

# **SCALE-UP: An implementation of Second-Principles DFT**

**Pablo García-Fernández, Jorge Íñiguez and  
Javier Junquera**

**Louvain May 2019**

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**Universidad de Cantabria**



# Collaborators

Cantabria University



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Jorge Iñiguez

Cantabria University Campus



Funding



RyC programme

# What can we do with first-principles simulations?

Predict material properties using just fundamental constants

Little input from user necessary to obtain reliable information

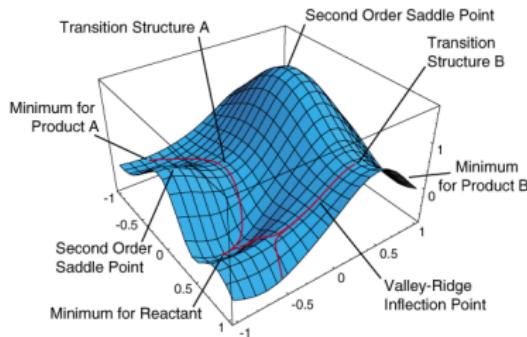
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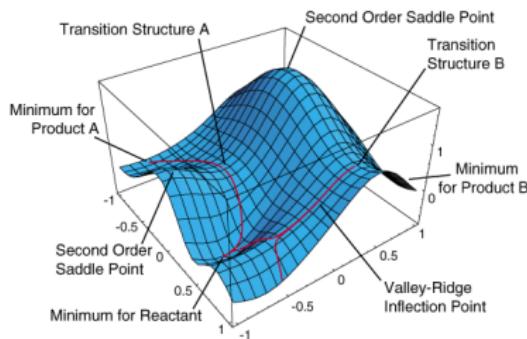
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- ✓ Accurate energies
- ✓ Equilibrium geometries

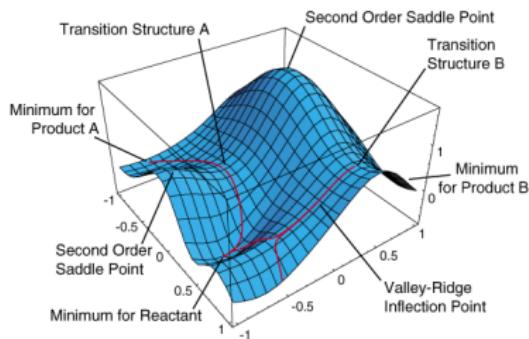
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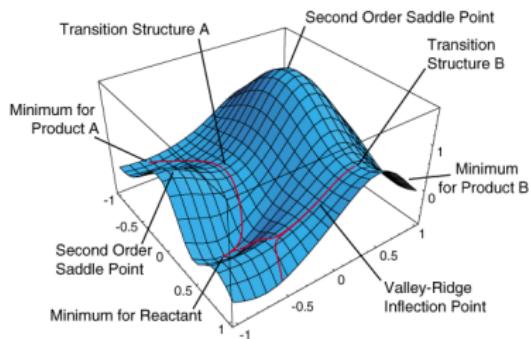
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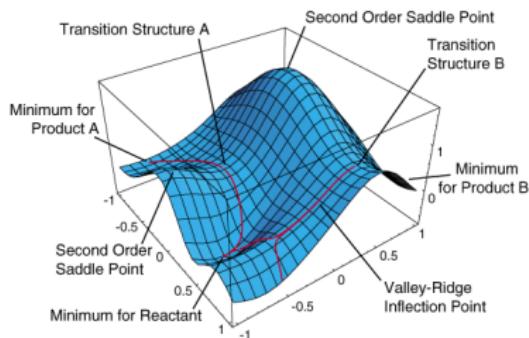
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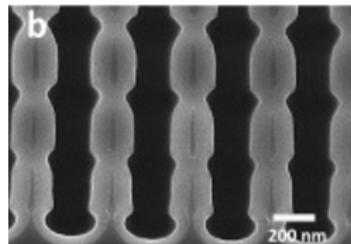
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Wealth of information that could be difficult to obtain experimentally

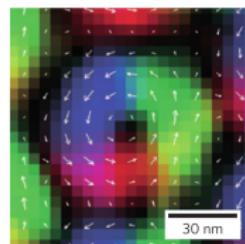
# The problem

Scale of interest

- ✓ Nanoscale ( $\approx 10 - 100\text{nm}$ )
- ✗ DFT  $\approx 1\text{ nm}$



Nanowires

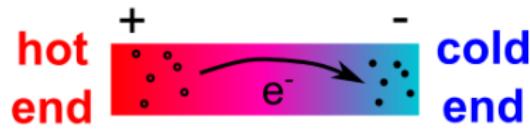


skyrmion in MnSi

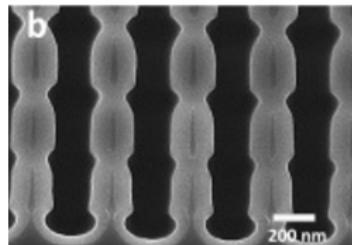
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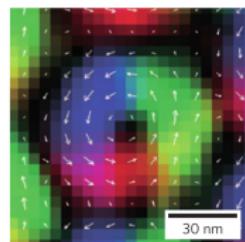


Thermoelectrics, polarons...



