Confidence Intervals: For proportions:

n = sample size

Y = num of samples in favor of an event

PDF

5 problems must do

α = level of significance

$1-\alpha$ = level of confidence

p = required proportion of the population

a) Find $z_{\alpha/2}$: ⇒

$$P\left(-z_{\alpha/2} \leq Z \leq z_{\alpha/2}\right) = 1-\alpha$$

$$\Rightarrow 2\phi\left(z_{\alpha/2}\right) - 1 = 1-\alpha$$

$$\therefore \phi\left(z_{\alpha/2}\right) = \frac{2-\alpha}{2}$$

b) Required Confidence Interval: ⇒

C.I. Proportion]

$$\frac{Y}{n} - z_{\alpha/2}\sqrt{\frac{\frac{Y}{n}\left(1-\frac{Y}{n}\right)}{n}} \leq p \leq \frac{Y}{n} + z_{\alpha/2}\sqrt{\frac{\frac{Y}{n}\left(1-\frac{Y}{n}\right)}{n}}$$

C.I. for mean(μ)]

$$\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

average

Problem (4)

In a forest, there are 200 birds under severe trouble of habitats, 75% of the birds are rescued from the forest. If 80% of the rescued birds survived after the attempt, find C.I. of proportion with an 85% confidence level.

$\nearrow \left(200 \times \frac{75}{100}\right)$

Soln:

$$n = 75\% \text{ of } 200 = 150$$

$$Y = 80\% \text{ of } 150 = 120$$

Prob^y of success in the samples,

$$\frac{Y}{n} = 0.8$$

$$\text{Confidence level} = 1 - \alpha = 0.85$$

$$\text{significance level} = 0.15 \quad \left\{ (1 - 0.85) \right.$$

$$\phi\left(Z_{\alpha/2}\right) = \frac{2-\alpha}{2}$$

$$= \frac{2 - 0.15}{2}$$

$$= 0.925$$

$$\therefore Z_{\alpha/2} = 1.44$$

So, Required C.I. is

$$\frac{Y}{n} - Z_{\alpha/2}\sqrt{\frac{\frac{Y}{n}\left(1-\frac{Y}{n}\right)}{n}} \le P \le \frac{Y}{n} + Z_{\alpha/2}\sqrt{\frac{\frac{Y}{n}\left(1-\frac{Y}{n}\right)}{n}}$$

$$\Rightarrow 0.8 - 1.44\sqrt{\frac{0.8(1-0.8)}{150}} \le P \le 0.8 + 1.44 \times \sqrt{\frac{0.8(1-0.8)}{150}}$$

$$\Rightarrow 0.8 - 0.047 \le P \le 0.8 + 0.047$$

$$\therefore \boxed{0.753 \le P \le 0.847}$$

[Ans.]

CamScanner

INTERVAL ESTIMATION

2. Let X equal to the amount of juice in milliliter per day consumed by a student. Suppose the variance of X is 36. To estimate the mean μ of X, a survey team took a random sample of 50 students and found they consumed on average 0.5 litter juice per day. Find an approximate 90% confidence interval for μ.

Solution:

Here, Sample size $n = 50$

Mean consumption $\bar{x} = 0.5$ litter or $\bar{x} = 500$ milliliter

Standard deviation $\sigma = 6$

Confidence $1 - \alpha = 0.90$

Significance $\alpha = 0.10$

Estimate the $z\alpha/_2$ as

$$\varphi\left(z\alpha/_2\right) = \frac{2-\alpha}{2}$$

$$or, \quad \varphi\left(z\alpha/_2\right) = \frac{2-0.10}{2}$$

$$or, \quad \varphi\left(z\alpha/_2\right) = 0.95$$

$$or, \quad z\alpha/_2 = 1.645$$

Now, the required confidence interval is

$$\bar{x} - z\alpha/_2 \, {}^{\sigma}/_{\sqrt{n}} \le \mu \le \bar{x} + z\alpha/_2 \, {}^{\sigma}/_{\sqrt{n}}$$

$$or, \quad 500 - 1.645 * \frac{6}{\sqrt{50}} \le \mu \le 500 + 1.645 * \frac{6}{\sqrt{50}}$$

$$or, \quad 500 - 1.396 \le \mu \le 500 + 1.396$$

$$or, \quad 498.604 \le \mu \le 501.396$$