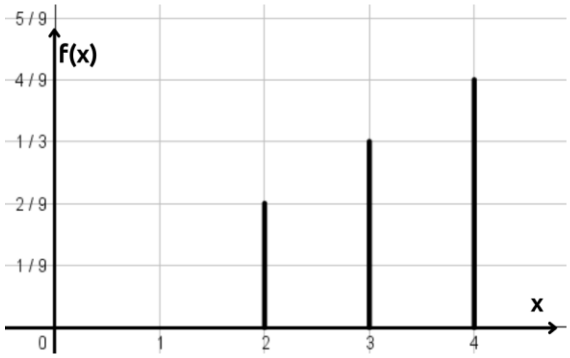
Chapter: 2.1

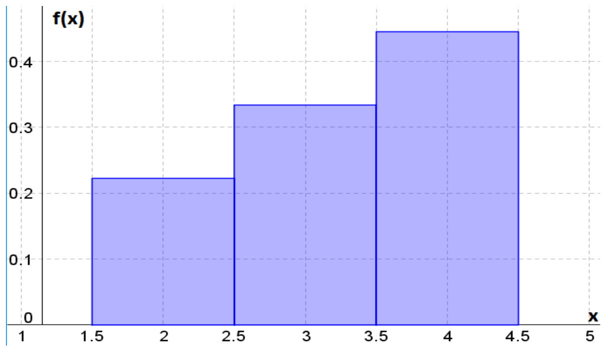
1. Here, $f(x) = \frac{x}{9}; \quad x = 2, 3, 4$

$P(X = 2) = \frac{2}{9}$

$P(X = 3) = \frac{3}{9}$

$P(X = 4) = \frac{4}{9}$





3. (a) $f(x) = \frac{x}{c}; \; x = 1, 2, 3, 4$

$\sum f(x) = 1$

$\Rightarrow \frac{1}{c} \times (1 + 2 + 3 + 4) = 1$

$\Rightarrow \frac{1}{c} \times \frac{4(4+1)}{2} = 1$

$\Rightarrow c = 10$

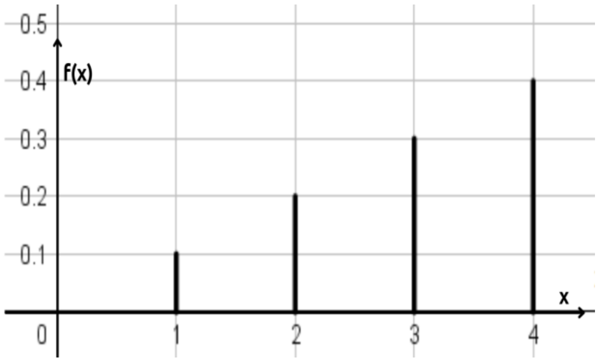
$f(x) = \frac{x}{10}$

$P(X = 1) = \frac{1}{10}$

$P(X = 2) = \frac{2}{10}$

$P(X = 3) = \frac{3}{10}$

$P(X = 4) = \frac{4}{10}$



(b) $f(x) = cx; \; x = 1, 2, 3, 4, \ldots\ldots., 10$

$\sum f(x) = 1$

$\Rightarrow c\,(1 + 2 + 3 + 4 + \ldots + 10) = 1$

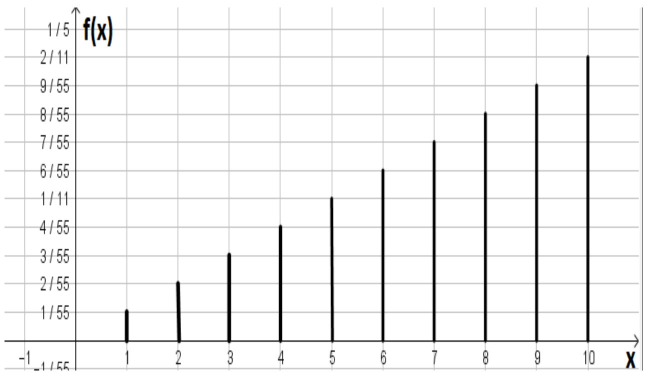
$\Rightarrow c \times \frac{10(10+1)}{2} = 1$

