

ORIE 5380, CS 5727: Optimization Methods
Homework Assignment 2
Due September 14, 12:00 pm

Please submit a single PDF document formatted to print. If you are doing any work in Excel, then please put screenshots of your spreadsheet in this PDF document. Make sure that your screenshots show the formulas in your spreadsheet. Note that CTRL and ~ key combination (Windows) or COMMAND and ~ key combination (OS X) displays the formulas in an Excel spreadsheet. Show all your work clearly.

Question 1

A call center operates 6 days a week from Monday to Saturday. The agents working at the call center either work 3 days a week or 4 days a week. Agents working 3 days a week work on 3 consecutive days in the week. Agents working 4 days a week work for 2 consecutive days, take at least 1 day of break and work for another 2 consecutive days.

There are two skill levels for the agents, high-skill and low-skill. High-skill agents can do the work that low-skill agents can do, but not the other way around. A high-skill agent working 3 days a week gets paid \$450 per week, whereas a high-skill agent working 4 days a week gets paid \$540 per week. A low-skill agent working 3 days a week gets paid \$300 per week, whereas a low-skill agent working 4 days a week gets paid \$360 per week.

The following table shows the number of high-skill agents that the call center needs on each day of the week.

Day	M	T	W	R	F	S
# High-Skill Agents	60	90	40	30	40	80

The following table shows the total number of agents (high-skill and low-skill) that the call center needs on each day of the week.

Day	M	T	W	R	F	S
# All Agents	100	120	90	140	150	120

- List all possible schedules that the agents could work in a particular week. For example, one schedule for an agent that works for 4 days a week is M-T-R-F, taking a break on Wednesday.
- Formulate a linear program to find the optimal hiring decisions for the call center that minimizes the labor cost per week. (Hint: Use decision variables that correspond to the number of high-skill and low-skill agents hired according to each schedule. Clearly state your decision variables, objective function and constraints.)
- Find the optimal solution by using Excel's solver. Report the optimal hiring decisions.

Question 2

(This problem is from Hillier and Lieberman.)

D 3.4-7. Consider the following problem, where the value of c_1 has not yet been ascertained.

$$\text{Maximize } Z = c_1x_1 + 2x_2,$$

subject to

$$4x_1 + x_2 \leq 12$$

$$x_1 - x_2 \geq 2$$

and

$$x_1 \geq 0, \quad x_2 \geq 0.$$

a) Plot the set of feasible solutions for this linear program.

b) Using the graphical method for solving linear programs, state the optimal solution to the linear program for all possible values of c_1 and indicate the appropriate ranges for c_1 that would render each one of these solutions optimal. (You may want to use the fact that the optimal solution to a linear program always occurs at a corner point of the set of feasible solutions.)

Question 3

Consider the linear program

$$\text{minimize } x_1 + x_2$$

$$\text{st } -x_1 + x_2 \geq 10$$

$$-2x_1 + x_2 \leq 20$$

$$x_1 \geq 0, x_2 \geq 0.$$

a) Plot the set of feasible solutions for this linear program and find the optimal solution by using the graphical method for solving linear programs.

b) Does the linear program have a finite optimal solution when we change the objective function to minimize $x_1 - x_2$? Explain your answer by using the graphical method for solving linear programs.

Question 4

Use row operations to compute the inverse of the matrix

$$\begin{bmatrix} 5 & 0 & -2 \\ 1 & -2 & 2 \\ 2 & -1 & 0 \end{bmatrix}$$

Show all your work.