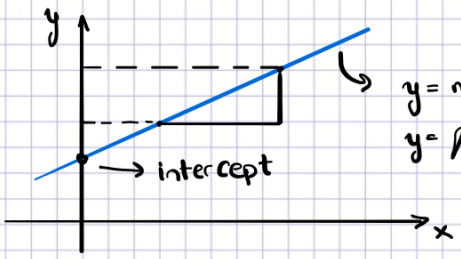
26.3 Equation of Line, 3D plane, and Hyperplane (n-Dimension)

Line



$y = mx + c$ (m = slope, c = intercept)

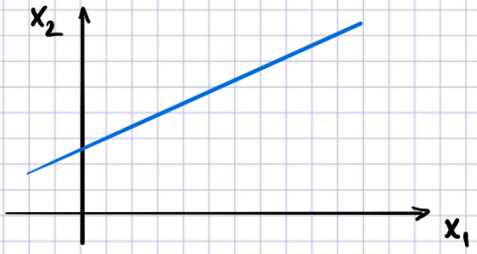
$y = \beta_0 + \beta_1 x$

$ax + by + c = 0$

$by + c = -ax$

$by = -ax - c$

$y = \frac{-a}{b}x - \frac{c}{b}$ ($\frac{-a}{b} = m$, $\frac{c}{b} = c$)



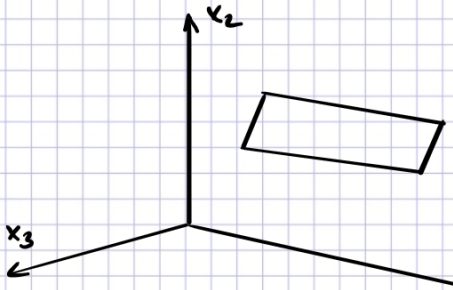
$a x + b y + c = 0$

$w_1 x_1 + w_2 x_2 + b = 0$

$w^T x + b = 0$

Equation of a straight line

3D plane



$w_1 x_1 + w_2 x_2 + w_3 x_3 + b = 0$

$w^T x + b = 0$

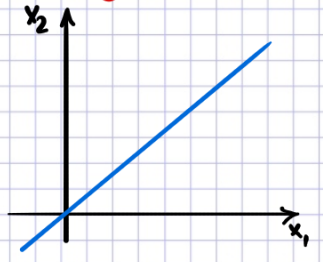
$w = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$ $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

n-Dimension plane

$w_1 x_1 + w_2 x_2 + w_3 x_3 + \dots + w_{n-1} x_{n-1} + w_n x_n = 0$

$w^T x + b = 0$

Straight line passing through an origin



$w_1 x_1 + w_2 x_2 + b = 0$ $\quad b = 0$

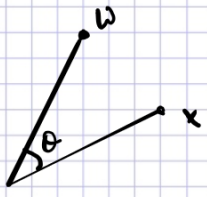
$w_1 x_1 + w_2 x_2 = 0$

$w^T x = 0$

Equation of a straight line passing through an origin

Equation of a plane π_n : $w^T x = 0$

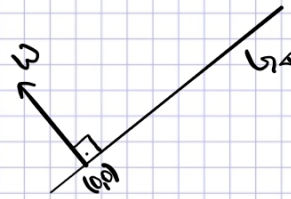
$w = \begin{bmatrix} w_1 \\ w_2 \\ \dots \\ w_{n-1} \\ w_n \end{bmatrix}$ $x = \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_{n-1} \\ x_n \end{bmatrix}$



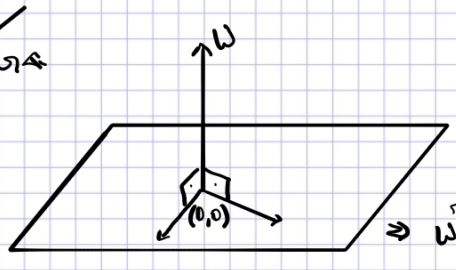
$w^T x = 0$

$w \cdot x = w^T x = \|w\| \|x\| \cos\theta = 0$, when $\theta = 90$ and $\cos\theta = 0$

↳ dot product



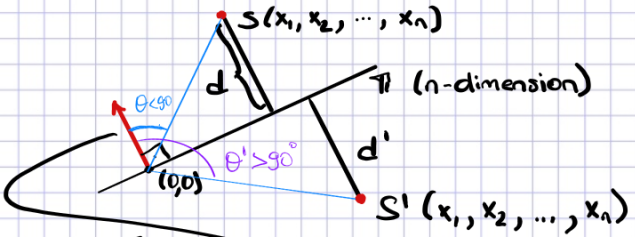
intercept = 0



$\Rightarrow w^T x = 0$, $w \perp \pi$

26.4 Distance of a point from plane

$w^T x = 0$



distance $d = \frac{w^T s}{\|w\|}$

$w^T s = \|w\| \|s\| \cos\theta$

since $\theta < 90 \Rightarrow \cos\theta$ is going to be positive

$d' = \frac{w^T s'}{\|w\|} \Rightarrow w^T s' = \|w\| \cdot \|s\| \cdot \cos\theta =$ will be negative value