Nanowires  
Non-equilibrium states

- ✓ Resistivity
- ✓ Charge diffusion
- ✓ Reaction yield

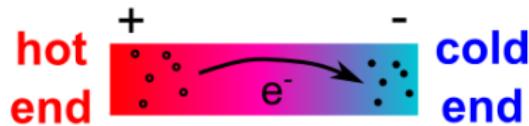


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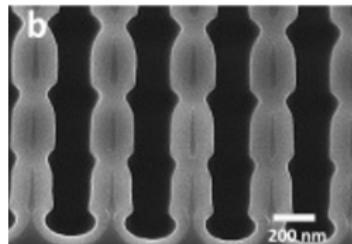
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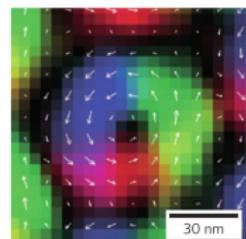


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Ferroelectric domains in  $\text{BiFeO}_3$

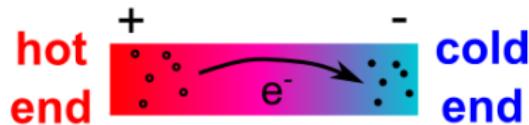


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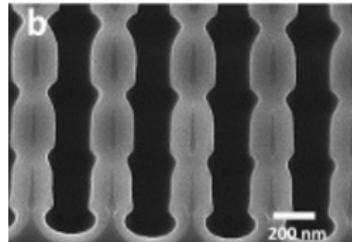


Thermoelectrics, polarons...

Disorder

- ✓ Domains
- ✓ Thermal
- ✓ Defects (polarons, impurities...)

- ▶ Perturbations/disorder are key elements in experiments.
- ▶ Room temperature is fundamental for applications.

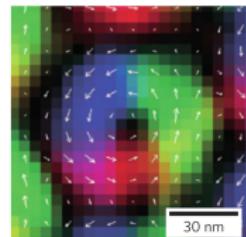


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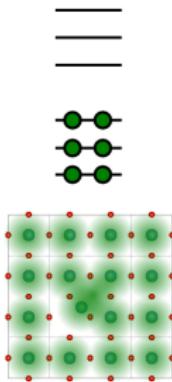
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# Are larger/faster FP simulations possible?

First principles simulations deal with all electrons in the system:

Number of electrons  
grows fast

Hamiltonian  $\sim N^2$   
Diagonalization  $\sim N^3$

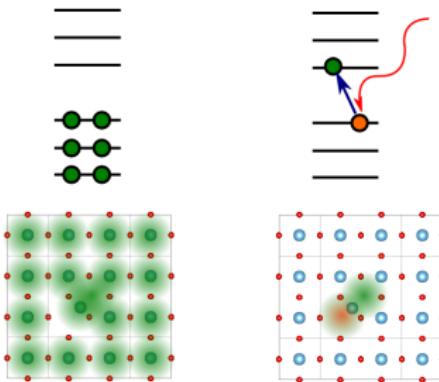


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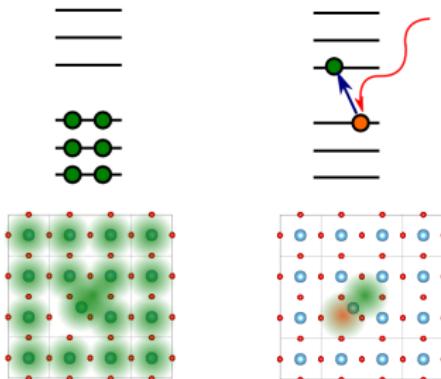
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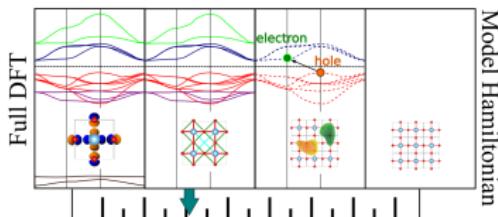
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Can we select the level of fidelity of our calculations?

Can we make it efficient?

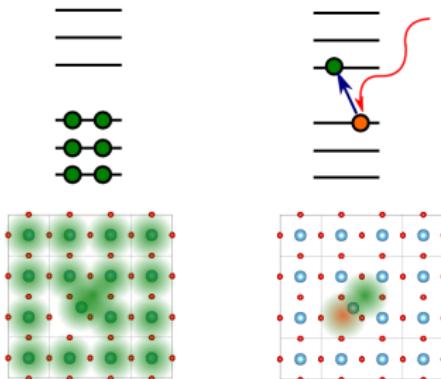
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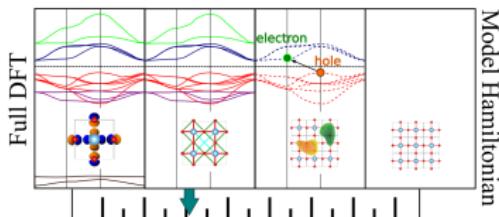
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## Second-principles Density Functional Methods

