ORIE 5380, CS 5727: Optimization Methods
Homework Assignment 11

Due November 26, 12:00 pm

Please submit a single PDF document formatted to print and show all your work clearly. Feel free to scan and submit handwritten work. Do not spend too much time on wordprocessing your answers.

Question 1

We are planning the locations of 3 fulfillment centers (FCs) that will be used by an online retailer over the next 20 years. The first FC will be opened immediately and will be the only operational FC for the next 6 years. After 6 years, a second FC will be opened and the online retailer will operate the two FCs concurrently for another 5 years. After a total of 11 years, a third FC will be opened and all three FCs will be operational for another 9 years. The retailer would like to choose the locations of these three FCs.

In the attached spreadsheet, the first tab shows the x and y coordinates of the demand points (DPs) that the online retailer serves. There are 20 DPs. Each DP is served by the closest FC. The yearly cost of serving a DP from a certain FC is simply the Euclidean distance between the DP and the FC. The second tab shows the x and y coordinates of the possible locations for the FCs. There are 10 possible locations for FCs.

a) Formulate an integer programming model to choose the locations of the first, second and third FCs so that the total cost over the next 20 years is minimized. State your decision variables, objective function and constraints.

b) Write a Python program that solves the integer programming model in Part a. Make sure that your Python program creates the decision variables, the objective function and the constraints in your model through loops. Solve your integer program. Accompanying this homework, there is a small Python program that you can use to plot the locations of your FCs and DPs, along with which DP is served by which FC. Use this Python program to give three plots. The first plot should show the location of the first FC and the assignment of DPs to the first FC. This is the situation faced by the company over the first 6 years. The second plot should show the locations of the first two FCs and the assignment of DPs to the first two FCs. This is the situation faced by the company over the next 5 years. Finally, the third plot should show the locations of all three FCs and the assignment of DPs to the three FCs. This is the situation faced by the company in the last 9 years.

State the optimal locations of the first, second and third FCs. Submit your Python code. After building your model in Python, output your model into a text file and submit a small representative portion of the text file for your integer programming model (e.g. first 10-20 lines). Also, submit the three plots as described above.

In Gurobi, to define a binary decision variable, you can use `vtype = GRB.BINARY` when adding a decision variable into the model. One idea to build your model is to use binary

decision variables of the form

$$x_{ik} = \begin{cases} 1 & \text{if location } i \text{ is used for the } k\text{th FC} \\ 0 & \text{otherwise,} \end{cases}$$

$$y_{ijk} = \begin{cases} 1 & \text{if FC at location } i \text{ serves DP } j \text{ when there are } k \text{ open FCs.} \\ 0 & \text{otherwise.} \end{cases}$$