What is Population?

In statistics, a population is the entire pool from which a statistical sample is drawn. A population may refer to an entire group of people, objects, events, hospital visits, or measurements. A population can thus be said to be an aggregate observation of subjects grouped together by a common feature.

Unlike a sample, when carrying out statistical analysis on a population, there are no standard errors to report, i.e., because such errors inform analysts using a sample how far their estimate may deviate from the true population value. But since you are working with the true population you already know the true value.

Population Samples

A sample is a random selection of members of a population. It is a smaller group drawn from the population that has the characteristics of the entire population. The observations and conclusions made against the sample data are attributed to the population.

The information obtained from the statistical sample allows statisticians to develop hypotheses about the larger population. In statistical equations, population is usually denoted with an uppercase N while the sample is usually denoted with a lowercase n.

Population Parameters

A parameter is data based on an entire population. Statistics such as averages and standard deviations, when taken from populations, are referred to as population parameters. The population mean and population standard deviation are represented by the Greek letters μ and σ, respectively.

The standard deviation is the variation in the population inferred from the variation in the sample. When the standard deviation is divided by the square root of the number of observations in the sample, the result is referred to as the standard error of the mean.

While a parameter is a characteristic of a population, a statistic is a characteristic of a sample. Inferential statistics enables you to make an educated guess about a population parameter based on a statistic computed from a sample randomly drawn from that population.

What is probability sampling?

Definition: Probability sampling is defined as a sampling technique in which the researcher chooses samples from a larger population using a method based on the theory of probability. For a participant to be considered as a probability sample, he/she must be selected using a random selection.

The most critical requirement of probability sampling is that everyone in your population has a known and equal chance of getting selected. For example, if you have a population of 100 people, every person would have odds of 1 in 100 for getting selected. Probability sampling gives you the best chance to create a sample that is truly representative of the population.

Probability sampling uses statistical theory to randomly select a small group of people (sample) from an existing large population and then predict that all their responses will match the overall population.

What is Sample Space?

A sample space is a collection or a set of possible outcomes of a random experiment. The sample space is represented using the symbol, "S". The subset of possible outcomes of an experiment is called events. A sample space may contain a number of outcomes which depends on the experiment. If it contains a finite number of outcomes, then it is known as discrete or finite sample spaces.

A samples space for a random experiment is written within curly braces { }. There is a difference between the sample space and the events. For rolling a die, we will get the sample space, S as $\{1, 2, 3, 4, 5, 6\}$ whereas the event can be written as $\{1, 3, 5\}$ which represents the set of odd numbers and $\{2, 4, 6\}$ which represents the set of even numbers. The outcomes of an experiment are random and the sample space becomes the universal set for some particular experiments. Some of the examples are as follows:

The meaning of probability is basically the extent to which something is likely to happen. This is the basic probability theory, which is also used in the probability distribution, where you will learn the possibility of outcomes for a random experiment. To find the probability of a single event to occur, first, we should know the total number of possible outcomes.

What are Events in Probability?

A probability event can be defined as a set of outcomes of an experiment. In other words, an event in probability is the subset of the respective sample space. So, what is sample space?

The entire possible set of outcomes of a random experiment is the sample space or the individual space of that experiment. The likelihood of occurrence of an event is known as probability. The probability of occurrence of any event lies between 0 and 1.

Impossible and Sure Events

If the probability of occurrence of an event is 0, such an event is called an impossible event and if the probability of occurrence of an event is 1, it is called a sure event. In other words, the empty set ϕ is an impossible event and the sample space S is a sure event.

Simple Events

Any event consisting of a single point of the sample space is known as a simple event in probability. For example, if $S = \{56, 78, 96, 54, 89\}$ and $E = \{78\}$ then E is a simple event.

Compound Events

Contrary to the simple event, if any event consists of more than one single point of the sample space then such an event is called a compound event. Considering the same example again, if $S = \{56, 78, 96, 54, 89\}$, $E_1 = \{56, 54\}$, $E_2 = \{78, 56, 89\}$ then, E_1 and E_2 represent two compound events.

Independent Events and Dependent Events

If the occurrence of any event is completely unaffected by the occurrence of any other event, such events are known as an independent event in probability and the events which are affected by other events are known as dependent events.

Mutually Exclusive Events

If the occurrence of one event excludes the occurrence of another event, such events are mutually exclusive events i.e. two events don't have any common point. For example, if $S = \{1 , 2 , 3 , 4 , 5 , 6\}$ and E_1, E_2 are two events such that E_1 consists of numbers less than 3 and E_2 consists of numbers greater than 4. So, $E_1 = \{1,2\}$ and $E_2 = \{5,6\}$. Then, E_1 and E_2 are mutually exclusive.