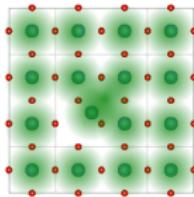
## Basic concepts

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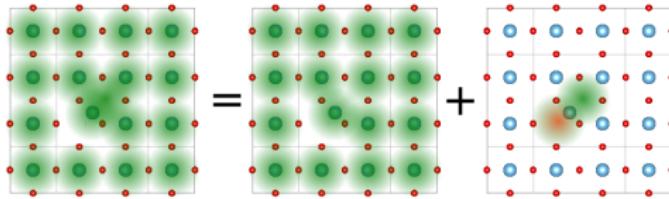
Suppose an insulator doped with electrons or holes:



# Basic concepts

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Suppose an insulator doped with electrons or holes:



The total density is separated in **reference** and **deformation** densities:

$$n(\vec{r}) = n_0(\vec{r}) + \delta n(\vec{r})$$

$n_0$  = reference density

$\delta n$  = deformation density

# Approximating the DFT energy

Our starting point is the DFT energy

$$E_{\text{DFT}} = \sum_{j\vec{k}} o_{j\vec{k}} \left\langle \psi_{j\vec{k}} \right| \hat{t} + v_{\text{ext}} \left| \psi_{j\vec{k}} \right\rangle + \frac{1}{2} \iint \frac{n(\vec{r})n'(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3rd^3r' + E_{\text{xc}}[n] + E_{\text{nn}}$$

We want to write the energy in terms of the reference and deformation densities.

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The only difficulty is the exchange-correlation energy that we expand in terms of  $\delta n$  (see e. g. M. Elstner et al., *Phys. Rev. B*, 58, 7260 (1998)):

$$E_{\text{xc}}[n] = E_{\text{xc}}[n_0] + \int \frac{\delta E_{\text{xc}}}{\delta n(\vec{r})} \Big|_{n_0} \delta n(\vec{r}) d^3r + \frac{1}{2} \iint \frac{\delta^2 E_{\text{xc}}}{\delta n(\vec{r}) \delta n(\vec{r}')} \Big|_{n_0} \delta n(\vec{r}) \delta n(\vec{r}') d^3r d^3r' + \dots$$

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As in usual TB-DFT approximations, we cut at second-order

$$E_{\text{DFT}} \approx E = E^{(0)} + E^{(1)} + E^{(2)}$$

However, we group the terms in a different way to TB-DFT.

## Second-principles DFT approach



Material simulations  
allow for various approaches

First principles methods are **atomistic** with **flexible detailed bonding**  
FP or TB-DFT



Based on atoms

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$$E_{\text{DFT}} \approx \underbrace{E^{(0)}_{\text{atomic cores}}}_{\text{full } 1e \text{ energy}} + \underbrace{E^{(1)}_{\text{full } 1e \text{ energy}}}_{\text{full } 2e \text{ energy}} + \underbrace{E^{(2)}_{\text{full } 2e \text{ energy}}} + \dots$$

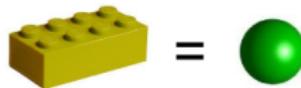
Atoms  $\Rightarrow$  FP  $\Rightarrow$  Materials

# Second-principles DFT approach



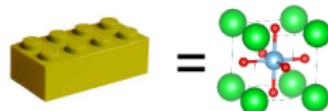
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Based on materials

$$E_{\text{DFT}} \approx \underbrace{E^{(0)}_{\text{lattice}}}_{\text{lattice}} + \underbrace{E^{(1)}_{\text{electron excitations}} + E^{(2)}_{\text{electron excitations}} + \dots}_{\text{electron excitations}}$$

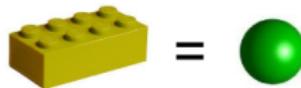
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Accurate properties do not require bond-breaking!

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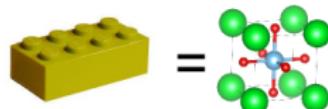
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Accurate properties do not require bond-breaking!

Precise, small, material-adapted basis  $\rightarrow$  Wannier-like functions

I. Souza et al., *Phys. Rev. B*, 65, 035109 (2001)

## Energy terms: $E = E^{(0)} + E^{(1)} + E^{(2)}$ Reference

This term is the full DFT energy for the reference state:

$$E^{(0)} = \sum_{j\vec{k}} o_{j\vec{k}}^0 \left\langle \psi_{j\vec{k}}^0 \right| \hat{t} + v_{\text{ext}} \left| \psi_{j\vec{k}}^0 \right\rangle + \frac{1}{2} \iint \frac{n_0(\vec{r}) n'_0(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3 r d^3 r' + E_{\text{xc}}[n_0] + E_{nn}$$

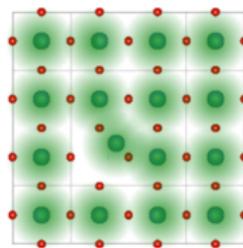
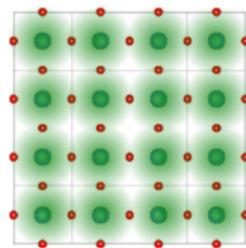
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No approximations

At difference with usual TB-DFT this term is very large and contains most of the total energy. It can be made really accurate.



$E_0(\eta, \{\vec{u}\})$  is the energy surface  
for the reference state

It can be represented by a  
high-quality model potential.

J. Wojdeł et al., *JPCM*, 25, 305401  
(2013)

The reference state is defined for  
all geometries

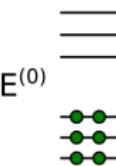
This lattice Hamiltonian is implemented in Multibinit!

