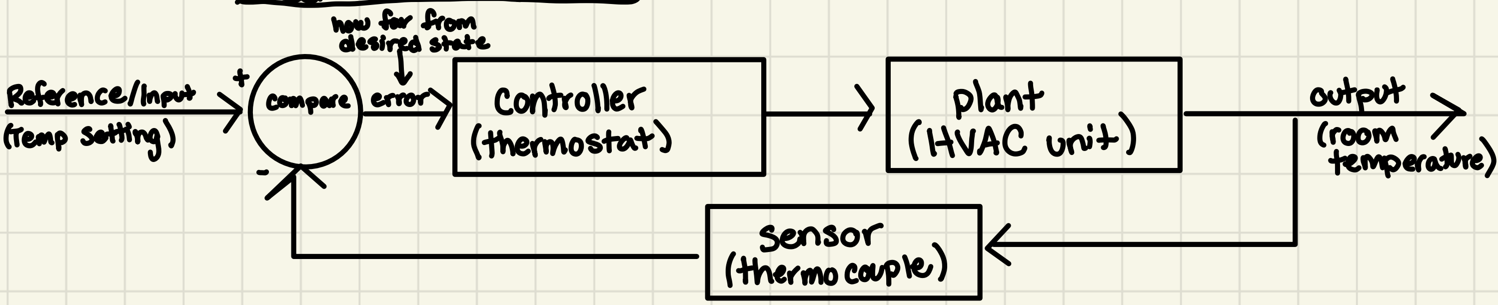
Feedback Control Systems

Let's think about a thermostat.



The thermostat measures the actual value of the temperature, compares it to the desired value of the temperature, and determines whether to turn on the air

↳ This is <u>feedback control</u>



Another term for feedback control is <u>closed-loop control</u>

Closed-loop control systems operate based on the <u>error</u> which is very simply the difference between the desired value and the actual value.

If we desire a temperature of 70°, but the actual temperature is 68°, we have 2° of error.

This brings up some controller design considerations:

1. How much error is ok?
2. How often should we check the error?

The answers depend on many things:

1) The purpose of the system
2) The responsiveness of the system (how long does it take to correct error)
3) The budget / resources / hardware available
4) The accuracy of the sensors

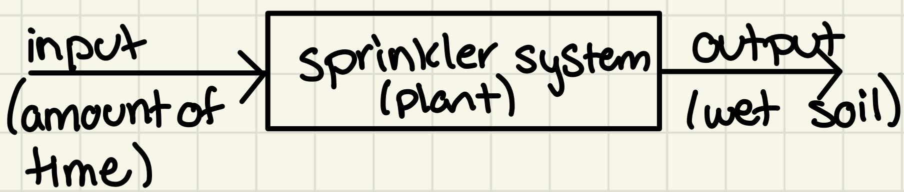
...

Open Loop Control Systems

A system in which the output has no affect on the control action is called an open-loop control system

An example is a washing machine. When you hit start, the system runs a pre-determined cycle (soak, wash, rinse) where each phase runs for a set amount of time with no feedback on the cleanliness of the clothes

A second example is a sprinkler system that always runs for 20 minutes at a pre-set time, regardless of the soil moisture, or whether it is currently raining



note that there is no sensor or controller

What do you think are pros/cons of open/closed loop?

Open-loop systems are generally simpler and cheaper, but are unable to respond to disturbances or changes in calibration

We will primarily focus on closed-loop systems in this course

A (really quick) review of differential equations

Differential Equations are a useful tool that can be used to represent the behavior of many real systems

- Newton's 2nd Law: $F \propto \frac{d(mv)}{dt}$
- Conservation Eqns (thermo): $\dot{m}_{in} = \dot{m}_{out}$

$$\left.\frac{d(m\vec{u})}{dt}\right|_{CM} = \sum F_{CV} \leftarrow \text{Newton}$$

$$\left.\frac{dE}{dt}\right|_{CM} = \dot{Q}_{in} - \dot{W}_{out}$$

- Kirchoff's voltage law $\quad V(t) = L\frac{di(t)}{dt}$
- Heat transfer
- Radioactive decay
- Lots more!

Differential Equations have two solution forms
- Exponetial e^{at}
- Sinusoidal $\sin(at)$ / $e^{-j\omega t}$

You will only have combinations of these in your solutions

how we often think about them

We can talk about these solutions in the time domain or in the frequency domain (since we have exponetial terms for our systems, we'll use the s-domain representaion of the frequency domain)

often useful for analysis

To analyze and control real systems, we first must be proficient at representing them in both the time and frequency domains and be comfortable switching between them

This course covers both analysis & design of control sysems

- analysis - determining the performance
- design - creating the desired performance

In both cases, we need to be able to predict a system's dynamic response

exponential piece

- transient response - The part of the response curve due to the way the system aquires and dissapates energy. In a stable system it is the part of the response that occurs before a steady state is reached
  - ⇒ this is what happens in between the initial state and the final state

- Steady state response - in a stable system, this is what remains after the transients have decayed. This is the state the system "settles into" after a disturbance

Total Response = Natural Response + Forced Response

(homogenous solns) (particular solutions)

⇑ a result of how the system aquires and dissipates energy. Dependant on system, not input

⇑ dependant on input

Control Systems need to stable. In other words, we need the natural response to dissapate (be transient) so that the steady state response is the forced response.

This is our design objective.

Linear, Time Invariant Systems

In this course, we will work often with Linear, Time Invariant (LTI) systems

There are two reasons for this:

1) Many real systems can be accurately approximated with LTI models
2) We can solve them mathematically!

Let's talk about what this really means:

Linear Systems: A system is said to be linear if it meets the criteria of

a) Homogeneity - If you scale the input by some factor c, the output will also be scaled by c

$$\text{if } x(t) \rightarrow \boxed{\text{LTI}} \rightarrow y(t) \quad \text{then } cx(t) \rightarrow \boxed{\text{LTI}} \rightarrow cy(t)$$

b) Superposition - the sum of the response to two input functions is the sum of the response to these inputs individually

$$\text{if } x_1(t) \rightarrow \boxed{\text{LTI}} \rightarrow y_1(t)$$

and

$$x_2(t) \rightarrow \boxed{\text{LTI}} \rightarrow y_2(t)$$

$$\text{then } x_1(t) + x_2(t) \rightarrow \boxed{\text{LTI}} \rightarrow y_1(t) + y_2(t)$$

this means coefficients are constants or functions only of the independent variable (time in our case)

but

Time Invariance: A system responds to an input the same way no matter when the input is given → the same input translated in time gives same output translated in time

$$\text{if } x(t) \rightarrow \boxed{\text{LTI}} \rightarrow y(t) \Rightarrow x(t+a) \rightarrow \boxed{\text{LTI}} \rightarrow y(t+a)$$

This means the coefficients of the differential equations don't vary with time

$$\boxed{\text{LTI System} \equiv \text{Constant Coefficient Diff Eq}}$$

Linear, time varying system

$$\dot{x}(t) = A(t)\,x(t) + B(t)u(t)$$
$$y(t) = C(t)\,x(t) + D(t)\,u(t)$$

⇐ can often linearize

LTI system

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

⇐ easier to solve