Question 1: HW3 Student: Ang Zhou

Question 1.

maximize $6x_1 + 8x_2 + 5x_3 + 9x_4$.

subject to
$$2x_1 + x_2 + x_3 + 3x_4 \le 5$$
$$x_1 + 3x_2 + x_3 + 2x_4 \le 3$$
$$x_1, x_2, x_3, x_4 \ge 0.$$

①
$$\begin{array}{l}
6x_1 + 8x_2 + 5x_3 + 9x_4 = z \qquad -9/2 \\
\hline
2x_1 + x_2 + x_3 + 3x_4 + w_1 = 5 \qquad -3/2 \\
x_1 + 3x_2 + x_3 + 2x_4 + w_2 = 3 \qquad \times 1/2
\end{array}$$

Initial feasible solution:
$x_1 = x_2 = x_3 = x_4 = 0$; $w_1 = 5$, $w_2 = 3$

Increase x_4 to increase z:
$x_4 = \min\{5/3, 3/2\} = 3/2$

Row operations:

②
$$\begin{array}{l}
3/2\, x_1 - 1/2\, x_2 + 1/2\, x_3 - 9/2\, w_2 = z - 27/2 \qquad -3 \\
\hline
1/2\, x_1 - 7/2\, x_2 - 1/2\, x_3 + w_1 - 3/2\, w_2 = 1/2 \qquad \times 2 \\
1/2\, x_1 + 3/2\, x_2 + 1/2\, x_3 + x_4 + 1/2\, w_2 = 3/2 \qquad -1
\end{array}$$

feasible solution: $w_1 = 1/2$, $x_4 = 3/2$, $z = 27/2$, others $= 0$.

Increase x_1 to increase z:
$x_1 = \min\{\frac{1}{2}/\frac{1}{2}, \frac{3}{2}/\frac{1}{2}\} = 1$.

Row operation:

③
$$\begin{array}{l}
5x_2 + 2x_3 - 3w_1 = z - 15 \qquad -1 \\
\hline
x_1 - 7x_2 - x_3 + 2w_1 - 3w_2 = 1 \qquad 7/5 \\
5x_2 + x_3 + x_4 - w_1 + 2w_2 = 1 \qquad \times 1/5
\end{array}$$

feasible solution: $x_1 = 1$, $x_4 = 1$, $z = 15$, others $= 0$.

Increase x_2 to increase z:
$x_2 = \min\{\infty, 1/5\} = 1/5$

Row operation:

④
$$\begin{array}{l}
x_3 - x_4 - 2w_1 - 2w_2 = z - 16 \qquad -5 \\
\hline
x_1 + 3/5\, x_3 + 7/5\, x_4 + 3/5\, w_1 - 1/5\, w_2 = 12/5 \qquad -2 \\
x_2 + 1/5\, x_3 + 1/5\, x_4 - 1/5\, w_1 + 3/5\, w_2 = 1/5 \qquad \times 5
\end{array}$$

feasible solution: $x_1 = 12/5$, $x_2 = 1/5$, $z = 16$, others $= 0$.

Increase x_3 to increase z:
$x_3 = \min\{\frac{12}{5}/\frac{3}{5}, \frac{1}{5}/\frac{1}{5}\} = 1$.

Row operation:

⑤
$$\begin{array}{l}
-5x_2 - 2x_4 - 3w_1 - 4w_2 = z - 17 \\
\hline
x_1 - 2x_2 + x_4 + w_1 - w_2 = 2 \\
5x_2 + x_3 + x_4 - w_1 + 2w_2 = 1
\end{array}$$

Feasible solution: $x_1 = 2$, $x_3 = 1$, $z = 17$, others $= 0$.

Since there is no variable with positive coefficient in the objective function any more, this feasible solution should be the optimal solution, providing the optimal objective value 17 for the L.P. where the solution is $x_1 = 2$, $x_2 = 0$, $x_3 = 1$, $x_4 = 0$.

Question 2:

Question 2 :

①

$$\begin{array}{rrrrrrrl}
x_1 & +2x_2 & +x_3 & & & & & = z \\
\hline
x_1 & & & +w_1 & & & & = 3 \\
 & x_2 & & & +w_2 & & & = 3 \\
x_1 & +x_2 & & & & +w_3 & & = 4 \\
-x_1 & & +x_3 & & & & +w_4 & = 1
\end{array}$$

Feasible solution:

$x_1=x_2=x_3=0$, $w_1=3, w_2=3, w_3=4, w_4=1$, $z=0$.

Increase x_2 to increase z.

$x_2 = \min\{3, 4\} = 3$.

Row operation:

②

$$\begin{array}{rrrrrrrl}
x_1 & & +x_3 & & -2w_2 & & & = z-6 \\
\hline
x_1 & & & +w_1 & & & & = 3 \\
 & x_2 & & & +w_2 & & & = 3 \\
x_1 & & & & -w_2 & +w_3 & & = 1 \\
-x_1 & & +x_3 & & & & +w_4 & = 1
\end{array}$$

Feasible solution:

$x_1=x_3=w_2=0$, $x_2=3, w_1=3, w_3=1, w_4=1, z=6$.

