Contents

1 Quadratics

A *quadratic (polynomial) in* X is an expression of the form aX^2+bX+c, where a, b, c are independent of X and $a \neq 0$. Collectively a, b, c are the *coefficients* of the quadratic while aX^2, bX, c are the *terms* of the quadratic.

- The term c is called the *constant term*, as it does not depend on X, or the *degree-0 term* (as it can be written as cX^0).
- The term bX is called the *linear term* or the *degree-1 term* (as it can be written as bX^1), and there are analogous names for the coefficient b.
- The term aX^2 is called the *quadratic term* or the *degree-2 term*, and there are analogous names for the coefficient a. Since this term contains the highest power of X in the whole expression, it is also called the *leading term*, with a correspondingly called the *leading coefficient*.

A *root* of the quadratic expression $aX^2 + bX + c$ is a value r for which $ar^2 + br + c = 0$.

1.1 Finding roots by factoring

One way that quadratic expressions arise is as a product of two linear expressions,

$$(X-2)(X+3) = X(X+3) - 2(X+3) = X^2 + 3X - 2X - 6 = X^2 + X - 6.$$

For a given quadratic, if we can find linear factors, identifying roots becomes straightforward.

Example 1.1. Find the roots of $X^2 + X - 6$.

Solution. Let r be a root, so by definition, we need $r^2 + r - 6 = 0$. By our calculation above, the left hand side is equal to $(r-2)(r+3)$. For a product of two (or more) factors to be equal to 0, at least one of them must be 0. Therefore, any root r must satisfy $r - 2 = 0$ or $r + 3 = 0$, and if r satisfies at least one of these two equations, it is a root. Hence the roots of $X^2 + X - 6$ are 2 and -3. □

Example 1.2. Find the roots of $X^2 - 8X + 12$.

Solution. This time, we need to find a factorisation of $X^2 - 8X + 12$ first. A reasonable guess is that the factorisation has the form $(X+A)(X+B)$, where A and B are constants to be determined. Expanding,

$$\begin{aligned}(X+A)(X+B) &= X(X+B) + A(X+B)\\ &= X^2 + BX + AX + AB\\ &= X^2 + (A+B)X + AB.\end{aligned}$$

The coefficients must match, □