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A computer simulation of the flying disc based on the wind tunnel test data

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Abstract

The purpose of this study was validation of a computer simulation on the flight of the flying disc. The flying disc, widely known as the Frisbee™, is thrown with rotation for its stable flight. The flight characteristics of the flying disc were identical in comparison with other flying sports materials. The flying disc can keep its lift and glide in the air by the wing effect. However, the traveling direction of the disc was steeply changed in particular conditions. In order to explain this phenomenon, the precise simulation of the disc movement is necessary. In this study, a computer simulation of the flying disc was accomplished. In advance of the simulation, both the three-axial forces and moments acting on the disc were measured in a wind tunnel test for the simulation. For the validation of the simulation results, the authors conducted an experiment using a motion capture system to obtain real flying disc kinematical parameters.

Keywords: Flying disc; computer simulation; aerodynamics

Nomenclature

m	Mass of the disc.
$\mathbf{I}$	Moment of inertia tensor of the disc.
Parameters related on the global coordinate system	
$\mathbf{P}$	Position vector of center of mass for disc on the global coordinate system.
$\mathbf{V}$	Velocity vector of center of mass for disc on the global coordinate system.

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$\mathbf{F}$	Force vector acting on the disc on the global coordinate system.
$\mathbf{M}$	Moment vector on the disc on the global coordinate system.
β	Elevation angle on the global coordinate system.
$\mathbf{g}$	Gravitational acceleration.
Parameters related on the disc fixed coordinate system.	
$\mathbf{F}_d$	Force vector on the disc on the disc coordinate system.
$\mathbf{M}_d$	Moment vector on the disc on the disc coordinate system.
$\mathbf{v}$	Velocity vector of center of mass for disc on the disc coordinate system.
$\boldsymbol{\omega}$	Angular velocity vector on the disc coordinate system.
α	Angle of attack on the disc coordinate system.
Parameters for coordinate transformation	
q_0, q_1, q_2, q_3	Quaternion parameters.

1. Introduction

A flying disc is used as a simple entertainment tool and is commonly known as a Frisbee^{TM}. The flight of the disc is different compared to other sport projectiles. The rotational axis is vertical to the surface of the disc. This provides stability during the flight. A wing effect provides lift force that enables the disc to glide in the air. However, the velocity vector of the disc changes steeply during its flight in some conditions. It is important to simulate accurately the movement and attitude of the disc to understand such a phenomenon. In this paper, we solved the equation of motion numerically to simulate the movement and attitude of the flying disc based on the wind tunnel test data. The results of the simulation were validated using the actual flight data. Tri-axial forces and moments acting on the disc were measured using the wind tunnel test for the simulation. The result of the simulation was compared with the actual flight data measured by motion capture system.

Several researchers have previously developed flight simulations of flying discs. Hubbard and Hummel [2] developed a numerical simulation model of flying disc flight with the data used in their research acquired from actual flight experiments. Hummel [3] compared simulated trajectories to experimental flight data. Crowther and Potts [1] developed a spinning disc wing and simulated straight flight. Based on previous work and in order to understand the phenomenon such as the steep change of the traveling direction of the disc, we validated the accuracy of the simulation based on the wind tunnel test data in this paper.

2. Mathematical model

2.1. Coordinate system

Two coordinate systems were used in this research. The first was a global Cartesian coordinate system whose Y axis is along with the throwing direction and Z axis is in the upward direction (Fig. 1). The second was defined on the disc (Fig. 2). This disc coordinate system is also a Cartesian coordinate system. The x- axis and the y-axis were fixed on the surface of the Disc while the z-axis was perpendicular to them. The direction of each axis was fixed on the disc. The origin of this system was fixed on the center of the disc. The disc axes were written in small letters to distinguish them from those of the global coordinates.

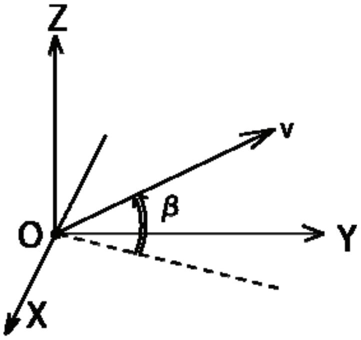

Fig. 1. The global coordinate system

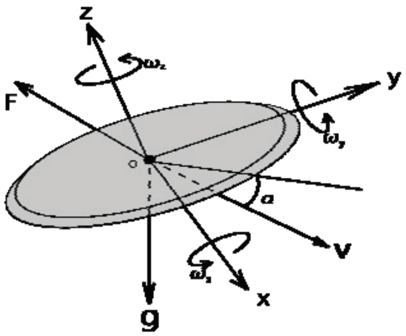

Fig. 2. The disc fixed coordinate system

The transformation of the coordinate systems is performed using a quaternion. The relationship between the forces in both the global and local coordinate systems is defined by the equation below,