Increase x_1 to increase z.

$x_1 = \min\{3, 1, \infty\} = 1$.

Row operation:

③

$$\begin{array}{rrrrrrrl}
 & & +x_3 & & -w_2 & -w_3 & & = z-7 \\
\hline
 & & & +w_1 & +w_2 & -w_3 & & = 2 \\
 & x_2 & & & +w_2 & & & = 3 \\
x_1 & & & & -w_2 & +w_3 & & = 1 \\
 & & x_3 & & -w_2 & +w_3 & +w_4 & = 2
\end{array}$$

Feasible solution:

$x_3=w_2=w_3=0$. $x_1=1$, $x_2=3$, $w_1=2$, $w_4=2$, $z=7$

Increase x_3 to increase z:

$x_3 = \min\{2\} = 2$.

Row operation:

④

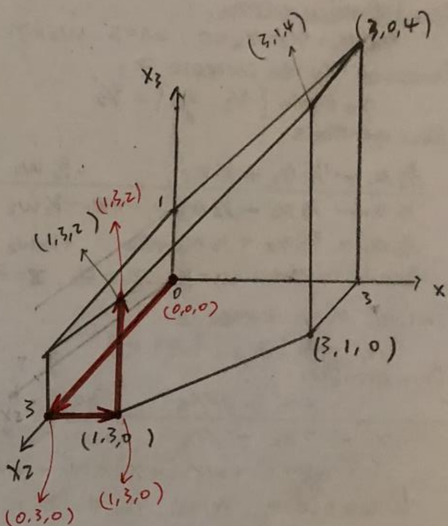
$$\begin{array}{rrrrrrrl}
 & & & & & -2w_3 & -w_4 & = z-9 \\
\hline
 & & & +w_1 & +w_2 & -w_3 & & = 2 \\
 & x_2 & & & +w_2 & & & = 3 \\
x_1 & & & & -w_2 & +w_3 & & = 1 \\
 & & x_3 & & -w_2 & +w_3 & +w_4 & = 2
\end{array}$$

Feasible solution:

$w_2=w_3=w_4=0$, $x_1=1, x_2=3, x_3=2, w_1=2, z=9$.

Since there is no more variables with positive coefficient in the objective function any more, we concludes that the optimal solution is:

$x_1=1$, $x_2=3$, $x_3=2$, with optimal objective value equals to 9.



Question 3:

Question 3.

(a) ①

$$\begin{array}{lllll}
5x_1 + 8x_2 & & & & = z \\
\hline
4x_1 + 2x_2 & + w_1 & & & = 80 \\
-x_1 + 2x_2 & & + w_2 & & = 20 \\
4x_1 - x_2 & & & + w_3 & = 40.
\end{array}
\qquad -4,\ -1,\ \times 1/2,\ +1/2$$

Feasible solution:

$x_1 = 0.\ x_2 = 0.\ w_1 = 80.\ w_2 = 20.\ w_3 = 40.\ z = 0.$

Increase x_2 to increase z:

$x_2 = \min\{80/2,\ 20/2,\ \infty\} = 10.$

Row operation:

②

$$\begin{array}{lllll}
9x_1 & & -4w_2 & & = z - 80 \\
\hline
5x_1 & + w_1 & - w_2 & & = 60. \\
-1/2\, x_1 + x_2 & & + 1/2\, w_2 & & = 10. \\
7/2\, x_1 & & + 1/2\, w_2 & + w_3 & = 50.
\end{array}
\qquad -9/5,\ \times 1/5,\ 1/10,\ -7/10$$

Feasible solution:

$x_1 = w_2 = 0.\ x_2 = 10,\ w_1 = 60,\ w_3 = 50,\ z = 80$

Increase x_1 to increase z:

$x_1 = \min\{60/5,\ \infty,\ 50/\tfrac{7}{2}\} = 12.$

Row operation:

③

$$\begin{array}{llll}
 & -9/5\, w_1 - 11/5\, w_2 & & = z - 188 \\
\hline
x_1 & + 1/5\, w_1 - 1/5\, w_2 & & = 12 \\
x_2 & + 1/10\, w_1 + 3/5\, w_2 & & = 16 \\
 & -7/10\, w_1 + 6/5\, w_2 & + w_3 & = 8
\end{array}$$

Feasible solution:

$x_1 = 12,\ x_2 = 16.\ w_1 = 0,\ w_2 = 0,\ w_3 = 8.\ z = 188$

Since there is no variables with positive coefficient in the objective function anymore, We conclude that the optimal solution is:

$x_1 = 12.\ x_2 = 16$, where the optimal objective value is 188.

(b)