Algebra 1 Practice Problems

Alan Zhou

2023

1 Exponents and Radicals

1. (Exponent rules)

   (a) If $3^4 \cdot 3^5 \cdot 3^7 = 3^n$, what is n?

   (b) Compute $\dfrac{2^{16} \cdot 2^{20}}{2^{11} \cdot 2^{23}}$.

   (c) Compute $58492^0 \cdot 6^3 + 2^8$.

   (d) Is 3^{-4} greater than or less than 0?

   (e) Is $(-1/2)^{-2}$ greater than or less than 1?

   (f) Compute $(-8)^{2/6}$, or if it is undefined, state so.

   (g) Simplify $\left(2^{\sqrt{2}}\right)^{\sqrt{2}}$.

   (h) Simplify $(4a^4 b^{-2} c^{-6})^{-3/2}$. Your final answer should have no negative exponents.

2. Put each of the following into simplest radical form.

   (a) $\sqrt{100}$

   (b) $\sqrt{8}$

   (c) $\sqrt{216}$

   (d) $\sqrt{2023}$

   (e) $\sqrt[3]{144}$

   (f) $\sqrt[4]{729}$

   (g) $\sqrt{50} + \sqrt{162}$

   (h) $\sqrt{\dfrac{2024}{2023}}$ (Your answer should not have any roots in the denominator.)

3. Solve for x in the equation
$$3^{8-2x} \cdot 27^{x-1} = 9^{2x+1}.$$

2 Expanding and Factoring

4. Expand each of the following with the distributive property.

   (a) $xy(6x + 7y)$

   (b) $5a^2 b(3ab - ab^2)$

   (c) $-3x(2 - 4x - 6x^2)$

   (d) $\dfrac{-8a^2 b - 6ab^2 + 4a^2 b^2}{2ab}$

5. (Some important expansions) Expand each of the following with the distributive property.

   (a) $(a + b)^2$

   (b) $(a - b)^2$

   (c) $(a - b)(a + b)$

6. Factor each of the following as much as possible.

   (a) $2x^2 - 18x$

   (b) $6x^2 y + 7xy^2$

   (c) $-63a^2 b - 54ab^2 + 45ab + 36b$

   (d) $a^2 - b^2$

   (e) $u^2 - 2uv + v^2$

   (f) $p^2 + 2pq + q^2$

7. Simplify each of the following expressions as much as possible. For expressions that start off as multiple fractions, you should end up with a single fraction.

   (a) $x + 1 + \dfrac{1}{x} - \dfrac{1}{x^2}$

   (b) $\dfrac{x}{y} + \dfrac{y}{x}$

   (c) $\dfrac{xy^2 - x^2 y^3}{3 - 3xy}$

   (d) $\dfrac{a^2 - 2ab + b^2}{a^2 - b^2} + \dfrac{ab + b^2}{a^2 + 2ab + b^2}$

8. Expand $(m + 2n)^2$.

9. (Sophie Germain's identity) Factor $a^4 + 4b^4$.

   Hint: $a^4 + 4b^4 = (a^4 + 4a^2 b^2 + 4b^4) - 4a^2 b^2$.

3   Solving Equations

10. Solve for the variable in each of the following equations:

    (a) $3x + 5 = 11$

    (b) $6(a - 2) = 3(-a + 4)$

    (c) $3(2c + 7) - 6(c + 1) = 11$

    (d) $-4(-t - 2) + t = 5(t + 5) - 17$

11. In each of the following equations, solve for the indicated variable in terms of the others:

    (a) $x + 7y = 5$ for x

    (b) $x + 7y = 5$ for y

    (c) $ax^2 + bx + c = 0$ for b

    (d) $pqr + pq + pr + qr = p + q + r + 1$ for p

12. Solve each of the following systems of equations for x and y:

    (a) $x + 2y = 3$ and $3x - y = 2$

    (b) $-2x + 4y = 7$ and $x - 2y = 3$

    (c) $4x + y = -2$ and $x + y = 1$

    (d) $7x - 5y = 1$ and $14x - 10y = 2$

    (e) $3x + 4y = 25$ and $4x + 3y = 24$

    Each of these systems is of the form

    $$Ax + By = E \quad \text{and} \quad Cx + Dy = F.$$

    How does the value of $|AD - BC|$ relate to the number of solutions to the system?