$$\mathbf{F} = \mathbf{Q}\mathbf{F}_d \quad (1)$$

Here, the transformation matrix ($\mathbf{Q}$) could be shown as follows,

$$\mathbf{Q} = \begin{bmatrix} 1-2(q_2^2+q_3^2) & 2(q_1q_2+q_0q_3) & 2(q_1q_3-q_0q_2) \\ 2(q_1q_2-q_0q_3) & 1-2(q_1^2+q_3^2) & 2(q_2q_3+q_0q_1) \\ 2(q_1q_3+q_0q_2) & 2(q_2q_3-p_0p_1) & 1-2(q_1^2+q_2^2) \end{bmatrix} \quad (2)$$

The differential calculus of quaternion and each quaternion parameter are shown as following equations,

$$q_0 = \cos\Theta/2, \quad [q_1 \quad q_2 \quad q_3]^T = \mathbf{u}\sin\Theta/2, \quad (3)$$

$$\begin{bmatrix} \dot{q}_0 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -q_1 & q_0 & -q_3 & q_2 \\ -q_2 & q_3 & q_0 & -q_1 \\ -q_3 & -q_2 & q_1 & q_0 \end{bmatrix}^T \boldsymbol{\omega} \quad (4)$$

Here, $\mathbf{u}$ was a unit vector of the rotational axis and Θ was the angle of rotation.

2.2. Equations of motion

The equations of motion were defined as follows,

$$m(\dot{\mathbf{v}} + \boldsymbol{\omega}\times\mathbf{v}) = \mathbf{F}_d - m\mathbf{Q}^T\mathbf{g}, \quad (5)$$

$$\mathbf{I}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega}\times\mathbf{I}\boldsymbol{\omega} = \mathbf{M}_d, \quad (6)$$

$$\mathbf{I} = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}, \quad I_{xx} = I_{yy} = 0.0046 \text{ (kg m}^2\text{)}, \quad I_{zz} = 0.0096 \text{ (kg m}^2\text{)}, \quad m = 0.235 \text{ (kg)}. \tag{7}$$

Here, these equations were written in the disc coordinate system. $\mathbf{F}_d$ and $\mathbf{M}_d$ were decided by $\|v\|$, ω_z and α . The equations above were numerically solved. The quaternion was used to transform the disc coordinate system to the global coordinate system.

3. Experiment

For the simulation, a wind tunnel experiment was conducted. A jig was attached on the center of the disc and rotated. The disc was rotated in the principal axis of inertia from 0 to 14 revolutions per second. We also changed the angle of attack of the disc α from +90°to -10°and changed the wind velocity from 0 m/s to 20 m/s. The wind tunnel test was shown in Fig. 3. The definition of the parameters was shown in Fig. 4.

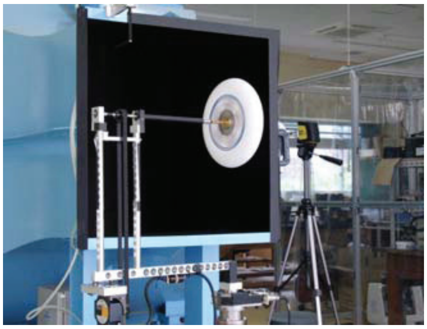

Fig. 3. wind tunnel test

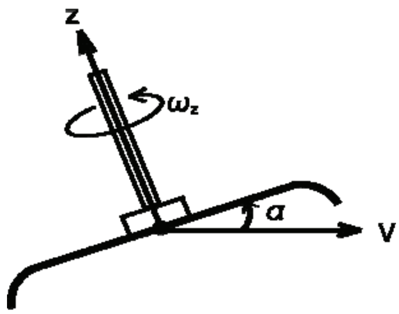

Fig. 4. Definition of parameters

The kinematical parameters were acquired using a motion capture system to validate the results of the simulation. The kinematical parameters were acquired from the release instant to about 10 (m). The measured flight time was 0.908 seconds. These measured parameters were compared with the results of the simulation.

