P3 A: Finding the Caplet

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CS 3451

Project 3 Part A Module 1

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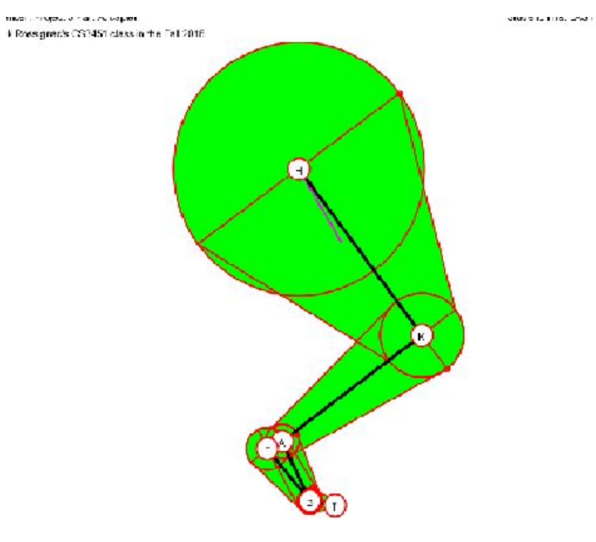
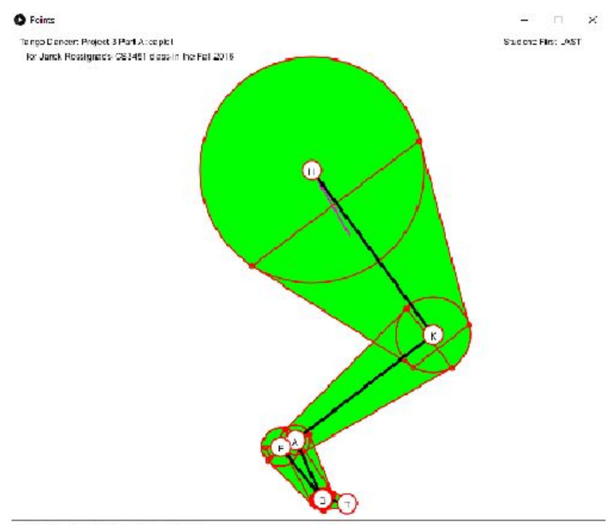
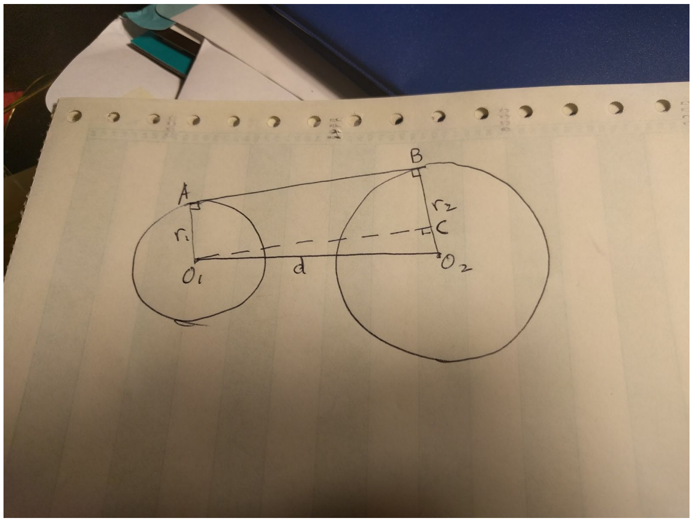
Summary

The problem addressed in this phase of the project is to find a solution to calculate the convex hull of two circles (or discs) of various sizes that replaces the cone function provided by the professor in the original source code. The cone function displays the lines of the caplet at points that do not meet at the respected discs tangentially. These points were calculated by rotating the vector between the two centers of the discs by 90 degrees counterclockwise or clockwise depending on the left side or the right side of the circle. The diagrams below demonstrates what is expected of the caplet and what was originally implemented.

Original Cone Function	New Cone Function
	

Solution



There are multiple solutions to finding the tangent points. Refer to the upper diagram for reference. The four points (O1, O2, A, B) form a right trapezoid. Given the distance between the two circles and the radii of each respective circle, angle between O1O2 and A, and angle between O2O1 and B can be calculated. From that, O1O2 or O2O1 can be rotated to get points A and B, the tangent points of the convex hull. The same procedure can be performed to produce the points on the opposing side of the discs.

