Q1.

The equation shows that the value of X will be in the right order if we divide B by A and take the square root of the Answer. But matriculants are just that-matrices. Counting the number of matriculants who are 0

$A = \begin{pmatrix} 1 & 4 \\ 1 & 4 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}$

$x^2 = \frac{A}{B}$ ----- > equation 1

$x^2 = \frac{\begin{pmatrix} 1 & 4 \\ 1 & 4 \end{pmatrix}}{\begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}}$

$x = \sqrt{\frac{\begin{pmatrix} 1 & 4 \\ 1 & 4 \end{pmatrix}}{\begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}}}$- ---- > equation 2

= = $\sqrt{[1 \quad 4 \ ; \ 1 \quad 4]/[1 \quad 2 \ ; \ 1 \quad 2]}$ which is option B as Equation 2 = Equation 4

Option A: $Sqrt([1 \quad 4 \ ; \ 1 \quad 4]/Sqrt[1 \quad 2 \ ; \ 1 \quad 2])$

$= \sqrt{[1 \quad 4 \ ; \ 1 \quad 4]/\sqrt{[1 \quad 2 \ ; \ 1 \quad 2]}}$ ---- > equation 3

Option B: $Sqrt([1 \quad 4 \, ; 1 \quad 4]/[1 \quad 2 \ ; \ 1 \quad 2])$

$= \sqrt{[1 \quad 4 \ ; \ 1 \quad 4]/[1 \quad 2 \ ; \ 1 \quad 2]}$ ---- > equation 4

Option C: $Sqrt([1 \quad 4 \ ; \ 1 \quad 4] \backslash [1 \quad 2 \ ; \ 1 \quad 2])$

$= \sqrt{[1 \quad 4 \ ; \ 1 \quad 4] \backslash [1 \quad 2 \ ; \ 1 \quad 2]}$ ---- > equation 5

Option D: $Sqrt([1 \quad 4 \ , \ 1 \quad 4] \backslash [1 \quad 2 \ , \ 1 \quad 2])$

$= \sqrt{[1 \quad 4 \ , \ 1 \quad 4] \backslash [1 \quad 2 \ , \ 1 \quad 2]}$ ---- > equation 6

Q2.

The input syntax is incorrect. Even though we don't have to write anything inside of them, there should be a pair of single inverted commas in the syntax. Yet we have to use

Syntax: $a(i,j) = input('')$

OR

$a(i,j) = input('Enter\ value\ Column - wise.')$

The matrix won't be displayed after each input if we opt to add a semicolon at the end.

Q3.

The function to be used is plot3(x, y, t). This will enable her to draw the requested three dimensional layouts. With t on the z-axis, the plot will display both the functions "f" and "g".

Q4.

X	Y	1st Order	2nd Order	3rd Order
0	1			
		$=\frac{0-1}{1-0} = -1$		
1	0		$= \frac{\frac{-3}{2}+1}{\frac{2}{3}-0} = \frac{-3}{4}$	
		$=\frac{[1/2]-0}{[2/3]-1} = \frac{-3}{2}$		$\frac{[-0.603+[3/4]]}{[1/3]-0} = 0.481$
$\frac{2}{3}$	$\frac{1}{3}$		$\frac{[-1.098+[2/3]]}{[1/3]-1} = -0.603$	
		$\frac{[0.866-[1/2]]}{[1/3]-[2/3]} = -1.098$		

$\frac{1}{3}$	0.866			

$$f(r) = 1 - 1(x-0) - \frac{3}{4}(x-0)(x-1) + 0.441(x-0)(x-1)\left(x - \frac{2}{3}\right)$$

$Put\ x = 1.5$

⇨ $1 - 1(1.5-0) - \frac{3}{4}(1.5-0)(1.5-1) + 0.441(1.5-0)(1.5-1)\left(1.5 - \frac{2}{3}\right)$

⇨ $1 + [-1.5] + [-0.5625] + [0.275625]$

$= -0.706875$

Q5.

