

# Trigonometriya.



$$1\pi = 180^\circ$$

$$1 \approx 57^\circ$$

1) Radiandan gradusga o'tish.

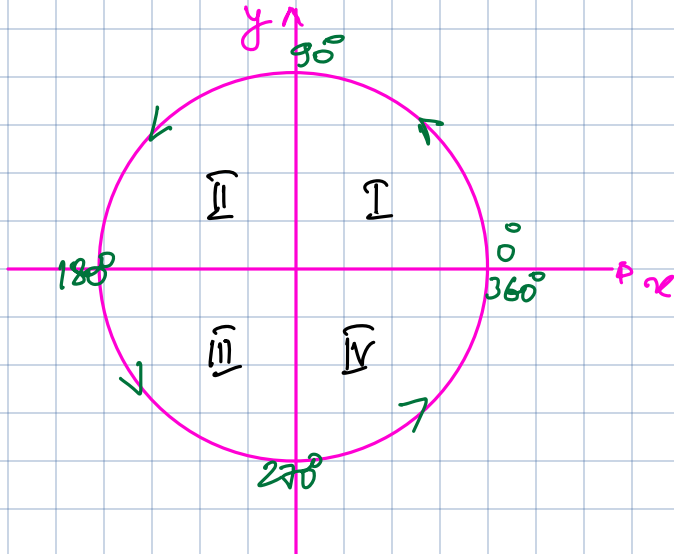
$$\frac{4\pi}{3} = \frac{4 \cdot 180^\circ}{3} = 240^\circ$$

2) Gradusdan radianga o'tish.

$$240^\circ = \frac{4}{3} \cdot 180^\circ = \frac{4\pi}{3}$$

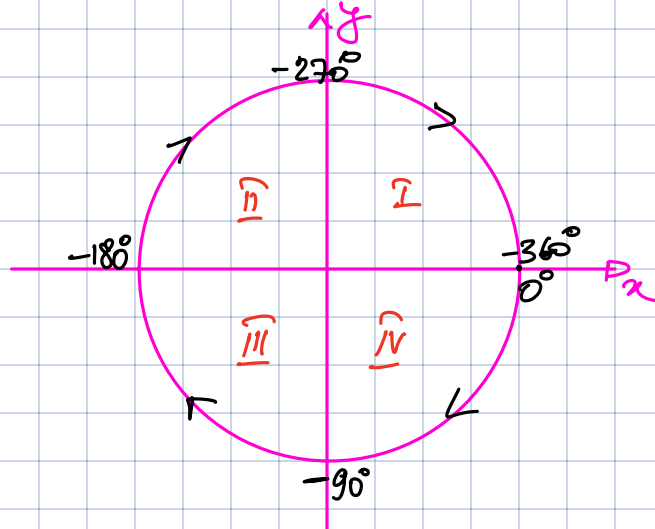
\*  $1^\circ = 60'$   $1' = 60''$   $\rightarrow$  burchak o'lchov birliklari.

Misol:  $25^\circ 30' = 25^\circ + \frac{30^\circ}{60} = 25,5^\circ$



$$I \in (0; 90^\circ) \quad II \in (90^\circ; 180^\circ)$$

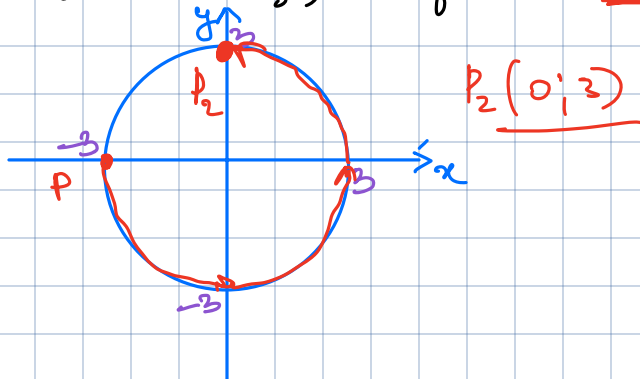
$$III \in (180^\circ; 270^\circ) \quad IV \in (270^\circ; 360^\circ)$$



