United International **University**

School of Science and Engineering

Mid Assessment   Trimester: Summer-2020

Course Title: Probability and Statistics

Course Code: Stat 205   Marks: 20   Time: 1 Hour

There are 3 questions. Answer question no. 1 and any <u>one</u> from 2 and 3.

1. **a)** During a visit to a doctor's chamber, the probability of having neither medical test, nor a referral to a specialist is 19%. Of those coming to that chamber, the probability of having a medical test is 37% and the probability of having referral is 54%. What is the probability of having both medical test and referral? **[3]**

b) Bowl $\boldsymbol{A}$ contains $\mathbf{7}$ red and $\mathbf{5}$ white chips, and bowl $\boldsymbol{B}$ contains $\mathbf{9}$ white and $\mathbf{6}$ red chips. A chip is drawn at random from bowl $\boldsymbol{B}$ and transferred to bowl $\boldsymbol{A}$. **Find** the probability of then drawing a **same color** chip from bowl $\boldsymbol{A}$. **[3]**

c) Bean seeds from supplier $\boldsymbol{A}$ have germination rate $\mathbf{76}\%$ and those from supplier $\boldsymbol{B}$ have germination rate $\mathbf{87}\%$. A seed-packaging company purchases $\mathbf{53}\%$ from supplier $\boldsymbol{A}$ and remaining from supplier $\boldsymbol{B}$ and mixes these seeds together. **[4]**

(i) If a seed germinates find the probability that it has been provided by supplier $\boldsymbol{A}$.

(ii) If a seed does not germinate find the probability that it has been provided by supplier $\boldsymbol{B}$.

2. **a)** Let a chip be taken at random from a bowl that contains ***seven*** white chips, ***four*** red chips, and ***one*** blue chip. Let the random variable $\boldsymbol{X} = \mathbf{1}$ if the outcome is a ***red chip***, let $\boldsymbol{X} = \mathbf{4}$ if the outcome is a ***white chip***, and let $\boldsymbol{X} = \mathbf{7}$ if the outcome is a ***blue chip***. **[6]**

(i) Find the ***pmf*** of $\boldsymbol{X}$.

(ii) Draw a ***line graph*** and ***probability histogram*** for this ***pmf***.

(iii) Find the ***mean*** and ***variance*** of $\boldsymbol{X}$.

b) The life $\boldsymbol{X}$ (in years) of a voltage regulator of a car has the ***pdf***; **[4]**

$$\boldsymbol{f(x) = \frac{3x^2}{9^3} e^{-\left(\frac{x}{9}\right)^3}; \quad x > 0}$$

(i) What is the probability that this regulator will last at least $\mathbf{9}$ years?

(ii) Find the **37**$^{\text{th}}$ percentile of $\boldsymbol{f(x)}$.

3. **a)** Show that the ***mgf*** of a random variable $\boldsymbol{X}$ is $\boldsymbol{M(t)} = \frac{e^{2t}}{(e^t-2)^2}$. Hence, find the ***mean*** and ***standard deviation*** of the corresponding probability distribution. **[4]**

b) Flaws in a certain type of drapery material appear on the average of ***two*** in $\mathbf{175}$ square feet. If we assume a Poisson distribution, find the probability of at most ***one*** flaw appearing in $\mathbf{250}$ square feet. **[3]**

c) It is claimed that $\mathbf{21}\%$ of the people in a certain region are infected by severe virus attack. Suppose $\mathbf{15}$ people are selected at random. Let $\boldsymbol{X}$ equal to the number of people infected by the virus. Assuming independence, how $\boldsymbol{X}$ is distributed. Find the probability that **(i)** at least **3** people are infected, **(ii)** exactly $\mathbf{12}$ people are infected. **[3]**

Formulae:

Distribution	***Pmf / pdf***
Hypergeometric	$f(x) = \dfrac{N_{1_{C_x}}\, N_{2_{C_{n-x}}}}{N_{C_n}}\ ;\quad N = N_1 + N_2, \qquad x = 1, 2, \ldots \ldots n$
Geometric	$f(x) = q^{x-1}p\ ;\ x = 0, 1, 2, \ldots.$
Binomial	$f(x) = n_{C_x} p^x q^{n-x}\ ;\ x = 0, 1, 2, \ldots.\, n$
Poisson	$f(x) = \dfrac{\lambda^x e^{-\lambda}}{x!}\ ;\ x = 0, 1, 2, \ldots ..$
Uniform	$f(x) = \dfrac{1}{b-a}\ ;\ a \le x \le b$
Exponential	$f(x) = \dfrac{1}{\theta} e^{-\frac{x}{\theta}}\ ;\ 0 \le x < \infty$
Gamma	$f(x) = \dfrac{1}{\Gamma(\alpha)\, \theta^{\alpha}} x^{\alpha-1} e^{-x/\theta}\ ;\ 0 \le x < \infty$
Normal	$f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\ ;\ -\infty < x < \infty$