Ball Ramp Theory

Kinematics

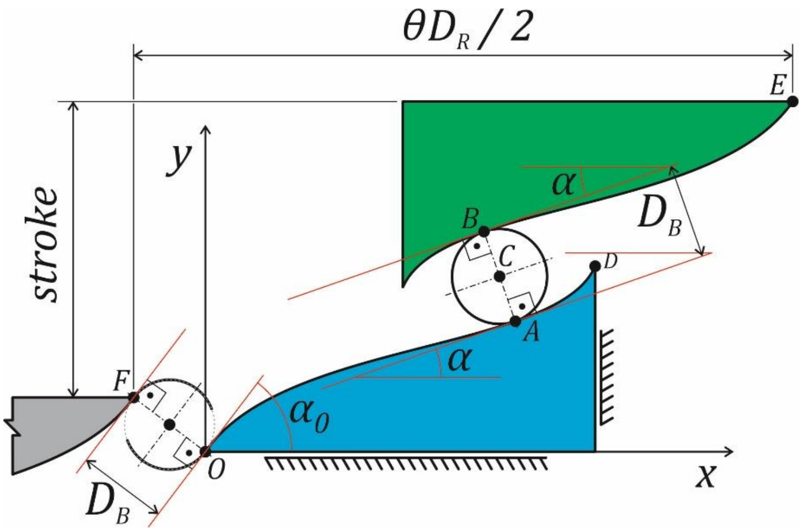

Fig.1 – Ball ramp kinematics diagram

Figure 1 depicts a ball ramp mechanics uncoil around the pitch diameter. The general slope ball ramp consists of two ramps with the same variable slope and a ball between them. The blue ramp (lower one) is fixed and the green ramp (upper one) can move in orthogonal directions. The gray slice of a ramp is the position of the moving ball ramp at the full closed position. The points *A* and *B* are the contact points between the ball and the ramp. The position on the point *A* is given by,

$$ {}_{O}\boldsymbol{r}_{A} = \begin{bmatrix} l_{OAx} \\ l_{OAy} \end{bmatrix}, \tag{1}$$

where the relationship between l_{OAx} and l_{OAy} is given by the function,

$$ slope(l_{OAy}) = l_{OAx}. \tag{2}$$

Due to symmetry,

$$ {}_{B}\boldsymbol{r}_{E} = {}_{O}\boldsymbol{r}_{A}\,. \tag{3}$$

The displacement of the moving ball ramp (from *F* to *E)* is calculated,

$$ {}_{F}\boldsymbol{r}_{E} = {}_{O}\boldsymbol{r}_{E} - {}_{O}\boldsymbol{r}_{F}\,, \tag{4}$$

$$ {}_{F}\boldsymbol{r}_{E} = {}_{O}\boldsymbol{r}_{A} + {}_{A}\boldsymbol{r}_{B} + {}_{B}\boldsymbol{r}_{E} - {}_{O}\boldsymbol{r}_{F} \tag{5}$$

$$ {}_{F}\boldsymbol{r}_{E} = \begin{bmatrix} l_{OAx} \\ l_{OAy} \end{bmatrix} + \begin{bmatrix} -D_B \sin\alpha \\ D_B \cos\alpha \end{bmatrix} + \begin{bmatrix} l_{OAx} \\ l_{OAy} \end{bmatrix} - \begin{bmatrix} -D_B \sin\alpha_0 \\ -D_B \cos\alpha_0 \end{bmatrix} \tag{6}$$

$$ {}_{F}\boldsymbol{r}_{E} = \begin{bmatrix} 2l_{OAx} + D_B(\sin\alpha_0 - \sin\alpha) \\ 2l_{OAy} + D_B(\cos\alpha - \cos\alpha_0) \end{bmatrix} \tag{7}$$

Hence, *stroke* and θ are given by,

$$ stroke = 2l_{OAy} + D_B(\cos\alpha - \cos\alpha_0). \tag{8}$$

$$ \theta = \frac{4l_{OAx} + 2D_B(\sin\alpha_0 - \sin\alpha)}{D_R}. \tag{9}$$

PROBLEM: find alpha

Trying these equations at the constant slope…

Constant Slope

In case of a constant slope,

$$\frac{l_{OAx}}{l_{OAy}} = \frac{\pi D_R}{L}, \tag{20}$$

where L in the constant lead (or pitch).