$\Rightarrow c = \frac{1}{55}$

$f(x) = \frac{x}{55}$

$P(X = 1) = \frac{1}{55}$

$P(X = 2) = \frac{2}{55}$



$P(X = 3) = \frac{3}{55}$

$P(X = 4) = \frac{4}{55}$

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$P(X = 10) = \frac{10}{55}$

(c) $f(x) = \frac{c}{4^x}; \quad x = 1, 2, 3, \ldots\ldots$

$\sum f(x) = 1$

$\Rightarrow c\left(\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \ldots\ldots\ldots\right) = 1$

$\Rightarrow \frac{c}{4} \times \left(1 + \frac{1}{4} + \frac{1}{4^2} + \ldots\ldots\right) = 1$

$\Rightarrow \frac{c}{4} \times \frac{1}{1 - \frac{1}{4}} = 1$

$\Rightarrow \frac{c}{4} \times \frac{1}{\frac{3}{4}} = 1$

$\Rightarrow c = 3$

$f(x) = \frac{3}{4^x}$

$P(X = 1) = \frac{3}{4}$

$P(X = 2) = \frac{3}{16}$

$P(X = 3) = \frac{3}{64}$

$P(X = 4) = \frac{3}{256}$

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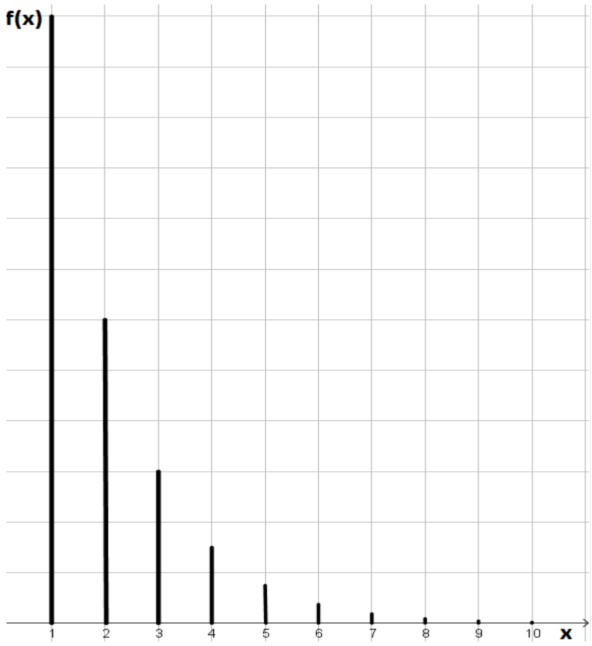


