Question1: (a) and (b)

Optimization Methods HW4. Student: Ang Zhou

Question 1:

(a) ①

$$\begin{array}{l}
\qquad\qquad\qquad\qquad u = z \\
\hline
-2x_1 - x_2 - x_3 + m_1 \qquad\qquad = -2 + u \\
2x_1 - x_2 + x_3 \qquad + m_2 \quad = 1 + u
\end{array}$$

②

$$\begin{array}{l}
\qquad\qquad\qquad\qquad\qquad u = z \\
\hline
-2x_1 - x_2 - x_3 + m_1 \qquad\qquad - u = -2 \\
2x_1 - x_2 + x_3 \qquad\qquad + m_2 - u = 1
\end{array}$$

$+1$, $\times(-1)$, -1

③ Since $-2 > 1$,

$$\begin{array}{l}
-2x_1 - x_2 - x_3 + m_1 \qquad\qquad = z - 2 \\
\hline
2x_1 + x_2 + x_3 - m_1 \qquad + u = 2 \\
4x_1 \qquad + 2x_3 - m_1 + m_2 \quad = 3
\end{array}$$

$(-\frac{1}{2})$, $+\frac{1}{2}$, $(\times\frac{1}{4})$

Feasible solution for phase-1 L.P.:

$u = 2,\ m_2 = 3,\ x_1 = x_2 = x_3 = m_1 = 0,\ z = 2$

Increase x_1 to decrease z:

$x_1 = \min\{2/2,\ 3/4\} = 3/4$.

④

$$\begin{array}{l}
-x_2 \qquad + (1/2)m_1 + (1/2)m_2 \qquad = z - \frac{1}{2} \\
\hline
+x_2 \qquad - (1/2)m_1 - (1/2)m_2 + u = 1/2 \\
x_1 \qquad + (\frac{1}{2})x_3 - (1/4)m_1 + (1/4)m_2 \quad = 3/4
\end{array}$$

1

Feasible solution for phase-1 L.P.:

$u = 1/2,\ x_1 = 3/4,\ x_2 = x_3 = m_1 = m_2 = 0.\ z = 1/2$

Increase x_2 to decrease z:

$x_2 = \min\{1/2,\ +\infty\} = 1/2$.

⑤

$$\begin{array}{l}
\qquad\qquad\qquad\qquad\qquad u = z \\
\hline
x_2 \qquad -(1/2)m_1 - (1/2)m_2 + u = 1/2 \\
x_1 \qquad + (\frac{1}{2})x_3 - (\frac{1}{4})m_1 + (1/4)m_2 \quad = 3/4
\end{array}$$

Feasible solution for phase-1 L.P.:

$x_1 = 3/4,\ x_2 = 1/2,\ x_3 = m_1 = m_2 = u = 0,\ z = 0$.

Since the objective function for phase-1 L.P. is 0, we find the feasible solution for the original L.P. that we want to solve:

$\boxed{x_1 = 3/4,\ x_2 = 1/2,\ x_3 = 0,\ m_1 = m_2 = 0}$ //end of part (a)

(b) phase-1 to original ⇒

⑥

$$\begin{array}{l}
2x_1 - 6x_2 + 2x_3 \qquad\qquad = z \\
\hline
x_2 \qquad -\frac{1}{2}m_1 - \frac{1}{2}m_2 = 1/2 \\
x_1 \qquad + \frac{1}{2}x_3 - \frac{1}{4}m_1 + \frac{1}{4}m_2 = 3/4
\end{array}$$

6, -2

Since x_1, x_2 are half basic variables, we make it to basic variables ⇒ table ⑦.

⑦

$$\begin{array}{l}
x_3 - \frac{5}{2}m_1 - \frac{7}{2}m_2 = z + 3/2 \\
\hline
x_2 \qquad -\frac{1}{2}m_1 - \frac{1}{2}m_2 = 1/2 \\
x_1 \qquad + \frac{1}{2}x_3 - \frac{1}{4}m_1 + \frac{1}{4}m_2 = 3/4
\end{array}$$

-2, $\times 2$

Feasible Solution: $x_1 = 3/4,\ x_2 = 1/2,\ x_3 = m_1 = m_2 = 0,\ z = -3/2$

Increase x_3 to increase z:

$x_3 = \min\{\infty,\ \frac{3}{4}/\frac{1}{2}\} = 3/2$

⑧

$$\begin{array}{l}
-2x_1 \qquad\quad -2m_1 - 4m_2 = z \\
\hline
\qquad x_2 \qquad -\frac{1}{2}m_1 - \frac{1}{2}m_2 = 1/2 \\
2x_1 \quad x_3 \quad -\frac{1}{2}m_1 + \frac{1}{2}m_2 = 3/2
\end{array}$$

Feasible Solution: $x_2 = 1/2,\ x_3 = 3/2,\ x_1 = m_1 = m_2 = 0,\ z = 0$.