# Energy terms: $E = E^{(0)} + E^{(1)} + E^{(2)}$ One electron

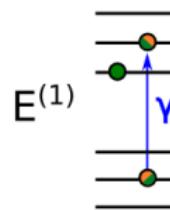
Reference

- ▶ Full DFT energy for  $n_0$
- ▶ Force field
- ▶ Multibin

$E^{(1)}$  contains differences in one-electron energies



$$E^{(1)} = \sum_{j\vec{k}} \left[ o_{j\vec{k}} \left\langle \psi_{j\vec{k}} \right| \hat{h}_0 \left| \psi_{j\vec{k}} \right\rangle - o_{j\vec{k}}^0 \left\langle \psi_{j\vec{k}}^0 \right| \hat{h}_0 \left| \psi_{j\vec{k}}^0 \right\rangle \right]$$



where  $\hat{h}_0$  is the Kohn-Sham Hamiltonian for the reference density:

$$\hat{h}_0 = \hat{t} + v_{\text{ext}} + v_{\text{H}}(n_0) + v_{\text{xc}}[n_0]$$

# Energy terms: $E = E^{(0)} + E^{(1)} + E^{(2)}$ One electron

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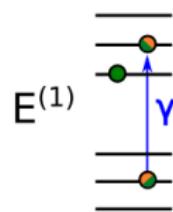
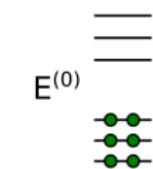
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$$= \sum_{ab} D_{ab} \gamma_{ab} \quad (\text{Wannier basis, } \chi_a)$$

$$\delta n(\vec{r}) = \sum_{ab} D_{ab} \chi_a^*(\vec{r}) \chi_b(\vec{r})$$



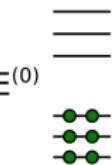
$\gamma_{ab}$  takes the role of the hopping constant in TB schemes.

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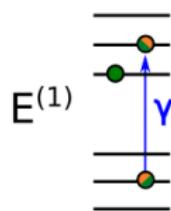


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Only depends on difference density!

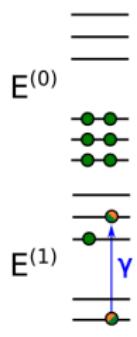
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Reference

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One-electron

- ▶ Depends only on difference density
- ▶ Tight-binding like



$E^{(2)}$  are interactions between 2 electrons ( $E^{(3)}$  3-electron, etc.):

$$E^{(2)} = \frac{1}{2} \int d^3r \int d^3r' g(\vec{r}, \vec{r}', s, s') \delta n(\vec{r}, s) \delta n(\vec{r}', s')$$

where  $g$  is a screened electron-electron interaction operator.

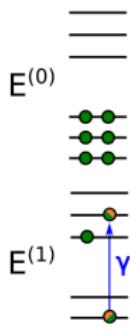
# Energy terms: $E = E^{(0)} + E^{(1)} + E^{(2)}$ Two electron

Reference

- ▶ Full DFT energy for  $n_0$
- ▶ Force field
- ▶ Multibinit

One-electron

- ▶ Depends only on difference density
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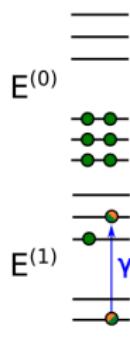
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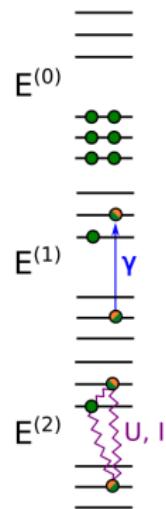
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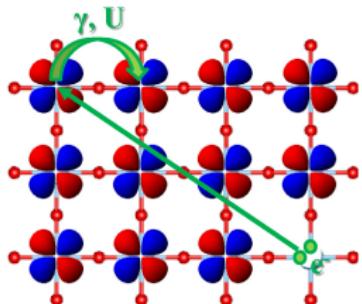
- ▶ Depends only on difference density
- ▶ Screened mean-field interactions

- ▶ Accurate
- ▶ Fast
- ▶ Valid for all kind of systems (magnetic, metallic, ...)



# Electrostatics/Electron-lattice interactions

All interactions occur between localized objects:



At long-range (far-field regime) shape of source density is unimportant

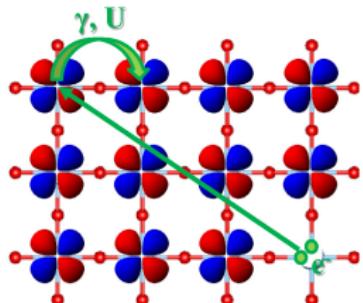
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$\gamma$  and  $U$  contain electrostatic (long-range) contributions

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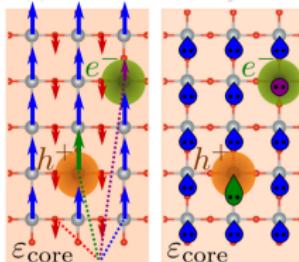
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Local dipoles:

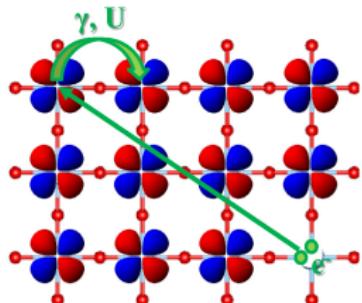
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Potential approximated by field of point charges and dipoles  
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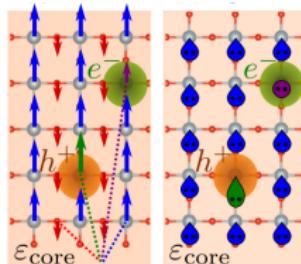
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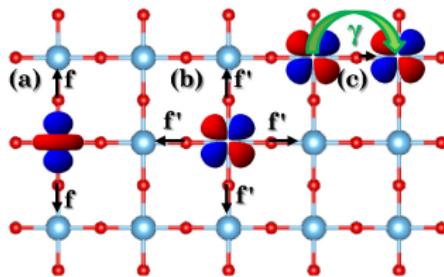


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Model parameters  $\longrightarrow$  long and short range contributions.

## Geometry dependence - Forces

Geometry is involved by expanding  $\gamma$  on the atomic positions:

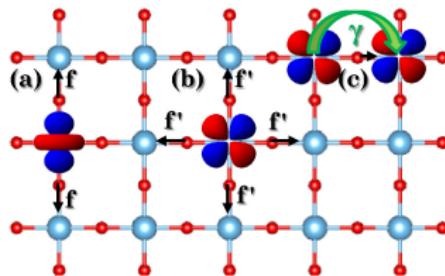


$$\gamma_{\mathbf{ab}}^{\text{sr}} = \gamma_{\mathbf{ab}}^0 + \sum_{\lambda v} \left[ \vec{f}_{\mathbf{ab}, \lambda v} \cdot \delta \vec{r}_{\lambda v} + \delta \vec{r}_{\lambda v} \cdot \overleftrightarrow{g}_{\mathbf{ab}, \lambda v} \cdot \delta \vec{r}_{\lambda v} + \dots \right]$$

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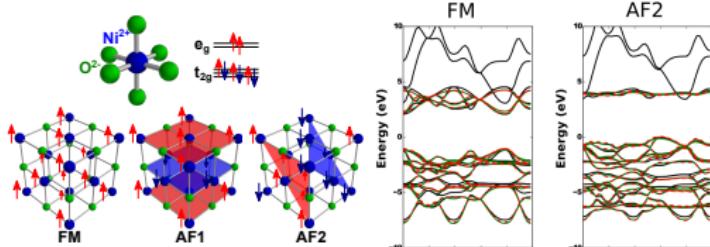
The forces are then obtained:

$$\vec{F}_\lambda = -\vec{\nabla}_\lambda E = -\vec{\nabla}_\lambda E^{(0)} - \sum_{\mathbf{ab}} D_{\mathbf{ab}} \vec{\nabla}_\lambda \gamma_{\mathbf{ab}}.$$

Electronic contribution corrects the force field

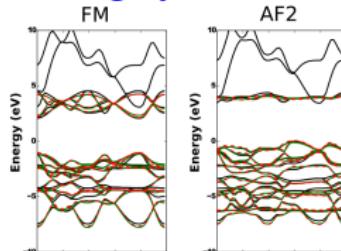
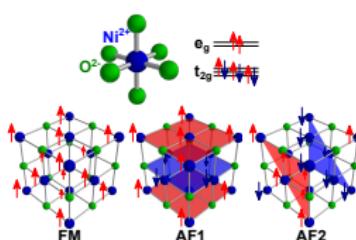
# Applications of SPDFT

NiO - Insulator with highly correlated electrons:



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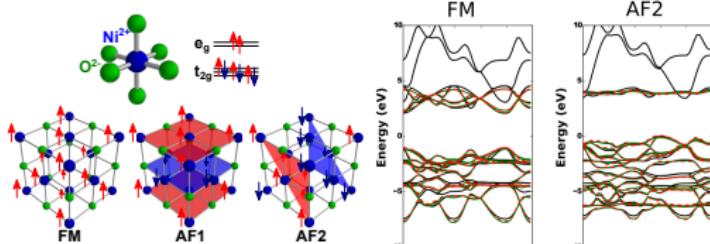
## Magnetic Properties

| Method            | $J_1$ (meV) | $J_2$ (meV) |
|-------------------|-------------|-------------|
| neutron           | 1.4         | -19.0       |
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Electronic/magnetic properties predicted at DFT level  
2000 atoms single point 1 hour in 1 Desktop cpu (full diag)

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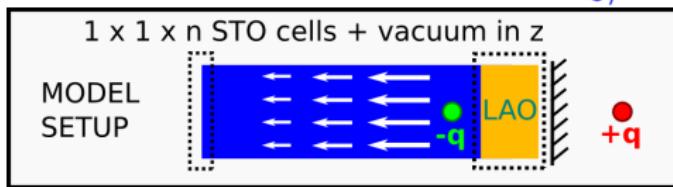


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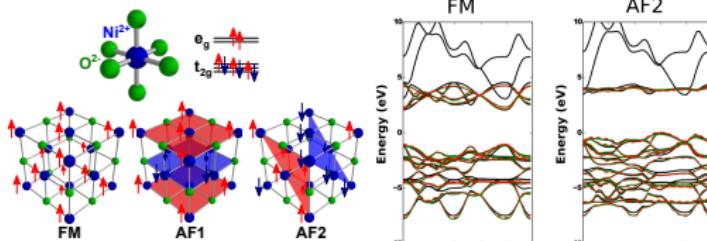
## 2DEG at SrTiO<sub>3</sub>/LaAlO<sub>3</sub> Interface



