Orbital Mechanics HW 4

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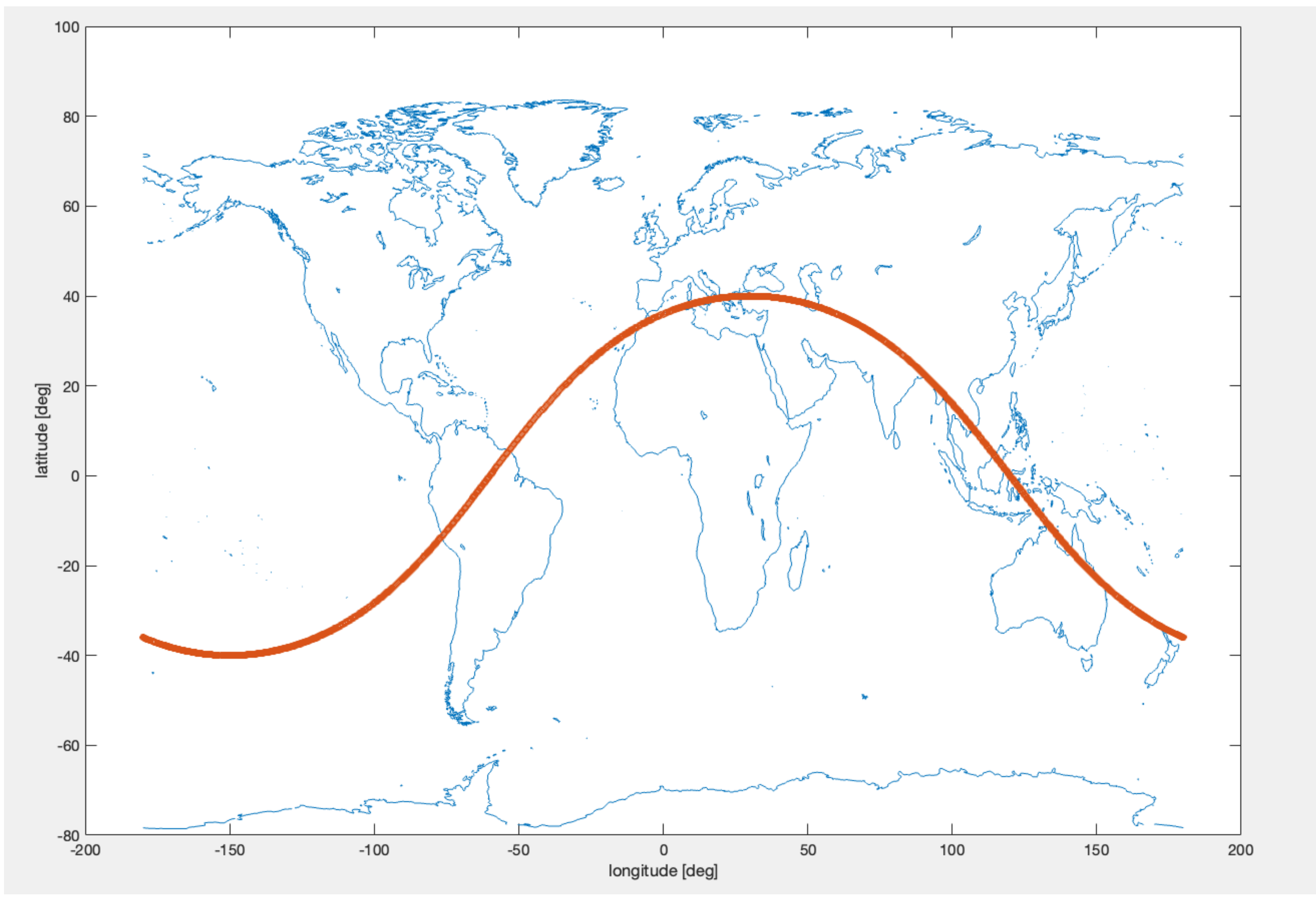
Problem 1:

Write a matlab code that generates the ground track for an orbit given orbital elements and a time interval. Superimpose the ground track to a coastline map of the world using the provided .txt file on Canvas. (Hint: Use ode45 to propagate through the orbit in time.) Remember to account for rotation of the Earth!
(a) Use this code to provide a plot of the ground track for an orbit with the following orbit elements:
a = 20, 000km
e = 0.25
i = 40 degrees
Ω = 300 degrees
ω = 0 degreees
f = 80 degrees

