

Algebra 1 Practice Problems II

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The focus of these review problems and the second midterm is on the material covered in Weeks 13 through 23, but keep in mind that prior material can still appear on the exam.

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1 Graphing

1.1 Review

1. Draw a coordinate plane and label the origin and the four quadrants.
2. Let $A = (3, 1)$. Find the coordinates of each of the following:
 - (a) the reflection of A across the x -axis
 - (b) the reflection of A across the y -axis
 - (c) the reflection of A across the line $y = x$
 - (d) the rotation of A around the origin by 180°
 - (e) the rotation of A around the origin by 90° counterclockwise
 - (f) the rotation of A around the point $(2, 2)$ by 90° clockwise
3. Quadrilateral $ABCD$ is positioned in the coordinate plane so that its vertices have coordinates

$$A = (5, 7); \quad B = (5, 6); \quad C = (3, 1); \quad D = (-4, -5).$$

Points E, F, G, H are the midpoints of segments $\overline{AB}, \overline{BC}, \overline{CD}, \overline{DA}$, respectively.

- (a) Find the coordinates of E, F, G , and H .
- (b) Compute the midpoints of segments \overline{EG} and \overline{FH} .

To check your work, the two midpoints computed in part (b) should be the same. Doing this calculation in general (rather than with specific numbers) and finding that the midpoints of the diagonals of $EFGH$ coincide proves the following:

The midpoints of the sides of any quadrilateral form a parallelogram.

4. Maurine needs to get from $(2, 3)$ to $(17, 11)$.
 - (a) If they take the shortest path possible, how much distance would they cover?
 - (b) Suppose Maurine gets distracted while pondering the meaning of life and goes from $(2, 3)$ to $(6, 6)$, then to $(11, 18)$, then to $(17, 10)$, and finally to $(17, 11)$. What is the minimum distance Maurine can cover which is consistent with this information?
5. Which of the following expressions correctly finds the slope between the points $(-1, 7)$ and $(3, -4)$? Circle all valid expressions.

$$\frac{3 - (-1)}{-4 - 7} \quad \frac{7 - (-4)}{-1 - 3} \quad \frac{-4 - 7}{3 - (-1)} \quad \frac{7 - (-4)}{3 - (-1)} \quad \frac{-4 - 3}{7 - (-1)}$$

6. A line is given by the point-slope form

$$y - 4 = \frac{1}{4}(x + 1).$$

- (a) Find the slope of the line and a point on the line.

- (b) Put the equation in slope-intercept form and find the y -intercept of the line.
- (c) Put the equation in standard form and find the x -intercept of the line.

7. (a) Of the equations

$$5x + 4y = 35; \quad (x + 4)^2 + (y - 1)^2 = 10; \quad x^2 + xy + y^2 = 49; \quad x - 2y = -7,$$

which one is an equation for the blue line below?

(b) Of the equations

$$5x + 4y = 35; \quad (x + 4)^2 + (y - 1)^2 = 10; \quad x^2 + xy + y^2 = 49; \quad x - 2y = -7,$$

which one is an equation for the red curve below?



8. A line is given by the slope-intercept form

$$y = -\frac{1}{4}x - 3.$$

- (a) Find the slope of the line and the y -intercept of the line.
- (b) Put the equation in standard form and find the x -intercept of the line.
- (c) Find the point on the line with x -coordinate 2024, then put the equation in point-slope form using this point.

9. A line is given by the standard form

$$3x + y = -4.$$

- (a) Find the x -intercept and y -intercept of the line.
 - (b) Put the equation in slope-intercept form and find the slope of the line.
 - (c) Find the point on the line for which the sum of the coordinates is 10, then put the equation in point-slope form using this point.
10. A line passes through the point $(-5, 2)$ and has slope $1/2$.
- (a) Write down an equation for this line in point-slope form.
 - (b) Find the slope-intercept form and the standard form of the line.
11. A line passes through the points $(-3, 4)$ and $(-3, -4)$. Find an equation for this line.
12. A line passes through the points $(-3, 3)$ and $(0, -4)$.
- (a) Find the slope of the line.
 - (b) For each of the two given points, find the point-slope form of the line using that point.
 - (c) Find the slope-intercept form and the standard form of the line.
- As a check of your answers, rearranging either of the point-slope equations from part (b) should give you the same slope-intercept form and standard form.
13. A line with equation $y = mx + b$ passes through the points $(5, 7)$ and $(8, -1)$. Find m and b .
14. Find the point at which the line with equation $2023x + 2022y = 2021$ intersects the line with equation $2024x + 2023y = 2022$.
15. Let ℓ be the line with equation $y = -4x - 2$ and let $P = (-4, 2)$. The point Q on line ℓ which is closest to P is the point for which $\overline{PQ} \perp \ell$. As such, if ℓ' is the line through P perpendicular to line ℓ , then Q is the intersection of ℓ and ℓ' .
- (a) Find the slope of ℓ' .
 - (b) Find an equation for ℓ' .
 - (c) Find the coordinates of Q .

