Sample Questions for Final Exam

1. During a visit to a doctor, the probability of having neither lab work, nor referral to a specialist is $\mathbf{0.19}$. Of those coming to the doctor, the probability of having lab work is $\mathbf{0.47}$ and the probability of having a referral is $\mathbf{0.51}$. What is the probability of having both lab work and referral?

2. A fair coin is tossed ***three*** times, and the sequence of heads and tails is observed. Let the events $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ be given by, $\boldsymbol{A} = \{\boldsymbol{at\ most\ 2\ tails}\}$, $\boldsymbol{B} = \{\boldsymbol{at\ least\ 2\ heads}\}$ and $\boldsymbol{C} = \{\boldsymbol{1\ head\ and\ 2\ tails}\}$. Find the sample space $\boldsymbol{S}$. Also, find $\boldsymbol{P(A \cap C)}$, $\boldsymbol{P(B \cup C)}$, $\boldsymbol{P(C')}$.

3. A certain test for identifying cancer had tested among $\mathbf{1}$ lac people, we can expect results similar to those given in the following table. If one of $\mathbf{1}$ lac people is selected randomly, find the following probabilities: $\boldsymbol{P(A_1 \backslash B_2)}$, $\boldsymbol{P(A_2 \backslash B_1)}$, $\boldsymbol{P(A_1 \cap B_1)}$ and $\boldsymbol{P(A_2 \cup B_2)}$.

	***A*1** **Carry cancer**	***A*2** **Do not carry cancer**	**Totals**
***B*1** **Test positive**	4500	75500	80000
***B*2** **Test negative**	500	19500	20000
Totals	5000	95000	100000

4. A shop contains $\mathbf{8}$ Nokia and $\mathbf{7}$ Samsung mobiles. Another shop contains an unknown number of Nokia and $\mathbf{11}$ Samsung mobiles. A mobile collected from each shop at random, and the probability of getting ***two*** mobiles of the same company is $\mathbf{\frac{151}{300}}$. How many Nokia mobiles are in the second shop?

5. Each of ***three*** bowlers will attempt to hit the wicket. Let $\boldsymbol{A_i}$ denote the event that the wicket is got by ***i***-th player, $\boldsymbol{i = 1, 2, 3}$. Assume that all of the events are mutually independent and that $\boldsymbol{P(A_1) = 0.35}$, $\boldsymbol{P(A_2) = 0.65}$ and $\boldsymbol{P(A_3) = 0.5}$. Find the probability that exactly two players are successful, and probability of no player is successful.

6. At a country fair carnival game there are $\mathbf{25}$ balloons on a board, of which $\mathbf{10}$ balloons are yellow, $\mathbf{8}$ are red, and $\mathbf{7}$ are green. A player throws darts at the balloons to win a prize and randomly hits one of them. If the first balloon hit is green, what is the probability of **(i)** next balloon is green, **(ii)** next balloon is not green.

7. A boy has ***three*** red coins and ***five*** white coins in his left hand, ***six*** red coins and ***four*** white coins in his right hand. If he shifts one coin at random from his left to right hand, what is the probability of his then drawing a ***white*** coin from his ***right*** hand?

8. Four inspectors look at a critical component of a product. Their probabilities of detecting an error are different, namely, $\mathbf{0.98, 0.95, 0.92, 0.89}$ respectively. If we in sections are independent, then find the probability of **(i)** no one detecting the error, **(ii)** at least one detecting the error, **(iii)** only one inspector detecting the error.

9. Let $\boldsymbol{P(A) = 0.3}$ and $\boldsymbol{P(B) = 0.6}$. Find $\boldsymbol{P(A \cup B)}$ when $\boldsymbol{A}$ and $\boldsymbol{B}$ are independent. Find $\boldsymbol{P(A|B)}$ when $\boldsymbol{A}$ and $\boldsymbol{B}$ are mutually independent.

Sample Questions for Final Exam

10. There is a new diagnostic test for a disease that occurs in about 1% of the population. The test is not perfect, but will detect a person with the disease 99% of the time. It will, however, say that a person without the disease has the disease about 3% of the time. A person is selected at random from the population, and the test indicates that this person has the disease. What are the conditional probabilities that **(i)** the person has the disease? **(ii)** the person does not have the disease?

