Mid Term Exam

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Section: A

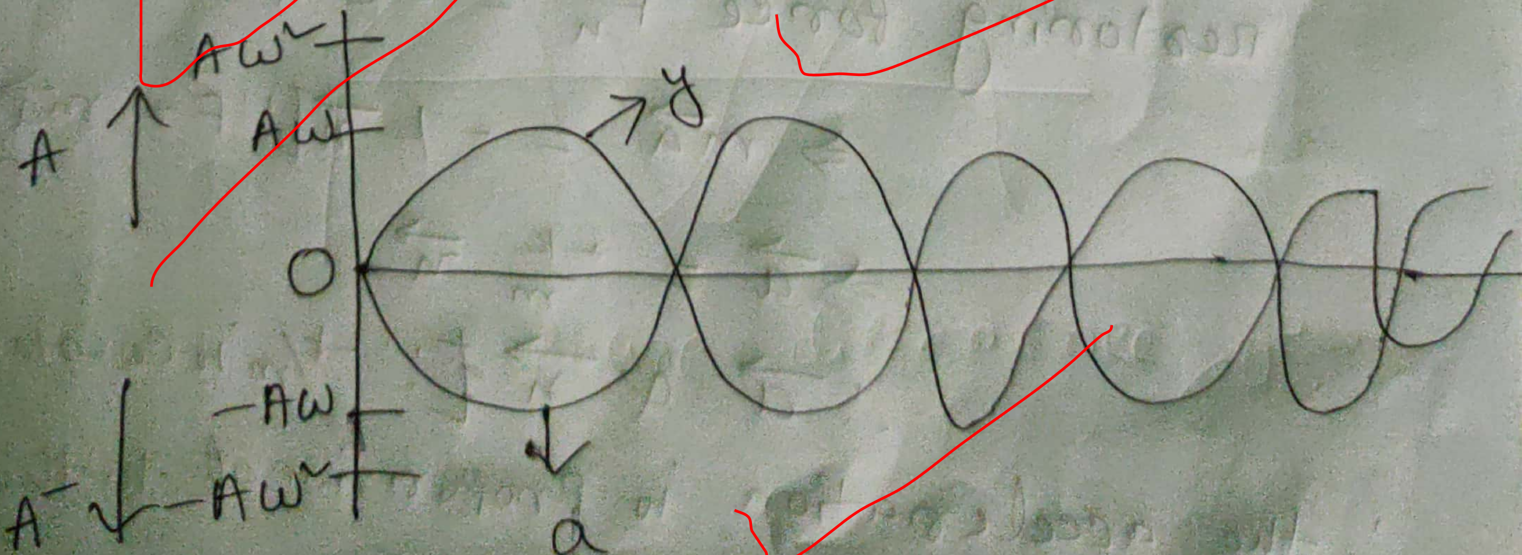
① ⓐ $y = A\sin(\omega t + \frac{\pi}{2})$

$\Rightarrow \frac{dy}{dt} = \frac{d}{dt}\{A\sin(\omega t + \frac{\pi}{2})\}$

$\Rightarrow \frac{dy}{dt} = A\omega\cos(\omega t + \pi/2)$

$\Rightarrow \frac{d^2y}{dt^2} = -A\omega^2\sin(\omega t + \frac{\pi}{2})$

$\therefore a = -A\omega^2\sin(\omega t + \frac{\pi}{2})$



(b) In Hook's law, we know,

restoring force $\vec{F}_n = -k\vec{n}$

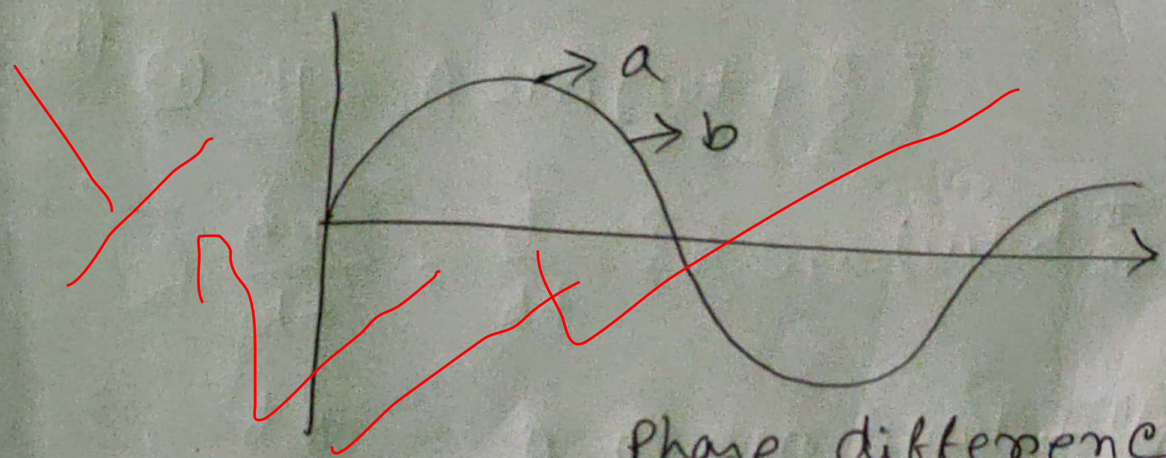
$\Rightarrow m\vec{a} = -k\vec{n}$ [$\because \vec{F} = m\vec{a}$]

