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Question 1:

(a) Dual of this L.P.

$$\min\ 240y_1 + 180y_2 + 60y_3$$
$$\text{st}\quad 2y_1 + y_2 \geq 3$$
$$y_1 + 2y_2 + y_3 \geq 4$$

(b) Prime problem: ① $x_1=0,\ x_2=0 \Rightarrow$ objective value $=0$.
② $x_1=120,\ x_2=0 \Rightarrow$ objective value $=360$
③ $x_1=0,\ x_2=60 \Rightarrow$ objective value $=240$.

Dual problem: ① $y_1=2,\ y_2=1,\ y_3=0 \Rightarrow$ objective value $=660$
② $y_1=1,\ y_2=2,\ y_3=0 \Rightarrow$ objective value $=600$
③ $y_1=0,\ y_2=3,\ y_3=0 \Rightarrow$ objective value $=540$

All three objective values of the dual are larger than all three objective values of the primal at the feasible solutions.

(c) ①

$$\begin{array}{l}
3x_1 + 4x_2 \qquad\qquad\qquad = z \\
\hline
2x_1 + x_2 + w_1 \qquad\quad = 240 \\
x_1 + 2x_2 \qquad + w_2 \quad = 180 \\
\qquad x_2 \qquad\qquad + w_3 = 60
\end{array}$$

-4, -1, -2

Increase x_2 to increase z:
$x_2 = \min\{240,\ 180/2,\ 60\} = 60$.

②

$$\begin{array}{l}
3x_1 \qquad\qquad -4w_3 = z - 240 \\
\hline
2x_1 \quad + w_1 \quad - w_3 = 180 \\
x_1 \qquad + w_2 - 2w_3 = 60 \\
\quad x_2 \qquad\quad + w_3 = 60.
\end{array}$$

-3, -2

Feasible solution: $x_2=60,\ w_1=180,\ w_2=60$
$x_1=w_3=0,\ z=240$

Increase x_1 to increase z.
$x_1 = \min\{180/2,\ 60,\ +\infty\} = 60$.

③

$$\begin{array}{l}
\qquad -3w_2 + 2w_3 = z - 420 \\
\hline
w_1 - 2w_2 + 3w_3 = 60 \\
x_1 \quad + w_2 - 2w_3 = 60 \\
x_2 \qquad\quad + w_3 = 60.
\end{array}$$

$-\frac{2}{3}$, $\times\frac{1}{3}$, $\frac{2}{3}$, $-\frac{1}{3}$

Feasible solution $x_1=60,\ x_2=60,\ w_1=120.$
$w_2=w_3=0,\ z=420$

Increase w_3 to increase z:
$w_3 = \min\{60/3,\ +\infty,\ 60\} = 20$

④

$$\begin{array}{l}
-\frac{2}{3}w_1 - \frac{5}{3}w_2 \qquad\quad = z - 460 \\
\hline
\frac{1}{3}w_1 - \frac{2}{3}w_2 + w_3 = 20 \\
x_1 \quad \frac{2}{3}w_1 - \frac{1}{3}w_2 \qquad = 100 \\
x_2 \quad -\frac{1}{3}w_1 + \frac{2}{3}w_2 \qquad = 40
\end{array}$$

$x_1=100,\ x_2=40,\ w_3=20,\ w_1=w_2=0,\ z=460$

Optimal solution: $(100, 40, 0, 0, 20)$ optimal value. 460

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Question 1.

(d) $\min 240y_1 + 180y_2 + 60y_3$

$$\begin{aligned} 240y_1 + 180y_2 + 60y_3 \quad &= z \\ -2y_1 - y_2 \quad + w_1 \quad &= -3 \\ -y_1 - 2y_2 - y_3 \quad + w_2 &= -4 \end{aligned}$$

① phase-1 L.P.:

$$\begin{aligned} u &= z' \\ -2y_1 - y_2 \quad + w_1 \quad - u &= -3 \\ -y_1 - 2y_2 - y_3 \quad + w_2 - u &= -4 \end{aligned}$$

$\times -1$

②
$$\begin{aligned} -y_1 - 2y_2 - y_3 \quad + w_2 \quad &= z' - 4 \\ -y_1 + y_2 + y_3 + w_1 - w_2 \quad &= 1 \\ y_1 + 2y_2 + y_3 \quad - w_2 + u &= 4 \end{aligned}$$

$w_1 = 1,\ u = 4,\ y_1 = y_2 = y_3 = w_2 = 0,\ z' = 4.$

Increase y_2 to $\min\{1, 4/2\} = 1$.