13. Eleven years ago Margot was five times as old as Preeti. In sixteen years, Margot will be twice as old as Preeti. How old is Preeti now? *We did this problem in Lesson 3 by using just one variable, but we can also solve it by defining two variables.*

14. Alex buys 3 pencils, 1 binder, and 1 calculator for $133. Bob buys 1 pencil, 3 binders, and 1 calculator for $135. Carol buys 1 pencil, 1 binder, and 3 calculators for $392.

    (a) How much would it cost to buy 1 pencil, 1 binder, and 1 calculator?

    (b) How much does each item cost?

4 Ratios, Proportions, and Percents

15. Compute each of the following:

    (a) 50% of 14

    (b) 75% of 24

    (c) 20% less than 65

    (d) 30% more than 70

    (e) 25% of 19% of 400

    (f) 8% of 25

    (g) 33% more than 300

16. A bag has two types of food, nuts and dry fruit. The ratio of the weight of nuts in the bag to the weight of dry fruit is $5 : 4$.

    (a) Let N be the weight of nuts in the bag, in grams, and let F be the weight of dry fruit in the bag, also in grams. Write down an equation involving N and F.

    (b) Suppose the total weight of food in the bag is 765 grams. How many grams of dry fruit are in the bag?

17. A recipe calls for a $7 : 6 : 5$ ratio of soy sauce to sugar to sesame oil. If Lena uses 14 teaspoons of sugar, then how much soy sauce and sesame oil should she use (in teaspoons)?

18. (Calculator) The speed of light is approximately 3×10^8 meters per second.

    (a) There are roughly 1.6 kilometers per mile. Find the speed of light in miles per second.

    (b) The approximation from part (a) is roughly 0.8% too high. Using this, adjust your answer to get a better approximation to the speed of light in miles per second.

19. It takes 3 identical robots 8 hours to paint 4 fences.

    (a) How much of a fence can one of these robots paint in one hour?

    (b) How long, in minutes, would it take 5 of these robots to paint 7 fences?

20. Pipe A takes 3 hours to fill a pool while pipe B takes 4 hours to fill the pool. Drain C takes 2 hours to drain the pool. The pool is initially empty, with drain C covered, when pipe A is turned on. One hour later, the cover of drain C is accidentally dislodged, so water begins to drain at the same time. Another hour after that, pipe B is turned on as well. From the point in time that pipe B is turned on, how many hours does it take to fill the pool?

21. On a road trip, Jerry drives for 200 miles at 50 miles per hour, then 108 miles at 36 miles per hour, then some number of miles at 60 miles per hour, and the rest of the trip at 41 miles per hour. The overall distance traversed was 450 miles, and the average speed for the entire trip was 45 miles per hour. How much time did Jerry spend going at 41 miles per hour?

22. (Simpson's paradox) In a certain country, operations are separated into two types, A and B. At Hospital X, type A operations succeed 90% of the time while type B operations succeed 60% of the time. At Hospital Y, type A operations succeed 80% of the time while type B operations succeed 40% of the time. The overall success rate of operations at Hospital X is 70% while the overall success rate at Hospital Y is 72%.

    (a) How is it possible that the overall success rate at Hospital Y is higher, even though it is lower for each individual type of operation?

    (b) If you had to go to one of these hospitals for an operation, which one would you prefer?

23. Katherine takes out a loan of $100 on January 1, 2000. On January 1 of each subsequent year, the loan balance (i.e. the amount that needs to be paid back) is increased by 20%.