11. A test indicates the presence of a particular disease 95% of the time when the disease is present and the presence of the disease 2% of the time when the disease is not present. If 0.2% of the population has the disease, calculate the conditional probability that a person selected at random has the disease if the test indicates the presence of the disease.

12. ***Die A*** has orange on ***one*** face and blue on ***five*** faces, ***Die B*** has orange on ***two*** faces and blue on ***four*** faces, ***Die C*** has orange on ***three*** faces and blue on ***three*** faces. If all are fair dice, picture the sample space.

- **(i)** If the ***three*** dice are rolled, find the probability that ***at least two*** of the three dice come up orange.
- **(ii)** If you are given, at least ***two*** of the three dice come up orange, find the probability that ***exactly two*** of the three dice come up orange.

13. Bowl $\boldsymbol{B_1}$ contains ***two*** white chips and ***one*** red chip, bowl $\boldsymbol{B_2}$ contains ***one*** white chip and ***two*** red chips, bowl $\boldsymbol{B_3}$ contains ***three*** white and ***two*** red chips and bowl $\boldsymbol{B_4}$ contains ***two*** white and ***three*** red chips. The probabilities of selecting bowl and $\boldsymbol{B_1, B_2, B_3}$ and $\boldsymbol{B_4}$ are $\frac{3}{8}, \frac{1}{4}, \frac{1}{4}$ and $\frac{1}{8}$ respectively. A bowl is selected using these probabilities and a chip is drawn at random. Find $\boldsymbol{P(B_3/R)}$, the conditional probability that bowl $\boldsymbol{B_3}$ had been selected, given that a **red** chip was drawn.

14. At an office, officials are classified and 30% of them efficient, 50% are moderate worker, and 20% are unfit for the work. Of efficient ones, 15% left the job; of the moderate workers, 20% left the job, and of unfit workers, 5% left the job. Given that an employee left the job, what is the probability that the employee is ***unfit one***? Consider independence for the employee classes.

15. Suppose there are 5 defective items in a lot of 100 items. A sample of size 15 is taken at random without replacement. Let $\boldsymbol{X}$ denote the number of defective items in the sample. Find the probability that the sample contains **(i)** at most one defective item, **(ii)** exactly three defective items.

16. If the ***mgf*** of $\boldsymbol{X}$ is $\boldsymbol{M(t) = \frac{4}{10}e^{t} + \frac{3}{10}e^{2t} + \frac{2}{10}e^{3t} + \frac{1}{10}e^{4t}}$, find the corresponding ***pmf***, mean and variance.

17. Flaws in a certain type of drapery material appear on the average of ***one*** in 120 square feet. If we assume a Poisson distribution, find the probability of no more than one flaw appearing in 60 square feet.

18. In the gambling game craps, the player wins $\$1$, $\$2$ and $\$3$ with probabilities 0.3, 0.2 and 0.1, and loses $\$1$ with probability 0.4 for each $\$1$ bet. What is the expected profit of the game for the player? Also, find the variance of the profit.

Sample Questions for Final Exam

19. Let the random variable $\boldsymbol{X}$ be the number of days that a certain patient needs to be in the hospital. Suppose $\boldsymbol{X}$ has the pmf $\boldsymbol{f(x) = \frac{5-x}{10}; x = 1, 2, 3, 4.}$
If the patient is to receive $\$\mathbf{200}$ from an insurance company for each of the first ***two*** days in the hospital and $\$\mathbf{100}$ for each day after the first ***two*** days, what is the expected payment for the hospitalization?

20. A boiler has five relief valves. The probability that each does not work is $\mathbf{0.05}$. Find the probability that **(i)** none of them work, **(ii)** at least four work.

21. If $\boldsymbol{X}$ has a Poisson distribution such that $\boldsymbol{P(X = 1) = 2P(X = 2)}$, evaluate $\boldsymbol{P(X = 5)}$. Also, find the standard deviation of the distribution.

22. Let $\boldsymbol{X}$ have a Poisson distribution with a standard deviation of $\mathbf{2}$. Find $\boldsymbol{P(X \geq 1)}$.