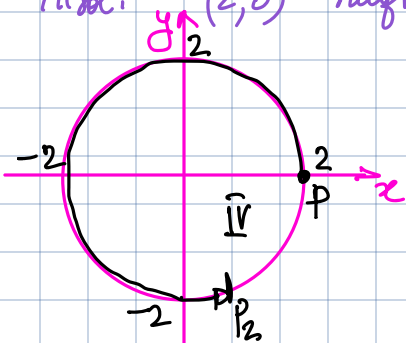
$$x \text{ o'qi} \rightarrow 0^\circ, 180^\circ, 360^\circ (\pi, 2\pi, \dots)$$

$$y \text{ o'qi} \rightarrow 90^\circ, 270^\circ (\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots)$$

Misol:  $P(-3; 0)$  nuqtani  $270^\circ$  ga burusak.



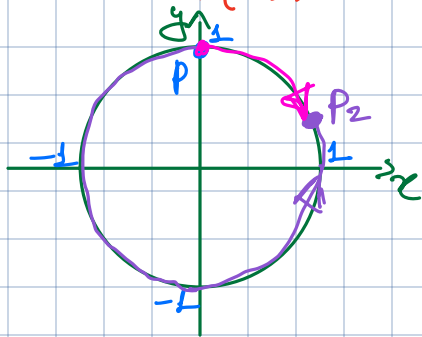
Misol:  $(2; 0)$  nuqtani  $100^\circ$  ga burusak qaysi chorak.



$$\begin{array}{r} 100^\circ \\ - 720^\circ \\ \hline 280^\circ \end{array}$$

IV chorak

Misol:  $P(0;1)$  nuqtani  $-800^\circ$  ga bursak.

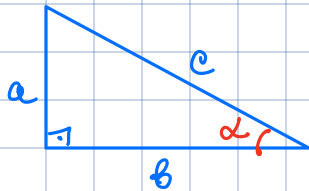


$$\begin{array}{r} -800^\circ / 360^\circ \\ -720^\circ / -2 \\ \hline -80^\circ \\ +360^\circ \\ \hline 280^\circ \end{array}$$

$$\begin{array}{r} -800^\circ / 360^\circ \\ -1080^\circ / -3 \\ \hline 280^\circ \end{array}$$

$P_2$  — I chorakda

Tisru' burchakli' uchburchak



$a, b \rightarrow$  katetlar  $c \rightarrow$  gipotenusa.

Pifagor teoremasi:

$$a^2 + b^2 = c^2$$

$$\sin \alpha = \frac{\text{b. g. k}}{\text{gipotenusa}}$$

$$\cos \alpha = \frac{\text{b. y. k}}{\text{gipotenusa}}$$

$$\tan \alpha = \frac{\text{b. g. k}}{\text{b. y. k}}$$

$$\cot \alpha = \frac{\text{b. y. k}}{\text{b. g. k}}$$

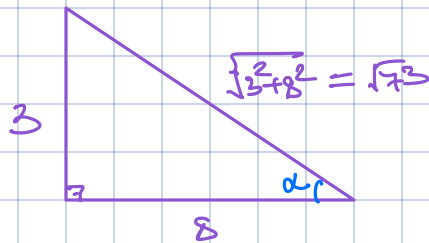
$$\sin \alpha = \frac{a}{c}$$

$$\cos \alpha = \frac{b}{c}$$

$$\tan \alpha = \frac{a}{b}$$

$$\cot \alpha = \frac{b}{a}$$

Misol:



$$\sqrt{3^2 + 8^2} = \sqrt{73}$$

$$\sin \alpha = \frac{3}{\sqrt{73}}$$

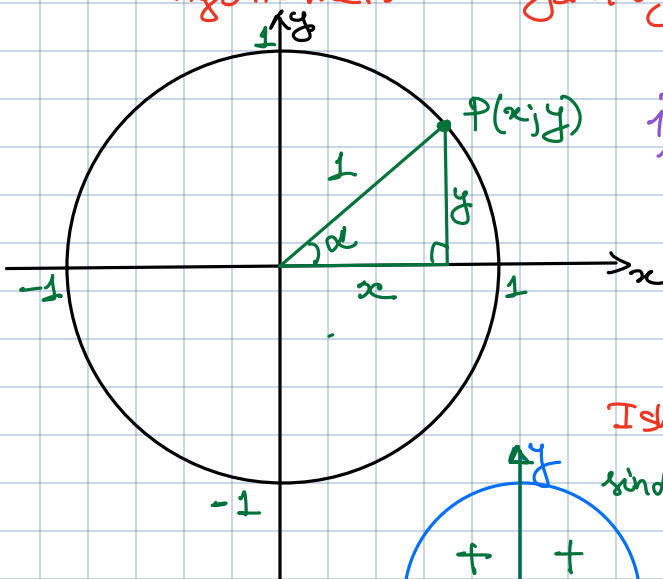
$$\cos \alpha = \frac{8}{\sqrt{73}}$$

$$\tan \alpha = \frac{3}{8}$$

$$\cot \alpha = \frac{8}{3}$$

Trigonometrik

funksiyalar.



$$1) \begin{cases} \sin \alpha = \frac{y}{1} = y \\ \cos \alpha = \frac{x}{1} = x \end{cases}$$

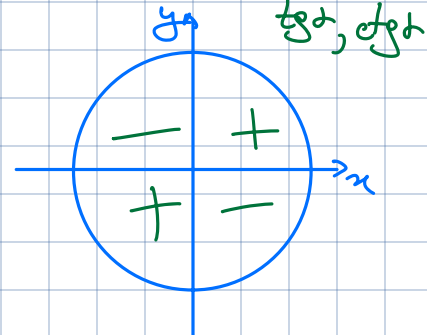
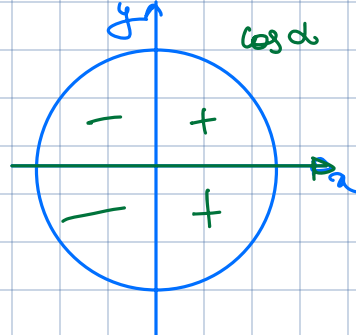
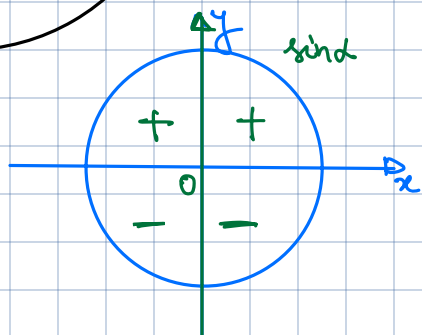
$$2) \tan \alpha = \frac{\sin \alpha}{\cos \alpha}$$

$$3) \cot \alpha = \frac{\cos \alpha}{\sin \alpha}$$

$$4) \tan \alpha \cdot \cot \alpha = 1$$

$$5) \tan \alpha = \frac{1}{\cot \alpha}$$

Ishoralari:



Misli:  $\frac{\sin 58^\circ \cdot \cos \frac{137}{5}}{\operatorname{tg} 5}$

$\sin 58^\circ = \sin 22^\circ \ominus$

$\cos \frac{137}{5} = \cos 46^\circ = \cos 108^\circ \ominus$

$\operatorname{tg} 5 = \operatorname{tg} 285^\circ \ominus$

$\frac{\ominus \cdot \ominus}{\ominus} = \ominus$

Misli:

~~$\sin^2 \alpha \cdot \operatorname{tg}^3 \alpha < 0$~~

$\operatorname{tg}^3 \alpha < 0$

$\operatorname{tg} \alpha < 0$

$\alpha \in \overline{\Pi}, \overline{IV}$

Misli:

$|\sin \alpha \cdot \operatorname{ctg} \alpha| = \sin \alpha \cdot \operatorname{ctg} \alpha$

$|a| = a$

$a \geq 0$

$\sin \alpha \cdot \operatorname{ctg} \alpha \geq 0$

$+\cdot+$   
 $-\cdot-$

$\textcircled{I}$   
 $\textcircled{IV}$

\*  $\cos(-\alpha) = \cos \alpha \rightarrow$  Jift funksiya

$\sin(-\alpha) = -\sin \alpha$

$\operatorname{tg}(-\alpha) = -\operatorname{tg} \alpha$

$\operatorname{ctg}(-\alpha) = -\operatorname{ctg} \alpha$

) Toq funksiya.

