Section 5: DISCRETE RANDOM VARIABLES

Discrete Probability

Discrete random variable: A variable that can only take on discrete values

Continuous random variable: A variable that can take on any value in a certain interval

Expected value: The mean of a discrete random variable

Variance of discrete random variable:

$$\sigma_x^2 = \sum_{i=1}^{n} (X_i - \mu)^2 P(X_i)$$

Transforming random variables

- Shifting the data set doesn't affect standard deviation
- Scaling the data set by k scales standard deviation by k

Combinations of random variables

Mean and variance of the combination of normally distributed variables:

	Combination	Mean	Variance
Sum	$S = X + Y$	$\mu_S = \mu_X + \mu_Y$	$\sigma_S^2 = \sigma_X^2 + \sigma_Y^2$
Difference	$D = X - Y$	$\mu_D = \mu_X - \mu_Y$	$\sigma_D^2 = \sigma_X^2 - \sigma_Y^2$

Permutations and Combinations

Permutation: The number of ways we can arrange a set of things, and the order of the arrangement matters.

$${}_nP_k = \frac{n!}{(n-k)!}$$

Combination: The number of ways we can arrange a set of things, but the order of the arrangement doesn't matter

$${}_nC_k = \frac{n!}{k!(n-k)!}$$

Binomial random variables

Binomial variable: A variable that can take on exactly two values, like a coin flip. In order for a variable X to be a binomial random variable,

- each trial must be independent,
- each trial can be called a "success" or "failure",
- there are a fixed number of trials, and
- the probability of success on each trial is constant.

Binomial probability:

$$P(k \text{ successes in } n \text{ attempts}) = \binom{n}{k} p^k (1-p)^{n-k}$$

Poisson distributions

Poisson process: Calculates the number of times an event occurs in a period of time, or in a particular area, or over some distance, or within any other kind of measurement.

1. The experiment counts the number of occurences of an event over some other measurement,
2. The mean is the same for each interval,
3. The count of events in each interval is independent of the other intervals,

4. The intervals don't overlap, and

5. The probability of the event occuring is proportional to the period of time.

Poisson probability:

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

Poisson probability for a binomial random variable:

$$P(x) = \frac{(np)^x e^{-np}}{x!}$$

"At least" and "at most", and mean, variance and standard deviation

Probability of at least one succes or failure:

P(at least 1 success) = 1 - P(all failures)

P(at least 1 failure) = 1 - P(all successes)

Mean, variance, and standard deviation of a binomial random variable:

Mean: $\mu_X = E(X) = np$

Variance: $\sigma_X^2 = np(1-p)$

Standard deviation: $\sigma_X = \sqrt{np(1-p)}$

Bernoulli random variables

Bernoulli random variable: A special category of binomial random variables, with exactly one trial, in which "success" is defined as a 1 and "failure" is defined as a 0

Mean, variance, and standard deviation of a Bernoulli random variable:

Mean: $\mu = p$

Variance: $\sigma^2 = p(1-p)$

Standard deviation: $\sigma = \sqrt{p(1-p)}$

Geometric random variables

Geometric random variable: We run an infinite number of trials until we get some defined "success".

- Each trial must be independent,
- Each trial can be called a "success" or "failure", and
- The probability of success on each trial is constant.

Probability of success on the n^{th} attempt:

$$P(S=n) = p(1-p)^{n-1}$$

Mean, variance, and standard deviation of a geometric random variable:

Mean: $\mu_X = E(X) = 1/p$

Variance: $\sigma_X^2 = \frac{1-p}{p^2}$

Standard deviation: $\sigma_X = \sqrt{\frac{1-p}{p^2}}$