23. For $\boldsymbol{f(x) = c(x + 1)^3 ; x = 0, 1, 2, \ldots \ldots \ldots 10}$, determine the constant $\boldsymbol{c}$ so that $\boldsymbol{f(x)}$ satisfies the conditions of being ***pmf*** for a random variable $\boldsymbol{X}$, and then depict ***pmf*** as line graph and histogram.

24. Given that $\boldsymbol{E[X + c] = 10}$ and $\boldsymbol{E[(X + c)^2] = 116}$. Find the mean and variance of $\boldsymbol{X}$.

25. It is claimed that $\mathbf{20}\%$ of the birds in a particular region have severe disease. Suppose that $\mathbf{15}$ birds are selected at random. Let $\boldsymbol{X}$ is the number of birds that are have the disease. Assuming independence, how $\boldsymbol{X}$ is distributed? Find $\boldsymbol{P(X \geq 2)}$ and $\boldsymbol{P(X \leq 14)}$.

26. Let a random experiment be casting of pair of fair ***six***-sided dice and let $\boldsymbol{X}$ equal the maximum of ***two*** outcomes. With reasonable assumptions, find the ***pmf*** of $\boldsymbol{X}$ and draw a probability histogram of the ***pmf*** of $\boldsymbol{X}$.

27. A random variable $\boldsymbol{X}$ has a binomial distribution with mean $\mathbf{10.5}$ and variance $\mathbf{3.15}$. How $\boldsymbol{X}$ is distributed and find $\boldsymbol{P(X \geq 1)}$.

28. Let the random variable $\boldsymbol{X}$ be the number of days that a certain patient needs to be in the hospital. Suppose $\boldsymbol{X}$ has the ***pmf*** $\boldsymbol{f(x) = 0.1(5 - x); x = 1, 2, 3, 4}$. If the patient is to receive $\$\mathbf{100}$ for the first day, $\$\mathbf{50}$ for the second day, $\$\mathbf{25}$ for the third day and have to return $\$\mathbf{25}$ for the fourth day, what is the expected payment for the hospitalization?

29. Suppose that in a region the probability of arresting an innocent person is $\mathbf{15}\%$. If $\mathbf{500}$ people are arrested, assuming Bernoulli experiment find the probability of arresting $\mathbf{35}$ innocent persons. Find the probability by Poisson process as well.

30. Suppose that $\mathbf{90}\%$ of UIU students are multi-taskers. In a random sample of $\mathbf{10}$ students are taken and let $\boldsymbol{X}$ is the number of multi-taskers. Assuming independence, how is $\boldsymbol{X}$ distributed? Find the standard deviation of $\boldsymbol{X}$. Also, compute $\boldsymbol{P(X \geq 2)}$ and $\boldsymbol{P(X > 8)}$.

31. Let the random variable $\boldsymbol{X}$ have the ***pmf*** $\boldsymbol{f(x) = \frac{(|x|-1)^2}{21}; x = -4, -2, 0, 2, 4}$. Compute the mean, variance, $\boldsymbol{E(X^2 - 3X + 4)}$ and $\boldsymbol{V(1 - 2X)}$.

32. Verify that $\boldsymbol{M(t) = (0.4 + 0.6e^t)^{15}}$ is a ***mgf*** of a binomial distribution and find the ***pmf*** of it. Evaluate the mean and variance of the binomial distribution?

33. Consider the ***mgf*** $\boldsymbol{M(t) = \frac{0.3e^t}{1-0.7e^t}}$ of random variable $\boldsymbol{X}$. How is $\boldsymbol{X}$ distributed? Find the mean and variance of $\boldsymbol{X}$.