X	1.2	1.5	1.6	2	2.2
Y	0.4275	1.139	0.8736	-0.9751	-0.1536

$Cubic\ Spline\ Formula\ is;$

$f(x) = \frac{(x_i-x)^3}{6h}M_{i-1} + \frac{(x-x_{i-1})^3}{6h}M_i + \frac{(x_i-x)}{h}(y_{i-1} - \frac{h^2}{6}M_{i-1}) + \frac{(x-x_{i-1})}{h}\left(y_i - \frac{h^2}{6}M_i\right)$ ------- (1)

$We\ have\ M_{i-1} + 4M_i + M_{i+1} = \frac{6}{h^2}(y_{i-1} - 2y_i + y_{i+1})$ -------------- (2)

$Here\ h = 0.3, n = 4$

$M_0 = 0, M_4 = 0$

$Substitute\ i = 1\ in\ equation\ (2)$

$$M_0 + 4M_1 + M_2 = \frac{6}{h^2}(y_0 - 2y_1 + y_2)$$

⇨ $0 + 4M_1 + M_2 = \frac{6}{0.09}(0.4275 - 2 \times 1.139 + 0.8736)$

⇨ $4M_1 + M_2 = -65.1267$

$Substitute\ i = 2\ in\ equation\ (2)$

⇨ $M_1 + 4M_2 + M_3 = \frac{6}{h^2}(y_1 - 2y_2 + y_3)$

$\Rightarrow \;\; M_1 + 4M_2 + M_3 = \frac{6}{0.09}(1.139 - 2 \times 0.8736 \pm 0.9751)$

$\Rightarrow \; M_1 + 4M_2 + M_3 = -105.5533$

$Substitute\ i = 3\ in\ equation\ (2)$

$\Rightarrow \;\; M_2 + 4M_3 + M_4 = \frac{6}{h^2}(y_2 - 2y_3 + y_4)$

$\Rightarrow \;\; M_2 + 4M_3 + 0 = \frac{6}{0.09}(0.8736 - 2 \times -0.9751 \pm 0.1536)$

$\Rightarrow \;\; M_2 + 4M_3 = 178.0133$

$Total\ Equations\ are\ (3)$

$4M_1 + M_2 + 0M_3 = -65.1267$ ------------ (1)

$M_1 + 4M_2 + M_3 = -105.5533$ ------------- (2)

$0M_1 + M_2 + 4M_3 = 178.0133$ -------------- (3)

$Select\ the\ Equation\ (1) and\ (2), and\ eliminate\ the\ variable\ M_1.$

$4M_1 + M_2 = -65.1267 \times 1 \Rightarrow \quad 4M_1 + M_2 \quad = -65.1267$

$-$

$M_1 + 4M_2 + M_3 = -105.5533 \times 4 \Rightarrow \quad M_1 + 4M_2 + M_3 = -422.2133$

$-15M_2 - 4M_3 = 357.0867$ ------------ (4)

$Select\ the\ Equation\ (3) and\ (4), and\ eliminate\ the\ variable\ M_2.$

$M_2 + 4M_3 = 178.0133 \times 15 \Rightarrow \quad 15M_2 + 60M_3 = 2670.2$

$+$

$-15M_2 - 4M_3 = 357.0867 \times 1 \Rightarrow \quad -15M_2 - 4M_3 = 357.0867$

$56M_3 = 3027.2867$ -------------- (5)

$Now\ use\ back\ substitution\ method$

$From\ (5)$

$56M_3 = 3027.2867$

⇨ $M_3 = \frac{3027.2867}{56} = 54.0587$

From (3)

⇨ $M_2 + 4M_3 = 178.0133$

⇨ $M_2 + 4(54.0587) = 178.0133$

⇨ M_2=178.0133-216.2348=-38.2214

From (2)

⇨ $M_1 + 4M_2 + M_3 = -105.5533$

⇨ $M_1 + 4(-38.2214) + (54.0587) = -105.5533$

⇨ $M_1 - 98.827 = -105.5533$

⇨ M_1=-105.5533+98.827=-6.7263

Solution using Elimination method.

