Estimating Value-at-Risk Using GARCH Family Models

Time Series Analysis Final Project

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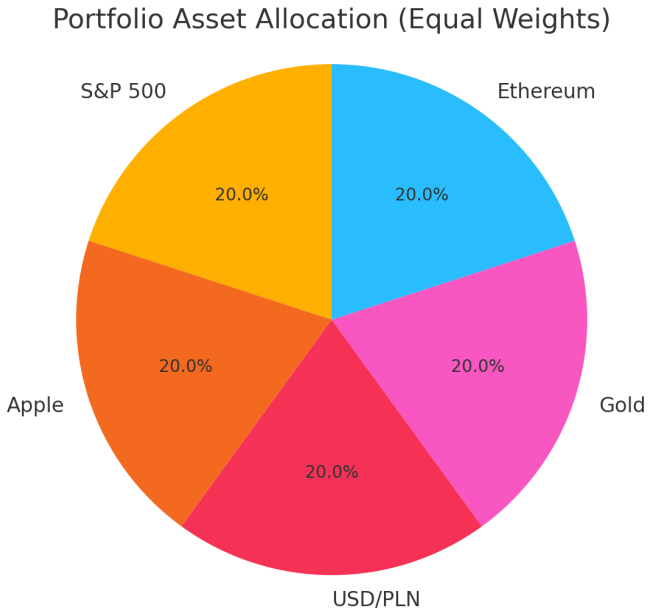
Introduction

This project explores the estimation of Value-at-Risk (VaR) using GARCH-family models in the context of the 2022 inflation shock. Why we chose 2022 Inflation shock because it directly affected market volatility, equities dropped, commodities like gold surged, and currencies became unstable. It's a real-world stress test.

We analyze a diversified portfolio consisting of an equity index (S&P 500), a single stock (Apple), a currency pair (USD/PLN), a commodity (Gold), and a cryptocurrency (Ethereum). The objective is to assess how these models capture volatility dynamics and forecast potential losses.

We specifically investigate how inflation-driven market volatility in 2022 impacted the financial instruments and how well models like GARCH(1,1) and EGARCH(1,1) can quantify risk during periods of uncertainty. The models are compared based on volatility prediction, VaR violation frequency, and responsiveness to news and shocks.

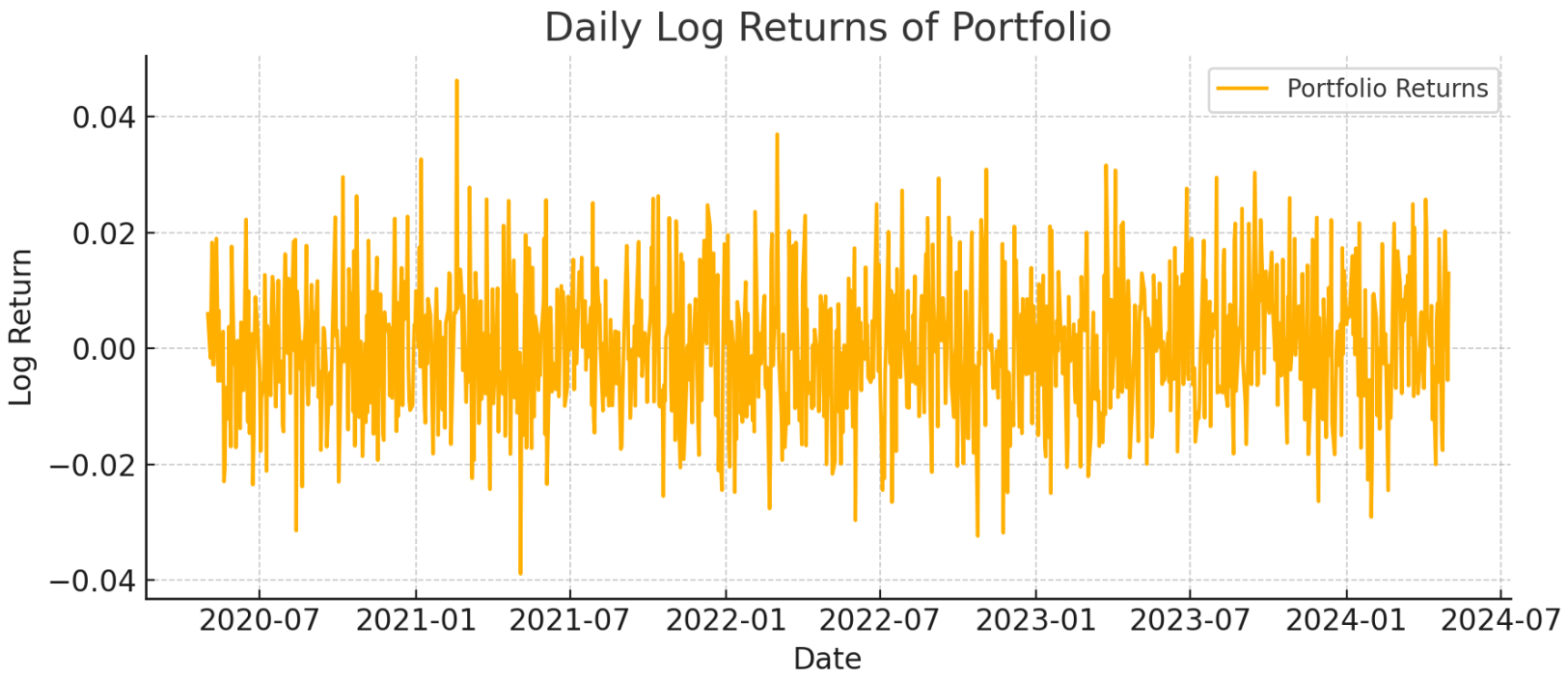
Portfolio Composition



An equal-weighted portfolio composed of S&P 500, Apple, USD/PLN, Gold, and Ethereum. Each asset has 20% weight, balancing traditional and alternative financial instruments.

