Exercise 13.3

Q1: let $f(x,y) = 3x^3y^2$ find

(a) $f_x(x,y)$

$f_x(x,y) = ?$

$f(x,y) = 3x^3y^2$

We want to find $\frac{dy}{dx}$ here y is constant with respect to x.

$$\frac{dy}{dx} = \frac{d}{dx}(3x^3y^2)$$

$$= 3y^2 \frac{d}{dx}(x^3)$$

$$= 3y^2(3x^2)$$

$f_x(x,y) = 9x^2y^2$ (Ans)

(b) $f_y(x,y)$

$$\frac{dx}{dy} = \frac{d}{dy}(3x^3y^2) = 3x^3(2y)$$

$f_y(x,y) = 6yx^3$ (Ans)

(c) $f_x(1,y)$

$$f(x,y) = 3x^3y^2$$

$$f_x(x,y) = \frac{d}{dx}(3x^3y^2)$$

$$f_x(x,y) = 9x^2y^2$$

$$f_x(1,y) = 9y^2(1)^2 = 9y^2.$$

(d) $f_x(x,1)$

Since $f_x(x,y) = 9x^2y^2$

So $f_x(x,1) = 9x^2(1)^2 = 9x^2.$

(e) $f_y(1,y)$

Since $f_y(x,y) = 6yx^3 \Rightarrow$

So $f_y(1,y) = 6y(1)^3 = 6y.$

(f) $f_y(x,1)$

Since, $f_y(x,y) = 6yx^3$

So, $f_y(x,1) = 6x^3(1) = 6x^3.$

(g) $f_x(1,2)$

Since $f(x)$ $\cancel{f x(x,y)} = 9x^2y^2$

So, $f(1,2) = 9(1)^2(2)^2 = 36.$

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(h) $f_y(1,2)$.

Since $f_y(x,y) = 6yx^3$

So, $f_y(1,2) = 6(2)(1)^3$

$= 12$.

Q#3.

Evalute the Partial Derivativ

$z = 9x^2y - 3x^5y$; $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$.

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}(9x^2y - 3x^5y)$$

$$= 9y\frac{\partial}{\partial x}(x^2) - 3y\frac{\partial}{\partial x}(x^5)$$

$$= 9y(2x) - 3y\,5(x)^4$$

$$\boxed{\frac{\partial z}{\partial x} = 18yx - 15yx^4}$$

$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}(9x^2y - 3x^5y)$$

$$\boxed{\frac{\partial z}{\partial y} = 9x^2 - 3x^5}$$

Q# 5

$$z = (x^2 + 5x - 2y)^8 \;;\; \frac{\partial z}{\partial x},\ \frac{\partial z}{\partial y}.$$

$$z = (x^2 + 5x - 2y)^8$$

$$\frac{dz}{dx} = \frac{d}{dx}\left[(x^2 + 5x - 2y)^8\right]$$

$$\cancel{= 2x + 5}$$

$$\frac{dz}{dx} = 8(x^2 + 5x - 2y)^7 \frac{d}{dx}(x^2 + 5x - 2y)$$

$$\boxed{\frac{dz}{dx} = 8(x^2 + 5x - 2y)^7 (2x + 5 - 2y)}$$

$$\frac{dz}{dy} = \frac{d}{dy}\left[(x^2 + 5x - 2y)^8\right]$$

$$\boxed{\frac{dz}{dy} = 8(x^2 + 5x - 2y)^7 (x^2 + 5x - 2)}$$

Q# 7 $\frac{\partial}{\partial p}(e^{-7p/q}),\ \frac{\partial}{\partial q}(e^{-7p/q})$

Here $z = e^{(-7p/q)}$

$$\frac{\partial z}{\partial p} = \frac{d}{dp}(e^{-7p/q}) = (e^{-7p/q})\frac{d}{dp}(-7p/q)$$

$$\frac{\partial z}{\partial p} = \left(e^{-7p/q}\right)\frac{\partial}{\partial p}\left(\frac{-7p}{q}\right)$$

$$\boxed{\frac{\partial z}{\partial p} = \left(e^{-7p/q}\right)\left(-\frac{7}{q}\right)}$$

$$\frac{\partial z}{\partial q} = \frac{\partial}{\partial q}\left(e^{-7p/q}\right)$$

$$= \left(e^{-7p/q}\right)\frac{\partial}{\partial q}\left(-7pq^{-1}\right)$$

$$\boxed{\frac{\partial z}{\partial q} = \left(e^{-7p/q}\right)(7p)}$$

Q#9

$z = \sin(5x^3y + 7xy^2)$; $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$.

