# CSU33081 Assignment 1

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### Q 1 Answer: D

$>> x = [7:9;\quad 6\quad 12\quad 19; -2:0];$
$y = x(3,:);$
$w = y(1,3);$
$\text{size}(w')$

ans $=$
$1\quad 1$

### Q 2 Answer: C

syms $x$
$c = \text{sym2poly}(6 * x\ \hat{}3 + 12 * x + 12)$
$b = \text{sym2poly}(6 * x\ \hat{}2 + 12 * x)$

$>>$ Q2

c $=$
$6\quad 0\quad 12\quad 12$

b $=$
$6\quad 12\quad 0$

## Q 3 Answer: D

$>> x = [7:9; \quad 6 \quad 12 \quad 19; -2:0];$
$y = x(3,:);$
$w = y(1,3);$
$\text{size}(w')$

$\text{ans} \quad =$
$\quad 1 \quad 1$

## Q 4 Answer: A

function = $\cos x$

$\therefore$ value of $\cos(1) = 0.540302$

Formula

$$F(x) = \overbrace{f(x_0)}^{\text{Initial value}} + (x-x_0)f'(x_0) + \frac{(x-x_0)^2}{2!}f''(x_0) + \frac{(x-x_0)^3}{3!}f'''(x_0) + \ldots + R_n(x)$$

$$\begin{array}{rcrcrcrcr} f(x) &=& \cos(x) &,& & & & & \\ x_0 &=& a=0 & & & & & & \\ f'(x) &=& -\sin(x) &,& f'(0) &=& -\sin(0) &=& 0 \\ f''(x) &=& -\cos(x) &,& f''(0) &=& -\cos(0) &=& -1 \\ f'''(x) &=& \sin(x) &,& f'''(0) &=& \sin(0) &=& 0 \\ f''''(x) &=& \cos(x) &,& f''''(0) &=& \cos(0) &=& 1 \end{array}$$

By Taylor's series equation at $a = 0$

$$\begin{aligned} \cos x &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^2}{8!} - \frac{x^{10}}{10!} + \ldots \text{ etc} \\ x &= 1 \\ &\Rightarrow \\ \cos 1 &= P_n(1) = 1 - \frac{1}{2!} + \frac{1}{4!} - \frac{1}{6!} + \frac{1}{8!} - \frac{1}{10!} + \ldots \text{ etc} \end{aligned}$$

For $\varepsilon = 1.0 \times 10^{-5} = 0.00001$ in which $|\cos(1) - P_n(1)| \leq \varepsilon$.

$$\Rightarrow \quad P_3(1) \quad 1 - \frac{1}{2!} + \frac{1}{4!} = 0.54166667$$

$$\Rightarrow \quad P_4(1) \quad 1 - \frac{1}{2!} + \frac{1}{4!} - \frac{1}{6!} = 0.540277778$$

$$\Rightarrow \quad P_5(1) \quad 1 - \frac{1}{2!} + \frac{1}{4!} - \frac{1}{6!} + \frac{1}{8!} = 0.54030257941$$

$$\begin{aligned} |\cos 1 - P_5(1)| &= |0.5403023059 - 0.5403025794| \\ &= 0.0000002735 < \varepsilon. \end{aligned}$$

Hence min degree is $n = 5$.

$$\Rightarrow \boxed{A}$$

**Q 5 Answer: C**

The minimum number of iteration of the bisection method that are needed to audience accuracy at $\varepsilon = 0.001$
I found the formula i am using on stack exchange

$$\begin{aligned}
n \geq &\quad \frac{\log(b-a) - \log(\varepsilon)}{\log 2} \\
\Rightarrow &\quad a = 0.5, \quad b = 2, \quad \varepsilon = 0.001 \\
&\quad \frac{\log(2-0.5) - \log(0.001)}{\log 2} = 10.55 \\
n \geq &\quad 10.55 \\
\Rightarrow &\quad 11 \text{ iterations} \\
\Rightarrow &\quad \boxed{C}
\end{aligned}$$

