



United International University
Department of Computer Science and Engineering

Course: CSI 227 Algorithms

Trimester: Spring 2019

Final Exam

Marks: 100

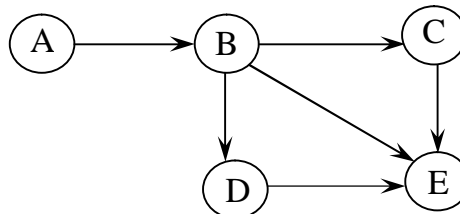
Time: 2 hours

There are *FOUR* questions. Answer *ALL* questions.

1. a) For a graph $G = (V, E)$, write down algorithms that **7 × 2**
- i) finds in-degrees and out-degrees of all vertices of G , when G is *directed* and represented by the *Adjacency Lists*;
 - ii) finds the maximum degree Δ and the minimum degree δ of G , when G is *undirected* and represented by the *Adjacency Matrix*.
- b) For a disconnected graph $G = (V, E)$, write an algorithm that finds the number of **7 + 4**
components of G , when G is *undirected* and represented by the *Adjacency Lists*. Analyze the running time of your algorithm.
2. a) Dijkstra's algorithm for the shortest path problem is shown below. What would be the time-complexity of the algorithm (show line by line analysis) for an input graph G if *adjacency list* is used for graph representation and *an array* is used for storing $d[]$ values. **10**

```
Dijkstra(G)
1   Q = V[G];
2   for each u ∈ Q
3       d[v] = ∞;
4   d[s] = 0; S = ∅;
5   while (Q ≠ ∅)
6       u = ExtractMin(Q);
7       S = S ∪ {u};
8       for each v ∈ u->Adj[]
9           if (v ∈ Q and d[v] > d[u] + w(u, v))
10              d[v] = d[u] + w(u, v);
```

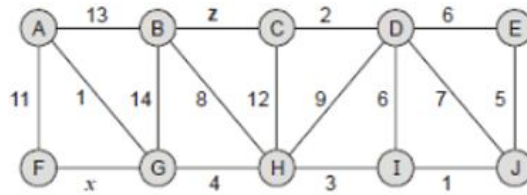
- b) Consider the graph shown below. Assuming A as the source vertex, assign weights to each of the edges such that the Dijkstra's algorithm always fails in this graph. Draw the wrong shortest-path tree of the Dijkstra's algorithm, and also draw the correct shortest-path tree. **8**



- c) Write an algorithm that can always find the shortest paths (if exist) in a graph with positive and negative edge weights. Analyze the running time of your algorithm. **7**

3. a) Write the Kruskal's algorithm that finds a minimum spanning tree of a graph. Analyze the running time of the algorithm. 10

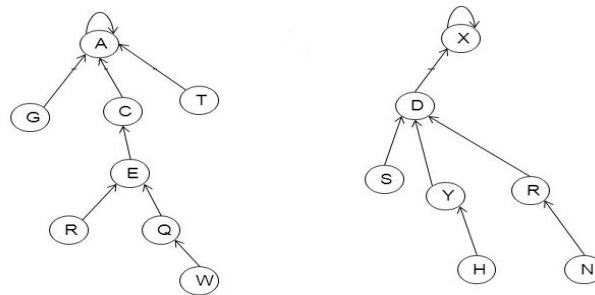
- b) Draw a minimum spanning tree of the graph G shown below such that the minimum spanning tree always consists the edges with weights x and z . 6



- c) Briefly explain how probe sequences are generated by Linear Probing, Quadratic Probing, and Double Hashing techniques of open addressing. 9

4. a) Write algorithms for UNION() and MAKE-SET() operations of *Disjoint-Set Forest* data structure assuming *union-by-rank* and the *path-compression* heuristics. 5

- b) What would the resultant forest be after calling UNION(W , Y) on the disjoint-sets forest of the following figure? You must use the *union-by-rank* and the *path-compression* heuristics. 5



- c) Consider an open-addressing hash table as shown below. The table already contains four data items. Assume that collisions are handled by the hash function 5

$$h(k, i) = (h'(k) + i h_2(k)) \bmod 13, \text{ where } h'(k) = (2k + 7) \bmod 13 \text{ and } h_2(k) = (k + 5) \bmod 13.$$

By showing calculations, redraw the table after (i) insert 90; (ii) insert 83.

0	1	2	3	4	5	6	7	8	9	10	11	12
	70			44	12				51			

- d) Using modulo $q = 13$, find out the *valid matches* and *spurious hits* that the Rabin-Karp algorithm encounters in the text $T = 17426564255$ when looking for the pattern $P = 265$. 5

- e) What do we mean when we say a problem is in P , NP , NP -Complete? Show the relationship among these three classes of problems. 5