$\Rightarrow \vec{a} = \frac{-k}{m}\vec{n}$

$\therefore \vec{a} \propto \vec{n}$ [$\because -k/m$ is constant]

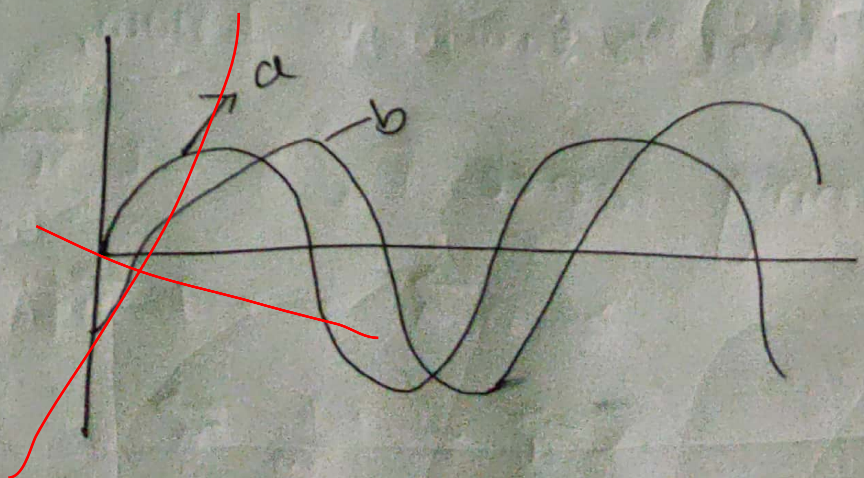
$\therefore$ the acceleration is proportional to the displacement.

ⓒ (i) In phase



Phase difference = 0°.

(ii) Out of phase.



phase difference > 0°.

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Question - 02

(a) Given,

$m = 0.4 kg$

maximum Distance $= 100 cm$

$\therefore A = 100 \times 10^{-2} m = 1 m$

and $T = 0.80 s$

(ii) At, $v = 100 cm/s$

$= 100 \times 10^{-2} m/s$

$= 1.9 m/s$

$\therefore$ we know,

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.80} = 7.85 \text{ rad/s}$$

Now, $v = \omega\sqrt{A^2 - x^2}$

$$\Rightarrow \frac{v^2}{\omega^2} = A^2 - x^2$$

$$\Rightarrow x = \pm\sqrt{A^2 - \frac{v^2}{\omega^2}}$$

$$= \pm\sqrt{(1)^2 - \frac{(1.9)^2}{(7.85)^2}}$$

$\therefore$ ~~x~~ $x = \pm 0.97 m.$

(iv) maximum accelaration,

$$a_{max} = A\omega^2$$
$$= 1 \times (7.85)^2$$
$$= 61.62 \ m/s^2.$$

(v) Instantaneous velocity;

$$v = \omega\sqrt{A^2 - x^2}$$
$$= 7.85\sqrt{1^2 - (0.9)^2}$$
$$= 3.42 \ m/s. \ \text{(Result)}$$

(i) spring constant b.

$$K = \frac{F_{max}}{A} = \frac{a_{max} m}{A}$$
$$= \frac{A\omega^2 m}{A} = \omega^2 m$$
$$= (7.85)^2 \times 0.4$$
$$= 24.65 \ N/m.$$

(2) (b) Here given,

$m = 1.68 \times 10^{-27}$ kg

$f = 10^{12}$ Hz

$A = 10^{-8}$ cm

$= 10^{-10}$ m.

$\therefore \omega = 2\pi f = 2\pi \times 10^{12}$

$= 6.28 \times 10^{12}$ rad/s.

(i) $F_{max} = a_{max}\, m$

$= (A\omega^2) \times m$

$= (10^{-10} \times (6.28 \times 10^{12})^2) \times 1.68 \times 10^{-27}$

$= 6.626 \times 10^{-12}$ N

(ii) spring constant;

$k = \frac{F_{max}}{A} = \frac{6.626 \times 10^{-12}}{10^{-10}}$

$= 0.066$ N/m.

