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An Improved Median Filtering Algorithm for Image Noise Reduction *

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Abstract

The median filtering algorithm has good noise-reducing effects, but its time complexity is not desirable. The paper proposed an improved median filtering algorithm. The algorithm uses the correlation of the image to process the features of the filtering mask over the image. It can adaptively resize the mask according to noise levels of the mask. The statistical histogram is also introduced in the searching process of the median value. Experimental results show that the algorithm reduces the noise and retains the details of the image. The complexity of the algorithm is decreased to O (N), and the performance of noise reduction has effectively improved.

Keywords: Image Processing; Median Filtering; Adaptive Filtering; Statistical Histogram

1. Introduction

Noises may be arisen in the capturing and transmission process of the image. The noise is usually divided into Gaussian noise, the balanced noise and the impulse noise. The arisen impulse noises display as light and dark noise pixels under random distribution on the image. This not only corrupts true information of the image, but also seriously affects the visual effects of the image. Therefore, the reduction of impulse noises has important significance to image processing and computer vision analysis.

For an image corrupted by noises, we can use linear or nonlinear filter methods to reduce noises. In the frequency domain, the details are high-frequency components of the image, which easily confused with high-frequency noises. Therefore, how to keep the image details and effectively filter random noises is the key to image filtering processing. The median filter is a nonlinear filter and it has widely used in digital image processing because of its good edge keeping characteristics and reducing impulse noise ability. The median filter is a rank-order filter. Its noise-reducing effects depend on the size and shape of the filtering

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mask; and its algorithm complexity mainly depends on how to get the median value. In order to improve the noise-reducing performance of the median filter, scholars proposed many improved methods to the conventional median filter [1-3]. To improve the searching speed of the median value, people proposed some fast algorithms based on the dividing-conquering strategy, and simplified the algorithm complexity of the conventional median filter from O (N^2) to O (n ln n) in references [4] and [5]. The prophase work [6] of the paper further simplified complexity to O (n(1+ln n) /2). Based on the prophase study, this paper proposed two improvements to the median filtering algorithm:

(1) To improve the noise-reducing performance, the mask may be adaptively resized according to noise levels of the mask;

(2) According to the median filtering theory, we only require quickly find the median value of the filtering mask, and not to rank all the pixels of the filtering mask. Therefore, the statistical histogram is introduced in the searching process of the median value to speed up the searching process.

2. Median filtering Theory

The median filter is a nonlinear signal processing technology based on statistics. The noisy value of the digital image or the sequence is replaced by the median value of the neighborhood (mask). The pixels of the mask are ranked in the order of their gray levels, and the median value of the group is stored to replace the noisy value. The median filtering output is $g(x,y) = med\{f(x-i, y-j), i, j \in W\}$, where $f(x,y)$, $g(x,y)$ are the original image and the output image respectively, W is the two-dimensional mask: the mask size is n$\times$ n(where n is commonly odd) such as 3×3, 5×5, and etc.; the mask shape may be linear, square, circular, cross, and etc.

The noise-reducing performance of the median filter:

Because the median filter is a nonlinear filter, its mathematical analysis is relatively complex for the image with random noise. For an image with zero mean noise under normal distribution, the noise variance of the median filtering is approximately [7]

$$\sigma_{med}^2 = \frac{1}{4n f^2(\overline{n})} \approx \frac{\sigma_i{}^2}{n + \frac{\pi}{2} - 1} \cdot \frac{\pi}{2}. \tag{1}$$

where σ_i^2 is input noise power (the variance), n is the size of the median filtering mask, $f(\overline{n})$ is the function of the noise density. And the noise variance of the average filtering is

$$\sigma_0^2 = \frac{1}{n}\sigma_i^2. \tag{2}$$

Comparing of (1) and (2), the median filtering effects depend on two things: the size of the mask, and the distribution of the noise. The median filtering performance of random noise reduction is better than the average filtering performance, but to the impulse noise, especially narrow pulses are farther apart and the pulse width is less than $n/2$, the median filter is very effective. The median filtering performance should be improved if the median filtering algorithm, combined with the average filtering algorithm, can adaptively resize the mask according to the noise density. Based on this, the paper proposed an improved median filtering algorithm.

