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Measurement of kinematics of a flying disc using an accelerometer

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Abstract

In this study, a tri-axial accelerometer was attached to a flying disc to measure the kinematics during flight. A subject performed disc throwing. High speed video cameras were used to capture release and end points to validate boundary measurements. Results of the acceleration wave showed a sinusoidal pattern during the flight phase. According to the results of an additional verification experiment, we classified components of the acceleration wave during the flight phase to estimate the angular velocity of the disc, and validated it using high speed video cameras.

Keywords: accelerometer, inertia sensor , flying disc, angular velocity ;

Nomenclature

$\sum XYZ$	Global coordinate system
$\sum xyz$	Local coordinate system
$\mathbf{a}:(a_x,\ a_y,\ a_z)$	Measured acceleration
$\mathbf{g}$	Gravitational acceleration in the global coordinate system
$\ddot{\mathbf{R}}$	Translational acceleration in the global coordinate system
$\boldsymbol{\omega}:(\omega_x,\ \omega_y,\ \omega_z)$	Angular velocity in the local coordinate system
$\dot{\boldsymbol{\omega}}:(\dot{\omega}_x,\ \dot{\omega}_y,\ \dot{\omega}_z)$	Angular acceleration in the local coordinate system
$\mathbf{r}:(r_x,\ r_y,\ r_z)$	Position vector in the local coordinate system

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1. Background

The purpose of this research is to estimate the angular velocity of a flying disc using a tri-axial accelerometer. The authors propose an original method for the estimation of the direction of the rotation. Understanding the aerodynamics of a flying object is one of the major topics in fluid mechanics in sports engineering. Wind tunnel tests and image analysis have been mainly used to measure fluid forces [1][2][6][7]. However, both of these methods have several difficulties in their application. The experimental conditions of the wind tunnel are different from those of actual flying objects in that the object must be fixed to the wind tunnel. As for image analysis, it is difficult to observe a flying object continuously. Essential kinematical parameters to understand the aerodynamic forces are the translational and angular velocities and the rotational axis of the object. For the actual flight phase, the measurement of these parameters is traditionally only accomplished by using image analysis. However this depends on camera performance and experimental conditions. Thus it is difficult to observe flying objects continuously during an entire flight phase. There was no previous work about the objects that were observed a drag boundary layer during flight. It is very difficult to determine and confirm such phenomena with the image analysis method. In this research, the authors propose a new method to measure the kinematical parameters by an object-mounted inertia sensor. The authors made a sensor baseball built-in an accelerometer and validated the acceleration components with controlled experiments in the laboratory in addition to a real pitching experiment. The authors concluded that the aero dynamic forces acting on the baseball might have the effect of shifting the rotational axis during the ball flight phase[4]. However, estimating the rotational axis of the ball is quite complicated and it is difficult to observe a small axis shift using image analysis. Therefore, in this research, the authors study flying disc behavior because of its simple rotation. The purpose of this research was to establish a kinematical measurement method for the flying disc.

2. Methods

2.1. Equipment

Figure 1 shows the wireless piezo-electric acceleration sensor, Cheviot (AR'S Co. Ltd). The measurement range of this accelerometer is ±12g. The sampling rate of the accelerometer is 1000Hz. The measured acceleration data are transmitted to a receiver by a wireless 2.4GHz transmitter. This accelerometer was attached beneath the flying disc. The distance between the center of the disc and sensor was 50mm. Balancing weights adjusted the center of gravity with the center of the disc (Fig. 2). Figure 3 shows a schematic illustration of the flying disc. The mass of the disc for measurement is 206 grams about 30 grams heavier than usual. For the analysis, right handed Cartesian coordinate system was adopted on the disc. The direction of each axis is along the accelerometer's axis. Each axial direction is described in the Fig. 3.