(Result).

② ⓒ $E = 2.00J$

$x_m = 13cm$

$= 0.13 m.$

$v_{max} = 1.20 m/s$

① $k = \frac{2E}{(x_m)^2} = \frac{2\times 2}{(0.13)^2}$

$= 236.686 N/m.$

ⓘ $E = \frac{1}{2} mv^2$

$\therefore m = \frac{2E}{v_{max}^2} = \frac{2\times 2}{(1.20)^2}$

$= 2.78 kg.$

(2) (c) (iii) frequency.

$$2\pi f = \omega$$

$$\Rightarrow f = \frac{\omega}{2\pi} = \frac{\sqrt{\frac{k}{m}}}{2\pi} = \frac{\sqrt{\frac{236.686}{2.78}}}{2\pi}$$

$$= 1.47 \text{ Hz}.$$

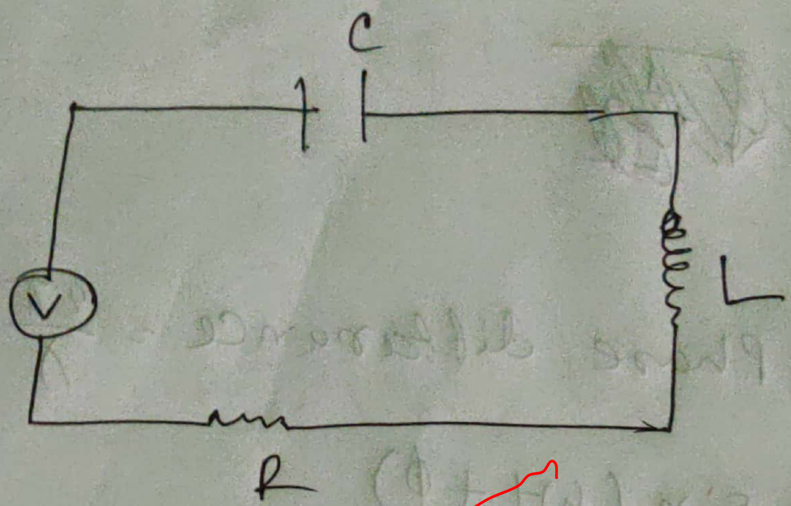
(a) (iii) maximum velocity; $= A\omega$

$$= 1 \times 7.85$$

$$= 7.85 \text{ m/s}.$$

(5)

(a)



V_{across} R, $V_R = IR$

V_{across}, C, $V_c = \frac{d}{C}$

V_{across}, L, $V_L = L\frac{di}{dt}$

$V_{gain} = V_{Lost}$

$\Rightarrow \quad V_L = -(V_c + V_R)$

$\Rightarrow \quad L\frac{di}{dt} + V_c + V_R = 0$

P.T.O

$$\Rightarrow L\frac{di}{dt} + \frac{\alpha}{c} \pm IR = 0$$

$$\Rightarrow \frac{di}{dt} + \frac{\alpha}{2C} + \frac{R}{L}i = 0$$

$$\Rightarrow \frac{d}{dt}\left(\frac{di}{dt}\right) + \frac{d}{dt}\left(\frac{\alpha}{LC}\right) + \frac{d}{dt}\left(\frac{R}{L}i\right) = 0$$

$$\Rightarrow \frac{d^2i}{dt^2} + \frac{d}{dt}\left(\frac{it}{LC}\right) + \frac{R}{L}\frac{di}{dt} = 0$$

$$\Rightarrow \frac{d^2i}{dt^2} + \frac{dt}{dt}\frac{i}{2C} + \frac{R}{L}\frac{di}{dt} = 0$$

$$\Rightarrow \frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{2C}i = 0$$

$$\Rightarrow \frac{d^2x}{dt^2} + \alpha\frac{dx}{dt} + \omega^2 x = 0 \quad \text{--- (1)}$$

$\therefore I = x$

$\frac{R}{L} = \alpha$

$\therefore \frac{1}{LC} = \omega^2$

Now,

$$\omega = \sqrt{\frac{1}{LC}}$$

(Result)

(b) ϕ = phase difference $= \frac{2\pi}{\lambda} x$

$y = A \sin(\omega t + \phi)$

$\Rightarrow y = A \sin\left(\frac{2\pi}{T} t + \frac{2\pi}{\lambda} x\right)$

$\left[\omega = \frac{2\pi}{T} ; \phi = \frac{2\pi}{\lambda} x\right]$

$\Rightarrow y = A \sin 2\pi \left(\frac{t}{T} + \frac{x}{\lambda}\right)$

$\left[v = \frac{\lambda}{T} ; T = \frac{\lambda}{v}\right]$

$\Rightarrow y = A \sin \frac{2\pi}{\lambda}(vt + x)$. which is the equation of progressive wave.