FP → M. Stengel, *PRL*, 106, 136803 (2011)

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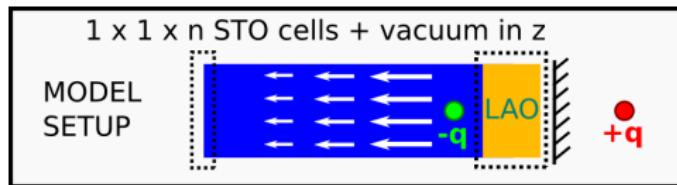


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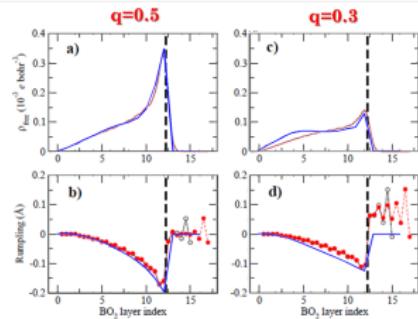
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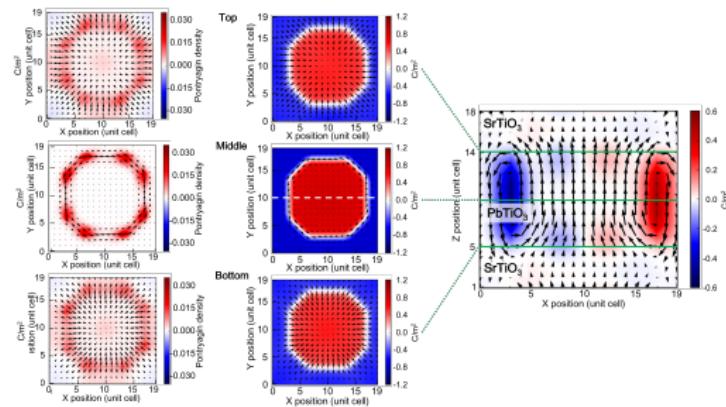
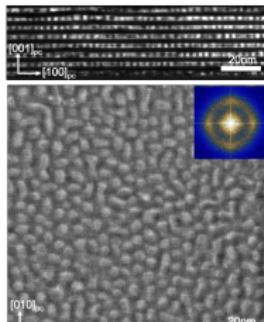
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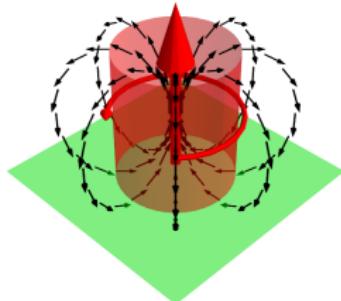
SPDFT captures doping and lattice screening!

# Ferroelectric bubbles

SP can simulate ferroelectric bubble skyrmions in  $\text{PbTiO}_3/\text{SrTiO}_3$



- ▶  $\sim 40000$  atoms
- ▶ Tangential polarization to bubble
- Makes bubbles chiral
- Explains XCD signal
- ▶ Bubble shows non-trivial topology
- Planes  $\rightarrow$  same topological charge



S. Das et al., *Nature*, 568, 368 (2019)

# The implementation of SP-DFT: SCALE-UP



P. Garcia-Fernandez, J. Wojdeł, J. Iñiguez and J. Junquera  
*Phys. Rev. B*, **93**, 195137 (2016)

- ▶ Fully integrated electron+lattice models
- ▶ Single-points, Dynamics (isokinetic, Langevin), Montecarlo
- ▶ SCF with convergence acelerators
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- ▶ Model building suite: MODELMAKER

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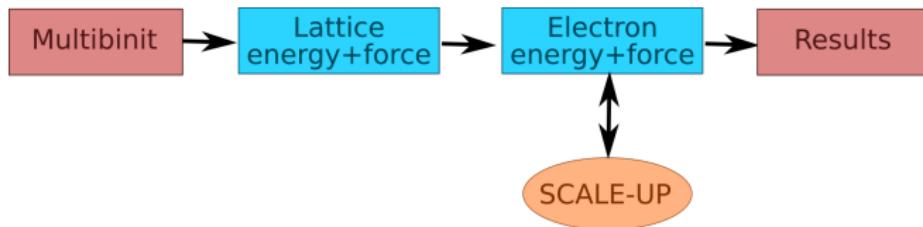


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- ▶ Future: Spin-orbit, defects (surfaces, interfaces, impurities...)
- ▶ Future: Pure density-matrix implementation

