Appendix A — Formal Properties and Full Proofs

This appendix provides formal statements and full proofs of the theoretical properties claimed in the paper. We use the same notation as in the main text and reproduce essential definitions for completeness.

A.1 Preliminaries and Notation

Evidence graph. Let $G = (V, E, \tau)$ be the typed co-occurrence graph built from the corpus, where $\tau: V \to \{\text{Outcome, Capability, Practice, Tool, Maturity}\}$ assigns each node a type. Allowed transitions are: Outcome → Capability → Practice → Tool and Outcome → Maturity. An edge $(u, v) \in E$ carries nonempty provenance $P(u, v) \subseteq \mathbb{N}$ (identifiers of studies supporting the co-occurrence).

Retrieval scores. For each node $u \in V$ and query q, let s_lex(u) and s_vec(u) denote respectively the lexical and vector retrieval scores computed from the node document. Let deg(u) be the degree of u within the subgraph used for reasoning.

Normalization. For a fixed per-query candidate pool $K \subseteq V$, define min–max normalization $z(x) = (x - \min_K) / (\max_K - \min_K)$ with the convention $z(x) = 0$ when $\max_K = \min_K$. This mapping is order-preserving on K when $\max_K > \min_K$ and is applied separately to s_lex, s_vec, and deg. All results assume K and the corresponding normalization bounds are fixed for the query.

Score fusion. The fused score is $s\_fuse(u) = \alpha \cdot z(s\_vec(u)) + (1 - \alpha) \cdot z(s\_lex(u))$, with $\alpha \in [0, 1]$. Outcome selection interpolates centrality: $s\_out(u) = (1 - \lambda) \cdot s\_fuse(u) + \lambda \cdot z(deg(u))$, with $\lambda \in [0, 1]$. Chain-constrained reasoning (CCR) expands only along allowed transitions and returns recommendations drawn from the expanded set.

A.2 Theorem A.1 (Chain soundness)

Statement. Every item recommended by CCR lies on a valid, type-correct path that starts at an Outcome node and follows only allowed transitions.

Proof. CCR performs layer-by-layer expansion from nodes typed Outcome, and each expansion step checks type admissibility against the transition schema. Formally, if L_0 is the set of selected Outcome nodes, CCR constructs $L_1 \subseteq \{v \in V : \exists u \in L_0, (u, v) \in E \text{ and } (\tau(u), \tau(v)) = (\text{Outcome, Capability})\}$, then L_2 from L_1 using (Capability, Practice), and so on, never following an edge that violates τ. Recommendations are selected from $\bigcup_{\{\ell\}} L_\ell$. By construction, any v in this union is reachable via a concatenation of allowed transitions, establishing soundness. ■

A.3 Proposition A.2 (Termination and time bound)

Statement. For fixed per-layer fan-out k and depth $d \leq 4$ (Outcome→…→Tool), CCR terminates and performs $O(k \cdot d)$ expansions per query.

Proof. CCR restricts expansion at each layer to the top-k neighbors by score. The chain schema imposes a strict layer order (Outcome, Capability, Practice, Tool/Maturity) with maximum depth $d \leq 4$ and prohibits cycles across layers. Hence, the total number of node expansions is bounded by k per layer, i.e., $O(k \cdot d)$. Since d is constant (≤ 4), CCR is linear in k. ■

A.4 Theorem A.3 (Score monotonicity)

Statement. Fix α, λ and the candidate pool K. If for some node u the normalized components do not decrease, i.e., $\Delta z(s\_vec(u)) \geq 0$, $\Delta z(s\_lex(u)) \geq 0$, and $\Delta z(deg(u)) \geq 0$, then $\Delta s\_out(u) \geq 0$.

Proof. We have $s\_fuse(u) = \alpha \cdot z(s\_vec(u)) + (1-\alpha) \cdot z(s\_lex(u))$, a convex combination of terms in [0, 1]. If each component is non-decreasing, then $\Delta s\_fuse(u) = \alpha \cdot \Delta z(s\_vec(u)) + (1-\alpha) \cdot \Delta z(s\_lex(u)) \geq 0$. Similarly, $s\_out(u) = (1-\lambda) \cdot s\_fuse(u) + \lambda \cdot z(deg(u))$ is a convex combination, so $\Delta s\_out(u) = (1-\lambda) \cdot \Delta s\_fuse(u) + \lambda \cdot \Delta z(deg(u)) \geq 0$. The assumption that K (and hence min/max bounds) is fixed ensures that z is order-preserving, so non-decreasing raw components imply non-decreasing normalized components. ■

A.5 Theorem A.4 (Lipschitz stability of scores)
Statement. Suppose $|\Delta z(s\_vec(u))| \leq \varepsilon\_vec$, $|\Delta z(s\_lex(u))| \leq \varepsilon\_lex$, and $|\Delta z(deg(u))| \leq \varepsilon\_deg$ for all $u \in K$. Then $|\Delta s\_out(u)| \leq (1-\lambda) \cdot (\alpha \cdot \varepsilon\_vec + (1-\alpha) \cdot \varepsilon\_lex) + \lambda \cdot \varepsilon\_deg$.