③
$$\begin{aligned} -3y_1 \quad + y_3 + 2w_1 - w_2 \quad &= z' - 2 \\ -y_1 + y_2 \quad + y_3 + w_1 - w_2 \quad &= 1 \\ 3y_1 \quad - y_3 - 2w_1 + w_2 + u &= 2 \end{aligned}$$

$\times \frac{1}{3}$

$y_2 = 1,\ u = 2,\ y_1 = y_3 = w_1 = w_2 = 0,\ z' = 2.$

Increase y_1 to $\min\{+\infty, 2/3\} = 2/3$

④
$$\begin{aligned} u &= z' \\ y_2 \quad + \tfrac{2}{3}y_3 + \tfrac{1}{3}w_1 - \tfrac{2}{3}w_2 + \tfrac{1}{3}u &= 5/3 \\ y_1 \quad - \tfrac{1}{3}y_3 - \tfrac{2}{3}w_1 + \tfrac{1}{3}w_2 + \tfrac{1}{3}u &= 2/3 \end{aligned}$$

$y_1 = \frac{2}{3},\ y_2 = \frac{5}{3},\ y_3 = w_1 = w_2 = 0,\ u = 0.$

The end of phase-1-L.P. ✓

⑤ Origin L.P.:
$$\begin{aligned} 240y_1 + 180y_2 + 60y_3 \quad &= z \\ y_2 + \tfrac{2}{3}y_3 + \tfrac{1}{3}w_1 - \tfrac{2}{3}w_2 &= 5/3 \\ y_1 \quad - \tfrac{1}{3}y_3 - \tfrac{2}{3}w_1 + \tfrac{1}{3}w_2 &= 2/3 \end{aligned}$$

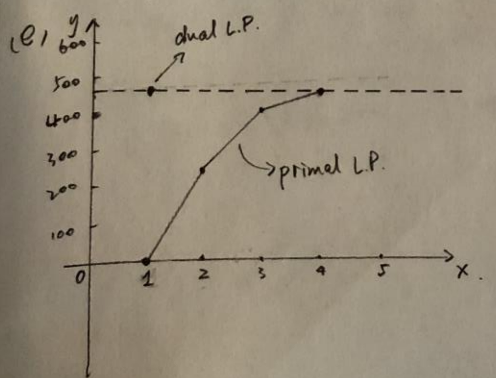
-180 -240.

⑥
$$\begin{aligned} 20y_3 + 100w_1 + 40w_2 &= z - 460 \\ y_2 + \tfrac{2}{3}y_3 + \tfrac{1}{3}w_1 - \tfrac{2}{3}w_2 &= \tfrac{5}{3} \\ y_1 \quad - \tfrac{1}{3}y_3 - \tfrac{2}{3}w_1 + \tfrac{1}{3}w_2 &= \tfrac{2}{3} \end{aligned}$$

$y_1 = \frac{2}{3},\ y_2 = \frac{5}{3},\ y_3 = w_1 = w_2 = 0,\ z = 460.$

The optimal solution for the dual: $(\frac{2}{3}, \frac{5}{3}, 0, 0, 0)$. optimal value: 460.

(e)



Conclusion: Objective value of the dual is always greater or equal than the objective value of the primal problem.

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Question 2 :

(a) $\min\ 7y_1 + 3y_2 + y_3 + y_4$ (dual)

s.t. $3y_1 + y_2 + y_3 + y_4 \geq 5$

$4y_1 + 2y_2 + \quad + y_4 \geq 9$

$5y_1 + y_2 + 2y_3 \geq 4$

$y_1, y_2, y_3, y_4 \geq 0.$

$\max\ 5x_1 + 9x_2 + 4x_3$ (primal)

st. $3x_1 + 4x_2 + 5x_3 \leq 7$

$x_1 + 2x_2 + x_3 \leq 3$

$x_1 + 2x_3 \leq 1$

$x_1 + x_2 \leq 1$

$x_1, x_2, x_3 \geq 0.$

(b) ① Rearrange the dual :

$\max\ -7y_1 - 3y_2 - y_3 - y_4$

s.t. $-3y_1 - y_2 - y_3 - y_4 \leq -5$

$-4y_1 - 2y_2 \quad - y_4 \leq -9$

$-5y_1 - y_2 - 2y_3 \leq -4$

$y_1, y_2, y_3, y_4 \geq 0$.

dual of dual: $\Longrightarrow$

$\min\ -5x_1 - 9x_2 - 4x_3$

st. $-3x_1 - 4x_2 - 5x_3 \geq -7$

$-x_1 - 2x_2 - x_3 \geq -3$

$-x_1 - 2x_3 \geq -1$

$-x_1 - x_2 \geq -1$

$x_1, x_2, x_3 \geq 0$.

(c) Rearrang the dual of the dual:

$\max\ 5x_1 + 9x_2 + 4x_3$

s.t. $3x_1 + 4x_2 + 5x_3 \leq 7$

$x_1 + 2x_2 + x_3 \leq 3$

$x_1 + 2x_3 \leq 1$

$x_1 + x_2 \leq 1$

$x_1, x_2, x_3 \geq 0$

$\Rightarrow$ Exactly the same as the orignal primal problem !!