1.2 Challenge

16. (Perpendicular bisector) Given any two points A and B , the *perpendicular bisector* of segment \overline{AB} is the line passing through the midpoint of \overline{AB} which is perpendicular to \overline{AB} .
- (a) Suppose $B = (14, 0)$ and $C = (5, 12)$. Find an equation for the perpendicular bisector of segment \overline{BC} .
 - (b) Show that for any point P on the perpendicular bisector of \overline{BC} , we have $BP = PC$.
 - (c) Conversely, show that if Q is a point in the plane for which $BQ = QC$, then Q lies on the perpendicular bisector of \overline{BC} .

17. (Circumcenter) Given any triangle ABC , the perpendicular bisectors of the sides of ABC all pass through a single point O , called the *circumcenter* of triangle ABC . It is the center of the unique circle passing through all three of A , B , and C (called the *circumcircle* of ABC).
- (a) Let $A = (0, 0)$, $B = (14, 0)$, and $C = (5, 12)$. Find the circumcenter of ABC .
 - (b) Find the radius of the circumcircle of ABC (i.e. the *circumradius* of ABC).
18. (Median and centroid) Given any triangle ABC , the *medians* are the lines that pass through one vertex and the midpoint of the side opposite that vertex. For example, the A -median is the line that passes through A and the midpoint of side \overline{BC} . The three medians of a triangle all pass through a single point G , called the *centroid* of triangle ABC .
- (a) Let $A = (0, 0)$, $B = (14, 0)$, and $C = (5, 12)$. Let D , E , and F be the midpoints of sides \overline{BC} , \overline{CA} , and \overline{AB} , respectively. Find the coordinates of D , E , and F .
 - (b) Find the coordinates of the centroid G of ABC . How do they relate to the coordinates of A , B , and C ?
 - (c) Compute the ratios AG/GD , BG/GE , and CG/GF .
19. (Altitude and orthocenter) Given any triangle ABC , the *altitudes* are the lines that pass through one vertex which are perpendicular to the opposite side. For example, the A -altitude is the line that passes through A and is perpendicular to line \overline{BC} . The three altitudes of a triangle all pass through a single point H , called the *orthocenter* of triangle ABC .
- (a) Let $A = (0, 0)$, $B = (14, 0)$, and $C = (5, 12)$. Find equations for the altitudes of ABC .
 - (b) Find the coordinates of the orthocenter H of ABC .
20. Let $A = (1, 1)$, $B = (5, 2)$, and $C = (-4, 3)$. There are three parallelograms in the plane for which A , B , and C are three of the four vertices. What are the possible coordinates for the fourth vertex?

1.3 Answers

1. In your drawing, the origin should be at $(0, 0)$, where the coordinate axes intersect. Quadrant I is in the upper right, quadrant II is in the upper left, quadrant III is in the lower left, and quadrant IV is in the lower right.
2. (a) $(3, -1)$
(b) $(-3, 1)$
(c) $(1, 3)$
(d) $(-3, -1)$
(e) $(-1, 3)$
(f) $(1, 1)$
3. (a) $E = (5, 13/2)$; $F = (4, 7/2)$; $G = (-1/2, -2)$; $H = (1/2, 1)$
(b) The midpoint of \overline{EG} is $(9/4, 9/4)$, which is also the midpoint of \overline{FH} .
4. (a) $\boxed{17}$
(b) $5 + 13 + 10 + 1 = \boxed{29}$
5. The second and third expressions are valid.
6. (a) The slope is $\boxed{1/4}$, and one of the points on the line is $\boxed{(-1, 4)}$.
(b) Distributing the right hand side gives us

$$y - 4 = \frac{1}{4}x + \frac{1}{4}.$$

Adding 4 to both sides gives us the slope-intercept form

$$\boxed{y = \frac{1}{4}x + \frac{17}{4}}.$$

The y -intercept is $\boxed{(0, 17/4)}$.

