CSE221: Data Structures
Sample Final Exam

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This sample midterm is for you to see what type of questions will be asked during the final exam.

Question 1:
(Textbook R-6.4) Draw a representation of an initially empty vector A after performing the following sequence of operations: insert(0, 4), insert(0, 3), insert(0, 2), insert(2, 1), insert(1, 5), insert(1, 6), insert(3, 7), insert(0, 8).

Question 2:
(Textbook C-6.5.) Describe a function for performing a card shuffle of an array of $2n$ elements, by converting it into two lists. A card shuffle is a permutation where a list L is cut into two lists, L_1 and L_2 , where L_1 is the first half of L and L_2 is the second half of L, and then these two lists are merged into one by taking the first element in L_1, then the first element in L_2, followed by the second element in L_1, the second element in L_2, and so on.

Question 3:
Consider the tree T shown in Figure 1. Give the order in which the nodes appear in a (i) preorder, (ii) inorder and (iii) postorder traversal of T.

Question 4:
Suppose that a full binary tree has p internal nodes and q leaves. Prove that $q = p + 1$.

Question 5:
(Textbook C7-6) Give an $O(n)$-time algorithm for computing the depth of all the nodes of a tree T , where n is the number of nodes of T.

Question 6:

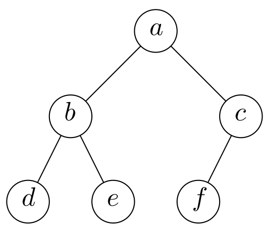

Figure 1:

(Textbook C-7.24) Let T be a tree with n nodes. Define the lowest common ancestor (LCA) between two nodes v and w as the lowest node in T that has both v and w as descendents (where we allow a node to be a descendent of itself). Given two nodes v and w, describe an efficient algorithm for finding the LCA of v and w. What is the running time of your method?

Question 7:
What is the most appropriate choice for implementing a priority queue:

(a) A singly-linked list

(b) A hash table with separate chaining

(c) A heap

(d) A hash table with open addressing

Question 8:
(Textbook R-8.5) Suppose you label each node v of a binary tree T with a key equal to the preorder rank of v. Under what circumstances is T a heap?

Question 9:
(Textbook C-8.4) Show how to implement the stack ADT using only a priority queue and one additional member variable.

Question 10:
(Textbook R-9.7) Draw the 11-entry hash table that results from using the hash function, $h(i) = (3i + 5) \bmod 11$, to hash the keys 12, 44, 13, 88, 23, 94, 11, 39, 20, 16, and 5, assuming collisions are handled by chaining.

Question 11:
(Textbook C-9.11) Suppose that each row of an $n \times n$ array A consists of 1's and 0's such that, in any row of A, all the 1's come before any 0's in that row. Assuming A is already in memory, describe a method running in $O(n \log n)$ time (not $O(n^2)$ time!) for counting the number of 1's in A.

Question 12:
Draw all the binary search trees that store the keys $\{1, 2, 3\}$.

Question 13:
(Textbook R-13.8) Would you use the adjacency list structure or the adjacency matrix structure in each of the following cases? Justify your choice.

a. The graph has 10,000 vertices and 20,000 edges, and it is important to use as little space as possible.

b. The graph has 10,000 vertices and 20,000,000 edges, and it is important to use as little space as possible.

c. You need to answer the query isAdjacentTo as fast as possible, no matter how much space you use.

Question 14:
(Textbook C-13.18) Design an efficient algorithm for finding a longest directed path from a vertex s to a vertex t of an acyclic weighted digraph G. Specify the graph representation used and any auxiliary data structures used. Also, analyze the time complexity of your algorithm.