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Maximum range of flying discs

Peter Lissaman[a], Mont Hubbard[b,*]

[a]Da Vinci Ventures, 1454 Miracerros Loop, Santa Fe, NM 87505, USA
[b]Sports Biomechanics Lab, Dept. Mechanical & Aerospace Engineeing, University of California, Davis, CA 95616 USA

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Abstract

Since the original 1963 Frisbee flying discs have evolved, becoming ubiquitous and far-ranging. Effects of drag, lift, pitching moment and spin on range are analysed. With lift, ranges significantly exceed the drag-free ballistic range. An upper limit wings level case, optimized for range, is compared with the realistic free case of a spinning, banking disc. For 30 m/s launch and 0.033 minimum drag coefficient the wings level range is 186 m, but the free range only 157 m. Tests report ranges between 77 - 170 m. The procedure is a powerful tool for designing superior discs.

Keywords: Flying discs: Frisbee: flight dynamics

1. Introduction

From time immemorial men and apes have hurled things (feces, rocks, sticks and spears) at other animals that displeased them. As missiles, these illustrate the military principle of action at a distance. Simian feces-slinging is reportedly quite accurate, so apes, as well as people, must have noted that range depends on the launch speed and angle (V,θ) as well as weight, size and shape. For a flattish projectile launched roughly horizontally, aerodynamic lift extends range significantly. The flying disc is a prolific and far-ranging example of a lifting projectile, with many variants. The original Frisbee, since 1963 a well-known commercial sport product, has evolved into the low drag, long-range professional, competition disc-golf version. The Aerobie flying disc, a ring-shaped advanced design, in 2003 achieved a range exceeding 400 m, the furthest of any hand thrown projectile. The present work gives a theoretical upper bound to range, assuming an optimized, controlled "wings level" trajectory and compares this with the predicted flight of a "free" spinning disc, banking in response to spin and pitching moment.

* Corresponding author. Tel.: 1-530-752-6450; fax: 1-530-752-4158.
E-mail address: mhubbard@ucdavis.edu.

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2. Fundamental ballistics

2.1. Ballistic trajectories

For an object with no external forces except gravity g the trajectory is called ballistic and range given by

$R = (V^2/g) \sin 2\theta$. Range varies with V^2, as expected, since it defines launch kinetic energy. Ballistic range is maximized for a 45^o launch. The effect of launch angle on range was well known to artillerymen in the 15th century; Leonardo drew a cartoon c. 1505 illustrating this for mortar trajectories.

2.2. Effect of drag

For a projectile with only aerodynamic drag, the range is less than the ballistic value. The effect of drag is shown in Fig. 1, plotting trajectories for a number of non-lifting projectiles launched at 24 m/s and 45^o. The cannon ball range is close to the normalized ballistic range of 6.25. It is little affected by drag; the ping-pong ball profoundly so. For a sport disc (Frisbee is a commercial trademark) at a minimum drag coefficient of 0.080 and zero lift the range is about 3.6, much less than the drag-free case. Disc range can increase significantly with lift.

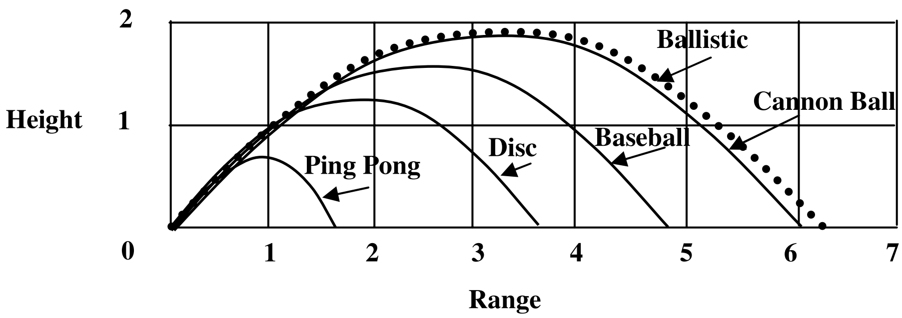

Fig. 1. Trajectories of common projectiles

2.3. Effect of lift

It was apparent to the original Olympic Greek athletes that the range of a classical symmetric discus can be increased by exploiting lift. Calculations for a standard discus with lift by Hubbard and Cheng [1] show a 10% increase over the ballistic range. But lift causes induced drag, so that the drag of a lifting system exceeds the minimum drag. Yet the lift, even with induced drag, produces longer range than the drag-free case. A near ballistic trajectory terminates almost at launch speed, so that little kinetic energy is lost. But with lift this kinetic energy can extend range, resulting in a low touch down speed, as described below