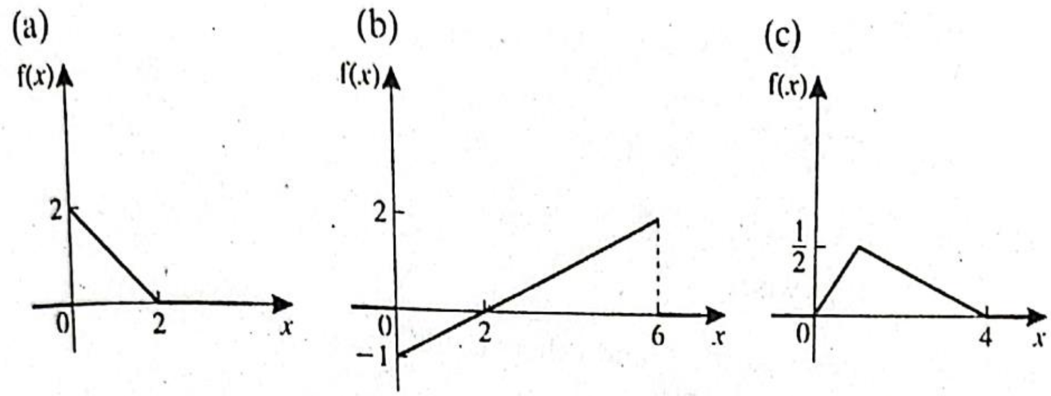
Sample Questions for Final Exam

34. Identify which of the following could represent a probability density function (pdf). If it could not be probability density function state why, and if it could then give the value of $\boldsymbol{k}$.

(a) $f(x) = \begin{cases} kx(x-2); & 1 \le x \le 3 \\ 0; & \text{otherwise} \end{cases}$

(b) $f(x) = \begin{cases} kx(4-x); & 2 \le x \le 4 \\ 0; & \text{otherwise} \end{cases}$

35. The following functions $f(x)$ are proposed as probability density functions. In each case state whether or not they could provide a suitable probability density function.



36. The time for which Lucy has to wait at a certain traffic light each day is $\boldsymbol{T}$ minutes, where $\boldsymbol{T}$ has probability density function (PDF) given by

$$f(t) = \begin{cases} k\left(t - \frac{t^2}{2}\right) & 0 \le t \le 2 \\ 0 & otherwise \end{cases}$$, where $\boldsymbol{k}$ is a constant.

(a) Find the value of $\boldsymbol{k}$
(b) Find the expected time that Lucy has to wait and $\mathrm{Var}(T)$.
(c) Find the probability that Lucy has to wait less than **0.5** minutes.
(d) Find the mode time.
(e) Construct the CDF for $\boldsymbol{T}$
(f) Find the median time that Lucy has to wait.

37. A continuous random variable $\boldsymbol{X}$ has cumulative distribution function

$$F(x) = \begin{cases} 0; & x < -2 \\ \dfrac{x+2}{6}; & -2 \le x \le 4 \\ 1; & x > 4 \end{cases}$$

(a) Find the probability density function $f(x)$ of $\boldsymbol{X}$.
(b) Write down the name of the distribution of $\boldsymbol{X}$.
(c) Find the mean and the variance of $\boldsymbol{X}$ and $P(X < 0)$.
(d) Write down the value of $P(X = 1)$.

38. Let $\boldsymbol{X}$ have the ***pdf*** $f(x) = 4x^3 e^{-x^4}; 0 \le x < \infty$. **Find** the ***cdf*** and hence **median** of $\boldsymbol{X}$. Also, find $P(X > -2)$.

39. Consider $f(x) = \frac{3x^2}{8}; 0 \le x \le 2$ as the ***pdf*** of $\boldsymbol{X}$. **Sketch** the graphs of ***pdf*** and ***cdf*** of $\boldsymbol{X}$.

40. Let $\boldsymbol{X}$ have the **pdf** $f(x) = e^{-x-1}; x > -1$. **Find** mean, median, variance, standard deviation and $P(X \ge 1)$.

Sample Questions for Final Exam

41. Let the ***mgf*** of the random variable X satisfies uniform distribution is $M(t) = \begin{cases} \frac{e^{4t}-1}{4t}; t \neq 0 \\ 1; \ t = 0 \end{cases}$. **Find** the ***pdf***, mean and variance of X. Also, find $P(X > 3.5)$.

42. The life X (in years) of a voltage regulator of a car has the **pdf** $f(x) = \frac{3x^2}{9^3} e^{-(x\backslash 9)^3}; x > 0$

(i) What is the probability that this regulator will last at most 7 years?

(ii) Find the 30^{th} percentile (in years) of voltage regulator.

