Assignment 02

1. A clothing store keeps track of customer purchases. They find that all customers buy at least one shirt. 60% of customers buy more than one item, 30% buy a blue shirt, and 10% buy more than one shirt including a blue shirt. What is the probability a random customer only buys a shirt that is not blue?

2. A fair coin and a five-sided dice are tossed at a times, and the sequence of outcomes are observed. Let the events $A = \{$head in the coin and 2 or 4 in the dice$\}$, find $P(A)$.

3. Consider $P(A) = 0.38$ and $P(B) = 0.42$ to find $P(A' \cup B')$ such that A and B are of the mutually exclusive events.

4. Complete the following table and compute $P(A_1|B_2)$, $P(B_2|A_1)$, and $P(A_1 \cup B_1)$.

	$n(A_1)$	$n(A_2)$	*Total*
$n(B_1)$	*25*		*65*
$n(B_2)$			
Total	*80*		*150*

5. A fair coin and a four-sided die are launched at a time. If $A = \{odd\ sides\ in\ the\ die\}$ and $B = \{head\ in\ the\ coin\}$, determine whether they are conditional or independent?

6. If 3 worker independently try to complete a task with the probability of success 0.7, 0.8, and 0.6, respectively. What is the probability of the (i) task will not be completed (ii) every worker will complete the task (iii) only the second worker will complete the work?

7. In a software development team, three programmers contribute to 35%, 40%, and 25% of the total code developed in a project cycle. Their respective coding efficiencies are 92%, 97%, and 95%. If a bug is detected in the code, what is the probability that it originated from the work of the second programmer?

8. The glucometer is a tool to rapidly test diabetes. Of the people appearing in the test, 12% of them false-positive while 9% of them false-negative. If the people in Bangladesh 5% have diabetes, find the probability of a person suffering from diabetes, when he/she tests negative in the test.

9. Suppose there are 5 defective items in a lot of 50 items. A sample of size 5 is taken at random without replacement. Let X denote the number of defective items in the sample. Find the probability that the sample contains (i) at most two defective item (ii) all defective items.

10. Let the random variable X have the *pmf* $f(x) = \frac{x}{10}; x = 1, 2, 3, 4$. Compute $P(X < 3)$, $E(X^2 + 5)$, and $V(1 - 3X)$. Also, sketch the line graph of X.

11. Suppose that 80% of employees in a certain company are proficient in using a particular software tool. In a random sample of 15 employees, let X be the number of proficient employees. Assuming independence, how is X distributed? Find the *mgf* of X. Also, compute $P(X < 3)$ and $P(X \geq 4)$.

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12. Let engaging in research is independently distributed. A group of 10 students are selected at random with probability of engaging in research 0.4. Find the probability that more than 2 students engaged in research. Also, evaluate the variance of the corresponding distribution?

13. Let X have the *pdf* $f(x) = 4x^3 e^{-x^4}; 0 \leq x < \infty$. Find the *cdf* and hence median of X. Also, find $P(X > -2)$.

14. Let $f(y) = \frac{3}{2} y^2$; $-1 < y < 1$ be the *pdf* of a continuous random variable Y. Find and sketch the *cdf of* Y. Find $P(Y \geq 0.5)$ and P_{65} of Y.

15. A random variable T that is representing the time in hour for the duration daily traffic jam in the Dhaka metropolitan is distributed $f(t) = k(3t - t^2); 0 \leq t \leq 3$.
    (a). Find the value of the constant k.
    (b). Find the standard deviation of the duration of the traffic jam.
    (c). Find the probability that the duration of the traffic jam more than 1 hour.
    (d). Find the mode time of duration of the traffic jam.

16. A continuous random variable X has the *cdf* as

$$F(x) = \begin{cases} 0; & x < 5 \\ \frac{x-5}{5}; & 5 \leq x < 10 \\ 1; & x \geq 10 \end{cases}$$

    (a). Identify the distribution and write the *pdf* of X.
    (b). Estimate $P(3 < X < 12)$.
    (c). Find the mean and the variance of X.

17. Let $M(t) = \begin{cases} \frac{e^{4t}-1}{4t} ; t \neq 0 \\ 1 ; t = 0 \end{cases}$ is the *mgf* of the random variable X satisfies uniform distribution. Find the *cdf* and hence the median of the distribution.

18. If the *mgf* of the normal variable X is $M(t) = e^{30t+18t^2}$, then (i) Find a constant k such that $P(|Z| \leq k) = 0.9544$ (ii) Evaluate $P(42.6 \leq X \leq 55.8)$. Also, find $-Z_{09656}$.

19. The average weight of the cement bags satisfied by a normal distribution with mean 50.5 Kg and standard deviation 1.25 Kg.
    (a). Determine the probability that a randomly selected bag of cement will weigh:
    (i) Less than 49 Kg. (ii) More than 51 Kg.
    (b). Determine, to two decimal places, exceeded by 60% of cement bags.

20. Assume $M(t) = e^{24t+50t^2}$ is the *mgf* of a normal variable X, then (i) find a constant k such that $P(Z \geq k) = 0.025$ (ii) evaluate $P(X < 40.2)$. Also, find $-Z_{0.05}$.

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21. In a certain machine-learning conference, an author delivered a talk to 400 people and half of them responded at the end of the session. If 120 responses were positive. Find an approximate 95% confidence interval for the fraction p of the people who are being encouraged by the speaker.

22. Let X equal the dirt in kg per day produced by a typical family in Dhaka city. Suppose the standard deviation of X is 2 kg. To estimate the mean μ of X, an agency took a random sample of 100 families and found they produced 0.5 metric ton of dirt every day. Find an approximate 95% confidence interval for μ.

23. A manufacturer produces a new cooker and they claimed that it will reduce the fuel cost in half with 90% accuracy. Now design decision rule for the process with significance 0.05 by testing the cooker to 50 customers.

24. Design a decision rule to test the hypothesis that a deck of playing cards is fair if we take a sample of 120 trials of the cards to get black/red and use 10% as the level of significance. Predict the acceptance and critical region.

25. A company produces mosquito killing bat whose average lifetime is $360.$ days and average variation 60 days. It is claimed that in a newly developed process the mean lifetime can be increased. Design a decision rule for 100 samples with 0.1 significance. If the new process has increased the mean lifetime to 375 days, assuming a sample of 120 bats with estimated lifetime 370 days, find α and β. Again, a sample of 80 bats is tested and it is found that the average lifetime is 368 days. Find the p $-$value of the test.