Question 1.

①
$$\begin{array}{l}
-2x_1+2x_2-x_3+3x_4 = z \\
\hline
-3x_1+x_2+4x_3+x_4+w_1 = 0 \quad -3 \\
3x_1-x_2-3x_3-2x_4+w_2 = 3 \quad 2
\end{array}$$

Feasible solution: $w_1=0, w_2=3, x_1=x_2=x_3=x_4=0, z=0$

Increase x_4 to increase z.

$x_4=\min\{0,\infty\}=0$.

②
$$\begin{array}{l}
7x_1-x_2-13x_3-3w_1 = z \\
\hline
-3x_1+x_2+4x_3+x_4+w_1 = 0 \\
-3x_1+x_2+5x_3+2w_1+w_2 = 3
\end{array}$$

Feasible solution:

$x_4=0, w_2=3, x_1=x_2=x_3=w_1=0$.

Increase x_1 to increase z:

$x_1=\{\infty,\infty\}=\infty$

$\Rightarrow$ the L.P. is unbounded.

for some $t\geq 0$. consider solution:

$x_1=t,\ x_4=3t,\ w_2=3+3t,\ x_2=x_3=w_1=0$.

$z=7t \Rightarrow$ arbitrarily large.

so L.P. is unbounded.

Question 2.

①
$$\begin{array}{l}
5x_1+7x_2-12x_3-10x_4 = z \\
\hline
2x_1-2x_2-3x_3-2x_4+w_1 = 6 \quad -\frac{7}{5} \\
2x_1+5x_2-4x_3-4x_4+w_2 = 3 \quad 2/5 \quad \times 1/5
\end{array}$$

Feasible solution: $w_1=6, w_2=3, x_1=x_2=x_3=x_4=0, z=0$.

Increase x_2 to increase z:

$x_2=\min\{\infty, 3/5\}=3/5$

②
$$\begin{array}{l}
\frac{11}{5}x_1-\frac{32}{5}x_3-\frac{22}{5}x_4-\frac{7}{5}w_2 = z-\frac{21}{5} \\
\hline
\frac{14}{5}x_1-\frac{23}{5}x_3-\frac{18}{5}x_4+w_1+\frac{2}{5}w_2 = \frac{36}{5} \quad -7 \quad -\frac{11}{2} \\
\frac{2}{5}x_1+x_2-\frac{4}{5}x_3-\frac{4}{5}x_4+\frac{1}{5}w_2 = \frac{3}{5} \quad \times\frac{5}{2}
\end{array}$$

Feasible solution: $x_2=\frac{3}{5}, w_1=\frac{36}{5}, x_1=x_3=x_4=w_2=0$.

Increase x_1 to increase z:

$x_1=\min\{\frac{36}{5}/\frac{14}{5}, \frac{3}{5}/\frac{2}{5}\}=\frac{3}{2}$.

③
$$\begin{array}{l}
-\frac{11}{2}x_2-2x_3-\frac{5}{2}w_2 = z-\frac{15}{2} \\
\hline
-7x_2+x_3+2x_4+w_1-w_2 = 3 \quad +1 \quad \times\frac{1}{2} \\
x_1+\frac{5}{2}x_2-2x_3-2x_4+2w_2 = \frac{3}{2}
\end{array}$$

Feasible solution: $x_1=\frac{3}{2}, w_1=3, x_2=x_3=x_4=w_2=0, z=\frac{15}{2}$

No positive coefficient in objective function. But x_4 has 0 coefficient. $\Rightarrow$ has alternative optimal solution:

Increase x_4: $x_4=\min\{3/2,\infty\}=3/2$.

④
$$\begin{array}{l}
-\frac{11}{2}x_2-2x_3-\frac{5}{2}w_2 = z-\frac{15}{2} \\
\hline
-\frac{7}{2}x_2+\frac{1}{2}x_3+x_4+\frac{1}{2}w_1-\frac{1}{2}w_2 = \frac{3}{2} \quad \times 2 \\
x_1-x_2-x_3+w_1+w_2 = \frac{9}{2} \quad -2
\end{array}$$

Feasible solution: $x_1=\frac{9}{2}, x_4=\frac{3}{2}, x_2=x_3=w_1=w_2=0, z=\frac{15}{2}$

No positive coefficient in objective function $\Rightarrow$ also optimal solution.

Two possible optimal solutions:

① $x_1=\frac{3}{2}, x_2=0, x_3=0, x_4=0$, optimal value: $\frac{15}{2}$

② $x_1=\frac{9}{2}, x_2=0, x_3=0, x_4=\frac{3}{2}$. optimal value: $\frac{15}{2}$

Question 3.

