United International **University**

School of Science and Engineering

Mid Assessment Trimester: Fall-2020

Course Title: Probability and Statistics

Course Code: Stat 205/Math 2205 Marks: 20 Time: 1 Hour

<u>There are 3 questions, answer any 2 of them.</u>

1. **a)** From an ordinary deck of playing cards, cards are to be drawn successively at random and **without replacement**. Find the probability of $\mathbf{4^{th}}$ red appears on the $\mathbf{11^{th}}$ draw. **[3]**

b) Candidates come to a certification authority at a mean rate **60** per day under a **Poisson process**. Find the probability of **(i) more than 2**, **(ii) at most 3** candidates arrive in a given hour. What is the **standard deviation** of the distribution? **[4]**

c) Let $\boldsymbol{X}$ have the ***pdf*** $\boldsymbol{f(x) = 4x^3e^{-x^4}; 0 \leq x < \infty}$. Find the ***cdf*** and hence **median** of $\boldsymbol{X}$. Also, find $\boldsymbol{P(X > 2)}$. **[3]**

2. **a)** **Three** dice are rolled **independently** and observed for the **faces multiples of 3** on the top. Find the probability of **any two** of them get success. What is the probability of **none** of them getting success? **[3]**

b) In a super-shop, there are **9** sales-persons with **3** of them **trained**. Company is going to give them annual increment in an **independent** process, find the probability that **at most 2 trained** sales-persons get the increment. What is the **mean** and **variance** of the distribution? **[4]**

c) Let $\boldsymbol{X}$ have the ***pdf*** $\boldsymbol{f(x) = \frac{1}{20}; 25 \leq x \leq 45}$. Find the ***cdf*** of $\boldsymbol{X}$ and hence $\boldsymbol{P(X \geq 27)}$. What are the **mean** and **variance** of $\boldsymbol{X}$? **[3]**

3. **a)** Rapid testing is a screening procedure to test Covid-19. The people appearing in the test, $\mathbf{19\%}$ of them **false-positive** while $\mathbf{16\%}$ of them **false-negative**. If the Covid-19 spreads among $\mathbf{2\%}$ people in bangladesh, find the probability of a person **not suffering** from Covid-19, when he/she **tested positive** in the test. **[3]**

b) Let a random experiment be the casting of a pair of fair six-sided dice and let $\boldsymbol{X}$ equal the **maximum of two outcomes**. With reasonable assumptions, find ***pmf*** of $\boldsymbol{X}$. Also, find the ***mgf*** and **variance** of $\boldsymbol{X}$. **[4]**

c) Consider $\boldsymbol{f(x) = \frac{3x^2}{8}; 0 \leq x \leq 2}$ as the ***pdf*** of $\boldsymbol{X}$. Sketch the graphs of ***pdf*** and ***cdf*** of $\boldsymbol{X}$. **[3]**

Formulae:

Distribution	*Pmf / pdf*
Hypergeometric	$f(x) = \frac{N_{1_{C_x}} N_{2_{C_{n-x}}}}{N_{C_n}}$; $N = N_1 + N_2,\quad x = 1, 2, \ldots \ldots n$
Geometric	$f(x) = q^{x-1}p$; $x = 0, 1, 2, \ldots.$
Binomial	$f(x) = n_{C_x} p^x q^{n-x}$; $x = 0, 1, 2, \ldots. n$
Poisson	$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}$; $x = 0, 1, 2, \ldots..$
Uniform	$f(x) = \frac{1}{b-a}$; $a \leq x \leq b$
Exponential	$f(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}$; $0 \leq x < \infty$
Gamma	$f(x) = \frac{1}{\Gamma(\alpha)\, \theta^\alpha} x^{\alpha-1} e^{-x/\theta}$; $0 \leq x < \infty$
Normal	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$; $-\infty < x < \infty$