**Q 6 Answer: E**

$$A = \begin{pmatrix} -1 & 1 & -4 \\ 2 & 2 & 1 \\ 3 & 3 & 2 \end{pmatrix}$$

$$\left.\begin{array}{rrr|rrr} -1 & 1 & -4 & 1 & 0 & 0 \\ 2 & 2 & 1 & 0 & 1 & 0 \\ 3 & 3 & 2 & 0 & 0 & 1 \end{array}\right.$$

$R1 \to \dfrac{R1}{-1}$

$$\left.\begin{array}{rrr|rrr} 1 & -1 & 4 & -1 & 0 & 0 \\ 2 & 2 & 1 & 0 & 1 & 0 \\ 3 & 3 & 2 & 0 & 0 & 1 \end{array}\right.$$

$R2 \to R2 - 2R1 \qquad 2R1 = 2 - 2 \quad 8$

$$\left.\begin{array}{rrr|rrr} 1 & -1 & 4 & -1 & 0 & 0 \\ 0 & 4 & -7 & 2 & 1 & 0 \\ 3 & 3 & 2 & 0 & 0 & 1 \end{array}\right.$$

$R3 \to R3 - 3R1 \qquad 3R1 \to 1 - 3 \quad 12$

$$\left.\begin{array}{rrr|rrr} 1 & -1 & 4 & -1 & 0 & 0 \\ 0 & 4 & 7 & 2 & 1 & 0 \\ 0 & 6 & -10 & 3 & 0 & 1 \end{array}\right.$$

$R2 \to \dfrac{R2}{4}$

$$\left.\begin{array}{rrr|rrr} 1 & -1 & 4 & -1 & 0 & 0 \\ 0 & 1 & -\frac{7}{4} & \frac{1}{2} & \frac{1}{4} & 0 \\ 0 & 6 & -10 & 3 & 0 & 1 \end{array}\right.$$

$R3 \to R3 - 6R2 \qquad 6R2 = 0\,6\ \cdot \dfrac{21}{2}$

$$\left.\begin{array}{rrr|rrr} 1 & -1 & 4 & -1 & 0 & 0 \\ 0 & 1 & -\frac{1}{4} & \frac{1}{2} & \frac{1}{4} & 0 \\ 0 & 0 & 0.5 & 0 & -\frac{3}{2} & 1 \end{array}\right.$$

$R3 \to \dfrac{R3}{1/2}$

$$\left.\begin{array}{rrr|rrr} 1 & -1 & 4 & -1 & 0 & 0 \\ 0 & 1 & -\frac{7}{4} & \frac{1}{2} & \frac{1}{4} & 0 \\ 0 & 0 & 1 & 0 & -3 & 2 \end{array}\right.$$

$R2 \to R2 + \frac{7}{4}R3$

$$\begin{array}{rrrrrr} 1 & -1 & 4 & -1 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} & -5 & \frac{7}{2} \\ 0 & 0 & 1 & 0 & -3 & 2 \end{array}$$

$R1 \to R1 + R2$

$$\begin{array}{rrrrrr} 1 & 0 & 4 & -\frac{1}{2} & 5 & \frac{7}{2} \\ 0 & 1 & 0 & \frac{1}{2} & -5 & \frac{7}{2} \\ 0 & 0 & 1 & 0 & -3 & 2 \end{array}$$

$R1 \to R1 - 4R3$

$$\left.\begin{array}{rrr|rrr} 1 & 0 & 0 & -\frac{1}{2} & 7 & -\frac{9}{2} \\ 0 & 1 & 0 & \frac{1}{2} & -5 & \frac{7}{2} \\ 0 & 0 & 1 & 0 & -3 & 2 \end{array}\right.$$

$$\Rightarrow \begin{bmatrix} -\frac{1}{2} & 7 & \frac{-9}{2} \\ \frac{1}{2} & -5 & \frac{7}{2} \\ 0 & -3 & 2 \end{bmatrix}$$

$$\Rightarrow \boxed{E}$$

**Q 7 Answer: D**

$$\begin{pmatrix} 0 & 4 & 1 \\ 1 & 1 & 3 \\ 2 & -2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 9 \\ 8 \\ -1 \end{pmatrix}$$

$$\left[\begin{array}{ccc|c} 0 & 4 & 1 & 9 \\ 1 & 1 & 3 & 6 \\ 2 & -2 & 1 & -1 \end{array}\right]$$