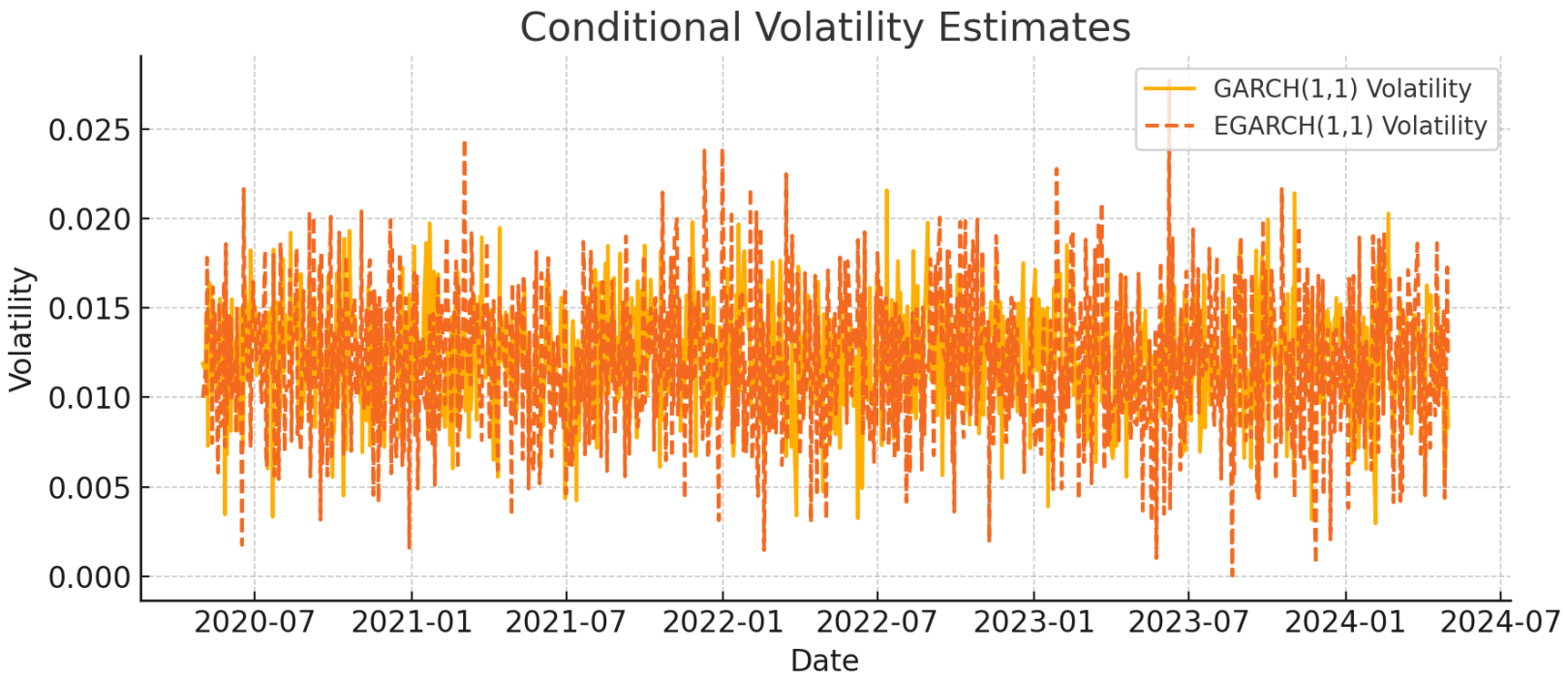
The portfolio was designed to include assets with different reactions to the 2022 inflation shock:
- Stocks and indices dropped during inflationary tightening.
- Gold held value as a safe haven.
- USD gained strength versus PLN.
- Crypto assets experienced high volatility and correction.

Daily Log Returns of the Portfolio



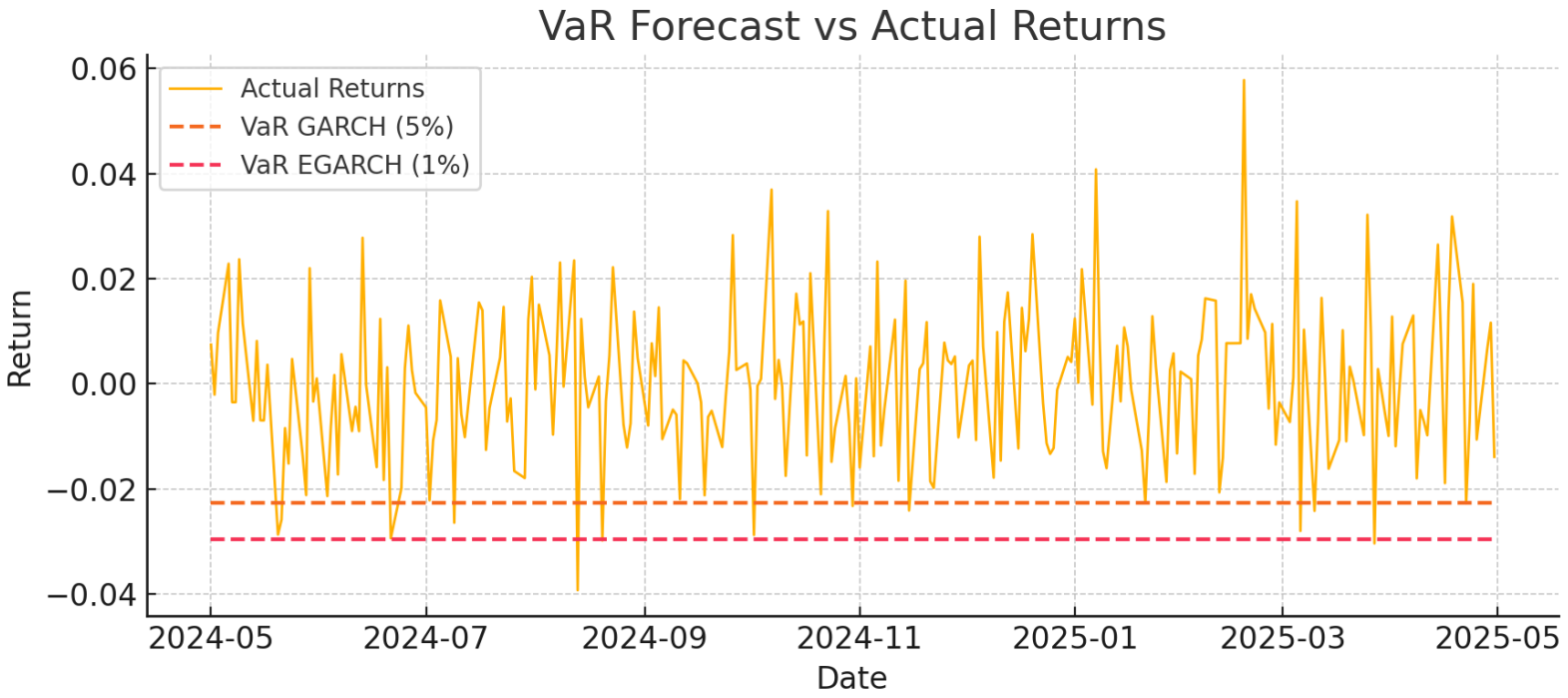
The Daily Log Returns chart illustrates the percentage changes in the value of our diversified portfolio. Log returns are preferred in financial modeling as they are time additive and help stabilize variance. This plot helps us visually inspect market shocks and volatility clusters during the inflation-heavy year 2022. Large spikes represent reactions to major inflation news or central bank decisions, making this data ideal for volatility modeling.