4. Results

The kinematical parameters such as the position, velocity on the global coordinate system, angular velocity of the z-axis on the disc coordinate system and angle of attack were compared with the measured data. The angle of attack was defined as the angle between the velocity vector and the plane of the disc. The elevation angle was defined as the angle between the X-Y plane on the global coordinate system and the velocity vector. The angle of attack α and the elevation angle β were defined as shown below,

$$\alpha = -\tan^{-1}(v_z / \sqrt{v_x^2 + v_y^2}), \quad \beta = -\tan^{-1}(V_z / \sqrt{V_x^2 + V_y^2}). \tag{8}$$

The initial conditions given in the simulation were measured by the motion capture system. $\omega_x(0)$ and $\omega_y(0)$ were assumed to be zero, in order to simplify the simulation. The origin of the global coordinate system at the initial instant was set at the center of the disc. These parameters were measured by the actual flight experiment. Other initial conditions were indicated as follows,

$$\alpha(0) = 1.4 \text{ (deg)}, \qquad \beta(0) = 6.7 \text{ (deg)}, \qquad \boldsymbol{\omega}(0) = [0 \quad 0 \quad 48.54]^T \text{ (rad/s)}, \tag{9}$$

$$\mathbf{P}(0) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T, \quad v_0 = 13.6 \text{ (m/s)}, \quad \mathbf{V}(0) = v_0 \begin{bmatrix} 0 & \cos\beta(0) & \sin\beta(0) \end{bmatrix}^T, \tag{10}$$

$$\begin{bmatrix} q_0(0) & q_1(0) & q_2(0) & q_3(0) \end{bmatrix}^T = \begin{bmatrix} 0.9863 & 0.0698 & -0.1492 & -0.0106 \end{bmatrix}^T. \tag{11}$$

The following figures indicate the comparison of the measured parameters and the results of the simulation. Each figure indicates $\mathbf{P}$ [Px, Py, Pz] (Fig. 5,6), $\mathbf{V}$ [Vx, Vy, Vz] (Fig. 7,8), ω_z (Fig. 9) and α (Fig. 10). A solid line expresses the results of the simulation and the dashed line expresses the measurement data, respectively.

