Probability

1. Tony is going to roll two standard six-sided dice and then add the numbers shown on the dice. What is the probability that the sum is a multiple of 5?

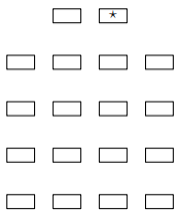
2. Alarm A works 90% of the time and Alarm B works 85% of the time. The two alarms are independent, meaning that each alarm's success or failure does not affect the other alarm. What percent of the time is there at least one alarm working? Express your answer as a decimal to the nearest tenth.

3. The diagram shows how the seats are arranged in a classroom. What is the probability of choosing the $\star$ seat if:

   (a) we label the seats from 1 to 18, and then randomly choose a seat by choosing an integer uniformly at random from 1 to 18 inclusive?

   (b) we select a column uniformly at random and then choose a seat in that column uniformly at random?

   (c) we select a row and then choose a seat in that row, both choices uniformly at random?



4. The 26 letters of the English alphabet are placed in a bag. Five of these letters are vowels and the rest are consonants. Mary draws one letter uniformly at random and writes it down. She doesn't put her letter back in the bag. Steph then draws a letter uniformly at random from the letters that remain in the bag. What is the probability that:

   (a) Mary's and Steph's letters are both vowels?

   (b) the two girls collectively have one vowel and one consonant?

5. (2011 State Sprint #29) A bag contains red balls and white balls. If five balls are to be pulled from the bag one at a time with replacement, the probability of getting exactly three red balls is 32 times the probability of getting exactly one red ball. What percent of the balls originally in the bag are red?

Extensions

1. _____ (2011 State Countdown)

2. _____ [common fraction]

3. _____

4. _____

5. _____ [common fraction]

6. (a) _____ [common fraction]

   (b) _____ [common fraction]

7. _____ [common fraction] (2011 State Sprint #27)

Extra Problems ($\star$)

1. Let A, B, C, D be integers and suppose

$$x^4 + Ax^3 + Bx^2 + Cx + D = 0$$

when $x = 2^{1/4} + 2^{1/2}$. Compute $A + B + C + D$.

2. For each positive integer N, let $P(N)$ be the probability that when a subset of $\{1, 2, \ldots, N\}$ is selected uniformly at random, the number of elements in the subset is a multiple of 4. For how many positive integers $N \leq 2025$ is it the case that $P(N) = 1/4$?

3. In triangle ABC, points E and F are the midpoints of $\overline{AC}$ and $\overline{AB}$, respectively. Lines $\overline{BE}$ and $\overline{CF}$ intersect at G. If $\angle GBC = 60^\circ$ and $\angle GCB = 45^\circ$ and $BG = 4$, then what is $(BF)^2$? Express your answer in simplest radical form.

4. How many (non-congruent) right triangles are there in which:

   (a) one of the leg lengths is 2025 and the other two side lengths are positive integers?

   (b) the hypotenuse has length 2025 and the two leg lengths are positive integers?