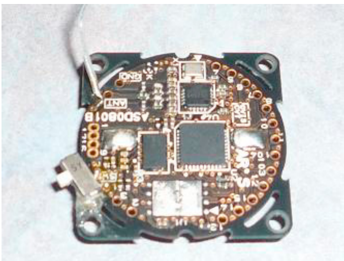

Fig. 1 : Wireless accelerometer

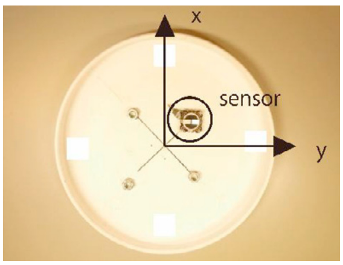

Fig. 2 : Sensor flying disc

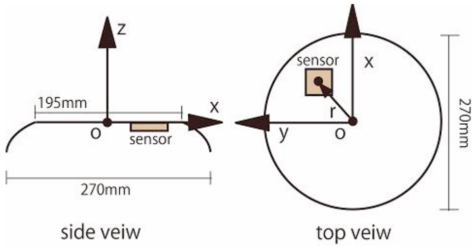

Fig. 3 : Schematic illustration of the flying disc

2.2. Experiment

A subject performed throwing using the illustrated flying disc. Close up release and end point images were acquired using two high speed cameras (1000Hz) both to obtain the disc trajectory and its rotation at the release and end instants. The authors adopted high speed videography with the DLT method. The distance of the disc's flight was about 20m. For image analysis, identification markers were applied on the surface of the disc. Figure 4 is a schematic illustration of the experiment. For image analysis, a right handed Cartesian coordinate system whose Y axis was along the throwing direction and with the Z axis upward was adopted as the global coordinate system.

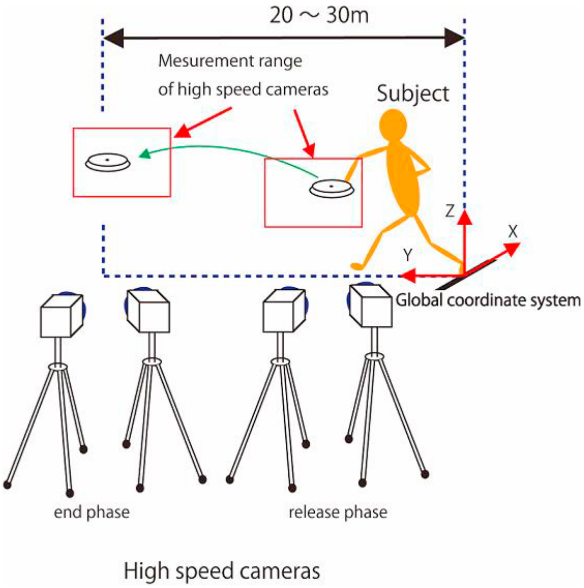

Fig. 4 : Schematic illustration of the experiment

2.3. Results of the experiment

Figure 5 shows experimental acceleration data during the flight phase. It showed a sinusoidal wave pattern in its x and y axes.

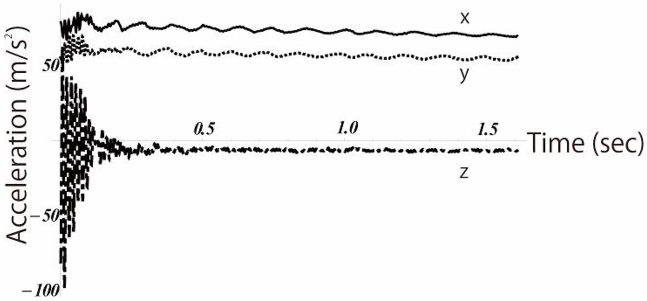

Fig. 5 : Acceleration data during the flight phase

3. Methodology of angular velocity estimation

3.1. Decomposition of the acceleration component

The acceleration data obtained in this experiment has four components, gravitational, translational, centrifugal and tangential accelerations. Ohta et al. demonstrated the angular velocity estimation using the accelerometer data which was attached to a moving object [5]. They proposed that the sensor acceleration can be expressed as follows.

