

Game Time! Continued

As a reminder, the game played at the start of class involves rolling a standard six-sided die with payouts according to the table below (not including initial cost to play).

Outcome	1	2	3	4	5	6
Payout	0	0	0	0	10	30

We found that the expected payout, i.e. the “fair” price for playing, is $20/3$ dollars.

Now instead, suppose the price for playing is set at \$7.

1. What is the expected value of the game master’s profit each time the game is played?
2. Lucas has \$100 to spend and decides to play the game repeatedly until they either make a net profit or is no longer able to play. Test this strategy several times, recording Lucas’s net profit in each trial (positive or negative).
 - (a) In what fraction of your trials does Lucas make a net profit?
 - (b) What is Lucas’s average net profit over all trials?
 - (c) (Open-ended) See what happens if they try a different strategy.

Extensions

1. _____

2. _____

3. _____ (2004 State Sprint #28)

4. _____ (2001 National Team)

5. (A): \$12 (B): \$30 (C): \$50 (2006 AMC 10A #13)
(D): \$60 (E): \$100

6. **Smallest:** _____ [common fraction]

Largest: _____ [common fraction]

7. _____ [common fraction]

Extra Problems (★)

1. Let A, B, C, D be integers and suppose

$$x^4 + Ax^3 + Bx^2 + Cx + D = 0$$

when $x = 2^{1/4} + 2^{1/2}$. Compute $A + B + C + D$.

2. For each positive integer N , let $P(N)$ be the probability that when a subset of $\{1, 2, \dots, N\}$ is selected uniformly at random, the number of elements in the subset is a multiple of 4. For how many positive integers $N \leq 2025$ is it the case that $P(N) = 1/4$?

3. In triangle ABC , points E and F are the midpoints of \overline{AC} and \overline{AB} , respectively. Lines \overline{BE} and \overline{CF} intersect at G . If $\angle GBC = 60^\circ$ and $\angle GCB = 45^\circ$ and $BG = 4$, then what is $(BF)^2$? Express your answer in simplest radical form.

4. How many (non-congruent) right triangles are there in which:
- (a) one of the leg lengths is 2025 and the other two side lengths are positive integers?
 - (b) the hypotenuse has length 2025 and the two leg lengths are positive integers?