A lifting vehicle has maximum glide range at the lift coefficient corresponding to maximum lift/drag ratio. The vehicle glides at constant flight speed and angle, extracting potential energy to offset drag losses. Glide should start from maximum height. To maximize height the vehicle should ascend at minimum drag (zero lift) so that most of the launch kinetic energy is converted into height (potential energy). This requires a roughly ballistic trajectory during ascent. The vehicle then glides at cruise speed until the ground is approached, when lift is increased to extend the range at the expense of forward speed. For aircraft landing this is called the flare-out.

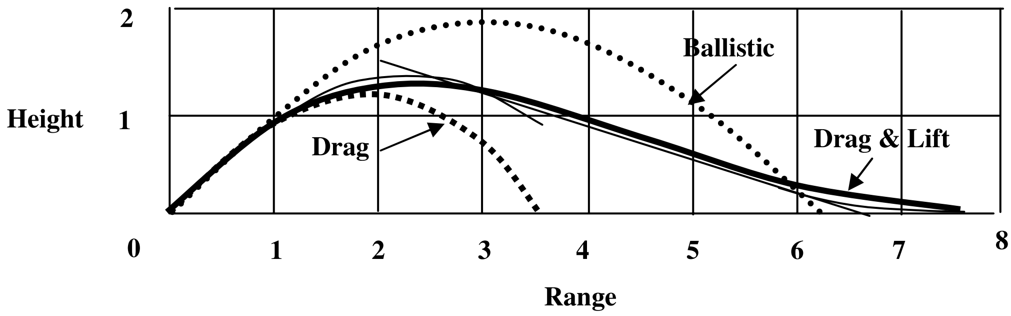

Fig. 2 Trajectories of sport disc with drag and lift

Some trajectories typical of a sport disc, described in 2.2, are shown above in Fig. 2. For a drag-free ballistic trajectory the normalized range is about 6.25; with drag alone it is only 3.6. Exploiting lift the range is about 7.2, exceeding the ideal drag-free case. The fine lines show the three asymptotic regimes, the low drag, almost parabolic ascent, the steady glide descent at optimum glide angle and the flare-out to horizontal as the ground is approached.

3. Controlled wings-level trajectories

3.1. Ideal dynamic flight equations

An ideal case of maximum range is obtained assuming that the disc flies wings level (without banking) and that the lift coefficient is controlled to provide the optimal schedule.. The flight dynamics are described by the equations of motion for an unpowered vehicle by Lissaman [2], where the dynamic flight trajectory of the albatross, soaring, turning and diving in a planetary boundary layer wind without wing flapping, is modelled and optimized. The case here is simpler, with no bank (wings level) and no ambient wind. The non-dimensional equations contain only one control variable, the normalized lift coefficient, that can be optimally scheduled to maximize the range.

3.2. Optimal trajectories

A typical optimized trajectory, using 50 equal spaced lift-coefficient control inputs, illustrating the above, for a sport disc with a minimum drag coefficient of 0.080 launched at 25 m/s is shown in Fig. 3. Normalized plots of the control lift, the flight speed, and the trajectory are shown.

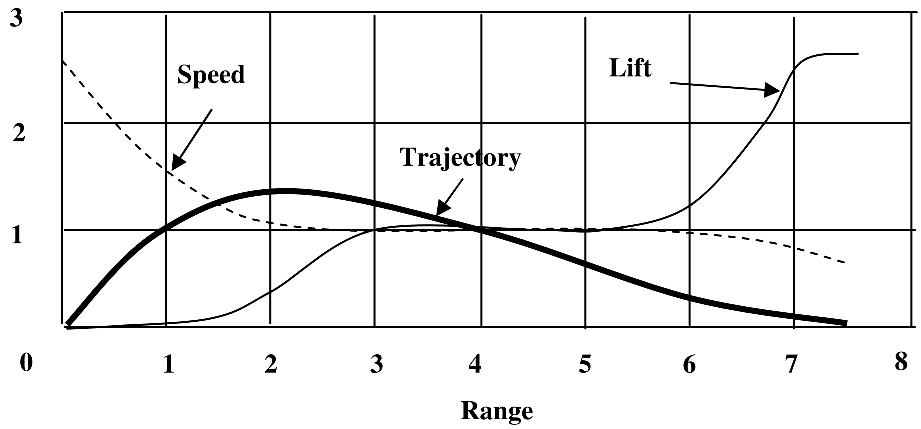

Fig. 3 Typical wings level sport disc trajectory with optimum lift

The lift starts at zero, increases smoothly to the cruise lift (normalized here as unity), holds constant for the glide and then rises to the maximum for flare out (the lift plateau represents the stall lift). During the glide portion the speed is constant (normalized as unity) at an approximately constant glide angle, as predicted.