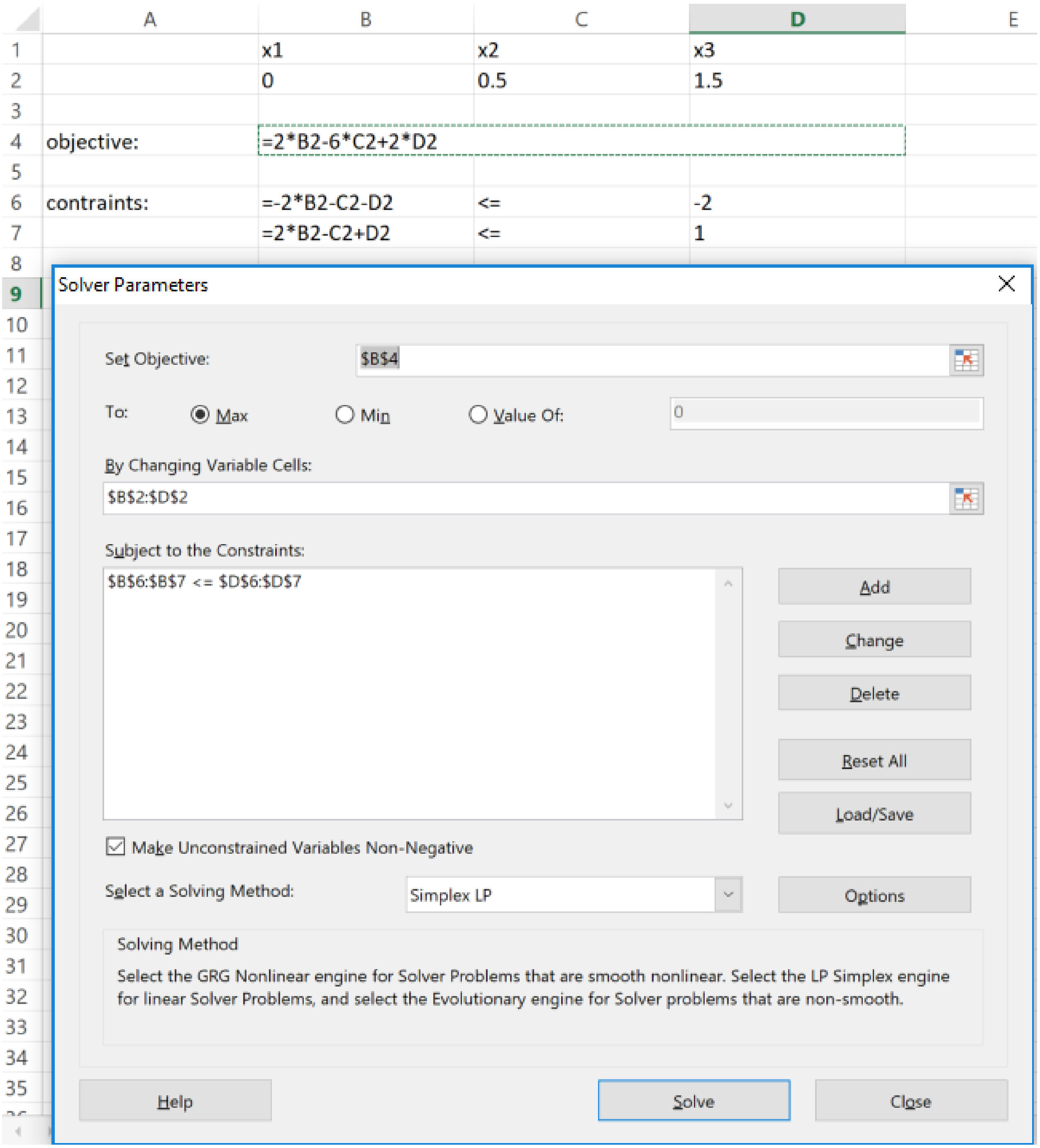
Since there is no varibles with possitive coefficient in the objective function anymore. We reach the optimal feasible solution to this L.P., which is:

$x_2 = 1/2,\ x_3 = 3/2,\ x_1 = m_1 = m_2 = 0$,

with optimal value: 0.