3. Improved Median Filtering Algorithm

A. Improvement of the filtering mask

The filtering mask is mainly $n \times n$ square mask or cross mask. Considering of the symmetry of the mask, n is commonly odd. The smaller the mask is, the better the image details are retained, the weaker the noise reduction performance is; the larger the mask is, the less the image details are retained, the stronger the noise reduction performance is. To solve the contradiction, we introduce the adaptive filtering algorithm. In the filtering process, it can adaptively resize the mask according to noise levels of the mask.

In the mask, *max* is the maximum value of gray levels, *min* is the minimum value of gray levels, *average* is the average value of gray levels, *med* is the median value of gray levels, $f(i, j)$ is the central value of the mask, n is the size of the mask. The adaptive filtering requires two steps:

Step 1: adaptively resizing the mask

(1) Initialization: let $n = 3$;

(2) Computation: $A1 = med - \min, A2 = med - \max$

(3) Judgment: if $A1 > 0$ and $A2 < 0$, then turn to the step 2; if not, then enlarge the size of the mask, let $n = n + 2$ and turn to (2).

Step 2: median filtering.

B. Improvements of the median algorithm

Because the average filter has better performance for filtering random noises, we combine the median filter with the average filter to certain size of the filtering mask. The improved method can reduce noises and retain the image details better.

Improving ideas:

For the natural image, neighboring pixels has strong correlation. The gray value of each pixel is quite close to neighboring pixels, and the edge pixels have the same property also. If the value of a pixel is greater or less than the value in the neighborhood, the pixel is contaminated by the noise; otherwise, the pixel is an available pixel. In the reducing-noise process, we sequentially check each pixel, if the value of a pixel is greater than the average value in the mask, then we judge that the pixel is contaminated by the noise and replace it with the median value of the mask; otherwise, we retain the original value of the pixel unchanged. This method not only reduces the computation time, but also retains the details of the image as far as possible. The original value of the pixel is replaced with the median value in the mask, and the next process of computation the average value may make full use of the new value of the pixel. This forms an iterative process; it not only decreases the time complexity, but also improves the noise-reducing effect better.

For example: reducing-noise of the pixel (i, j), the neighborhood size is 3×3.

If $f - average > 0$, then the median value is $f^{/}(i, j)$. If $f^{/}(i, j) < f(i, j)$, then $f^{/}(i, j)$ is the noise. By the conventional algorithm, the average and median value of the pixel $(i, j+1)$ are respectively

$$\begin{aligned} Average = \{ & f(i-1, j) + \cdots f(i, j) \\ & + f(i, j+1) + \cdots f(i+1, j+2)\} / 9 \end{aligned} \tag{3}$$

$$Med = \{f(i-1,j) + \cdots f(i,j) + f(i,j+1) + \cdots f(i+1,j+2)\} \tag{4}$$

If $f(i,j)$ is replaced by the factor of improved algorithm $f^{'}(i,j)$, the average and median value are respectively

$$average = \{f(i-1,j) + \cdots + f^{/}(i,j) + f(i,j+1) + \cdots f(i+1,j+2)\}/9 \tag{5}$$

$$med = \{f(i-1,j) + \cdots + f^{/}(i,j) + f(i,j+1) + \cdots f(i+1,j+2)\} \tag{6}$$

According to $f^{/}(i,j) < f(i,j)$, *average* is less than *Average*. Thus, the spatial extent of the noise reduction is increase, and the time complexity of the improved algorithm is less than the conventional algorithm. Steps of the improved algorithm are shown as below.