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Question - 03

ⓐ Here given,

$C = 1\,\mu F = 10^{-6}\,F$

$L = 5\,mH = \frac{5}{1000} = 5 \times 10^{-3}\,H.$

$R = 300\,\Omega$

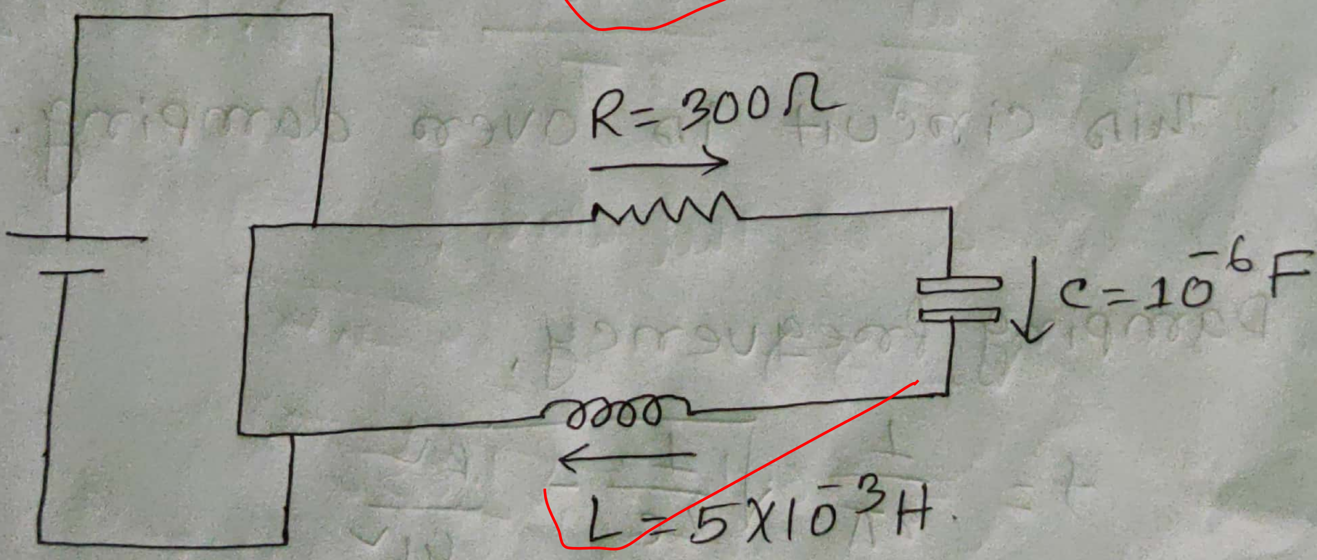

Fig: In series.

$\therefore \omega^2 = \frac{1}{LC} = \frac{1}{5 \times 10^{-3} \times 10^{-6}}$

$= 200000000.00$

$= 2 \times 10^{8}$

P.T.O

$$\therefore \left(\frac{r}{2}\right)^2 = \frac{R^2}{4L^2} = \frac{(300)^2}{4\times(5\times10^{-3})^2}$$

$$= 900000000$$

$$= 9\times10^8.$$

$$\therefore \frac{1}{LC} < \frac{R^2}{4L^2}$$

$\therefore$ This circuit is over damping.

Damping frequency,

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

$$= \frac{1}{2\pi}\sqrt{(2\times10^8) - (9\times10^8)}$$

$$= \text{undefined}.$$

There will be no damping frequency.

∴ resonant frequency;

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{5\times10^{-3}\times10^{-6}}}$$

$$= 2250.79\ H$$

(Result)

(b) $\omega = 240$ rad/s

$\lambda = 185$ cm.

$= 185 \times 10^{-2}$ m

$= 1.85$ m

We know;

$\omega = 2\pi f$

$\Rightarrow f = \frac{\omega}{2\pi} = \frac{240}{2\pi} = 38.197$ HZ.

(i) speed of the wave,

$v = f\lambda$

$= 38.197 \times 1.85$

$= 70.66$ m/s.

(ii) frequency,

$f = \frac{\omega}{2\pi} = \frac{240}{2\pi} = 38.197$ HZ.

3 (b)

(iii) Time Period,

$$T = \frac{1}{f} = \frac{1}{38.197} = 0.03 \text{ sec}$$

(iv) Equation of the wave;

$$x = A \sin(\omega t + \phi)$$

$$= A \sin(240t + \phi)$$

(Result)