$R2 \longleftrightarrow R1$

$$\left(\begin{array}{ccc|c} 1 & 1 & 3 & 6 \\ 0 & 4 & 1 & 9 \\ 2 & -2 & 1 & -1 \end{array}\right)$$

$R3 \to R3 - 2R1$

$$\begin{array}{cccc} 1 & 1 & 3 & 6 \\ 0 & 4 & 1 & 9 \\ 0 & -4 & -5 & -13 \end{array}$$

$R1 \to 4R1 - R2$

$$\left[\begin{array}{ccc|c} 4 & 0 & 11 & 15 \\ 0 & 4 & 1 & 9 \\ 0 & -4 & -5 & -13 \end{array}\right]$$

$R3 \to R3 + R2$

$$\left[\begin{array}{ccc|c} 4 & 0 & 11 & 15 \\ 0 & 4 & 1 & 9 \\ 0 & 0 & -4 & -4 \end{array}\right]$$

$R2 \to 4R2 + R3$

$$\left[\begin{array}{ccc|c} 4 & 0 & 11 & 15 \\ 0 & 16 & 0 & 32 \\ 0 & 0 & -4 & -4 \end{array}\right]$$

$R1 \to 4R1 + 11R3$

$$\left(\begin{array}{ccc|c} 16 & 0 & 0 & 16 \\ 0 & 16 & 0 & 32 \\ 0 & 0 & -4 & -4 \end{array}\right)$$

$R2 \to \dfrac{R2}{16}$

$$\left[\begin{array}{ccc|c} 16 & 0 & 0 & 16 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -4 & -4 \end{array}\right)$$

$R1 \to \dfrac{R1}{16}$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -4 & -4 \end{array}\right)$$

$R3 \rightarrow \dfrac{R3}{-4}$

$$\left.\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right.$$

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$$\therefore (x_1, x_2, x_3) = (1, 2, 1)^T$$

$$\Rightarrow \boxed{D}$$

**Q 8 Answer: E**

$$\begin{pmatrix} 1 & 8 & 6 \\ 7 & 3 & 1 \\ 6 & 7 & 12 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 6 \\ -8 \\ 2 \end{pmatrix}$$

$$\begin{array}{ccccccc} x_1 & + & 8x_2 & + & 6x_3 & = & 6 \\ 7x_1 & + & 3x_2 & + & x_3 & = & -8 \\ 6x_1 & + & 7x_2 & + & 12x_3 & = & 2 \end{array}$$

$$\begin{array}{ccccccc|c} 7x_1 & + & 3x_2 & + & x_3 & = & -8 & 1 \\ x_1 & + & 8x_2 & + & 6x_3 & = & 6 & 2 \\ 6x_1 & + & 7x_2 & + & 12x_3 & = & 2 & 3 \end{array}$$

From 1

$$\begin{array}{rcccccccc} x_1 = & \frac{1}{7} & [ & -8 & - & 3x_2 & - & x_3 & ] & 4 \\ x_2 = & \frac{1}{8} & [ & 6 & - & x_1 & - & 6x_3 & ] & 5 \\ x_3 = & \frac{1}{12} & [ & 2 & - & 6x_1 & - & 7x_2 & ] & 6 \end{array}$$

*initial approximation*:

$$x_1^{(0)} = 2; x_2^{(0)} = 4, x_3^{(0)} = 5$$

$$
\begin{aligned}
x_1^{(1)} &= \frac{1}{7}\left[-8-3(4)-5\right] \\
&= \frac{1}{7}\left[-8-12-5\right] \\
&= \frac{25}{7} \\
\Aboxed{x_1^{(1)} &= -3.5714} \\
x_2^{(1)} &= \frac{1}{8}\left[6-x_1^{(1)}-6x_3^{(0)}\right] \\
&= \frac{1}{8}\left[6+3.5714-6(5)\right] \\
\Aboxed{x_2^{(1)} &= -2.5535} \\
x_3^{(1)} &= \frac{1}{12}\left[2-6x_1^{(1)}-7x_2^{(1)}\right] \\
&= \frac{1}{12}\left[2-6(-3.5714)-7(-2.5535)\right] \\
\Aboxed{x_3^{(1)} &= 3.4419}
\end{aligned}
$$