(b) Label the locations of the periapsis and apoapsis on the plot.
(c) Include your code

a) Process:
1) Use orbital elements to get perifocal position
2) Transform perifocal frame to ECI frame using DCM
3) Transform ECI to ECEF using R_3 rotation
4) account for earth rotation for latitude values
5) Plot ground tracks

Functions:
1) Orbital elements to Eci
2) Eci to ECEF
3) ECEF to latitude and longitude 4) Ground track plotter

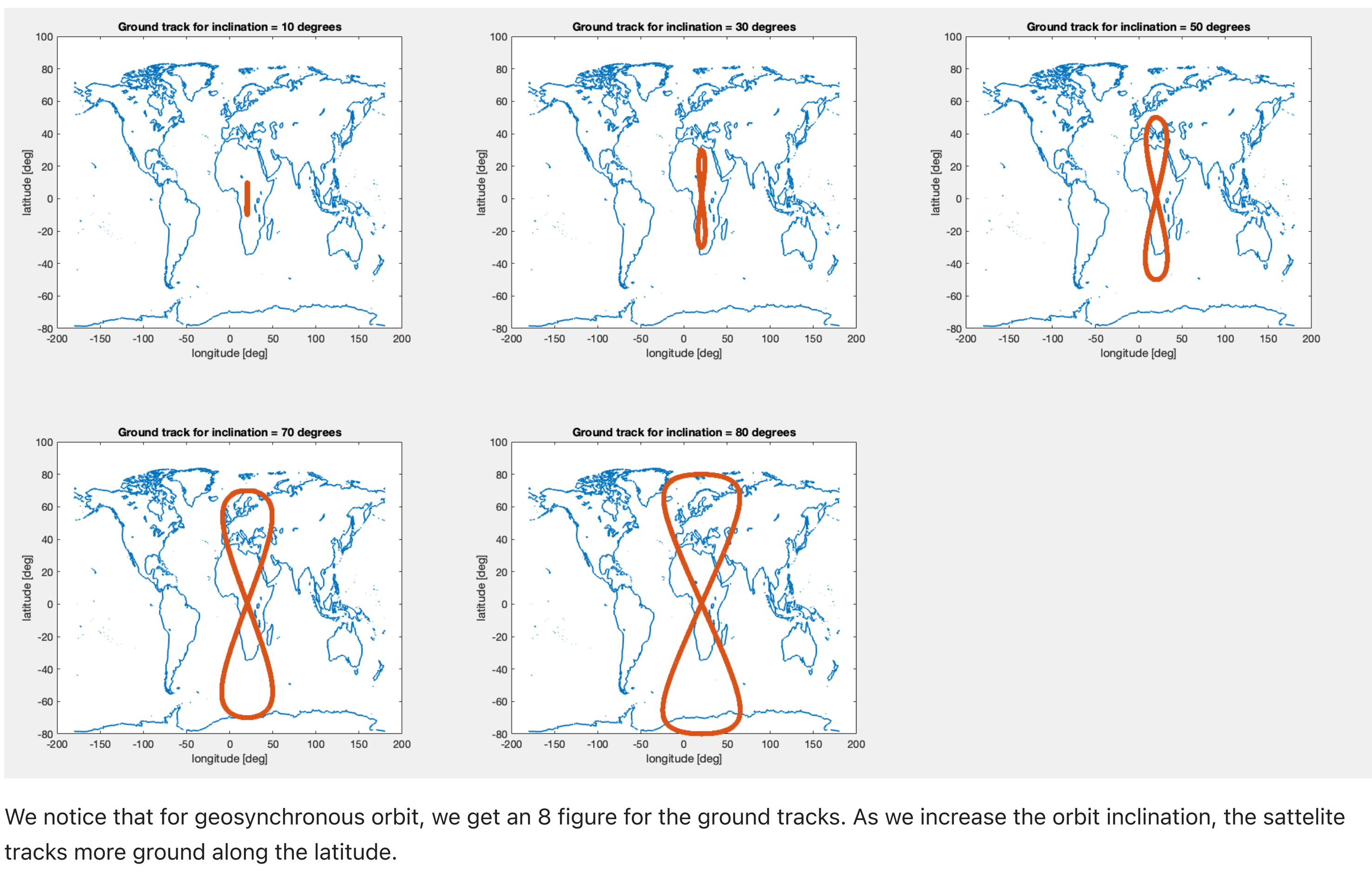


Periapsis occurs at the peak of the ground track, which is around 30 degrees longitude and 40 degrees latitude. The apoapsis occurs at the trough of the ground track, which is around -150 degrees longitude and 40 degrees latitude.

