Quartiles, Deciles & Percentiles for Ungrouped Data

Arrange the data in ascending order, then

1. **Quartiles:** $\boldsymbol{Q_i = \left(\frac{i.(n+1)}{4}\right)^{th}}$ value of the observation, where, $\boldsymbol{i = 1, 2, 3}$.
2. **Deciles:** $\boldsymbol{D_i = \left(\frac{i.(n+1)}{10}\right)^{th}}$ value of the observation, where, $\boldsymbol{i = 1, 2, \dots, 9}$.
3. **Percentiles:** $\boldsymbol{P_i = \left(\frac{i.(n+1)}{100}\right)^{th}}$ value of the observation, where, $\boldsymbol{i = 1, 2, \dots, 99}$.

Examples:

a) Calculate Quartile-1, Deciles-3, Percentiles-20 from the following data: 3,13,11,11,5,4,2.

Solution: Arranging Observations in the ascending order, We get: 2,3,4,5,11,11,13.

Here, $\boldsymbol{n = 7}$ (Odd)

$\boldsymbol{Q_1 = \left(\frac{1.(7+1)}{4}\right)^{th}}$ value of the observation
$= \boldsymbol{\left(\frac{8}{4}\right)^{th}}$ value of the observation $= \boldsymbol{2^{nd}}$ value of the observation $= \boldsymbol{3}$

$\boldsymbol{D_3 = \left(\frac{3.(7+1)}{10}\right)^{th}}$ value of the observation
$= \boldsymbol{\left(\frac{24}{10}\right)^{th}}$ value of the observation $= \boldsymbol{2.4^{th}}$ value of the observation
$= \boldsymbol{2^{nd}}$ observation + $\boldsymbol{0.4 \times (3^{rd} - 2^{nd})}$
$\boldsymbol{= 3 + 0.4 \times (4 - 3) = 3 + 0.4 \times 1 = 3.4}$

$\boldsymbol{P_{20} = \left(\frac{20.(7+1)}{100}\right)^{th}}$ value of the observation
$= \boldsymbol{\left(\frac{160}{100}\right)^{th}}$ value of the observation $= \boldsymbol{1.6^{th}}$ value of the observation
$= \boldsymbol{1^{st}}$ observation + $\boldsymbol{0.6 \times (2^{nd} - 1^{st})}$
$\boldsymbol{= 2 + 0.6 \times (3 - 2) = 2 + 0.6 \times 1 = 2.6}$

b) Calculate Quartile-2, Deciles-6, Percentiles-45 from the following data: 85,96,76,108,85,80,100,85,70,95.

Solution: Arranging Observations in the ascending order, We get: 70,76,80,85,85,85,95,96,100,108

Here, $\boldsymbol{n = 10}$ (Even)

$\boldsymbol{Q_2} = \left(\frac{\mathbf{2.(10+1)}}{\mathbf{4}}\right)^{\boldsymbol{th}}$ value of the observation
$= \left(\frac{\mathbf{22}}{\mathbf{4}}\right)^{\boldsymbol{th}}$ value of the observation $= \mathbf{5.5}^{\boldsymbol{th}}$ value of the observation
$= \mathbf{5}^{\boldsymbol{th}}$ observation + $\mathbf{0.5} \times (\mathbf{6}^{\boldsymbol{th}} - \mathbf{5}^{\boldsymbol{th}})$
$= \mathbf{85 + 0.5} \times (\mathbf{85 - 85}) = \mathbf{85 + 0.5} \times \mathbf{0 = 85}$

$\boldsymbol{D_6} = \left(\frac{\mathbf{6.(10+1)}}{\mathbf{10}}\right)^{\boldsymbol{th}}$ value of the observation
$= \left(\frac{\mathbf{66}}{\mathbf{10}}\right)^{\boldsymbol{th}}$ value of the observation $= \mathbf{6.6}^{\boldsymbol{th}}$ value of the observation
$= \mathbf{6}^{\boldsymbol{th}}$ observation + $\mathbf{0.6} \times (\mathbf{7}^{\boldsymbol{th}} - \mathbf{6}^{\boldsymbol{th}})$
$= \mathbf{85 + 0.6} \times (\mathbf{95 - 85}) = \mathbf{85 + 0.6} \times \mathbf{10 = 91}$

$\boldsymbol{P_{45}} = \left(\frac{\mathbf{45.(10+1)}}{\mathbf{100}}\right)^{\boldsymbol{th}}$ value of the observation
$= \left(\frac{\mathbf{495}}{\mathbf{100}}\right)^{\boldsymbol{th}}$ value of the observation $= \mathbf{4.95}^{\boldsymbol{th}}$ value of the observation
$= \mathbf{4}^{\boldsymbol{th}}$ observation + $\mathbf{0.95} \times (\mathbf{5}^{\boldsymbol{th}} - \mathbf{4}^{\boldsymbol{th}})$
$= \mathbf{85 + 0.95} \times (\mathbf{85 - 85}) = \mathbf{85 + 0.95} \times \mathbf{0 = 85}$