$$\mathbf{a} = \ddot{\mathbf{R}} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + \dot{\boldsymbol{\omega}} \times \mathbf{r} - \mathbf{g} \tag{1}$$

The authors conducted some controlled experiments to validate the acceleration components for the estimating the angular velocity and rotational axis of a ball in flight[3]. The controlled experiments were a rotational table experiment and a free fall experiment. The results of the controlled experiment showed that the gravitational and translational accelerations were indicated as the amplitude of the acceleration wave. On the contrary, the centrifugal force was indicated as the DC (0Hz) component of the acceleration wave (Fig. 6). In our experiment, translational acceleration was caused by aerodynamic forces such as drag and lift acting on the flying object.

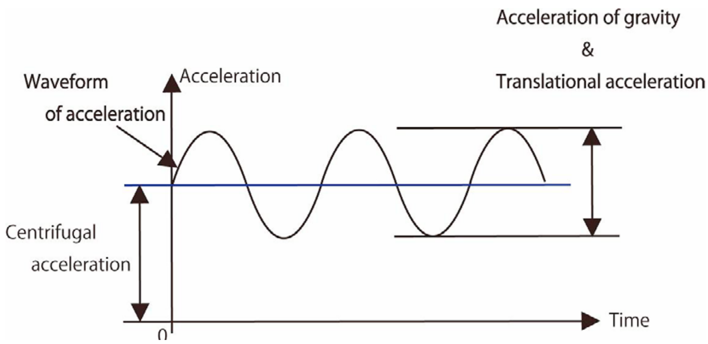

Fig. 6 : Acceleration components of the flying ball

3.2. Estimation of the angular velocity

Since gravity and the aerodynamic forces, such as drag and lift, always act in a constant direction with respect to the global coordinate system, the illustrated disc could have sinusoidal force effects during the flight phase, because their directions always change in the local coordinate system. On the other hand, the direction of the centrifugal acceleration is constant in the local coordinate system of the flying disc. Therefore, the orientation of the resultant vector of the aerodynamic forces and the gravitational acceleration change continuously in the local coordinate system. The direction change of the gravitational and translational accelerations in the local coordinate system causes a periodic change of the acceleration in the experiment. As the result, the accelerometer of the x and y axes attached to the disc, outputs a sinusoidal wave pattern.

In order to reveal the frequency characteristics of these wave patterns and eliminate the gravitational and translational accelerations, a discrete Fourier transformation was conducted. Figure 7 shows the power spectra of the x and y accelerations shown in Fig. 5. From Fig. 7, the power spectrum of the acceleration data has its peak at about 10Hz. Thus, this peak frequency component was from the observed acceleration.

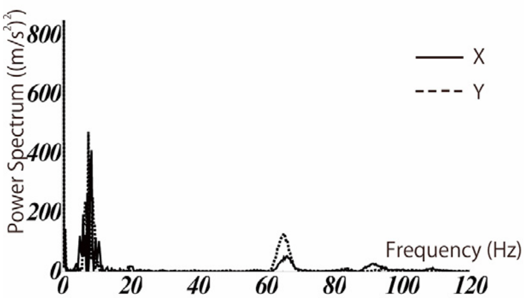

Fig. 7: Power spectrum of x and y axis of the acceleration

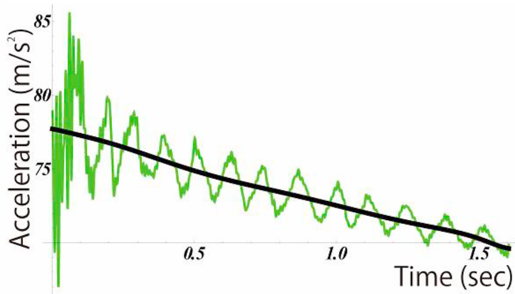

Fig. 8 : Acceleration after smoothing

Using the DFT as a filtering method, the gravitational and translational components were eliminated from the obtained raw acceleration. Figure 8 shows the result of the inverse DFT procedure. The thick line represents post DFT filtering after eliminating the gravitational and the translational components. The remaining acceleration ($\mathbf{a}'$) can be written as follows.