Problem 2:

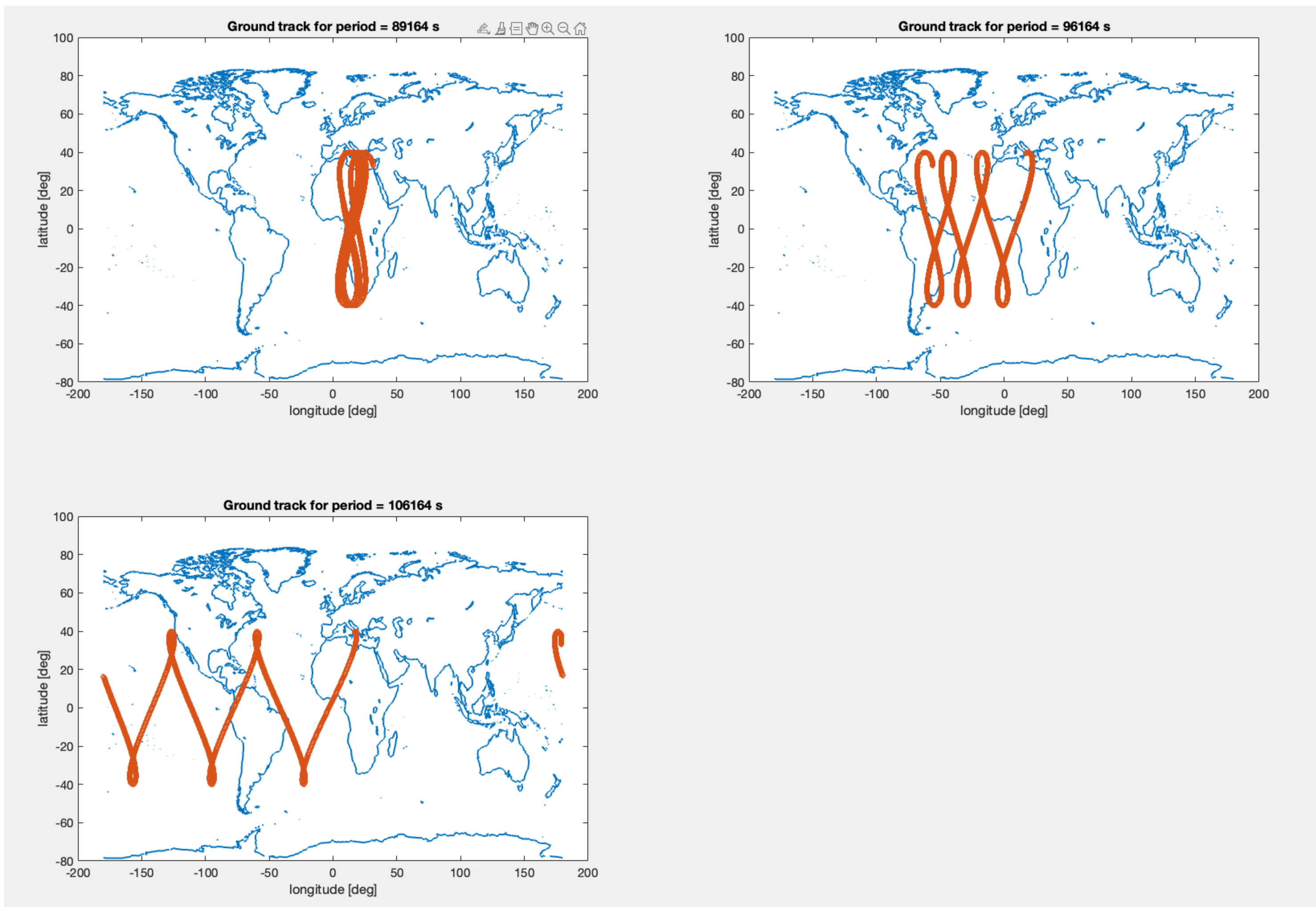
Use the code in this assignment to produce ground tracks for a geosynchronous orbit. In particular, plot the ground tracks for a geosynchronous orbit with the following variations. Include at lease five variations in the specified parameter.
(a) Plot the ground tracks for this orbit with variations in inclination and explain the trend. Include at least five variations in inclination.
(b) Plot the ground tracks for this orbit with variations in orbital period and explain the trend. Include at least three variations in the orbital period.
(c) Plot the ground tracks for this orbit with variations in eccentricity, and explain the trend. Include at least five variations in the eccentricity.

