220522

# Decision Tree Classifier and Decision Tree Regressor

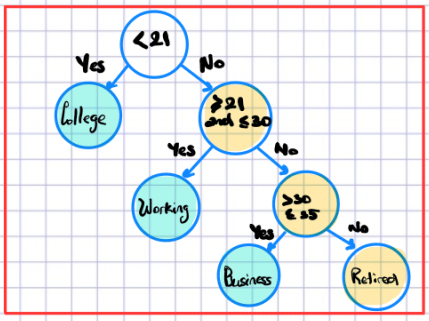
DT Classifier
1. Purity (Impurity)
   1.1 Entropy
   1.2 Gini Index
2. Information Gain
   2.1 Categorical Features
   2.2 Numerical Features

DT Regressor

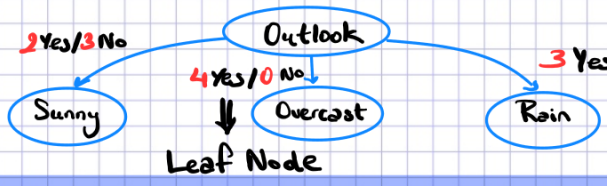
## Decision Tree Classifier

Ex. ID3 is nested if-else clause:

```
if (p_age < 21):
    print ('college')
elif (p_age ≥ 21 and p_age ≤ 30):
    print ('work')
elif (p_age ≥ 30 and p_age ≤ 35):
    print ('business')
```




Decision Tree: 1. ID3 Algorithm
2. CART (Classification and Regression Tree)




*PlayTennis*: training examples

| Day | Outlook | Temperature | Humidity | Wind | PlayTennis |
|---|---|---|---|---|---|
| D1 | Sunny | Hot | High | Weak | No |
| D2 | Sunny | Hot | High | Strong | No |
| D3 | Overcast | Hot | High | Weak | Yes |
| D4 | Rain | Mild | High | Weak | Yes |
| D5 | Rain | Cool | Normal | Weak | Yes |
| D6 | Rain | Cool | Normal | Strong | No |
| D7 | Overcast | Cool | Normal | Strong | Yes |
| D8 | Sunny | Mild | High | Weak | No |
| D9 | Sunny | Cool | Normal | Weak | Yes |
| D10 | Rain | Mild | Normal | Weak | Yes |
| D11 | Sunny | Mild | Normal | Strong | Yes |
| D12 | Overcast | Mild | High | Strong | Yes |
| D13 | Overcast | Hot | Normal | Weak | Yes |
| D14 | Rain | Mild | High | Strong | No |

9Yes/5No

1. Purity (Impurity Criterion)
   - 1.1 Entropy $I_H = -\sum_{j=1}^{c} p_j \log_2(p_j)$
   - 1.2 Gini Index $I_G = 1 - \sum_{j=1}^{c} p_j^2$
2. Information Gain [How features are selected]

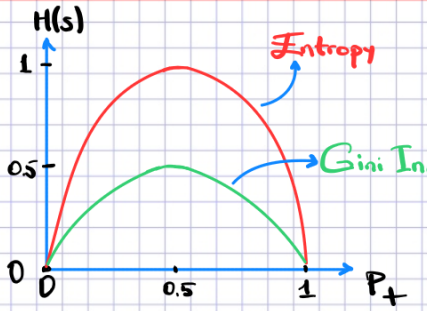
C: number of classes
$p_j$: probability of randomly picking an element of class $i$.

## 1. Purity

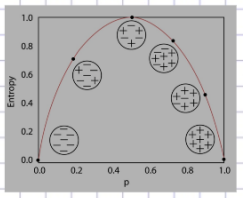
### 1.1 Entropy

binary $H(s) = -p_+ \log_2 p_+ - p_- \log_2 p_-$

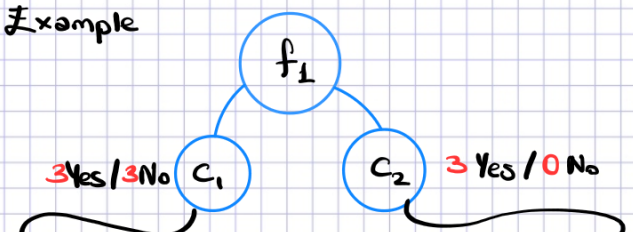
multi $H(s) = -p_{c_1} \log_2 p_{c_1} - p_{c_2} \log_2 p_{c_2} - p_{c_3} \log_2 p_{c_3}$




$\log_b a = x \Leftrightarrow b^x = a$




Example




$H(s) = -\frac{3}{6}\log_2\left(\frac{3}{6}\right) - \left(\frac{3}{6}\right)\log_2\left(\frac{3}{6}\right)$

$H(s) = 1$ Very impure split

$H(s) = -\frac{3}{3}\log_2\left(\frac{3}{3}\right) =$

$= -\log_2(1) = 0$ Pure Split

$H(s) = -\frac{Y}{Y+N}\log_2\left(\frac{Y}{Y+N}\right) - \frac{N}{Y+N}\log_2\left(\frac{N}{Y+N}\right)$

### 1.2 Gini Index

$G.I. = 1 - \sum_{i=1}^{n} (p)^2$

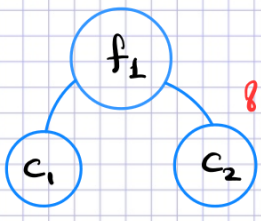
p stands for "proportion" or probability of randomly picking an element of class $i$.