Misli:

$\cos(-300^\circ) \cdot \operatorname{ctg}(-510^\circ) = \ominus \cos 300^\circ \cdot \operatorname{ctg} 510^\circ = \ominus \cdot \ominus \cdot \ominus = \ominus$

$\cos 300^\circ = \cos 120^\circ \ominus$

$\operatorname{ctg} 510^\circ = \operatorname{ctg} 150^\circ \ominus$

Alohiy qiymatlar.

$\alpha$	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$
$\sin$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\operatorname{tg}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	—
$\operatorname{ctg}$	—	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Qiymatlar tohavi.

1)  $-1 \leq \sin x \leq 1$

$-1 \leq \cos x \leq 1$

2)  $0 \leq \sin^2 x \leq 1$

$0 \leq \cos^2 x \leq 1$

Misli:

$y = 5 \sin 3x - 2$

$5 \cdot 1 - 2 = 3$

$5 \cdot (-1) - 2 = -7$

qiymatlar tohavi:  $[-7; 3]$

Misli:

$y = 5 - 2 \cos^2 4x$

$5 - 2 \cdot 1 = 3$

$5 - 2 \cdot 0 = 5$

qiymatlar tohavi:  $[3; 5]$

Misol

$$\frac{5\cos\alpha + 2}{3\sin\alpha + 5} + \frac{4}{\tan^2\alpha + \cot^2\alpha} \rightarrow \text{eng katta qiymati}$$

koshi tengsizligi  
 $a+b \geq 2\sqrt{ab}$

$$\tan^2\alpha + \cot^2\alpha \geq 2\sqrt{\tan^2\alpha \cdot \cot^2\alpha} = 2$$

$$\frac{5 \cdot 1 + 2}{3 \cdot (-1) + 5} + \frac{4}{2} = \frac{7}{2} + \frac{4}{2} = \frac{11}{2} = 5,5.$$

Misol:

$$(\sin x - 2)(\lg(x+2) - 2) > 0$$

$$\begin{aligned} 1-2 &= -1 \\ -1-2 &= -3 \end{aligned}$$

$$[-3; -1]$$

daimo manfiy

$$\lg(x+2) - 2 < 0$$

$$\lg(x+2) < 2$$

$$\begin{cases} x+2 < 10^2 \\ x+2 > 0 \end{cases}$$

$$\begin{cases} x < 98 \\ x > -2 \end{cases}$$

$$y: (2; 98)$$

Asosiy trigonometrik ayniqatlar.

$$1) \sin^2\alpha + \cos^2\alpha = 1$$

$$2) \sin^2\alpha = 1 - \cos^2\alpha$$

$$3) \cos^2\alpha = 1 - \sin^2\alpha$$

$$4) 1 + \tan^2\alpha = \frac{1}{\cos^2\alpha}$$

$$5) 1 + \cot^2\alpha = \frac{1}{\sin^2\alpha}$$

Misol:

$$\frac{\cos^3\alpha + \sin^3\alpha}{1 - \sin\alpha \cos\alpha} = \frac{(\cos\alpha + \sin\alpha)(\cos^2\alpha - \cos\alpha \sin\alpha + \sin^2\alpha)}{1 - \sin\alpha \cos\alpha}$$

$$= \frac{(\cos\alpha + \sin\alpha)(1 - \cos\alpha \sin\alpha)}{1 - \sin\alpha \cos\alpha} = \cos\alpha + \sin\alpha.$$

Misol:

$$\cot\alpha - \frac{\sin(-\alpha)}{1 + \cos(-\alpha)} = \frac{\frac{1 + \cos\alpha}{\sin\alpha}}{\frac{\sin\alpha}{1 + \cos\alpha}} = \frac{\cos\alpha + \cos^2\alpha + \sin^2\alpha}{\sin\alpha(1 + \cos\alpha)}$$

$$= \frac{\cos\alpha + 1}{\sin\alpha(1 + \cos\alpha)} = \frac{1}{\sin\alpha}.$$

Misol:

$$\tan\alpha = 4$$

$$\frac{3\sin\alpha + 2\cos\alpha}{4\sin\alpha - \cos\alpha}$$

$$= \frac{\frac{3\sin\alpha}{\cos\alpha} + \frac{2\cos\alpha}{\cos\alpha}}{\frac{4\sin\alpha}{\cos\alpha} - \frac{\cos\alpha}{\cos\alpha}}$$

$$= \frac{3\tan\alpha + 2}{4\tan\alpha - 1} = \frac{3 \cdot 4 + 2}{4 \cdot 4 - 1} = \frac{14}{15}.$$