$$\mathbf{a}' = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + \dot{\boldsymbol{\omega}} \times \mathbf{r} \quad (2)$$

Here, it was assumed that $r_z = 0$ and $\dot{\omega}_x = \dot{\omega}_y = 0$. Thus, the x and y accelerations can be written as equation *(3)* and *(4)*.

$$a'_x = r_y \omega_x \omega_y - r_x(\omega_y^2 + \omega_z^2) - r_y \dot{\omega}_z \quad (3)$$

$$a'_y = -r_x \omega_x \omega_y - r_y(\omega_x^2 + \omega_z^2) + r_x \dot{\omega}_z \quad (4)$$

Here, if the rotational axis of flight disc were changed, the ω_x and ω_y would have some periodicity. Note that ω_x and ω_y have the same frequency as gravitational and translational acceleration. Therefore, once gravitational and translational acceleration were eliminated using DFT filtering, both ω_x and ω_y are also eliminated. Thus equations *(3)* and *(4)* are rewritten as follows:

$$a'_x = -r_x \omega_z^2 - r_y \dot{\omega}_z \quad (5)$$

$$a'_y = -r_y \omega_z^2 + r_x \dot{\omega}_z \quad (6)$$

The angular velocity ω_z was algebraically calculated from equations *(5)* and *(6)*. The angular velocity ω_z can be written as follows:

$$\omega_z(t) = \sqrt{\frac{a'_x(t)}{r_x}} \tanh\left(\sqrt{a'(t)_x r_x}\, \omega_z(0) - \frac{\sqrt{a'(t)_x r_x}}{r_y} t\right) \quad (7)$$

Here, $r_x = 36.7mm$, $r_y = 28.7mm$, and $\omega_z(0)$ is calculated from the value of the acceleration. For calculation of $\omega_z(0)$, it was assumed that $\dot{\omega}_z(0) = 0$.

4. Results

Figure 9 shows the angular velocity ω_z estimated using our method (eq. 7) from the accelerometer data. Table 1 shows a comparison of the angular velocity calculated by both the DLT method with high speed cameras and the accelerometer. The mean of the velocities of 40 frames by the high speed cameras just after the disc's release and at the end phase were compared. The difference between the high speed cameras and accelerometer during the release phase and the end phase was about 0.54 (rad/s), and -0.40(rad/s) respectively.

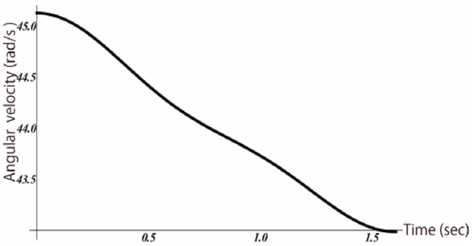

Fig. 9 : Estimated angular velocity of the flying disc by the accelerometer during its flight phase

Table 1 : Comparison of the angular velocity

Measurement method	Release Phase *(rad/s)*	End Point *(rad/s)*
High speed cameras	46.06	42.69
Accelerometer	45.52	43.09
Difference (Camera - Accelerometer)	0.54	-0.40

5. Conclusions

For understanding of behavior of sporting flying objects, we must establish our observation methodology for the whole flying phase. Due to a lack of time and space, we have proposed a sensor-based measurement method to acquire and estimate the kinematics of a flying object. In this paper, the authors proposed and demonstrated the angular velocity estimation of the flying disc by using a disc-mounted accelerometer. Then the estimated data was validated by the traditional DLT method with high speed cameras. The estimation was shown to be accurate and it guaranteed total flight phase quantification. Estimation of the rotational axis of the disc and its shift during flight will be our future work. In order to quantify the rotational axis of the flying disc, we must have the exact magnitudes of both the ω_x and ω_y . For this purpose, the authors are planning to adopt a bi-axial gyroscope for the next experiments.

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