

ORIE 5380, CS 5727: Optimization Methods

Homework Assignment 8

Due November 2, 12:00 pm

Please submit a single PDF document formatted to print and show all your work clearly.

Feel free to scan and submit handwritten work. Do not spend too much time on wordprocessing your answers.

Question 1

Consider the linear program

$$\begin{array}{ll}\max & 3x_1 + 4x_2 \\ \text{st} & 2x_1 + x_2 \leq 240 \\ & x_1 + 2x_2 \leq 180 \\ & x_2 \leq 60 \\ & x_1, x_2 \geq 0.\end{array}$$

- Write the dual of this linear program.
- By inspection, give three feasible solutions to the primal problem and compute the objective value of the primal problem at these three feasible solutions. Similarly, by inspection, give three feasible solutions to the dual problem and compute the objective value of the dual problem at these three feasible solutions. How does the objective value of the dual at the feasible solutions compare to the objective value of the primal at the feasible solutions?
- Use the simplex method to obtain the optimal solution to the primal problem above.
- Consider the dual problem you wrote in Part a. Use the simplex method to obtain the optimal solution this dual problem. (Hint: You may need to use the phase-1 linear program to obtain a feasible solution to the dual problem. Also, since we are minimizing the objective function in the dual problem, we need to choose the variable with the most negative objective function row coefficient as the entering variable.)
- Consider the solutions that the simplex method visited in Part c. Give a plot where the x -axis shows the iteration number and the y -axis shows the objective value obtained at each iteration. Similarly, consider the solutions that the simplex visited in Part d after finding a feasible solution to the dual problem by using the phase-1 linear program. On the same chart, show the objective values for the dual problem that you obtain at each iteration. Verify that the objective values that you obtain for the dual problem are always greater than or equal to the objective values that you obtain for the primal problem.

(There is one more problem on the next page.)

Question 2

In this problem, you will show that the dual of the dual gives us the primal problem back. Consider the linear program

$$\begin{array}{ll}\max & 5x_1 + 9x_2 + 4x_3 \\ \text{st} & 3x_1 + 4x_2 + 5x_3 \leq 7 \\ & x_1 + 2x_2 + x_3 \leq 3 \\ & x_1 + 2x_3 \leq 1 \\ & x_1 + x_2 \leq 1 \\ & x_1, x_2, x_3, x_4 \geq 0.\end{array}$$

- a) Write the dual of this linear program.
- b) Write the dual of the dual. (Hint: We know how to write the dual of a linear program that maximizes its objective function with less-than-or-equal-to constraints. The dual in Part a minimizes its objective function and it has greater-than-or-equal-to constraints. So, rearrange the dual so that it maximizes its objective function and it has less-than-or-equal-to constraints. After rearranging, you can write the dual of the dual.)
- c) Write the dual of the dual in Part b as a linear program, where we maximize the objective function with less-than-or-equal to constraints. Compare this form of the dual of the dual with the primal linear program given at the very beginning.