a)



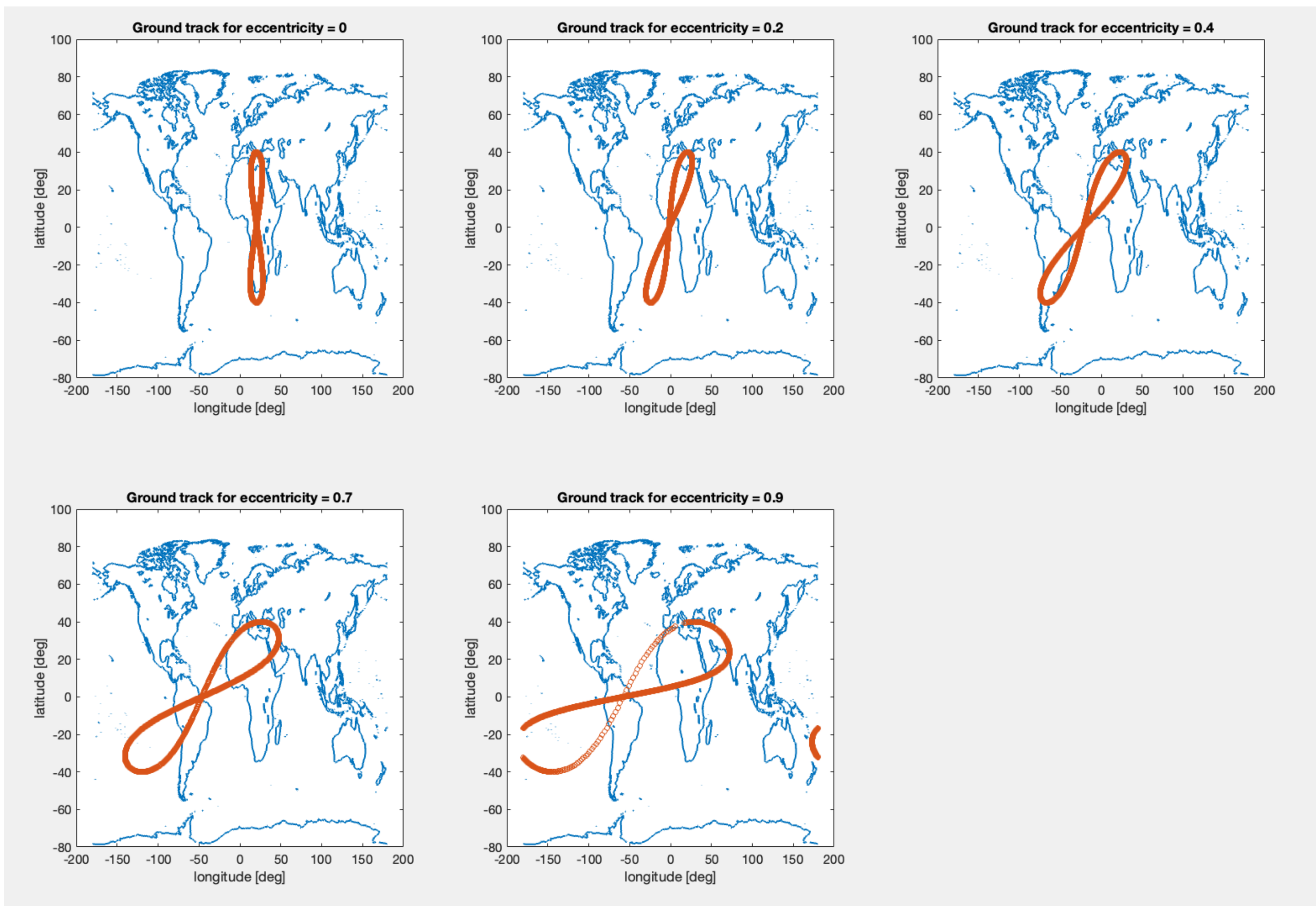
We notice that for geosynchronous orbit, we get an 8 figure for the ground tracks. As we increase the orbit inclination, the sattelite tracks more ground along the latitude.

b)



As we increase the orbit slightly from the geosynchronous value, we notice the ground track 8 figure breaks. As we increase the period further, the ground track near the apoapsis stays relatively the same but the periapsis gets spread apart further. This is no longer the nice 8 figure of a geosynchronous orbit. They also start to drift away from the starting position.

c)



If we have a geosynchronous and circular orbit (e=0), we get a nice veritcal 8 figure. As we increase the eccentricity, we notice the ground track rotates and gets larger. If the orbit is highly eccentric (e = 0.9), the ground track traces the biggest ground area compared to less eccentric orbits.