Exhaustive Events

A set of events is called exhaustive if all the events together consume the entire sample space.

Complementary Events

For any event E_1 there exists another event E_1', which represents the remaining elements of the sample space S. So, the complementary event of E_1 is $E_1' = S - E_1$.

If a dice is rolled then the sample space S is given as $S = \{1, 2, 3, 4, 5, 6\}$. If event E_1 represents all the outcomes which is greater than 4, then $E_1 = \{5,6\}$ and $E_1' = \{1,2,3,4\}$. Thus E_1' is the complement of the event E_1.

Events Associated with "OR"

If two events E_1 and E_2 are associated with OR then it means that either any of E_1, E_2 or both. The union symbol ($\cup$) is used to represent OR in probability. Thus, the event $E_1 \cup E_2$ denotes $E_1\ OR\ E_2$.

If we have mutually exhaustive events $E_1, E_2, E_3, \dots \dots, E_n$ associated with sample space S then the Sample Space $S = E_1 \text{U}\, E_2 \text{U}\, E_3 \text{U} \dots \dots \text{U}\, E_n$.

Events Associated with "AND"

If two events E_1 and E_2 are associated with AND then it means the intersection of elements which is common to both the events. The intersection symbol $(\cap)$ is used to represent AND in probability. Thus, the event $E_1 \cap E_2$ denotes E_1 AND E_2.

Difference of the Events

It represents the difference between both the events. Event E_1 but not E_2 represents all the outcomes which are present in E_1 but not in E_2. Thus, the event E_1 but not E_2 is represented as $E_1 - E_2$.

Probability Definition in Math

Probability is a measure of the likelihood of an event to occur. Many events cannot be predicted with total certainty. We can predict only the chance of an event to occur i.e. how likely they are to happen, using it. Probability can range in from 0 to 1, where 0 means the event to be an impossible one and 1 indicates a certain event. For example, when we toss a coin, either we get Head or Tail; only two possible outcomes are possible (H, T). But if we toss two coins in the air, there could be three possibilities of events to occur, such as both the coins show heads or either shows tails or one shows heads and one tail, i.e., $(H, H), (H, T), (T, T)$.

Formula for Probability

The probability formula is defined as the possibility of an event to happen is equal to the ratio of the number of favorable outcomes and the total number of outcomes.

Probability of event E is $P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total Number of outcomes}}$.

Sometimes students get mistaken for "favorable outcome" with "desirable outcome". This is the basic formula. But there are some more formulas for different situations or events.

Theoretical Probability

It is based on the possible chances of something to happen. The theoretical probability is mainly based on the reasoning behind probability. For example, if a coin is tossed, the theoretical probability of getting a head will be $\frac{1}{2}$.

Experimental Probability

It is based on the basis of the observations of an experiment. The experimental probability can be calculated based on the number of possible outcomes by the total number of trials. For example, if a coin is tossed 10 times and heads is recorded 6 times then, the experimental probability for heads is $\frac{6}{10}$.

Probability Terms and Definition

Some of the important probability terms are discussed here:

Term	Definition	Example
Sample Space	The set of all the possible outcomes to occur in any trial.	Tossing a coin, the Sample Space $S = \{H, T\}$ Rolling a die, the Sample Space $S = \{1,2,3,4,5,6\}$
Sample Point	It is one of the possible results.	In a deck of Cards: 4 of hearts is a sample point. The queen of clubs is a sample point.
Experiment or Trial	A series of actions where the outcomes are always uncertain.	The tossing of a coin, Selecting a card from a deck of cards, throwing a dice.
Event	It is a single outcome of a trial or an experiment.	Getting a Heads while tossing a coin is an event.
Outcome	Possible result of a certain trial or experiment.	Head or Tail is a possible outcome when a coin is tossed.
Complimentary event	The non-happening events. The complement of an event A is the event, not A (or A')	Standard 52 card deck, A = Draw a heart, then A' = Don't draw a heart
Impossible Event	The event cannot happen	In tossing a coin, impossible to get both head and tail at the same time

Probability of an Event

Assume an event E can occur in r ways out of a sum of n probable or possible equally likely ways. Then the probability of happening of the event or its success is expressed as $P(E) = \frac{r}{n}$.

The probability that the event E will not occur (E', complement of E) or known as its failure is expressed by $P(E') = \frac{n-r}{n} = 1 - \frac{r}{n}$.