Example:

$G.I. = 1 - \left[(P_+)^2 + (P_-)^2\right]$

$= 1 - \left[\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2\right]$

$= 1 - 1/2 = 0.5$




8 Yes / 2 No

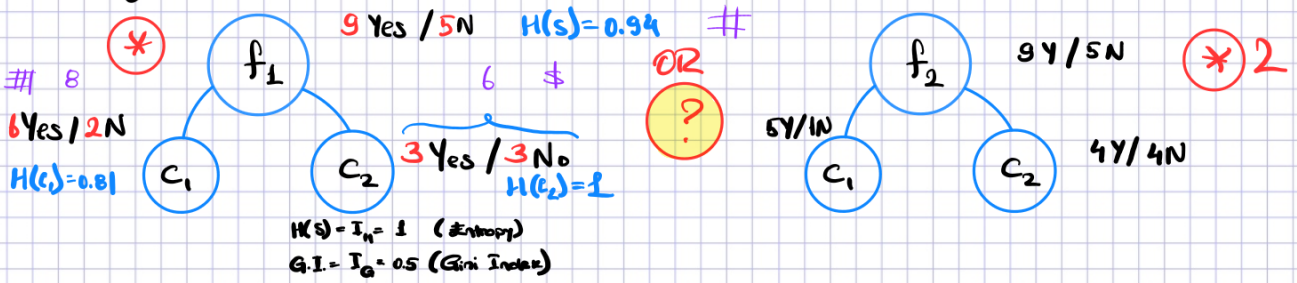
$G.I. = 1 - \left[\left(\frac{8}{10}\right)^2 + \left(\frac{2}{10}\right)^2\right] =$

$= 1 - \left(\frac{64}{100} + \frac{4}{100}\right)$

$= 1 - 0.68 = 0.32$

# 2. Information Gain

## 2.1 Categorical Features




$H(S) = I_H = 1$ (Entropy)

$G.I. = I_G = 0.5$ (Gini Index)

Feature, that comes first into order of splitting, can be defined by Information Gain.

Gain $(s, f_i)$: Gain of '$f_i$' with respect to the sample 's'

$$Gain(s, f_i) = H(s) - \sum_{v \in values} \frac{|s_v|}{|s|} \cdot H(s_v)$$

Information Gain is calculated for the entire tree, not 1 Node

Example (*): $H(s) = -p_+ \log_2 p_+ - p_- \log_2 p_-$

$$= -\left(\frac{9}{14}\right) \cdot \log_2\left(\frac{9}{14}\right) - \left(\frac{5}{14}\right) \cdot \log_2\left(\frac{5}{14}\right) \approx 0.94$$

$$H(C_1) = -(0.75) \cdot \log_2(0.75) - 0.125 \cdot \log_2(0.125) = 0.81$$

$$H(C_2) = -\left(\frac{3}{6}\right) \cdot \log_2\left(\frac{3}{6}\right) - \left(\frac{3}{6}\right) \cdot \log_2\left(\frac{3}{6}\right) = \log_2\left(\frac{1}{2}\right) \cdot (-1) = -\log_2\left(\frac{1}{2}\right) = 1$$

$$Gain(s, f_1) = 0.94 - \left[\frac{8}{14}(0.81) + \frac{6}{14} \cdot 1\right] = 0.0486$$

Example (*)2: $H(s) = 0.94$

$$H(C_1) = -\frac{5}{6}\log_2\left(\frac{5}{6}\right) - \frac{1}{6}\log_2\left(\frac{1}{6}\right) = -\frac{5}{6} \cdot (-0.23) - \frac{1}{6} \cdot (-2.58) = 0.19 + 0.43 = 0.62$$

$$H(C_2) = 1$$

$$Gain(s, f_2) = 0.94 - \left[\frac{6}{14} \cdot 0.62 + \frac{8}{14}\right] = 0.94 - 0.837 = 0.103$$