Problem 3:

Starting with an initially circular LEO orbit (i = 28.5 deg, h = 300km), your objective is to place a payload in geosynchronous orbit using as little Δv as possible.

(a) You believe the inclination change maneuver should take place once geosynchronous altitude has been achieved. Prove to your boss that this is true - support your statement by deriving an analytical inequality.
(b) Should the inclination maneuver be combined with the circularization maneuver at geosynchronous altitude? Support your statement by deriving an analytical inequality
(c) What is the total Δv necessary to reach a geosynchronous orbit with a zero inclination.

a) To change the inclination of an orbit, we use a delta V equal to:

$$\Delta V = 2 \cdot V_c \cdot sin(\frac{\Delta i}{2})$$

Expanding the circular velocity:

$$\Delta V = 2 \cdot \sqrt{\frac{\mu}{r_c}} \cdot sin(\frac{\Delta i}{2})$$

Let's say the LEO orbit is defined as 1 and the geosynchronous orbit as 2. I argue that:

$$\Delta V_1 \geq \Delta V_2$$

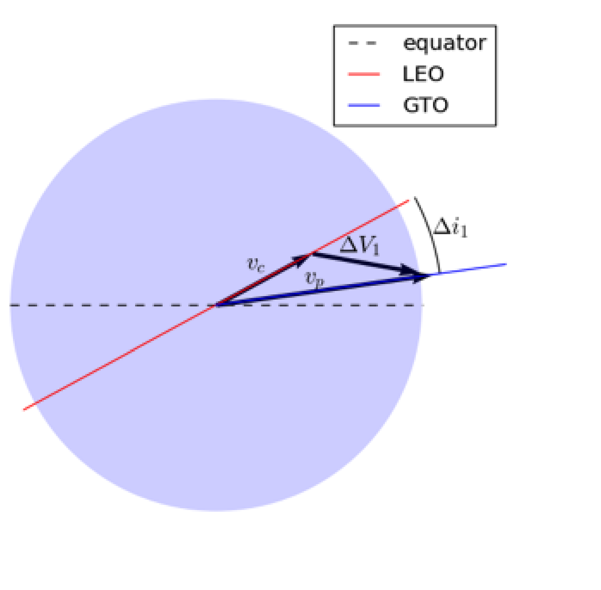
$$2 \cdot \sqrt{\frac{\mu}{r_1}} \cdot sin(\frac{\Delta i}{2}) \geq 2 \cdot \sqrt{\frac{\mu}{r_2}} \cdot sin(\frac{\Delta i}{2})$$

$$\frac{\mu}{r_1} \geq \frac{\mu}{r_2}$$

$$r_2 \geq r_1$$

Since we know that the geosynchronous orbit has higher altitude, it means $r_2 \geq r_1$, which we just proved. This means a burn at higher altitude for inclination change will be lower that doing that burn on the journey before reaching the altitude.

b)



Vector diagram of the first impulsive ΔV

We can use geometric argument to show that the dV for the combination maneuver is less than the dV for inclination change and the dV for circularization, using Pythagoras theorem. We know that:

$$V_p^2 = V_c^2 + \Delta V_1{}^2$$

$$\Delta V_1{}^2 = V_p^2 - V_c^2$$

$$\Delta V_{simple} = 2V sin(\frac{\Delta i}{2}) + \sqrt{\frac{\mu}{R_2}} - \sqrt{\frac{2\mu}{R_2} - \frac{\mu}{a_T}}$$

$$\Delta V_{combined} = \sqrt{V_{T2}^2 + V_2^2 - V_{T2} V_2 cos(\Delta i)}$$

$$\Delta V_{simple} \geq \Delta V_{combined}$$

Thus, doing a combined manuever requires less delta v compared when doing both manuevers sperately.