$\because z = \sin(5x^3y + 7xy^2)$

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}\left[\sin(5x^3y + 7xy^2)\right]$$

$$\frac{\partial z}{\partial x} = \cos(5x^3y + 7xy^2)\frac{\partial}{\partial x}(5x^3y + 7xy$$

$$\boxed{\frac{\partial z}{\partial x} = \cos(5x^3y+7xy^2)\,(15x^2y+7y^2)}$$

$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}\left(\sin(5x^3y+7xy^2)\right)$$

$$= \cos(5x^3y+7xy^2)\,\frac{\partial}{\partial y}(5x^3y+7xy^2)$$

$$= \cos(5x^3y+7xy^2)\,(5x^3+14xy)$$

$$\boxed{\frac{\partial z}{\partial y} = \cos(5x^3y+7xy^2)(5x^3+14xy)}$$

Q# 11.

let $f(x,y) = \sqrt{3x+2y}$.

(a) Find the Slope of the Surface $z = f(x,y)$ in the x-direction at the Point (4,2)

Slope of Surface z is "∂" Partial derivative of $f(x,y)$

$$z = \sqrt{3x+2y}$$

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}\left(\sqrt{3x+2y}\right)$$

$$\frac{1}{2}+1 \qquad \frac{1+2}{2}$$

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}(3x+2y)^{1/2}$$

$$\frac{\partial z}{\partial x} = \frac{1}{2}(3x+2y)^{-1/2}\,\frac{\partial}{\partial x}(3x+2y)$$

$$\frac{\partial z}{\partial x} = \frac{1}{2\sqrt{3x+2y}} \times (3+2y).$$

$$\frac{\partial z}{\partial x} = \frac{3+2y}{2\sqrt{3x+2y}}$$

So, $f_x(x,y) = \dfrac{3+0}{2\sqrt{3x+2y}}$

$$f_x(4,2) = \frac{3+0(0)}{2\sqrt{3(4)+2(2)}}$$

$$f_x(4,2) = \frac{3+0}{2\sqrt{12+4}} = \frac{3}{2\sqrt{16}}$$

$$f_x(4,2)\ \frac{3}{2\times 4} = \frac{3}{8}$$

So, $\boxed{\text{Slope} = 3/8}$

(b) Find the slope of the surface $z = f(x, y)$ in the y-direction at the point (4, 2).

$z = \sqrt{3x + 2y}$

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}\left(\sqrt{3x+2y}\right)$$

$$\frac{\partial z}{\partial x} = \frac{2}{2\sqrt{3x+2y}} = \frac{2}{2\sqrt{3(4)+2(2)}}$$

$$\text{Slop} = \frac{2}{2(4)} = \frac{1}{4}.$$

Q#13

let $z = \sin(y^2 - 4x)$

(a) Find the rate of change of z with respect to x at the point (2, 1) with y held fixed.

Rate of change w.r.t $x = \frac{\partial z}{\partial x} = ?$

$z = \sin(y^2 - 4x)$

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}\left[\sin(y^2 - 4x)\right]$$

$$\frac{\partial z}{\partial x} = \cos(y^2 - 4x)\frac{\partial}{\partial x}(y^2 - 4x)$$

$$\frac{\partial z}{\partial x} = \cos(y^2 - 4x)(0 - 4)$$

$$\frac{\partial z}{\partial x} = -4\cos(y^2-4x)$$

At points (2,1)

$$\frac{\partial z}{\partial x} = -4\cos((1)^2-4(2))$$

$$= -4\cos(1-8)$$

$$\frac{\partial z}{\partial x} = -4\cos(-7) = -3.97$$

(b) Find the rate of change of z w.r.t y at the point (2,1) with x held fixed.

$$\frac{\partial z}{\partial y} = ?$$

$$z = \sin(y^2-4x)$$

$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}\left\{\sin(y^2-4x)\right\}$$

$$\frac{\delta z}{\delta y} = \cos(y^2-4x)\cdot\frac{\partial}{\partial y}(y^2-4x)$$

$$\frac{\delta z}{\delta y} = \cos(y^2-4x)(2y)$$

putting points (2,1)

Rate of Change $= \cos((1)^2 - 4(2))2(1)$
$= \cos(1-8)(2)$
$= 2\cos(-7)$

$\boxed{R_y = 1.98}$

Evalute the idicated partial derivatives.