For a low drag competition disc (C_{D0} = 0.033) launched at 30 m/s the wings level model predicts a range of about 186 m, implying an augmentation above the ballistic of about 101%. Pozzy (2000) presents data for 26 launches of different discs by experts at speeds between 29 and 31 m/s, reporting an average range of about 123 m with a maximum of about 170 m.

3.3. Disc attitude

The angle of attack (angle of the velocity with respect to the disc plane) for an optimal wings level trajectory is that required for optimal lift. The disc ascends in a roughly ballistic parabola, so that, for minimum drag, the angle is that for zero lift, approximately parallel to the flight path. The descent is at cruise conditions, requiring a constant attitude (disc angle to the horizontal) and lift, while the final stage calls for a high angle of attack to extend the flare-out. So the disc must start ascending at an attitude of about 45°, level off at the apex and pitch over to cruise attitude for the glide descent, with a final pitch up for the flare. A projectile with stabilizing fins, like a bomb, does not execute this trajectory, since the fins force a near zero angle of attack and lift throughout the flight.

The free disc will not execute this schedule. Rather it pitches and banks according to the coupling between its aerodynamic pitching moment and the gyroscopic effects due to spin. This free trajectory is discussed next.

4. Free trajectories

4.1. Mechanics of free trajectory

For the real case, the disc is released with an initial launch speed, spin rate and attitude. Discs are normally unstable due to displacement of the center of lift ahead of the center of mass (c.m.), and, to avoid tumbling, must be stabilized by spin. The pitching moment couples gyroscopically with the spin to induce a roll rate. This rate is right side upwards for a pitch up moment on a disc spinning clockwise viewed from above. Long range is achieved by exploiting the gyroscopic terms so that the disc acquires a roughly wings level state near the apex and thereafter glides at approximately constant attitude, giving the optimum lift/drag ratio. This requires that the disc be launched at the correct ascentangle, usually with a pronounced bank, calling for skill by the thrower.

It is not possible to design the pitching moment curve so that the disc actually follows the ideal orientation calculated in the optimal case. In fact, the variation of the pitching moment at disc angles of attack between -5° and 10° has a very pronounced effect on the free trajectory. To calculate the trajectory, a Matlab code following Hubbard and Hummel [3] has been written and is exercised for different launch conditions and pitching moment characteristics. The former are controlled by the disc thrower, the latter by the disc designer. The code requires input of the disc physical properties and launch conditions as well as fundamental aerodynamic force and moment derivatives. The latter are not well known since the disc experiences separated flow over the top, and cavity flow on the lower surface; a complicated flow state, beyond the capability of prediction by computational methods. Wind tunnel tests on rotating discs by Potts reported by Hubbard and Hummel [3] indicate that the aerodynamic center of the disc changes significantly with angle of attack for attack angles below 10°. This is expected from the complex upper and lower surface separated flow for disc-like shapes. Computational fluid dynamics methods cannot correctly predict this flow, so that testing is required for accurate drag and pitching characteristics.

4.2. Maximum range cases

The disc is launched at a speed and spin of 30 m/s and 50 rad/s. Constants are 0.175 kg mass; 0.27 m diameter; spin axis moment of inertia, I = 0.0025 kg m^2; lift at zero angle of attack, C_{L0} = 0.15; lift curve slope, C_{LA} = 2.91; minimum drag, C_{D0} = 0.033; span effectiveness, e = 0.85; pitch damping, C_{Mq} = - 0.10; roll damping, C_{Lp} = - 0.15.

Bank angles and trajectories for best range for a typical high performance disc for wings level and for free flight are shown in Figs. 4 and 5. The wings level and free trajectories are similar except for the flare-out, even though the free case is far from wings level, involving large, near vertical, bank angles during the initial ascent.

The range increase over ballistic compared with the sport disc is apparent. The free case starts with a large – 70° bank (right side up) levels off before mid-flight and does not flare. The pitching moment characteristics for the longest range are those of Case 1, close to those measured on a sport disc.

