Probability Distribution

1.

The probability distribution

The probability distribution for a random variable describes how the probabilities are distributed over the values of the random variable. In the development of the probability function for a discrete random variable, two conditions must be satisfied: (1) p(x) must be nonnegative for each value of the random variable, and (2) the sum of the probabilities for each value of the random variable must equal one.

X	x_1	x_2	x_3	- - -	x_n
P(X= x)	p_1	p_2	p_3	- - -	p_n

Mean = Expected value $\mu = E(x) = \sum x\, p(x) = x_1p_1 + x_2p_2 - - - + x_np_n$

Var(x) = $\sum x^2\, p(x) - (E(x))^2$

Exercise:

1. A fair 4 sided die, numbered 1, 2,3, and 5 is rolled twice. The random variable X is the sum of the two outcomes on which the die comes to rest.

(i) Show that P(x = 8) = $\frac{1}{8}$

(ii) Draw up the probability distribution table for X, and find p(x >6)

2.

A bag contains 7 orange balls and 3 blue balls. 4 balls are selected at random from the bag, without replacement. Let X denote the number of blue balls selected.

(i) Show that $P(X = 0) = \frac{1}{6}$ and $P(X = 1) = \frac{1}{2}$.

(ii) Construct a table to show the probability distribution of X.

(iii) Find the mean and variance of X.

3.

A fair cubical die with faces numbered 1, 1, 1, 2, 3, 4 is thrown and the score noted. The area A of a square of side equal to the score is calculated, so, for example, when the score on the die is 3, the value of A is 9.

(i) Draw up a table to show the probability distribution of A.

(ii) Find E(A) and Var(A).

4.

The discrete random variable X has the following probability distribution.

x	1	3	5	7
$P(X = x)$	0.3	a	b	0.25

(i) Write down an equation satisfied by a and b.

(ii) Given that $E(X) = 4$, find a and b.

5.

Two fair dice are thrown. Let the random variable X be the smaller of the two scores if the scores are different, or the score on one of the dice if the scores are the same.

(i) Copy and complete the following table to show the probability distribution of X.

x	1	2	3	4	5	6
$P(X = x)$						

(ii) Find $E(X)$.

Binomial /Bernoulli Distribution:

A binomial distribution is one kind of discrete probability distribution that has **two possible outcomes** (Success or failure / Pass or Fail)

Properties/Criteria

Binomial distributions must also meet the following three criteria:

- The number of observations or trials is fixed
- Trails are independent
- The probability of success or pass) is exactly the same from one trial to another.

When the random variable X, satisfies these conditions we denote it by

$$X \sim \beta(n, p)$$

The random variable X, which represents the number of successes in the ***n*** trials of this experiment, has a probability distribution given by

$P(X = r) = nC_r p^r q^{n-r}$ $\quad$ $where\ n = 0, 1,2,3, \ldots\ldots. n$ (numbers of trails)

$$p = probability\ of\ success$$

$$q = 1 - p \text{ (probability of failure)}$$

Mean (Expected Value) and Variance of Binomial Distribution

If $X \sim \beta(n, p)$, then mean $\mu = E(x) = n \times p$ Variance $\sigma^2 = npq$

Exercise:

1. A driving test is passed by 70% of people at their attempt. Find the probability that
   (i) exactly 5 people out of 10 people will passed the driving test.
   (ii) More than 1 people out of 8 people will passed the driving test.

2.

65% of all watches sold by a shop have a digital display and 35% have an analog display.

(i) Find the probability that, out of the next 12 customers who buy a watch, fewer than 10 choose one with a digital display. [4]

3.

(i) A garden shop sells polyanthus plants in boxes, each box containing the same number of plants. The number of plants per box which produce yellow flowers has a binomial distribution with mean 11 and variance 4.95.

(a) Find the number of plants per box. [4]

(b) Find the probability that a box contains exactly 12 plants which produce yellow flowers. [2]

Poisson Distribution

In probability theory and statistics, the Poisson distribution is a discrete probability distribution that expresses the probability of a given number of events occurring in a fixed interval of time or space if these events occur with a known constant mean rate and independently of the time since the last event.

Conditions /Properties

- Occurs singly
- The average rate at which events occur is always the same
- Events are independent

The random variable X, satisfying Poisson distribution $X \sim P_0()$, then probability distribution given by

$$P(X = r) = \frac{e^{-\mu} \times \mu^r}{r!}$$

Mean (Expected Value) and Variance of Poisson Distribution

$E(x)$ = Variance $\sigma^2 = \mu$

Exercise:

1.

Computer breakdowns occur randomly on average once every 48 hours of use.

(i) Calculate the probability that there will be fewer than 4 breakdowns in 60 hours of use.

(ii) there will be no breakdowns in 24 hours of use.

2.

Between 7 p.m. and 11 p.m., arrivals of patients at the casualty department of a hospital occur at random at an average rate of 6 per hour.

(i) Find the probability that, during any period of one hour between 7 p.m. and 11 p.m., exactly 5 people will arrive. [2]

(ii) A patient arrives at exactly 10.15 p.m. Find the probability that at least one more patient arrives before 10.35 p.m. [3]

3.

The number of radioactive particles emitted per second by a certain metal is random and has mean 1.7. The radioactive metal is placed next to an object which independently emits particles at random such that the mean number of particles emitted per second is 0.6. Find the probability that the total number of particles emitted in the next 3 seconds is 6, 7 or 8. [4]