Solution!! for part (b)

Question1: (c)


A
B
C
D
E
x1
x2
x3
0
0.5
1.5
objective:
=2*B2-6*C2+2*D2
contraints:
=-2*B2-C2-D2
<=
-2
=2*B2-C2+D2
<=
1
Solver Parameters
Set Objective:
$B$4
To:
Max
Min
Value Of:
0
By Changing Variable Cells:
$B$2:$D$2
Subject to the Constraints:
$B$6:$B$7 <= $D$6:$D$7
Add
Change
Delete
Reset All
Load/Save
Make Unconstrained Variables Non-Negative
Select a Solving Method:
Simplex LP
Options
Solving Method
Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.
Help
Solve
Close

Question 2: (a) and (b)

Question 2

(a) ①

$$\begin{array}{lr}
 & u = z \quad \\
\hline
4x_1 + x_2 + w_1 \qquad\qquad - u = 10 & +1 \\
-x_1 + x_2 \quad + w_2 \qquad - u = -1 & -1 \\
-x_1 - x_2 \qquad\quad + w_3 - u = -3 & -1 \quad \times(-1)
\end{array}$$

$x_1, x_2, w_1, w_2, w_3, u \geq 0.$

②

$$\begin{array}{lr}
-x_1 - x_2 \qquad + w_3 \qquad = z - 3 & \\
\hline
5x_1 + 2x_2 + w_1 \quad - w_3 \qquad = 13 & -1 \quad \tfrac{1}{2} \\
2x_2 \quad + w_2 - w_3 \qquad = 2 & -\tfrac{1}{2} \quad \times(1/2) \\
x_1 + x_2 \qquad - w_3 + u = 3 &
\end{array}$$

Increase x_2 to decrease z:

$x_2 = \min\{13/2, 2/2, 3/1\} = 1.$

③

$$\begin{array}{lr}
-x_1 \qquad + (1/2)w_2 + \tfrac{1}{2}w_3 \qquad = z - 2 & \\
\hline
5x_1 \quad + w_1 - w_2 \qquad = 11 & +1 \\
x_2 \quad + (\tfrac{1}{2})w_2 - \tfrac{1}{2}w_3 \qquad = 1 & -5 \\
x_1 \qquad - \tfrac{1}{2}w_2 - \tfrac{1}{2}w_3 + u = 2 &
\end{array}$$

Increase x_1 to decrease z:

$x_1 = \min\{11/5, \infty, 2\} = 2$

④

$$\begin{array}{l}
\qquad\qquad\qquad u = z \\
\hline
+ w_1 + \tfrac{3}{2}w_2 + \tfrac{5}{2}w_3 - 5u = 1 \\
x_2 \quad + \tfrac{1}{2}w_2 - \tfrac{1}{2}w_3 \qquad = 1 \\
x_1 \quad - \tfrac{1}{2}w_2 - \tfrac{1}{2}w_3 + u = 2
\end{array}$$

Feasible Solution:

$x_1 = 2,\ x_2 = 1,\ w_1 = 1,\ w_2 = w_3 = 0,\ u = z = 0.$

Since $u = 0$, we find the ~~feasible solution for phase~~ optimal solution for phase-1 L.P. therefore, we find the feasible solution for the original L.P., which is:

$x_1 = 2,\ x_2 = 1,\ w_1 = 1,\ w_2 = w_3 = 0.$

(b) ⑤

$$\begin{array}{lr}
x_2 \qquad\qquad = z & \\
\hline
w_1 + \tfrac{3}{2}w_2 + \tfrac{5}{2}w_3 = 1 & -1 \\
x_2 \quad + \tfrac{1}{2}w_2 - \tfrac{1}{2}w_3 = 1 & \\
x_1 \quad - \tfrac{1}{2}w_2 - \tfrac{1}{2}w_3 = 2 &
\end{array}$$

Since x_2 is half basic variable, we make it into basic variable.

⑥

$$\begin{array}{lr}
-\tfrac{1}{2}w_2 + \tfrac{1}{2}w_3 = z - 1 & -1/5 \quad \times\tfrac{2}{5} \\
\hline
w_1 + \tfrac{3}{2}w_2 + \tfrac{5}{2}w_3 = 1 & 1/5 \quad 1/5 \\
x_2 \quad + \tfrac{1}{2}w_2 - \tfrac{1}{2}w_3 = 1 & \\
x_1 \quad - \tfrac{1}{2}w_2 - \tfrac{1}{2}w_3 = 2 &
\end{array}$$

Increase w_3 to increase z:

$w_3 = \min\{1/\tfrac{5}{2}, \infty, \infty\} = \tfrac{2}{5}$

⑦

$$\begin{array}{l}
-\tfrac{1}{5}w_1 - \tfrac{4}{5}w_2 \qquad = z - \tfrac{6}{5} \\
\hline
\tfrac{2}{5}w_1 + \tfrac{3}{5}w_2 + w_3 = \tfrac{2}{5} \\
x_2 + \tfrac{1}{5}w_1 + \tfrac{4}{5}w_2 \qquad = \tfrac{6}{5} \\
x_1 \quad + \tfrac{1}{5}w_1 - \tfrac{1}{5}w_2 \qquad = \tfrac{11}{5}
\end{array}$$