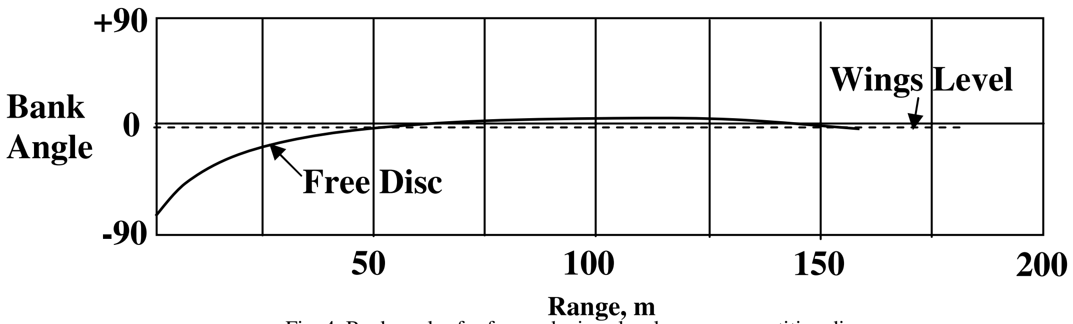

Fig. 4 Bank angles for free and wings level cases, competition disc

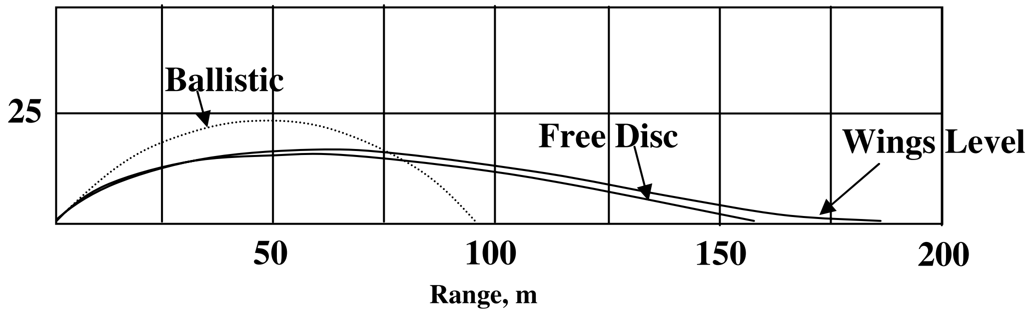

Fig. 5 Trajectories for ballistic, free and wings level cases, competition disc

The pitching moment characteristics have a dominant effect on the flight. Fig. 6 shows the pitching moment coefficients used in the cases analyzed in the flight dynamics model.