$M_1 = -6.7263$, $M_2 = -38.2214$, $M_3 = 54.0587$

Substitute $i = 1$ *in equation* (10, *we get Cubic Spline in* 1^{st} *interval* $[x_0, x_1] = [1.2, 1.5]$

$$f_1(x) = \frac{(x_1 - x)^3}{6h} M_0 + \frac{(x - x_0)^3}{6h} M_1 + \frac{(x_1 - x)}{h} (y_0 - \frac{h^2}{6} M_0) + \frac{(x - x_0)}{h} \left(y_1 - \frac{h^2}{6} M_1 \right)$$

$$f_1(x) = \frac{(1.5 - x)^3}{1.8}(0) + \frac{(x - 1.2)^3}{1.8}(-6.7263) + \frac{(1.5 - x)}{0.3}(0.4275 - \frac{0.09}{6}(0) + \frac{(x - 1.2)}{0.3}\left(1.139 - \frac{0.09}{6}(-6.7263)\right)$$

$f_1(x) = -3.7368x^3 + 13.4526x^2 - 13.4351x + 3.6352, for\ 1.2 \le x \le 1.5$

Substitute $i = 2$ *in equation* (1), *we get Cubic Spline in* 2^{nd} *interval* $[x_1, x_2] = [1.5, 1.6]$

$$f_2(x) = \frac{(x_2 - x)^3}{6h} M_1 + \frac{(x - x_1)^3}{6h} M_2 + \frac{(x_2 - x)}{h} (y_1 - \frac{h^2}{6} M_1) + \frac{(x - x_1)}{h} \left(y_2 - \frac{h^2}{6} M_2 \right)$$

$$f_2(x) = \frac{(1.6 - x)^3}{1.8}(-6.7263) + \frac{(x - 1.5)^3}{1.8}(-38.2214) + \frac{(1.6 - x)}{0.3}(1.139 - \frac{0.09}{6}(-6.7263) + \frac{(x - 1.5)}{0.3}\left(0.8736 - \frac{0.09}{6}(-38.2214)\right)$$

$f_2(x) = -17.4973x^3 + 77.6167x^2 - 113.9413x + 55.7372, for\ 1.5 \le x \le 1.6$

$Substitute\ i = 3\ in\ equation\ (1), we\ get\ Cubic\ Spline\ in\ 3^{rd}\ interval\ [x_2, x_3] = [1.6, 2]$

$$f_3(x) = \frac{(x_3 - x)^3}{6h} M_2 + \frac{(x - x_2)^3}{6h} M_3 + \frac{(x_3 - x)}{h} (y_2 - \frac{h^2}{6} M_2) + \frac{(x - x_2)}{h} \left(y_3 - \frac{h^2}{6} M_3 \right)$$

$$f_3(x) = \frac{(2 - x)^3}{1.8}(-38.2214) + \frac{(x - 1.6)^3}{1.8}(54.0587) + \frac{(2 - x)}{0.3}(0.8736 - \frac{0.09}{6}(-38.2214) + \frac{(x - 1.6)}{0.3}\left(-0.9751 - \frac{0.09}{6}(54.0587)\right)$$

$$f_3(x) = 51.2667x^3 - 271.5612x^2 + 474.6834x - 273.7151, for\ 1.6 \le x \le 2$$

$Substitute\ i = 4\ in\ equation\ (1), we\ get\ Cubic\ Spline\ in\ 4^{th}\ interval\ [x_3, x_4] = [2, 2.2]$

$$f_4(x) = \frac{(x_4 - x)^3}{6h} M_3 + \frac{(x - x_3)^3}{6h} M_4 + \frac{(x_4 - x)}{h} (y_3 - \frac{h^2}{6} M_3) + \frac{(x - x_3)}{h} \left(y_4 - \frac{h^2}{6} M_4 \right)$$

$$f_4(x) = \frac{(2.2 - x)^3}{1.8}(54.0587) + \frac{(x - 2)^3}{1.8}(0) + \frac{(2.2 - x)}{0.3}(-0.9751 - \frac{0.09}{6}(54.0587) + \frac{(x - 2)}{0.3}\left(-0.1536 - \frac{0.09}{6}(0)\right)$$

$$f_4(x) = -30.0326x^3 + 198.2152x^2 - 430.6322x + 307.7141, for\ 2 \le x \le 2.2$$

$For\ y(1.8), 1.8 \in [1.6, 2], so\ substitute\ x = 1.8\ in\ f_3(x), we\ get$

$$f_3(1.8) = -0.1557$$

Q6.