(d) $f(x) = c\,(x + 1)^2; \ x = 0, 1, 2, 3$

$\sum f(x) = 1$

$\Rightarrow c\,(1^2 + 2^2 + 3^2 + 4^2) = 1$

$\Rightarrow c \times \frac{4(4+1)(2\times4+1)}{6} = 1$

$\Rightarrow 30c = 1$

$\Rightarrow c = \frac{1}{30}$

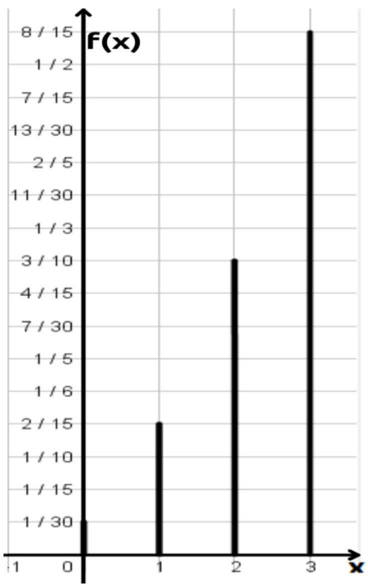
$f(x) = \frac{(x+1)^2}{30}$

$P(X = 0) = \frac{1}{30}$

$P(X = 1) = \frac{4}{30}$

$P(X = 2) = \frac{9}{30}$

$P(X = 3) = \frac{16}{30}$



(e) $f(x) = \frac{x}{c}$; $x = 1, 2, \ldots, n$

$$\sum f(x) = 1$$

$$\Rightarrow \frac{1}{c} \times (1 + 2 + 3 + \ldots\ldots + n) = 1$$

$$\Rightarrow \frac{1}{c} \times \frac{n(n+1)}{2} = 1$$

$$\Rightarrow c = \frac{n(n+1)}{2}$$

$$f(x) = \frac{2x}{n(n+1)}$$

$$P(X = 1) = \frac{2}{n(n+1)}$$

$$P(X = 2) = \frac{4}{n(n+1)}$$

$$P(X = 3) = \frac{6}{n(n+1)}$$

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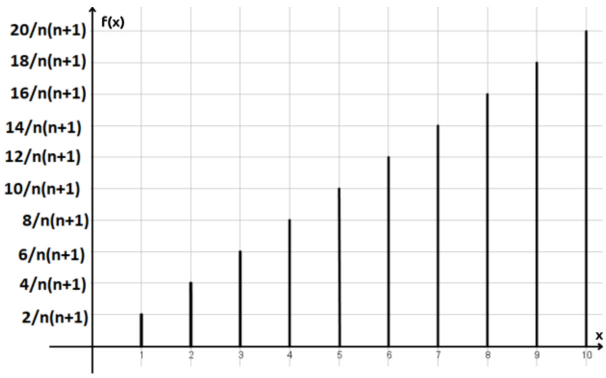
$$P(X = n) = \frac{2}{(n+1)}$$



(f) $f(x) = \frac{c}{(x+1)(x+2)} = c\left(\frac{1}{x+1} - \frac{1}{x+2}\right)$; $x = 0, 1, 2, 3, \ldots\ldots$

$$\sum f(x) = 1$$

$$\Rightarrow c\left(\sum \frac{1}{x+1} - \sum \frac{1}{x+2}\right) = 1$$

$$\Rightarrow c \times \left[\left(1 + \frac{1}{2} + \frac{1}{3} + \ldots.\right) - \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots.\right)\right] = 1$$

$$\Rightarrow c = 1$$

$$f(x) = \frac{1}{(x+1)(x+2)}$$

$$P(X = 0) = \frac{1}{2}$$

$$P(X = 1) = \frac{1}{6}$$

$$P(X = 2) = \frac{1}{12}$$

$$P(X = 3) = \frac{1}{20}$$

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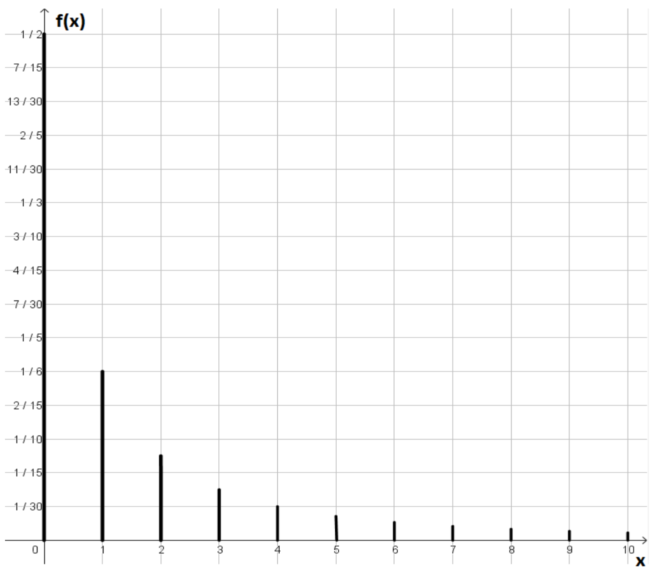


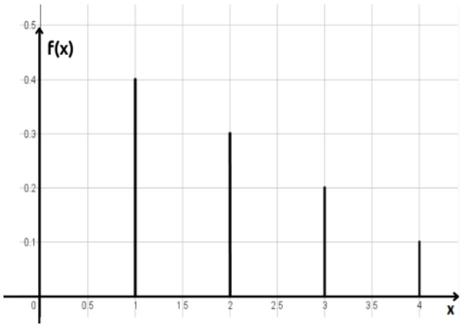
5. (a-c) Given, $f(x) = \frac{5-x}{10}$; $x = 1, 2, 3, 4$