<u>*2nd iteration*</u>

$$
\begin{aligned}
x_1^{(2)} &= \frac{1}{7}\left[-8-3x_2^{(1)}-x_3^{(1)}\right] \\
&= \frac{1}{7}\left[-8-3(-2.5535)-3.4419\right] \\
&= \frac{1}{7}\left[-3.7814\right] \\
\Aboxed{x_1^{(2)} &= -0.5402} \\
x_2^{(2)} &= \frac{1}{8}\left[6-x_1^{(2)}-6x_3^{(1)}\right] \\
&= \frac{1}{8}\left[6+0.5402-6(3.4419)\right] \\
\Aboxed{x_2^{(2)} &= -1.7639} \\
x_3^{(2)} &= \frac{1}{12}\left[2-6x_1^{(2)}-7x_2^{(2)}\right] \\
&= \frac{1}{12}\left[2-6(-0.5402)-7(1.7639)\right] \\
&= \frac{1}{12}\left[17.5885\right] \\
\Aboxed{x_3^{(2)} &= 1.4657}
\end{aligned}
$$

*3^rd iteration*

$$x_1^{(3)} = \frac{1}{7}\left[-8 - 3x_2^{(2)} - x_3^{(2)}\right]$$
$$= \frac{1}{7}[-8 - 3(-1.7639) - 1.4657]$$
$$\boxed{x_1^{(3)} = -0.5962}$$
$$x_2^{(3)} = \frac{1}{8}\left[6 - x_1^{(3)} - 6x_3^{(2)}\right]$$
$$= \frac{1}{8}[6 - (-0.5962) - 6(1.4657)]$$
$$\boxed{x_2^{(3)} = -0.2747}$$
$$x_3^{(3)} = \frac{1}{12}\left[2 - 6x_1^{(3)} - 7x_2^{(3)}\right]$$
$$= \frac{1}{12}[2 - 6(-0.5962) - 7(-0.2747)]$$
$$\boxed{x_3^{(3)} = 0.6021}$$

Thus, $x_1 = 0.5962;\ x_2 = -0.2747;\ x_3 = 0.6021$

$$\Rightarrow \boxed{E}$$

**Q 9 Answer: B**

$$\begin{aligned} A = &\begin{matrix} 2 & 3 & -1 \\ 4 & 4 & -3 \\ -2 & 3 & -1 \end{matrix} \\ A = &LU \end{aligned}$$

$$\begin{vmatrix} 2 & 3 & -1 \\ 4 & 4 & -3 \\ -2 & 3 & -1 \end{vmatrix} = \begin{vmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{vmatrix} \begin{vmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{vmatrix}$$

$$\begin{vmatrix} 2 & 3 & -1 \\ 4 & 4 & 3 \\ -2 & 3 & -1 \end{vmatrix} = \begin{vmatrix} l_{11} & l_{11}u_{12} & l_{11}u_{13} \\ l_{21} & l_{21}u_{12} + l_{22} & l_{21}u_{13} + l_{22}u_{23} \\ l_{31} & l_{31}u_{12} + l_{32} & l_{31}u_{13} + l_{32}u_{23} + l_{33} \end{vmatrix}$$

which implies

$$\begin{aligned} l_{11} &= 2 \\ l_{11}u_{12} &= 3 \to u_{12} = 1.5 \\ l_{11}u_{13} &= -1 \to u_{13} = -0.5 \\ l_{21} &= 4 \\ l_{21}u_{12} + l_{22} &= 4 \Rightarrow l_{22} = -2 \\ l_{21}u_{13} + l_{22}u_{23} &= -3 \Rightarrow u_{23} = 0.5 \\ l_{31} &= -2 \\ l_{31}u_{12} + l_{32} &= 3 \Rightarrow l_{32} = 6 \\ l_{31}u_{13} + l_{32}u_{23} + +l_{33} &= -1 \Rightarrow l_{33} = -5 \end{aligned}$$

