UNIT 07

Probability

MATH -2205

Measures of Location /Central Tendency

Probability is a measure of the likelihood or chance that a particular event will occur. It is usually expressed as a number between 0 and 1, where 0 indicates an impossible event and 1 indicates a certain event. Mathematically, the probability of an event A is defined as:

$$P(A) = \frac{\text{Number of favorable outcomes for event } A}{\text{Total number of possible outcomes}}$$

In this context, "favorable outcomes" are the outcomes where the event happens, and "possible outcomes" are all outcomes that could happen in a given scenario.

$$P(A') = 1 - P(A)$$

Random Experiment: A random experiment is a process or action that leads to one or more outcomes, where the outcome cannot be predicted with certainty ahead of time.

Example:

Rolling a six-sided die is a random experiment because we do not know which number will come up.

Sample Space (S): The sample space of a random experiment is the set of all possible outcomes of that experiment.

Addition Rule

If events are not mutually exclusive, (they can happen at the same time). The probability of either A or B occurring is the sum of their individual probabilities minus the probability of both A and B.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Mutually Exclusive Events:

Events are **mutually exclusive** if they cannot happen at the same time.

$$P(A \cap B) = 0. \qquad P(A \cup B) = P(A) + P(B)$$

Independent Events:

Two events *A* and *B* are independent if the occurrence of one does not affect the occurrence of the other. The probability of both *A* and *B* occurring is the product of their individual probabilities.

$$P(A \cap B) = P(A) \times P(B)$$

Dependent Events

Dependent Events are events where the occurrence of one event affects the probability of the occurrence of another event. In other words, the events are linked such that the outcome of one alters the probability of the other.

$$P(B|A) \neq P(B)$$

Conditional Probability

Conditional Probability is the probability of an event occurring given that another event has already occurred.

Conditional Probability Formula: $P(B|A) = \frac{P(A \cap B)}{P(A)}$.

Tree diagram

A tree diagram in probability is a visual representation that helps illustrate all possible outcomes of a sequence of events, along with their probabilities. It is a useful tool for organizing and calculating probabilities in situations involving multiple stages or steps, where each stage depends on the outcome of the previous one.

Bayes' Theorem

Bayes' Theorem is a fundamental concept in probability theory and statistics that describes how to update the probability of a hypothesis or event based on new evidence. It provides a way to revise existing beliefs or probabilities in light of new data, making it a powerful tool for reasoning under uncertainty.

Bayes' Theorem is expressed mathematically as:

$$P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}$$