- (c) Subtracting y from both sides, then subtracting $17/4$ from both sides,

$$\frac{1}{4}x - y = -\frac{17}{4}.$$

To clear denominators, we multiply both sides by 4 to get the standard form

$$\boxed{x - 4y = -17}.$$

When $y = 0$, we have $x = -17$, so the x -intercept is $\boxed{(-17, 0)}$.

7. (a) The blue line passes through $(0, 3.5)$, which only satisfies the equation $\boxed{x - 2y = -7}$.

- (b) The first and last equation describe lines, so the answer cannot be those. Of the remaining two equations, the point $(7, 0)$ on the red curve only satisfies $x^2 + xy + y^2 = 49$.

8. (a) The slope is $-1/4$ and the y -intercept is $(0, -3)$.
 (b) To get standard form, we start by moving all of the variables to one side so that x has a positive coefficient and moving all of the constants to the other. Adding $\frac{1}{4}x$ to both sides gives us

$$\frac{1}{4}x + y = -3.$$

Multiplying both sides by 4 gives the standard form

$$x + 4y = -12.$$

To get the x -intercept, we plug in $y = 0$ to get $x = -12$, so the x -intercept is $(-12, 0)$.

- (c) Substituting $x = 2024$ into the slope-intercept form given to us,

$$y = \left(-\frac{1}{4}\right) \cdot 2024 - 3 = -509.$$

The point-slope form with slope $-1/4$ and point $(2024, -509)$ is

$$y + 509 = -\frac{1}{4}(x - 2024).$$

9. (a) Letting $x = 0$, the y -intercept is $(0, -4)$.

Letting $y = 0$, the x -intercept is $(-4/3, 0)$.

- (b) Solving for y in terms of x , the slope-intercept form is

$$y = -3x - 4.$$

The slope of the line is -3 .

- (c) We want the point (x, y) to lie on the line, i.e. to satisfy $3x + y = -4$, and to have sum of coordinates 10, i.e. $x + y = 10$. Solving the system of equations, $(x, y) = (-7, 17)$. The desired point-slope form is then

$$y - 17 = -3(x + 7).$$

10. (a) $y - 2 = \frac{1}{2}(x + 5)$

- (b) Slope-intercept form: $y = \frac{1}{2}x + \frac{9}{2}$
 Standard form: $x - 2y = -9$

11. The line through the two points is the vertical line $x = -3$.

12. (a) $\frac{3 - (-4)}{(-3) - 0} = \boxed{-\frac{7}{3}}$

(b) Using $(-3, 3)$, we get $\boxed{y - 3 = -\frac{7}{3}(x + 3)}$.

Using $(0, -4)$, we get $\boxed{y + 4 = -\frac{7}{3}(x - 0)}$.

(c) Slope-intercept form: $\boxed{y = -\frac{7}{3}x - 4}$

Standard form: $\boxed{7x + 3y = -12}$

13. Substituting the two given points, we get the system of equations

$$5m + b = 7 \quad \text{and} \quad 8m + b = -1.$$

Solving yields $\boxed{(m, b) = \left(-\frac{8}{3}, \frac{61}{3}\right)}$.

14. The point of intersection is given by the solution to the system of equations

$$2023x + 2022y = 2021, \tag{1}$$

$$2024x + 2023y = 2022. \tag{2}$$

Taking the difference of equations (2) $-$ (1) gives us

$$x + y = 1. \tag{3}$$

Then, (1) $- 2022 \cdot$ (3) gives us $x = -1$. Substituting back into (3) gives us $y = 2$, so the point of intersection is $\boxed{(-1, 2)}$.

15. (a) Given a line with slope $m \neq 0$, the slope of any line perpendicular to it is given by $-1/m$. Applying this formula with $m = -4$, the slope of ℓ , tells us the slope of ℓ' must be $\boxed{1/4}$.

(b) Using point-slope form, since ℓ' passes through P , one equation for ℓ' is

$$\boxed{y - 2 = \frac{1}{4}(x + 4)}.$$

(c) Solving the system of equations

$$y = -4x - 2 \quad \text{and} \quad y - 2 = \frac{1}{4}(x + 4),$$

we find that $Q = \boxed{(-20/17, 46/17)}$.

16.

2 Linear Inequalities