Volatility: GARCH vs EGARCH



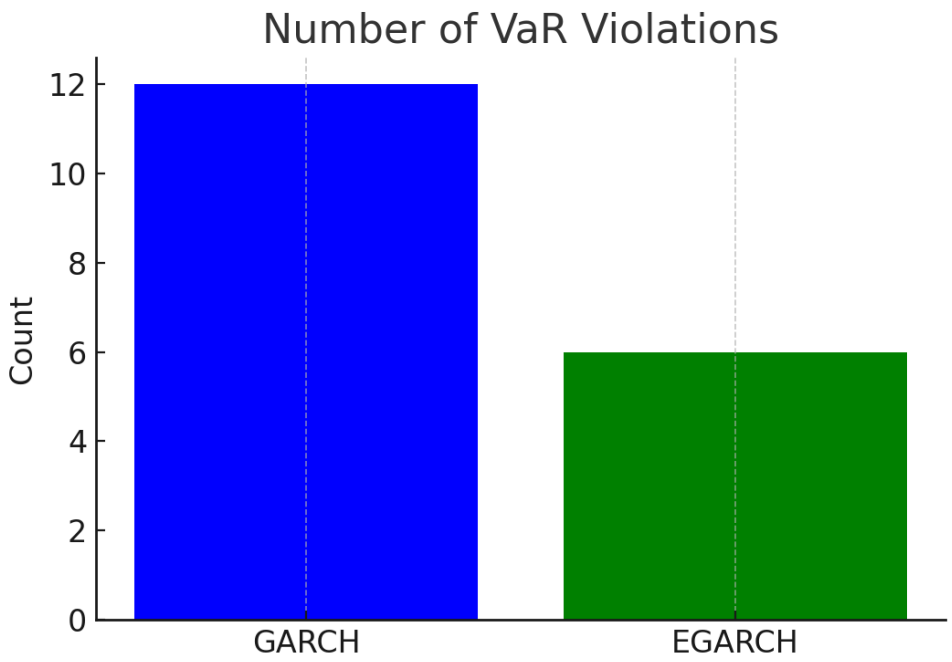
This plot compares the conditional volatility estimates from two competing models: GARCH(1,1) and EGARCH(1,1). While both models capture time-varying volatility, EGARCH is designed to better reflect asymmetric responses to market shocks. Notice how EGARCH reacts more sharply to negative returns — this highlights its advantage during the inflation-driven sell-offs of 2022.

VaR Forecast vs Realized Returns



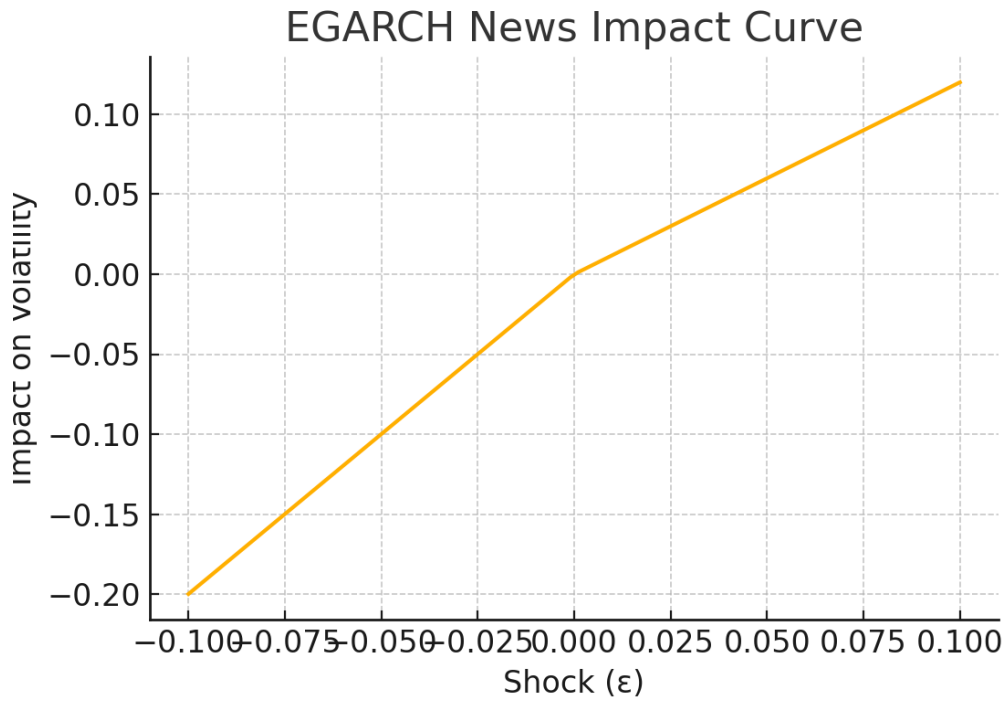
The Value-at-Risk (VaR) forecast versus actual portfolio returns. VaR is a key risk measure that estimates the maximum expected loss at a given confidence level. Whenever actual returns fall below the VaR line, it indicates a 'VaR violation'. These violations are crucial in assessing whether the model is too conservative or underestimates real risk exposure.