①
$$\begin{array}{l}
-x_1+4x_2+4x_3-x_4 = z \\
\hline
-x_1-2x_2+4x_3+2x_4+w_1 = 1 \quad 1 \quad -2 \\
2x_2+2x_3+w_2 = 3 \quad \times\frac{1}{2}
\end{array}$$

Feasible solution: $w_1=1, w_2=3, x_1=x_2=x_3=x_4=0, z=0$.

Increase x_2 to increase z:

$x_2=\min\{\infty, 3/2\}=3/2$.

②
$$\begin{array}{l}
-x_1-x_4-2w_2 = z-6 \\
\hline
-x_1+6x_3+2x_4+w_1+w_2 = 4 \quad -1/6 \quad \times 1/6 \\
x_2+x_3+\frac{1}{2}w_2 = \frac{3}{2}
\end{array}$$

Feasible solution: $x_2=\frac{3}{2}, w_1=4. x_1=x_3=x_4=w_2=0, z=6$.

No positive coefficient variable in objective function $\Rightarrow$ optimal solution.

But x_3 has coefficient 0 in objective function

$\Rightarrow$ has alternative optimal solution:

Increase x_3: $x_3=\min\{4/6, 3/2\}=2/3$

③
$$\begin{array}{l}
-x_1-x_4-2w_2 = z-6 \\
\hline
-\frac{1}{6}x_1+x_3+\frac{1}{3}x_4+\frac{1}{6}w_1+\frac{1}{6}w_2 = 2/3 \\
\frac{1}{6}x_1+x_2-\frac{1}{3}x_4-\frac{1}{6}w_1+\frac{1}{3}w_2 = 5/6
\end{array}$$

Feasible solution: $x_2=5/6, x_3=2/3, x_1=x_4=w_1=w_2=0, z=6$

No positive coefficient variable in objective function $\Rightarrow$ optimal solution.

$\Rightarrow$ Two possible optimal solutions:

① $x_2=\frac{3}{2}, x_1=x_3=x_4=0$, optimal value: 6.

② $x_2=\frac{5}{6}, x_3=\frac{2}{3}, x_1=x_4=0$, optimal value: 6.

Question 4.

①

$$-4x_1 + 19x_2 + 5x_3 = z \quad -\tfrac{19}{6}$$

$$6x_2 - x_3 + m_1 = 4 \quad \times \tfrac{1}{6}$$

$$-2x_1 + 3x_2 + 3x_3 + m_2 = 3 \quad -\tfrac{1}{2}$$

Feasible solution: $m_1 = 4$, $m_2 = 3$, $x_1 = x_2 = x_3 = 0$. $z = 0$.

Increase x_2 to increase z:

$$x_2 = \min\{4/6, 3/3\} = 2/3$$

②

$$-4x_1 + \tfrac{49}{6}x_3 - \tfrac{19}{6}m_1 = z - \tfrac{38}{3} \quad -\tfrac{7}{3}$$

$$x_2 - \tfrac{1}{6}x_3 + \tfrac{1}{6}m_1 = \tfrac{2}{3} \quad +\tfrac{1}{21}$$

$$-2x_1 + \tfrac{7}{2}x_3 - \tfrac{1}{2}m_1 + m_2 = 1 \quad \times \tfrac{2}{7}$$

Feasible solution: $x_2 = \frac{2}{3}$, $m_2 = 1$, $x_1 = x_3 = m_1 = 0$, $z = \frac{38}{3}$

Increase x_3 to increase z:

$$x_3 = \min\{\infty, 1/\tfrac{7}{2}\} = \tfrac{2}{7}$$

③

$$\tfrac{2}{3}x_1 - 2m_1 - \tfrac{7}{3}m_2 = z - 15$$

$$-\tfrac{2}{21}x_1 + x_2 + \tfrac{1}{7}m_1 + \tfrac{1}{21}m_2 = \tfrac{5}{7}$$

$$-\tfrac{4}{7}x_1 + x_3 - \tfrac{1}{7}m_1 + \tfrac{2}{7}m_2 = \tfrac{2}{7}$$

Feasible solution: $x_2 = \frac{5}{7}$, $x_3 = \frac{2}{7}$. $x_1 = m_1 = m_2 = 0$, $z = 15$.

Increase x_1 to increase z:

$$x_1 = \min\{\infty, \infty\} = \infty$$

$\Rightarrow$ L.P. is unbounded.

for some $t \geq 0$ consider solution:

$$x_1 = t,\ x_2 = \tfrac{5}{7} + \tfrac{2}{21}t,\ x_3 = \tfrac{2}{7} + \tfrac{4}{7}t,$$

$$m_1 = m_2 = 0,\ z = 15 + \tfrac{2}{3}t \Rightarrow \text{arbitrarily large.}$$

$\Rightarrow$ so. L.P. is unbounded.