Here is a breakdown of what needs to be done:

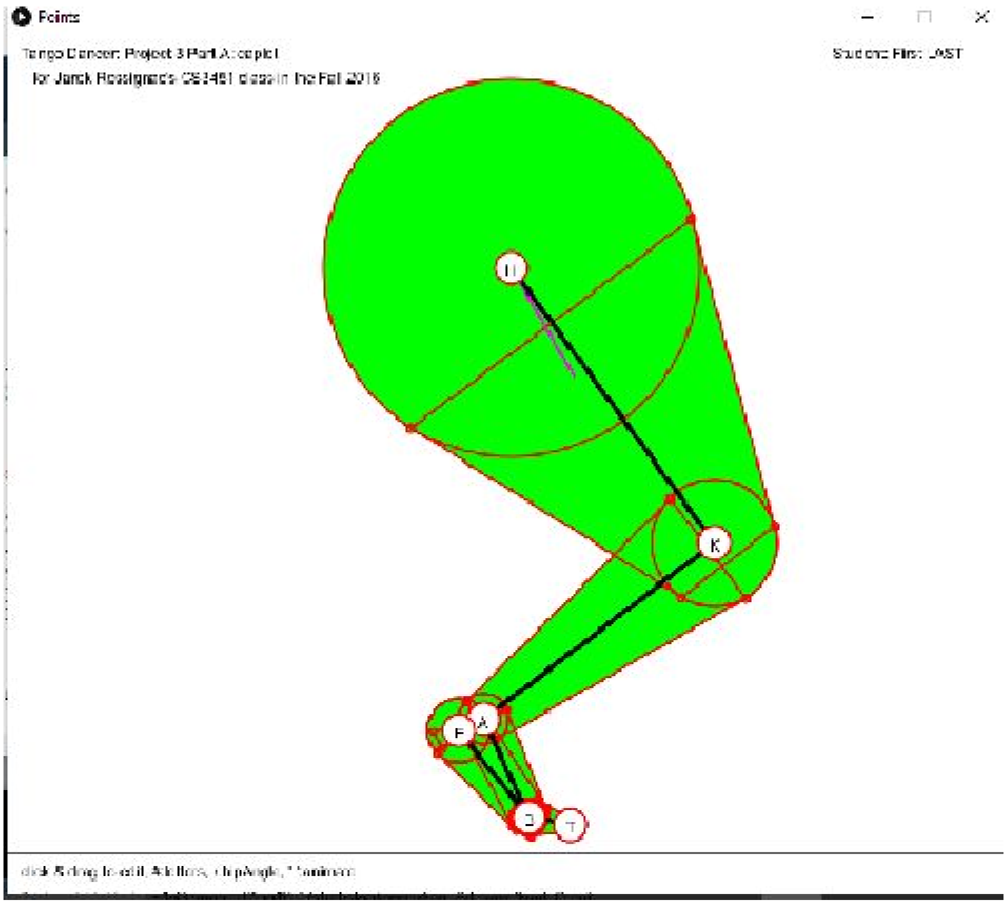
- Find the angles of the right triangle produced inside of the right trapezoid (refer to the image).
- Rotate unit vector O1O2 of magnitude r1 by an angle of (alpha + 90) in the counterclockwise direction to get point A.
- Rotate unit vector O2O1 of magnitude r2 by an angle of beta in the clockwise direction to get point B.
- Rotate unit vector O1O2 of magnitude r1 by an angle of -(alpha+90) in the counterclockwise direction to get point on the opposite side of A.
- Rotate unit vector O2O1 of magnitude r2 by an angle of -beta in the clockwise direction to get the point on the opposite side of B.

In order to solve for the triangle angles in the right trapezoid, use the given length between O1 and O2 and the difference between r2 and r1 that make up two sides of the triangle as well as elementary trigonometric functions. To find alpha, take the inverse cosine of [(r2-r1)/d(O1, O2)]. To find beta, subtract alpha from 90 based on the triangle identities and rules of complementary angles. Notice that the trapezoid is constructed from the triangle and a rectangle. Each side of the rectangle is 90 degrees.

Therefore in order to find A, rotate vector O1O2 of magnitude r1 by an angle of (alpha + 90) in the counterclockwise direction. Rotate it by an angle of -(alpha+90) to find the tangent point opposite of A. To find B, rotate unit vector O2O1 of magnitude r2 by an angle of beta in the clockwise direction. Rotate it by an angle of -beta to find the tangent point opposite of B.

Justification

This solution calculates the tangent points of respective discs along a common line whereas the original solution does not. The original source code simply determines the point on the circumference normal to distance between centers and then connects those points. That connecting line does not represent a tangent line. The proposed solution is accurate because the points are calculated based off of a the rotation of an accurate angle measurement as demonstrated by the above calculations. Final result is as shown below:


Points
for Jarek Rossignac's CS3451 class in the Fall 2018
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