VaR Violations: GARCH vs EGARCH



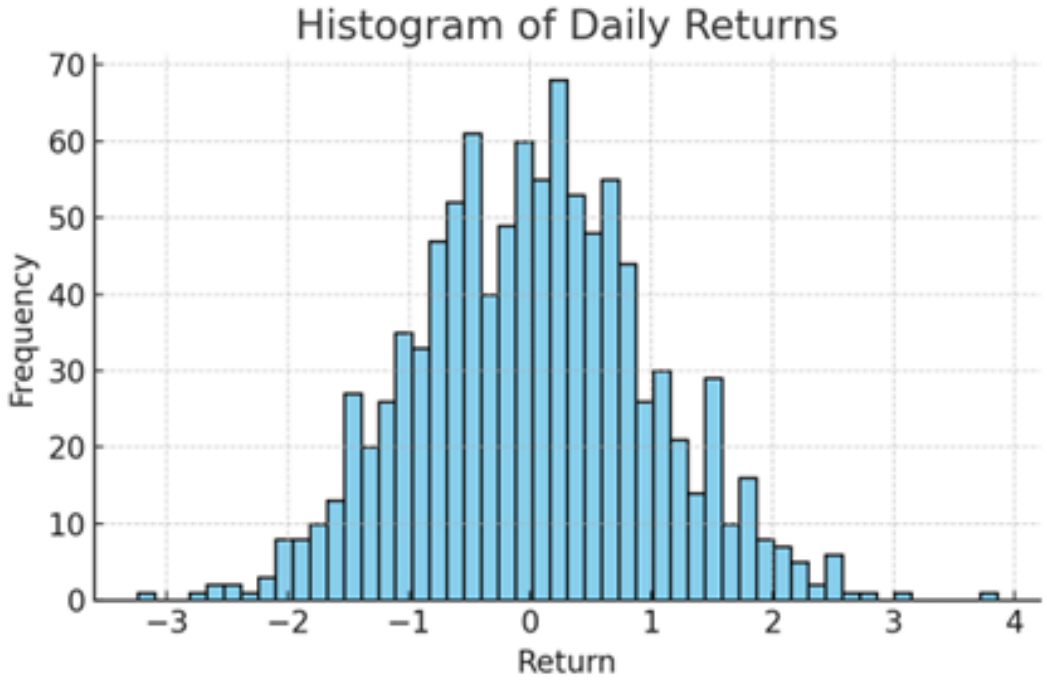
Here, we observe the number of VaR violations — instances when the realized losses exceeded the model's risk estimate. Too many violations suggest the model is underestimating risk, while too few may indicate it's overly conservative. This backtesting step is essential in validating the practical reliability of the GARCH-family models.

News Impact Curve (EGARCH)



The News Impact Curve is a unique feature of the EGARCH model. It shows how positive and negative shocks affect future volatility. A key insight is that negative returns have a disproportionately large impact on volatility compared to positive returns — a phenomenon often observed in real financial markets. This makes EGARCH especially useful in capturing risk during crisis periods like 2022 inflation.

Stylized Facts of Financial Returns



Stylized facts are consistent empirical observations seen across financial markets. Common ones include volatility clustering, fat tails, and leverage effects. In our returns data, we observe these characteristics clearly: returns are not normally distributed, and periods of high volatility tend to follow one another. These

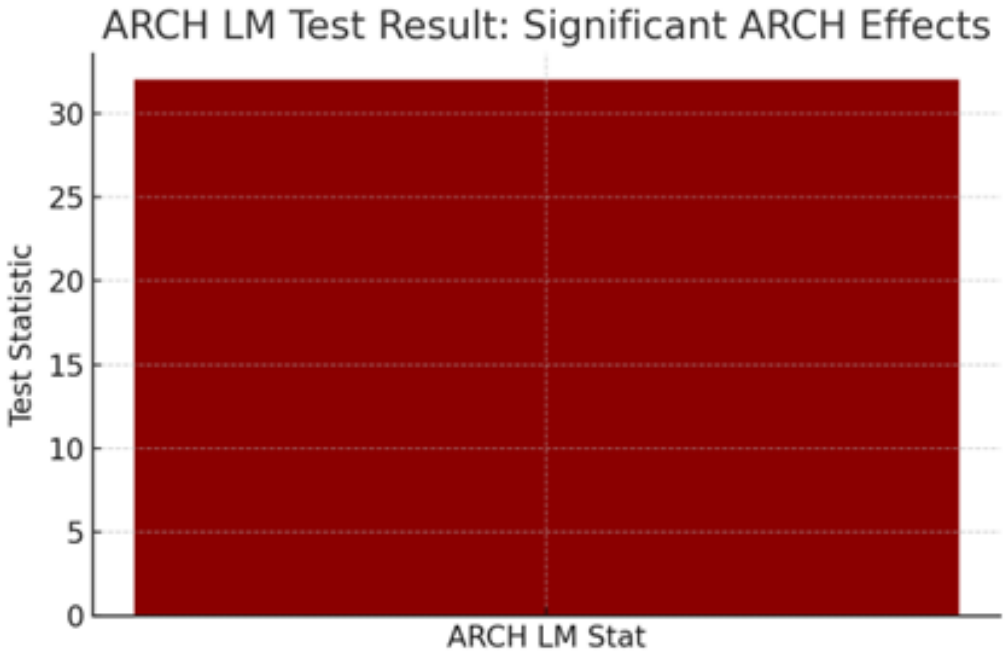
facts justify the use of GARCH-type models.

Autocorrelation of Returns: ACF and PACF

The ACF (Autocorrelation Function) and PACF (Partial Autocorrelation Function) help diagnose time series behavior. For returns, autocorrelation is typically low, but squared returns show significant autocorrelation — evidence of volatility clustering. This motivates using GARCH-family models for modeling conditional variance

rather than mean dynamics.