Quartiles, Deciles & Percentiles for Grouped Data

$First\ Quartile\ Q_1 = P_{25}$ $\qquad$ $First\ Decile\ D_1 = P_{10}$

$Second\ Quartile\ Q_2 = P_{50}$ $\qquad$ $Second\ Decile\ D_2 = P_{20}$

$Third\ Quartile\ Q_3 = P_{75}$ $\qquad$ $Fifth\ Decile\ D_5 = P_{50}$ *and so on*

$$Second\ Quartile = Fifth\ Decile = 50th\ Percentile = Median$$

$$Q_2 = D_5 = P_{50} = Median$$

Time taken (min)	8 -- 10	11 -- 13	14 -- 16	17 -- 19	20 -- 22	23 -- 25
Frequencies	2	4	6	4	3	1

x	f	*Class Boundaries*	$c.f$
8 – 10	2	7.5 – 10.5	2
11 – 13	4	10.5 – 13.5	6
14 – 16	6	13.5 – 16.5	12
17 – 19	4	16.5 – 19.5	16
20 – 22	3	19.5 – 22.5	19
23 – 25	1	22.5 – 25.5	20
	20		

$$Qi = l + \frac{h}{f}\left(\frac{iN}{4} - c\right);\ i = 1,2,3$$

Where:

l = lower boundary of Quartile group

h = Width of Quartile group

f = Frequency of Quartile group

N = Total number of observations i.e. sum of the frequencies

c = Cumulative frequency preceding Quartile group

$$Q_i = \frac{i(N)}{4}\ th\ value$$

$$Q_2 = \frac{2(20)}{4}\ th\ value$$

$$Q_2 = 10\ th\ value$$

Since 10th value is in the interval (13.5 – 16.5)

Therefore Group of Q_2 is (13.5 – 16.5)

$$Qi = l + \frac{h}{f}\left(\frac{iN}{4} - c\right)$$

$$Q_2 = 13.5 + \frac{3}{6}\left(\frac{2(20)}{4} - 6\right)$$

$$Q_2 = 13.5 + 2$$

$$Q_2 = 15.50$$

$$Di = l + \frac{h}{f}\left(\frac{iN}{10} - c\right);\ i = 1,2,3\ldots,9$$

Where:

l = lower boundary of Deciles group

h = Width of Deciles group

f = Frequency of Deciles group

N = Total number of observations i.e. sum of the frequencies

c = Cumulative frequency preceding Deciles group

$$D_i = \frac{i(N)}{10}\ th\ value$$

$$D_5 = \frac{5(20)}{10}\ th\ value$$

$$D_5 = 10^{th}\ value$$

Since 10th value is in the interval (13.5 – 16.5)

Therefore Group of D_2 is (13.5 – 16.5)

$$Di = l + \frac{h}{f}\left(\frac{iN}{10} - c\right)$$

$$D_5 = 13.5 + \frac{3}{6}\left(\frac{5(20)}{10} - 6\right)$$

$$D_5 = 13.5 + 2$$

$$D_5 = 15.5$$

$$Pi = l + \frac{h}{f}\left(\frac{iN}{100} - c\right);\ i = 1, 2, 3 \ldots, 99$$

Where:

l = lower boundary of Percentile group

h = Width of Percentile group

f = Frequency of Percentile group

N = Total number of observations i.e. sum of the frequencies

c = Cumulative frequency preceding Percentile group

$$P_i = \frac{i(N)}{100}\ th\ value$$

$$P_{50} = \frac{50(20)}{100}\ th\ value$$

$$P_{50} = \frac{1000}{100}\ th\ value$$

$$P_{50} = 10\ th\ value$$

Since 10th value is in the interval (13.5 – 16.5)

Therefore Group of P_{50} is (13.5 – 16.5)

$$Pi = l + \frac{h}{f}\left(\frac{iN}{100} - c\right)$$

$$P_{50} = 13.5 + \frac{3}{6}\left(\frac{50(20)}{100} - 6\right)$$

$$P_{50} = 13.5 + 2$$

$$P_{50} = 15.50$$

$$Median = l + \frac{h}{f}\left(\frac{N}{2} - c\right)$$

Where:

l = lower class boundary of the median class

h = Size of the median class interval

f = Frequency corresponding to the median class

N = Total number of observations i.e. sum of the frequencies

c = Cumulative frequency preceding median class.

$$Median = \ Size\ of\ \frac{N}{2}th\ value$$

$$Median = \ Size\ of\ \frac{20}{2} = 10th\ value$$

$10th\ value\ lies\ in\ the\ interval\ 13.5 - 16.5$

$Therefore\ 13.5 - 16.5\ is\ called\ median\ class$

$$Median = l + \frac{h}{f}\left(\frac{N}{2} - c\right)$$

$$Median = 13.5 + \frac{3}{6}\left(\frac{20}{2} - 6\right)$$

$$Median = 13.5 + 2$$

$$Median = 15.50$$