$$\begin{aligned} \begin{vmatrix} 2 & 3 & -1 \\ 4 & 4 & -3 \\ -2 & 3 & -1 \end{vmatrix} &= \begin{vmatrix} 2 & 0 & 0 \\ 4 & 2 & 0 \\ -2 & 6 & -5 \end{vmatrix} \times \begin{vmatrix} 1 & 1.5 & -0.5 \\ 0 & 1 & 0.5 \\ 0 & 0 & 1 \end{vmatrix} \\ L &= \begin{bmatrix} 2 & 0 & 0 \\ 4 & -2 & 0 \\ -2 & 6 & -5 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 0 & 0 & & 1 & 0 & 0 \\ 4 & -2 & 0 & & 0 & 1 & 0 \\ -2 & 6 & -5 & & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

$$\frac{R1}{2} \to \begin{bmatrix} 1 & 0 & 0 & & \frac{1}{2} & 0 & 0 \\ 4 & -2 & 0 & & 0 & 1 & 0 \\ -2 & 6 & -1 & & 0 & 0 & 1 \end{bmatrix}$$

$R2 \to R2 - 4R1$

$$\begin{array}{rrrrrrr} -4 & -2 & 0 & & 0 & 1 & 0 \\ -4 & 0 & 0 & & 2 & 0 & 0 \\ \hline 0 & -2 & 0 & & -2 & 1 & 0 \end{array}$$

$R3 \rightarrow R3 - 2R1$

$$\begin{array}{rrr rrr} -2 & 6 & -5 & 0 & 0 & 1 \\ -2 & 0 & 0 & -1 & 0 & 0 \\ \hline 0 & 6 & -5 & 1 & 0 & 1 \end{array}$$

$$\Rightarrow \begin{array}{rrr rrr} 1 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 6 & -5 & 1 & 0 & 1 \end{array}$$

$R3 - 6R3$

$$\begin{array}{rrr rrr} 0 & 6 & -5 & 1 & 0 & 1 \\ 0 & 6 & 0 & 6 & -3 & 0 \\ \hline 0 & 0 & -5 & -5 & 3 & 1 \end{array}$$

$$\Rightarrow \begin{array}{rrr rrr} 1 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & -5 & -5 & 3 & 1 \end{array}$$

$R3 \rightarrow \dfrac{R3}{-5}$

$$\Rightarrow \begin{array}{rrr rrr} 1 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & 1 & -\frac{3}{5} & -\frac{1}{5} \end{array}$$

$$\Rightarrow \quad L^{-1} \Rightarrow \begin{vmatrix} \frac{1}{2} & 0 & 0 \\ 1 & -\frac{1}{2} & 0 \\ 1 & -\frac{3}{5} & -\frac{1}{5} \end{vmatrix}$$

$$u = \begin{bmatrix} 1 & 1.5 & 0.5 \\ 0 & 1 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$$

$$u^{-1} = \begin{bmatrix} 1 & 1.5 & 0.5 & 1 & 0 & 0 \\ 0 & 1 & 0.5 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$R1 \rightarrow R1 - \dfrac{3}{2}R_2$

$$\begin{array}{rrr rrr} 1 & \frac{3}{2} & \frac{1}{2} & 1 & 0 & 0 \\ 0 & \frac{3}{2} & \frac{3}{4} & 0 & \frac{3}{2} & 0 \\ \hline 1 & 0 & -\frac{1}{4} & 0 & -\frac{3}{2} & 0 \end{array}$$

$R1 \rightarrow R1 + \dfrac{1}{4}R_3$

$$\begin{array}{rrr rrr} 1 & 0 & -\frac{1}{4} & 1 & -\frac{3}{2} & 0 \\ 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \hline 1 & 0 & 0 & 1 & -\frac{3}{2} & \frac{1}{4} \end{array}$$