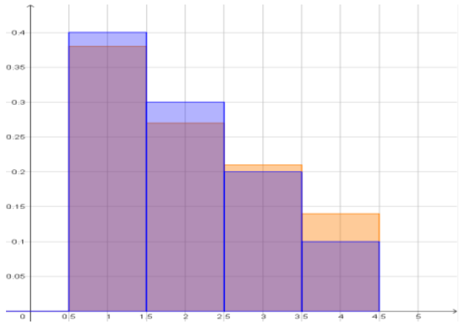
$$P(X = 1) = \frac{4}{10}$$

$$P(X = 2) = \frac{3}{10}$$

$P(X=3) = \frac{2}{10}$

$P(X=4) = \frac{1}{10}$





6. (a-b) From the given statement, we have

$P(X=2) = \frac{1}{36}$

$P(X=3) = \frac{2}{36}$

$P(X=4) = \frac{3}{36}$

$P(X=5) = \frac{4}{36}$

$P(X=6) = \frac{5}{36}$

$P(X=7) = \frac{6}{36}$

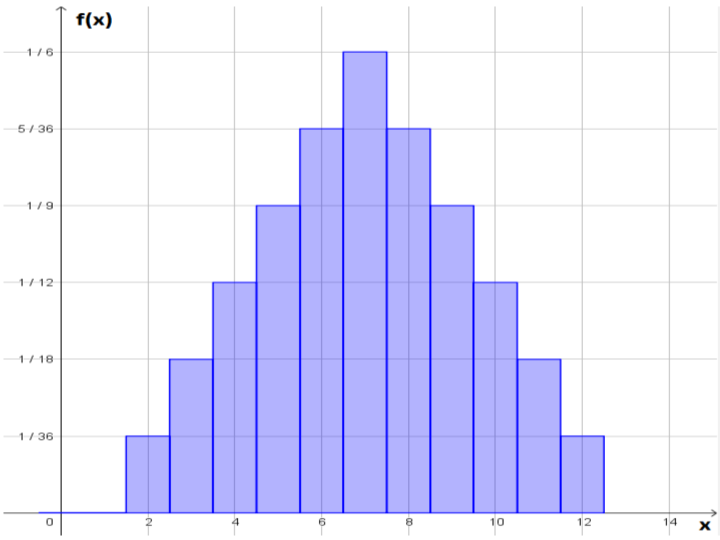
$P(X=8) = \frac{5}{36}$

$P(X=9) = \frac{4}{36}$

$P(X=10) = \frac{3}{36}$

$P(X=11) = \frac{2}{36}$

$P(X=12) = \frac{1}{36}$



Now, $f(x) = \frac{y_{max} - |x - x_{max}|}{m}$

$\Rightarrow f(x) = \frac{6 - |x-7|}{36}; \ x = 2, 3, \dots \dots \dots, 12$

7. (a-b). From the given statement, we have

$P(X=1) = \frac{11}{36}$

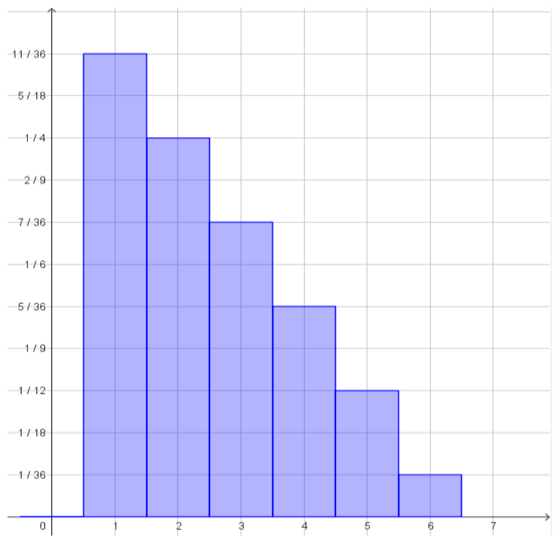
$P(X=2) = \frac{9}{36}$

$P(X=3) = \frac{7}{36}$

$P(X=4) = \frac{5}{36}$

$P(X=5) = \frac{3}{36}$

$P(X=6) = \frac{1}{36}$



Now, $\frac{f(x)-\frac{11}{36}}{\frac{1}{36}-\frac{11}{36}} = \frac{x-1}{6-1}$

$\Rightarrow \frac{36f(x)-11}{-10} = \frac{x-1}{5}$

$\Rightarrow f(x) = \frac{13-2x}{36}$

$\Rightarrow f(x) = \frac{13-2x}{36};\ x = 1, 2, \dots \dots \dots, 6$

9. Here, $f(x) = \frac{1+|x-3|}{11};\quad x = 1, 2, 3, 4, 5$

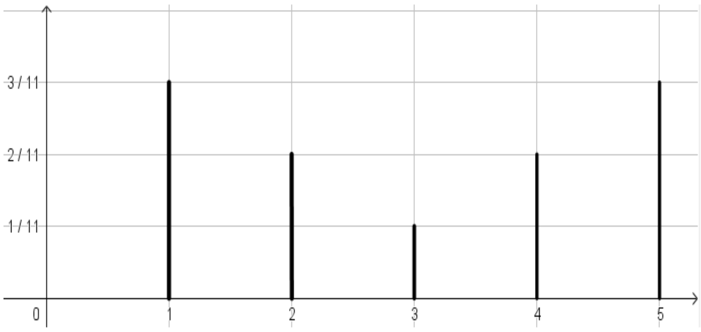
$P(X = 1) = \frac{3}{11}$

$P(X = 2) = \frac{2}{11}$

$P(X = 3) = \frac{1}{11}$

$P(X = 4) = \frac{2}{11}$

$P(X = 5) = \frac{3}{11}$



10. Here, $N = 50, N_1 = 3, N_2 = 47, n = 10$. So, $f(x) = \frac{{}^{3}C_x \times {}^{47}C_{10-x}}{{}^{50}C_{10}}$.

(a) *P(Exactly one defective item)* $= P(X = 1) = \frac{{}^{3}C_1 \times {}^{47}C_9}{{}^{50}C_{10}} = \frac{39}{98}$

(b) *P(At most one defective item)*$= P(X \le 1) = P(X = 0) + P(X = 1)$

