Stat-205-Final

1.(a)

$$M(t) = \begin{cases} \frac{e^{2t}-1}{2t} & ; t \neq 0 \\ 1 & ; t = 0 \end{cases}$$

$$= \begin{cases} \frac{e^{2t}-e^{0}}{t(2-0)} & ; t \neq 0 \\ 1 & ; t = 0 \end{cases}$$

So, $a=0$, $b=2$

1+1+1+1

$$f(x) = \frac{1}{2-0} = \frac{1}{2} \quad ; \quad 0 \leq x \leq 2$$

$$F(x) = \begin{cases} 0 & ; x < 0 \\ \frac{x}{2} & ; 0 \leq x < 2 \\ 1 & ; x \geq 2 \end{cases}$$

$$\mu = \frac{0+2}{2} = 1$$

$$\sigma^2 = \frac{(2-0)^2}{12} = \frac{1}{3}$$

(b)

$$M(t) = (1-7t)^{-2} = \frac{1}{(1-7t)^2}$$

$\therefore \quad \theta = 7, \ \alpha = 2$

$$f(x) = \frac{1}{\Gamma 2 \ 7^2} x^{2-1} e^{-x/7}$$

$$= \frac{1}{49} x e^{-x/7} \quad ; \quad 0 \leq x < \infty$$

$\mu = 2\times 7 = 14$

$\sigma^2 = 2\times 7^2 = 98$

$$P(X>4) = \int_4^\infty \frac{1}{49} x e^{-x/7}\,dx$$

1+1+1+2

$$= \frac{1}{49}\left[-7xe^{-x/7} - 49e^{-x/7}\right]_4^\infty$$

$x \xrightarrow{(+)} e^{-x/7}$

$1 \xrightarrow{(-)} -7e^{-x/7}$

$0 \quad 49e^{-x/7}$

$$= \frac{1}{49}\left[-0-0+28e^{-4/7}+49e^{-4/7}\right]$$

$$= \frac{77}{49}e^{-4/7} = 0.8874$$

(c)

$P(0 \le Z \le Z_0) = 0.3621$

$\Rightarrow \Phi(z_0) - \Phi(0) = 0.3621$

4

$\Rightarrow \Phi(z_0) - 0.5 = 0.3621$

$\Rightarrow \Phi(z_0) = 0.8621$

$\Rightarrow z_0 = 1.09$

2.

$$f(x) = \frac{1}{3} e^{-x/3} \; ; \; 0<x<\infty$$

$$F(x) = 1-e^{-x/3} \; ; \; 0<x<\infty$$

$$f(y) = \frac{1}{3} e^{-y/3} \; ; \; 0<y<\infty$$

$$F(y) = 1-e^{-y/3} \; ; \; 0<y<\infty$$

1+2+1+1.5+1.5

$$G_4(y) = \sum_{k=4}^{5} {}^5C_k \, (1-e^{-y/3})^k (e^{-y/3})^{5-k}$$

$$= 5(1-e^{-y/3})^4 (e^{-y/3}) + (1-e^{-y/3})^5$$

$$P(Y_4<5) = G_4(5) = 5(1-e^{-5/3})^4 (e^{-5/3}) + (1-e^{-5/3})^5$$

$$= 0.7599$$

$$g_3(y) = \frac{5!}{2!\,2!} (1-e^{-y/3})^2 (e^{-y/3})^2 \left(\frac{1}{3} e^{-y/3}\right)$$

$$= 10(1-e^{-y/3})^2 e^{-y}$$

$$\mu_3(y) = \int_0^\infty y \left[10(1-e^{-y/3})^2 e^{-y}\right] dy$$

$$= 10\int_0^\infty \left(ye^{-y} - 2ye^{-\frac{4}{3}y} + ye^{-\frac{5}{3}y}\right) dy$$

$$= 10\left[-ye^{-y} - e^{-y} + \frac{3}{2} y e^{-\frac{4}{3}y} + \frac{9}{8} e^{-\frac{4}{3}y} - \frac{3}{5} y e^{-\frac{5}{3}y} - \frac{9}{25} e^{-\frac{5}{3}y}\right]_0^\infty$$

$$= 10\left[1 - \frac{9}{8} + \frac{9}{25}\right]$$

$$= \frac{47}{20} = 2.35$$

3. Here, $n = \frac{2}{3} \times 300 = 200$

$r = 0.75 \times 200 = 150$

$\Rightarrow \frac{r}{n} = 0.75$

$1-\alpha = 0.9$

$\Rightarrow \alpha = 0.1$

$\phi(z_{\alpha/2}) = \frac{2-\alpha}{2} = \frac{2-0.1}{2} = 0.95$

$\Rightarrow z_{\alpha/2} = 1.645$

4

CI:

$0.75 \pm 1.645\sqrt{\frac{0.75 \times 0.25}{200}}$

$= 0.75 \pm 0.0504$

So the CI is $[0.6996, 0.8004]$

or, $0.6996 \leq p \leq 0.8004$

4. Here, $H_0: p = 0.90$

$H_1: p < 0.90$

Given by $n = 100$

$\alpha = 0.1$

$p = 0.9$

$q = 0.1$

For a new cooker the favor will go for the manufacturer. So, the problem is of left tailed.

Now, $P(Z > -z_\alpha) = 1 - 0.1$

$\Rightarrow P(Z < z_\alpha) = 0.9$

$\Rightarrow \phi(z_\alpha) = 0.9$

$\Rightarrow z_\alpha = 1.28$

$\mu = 100 \times 0.9 = 90$

$\sigma = \sqrt{100 \times 0.9 \times 0.1} = 3$

H_0 will be accepted for

$Z > -z_\alpha$

$\Rightarrow \frac{x-\mu}{\sigma} > -z_\alpha$

$\Rightarrow x > \mu - z_\alpha \sigma$

$\Rightarrow x > 90 - 1.28 \times 3$

$\Rightarrow x > 86.16$

So, if at least 87 cooker work well, we will accept the claim of the manufacturer.

5.

Class	61–65	66–70	71–75	76–80	81–85
Original class	60.5–65.5	65.5–70.5	70.5–75.5	75.5–80.5	80.5–85.5
Frequency	8	15	19	9	5
Cumulative frequency	8	23	42	51	56

2+2+1.5×4

Draw the cumulative frequency polygon and histogram. Then from the graph we have

$D_3 = 68.43$

$Q_3 = 75.50$

$P_{95} = 82.70$

$M_0 = 71.93$

Graphs are mandatory.