Therefore, now we can say $P(E) + P(E') = 1$. This means that the total of all the probabilities in any random test or experiment is equal to1.

What are Equally Likely Events?

When the events have the same theoretical probability of happening, then they are called equally likely events. The results of a sample space are called equally likely if all of them have the same probability of occurring. For example, if you throw a die, then the probability of getting 1 is $\frac{1}{6}$. Similarly, the probability of getting all the numbers from 2,3,4,5 and 6, one at a time is $\frac{1}{6}$.

Complementary Events

The possibility that there will be only two outcomes considers that an event will occur or not. Like a person will come or not come to your house, getting a job or not getting a job, etc. are examples of complementary events. Basically, the complement of an event occurring in the exact opposite that the probability of it is not occurring.

Conditional Probability

The conditional probability of an event B is the probability that the event will occur given the knowledge that an event A has already occurred. This probability is written $P(B|A)$, notation for the probability of B given A. In the case where the event A has no effect on the probability of event B, the conditional probability of event B given event A is simply the probability of event B, that is $P(B)$.

If any of the events A and B is effects of occurring other, then the probability of the intersection of A and B (the probability that both events occur) is defined by $P(A \cap B) = P(A)P(B|A)$. So, the conditional probability $P(B|A)$ is easily obtained by $P(B|A) = \frac{P(A \cap B)}{P(A)}$, where $P(A) > 0$.

What are Independent Events?

Independent events are those events whose occurrence is not dependent on any other event. For example, if we flip a coin in the air and get the outcome as Head, then again if we flip the coin but this time we get the outcome as Tail. In both cases, the occurrence of both events is independent of each other. It is one of the types of events in probability.

In Probability, the set of outcomes of an experiment is called events. There are different types of events such as independent events, dependent events, mutually exclusive events, and so on. If the probability of occurrence of an event A is not affected by the occurrence of another event B, then A and B are said to be independent events.

If A and B are independent events, then $P(B|A) = P(B)$. Using the multiplication rule of the probability, it can be written as $P(A \cap B) = P(A).P(B|A) = P(A).P(B)$.

What are Mutually Exclusive Events?

Two events A and B are said to be mutually exclusive events if they cannot occur at the same time. Mutually exclusive events never have an outcome in common.

In probability theory, two events are said to be mutually exclusive if they cannot occur at the same time or simultaneously. In other words, mutually exclusive events are called disjoint events. If two events are considered disjoint events, then the probability of both events occurring at the same time will be zero. If A and B are the two events, then the probability of disjoint of event A and B is written by $P(A \cap B) = 0$.

In probability, the specific addition rule is valid when two events are mutually exclusive. It states that the probability of either event occurring is the sum of probabilities of each event occurring. If A and B are said to be mutually exclusive events then the probability of an event A occurring or the probability of event B occurring is given as $P(A) + P(B)$, thus it can be expressed as $P(A \cup B) = P(A) + P(B)$.

Independent Events vs. Mutually Exclusive Events

The difference between the independent events and mutually exclusive events are given below:

Independent Events	Mutually exclusive events
They cannot be specified based on the outcome of a maiden trial.	They are independent of trials
Can have common outcomes	Can never have common outcomes
If A and B are two independent events, then $P(A \cap B) = P(B).P(A)$	If A and B are two mutually exclusive events, then $P(A \cap B) = 0$

Dependent and Independent Events

Two events are said to be dependent if the occurrence of one event changes the probability of another event. Two events are said to be independent events if the probability of one event that does not affect the probability of another event. If two events are mutually exclusive, they are not independent. Also, independent events cannot be mutually exclusive.

Conditional Probability for Mutually Exclusive Events

Conditional probability is stated as the probability of an event A, given that another event B has occurred. Conditional Probability for two independent events B has given A is denoted by the expression $P(\ B|A)$ and it is defined using the equation $P(B|A) = \frac{P(A\cap B)}{P(A)}$.

Redefine the above equation using multiplication rule, $P(A \cap B) = 0$, then $P(B|A) = \frac{0}{P(A)} = 0$.

So the conditional probability formula for mutually exclusive events is $P(B|A) = P(A|B) = 0$.

Probability Density Function

The Probability Density Function (PDF) is the probability function which is represented for the density of a continuous random variable lying between a certain ranges of values. Probability Density Function explains the normal distribution and how mean and deviation exists. The standard normal distribution is used to create a database or statistics, which are often used in science to represent the real-valued variables, whose distribution are not known.

https://byjus.com/maths/probability/

https://courses.lumenlearning.com/introstats1/chapter/the-terminology-of-probability/