Using (9),

$$\theta_{cons} = \frac{4\pi l_{OAy}}{L}, \tag{20}$$

$$l_{OAy} = \frac{\theta_{cons} L}{4\pi}, \tag{21}$$

and then from (8),

$$stroke = \frac{\theta_{cons} L}{2\pi}. \tag{22}$$

Quadratic Slope

In case of a quadratic slope,

$$l_{OAx} = c_1 {l_{OAy}}^2 + c_2 l_{OAy}, \tag{20}$$

where c_1 and c_2 are constants.

Spline Slope

Cubic, 5[th] or higher

Statics

In case of quasi-statics forward motion, infinitesimals segments of the ramp are depicted at Fig. 2. Where F_R and T_R is the force done on the ball ramp, respectively

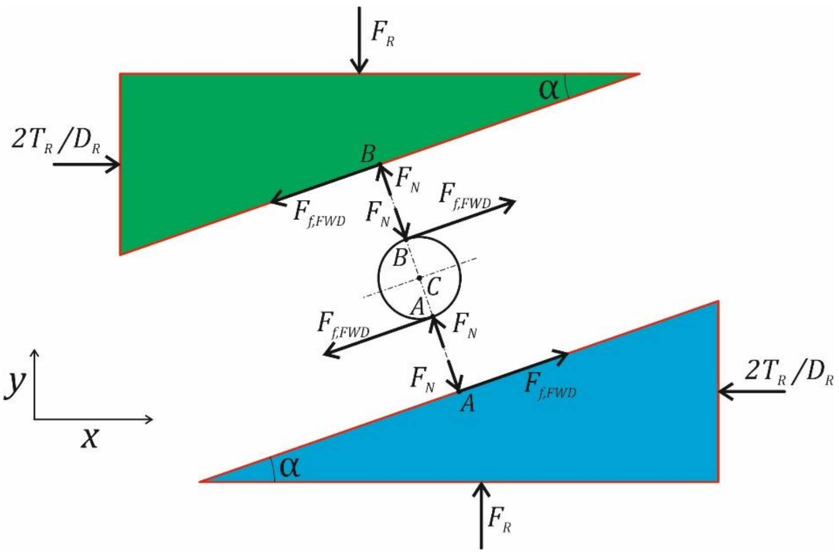

Fig.2 – Ball ramp forward statics

So, computing the forces of the blue segment (lower one),

$$F_R - F_N \cos\alpha + F_{f,FWD} \sin\alpha = 0, \tag{30}$$

$$F_N \sin\alpha - \frac{2T_R}{D_R} + F_{f,FWD} \cos\alpha = 0, \tag{31}$$

Friction is considered as,

$$F_{f,FWD} = \mu F_N. \tag{32}$$

Hence, equations (30-32) gives the following relation between force and torque,

$$F_R = \frac{2T_R}{D_R} \frac{(\cos\alpha - \mu \sin\alpha)}{(\sin\alpha + \mu \cos\alpha)} \tag{33}$$

Since locally, for the statics,

$$\tan\alpha = \frac{L}{\pi D_R} \tag{34}$$

Adding the index for forward force and torque to (33) and reorganizing using (34),

$$F_{R,FWD} = \frac{2\pi}{L} T_{R,FWD} \underbrace{\frac{(1 - \mu \tan\alpha)}{(1 + \mu/\tan\alpha)}}_{\eta_{FWD}}. \tag{35}$$

In backward motion,

$$F_{f,BWD} = -F_{f,FWD}. \tag{40}$$

Hence,

$$F_{R,BWD} = \frac{2\pi}{L} T_{R,BWD} \underbrace{\frac{(1 - \mu / \tan\alpha)}{(1 + \mu \tan\alpha)}}_{1/\eta_{BWD}}. \tag{41}$$