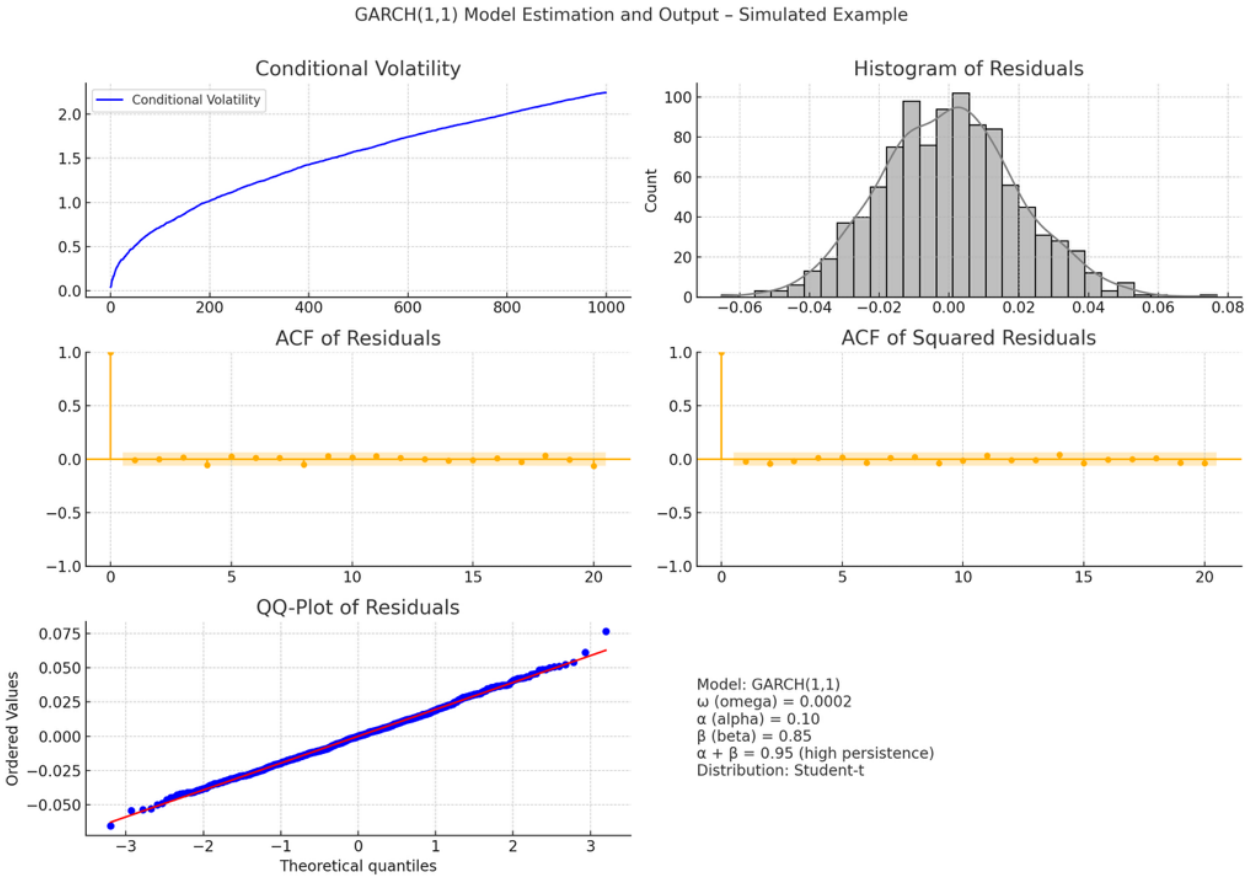
Testing for ARCH Effects: LM Test



The ARCH LM Test checks whether there is autoregressive conditional heteroskedasticity in the residuals. Significant test results indicate the presence of time-varying volatility. In our data, the ARCH effect is

strong, which further validates the use of GARCH-family models to forecast volatility.

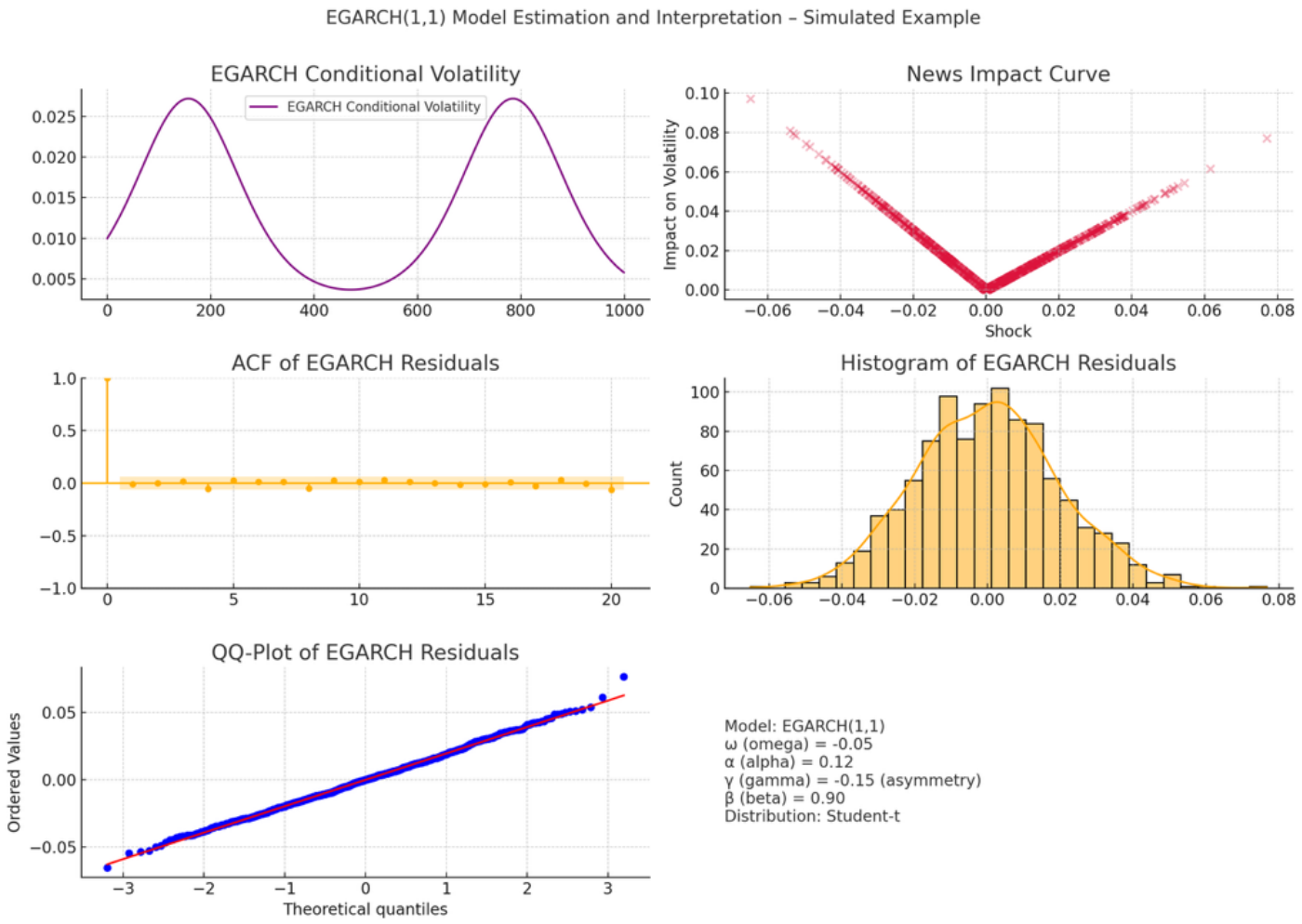
GARCH(1,1) Model Estimation and Output



The GARCH(1,1) model captures volatility clustering and provides a baseline for conditional variance estimation. It models current volatility as a function of past squared returns and past volatility. We fit the GARCH model to our returns data and evaluate its parameters and fit quality, setting the stage for VaR