Misal:  $\boxed{\operatorname{ctg} \alpha = -2}$

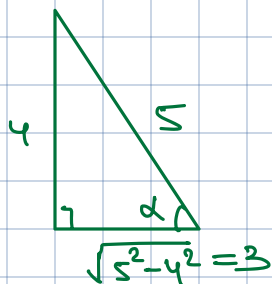
$$\frac{3 \cos \alpha}{2 \sin^3 \alpha - \cos^3 \alpha} = \frac{3 \cos \alpha}{\sin \alpha} \cdot \left( \frac{1}{\sin^2 \alpha} \right)$$

$$= \frac{3 \operatorname{ctg} \alpha \cdot (1 + \operatorname{ctg}^2 \alpha)}{2 - \operatorname{ctg}^3 \alpha} = \frac{-6(1+4)}{2 - (-8)} = \frac{-30}{10} = -3$$

Uchburchak usuli.

1)  $\sin \alpha = \frac{4}{5}$

$\boxed{0^\circ < \alpha < 90^\circ}$



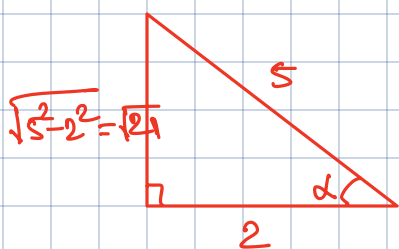
$\cos \alpha = \frac{3}{5}$

$\operatorname{tg} \alpha = \frac{4}{3}$

$\operatorname{ctg} \alpha = \frac{3}{4}$

2)  $\cos \alpha = -\frac{2}{5}$

$\boxed{90^\circ < \alpha < 180^\circ}$



$\sin \alpha = \frac{\sqrt{21}}{5}$

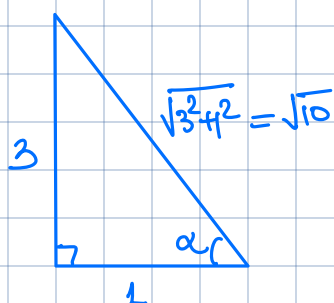
$\operatorname{tg} \alpha = -\frac{\sqrt{21}}{2}$

$\operatorname{ctg} \alpha = -\frac{2}{\sqrt{21}}$

3)  $\operatorname{tg} \alpha = 3 = \frac{3}{1}$

$\boxed{180^\circ < \alpha < 270^\circ}$

III chorak.



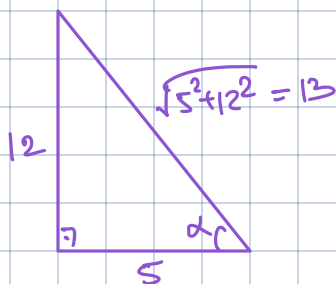
$\sin \alpha = -\frac{3}{\sqrt{10}}$

$\cos \alpha = -\frac{1}{\sqrt{10}}$

$\operatorname{ctg} \alpha = \frac{1}{3}$

4)  $\operatorname{ctg} \alpha = -\frac{5}{12}$

$\boxed{270^\circ < \alpha < 360^\circ}$



$\sin \alpha = -\frac{12}{13}$

$\cos \alpha = \frac{5}{13}$

$\operatorname{tg} \alpha = -\frac{12}{5}$

Nisoli:

$$\sin x - \cos x = a \quad \uparrow^2$$

$$\sin^2 x - 2 \sin x \cos x + \cos^2 x = a^2$$

$$1 - 2 \sin x \cos x = a^2$$

$$1 - a^2 = 2 \sin x \cos x$$

$$\sin x \cos x = \frac{1 - a^2}{2}$$

$$\sin^3 x - \cos^3 x = (\sin x - \cos x) (\sin^2 x + \sin x \cos x + \cos^2 x) = a \left( 1 + \frac{1 - a^2}{2} \right) = a \cdot \frac{3 - a^2}{2}$$