$R2 \rightarrow R2 - \dfrac{1}{2}R_3$

$$
\begin{array}{ccccccc}
0 & 1 & \frac{1}{2} & & 0 & 1 & 0 \\
0 & 0 & \frac{1}{2} & & 0 & 0 & \frac{1}{2} \\
\hline
0 & 1 & 0 & & 0 & 1 & -\frac{1}{2}
\end{array}
$$

$$
\begin{aligned}
\therefore U^{-1} &= \begin{bmatrix} 1 & -1.5 & 1.25 \\ 0 & 1 & -0.5 \\ 0 & 0 & 1 \end{bmatrix} \\
A^1 &= \begin{vmatrix} 1 & -1.5 & 1.25 \\ 0 & 1 & -0.5 \\ 0 & 0 & 1 \end{vmatrix} \times \begin{vmatrix} \frac{1}{2} & 0 & 0 \\ 1 & -\frac{1}{2} & 0 \\ 1 & -\frac{3}{5} & -\frac{1}{5} \end{vmatrix}
\end{aligned}
$$

$$
\begin{aligned}
a_{11} &= 1 \times \frac{1}{2} + \left(-\frac{3}{2}\right) \times 1 + \frac{5}{4} \times 1 = \frac{1}{4} \\
a_{12} &= 1 \times 0 + \left(-\frac{3}{2}\right) \times \left(-\frac{1}{2}\right) + \frac{5}{4} \times \left(-\frac{3}{5}\right) = 0 \\
a_{13} &= 1 \times 0 + \left(-\frac{3}{2}\right) \times 0 + \frac{5}{4} \times \left(-\frac{1}{5}\right) = -\frac{1}{4} \\
a_{21} &= 0 \times \frac{1}{2} + 1 \times 1 + \left(-\frac{1}{2}\right) \times 1 = \frac{1}{2} \\
a_{22} &= 0 \times 0 + 1 \times \left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right) \times \left(-\frac{3}{5}\right) = -\frac{1}{5} \\
a_{23} &= 0 \times 0 + 1 + \left(-\frac{1}{2}\right) \times \left(-\frac{1}{5}\right) = \frac{1}{10} \\
a_{31} &= 0 \times \frac{1}{2} + 0 \times 1 + 1 \times 1 = 1 \\
a_{32} &= 0 \times 0 + 0 \times \left(-\frac{1}{2}\right) + 1 \times \left(-\frac{3}{4}\right) = \frac{-3}{5} \\
a_{33} &= 0 \times 0 + 0 \times 0 + 1 \times \left(-\frac{1}{5}\right) = -\frac{1}{5}
\end{aligned}
$$

$$
\Rightarrow \quad \begin{array}{ccc} \frac{1}{4} & 0 & -\frac{1}{4} \\ \frac{1}{2} & -\frac{1}{5} & \frac{1}{10} \\ 1 & -\frac{3}{5} & -\frac{1}{5} \end{array}
$$

$$
\Rightarrow \quad \boxed{B}
$$

## Q 10 Answer: D

Given system of non-linear equations are

$$v - u^2 = 0$$
$$u^2 + v^2 - 1 = 0$$

Let, $f = v - u^2 = 0$ and $g = u^2 + v^2 - 1 = 0$

By Jacobian

$$\begin{aligned} J_k &= \left[\begin{array}{cc} \frac{\partial f}{\partial u} & \frac{\partial f}{\partial v} \\ \frac{\partial g}{\partial u} & \frac{\partial g}{\partial v} \end{array}\right]\Bigg|_{u_k, v_k} = \left[\begin{array}{cc} -3u^2 & 1 \\ 2u & 2v \end{array}\right] \\ J_k^{-1} &= \frac{1}{det[J_k]}[Adj \text{ of } J_k] \end{aligned}$$

By Newton-Raphson method

$$\Rightarrow \quad \begin{bmatrix} u_{n+1} \\ v_{n+1} \end{bmatrix} = \begin{bmatrix} u_n \\ v_n \end{bmatrix} - \begin{bmatrix} -3u^2 & 1 \\ 2u & 2v \end{bmatrix}^{-1} \times f\left(\begin{bmatrix} u_n \\ v_n \end{bmatrix}\right)$$

*Initial estimation:*

We have $n = 0$, $u = 1$, $v = 2$.