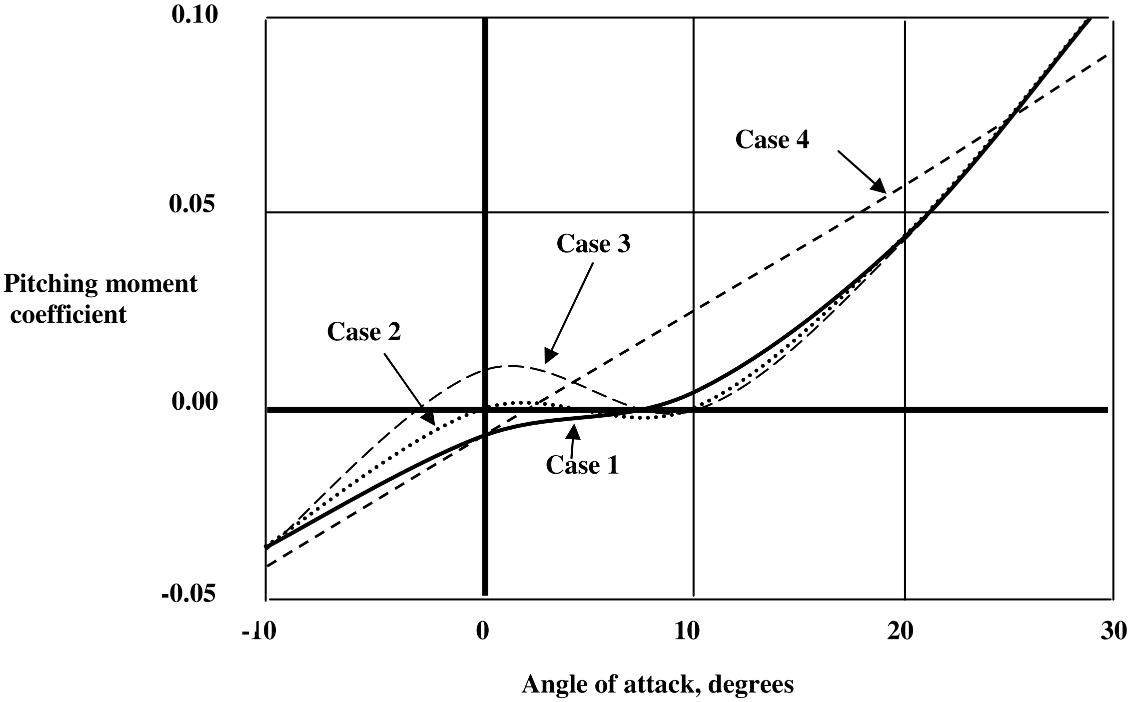

Fig. 6 Representative pitching moment characteristics

Four cases have been analysed, each having representative pitching moments. They are: Case 1: from measurements on an actual disc, with a slightly unstable moment, as given by Potts wind tunnel tests of a sport disc, described by Hubbard and Hummel [3], Case 2: a neutral case, where the moment gradient between 0^o and 10^o is zero, Case 3: where the moment has a slight stable gradient in this range and Case 4, where the moment is linear and unstable, corresponding to a fixed aerodynamic center at 6.3% of the diameter ahead of the c.m. The four cases were analyzed, optimum initial conditions for each determined, and maximum range calculated, as tabulated in Table I. Case 4 gave a very low range and results have not been listed. It is of interest that the Case 4 “normal”, unstable linear pitching moment, associated with a shape with a fixed aerodynamic center, does not provide suitable moment characteristics for good performance from a free, spinning disc.

Sensitivity is given by calculating the range reduction due to a 5^o change in both launch bank and angle. The stable case, 3, is very sensitive to small errors in launch, and would require much skill to throw consistently.

Five cases for the competition disc are summarized in Table I. Spin is clockwise viewed from above and positive bank angle right wing down. Case 3 has a slightly stable pitching moment and initial bank is in the opposite direction to that of Cases 1 and 2. In Case 3 there is a large change associated with a small reduction in initial optimum bank angle. For launch at 36^o ascent angle and 52^o bank, Case 3 range drops by more than 66 m from the optimum of 135 m listed in the table. This needs interpretation.

Table I. Range for varying cases

Case	Stability	Ascent Angle, ^{o}	Bank, ^{o}	Range, m	Sensitivity, m
Free, Case 1	Unstable	41	-75	157	2.0
Free, Case 2	Neutral	8	-5	144	1.3
Free, Case 3	Stable	31	57	135	66.7
Pozzy	NA	NA	NA	184	NA
Wings level	NA	44	0	186	NA

5. Field tests

On Dec. 20, 2009 field tests were conducted using competition disc-golf discs thrown by professional disc golfers (G. Lissaman and J. Lissaman) and by amateurs (D. Peterson and R. McMurray) Range and launch speeds were measured and trajectories recorded with 30 Hz video for 100 flights. Observed ranges compare well with those of Pozzy [4]. Although the professional throwers had mean release velocities slightly, but not significantly, less than those of the amateurs (V_p=22.9, V_a=23.3m/s), their velocities were more repeatable (σ_{vp}=0.7, σ_{va}=1.2 m/s) and they achieved larger (and again more repeatable) mean ranges (R_p=81.8, R_a=69.9 m; σ_{Rp}=10.8, σ_{Ra}=16.6 m). Detailed records of trajectories and velocity will permit estimation of disc aerodynamic properties in the future.

6. Conclusions

1. Methods of calculating the range of a disc for the ideal wings level case and for the spinning and banking free case have been developed and exercised.
2. Range depends strongly on the minimum drag coefficient. For competition discs with minimum drag coefficient of 0.033 launched at 30 m/s the ideal wings level case range is about 186 m. Free disc range is 157 m.
3. The free range depends significantly on the pitching moments in the angle of attack range $0^o < \alpha < 10^o$.
4. The slightly unstable pitching moment characteristics obtaining in Free Case 1 give the best range for the free cases considered.
5. A field test procedure has been developed and exercised. Test results support the free disc analysis, but the theory cannot be exactly correlated with the experiment without accurate values of aerodynamic characteristics . The data will be reduced and these characteristics will be reported later.

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