43. Assume the ***pdf*** of X be $f(x) = 3e^{-3x}$; $0 \le x < \infty$. Then, **(i)** Estimate the ***cdf*** of X, **(ii)** Calculate the mean and variance of X, (iii) Find $P(X \ge 2)$.

44. For $(x) = 4x^c$; $0 \le x \le 1$, find the constant c so that $f(x)$ is a ***pdf*** of a random variable X. Find μ, σ^2 and ***cdf*** of X. Also, ***sketch*** the graph of ***pdf*** and ***cdf***.

45. Let the random variable X have the ***pdf*** $f(x) = 2e^{1-2x}$; $x \ge \frac{1}{2}$, find the ***cdf*** and hence the 3^{rd} decile of the distribution.

46. Telephone calls arrive at a physician's office according to the Poisson process on average 2 every 5 minute. Let X denote the waiting time in minutes until 3 calls arrive. Find the ***pdf*** and compute $P(X > 2)$.

47. Customers arrive at a travel agency at a mean rate of 48 per day. Assuming that the number of arrivals per hour has a Poisson process, find the probability that more than 3 customers arrive in a given hour.

48. If X has a gamma distribution with $\theta = 5$ and $\alpha = 2$. Find $P(X \ge 6)$. What are the mean and variance of the gamma distribution?

49. If the ***mgf*** of a Gamma distribution of a random variable X is $M(t) = (1 - 5t)^{-3}$, find the ***pdf***, mean and variance of X. Also, find $P(X > 4)$.

50. Telephone calls arrive at a department according to a Poisson process on the average of three every ***four*** minutes. Let X denote the waiting time until the ***first*** call arrives at a certain time duration. What is the ***pdf*** of X? Find $P(X \ge 3)$. Also, find the ***mgf*** of X.

51. Consider the ***pdf*** of a Normal variable X is defined as $f(x) = \frac{1}{\sqrt{98\pi}} e^{-\frac{(x+11)^2}{98}}$

(i) Evaluate $P(-3 > -X > 13)$ and $P(-5 < X < 2)$.

(ii) Find $P(|X| \ge 1.7)$.

(iii) Find the value of C such that $P(|X + 11| \ge C) = 0.0614$.

(iv) Find the value of C such that $P(X \ge C) = 0.9949$ and, find $P(X > 9)$.

52. The weights of bags of red gravel may be modeled by a normal distribution with mean 25.8 Kg and standard deviation 0.5 Kg.

(a) Determine the probability that a randomly selected bag of red gravel will weigh:

(i) less than 25 Kg.

(ii) between 25.5 Kg and 26.5 Kg.

(b) Determine, to two decimal places, exceeded by 75% of bags.

Sample Questions for Final Exam

53. **(a)** The time, X minutes, taken by Fred Fast to install a satellite dish may be assumed to be a normal random variable with mean 134 and standard deviation 16.

(i) Determine $P(X < 150)$.

(ii) Determine, to one decimal place, the time exceeded by 10 percent of installations.

(b) The time, Y minutes, taken by Sid Slow to install a satellite dish may be assumed to be a normal random variable, but with

$$P(Y < 170) = 0.14 \text{ and } P(Y > 200) = 0.03$$

Determine, to the nearest minute, values for the mean and standard deviation of Y.

54. The volume, L liters, of emulsion paint in a plastic tub may be assumed to be normally distributed with mean 10.25 and variance σ^2.

(a) Assuming that $\sigma^2 = 0.04$, determine $P(L < 10)$.

(b) Find the value of σ so that 98% of tubs contain more than 10 liters of emulsion paint.

(c) Find the probability that within 7% of mean liter

55. The lengths of pine needles, in cm, are Normally distributed. It is further given that 11.51% of these pine needles are shorter than 6.2 cm and 3.59% are longer than 9.5 cm. Find the mean and the standard deviation of the length of these pine needles.

56. The volume of shower gel bottles, V ml, is Normally distributed with a mean of 250 and a variance of 10.

(a) Find the probability that the volume of one of these shower gel bottles picked at random will be between 249 ml and 254 ml.

(b) Determine the value of V exceeded by 1% of the shower gel bottles.

Three shower gel bottles are picked at random.

