ORIE 5380, CS 5727: Optimization Methods
Homework Assignment 3
Due September 21, 12:00 pm

Please submit a single PDF document formatted to print and show all your work clearly.

Question 1

(This problem is from Vanderbei)

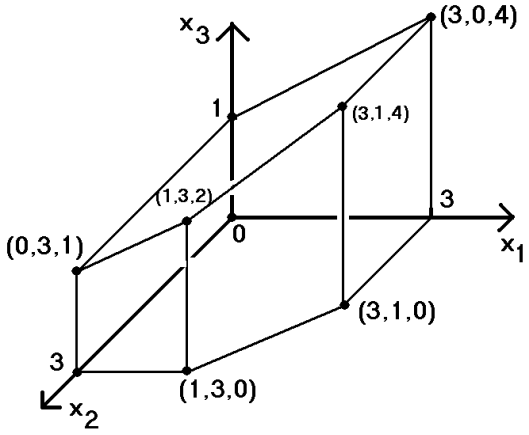
Solve the linear program below by using the simplex method. Show all systems of equations that the simplex method visits and state the optimal solution you reach.

$$\begin{aligned} \text{maximize} \quad & 6x_1 + 8x_2 + 5x_3 + 9x_4 \\ \text{subject to} \quad & 2x_1 + x_2 + x_3 + 3x_4 \leq 5 \\ & x_1 + 3x_2 + x_3 + 2x_4 \leq 3 \\ & x_1,\ x_2,\ x_3,\ x_4 \geq 0. \end{aligned}$$

Question 2

Below are a linear program and a graph for its set of feasible solutions.

$$\begin{array}{lrcrcrcl} \text{maximize} & x_1 & + & 2x_2 & + & x_3 & & \\ \text{subject to} & x_1 & & & & & \leq & 3 \\ & & & x_2 & & & \leq & 3 \\ & x_1 & + & x_2 & & & \leq & 4 \\ - & x_1 & & & + & x_3 & \leq & 1 \\ & x_1 & , & x_2 & , & x_3 & \geq & 0. \end{array}$$



Use the simplex method to solve the linear program. Show all systems of equations that the simplex method visits and state the optimal solution you reach. Use the figure above to mark *each* solution that the simplex method visits on its way to the optimal solution. (If there is a tie in terms of which variable you want to increase in the simplex method, you can pick one arbitrarily.)

(There is one more question on the next page.)

Question 3

(This problem is partially from Hillier and Lieberman)

a) Solve the linear program below by using the simplex method. Show all systems of equations that the simplex method visits and state the optimal solution you reach.

maximize	$5\,x_1 + 8\,x_2$
st	$4\,x_1 + 2\,x_2 \leq 80$
	$-x_1 + 2\,x_2 \leq 20$
	$4\,x_1 - x_2 \leq 40$
	$x_1 \geq 0,\ x_2 \geq 0.$

b) Plot the set of feasible solutions to the linear program above and mark *each* solution that the simplex method visits on its way to the optimal solution.