    (a) If Katherine waits until December 1, 2002 to pay back the loan, then how much does she end up paying?

    (b) Suppose Katherine pays back $50 on December 1, 2000, another $50 on December 1, 2001, and the rest on December 1, 2002. How much does she end up paying in total?

    (c) (Calculator) Suppose Katherine pays back X on December 1 of 2000, 2001, and 2002, and this pays back the loan in full. To the nearest hundredth, what is X?

24. (Classic) An orchestra of 120 players takes 75 minutes to play Beethoven's ninth symphony. How long would it take an orchestra of 60 players to play the symphony?

5 Graphing

25. Draw graphs for each of the following equations:

    (a) $2x + 3y = 6$

    (b) $y = (1/2)x + 3$

    (c) $x = -y - 1$

    (d) $y = |x|$

    (e) $x = y^2 + 2$

    (f) (Challenge) $y^2 = x(x + 1)(x - 1)$

26. (a) When solving a system of two equations for variables x and y, Polly finds that the graphs of the equations are parallel lines. How many solutions are there to the system?

    (b) For another system, Quaresma finds that the graphs of the equations are the same line. How many solutions are there to this second system?

27. What are the slope and the intercepts of the line given by the equation $4x - 7y = 84$?

28. Let $O = (0, 0)$, $P = (14, 0)$, and $Q = (5, 12)$.

    (a) What are the side lengths of triangle OPQ?

    (b) What are the slopes of the sides of triangle OPQ?

29. (Centroid) Let $A = (5, 1)$, $B = (-1, 3)$, and $C = (3, 7)$.

    (a) Suppose D, E, and F are the midpoints of $\overline{BC}$, $\overline{CA}$, and $\overline{AB}$, respectively. What are the coordinates of D, E, and F?

    (b) Write down equations for lines $\overleftrightarrow{AD}$, $\overleftrightarrow{BE}$, and $\overleftrightarrow{CF}$.

    (c) These three lines intersect at a single point (draw them!), called the *centroid* of triangle ABC. What are the coordinates of the centroid?

30. Let $A = (0, 2)$ and $B = (12, 8)$.

    (a) Let $P = (x, y)$. Write down expressions for $(AP)^2$ and $(BP)^2$.

    (b) If $AP = BP$, then P must lie on a certain line. What are the slope and y-intercept of that line?

    (c) Where does that line intersect $\overline{AB}$?

6 Answers

1. (a) $n = 16$
   (b) 4
   (c) 472
   (d) greater than 0
   (e) greater than 1
   (f) -2
   (g) 4
   (h) $\dfrac{b^3 c^9}{8a^6}$

2. (a) 10
   (b) $2\sqrt{2}$
   (c) $6\sqrt{6}$
   (d) $17\sqrt{7}$
   (e) $2\sqrt[3]{18}$
   (f) $3\sqrt{3}$
   (g) $14\sqrt{2}$
   (h) $\dfrac{2\sqrt{3542}}{119}$

3. $x = 1$

4. (a) $6x^2 y + 7xy^2$
   (b) $15a^3 b^2 - 5a^3 b^3$
   (c) $-6x + 12x^2 + 18x^3$
   (d) $-4a - 3b + 2ab$

5. (a) $a^2 + 2ab + b^2$
   (b) $a^2 - 2ab + b^2$
   (c) $a^2 - b^2$

6. (a) $2x(x - 9)$
   (b) $xy(6x + 7y)$
   (c) $9b(-7a^2 - 6ab + 5a + 4)$
   (d) $(a - b)(a + b)$
   (e) $(u - v)^2$ or $(v - u)^2$
   (f) $(p + q)^2$

7. (a) $\dfrac{x^3 + x^2 + x - 1}{x^2}$
   (b) $\dfrac{x^2 + y^2}{xy}$
   (c) $\dfrac{xy^2}{3}$
   (d) $\dfrac{a}{a + b}$