Proof. By the triangle inequality, $|\Delta s\_fuse(u)| = |\alpha \cdot \Delta z(s\_vec(u)) + (1-\alpha) \cdot \Delta z(s\_lex(u))| \leq \alpha \cdot |\Delta z(s\_vec(u))| + (1-\alpha) \cdot |\Delta z(s\_lex(u))| \leq \alpha \cdot \varepsilon\_vec + (1-\alpha) \cdot \varepsilon\_lex$. Then $|\Delta s\_out(u)| = |(1-\lambda) \cdot \Delta s\_fuse(u) + \lambda \cdot \Delta z(deg(u))| \leq (1-\lambda) \cdot |\Delta s\_fuse(u)| + \lambda \cdot |\Delta z(deg(u))| \leq (1-\lambda) \cdot (\alpha \cdot \varepsilon\_vec + (1-\alpha) \cdot \varepsilon\_lex) + \lambda \cdot \varepsilon\_deg$. ■

A.6 Corollary A.5 (Top-k stability by margin)
Statement. Let δ be the minimum score margin between the k-th and (k+1)-th ranked items under s_out. If $\delta > (1-\lambda) \cdot (\alpha \cdot \varepsilon\_vec + (1-\alpha) \cdot \varepsilon\_lex) + \lambda \cdot \varepsilon\_deg$, then the top-k set is invariant under the perturbations bounded as in Theorem A.4.

Proof. By Theorem A.4, the maximum score change per item is bounded by $B = (1-\lambda) \cdot (\alpha \cdot \varepsilon\_vec + (1-\alpha) \cdot \varepsilon\_lex) + \lambda \cdot \varepsilon\_deg$. If the margin δ exceeds B, then no item below rank k can overtake the k-th item, and no item in the top-k can fall below rank k. Thus, the set membership is preserved. ■

A.7 Proposition A.6 (Robustness to synonym merges)
Statement. Under study-level counting (a study contributes at most once per normalized label), merging synonyms into a canonical label cannot decrease that label's document frequency or degree. Consequently, with fixed normalization bounds, its s_out cannot decrease.

Proof. Let S be the set of studies mentioning any alias of the concept; study-level deduping counts a study once regardless of how many aliases appear. Merging aliases replaces multiple nodes $\{v\_i\}$ by a single canonical node v^*, with document frequency $|S|$, which is $\geq$ the maximum individual alias count. All incident edges to the aliases are rewired to v^*, so the degree of v^* is at least the maximum degree among the aliases (and typically larger due to edge union). With K fixed, z remains order-preserving, and by Theorem A.3 the score $s\_out(v^*)$ is non-decreasing after the merge. ■

A.8 Proposition A.7 (Continuity in α and λ)
Statement. For fixed $z(\cdot)$, s_out(u) is continuous and piecewise linear in α and λ. Moreover, $|\partial s\_out/\partial\alpha| \leq |z(s\_vec(u)) - z(s\_lex(u))|$ and $|\partial s\_out/\partial\lambda| \leq |z(deg(u)) - s\_fuse(u)|$.

Proof. Since $s\_fuse(u) = \alpha \cdot z(s\_vec(u)) + (1-\alpha) \cdot z(s\_lex(u))$, we have $\partial s\_fuse/\partial\alpha = z(s\_vec(u)) - z(s\_lex(u))$. Then $s\_out(u) = (1-\lambda) \cdot s\_fuse(u) + \lambda \cdot z(deg(u))$ implies $\partial s\_out/\partial\alpha = (1-\lambda) \cdot (z(s\_vec(u)) - z(s\_lex(u)))$ and $\partial s\_out/\partial\lambda = z(deg(u)) - s\_fuse(u)$. Each term lies in $[-1, 1]$, yielding the stated bounds. Because s_out is an affine function of α and λ when z is fixed, it is continuous and piecewise linear over $\alpha, \lambda \in [0, 1]$. ■

A.9 Proposition A.8 (Provenance completeness)
Statement. Every recommended item has nonempty supporting provenance (study identifiers).

Proof. Nodes and edges are created only from extracted co-occurrences that carry at least one study identifier. CCR returns items reachable via edges in E; hence each returned node lies on at least one path composed of edges with nonempty provenance sets. By concatenation, the provenance for the node is nonempty. ■

A.10 Remarks on Assumptions and Practical Implications

Fixed candidate pool. The monotonicity and stability results rely on a fixed per-query candidate pool K and its corresponding min–max bounds. This is consistent with the evaluation protocol (depth-20 pooling per system with deduplication). If K changes, the normalization mapping z may change; the results then hold with respect to the z induced by the chosen K.

Diversity constraints. If CCR applies per-vendor or per-type quotas during expansion, chain soundness and termination remain unaffected; quotas only restrict the selected subset at each layer and preserve type-correctness by construction.