(c) Find the probability that the volume of only one of these three bottles will be between 249 ml and 254 ml

57. The lifetimes, in hours, of a certain make of light bulbs are assumed to be Normally distributed with a mean of $\mathbf{5500}$ hours and a standard deviation of $\mathbf{120}$.

(i) **Find** the probability that the lifetime of a light bulb picked at random will exceed $\mathbf{5764}$ hours.

(ii) **Determine** the lifetime **not** achieved by $\mathbf{0.4\%}$ of these light bulbs.

(iii) **Find** the probability that ***two*** out of these ***thirty*** light bulbs will have a lifetime exceeding $\mathbf{5764}$ hours.

58. The weights of marmalade jars are Normally distributed with a mean of 250 grams.

(a) Calculate, correct to 1 decimal place, the standard deviation of these jars if 1% of the jars are heavier than 256 grams.

(b) Using the answer of part **(a)**, determine the probability that the weight of one such marmalade jar is between 249 and 253 grams.

(c) Given that the weight of a randomly picked marmalade jar is between 249 and 253 grams, find the probability that the jar weighs more than 250 grams.

59. If the ***mgf*** of the normal variable $\boldsymbol{X}$ is $\boldsymbol{M(t) = e^{30t+18t^2}}$, then **(i)** Find a constant $\boldsymbol{k}$ such that $\boldsymbol{P(|Z| \le k) = 0.9544}$ and **(ii)** Evaluate $\boldsymbol{P(42.6 \le X \le 55.8)}$.

Sample Questions for Final Exam

60. Let $\boldsymbol{X}$ equal to the amount of fruits in **kg per day** consumed by a student. Suppose the standard deviation of $\boldsymbol{X}$ is $\mathbf{0.1}$ **kg**. To estimate the mean $\boldsymbol{\mu}$ of $\boldsymbol{X}$, an agency took a random sample of $\mathbf{25}$ students and found they consumed $\mathbf{10}$ **kg** of fruits per day. Find an approximate $\mathbf{95}\%$ confidence interval for $\boldsymbol{\mu}$.

61. In a forest there are $\mathbf{1300}$ animal under severe virus infection, $\mathbf{77}\%$ of the animals are rescued from the forest. If half of the total animals survived after the attempt, find the confidence interval of the proportion with $\mathbf{5}\%$ significance level. Is the rescue process effective? Why?

62. A company produces electric bulbs whose average life time is $\mathbf{180}$ days and average variation $\mathbf{10}$ days. It is claimed that, in a newly developed process the mean life time can be increased.

(a) Design a decision rule for the process at the $\mathbf{0.05}$ significance to test $\mathbf{100}$ bulbs.
(b) What about the decision if the average life time of a bulb **(i)** $\mathbf{184}$ days **(ii)** $\mathbf{187}$ days?
(c) If the new process has increased the mean life time to $\mathbf{185}$ days. Find $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ for the estimated mean $\mathbf{183}$ days for $\mathbf{80}$ samples.
(d) If the estimated average life time for $\mathbf{55}$ samples is $\mathbf{184}$ days, find the $\boldsymbol{p}$-value of the claim of the manufacturer.

63. Design a decision rule to test the hypothesis that a die is fair if we take a sample of $\mathbf{150}$ trials for the die to get even/odd faces and use $\mathbf{0.01}$ as the significance level. **Predict** the acceptance and critical region.

64. Design a decision rule to test the hypothesis that a coin is fair if we take a sample of $\mathbf{120}$ trials of the die to get head/tail and use $\mathbf{0.1}$ as the significance level. **Predict** the acceptance and critical region.

65. A company produces an electric tool whose average life time is $\mathbf{260}$ days and variance $\mathbf{169}$ days. It is claimed that, in a newly developed process the mean life time can be increased. If the new process has increased the mean life time to $\mathbf{276}$ days, assuming a sample of $\mathbf{80}$ bulbs with estimated life time $\mathbf{269}$ days, find $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$.

66. A pharmaceutical company produces a new medicine and they claimed that it will reduce the migraine pain very fast with $\mathbf{85}\%$ accuracy. Design a decision rule for the process with the significance $\mathbf{0.01}$ by apply the medicine to $\mathbf{150}$ people.

67. ***Practice Problems of Hypothesis Testing Sheet***