(1) The mask slides over the image, overlaps the center of the mask with the pixel on the image to search the center element $f(i,j)$;

(2) To read the values of the corresponding pixels of the mask;

(3) To compute the average value (*average*) of the mask;

(4) To compare the value of each pixel with *average*, if the value of each pixel is greater than *average*, then searching the median value and let $f(i,j) = med$; otherwise, retaining the original value of the pixel unchanged;

(5) Repeating the step (4), until $i = j = n$.

Fast computation of the median value:

The complexity of the algorithm is mainly decided by the computation of the median value on above steps. The paper introduces the statistical histogram to improve the searching speed of the median value. The method requires below steps:

(1) To compute the gray histogram $hist[i]$ ($0<i<G$, where G is the range of the gray) of the $n \times n$ mask, let $N = n \times n$, find the median value med, and record $ltmed$ (the number of the pixel value which is less than med);

(2) To let the left row shift out of the histogram, if the value of the shifting out pixels is less than med, then $ltmed - 1$;

(3) To let the right row shift in the histogram, if the value of the shifting in pixels is less than med, then $ltmed + 1$;

(4) If $ltmed < N/2$, then repeat $med + 1$, $ltmed + hist[med]$, until $ltmed = N/2$;

(5) If $ltmed > N/2$, then repeat $med - 1$, $ltmed - hist[med]$, until $ltmed = N/2$;

(6) To return the median value med.

The improved algorithm has two improvements compared to the conventional median filtering algorithm. One is to make the number of the compared pixels equal to N by using the historical information of the sliding mask, and the value of each pixel always compares to the original median value of the mask. Another is to make the complexity of the median algorithm decrease greatly by using the characteristic of the limited gray-level distribution range.

C. Analysis of the complexity of the algorithm

Suppose $X = \{X_i\}$ (i=1, 2, … , N) is the array to solve the median value, where $0 \le x_i \le M$ and x_i is integer. By using the statistical histogram method to find the median value, the required maximum number is N, and the complexity of the algorithm is approximately O (N).

TABLE I Comparison of the Complexities of Three Algorithms

Size of the mask	the standard median filtering algorithm N ln N	the fast median filtering algorithm based on average N (1+lnN)/2	the improved algorithm in the paper N
3×3	19	14	9
5×5	80	52	25
7×7	190	119	49
9×9	355	218	81
11×11	580	350	121

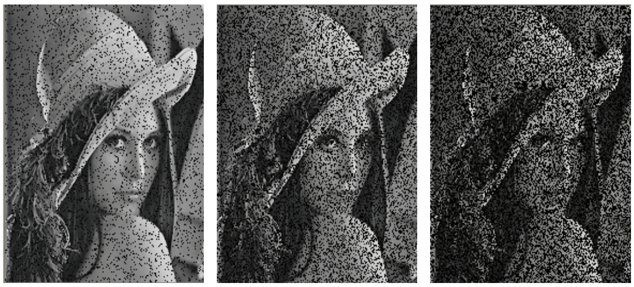
According to TABLE I, the computation complexity of the improved algorithm is obviously reduced.

4. Simulation Experiments

Experiment 1: comparative experiment among the standard median filtering algorithm, the fast median filtering algorithm based on average and the improved algorithm in the paper.

10%, 35%, and 45% density impulse noises are respectively added to the original image of Lena. With VC++6.0, results of the comparative experiment are shown as Fig. 1.

(a) Original image of Lena



(b1) 10% (b2) 35% (b3) 45%

Lena image with the impulse noises

(c1) (c2) (c3)

Results of the standard median filtering algorithm

(d1) (d2) (d3)

Results of the fast median filtering algorithm based on average

(e1) (e2) (e3)

Results of the improved algorithm in the paper

Fig. 1 Comparative experiment

Experiment 2: low signal to noise ratio experiment.

60%, 70%, and 80% density impulse noises are respectively added to the original image of Lena. Results of the improved algorithm in the paper are shown as Fig. 2.

