Algebra 1 Practice Problems III

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The focus of these review problems is on the material covered in Weeks 25 through 35, but keep in mind that prior material can still appear on the exam.

Contents

1 Functions

Throughout, the notation $f : D \to \mathbb{R}$ means that f is a function whose domain is D and whose output values are real numbers. By the "natural domain" of a formula, we mean the largest subset of the reals which can be the domain of a function given by that formula. For example, the natural domain of $1/x$ is the set of all real numbers other than 0.

1.1 Review problems

1. Let $f : \mathbb{R} \to \mathbb{R}$ be the function given by $f(x) = x^2 - 6$. Evaluate each of the following:
   (a) $f(0)$
   (b) $f(f(1))$
   (c) $f^6(3)$
   (d) $(f(3))^6$
   (e) $f(3+4) - (f(3) + f(4))$
2. Let $f : \mathbb{R} \to \mathbb{R}$ be the function given by $f(x) = x^2 - 6$ and let $g : \mathbb{R} \to \mathbb{R}$ be the function given by $g(x) = 2x$. Evaluate each of the following:
   (a) $(f + g)(2)$
   (b) $(f \cdot g)(2)$
   (c) $(f \circ g)(2)$
   (d) $(g \circ f)(2)$
   (e) $f(g(f(2)))$
   (f) $(f^2 \circ g)(2)$
3. Determine the natural domains of each of the following formulas.
   (a) $x^3 - x$
   (b) $\frac{x+2}{(x-2)(x-3)}$
   (c) $\sqrt{2x - 8}$
   (d) $\sqrt[4]{x^2 - 5x - 14}$
4. For each of the following functions, determine the range. If the domain is unspecified and only a formula is given, assume that the corresponding function has the natural domain.
   (a) x^2
   (b) x^3
   (c) $\frac{1}{x-1} + 4$
   (d) $\sqrt{x+3} - 2$
   (e) x^2 with domain $(-2, 3]$
   (f) $\frac{2x}{x^2+1}$

5. For each of the following functions, determine whether the function is invertible. If so, find the inverse function (including domain specification as needed). If not, find two input values which produce the same output.

   (a) x^2

   (b) x^2 with domain $(-2, 3]$

   (c) $x^2 - 6x + 8$ with domain $(3, +\infty)$

   (d) $\frac{1}{x-1} + 4$

   (e) $\sqrt{x+3} - 2$

   (f) $\frac{2x}{x^2+1}$ with domain $[-1, 1]$

6. Let $f : \mathbb{R} \to \mathbb{R}$ be a function. Describe a sequence of transformations that would transform the graph of f into the graph of the given equation.

   (a) $y = f(x) - 3$

   (b) $y = -2f(x)$

   (c) $y = f(x - 5)$

   (d) $y = f(x/4)$

   (e) $y = 3f(2x - 1) + 4$

   (f) $(y + 1)/2 = f(-x + 6)$

7. Let $f : \mathbb{R} \to \mathbb{R}$ be a function satisfying $f(4) = 7$ and let $g : \mathbb{R} \to \mathbb{R}$ be given by the formula $g(x) = 3f(x^2) - 4$.

   (a) Find two points on the graph of g.

   (b) Does g have an inverse?

1.2 Challenge problems

8. Let $f : \{1, 2, 3, 4, 5, 6, 7, 8\} \to \{1, 2, 3, 4, 5, 6, 7, 8\}$ be given by

$$\begin{array}{llll} f(1) = 6, & f(2) = 4, & f(3) = 2, & f(4) = 3, \\ f(5) = 7, & f(6) = 8, & f(7) = 1, & f(8) = 5. \end{array}$$

   Find the smallest positive integer n such that $f^n(k) = k$ for all valid inputs k.

9.

10. (Even more challenging than usual) Describe a function $f : \mathbb{R} \to \mathbb{R}$ with the property that for any real numbers a, b, and c with $a < b$, there is a real number x between a and b such that $f(x) = c$. *Hint: Digits and base number arithmetic.*

1.3 Answers