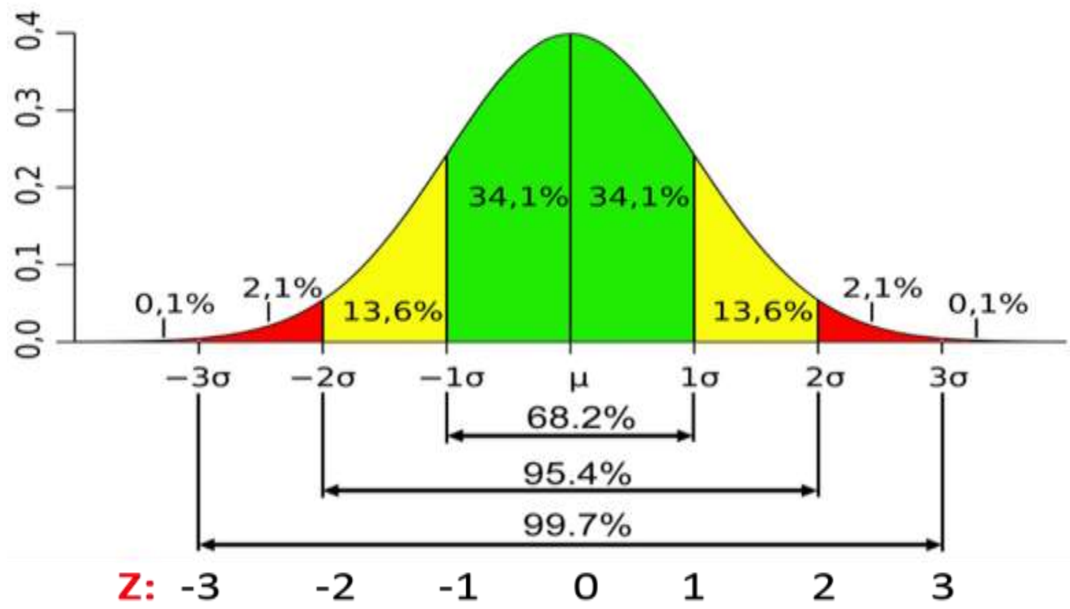
Normal Distribution

The normal distribution is a continuous probability distribution that is symmetrical around its mean, most of the observations cluster around the central peak, and the probabilities for values further away from the mean taper off equally in both directions.

In graphical form, the normal distribution appears as a "bell curve".



Properties of a normal distribution

- The mean, mode and median are all equal.
- The curve is symmetric at the center (i.e. around the mean, μ).
- Exactly half of the values are to the left of center and exactly half the values are to the right.
- The total area under the curve is 1.

To standardize your data, you need to convert the raw measurements into Z-scores.

To calculate the standard score for an observation, take the raw measurement, subtract the mean, and divide by the standard deviation. Mathematically, the formula for that process is the following:

The z-score formula that we have been using is: $Z = \frac{X - \mu}{\sigma}$

The probability distribution function for Normal Distribution is defined as

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \qquad -\infty < x < \infty$$

$\mu =$ Mean
$\sigma =$ Standard Deviation
$\pi \approx 3.14159\cdots$
$e \approx 2.71828\cdots$

$$P(a \leq x \leq b) = \int_a^b f(x)dx \geq 0$$

Exercise:

1.

The distance in metres that a ball can be thrown by pupils at a particular school follows a normal distribution with mean 35.0 m and standard deviation 11.6 m.

(i) Find the probability that a randomly chosen pupil can throw a ball between 30 and 40 m. [3]

(ii) The school gives a certificate to the 10% of pupils who throw further than a certain distance. Find the least distance that must be thrown to qualify for a certificate. [3]

2.

(i) In a normal distribution with mean μ and standard deviation σ, $P(X > 3.6) = 0.5$ and $P(X > 2.8) = 0.6554$. Write down the value of μ, and calculate the value of σ. [4]

(ii) If four observations are taken at random from this distribution, find the probability that at least two observations are greater than 2.8. [4]

3.

The waiting time in a doctor's surgery is normally distributed with mean 15 minutes and standard deviation 4.2 minutes.

