Optimization Method HW9   Student: Ang Zhou

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Question 1.

(a) Primal problem:

$$\max\ 4x_1+3x_2$$
$$\text{s.t. } x_1+4x_2 \le 140 \quad (y_1)$$
$$6x_1+3x_2 \le 180 \quad (y_2)$$
$$x_2 \le 30 \quad (y_3)$$
$$x_1, x_2 \ge 0$$

$\Rightarrow$ dual problem:

$$\min\ 140y_1+180y_2+30y_3$$
$$\text{s.t. } y_1+6y_2 \ge 4$$
$$4y_1+3y_2+y_3 \ge 3$$
$$y_1, y_2, y_3 \ge 0.$$

(b) ①

$$\begin{array}{l}
4x_1+3x_2 = z \\ \hline
x_1+4x_2+w_1 = 140 \\
6x_1+3x_2 + w_2 = 180 \\
x_2 + w_3 = 30
\end{array}
\xrightarrow{-\frac{2}{3},\ -\frac{1}{6}}
\begin{array}{l} w_1=140,\ w_2=180,\ w_3=30 \\ x_1=x_2=0,\ z=0. \end{array}$$

increasing x_1 to $\min\{140, 180/6\}=30$.

②

$$\begin{array}{l}
x_2 - \frac{2}{3}w_2 = z-120 \\ \hline
\frac{7}{2}x_2 + w_1 - \frac{1}{6}w_2 = 110 \\
x_1+\frac{1}{2}x_2 + \frac{1}{6}w_2 = 30 \\
x_2 + w_3 = 30
\end{array}
\xrightarrow{-\frac{2}{7},\ -\frac{1}{2}}
\begin{array}{l} w_1=110,\ x_1=30 \\ w_3=30 \\ x_2=w_2=0,\ z=120 \end{array}$$

increasing x_2 to $\min\{110/\frac{7}{2}, 30/\frac{1}{2}, 30\}=30$.

③

$$\begin{array}{l}
-\frac{2}{3}w_2-w_3 = z-150 \\ \hline
w_1 - \frac{1}{6}w_2 - \frac{7}{2}w_3 = 5 \\
x_1 \quad + \frac{1}{6}w_2 - \frac{1}{2}w_3 = 15 \\
x_2 \quad + w_3 = 30
\end{array}$$

$w_1=5,\ x_1=15,\ x_2=30,\ w_2=w_3=0,\ z=150.$

$\Rightarrow$ The optimal solution is $(15, 30, 5, 0, 0)$.
the optimal value is 150.

(c) According to the last system of equations. the optimal of the dual problem is $(0, \frac{2}{3}, 1, 0, 0)$

(d) Feasible solution for the primal: $(15, 30, 5, 0, 0)$
with optimal value: 150

Feasible solution for the dual: $(0, \frac{2}{3}, 1, 0, 0)$
with optimal value: $140\cdot 0+180\cdot\frac{2}{3}+30$
$=120+30=150$

equal

Because of weak duality,
a_1 = any feasible objective value of the primal
$\le$ optimal value of the primal
$\le$ optimal value of the dual
$\le$ any feasible objective value of the dual $= a_2$

Since $a_1=a_2=150 \Rightarrow$ The solution in part C is indeed the optimal solution to the dual.

(e) Increasing right hand side of second constraint by a small ε, is equivalent to replace w_2 with $(w_2-\varepsilon)$.

$\Rightarrow$ End with following L.P:

$$\begin{array}{l}
-\frac{2}{3}(w_2-\varepsilon)-w_3 = z-150 \\ \hline
w_1 - \frac{1}{6}(w_2-\varepsilon) - \frac{7}{2}w_3 = 5 \\
x_1 \quad + \frac{1}{6}(w_2-\varepsilon) - \frac{1}{2}w_3 = 15 \\
x_2 \quad + w_3 = 30
\end{array}$$

$$\Rightarrow \begin{array}{l}
-\frac{2}{3}w_2-w_3 = z-150-\frac{2}{3}\varepsilon \\ \hline
w_1 - \frac{1}{6}w_2 - \frac{7}{2}w_3 = 5-\frac{1}{6}\varepsilon \\
x_1 \quad + \frac{1}{6}w_2 - \frac{1}{2}w_3 = 15+\frac{1}{6}\varepsilon \\
x_2 \quad + w_3 = 30
\end{array}$$

$w_1=5-\frac{1}{6}\varepsilon,\ x_1=15+\frac{1}{6}\varepsilon,\ x_2=30$
$w_2=w_3=0,\ z=150+\frac{2}{3}\varepsilon$

$\therefore$ the optimal objective value change is $\frac{2}{3}\varepsilon$

(f)
$$\left.\begin{array}{l}
w_1 = 5-\frac{1}{6}\varepsilon \ge 0 \\
x_1+4x_2 = 15+\frac{1}{6}\varepsilon+120 \le 140 \\
6x_1+3x_2 = 6(15+\frac{1}{6}\varepsilon)+90 \le 180+\varepsilon
\end{array}\right\} \varepsilon \le 30$$

$\Rightarrow$ The largest value of ε is 30.

(g) ε = 6, set the second constrain from 180 to 180+6 = 186.

vars	x1	x2	
	16	30	
obj(max)	=4*B2+3*C2		
Constraints	=B2+4*C2	<=	140
	=6*B2+3*C2	<=	186
	=C2	<=	30

Get optimal solution: (16,30) with objective value goes to 154.

The optimal objective value increased by 4, which matches the conclusion from Part e. (2/3)*6 = 4.

(h) ε = 36, set the second constrain from 180 to 180+36 = 216.

vars	x1	x2	
	21.1428571428571	29.7142857142857	
obj(max)	=4*B2+3*C2		
Constraints	=B2+4*C2	<=	140
	=6*B2+3*C2	<=	216
	=C2	<=	30

Get optimal solution: (21.14286, 29.71429) with optimal objective value: 173.7143

vars	x1	x2	
	21.14286	29.71429	
obj(max)	173.7143		
Constraints	140	<=	140
	216	<=	216
	29.71429	<=	30

The objective value increased by 23.7143.

Since as what is said in part f, the largest ε is 30 in order to follow the conclusion of part e. When ε is larger than 30, in this case is 36, then it will not follow the rule of the optimal objective value change in part e.