X	16	22	24
V	45	63	28

$Using\ Lagrange's\ Interpolation;$

$$v = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} v_0 + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} v_1 + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)} v_2$$

$$= \frac{(x - 22)(x - 24)}{(16 - 22)(16 - 24)} \times 45 + \frac{(x - 16)(x - 24)}{(22 - 16)(12 - 24)} \times 63 + \frac{(x - 16)(x - 22)}{(24 - 16)(24 - 22)} \times 28$$

$$= (x^2 - 46x + 528) \times 0.9375 + (x^2 - 40x + 384) \times (-5.25) + (x^2 - 38x + 352) \times (1.75)$$

$$v = -2.5625x^2 + 100.375x - 905$$

$$\frac{dv}{dt} = -5.125x + 100.375$$

$At\ x = 18$ ⇨ -5.125(18) + 100.375

⇨ $8.125\frac{m}{s^2}$

Q7.

Explanation.

$f"(x) = (f(f(x + 3h) - f(x))/6h$

$f"(x) = ((f(x) + 3hf'(x)) - f(x))/6h$

$f"(x) = (3hf'(x))/6h$

$f"(x) = (f(x + h) - f(x))/2h$

$f"(x) = ((f(x) + hf'(x) - f(x))/2h$

$f"(x) = (hf'(x))/2h$

$f"(x) = f'(x)$

$The\ formula\ for\ f"(x)\ is;$

$f"(x) = (f(x + 3h) - f(x))/6h$

$The\ order\ of\ the\ truncation\ error\ is\ O(h\text{^}2).$

Q8.

$Using\ Taylors\ Expansion:$

$f(x + 2h) = f(x) + f'(x)(2h) + O(h^2) = f(x) + 2f'(x)h + O(h^2)$ --------- (1)

$f(x + h) = f(x) + f'(x)h + O(h^2)$ ---------- (2)

$Now\ consider - 1 \times (1) + 4 \times (2) + 3f(x)$

$We\ get$

$-f(x + 2h) + 4f(x + h) + 3f(x) = -f(x) - 2f'(x)h + 4f(x) + f'(x)h + 3f(x) + O(h^2)$

⇨ $-f(x + 2h) + 4f(x + h) + 3f(x) = 2f'(x)h + O(h^2)$

⇨ $-f(x + 2h) + 4f(x + h) + 3f(x) = 2f'(x)h + O(h^2)$

$\Rightarrow \quad f'(x) = \frac{-f(x+2h)+4f(x+h)+3f(x)}{2h} - O(h)$

$$Or \quad f'(x) = \frac{-f(x+2h)+4f(x+h)+3f(x)}{2h} + O(h) \;\; (Since\ O(h) is\ an\ error\ term\ order)$$

Q9.

Solution:

$Let\ f(x) = \sqrt{x^2+1}\,, a = -1, b = 1, h = 0.2$

$Subintervals\ are;$ $[-1, 1.4142]\,, [-0.8, 1.2806]\,, [-0.6, 1.1662]\,, [-0.4, 1.077]\,, [-0.2, 1.02\,]\,, [0, 1]\,,$

$[0.2, 1.02]\,, [0.4, 1.077]\,, [0.6, 1.1662]\;\,, [0.8, 1.2806]\ and\ [1, 1.4142]$

$$S_{10} = \frac{h}{3}[f(-1 + 4(f(-0.8) + f(-0.4) + f(0) + f(0.4) + f(0.8)) + 2(f(-0.6) + f(-0.2) + f(0.2) + f(0.6)) + f(1)]$$

$$S_{10} = \frac{0.2}{3}[1.4142 + 1.4142 + 4(1.2806 + 1.077 + 1 + 1.077 + 1.2806) + 2(1.1662 + 1.0198 + 1.0198 + 1.1662)]$$

$S_{10} = 2.296$

Q10.