# The connection between SCALE-UP and MULTIBINIT

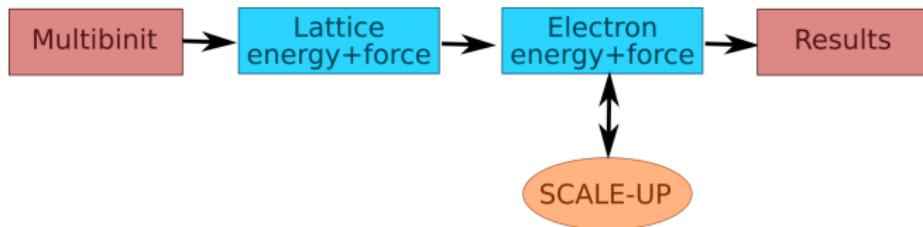
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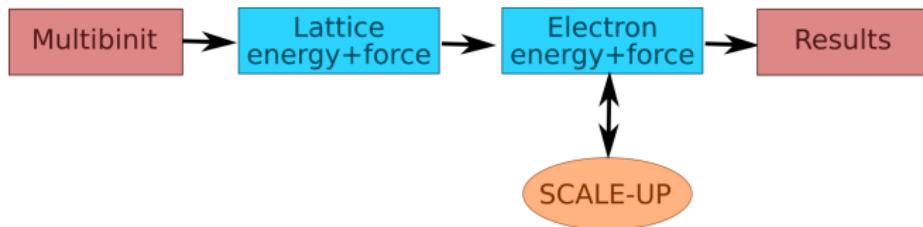
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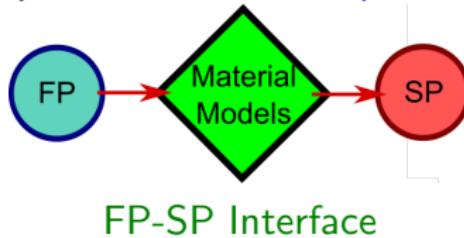
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Distribution SCALE-UP → Attendance to workshop

Rationale: Forming community, second-principles are not blackbox

# Model construction

Second Principles depend on First Principles to create models

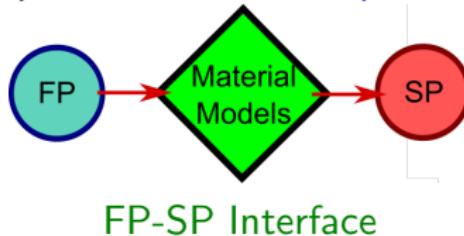


Very delicate process, we need:

- ▶ **Accuracy:** SP model needs to reproduce FP
- ▶ **Lightweight:** SP model needs to be efficient

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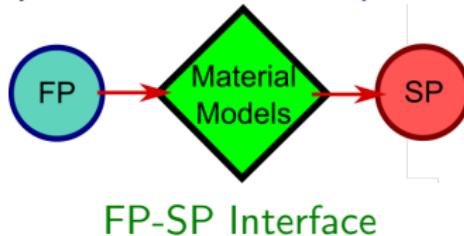


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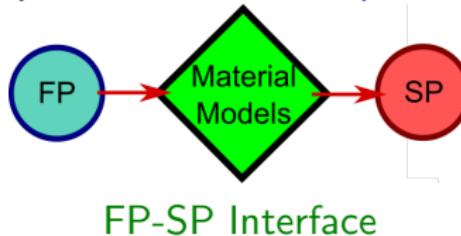


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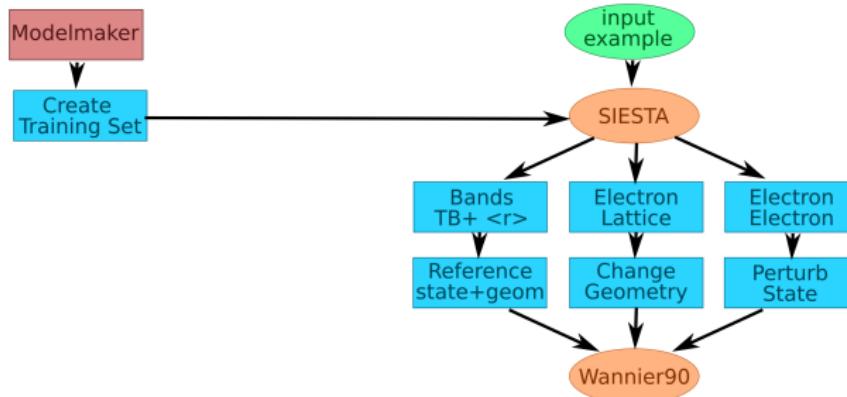
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  - ③ Systematically improvable models
  - ④ Produces hopping, electron-lattice, electron-electron-parameters

Currently we have a SCALE-UP - SIESTA - WANNIER90 Interface

# Modelmaker

The first step to create a model is to create a FP **training set**



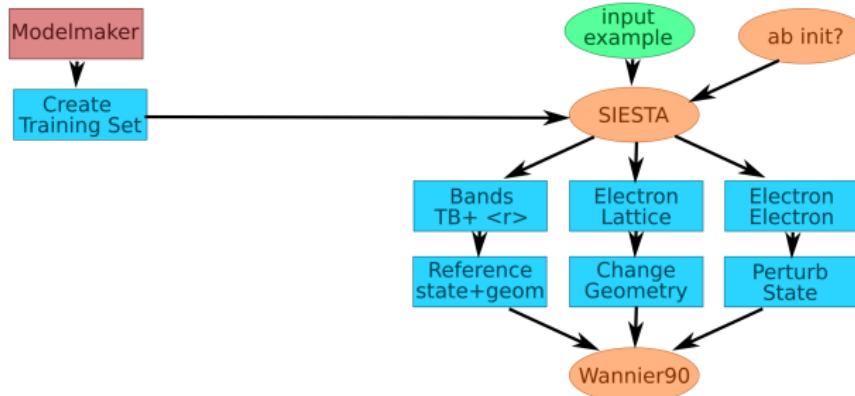
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Model Hamiltonian → capture FP physics