8. $m^2 + 4mn + 4n^2$

9. $(a^2 + 2b^2 - 2ab)(a^2 + 2b^2 + 2ab)$

10. (a) $x = 2$
    (b) $a = 8/3$
    (c) no solution
    (d) all real numbers

11. (a) $x = 5 - 7y$
    (b) $y = \dfrac{5 - x}{7}$
    (c) $b = \dfrac{-ax^2 - c}{x}$
    (d) $p = \dfrac{-qr + q + r + 1}{qr + q + r - 1}$

12. (a) $(x, y) = (1, 1)$
    (b) no solution
    (c) $(x, y) = (-1, 2)$
    (d) all pairs (x, y) such that $7x - 5y = 1$
    (e) $(x, y) = (3, 4)$

    The system has a unique solution if and only if $AD - BC \neq 0$.

13. 20

14. (a) \$132 (Add up all of the purchases)
    (b) Pencils are \$0.50, binders are \$1.50, calculators are \$130

15. (a) 7
    (b) 18
    (c) 52
    (d) 91
    (e) 19
    (f) 2

(g) 399

16. (a) $N/F = 5/4$

    (b) 340

17. 49/3 tsp soy sauce, 35/3 tsp sesame oil

18. (a) 187 500 miles per second

    (b) $\approx 186\,012$ miles per second

19. (a) 1/6

    (b) 504

20. After one hour, 1/3 of the pool is full. For the second hour, the pool fills at a rate of $(1/3) - (1/2) = -1/6$ of the pool per hour, so after the second hour, 1/6 of the pool is full. Afterwards, the fill rate is $(1/3) + (1/4) - (1/2) = 1/12$ of the pool per hour, and we have 5/6 of the pool left, so the answer is $(5/6)/(1/12) = \boxed{10}$ hours.

21. Let F be the time spent at 60 miles per hour and S be the time spent at 41 miles per hour. The total time for the trip was $450/45 = 10$ hours, so $F + S = 3$. Also, we need $60F + 41S = 142$. Solving yields $F = 1$ and $S = \boxed{2}$ hours.

22. (a) This can happen when a much higher proportion of Hospital Y's operations are type A (lower risk) compared to Hospital X. Given these numbers, only 50% of Hospital X's operations are type A, vs 80% of Hospital Y's.

    (b) Hospital X

23. (a) $144

    (b) $112

    (c) On January 1, 2001, the balance is $1.2(100 - X) = 120 - 1.2X$. Then, on January 1, 2002, the balance is

$1.2(120 - 1.2X - X) = 144 - 2.64X$. This is paid off in full by the last X, so $144 - 2.64X = X$, so $X \approx \boxed{39.56}$.

24. 75

25. See Desmos

26. (a) none

    (b) infinitely many

27. Slope 4/7

    x-intercept $(21, 0)$

    y-intercept $(0, -12)$

28. (a) $OP = 14$, $PQ = 15$, $QO = 13$

    (b) $\overline{OP}$ has slope 0

    $\overline{PQ}$ has slope $-4/3$

    $\overline{QO}$ has slope $12/5$

29. (a) $D = (1, 5)$

    $E = (4, 4)$

    $F = (2, 2)$

    (b) $\overleftrightarrow{AD}$ is $y = -x + 6$

    $\overleftrightarrow{BE}$ is $y - 3 = (1/5)(x + 1)$

    $\overleftrightarrow{CF}$ is $y = 5x - 8$

    (c) $(7/3, 11/3)$

30. (a) $(AP)^2 = x^2 + (y - 2)^2$

    $(BP)^2 = (x - 12)^2 + (y - 8)^2$

    (b) Expand $(AP)^2 = (BP)^2$ and cancel squares: $-4y + 4 = -24x - 16y + 208$. This means $12y = -24x + 204$, so $y = -2x + 17$. The slope is $\boxed{-2}$ and the y-intercept is $\boxed{(0, 17)}$.

    (c) $(6, 5)$ (This is the midpoint of $\overline{AB}$.)