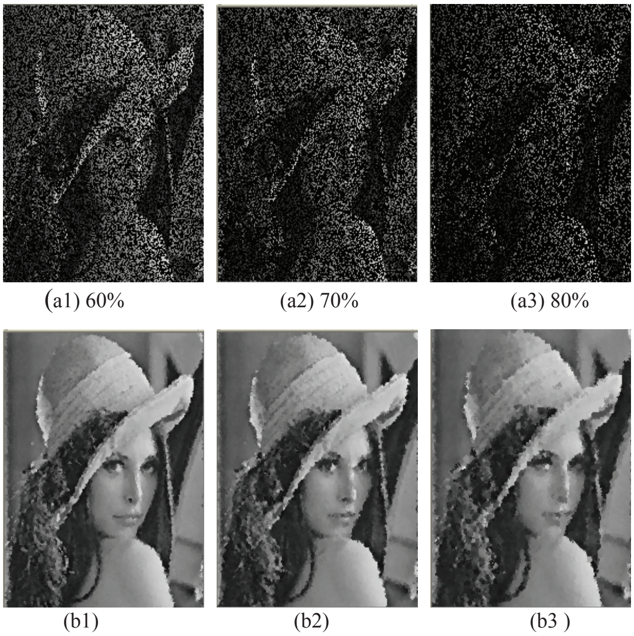


Fig. 2 low signal to noise ratio experiment results of the improved algorithm

Performance estimation:

The effect of the image noise reduction may estimated by the subjective visual effect or the objective estimation method. The paper takes the peak signal to noise ratio (*PSNR*) and the signal to noise ratio (*SNR*) as the performance estimation standard.

Suppose an original image is $f(i,j)$ and its size is $M \times N$, the processed image is $f_{OUT}(i,j)$ and its size is $M \times N$, where $i = 1,2,...,M$, $j = 1,2,...,N$, then we have

$$MSE = \frac{1}{M \times N} \sum_{i}^{M} \sum_{j}^{N} (f_{OUT}(i,j) - f(i,j))^2. \tag{7}$$

$$PSNR = 10 \lg(\frac{a_{\max}^2}{MSE})(dB). \tag{8}$$

$$SUM = \frac{\sum_{i}^{M} \sum_{j}^{N} f(i,j)^2}{\sum_{i}^{M} \sum_{j}^{N} (f_{OUT}(i,j) - f(i,j))^2} \tag{9}$$

$$SNR = 10 \lg(SUM)(dB). \tag{10}$$

where $a_{\max} = 2^k - 1$, k denotes the number of a pixel binary bit. If k =8, then $a_{\max} = 255$. The results of two experiments are shown in the TABLE II.

Experimental results show that the performance of the improved algorithm in the paper is better than the standard median filtering algorithm and the fast median filtering algorithm based on average. Especially in low signal to noise ratio condition, the improved algorithm has obvious advantages.

TABLE II Comparison of the Performance of Three Algorithms

Noise density	the standard median filtering algorithm		the fast median filtering algorithm based on average		the improved algorithm in the paper	
	PSNR(dB)	SNR(dB)	PSNR(dB)	SNR(dB)	PSNR(dB)	SNR(dB)
10%	31.2439	25.5618	31.3804	25.6983	32.1180	26.1059
35%	28.7432	22.0611	28.8262	22.1441	31.8541	25.1720
45%	16.9419	11.2598	17.3371	11.6550	30.5521	23.8699
60%	—	—	—	—	28.5861	21.6371
70%	—	—	—	—	27.2651	20.3327
80%	—	—	—	—	26.1206	19.1103

5. Conclusions

The paper proposed an improved median filtering algorithm for image noise reduction. It can adaptively resize the mask according to noise levels of the mask. Combined the median filtering with the average filtering, the improved algorithm can reduce the noise and retain the image details better. The statistical histogram is introduced to improve the searching speed of the median value and the correlation of image has been fully used. Thus, the complexity of the improved algorithm is decreased to O (N). Experimental results show that the improved algorithm can well do with the relationship between the effect of the noise reduction and the time complexity of the algorithm. Therefore, it has a good application prospect in image processing.

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