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- ▶ **Electron-lattice terms:** Change geometry on a supercell
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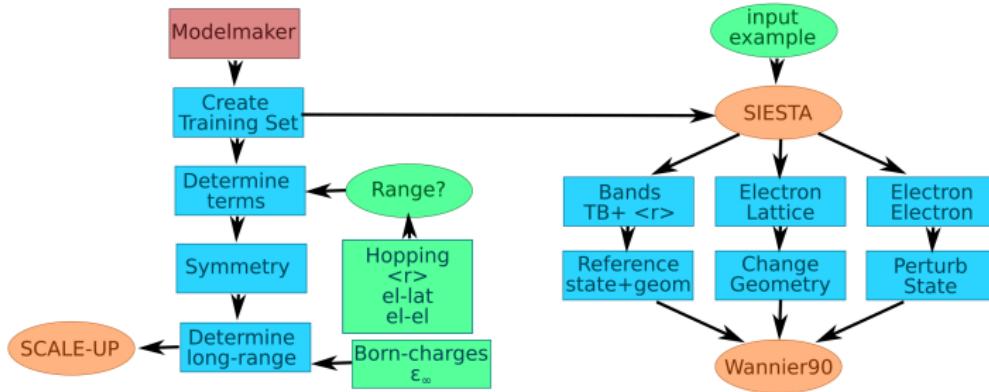
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# The SIESTA-WANNIER90 interface...

Javier Junquera

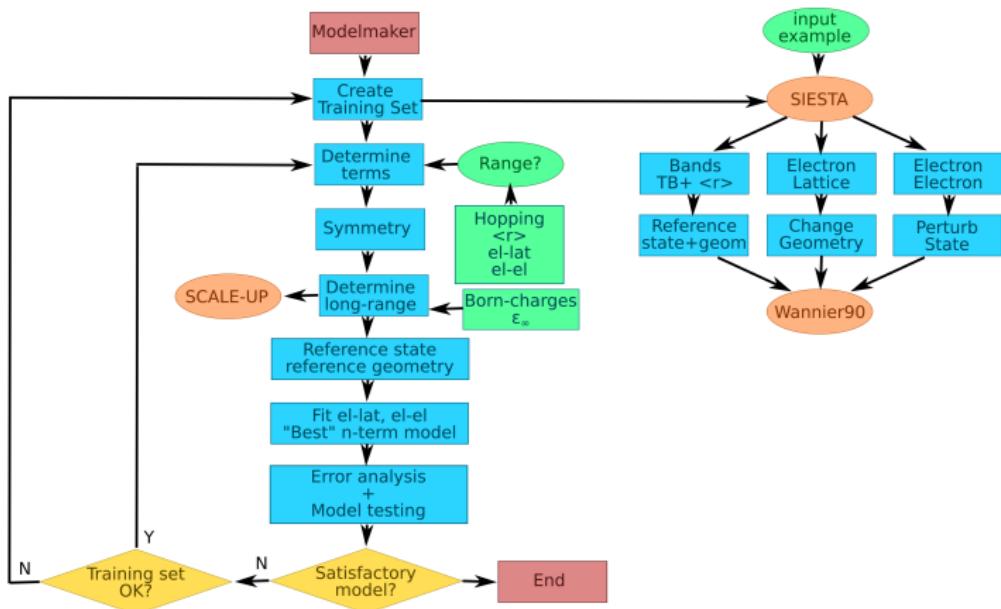
# Modelmaker

Next we need to **determine the terms** that form the model



- ▶ The input is just 4 distances that determine the range of **hopping**, **position**, **electron-lattice** and **electron-electron** matrix elements.
- ▶ The code then filters the created terms using **symmetry**
- ▶ Finally, modelmaker calls **SCALE-UP** to determine the long-range corrections

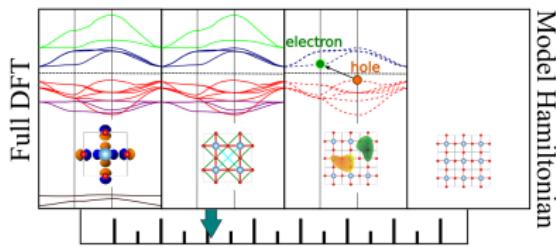
# Modelmaker



- ▶ The code determines iteratively "**best n-terms**" models
- ▶ It provides with tools to check accuracy of the model and systematic error detection
- ▶ The user decides if model is **good enough** and how to improve

# Summary

- ▶ Second-Principles DFT bridges the gap between first-principles and model Hamiltonians
- ▶ The goal is getting closer to "Computational experiments"  
P. García-Fernández et al., *PRB*, 93, 195137 (2016)  
J. Wojdeł et al., *JPCM*, 25, 305401 (2013)



- ▶ Current applications show the accuracy and versatility of method  
S. Das et al., *Nature*, 568, 368 (2019)
- ▶ Their use requires still a lot of user input  
**Strong interactions with FP codes**

**Thank you for your attention!**