## Question 1

(a)

To formulate the problem into a min-cost network flow problem, I decided to assign costs on each combination of referees and paper. Use 'i' to denote 71 papers, and 'j' to denote 21 papers. If the referees are assigned to papers that they said "yes, I can definitely review this paper", the costs are 1. If the referees are assigned to papers that they said "I can maybe review this paper", the costs are 10. If the referees are assigned to papers that they said "No, I do not want to review this paper" the costs are 100. If the referees are assigned to papers that they said "I have a conflict of interest with this paper, so it is unethical for me to review this paper" the costs are 10^6, since we definitely do not want a referee to be assigned to a paper if he/she said that.

The cost table would be look like this:

	Yes	Maybe	No	Definitely Not
Costs	1	10	100	10^6

Let Cij denote the cost that assign paper 'i' to referee 'j'.

Let Xij denote the true/false that paper 'i' is assigned to referee 'j', Xij=1 means paper 'i' is assigned to referee 'j'.

Our objective function would be:

Min 
$$\sum_{i=1}^{71} \sum_{j=1}^{21} Cij * Xij$$

Since each paper need to have 3 referees and each referee need to review about 10~11 papers Our constraints are represented as following:

- 1)  $\sum_{j=1}^{21} Xij = 3$ , where i belongs to [1,71]
- 2)  $\sum_{i=1}^{71} Xij \ge 10$ , where j belongs to [1,21]
- 3)  $\sum_{i=1}^{71} Xij \le 11$ , where j belongs to [1,21]
- 4)  $Xij \ge 0$
- 5) Xij ≤ 1

(b) After using the gurobi to solve this optimization problem, we got the following result:



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Code:
from gurobipy import *
import xlwt
import xlrd
# loading data
f = xlrd.open workbook('paper data.xlsx')
sheet = f.sheet_by_index(0)
# create a new model
myModel = Model("HW10_Q1")
# create decision vats and integrate them into the model
nrows = sheet.nrows-1
ncols = sheet.ncols-1
cost = [[0 for i in range(ncols)] for j in range(nrows)]
myVars = [[0 for i in range(ncols)] for j in range(nrows)]
for i in range(nrows):
  for j in range(ncols):
    curVar = myModel.addVar(vtype=GRB.INTEGER, name='X' + str(i+1) + ',' + str(j+1))
    myVars[i][j] = curVar
    arc = sheet.cell(i+1,j+1).value
    if arc == "yes":
      cost[i][j] = 1
    if arc == "maybe":
      cost[i][j] = 10
    if arc == "no":
      cost[i][j] = 100
    if arc == "conflict":
      cost[i][i] = 1000000
myModel.update()
# create a linear expression for the objective
objExpr = LinExpr()
for i in range(nrows):
  for j in range(ncols):
    curVar = myVars[i][j]
    objExpr += cost[i][j]*curVar
    myModel.setObjective(objExpr, GRB.MINIMIZE)
myModel.update()
# Constraint for each paper having exactly three referees to review
for i in range(nrows):
  constExpr = LinExpr()
  for j in range(ncols):
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curVar = myVars[i][j]
    constExpr += curVar
  myModel.addConstr(lhs=constExpr, sense=GRB.EQUAL, rhs=3, name="paper" + str(i+1))
# Constraint for each referee having at least 10 paper to review
for j in range(ncols):
  constExpr = LinExpr()
  for i in range(nrows):
    curVar = myVars[i][j]
    constExpr += curVar
  myModel.addConstr(lhs=constExpr, sense=GRB.GREATER_EQUAL, rhs=10,
name="referee_lowerbound" + str(j+1))
# Constraint for each referee having at most 11 paper to review
for j in range(ncols):
  constExpr = LinExpr()
  for i in range(nrows):
    curVar = myVars[i][j]
    constExpr += curVar
  myModel.addConstr(lhs=constExpr, sense=GRB.LESS_EQUAL, rhs=11, name="referee_upperbound" +
str(j+1))
# boundary
for j in range(ncols):
 for i in range(nrows):
    constExpr = LinExpr()
    constExpr = myVars[i][j]
    myModel.addConstr(lhs=constExpr, sense=GRB.LESS EQUAL, rhs=1, name="boundary" + str(i+1) +
',' + str(j+1))
# integrate objective and constraints into the model
myModel.update()
# write the model in a file to make sure it is constructed correctly
myModel.write(filename="HW10 Q1.lp")
# optimize the model
myModel.optimize()
# save result to xls file
allVars = myModel.getVars()
writebook = xlwt.Workbook()
sheet1 = writebook.add_sheet('test')
for i in range(71):
  for j in range(21):
    if(allVars[21*i+j].x == 1):
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sheet1.write(i+1,j+1,sheet.cell(i+1,j+1).value)

writebook.save('result.xls')