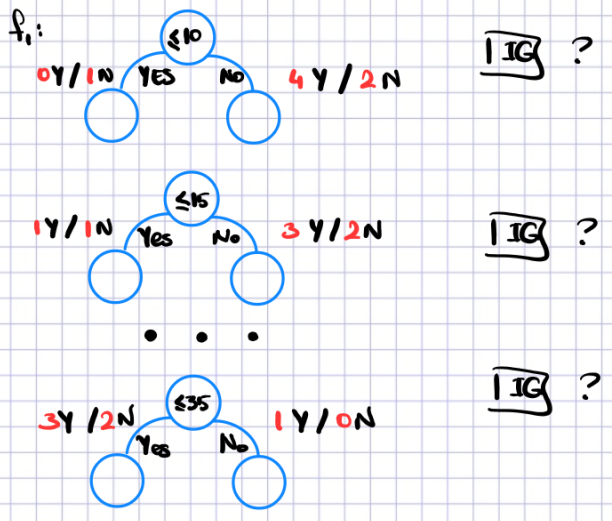
$I.G.(f_2) > I.G.(f_1)$

## 2.2 Numerical Features

CART

Example:

| $f_1$ | $f_2$ | o/p |
|---|---|---|
| 10 | - | 0 |
| 15 | - | 1 |
| 20 | - | 1 |
| 25 | . | 0 |
| 30 | . | 1 |
| 35 | . | 0 |
| 40 | . | 1 |

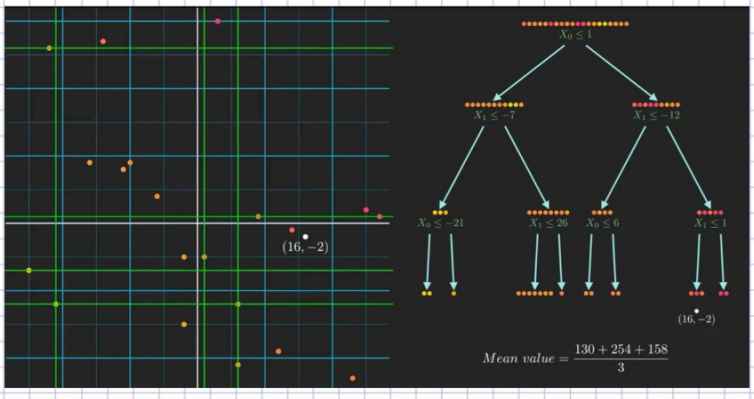


# Decision Tree Regressor

| $f_1$ | $f_2$ | Price | |
|---|---|---|---|
| - | - | 20 | } Mean |
| - | - | 24 | |
| - | - | 28 | |
| - | - | 14 | |
| - | - | 16 | |
| - | - | 20 | |

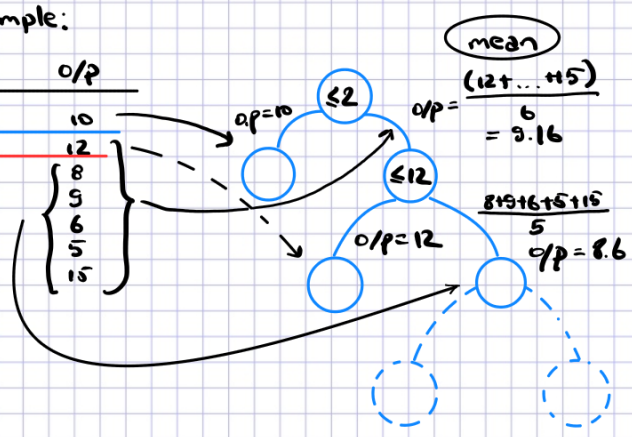
$$MSE = \frac{1}{2m}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

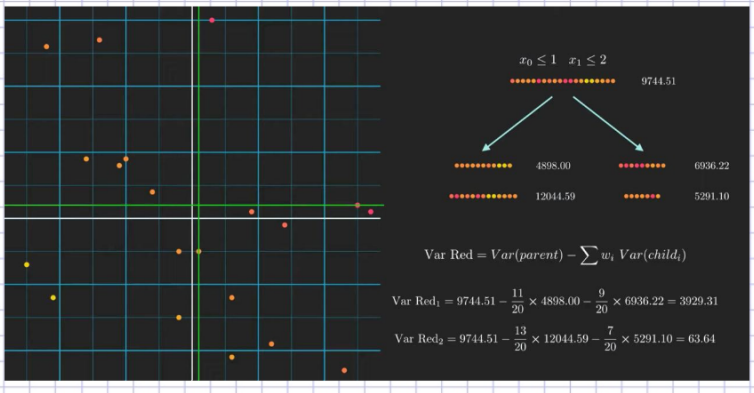
$\hat{y}_i$ → mean




Example:

| $f_1$ | o/p |
|---|---|
| 2 | 10 |
| 4 | 12 |
| 6 | 8 |
| 8 | 9 |
| 10 | 6 |
| 20 | 5 |
| 40 | 15 |




The main goal, while splitting is to minimize MSE or MAE

Overfitting: Training accuracy ↑ } Low bias
Test Accuracy ↓ } High variance

To avoid overfitting in Decision Tree Algorithm we can use:
1. post pruning
2. pre pruning