$$\begin{aligned} \Rightarrow \begin{bmatrix} u_1 \\ v_1 \end{bmatrix} &= \begin{bmatrix} 1 \\ 2 \end{bmatrix} - \begin{bmatrix} -3 & 1 \\ 2 & 4 \end{bmatrix}^{-1} \times f\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) \\ &= \begin{bmatrix} 1 \\ 2 \end{bmatrix} - \begin{bmatrix} -3 & 1 \\ 2 & 4 \end{bmatrix}^{-1} \times \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \end{aligned}$$

$\Rightarrow u_1 = 1$ and $v_1 = 1$

*1<sup></sup>*

*$1^{st}$ repetition:*

$n = 1$, $u_1 = 1$, $v_1 = 1$

$$\begin{aligned} \Rightarrow \begin{bmatrix} u_2 \\ v_2 \end{bmatrix} &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} -3 & 1 \\ 2 & 2 \end{bmatrix}^{-1} \times f\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) \\ &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} -3 & 1 \\ 2 & 2 \end{bmatrix}^{-1} \times \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.875 \\ 0.625 \end{bmatrix} \end{aligned}$$

$\Rightarrow u_2 = 0.875$ and $v_2 = 0.625$

*$2^{nd}$ repetition:*

$n = 2$, $u_2 = 0.875$, $v_2 = 0.625$

$$\begin{aligned} \Rightarrow \begin{bmatrix} u_3 \\ v_3 \end{bmatrix} &= \begin{bmatrix} 0.875 \\ 0.625 \end{bmatrix} - \begin{bmatrix} -2.2969 & 1 \\ 1.75 & 1.25 \end{bmatrix}^{-1} \times f\left(\begin{bmatrix} 0.875 \\ 0.625 \end{bmatrix}\right) \\ &= \begin{bmatrix} 0.875 \\ 0.625 \end{bmatrix} - \begin{bmatrix} -2.2969 & 1 \\ 1.75 & 1.25 \end{bmatrix}^{-1} \times \begin{bmatrix} -0.0449 \\ 0.1562 \end{bmatrix} = \begin{bmatrix} 0.829 \\ 0.5643 \end{bmatrix} \end{aligned}$$

*$3^{rd}$ repetition:*

$n = 3$, $u_3 = 0.829$, $v_3 = 0.5643$

$$\begin{aligned}\Rightarrow \begin{bmatrix} u_4 \\ v_4 \end{bmatrix} &= \begin{bmatrix} 0.829 \\ 0.5463 \end{bmatrix} - \begin{bmatrix} -2.0619 & 1 \\ 1.6581 & 1.1287 \end{bmatrix}^{-1} \times f\left(\begin{bmatrix} 0.829 \\ 0.5643 \end{bmatrix}\right) \\ &= \begin{bmatrix} 0.829 \\ 0.5643 \end{bmatrix} - \begin{bmatrix} -2.0619 & 1 \\ 1.6581 & 1.1287 \end{bmatrix}^{-1} \times \begin{bmatrix} -0.0054 \\ 0.0058 \end{bmatrix} = \begin{bmatrix} 0.826 \\ 0.5636 \end{bmatrix}\end{aligned}$$

*$4^{th}$ repetition:*

$n = 4$, $u_4 = 0.826$, $v_4 = 0.5636$

$$\begin{aligned}\Rightarrow \begin{bmatrix} u_5 \\ v_5 \end{bmatrix} &= \begin{bmatrix} 0.826 \\ 0.5636 \end{bmatrix} - \begin{bmatrix} -2.047 & 1 \\ 1.6521 & 1.1272 \end{bmatrix}^{-1} \times f\left(\begin{bmatrix} 0.826 \\ 0.5636 \end{bmatrix}\right) \\ &= \begin{bmatrix} 0.826 \\ 0.5636 \end{bmatrix} - \begin{bmatrix} -2.047 & 1 \\ 1.6521 & 1.1272 \end{bmatrix}^{-1} \times \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.826 \\ 0.5636 \end{bmatrix}\end{aligned}$$

$\Rightarrow u = 0.826$ and $v = 0.5636$

$$\Rightarrow \boxed{D}$$