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# 1 Quadratics

## 1.1 Vocabulary

A *quadratic (polynomial) in  $X$*  is an expression of the form  $aX^2 + bX + c$ , where  $a, b, c$  are independent of  $X$  and  $a \neq 0$ . Collectively  $a, b, c$  are the *coefficients* of the quadratic while  $aX^2, bX, c$  are the *terms* of the quadratic.

- The term independent of  $X$ , namely  $c$ , is the *constant term* or the *degree-0 term* (as it can be written as  $cX^0$ ).
- The term with just  $X$ , namely  $bX$ , is the *linear term* or the *degree-1 term* (as it is  $bX^1$ ).
- The term with  $X^2$ , namely  $aX^2$ , is the *quadratic term* or the *degree-2 term*. Since this term contains the highest power of  $X$  in the whole expression, it is also called the *leading term*.

Specific coefficients can be referred to by borrowing the names of their corresponding terms. For example, in the above,  $a$  can be called the *leading coefficient*.

A *root* of the quadratic expression  $aX^2 + bX + c$  is a value  $r$  for which the expression is 0 when evaluated at  $X = r$ , i.e.  $ar^2 + br + c = 0$ .

## 1.2 Factoring quadratics

### 1.3 Finding roots by factoring

One way that quadratic expressions arise is as a product of two linear expressions,

$$(X - 2)(X + 3) = X(X + 3) - 2(X + 3) = X^2 + 3X - 2X - 6 = X^2 + X - 6.$$

For a given quadratic, if we can find linear factors, identifying roots becomes straightforward.

**Example 1.1.** Find the roots of  $X^2 + X - 6$ .

*Solution.* Let  $r$  be a root, so by definition, we need  $r^2 + r - 6 = 0$ . By our calculation above, the left hand side is equal to  $(r - 2)(r + 3)$ . For a product of two (or more) factors to be equal to 0, at least one of them must be 0. Therefore, any root  $r$  must satisfy  $r - 2 = 0$  or  $r + 3 = 0$ , and if  $r$  satisfies at least one of these two equations, it is a root. Hence the roots of  $X^2 + X - 6$  are 2 and  $-3$ .  $\square$

**Example 1.2.** Find the roots of  $X^2 - 8X + 12$ .

*Solution.* This time, we need to find a factorisation of  $X^2 - 8X + 12$  first. A reasonable guess is that the factorisation has the form  $(X + A)(X + B)$ , where  $A$  and  $B$  are constants to be determined. Expanding,

$$\begin{aligned} (X + A)(X + B) &= X(X + B) + A(X + B) \\ &= X^2 + BX + AX + AB \\ &= X^2 + (A + B)X + AB. \end{aligned}$$

The coefficients must match,  $\square$