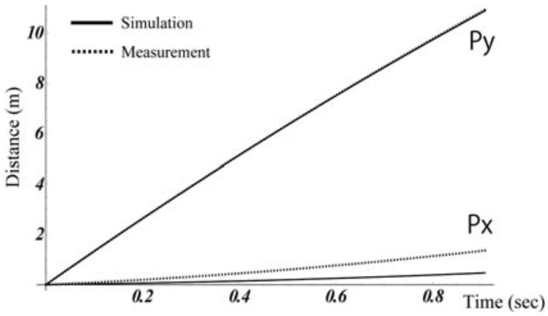

Fig. 5. Comparison of the Px, Py

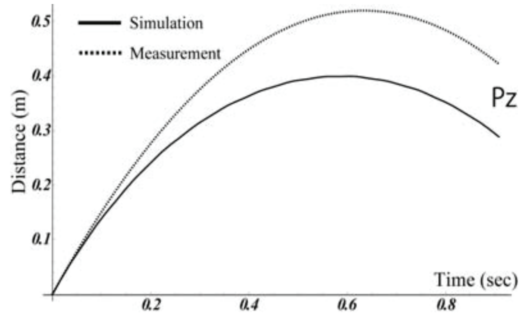

Fig. 6. Comparison of the Pz

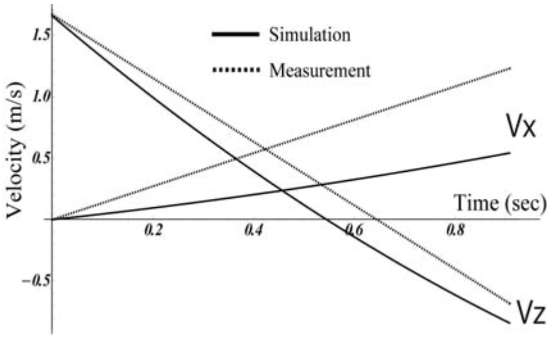

Fig. 7. Comparison of the Vx, Vz

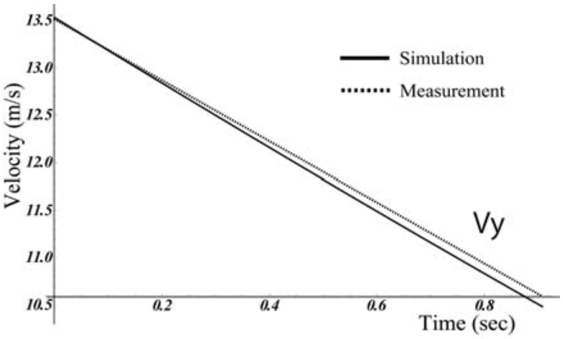

Fig. 8. Comparison of the Vy

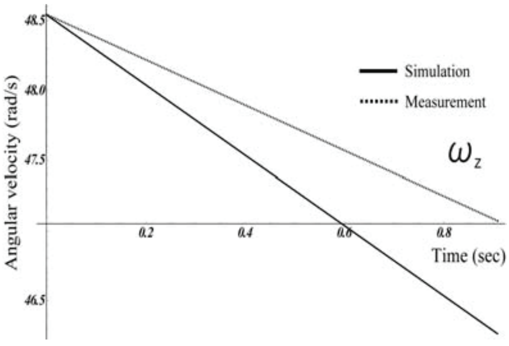

Fig. 9. Comparison of the ω_z

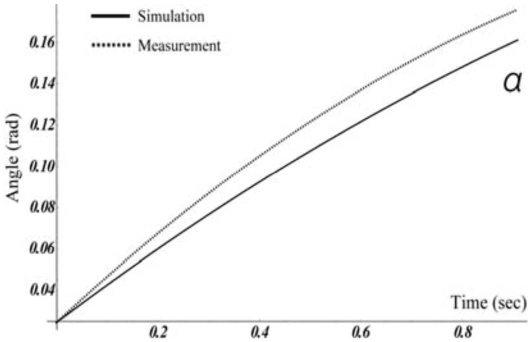

Fig. 10. comparison of the angle of attack

The difference of each parameter at t= 0.908 (s) was shown in Table 1.

Table 1. Difference between the result of the simulations and actual disc flight data

Parameters [t= 0.908 (sec)]	Px (m)	Py (m)	Pz (m)	Vx (m/s)	Vy (m/s)	Vz (m/s)	ω_z (rad/s)	α (rad)
Actual flight	1.363	10.925	0.422	1.229	10.595	-0.683	47.071	0.176
Simulation	0.521	10.886	0.289	0.521	10.522	-0.703	46.375	0.161
Difference	0.842	0.029	0.133	0.707	0.073	-0.020	0.696	0.015

The results of the comparison indicate that the rate of change of simulated Vx was small compared with the actual flight. As for the Vz, the rate of change of Vz was large compared with the actual flight. The result of the simulated Vy was almost equal to the actual flight data. The rate of the change of the simulated ω_z was large compared with the measured data. As for the angle of attack, the simulated α was almost similar to the actual angle.

5. Discussion

Differences were observed between the results of the simulation and measured data in the Vx and Vz. The rate of change of the simulated Vx was small compared with the actual flight while that of Vz was large. On the other hand, the results of the simulated Y-axis parameters (Py and Vy) were almost equal to the actual flight data. As for ω_z, the rate of change of the simulated ω_z was large compared with the measured data. It is thought that the initial conditions of ω_x and ω_y produced the error in the simulation. In order to simplify the simulation, the initial conditions of $\omega_x(0)$ and $\omega_y(0)$ were inputted as zero. This simplification may have caused the error of the X-axis parameters. Alternatively, it is also thought that the cause of the error might be the difference between the normal disc and the disc used in the actual flight experiment. The mass of the disc used in the actual flight experiment is 60 grams heavier than a normal disc and the inertia of the disc is large compared with that of a normal disc. It is thought that difference of mass might produce the error of the parameters of the Z-axis which were mainly affected on the lift force. Furthermore, it is also believed that a difference of inertia might cause the error of the parameters in the X-axis.

6. Conclusion

The equation of motion was numerically solved based on the wind tunnel test data. The result of the simulation was compared with the actual flight data. The simulated angle of attack and Py were similar to the actual flight data. On the contrary, the differences in Px and Pz were large compared with the Py. As for ω_z, the rate of the change of the simulated ω_z was large compared with the measured data. It was suggested that these differences might be caused by the initial conditions of the angular velocities and the difference of physical properties between two discs used in the wind tunnel test and actual flight experiment. For further study, it is necessary to improve the accuracy of the simulate on and conduct an experiment with the remarkable and unique changes of flying disc movements.

Reference

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