(i) Find the probability that a patient has to wait less than 10 minutes to see the doctor. [3]

(ii) 10% of people wait longer than T minutes. Find T. [3]

(iii) In a given week, 200 people attend the surgery. Estimate the number of these who wait more than 20 minutes. [3]

4.

In a certain country the time taken for a common infection to clear up is normally distributed with mean μ days and standard deviation 2.6 days. 25% of these infections clear up in less than 7 days.

(i) Find the value of μ. [4]

In another country the standard deviation of the time taken for the infection to clear up is the same as in part **(i)**, but the mean is 6.5 days. The time taken is normally distributed.

(ii) Find the probability that, in a randomly chosen case from this country, the infection takes longer than 6.2 days to clear up. [3]

5.

The lengths, in cm, of the leaves of a particular type are modelled by the distribution $N(5.2,\ 1.5^2)$.

(a) Find the probability that a randomly chosen leaf of this type has length less than 6 cm. [2]

The lengths of the leaves of another type are also modelled by a normal distribution. A scientist measures the lengths of a random sample of 500 leaves of this type and finds that 46 are less than 3 cm long and 95 are more than 8 cm long.

(b) Find estimates for the mean and standard deviation of the lengths of leaves of this type. [5]

6.

The random variable X has the following probability distribution shown below.

x	1	2	3	4
P(X = x)	0.1	a	0.3	b

Given that E(x) = 3. Find

(a) The values of ***a*** and ***b***
(b) Var(x)
(c) Hence fine E(2x+3) and Var (2x+3)

7.

(a) Jamia has 75% chance to attend a training session before a football match. If he attends, he is certain to be chosen for the team which plays in the match. If he does not attend, there is a probability of 0.6 that he is certain for the team

(i) Find the probability that Jamia is chosen for the team

(ii) Find the probability that Jamia attended the training session, given that he was chosen for the team.

(b) A typist makes, on average, 1 error for every 200 keyboard strokes. Assuming the error occur independently and random, find the probability that

(i) in a document requiring 400 keyboard strokes there is no error

(ii) in a document requiring 1000 keyboard strokes there is, at most one error.

8.

Two sisters make contact using an internet messaging service. The length of time for which they are logged on, in minutes is meddled by the random variable T with the probability density function (PDF)given by

$$f(t) = \begin{cases} \frac{1}{k}(40 - t) & 10 \leq t \leq 30 \\ 0 & otherwise \end{cases}$$, where k is a constant.

(a) Show that $k = 400$
(b) Find the expected time that they logged in
(c) Find the probability that the time that they logged on for is less than 15 minutes.
(d) Find the median time.

9.

(a) The lifetime of the Powerhouse battery has a normal distribution with mean 210 hours. It is found that 4% of these batteries operate for more than 222 hours. Find the variance of the distribution, correct to 2 decimal places.

(b) If $X \sim B(n, p)$ and mean and variance of X are 6 and 3.6 respectively . Find

(i) the values of *n* and *p*.

(ii) $p(x \geq 2)$

10.

(a) In Europe the diameters of women's rings have mean 18.5 mm. Researchers claim that women in Jakarta have smaller fingers than women in Europe. The researchers took a random sample of 20 women in Jakarta and measured the diameters of their rings. The mean diameter was found to be 18.1 mm. Assuming that the diameters of women's rings in Jakarta have a normal distribution with standard deviation 1.1 mm, carry out a hypothesis test at the 5% level of significance to determine whether the researchers' claim is justified.

(b) Studies suggest that 10% of the world population is left- handed. Barlin suspect that being left-handed is less common amongst basketball players and plan to test this by asking a random sample 50 basketball players if they are left-handed. She found 1 player is left-handed. Use a 5% level of significance whether the claim is justified.

[4+4=8]

Distribution	***pmf /pdf***
Binomial	$\boldsymbol{f(x) = n_{C_x} p^x (1-p)^{n-x}\ ;\ x = 0, 1, 2, \ldots. n}$
Poisson	$\boldsymbol{f(x) = \frac{\lambda^x e^{-\lambda}}{x!}\ ;\ x = 0, 1, 2, \ldots..}$
Uniform	$\boldsymbol{f(x) = \frac{1}{b-a}\ ;\ a \leq x \leq b}$
Normal	$\boldsymbol{f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\ ;\ -\infty < x < \infty}$