Section 4: PROBABILITY

Simple probability

Probability: How likely it is that some event will occur. All probabilities are numbers equal to or between 0 and 1. This formula applies when all possible outcomes are equally likely.

$$P(\text{event}) = \frac{\text{outcomes that meet our criteria}}{\text{all possible equally likely outcomes}}$$

Sample space: The collection of 'all possible outcomes' from the denominator of the simple probability formula

Experiment: One event in the sample space

Experimental/empirical probability: The probability we find when we run experiments. Experimental probability changes as we run experiments over time. If the experiment is a good one, the experimental probability should get very close to the theoretical probability as we run more and more experiments

Theoretical/classical probability: The probability that an event will occur, based on an infinite number of experiments. This is the probability we get from the simple probability formula.

Law of large numbers: This law tells us that if we could run an infinite number of experiments, the experimental probability would eventually equal the theoretical probability.

The addition rule, and union vs intersection

Event: A specific collection of outcomes from the sample space

Addition rule, sum rule:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Mutually exclusive, disjoint events: Events that can't both occur. In this case, $P(A \text{ and } B) = P(A \cap B) = 0$. Therefore, for mutually exclusive events, the addition rule simplifies to $P(A \text{ or } B) = P(A) + P(B)$, or $P(A \cup B) = P(A) + P(B)$.

Union of events: $P(A \cup B)$ is the union of A and B, and it means the probability of either A or B or both occuring.

Intersection of events: $P(A \cap B)$ is the intersection of A and B, and it means the probability of A and B both occuring.

Independent and dependent events and conditional probability

Independent events: Events that don't affect one another, like two separate coin flips

Multiplication rule:

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

Dependent events: Events that affect one another, like pulling two cards from a deck without replacing the first card before pulling the second

Conditional probability: The probability that multiple dependent events occur

Bayes' Theorem

Bayes' Theorem/Law/Rule: Tells us the probability of an event, given prior knowledge of related events that occured earlier. To solve problems with Bayes' Theorem, write out what you know, build a tree diagram that includes all possibilities, and then 'trim the branches' of your tree that aren't relevant to the question being asked.

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$