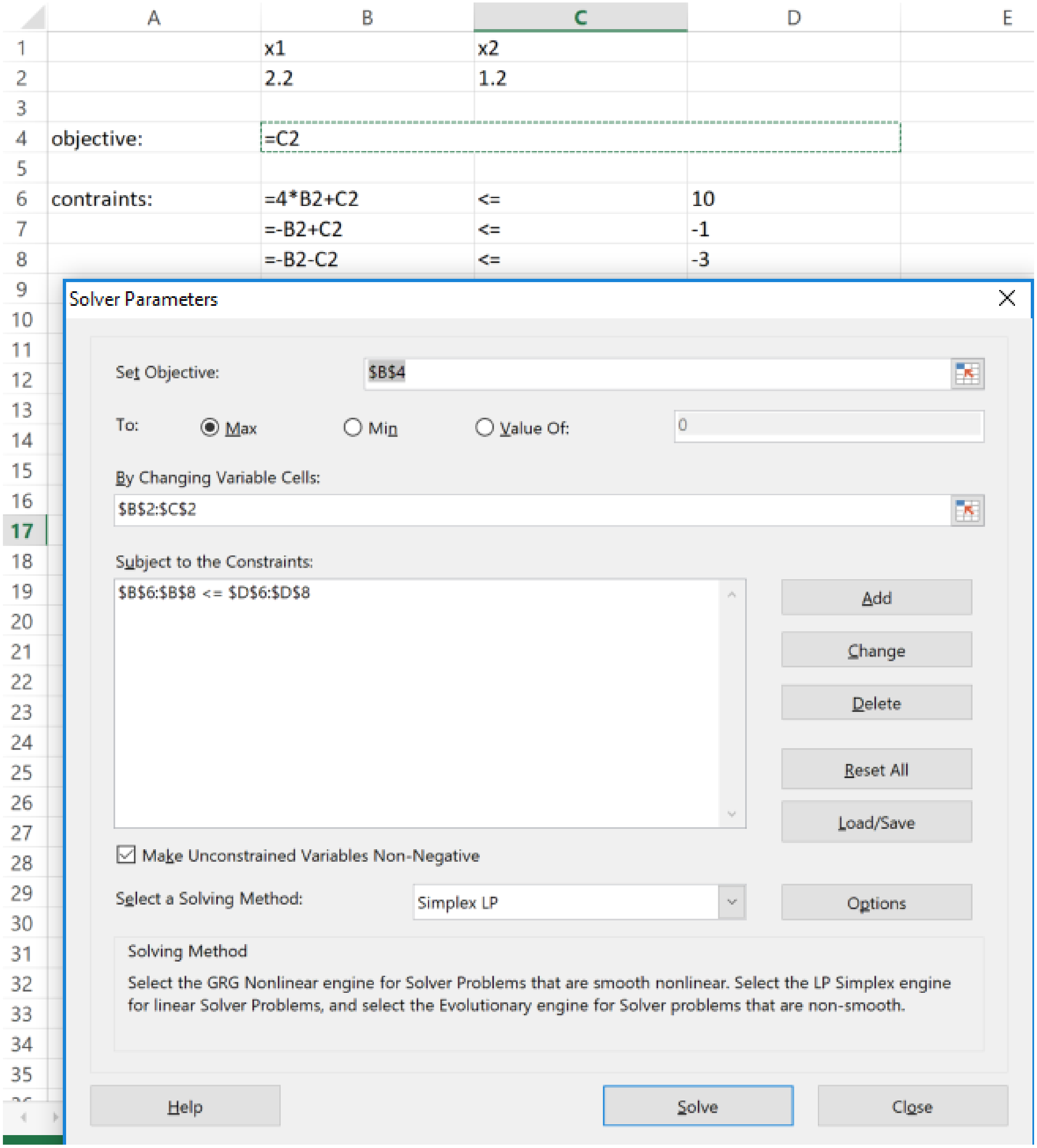
Feasible solution:

$x_1 = \tfrac{11}{5},\ x_2 = \tfrac{6}{5},\ w_3 = \tfrac{2}{5},\ w_1 = w_2 = 0,\ z = \tfrac{6}{5}$

Since there is no variables with positive coefficient in the objective function any more, we conclude that we find the optimal feasible solution:

$x_1 = \tfrac{11}{5},\ x_2 = \tfrac{6}{5}$, where the optimal value is $\tfrac{6}{5}$.

Question (c):


A
B
C
D
E
x1
x2
2.2
1.2
objective:
=C2
contraints:
=4*B2+C2
<=
10
=-B2+C2
<=
-1
=-B2-C2
<=
-3
Solver Parameters
Set Objective:
$B$4
To:
Max
Min
Value Of:
0
By Changing Variable Cells:
$B$2:$C$2
Subject to the Constraints:
$B$6:$B$8 <= $D$6:$D$8
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