In [28]:
```
## Part C:
mu = 398600
r_1 = 300 + 6378
period = 23*3600 + 56*60 + 4
a = (period**2/(4*pi**2)*mu)**(1/3)
e = 0
p = a*(1-e**2)
r_2 = p
Vc = sqrt(mu/r)

a_t = 0.5*(r_1+r_2)
e_t = 1 - r_1/a_t
print("r_2 = {:.2f} km  a_t = {:.2f} km and e_t = {:.2f}".format(r_2, a_t, e_t))
```

```
r_2 = 42164.12 km  a_t = 24421.06 km and e_t = 0.73
```

In [29]:
```
deltai = 28.5/180*pi
dVplane = 2*Vc*sin(deltai)

dV1 = sqrt(2*mu/r_1 - mu/a_t) - sqrt(mu/r_1)
dV2 = sqrt(mu/r_2) - sqrt(2*mu/r_2 - mu/a_t)
dV = dV1+dV2
print("The total dV required to reach geosynchronous orbit with zero inclination is {:.2f} km/s".format(dVplane
```

```
The total dV required to reach geosynchronous orbit with zero inclination is 6.83 km/s
```

In [25]:
```
v_t2 = sqrt(2*mu/r_2 - mu/a_t)
v2 = sqrt(mu/r_2)
dVcombined = sqrt(v_t2**2 + v2**2 - v_t2*v2*cos(deltai))
dVcombined
```

Out[25]:
```
2.7738400022847527
```

Problem 4:

Momentum transfer and orbital maneuvers. You are an astronaut with mass m stranded in a circular orbit with radius r about an asteroid with a gravitational parameter μa. You need to get in to your spacecraft at a higher circular orbit (radius r_{SC}) so you can return to Earth (see Figure 1). To accomplish this, all you have is your satchel of sample rocks that you're carrying with you. Your plan is to throw one of the rocks (mass mr) to adjust your orbit so you can just reach the spacecraft.

In what direction and how fast (v_{th}) do you throw the rock? Express vth in terms of μ_a, r, r_{SC}, m, and m_r.

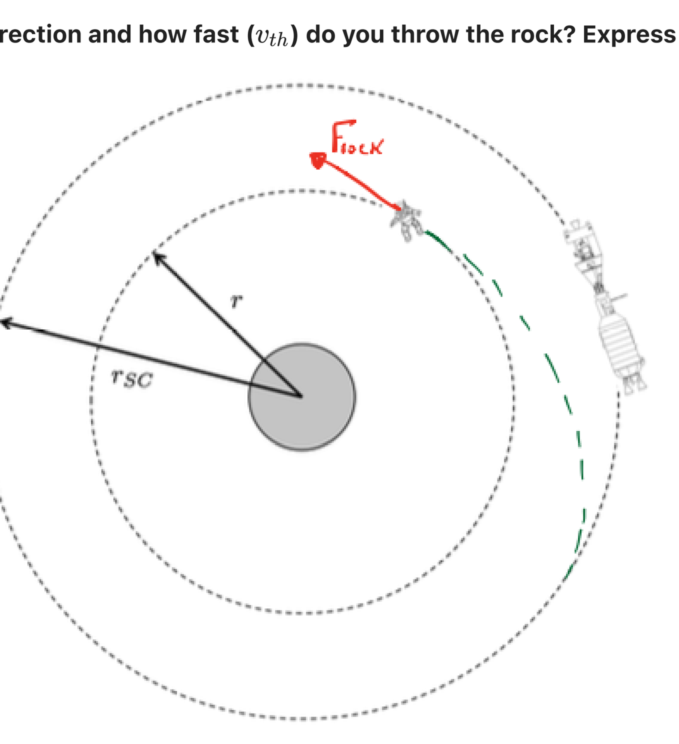


Figure 1: Stranded Astronaut Scenario

Since we want to increase the orbit size, the impulse thrust needs to be in the same direction as the velocity. Thus the rock should be thrown opposite to velocity to get a reaction impulse that increases the orbit energy.

$$V_1 = \sqrt{\frac{\mu_a}{r}}$$

$$a_T = 0.5 \cdot (r + r_{SC})$$

$$V_{T1} = \sqrt{\frac{2\mu_a}{r} - \frac{\mu_a}{a_T}}$$

Using coservation of angular momentum, we know:

$$m \cdot V_1 \cdot r_1 = (m - m_r) \cdot V_{th} \cdot r_2$$

$$m\sqrt{\frac{\mu_a}{r_1}} r_1 = (m - m_r) V_{th} r_2$$

$$V_{th} = \frac{m}{m - m_r} \frac{r}{r_{SC}} \sqrt{\frac{\mu_a}{r}}$$

In [ ]: