University of Tunis
Tunis Business School **Spring 2023**

Assignment 7

Question 1

The turbines of a power plant are inspected daily. At the end of each day, a turbine is classified in one of the following three states: state 1: efficient, state 2: inefficient, state 3: total failure. The daily operating costs of a type 1, 2 and 3 turbine are estimated at 100, 200 and 1000 TD respectively. The relatively high operating cost of state 3 is explained by the fact that in the case of a total failure of a turbine, the plant must cover the lack of production by having recourse to another power plant to meet its needs.

The maintenance policy followed by the plant is to repair the turbine only if it is in total failure. It is assumed that the repair takes a day and that the turbine, once repaired, becomes efficient. The repair cost is estimated at 500 TD. The transition matrix is as follows:

$$P = \begin{pmatrix} 0{,}8 & 0{,}2 & 0 \\ 0 & 0{,}9 & 0{,}1 \\ 1 & 0 & 0 \end{pmatrix}$$

1. Consider an efficient turbine,
   a. What is the probability that the first total failure will not occur for 3 days?
   b. What is the average time that elapses before the first total failure?
2. The plant has 16 identical turbines, of which currently 4 are inefficient and 2 are in total failure.
   a. Determine the proportion of turbines in total failure after 2 days.
   b. What is the average number of turbines that will be completely out of order at the end of a given day? Justify your answer.
   c. Calculate the average total cost per day.

Question 2

A machine breaks down with a probability *p*. It takes one day to be fixed. The following three states are considered: state 0 when the machine operates normally all the day, state 1 when the machine breaks down during the day, and state 2 when the machine spends the day in repair.

1. Show that this stochastic process forms a Markov chain and provide its transition matrix.
2. Calculate the probability that an operational machine will break for the first time in 5 days.
3. Calculate the probability that an operational machine will break down.
4. Find the fraction of time during which the machine is operational?

Now, it is assumed that another back-up machine is placed and begins to operate as soon as the first fails. We denote by (x,y) the state of the system where x and y take the values 1 and 0 depending on whether the machines are working or not at the end of the day.

5. Knowing that this process forms a Markov chain give its transition matrix.
6. Find the steady-state distribution.

Question 3

University regulations state that a student cannot fail the same course more than 3 times. Otherwise, the student will quit the university. In average a student has a 60% chance to pass the stochastic processes course if taken for the first time. This chance increases to 70% and 80% if the student will have to take it for the second and third time respectively. The course is offered every semester.

1. Represent the problem using Markov chain and specify the states and their classes.
2. How many semesters on the average it takes a student to pass the course?
3. If an average of 15 students takes the course every semester, how many are expected to quit school because of this course every year?