$$= \frac{{}^{3}C_0 \times {}^{47}C_{10}}{{}^{50}C_{10}} + \frac{{}^{3}C_1 \times {}^{47}C_9}{{}^{50}C_{10}} = \frac{247}{490} + \frac{39}{98} = \frac{221}{245}$$

11. Here, $N = 100, N_1 = 5, N_2 = 95, n = 10$. So, $f(x) = \frac{{}^{5}C_x \times {}^{95}C_{10-x}}{{}^{100}C_{10}}$.

P(At least one defective bulb) $= P(X \ge 1) = 1 - P(X = 0)$

$$= 1 - \frac{{}^{5}C_0 \times {}^{95}C_{10}}{{}^{100}C_{10}} = 1 - 0.5837 = 0.4162$$

13. Here, $N = 6, N_1 = 3, N_2 = 3, n = 3$. So, $f(x) = \frac{{}^{3}C_x \times {}^{3}C_{3-x}}{{}^{6}C_3}$.

P(At least one is selected) $= P(X \ge 1) = 1 - P(X = 0) = 1 - \frac{{}^{3}C_0 \times {}^{3}C_3}{{}^{6}C_3} = \frac{19}{20}$

P(All three are selected) $= P(X = 3) = \frac{{}^{3}C_3 \times {}^{3}C_0}{{}^{6}C_3} = \frac{1}{20}$

P(Exactly two are selected) $= P(X = 2) = \frac{{}^{3}C_2 \times {}^{3}C_1}{{}^{6}C_3} = \frac{9}{20}$

14. Here, $N = 20, N_1 = 3, N_2 = 17, n = 5$. So, $f(x) = \frac{{}^{3}C_x \times {}^{17}C_{5-x}}{{}^{20}C_5}$.

P(At least one) $= P(X \ge 1) = P(X = 1) + P(X = 2) + P(X = 3)$

$$= \frac{{}^{3}C_1 \times {}^{17}C_4}{{}^{20}C_5} + \frac{{}^{3}C_2 \times {}^{17}C_3}{{}^{20}C_5} + \frac{{}^{3}C_3 \times {}^{17}C_2}{{}^{20}C_5}$$
$$= 0.4605 + 0.1316 + 0.0088 = 0.6009$$

Chapter: 2.2

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2. Given, $f(x) = \frac{(|x|+1)^2}{9};\ \ x = -1,0,1$

$P(X = -1) = \frac{4}{9}$, $\quad P(X = 0) = \frac{1}{9}$, $\quad P(X = 1) = \frac{4}{9}$,

Mean, $\mu = E(X) = \sum x f(x) = -1 \times \frac{4}{9} + 0 \times \frac{1}{9} + 1 \times \frac{4}{9} = 0$

$E(X^2) = \sum x^2 f(x) = (-1)^2 \times \frac{4}{9} + 0^2 \times \frac{1}{9} + 1^2 \times \frac{4}{9} = \frac{8}{9}$