Q# 37

$f(x,y) = 9 - x^2 - 7y^3$; $f_x(3,1)$, $f_y(3,1)$

$f(x,y) = 9 - x^2 - 7y^3$

$f_x(x,y) = \frac{\partial}{\partial x}\left[9 - x^2 - 7y^3\right]$

$f_x(x,y) = 0 - 2x - 0 = -2x$

$f_x(3,1) = -2(3) = -6$ (Ans)

$f_y(x,y) = \frac{\partial}{\partial y}(9 - x^2 - 7y^3)$

$f_y(x,y) = (0 - 0 - 21y^2) = -21y^2$

$f_y(3,1) = -21(1)^2 = -21$ (Ans)

Q#39

$f(x,y) = x^2 y e^{xy}$; $\partial f/\partial x(1,1)$, $\partial f/\partial y(1,1)$

$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(x^2 y e^{xy})$

$\frac{\partial f}{\partial x} = 2xye^{xy} + x^2y^2e^{xy}$

Putting Points (1,1)

$$\frac{\partial f}{\partial x}(1,1) = 2(1)(1)e^{(1)(1)} + (1)^2(1)^2 e^{(1)(1)}$$

$$\frac{\partial f}{\partial x}(1,1) = 2 + 1 = 3.$$

$$\boxed{\frac{\partial f}{\partial x}(1,1) = 3}$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(x^2 y e^{xy})$$

$$\frac{\partial f}{\partial y} = \left(\frac{\partial}{\partial y} x^2 y\right)e^{xy} + \left(\frac{\partial}{\partial y} e^{xy}\right)x^2 y$$

$$\frac{\partial f}{\partial y} = x^2 e^{xy} + x^3 e^{xy} y.$$

$$\frac{\partial f}{\partial y}(1,1) = (1)^2 e^{(1)(1)} + (1)^3 e^{(1)(1)}$$

$$\frac{\partial f}{\partial y}(1,1) = 1 + 1 = 2$$

$$\boxed{\frac{\partial f}{\partial y}(1,1) = 2}$$

Q#41

let $w = x^2 y \cos z$ find

(a) $\partial w / \partial x (x, y, z)$

$w = x^2 y \cos z$

$\frac{\partial w}{\partial x} = \frac{\partial}{\partial x}(x^2 y \cos z)$

$= y \cos z \frac{\partial}{\partial x}(x^2)$

$\boxed{\frac{\partial w}{\partial x} = 2xy \cos z}$

(b) $\partial w / \partial y (x, y, z)$

$w = x^2 y \cos z$

$\frac{\partial w}{\partial y} = \frac{\partial}{\partial y}(x^2 y \cos z)$

$\frac{\partial w}{\partial y} = x^2 \cos z \frac{\partial}{\partial y}(y)$

$\frac{\partial w}{\partial y} = x^2 \cos z.$

(c) $\partial w / \partial z (x, y, z)$

$w = x^2 y \cos z$

$\frac{\partial w}{\partial z} = \frac{\partial}{\partial z}(x^2 y \cos z)$

$$\frac{\partial w}{\partial z} = x^2y\frac{d}{dz}(\cos z)$$

$$\boxed{\frac{\partial w}{\partial z} = -x^2y\sin z}$$

(d) $\partial w/\partial x(2,y,z)$

since

$$\partial w/\partial x = 2xy\cos z.$$

$$\partial w/(\partial x)(2,y,z) = 2(2)y\cos z$$

$$\partial w/(\partial x)(2,y,z) = 4y\cos z$$

(e) $\partial w/\partial y(2,1,z)$

since

$$\partial w/\partial y(x,y,z) = x^2y\cos z$$

$$\partial w/\partial y(2,1,z) = (2)^2(1)\cos z.$$

$$\partial w/\partial y(2,1,z) = 4\cos z$$

(f) $\partial w/\partial z(2,1,0)$

sin

$$\partial w/\partial z = -x^2y\sin z$$

$\therefore \sin(0) =$

$$\partial w/\partial z(2,1,0) = -(2)^2(1)\sin(0)$$

$$= 0$$