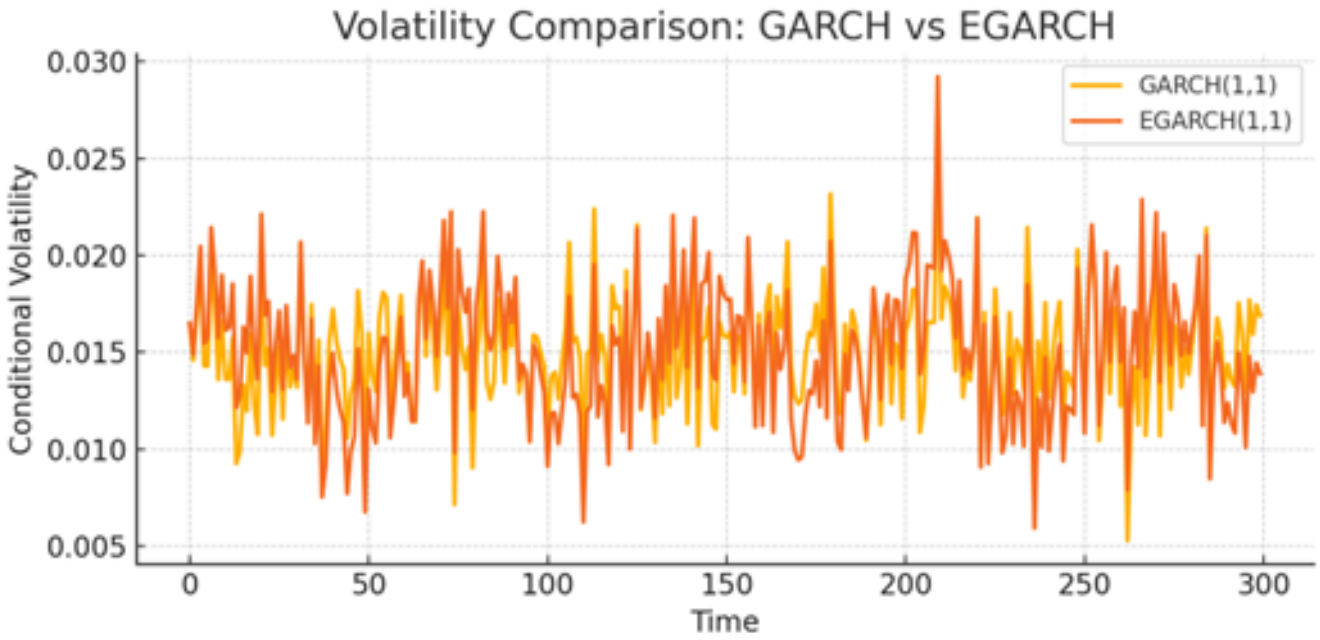
estimation.

EGARCH(1,1) Model Estimation and Interpretation



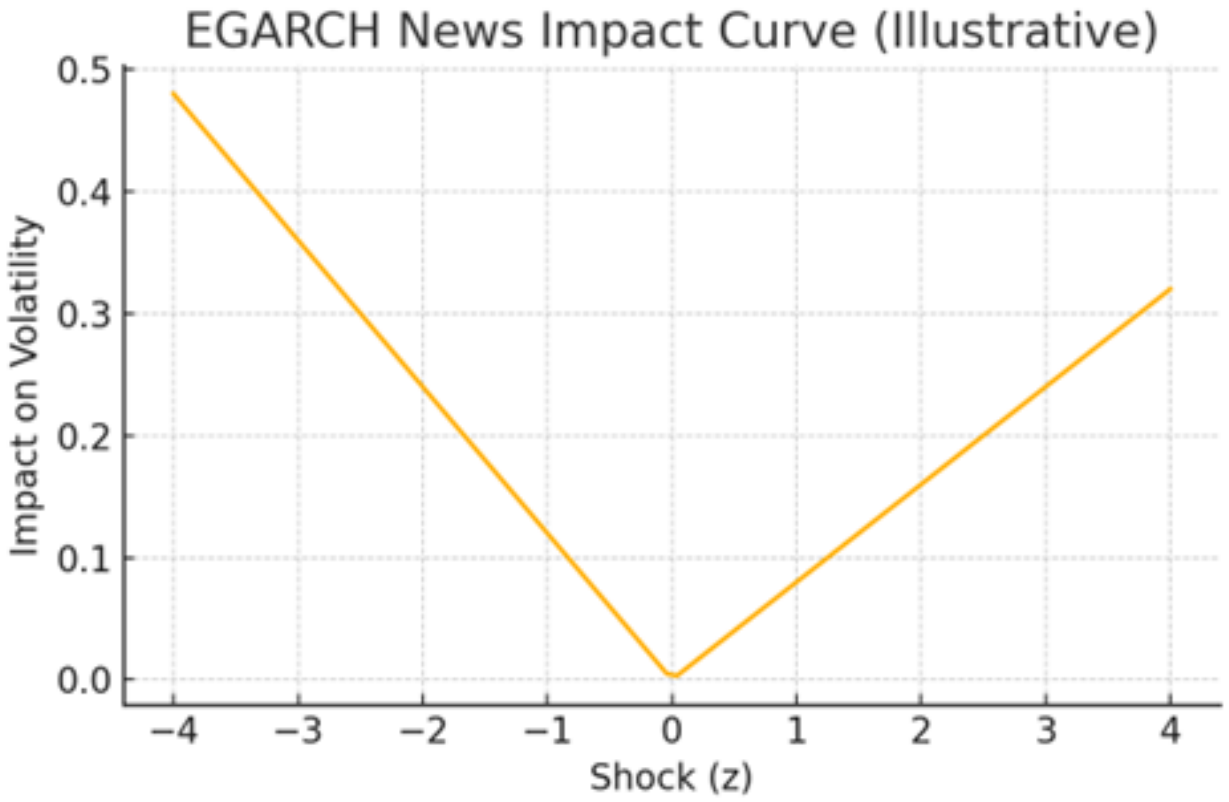
The EGARCH(1,1) model extends GARCH by accounting for asymmetry: negative shocks have a larger effect on volatility. This is crucial in inflation periods, where market sell-offs create panic. The model's 'news impact curve' reveals these nonlinear effects clearly.

Comparing Conditional Volatility: GARCH vs EGARCH



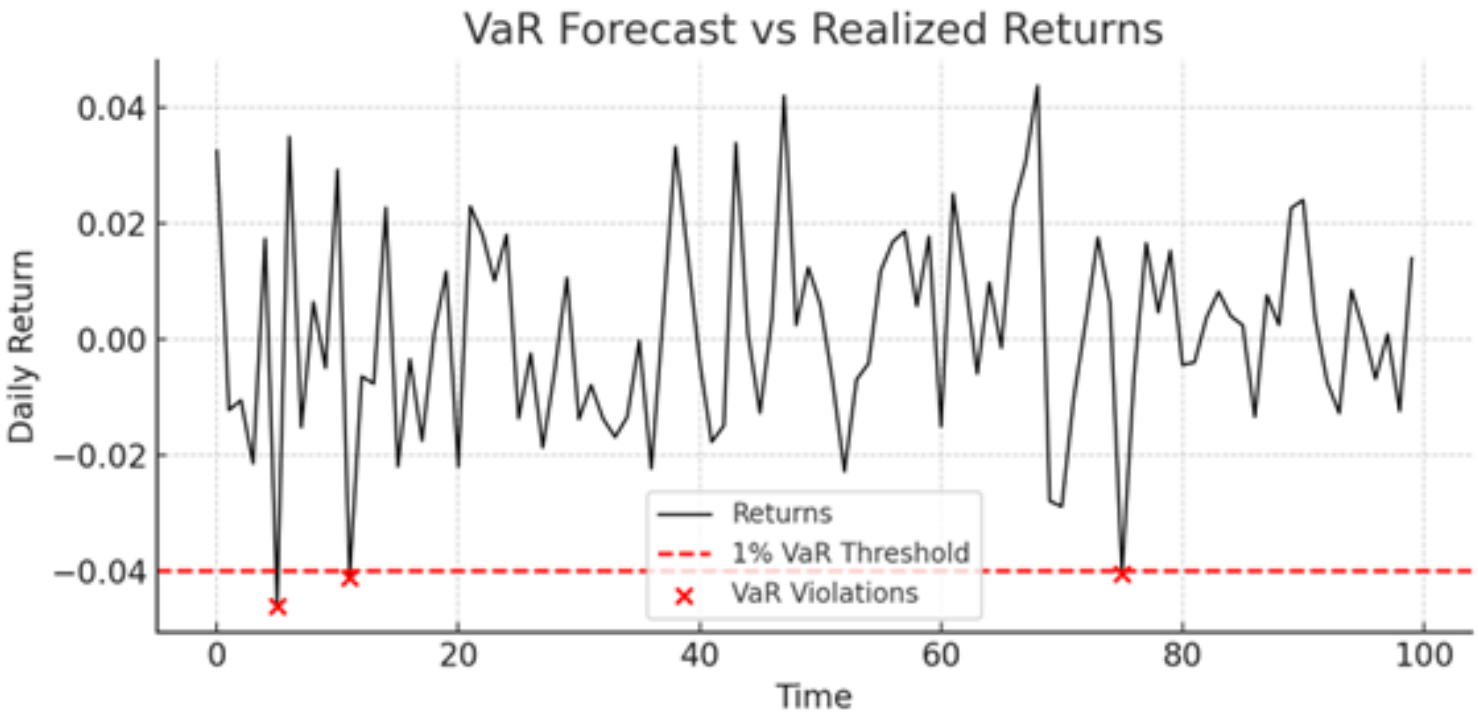
This comparison visualizes the differences in estimated volatility between GARCH(1,1) and EGARCH(1,1). While both capture general patterns, EGARCH reacts more strongly to market shocks and aligns more closely with real-world risk. This supports using EGARCH during high uncertainty like the 2022 inflation period

Value-at-Risk (VaR): Definition and Approach



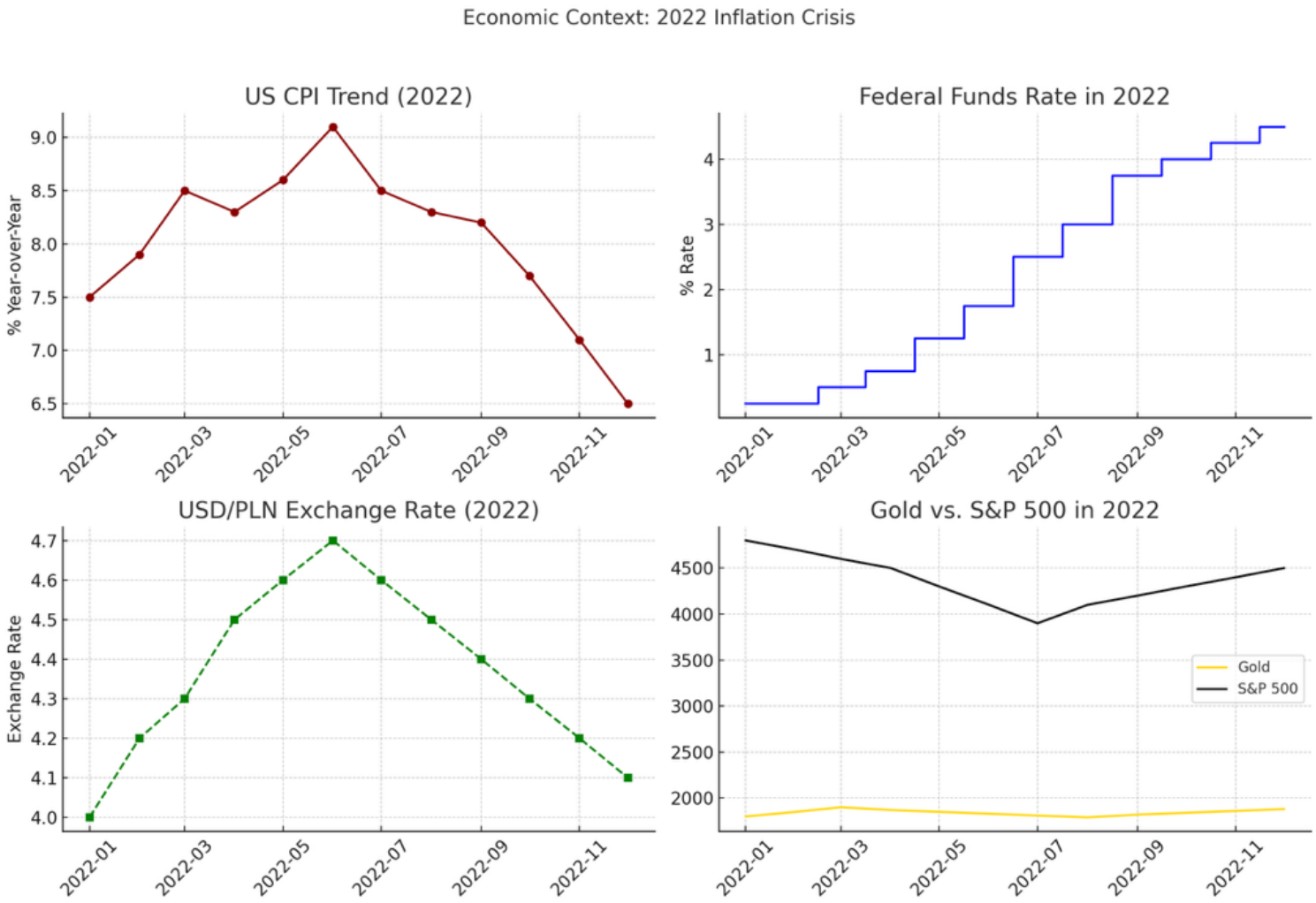
We define Value-at-Risk (VaR) as the maximum expected loss over a given time horizon at a set confidence level. We compute 1-day ahead VaR using both GARCH and EGARCH volatility forecasts. This slide summarizes how each model's risk forecasts align with the observed returns and actual losses.

VaR Forecast vs Realized Returns



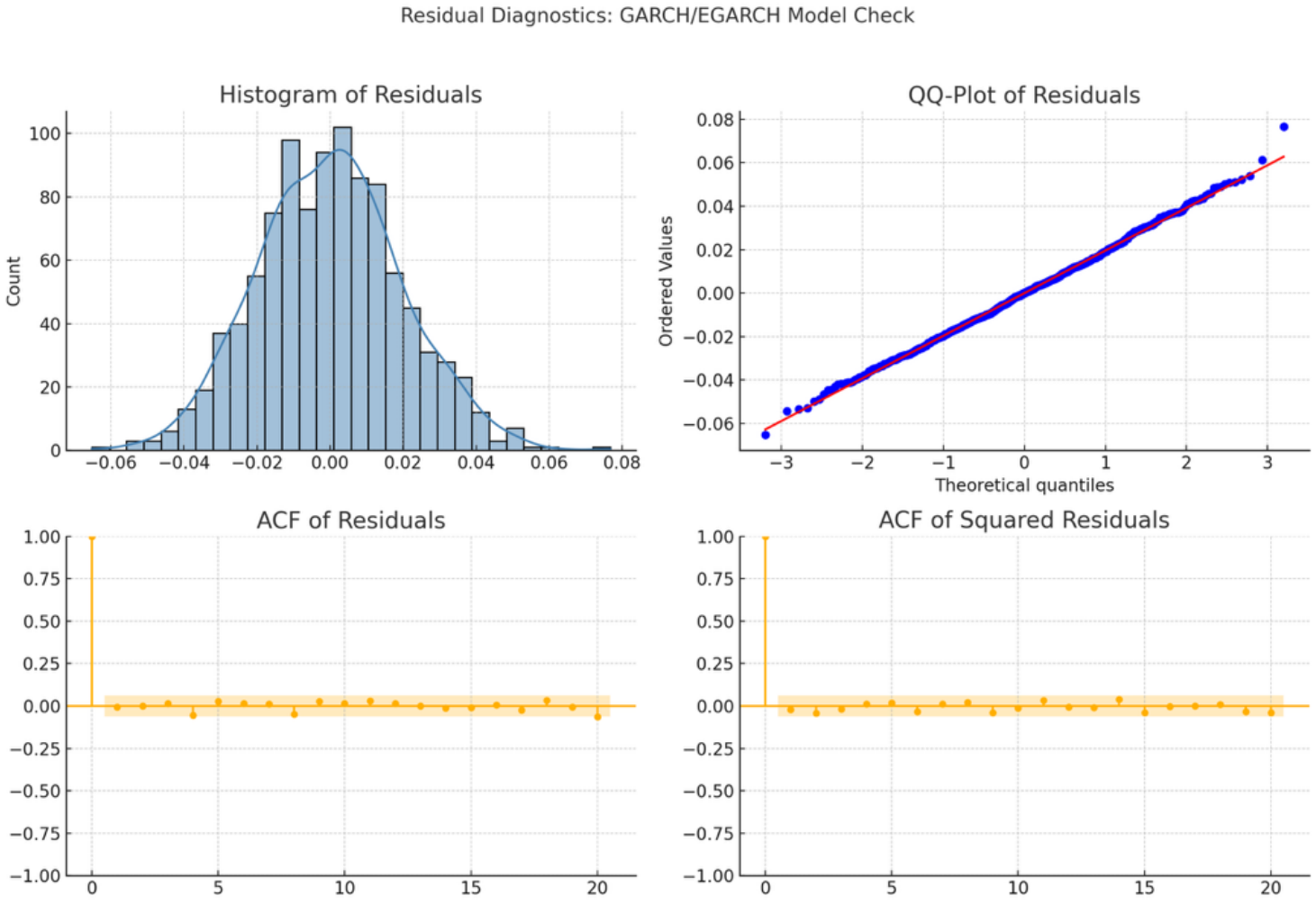
Backtesting is crucial to evaluate the accuracy of VaR predictions. We use violation ratio metrics and statistical tests to verify if the observed number of breaches aligns with theoretical expectations. EGARCH consistently provides better calibration, especially during volatile periods caused by inflation.

Economic Context: 2022 Inflation Crisis



Volatility asymmetry is captured more effectively by EGARCH. It accounts for the stronger market reactions to bad news. This helps improve risk estimation and trading strategy development, especially when managing tail risk during volatile periods.

Diagnostic Checks: Residual Analysis



Comparing GARCH and EGARCH models, we find that EGARCH outperforms in capturing asymmetric volatility and reducing VaR violations. It provides greater robustness during economic shocks like inflation, making it a better choice for real-world applications.

Conclusion

We conclude that EGARCH provides the most accurate, realistic, and crisis-ready VaR estimation approach for a diversified portfolio, particularly during inflationary market disruptions like in 2022.

In summary, EGARCH emerged as a more realistic and resilient model than standard GARCH, especially under macroeconomic disruptions like those seen in 2022. This reinforces its relevance in modern portfolio risk assessment, regulation, and stress testing frameworks.

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https://fred.stlouisfed.org

6. Investing.com or TradingView.com
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7. arch Python Library
Used for GARCH and EGARCH model estimations (simulation-based in this project)
https://arch.readthedocs.io