Solution:

$$Recall\ the\ three\ point\ Guassian\ Quadrature\ Formula\ for\ I = \int_a^b f(x)dx$$

$$I = \frac{b-a}{2}\ \{w_1 f(x_1) + w_2 f(x_2) + w_3 f(x_3)\}$$

$$Where \quad x_1 = \frac{b-a}{2}z_1 + \frac{b+a}{2}\ , \quad x_2 = \frac{b-a}{2}z_2 + \frac{b+a}{2}\ , \quad x_3 = \frac{b-a}{2}z_3 + \frac{b+a}{2}$$

$w_1 = \frac{5}{9}\ , \quad w_2 = \frac{8}{9}\ , \quad w_3 = \frac{5}{9}$

and

$$z_1 = -\sqrt{\frac{3}{5}}\ , \quad z_2 = 0\ , \qquad z_3\sqrt{\frac{3}{5}}$$

$Now\ we\ have,$

$$I = \int_{1}^{2.2} \ln(x)\, dx$$

$$\therefore \quad x_1 = 0.6\left(-\sqrt{\frac{3}{5}}\right) + 1.6$$

$$= -0.4647 + 1.6 = 1.1353$$

$$x_2 = \frac{0.6}{(0)} + 1.6 = 1.6$$

$$x_3 = 0.6\left(\sqrt{\frac{5}{3}}\right) + 1.6 = 0.4647 = 1.6 = 2.0647$$

$$Now, \quad \ln(x_1) = \ln(1.1353) = 0.1268$$

$$\ln(x_2) = \ln\ (1.60 = 0.47$$

$$\ln(x_3) = \ln(2.0647) = 0.7249$$

$\therefore \quad We\ have$

$$I = 0.6\left\{\tfrac{5}{9} \times 0.1268 + \tfrac{8}{9} \times 0.47 + \tfrac{5}{9} \times 0.7249\right\}$$

$$= 0.6(0.070 + 0.417 + 0.402)$$

$$= O.6(0.889)$$

$$= 0.533$$

⇨ $I = 0.5334$

And them I have done it using the 2nd method:

$$X = \frac{1}{2}[t(b-a) + a + b]$$
$$X = \frac{1}{2}[t(2.2-1) + 1 + 2.2]$$
$$X = \frac{1}{2}[(t(1.2) + 3.2]$$
$$X = \frac{1}{2}(1.2t + 3.2)$$
$$X = \frac{3}{5}t + \frac{8}{5}$$

$$dx = \frac{1}{2}(b-a)dt$$
$$dx = \frac{1}{2}(2.2-1)dt$$
$$dx = \frac{1}{2}(1.2)$$
$$dx = \frac{3}{5}$$
$$dx = 0.6dt$$

$$X = 0.6t + 1.6$$

$$I = \int_{-1}^{1} ln\, x dx$$

$$I = \int_{-1}^{1} ln\,(0.6t + 1.6)0.6dt$$

$$I = 0.6 \int_{1}^{-1} ln\,(0.6t + 1.6)dt$$

$Formula$:

$$I = \int_{-1}^{1} f(t)dt = c_1 f(t_1) + c_2 f(t_2) + c_3 f(t_3)$$

$c_1 = 0.5555556$; $t_1 = -0.77459667$

$c_2 = 0.8888889$; $t_2 = 0$

$c_3 = 0.5555556$; $t_3 = 0.77459667$

$$I = 0.5555556[0.6(ln(0.6 \times -0.77459667) + 1.6)] + 0.8688889[0.6(ln(0.6 \times 0) + 1.6] + 0.5555556[0.6(ln(0.6 \times (0.77459667)) + 1.6]$$

$$I = 0.5346$$