$E(3X^2 - 2X + 4) = 3E(X^2) - 2E(X) + 4 = 3 \times \frac{8}{9} - 2 \times 0 + 4 = \frac{20}{3}$

3. Given, $f(x) = \frac{5-x}{10};\ x = 1, 2, 3, 4$

$P(X = 1) = \frac{4}{10}$, $\quad P(X = 2) = \frac{3}{10}$, $\quad P(X = 3) = \frac{2}{10}$, $\quad P(X = 4) = \frac{1}{10}$

Expected payment $= 200 \times \frac{4}{10} + 400 \times \frac{3}{10} + 500 \times \frac{2}{10} + 600 \times \frac{1}{10}$
$= 80 + 120 + 100 + 60 = \$360$

4. Given, $f(x) = \begin{cases} 0.9\ ;\ when\ x = 0, \\ \frac{c}{x}\ ;\ when\ x = 1, 2, 3, 4, 5, 6 \end{cases}$

We know, $\sum_{x=0}^{6} f(x) = 1$

$$\Rightarrow \frac{9}{10} + c\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}\right) = 1$$
$$\Rightarrow c = \frac{2}{49}$$

So, $f(x) = \begin{cases} 0.9\ ;\ when\ x = 0, \\ \frac{2}{49x}\ ;\ when\ x = 1, 2, 3, 4, 5, 6 \end{cases}$

Expected payment $= E(u(X)) = \sum_{x=0}^{6} u(x) f(x)) = \sum_{x=0}^{6} (x-1) f(x)$
$$= 0 \times 0.9 + \frac{2}{49}\left(0 \times 1 + 1 \times \frac{1}{2} + 2 \times \frac{1}{3} + 3 \times \frac{1}{4} + 4 \times \frac{1}{5} + 5 \times \frac{1}{6}\right)$$
$$= \frac{71}{490}$$

7. Probability of winning $\$1, \$2, \$3$ are $\frac{75}{216}, \frac{15}{216}, \frac{1}{216}$, respectively. Also, probability of losing $\$1$ is $\frac{125}{216}$.

$E(X) = 1 \times \frac{75}{216} + 2 \times \frac{15}{216} + 3 \times \frac{1}{216} + (-1) \times \frac{125}{216} = -\frac{17}{216}$

His expected loss is $\$\frac{17}{216}$.

11. Probability of winning $\$1 = 0.49293$
    Probability of losing $\$1 = 0.50707$

$E(X) = 1 \times 0.49293 + (-1) \times 0.50707 = -0.01414$
His expected loss is $\$0.01414$.

12. (a) μ = average = mean
Average class size, $\mu = \frac{16\times25+3\times100+1\times300}{20} = 50$

(b) $X = 25, 100, 300$
$P(X = 25) = \frac{16\times25}{1000} = \frac{4}{10}$
$P(X = 100) = \frac{3\times100}{1000} = \frac{3}{10}$
$P(X = 300) = \frac{1\times300}{1000} = \frac{3}{10}$

(c) $E(X) = \sum xf(x) = 25 \times \frac{4}{10} + 100 \times \frac{3}{10} + 300 \times \frac{3}{10} = 130$

Chapter: 2.3

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1. (a) Given, $f(x) = \frac{1}{5}$, $\quad x = 5, 10, 15, 20, 25$
Mean, $\mu = E(X) = \sum xf(x) = 5 \times \frac{1}{5} + 10 \times \frac{1}{5} + 15 \times \frac{1}{5} + 20 \times \frac{1}{5} + 25 \times \frac{1}{5} = 15$

$E(X^2) = \sum x^2 f(x) = 5^2 \times \frac{1}{5} + 10^2 \times \frac{1}{5} + 15^2 \times \frac{1}{5} + 20^2 \times \frac{1}{5} + 25^2 \times \frac{1}{5} = 275$
Variance, $\sigma^2 = E(X^2) - [E(X)]^2 = 275 - 15^2 = 50$

(b) Given, $f(x) = 1$, $\quad x = 5$
Mean, $\mu = E(X) = \sum xf(x) = 5 \times 1 = 5$

$E(X^2) = \sum x^2 f(x) = 5^2 \times 1 = 25$
Variance, $\sigma^2 = E(X^2) - [E(X)]^2 = 25 - 5^2 = 0$

(c) Given, $f(x) = \frac{4-x}{6}$, $\quad x = 1, 2, 3$
$P(X = 1) = \frac{3}{6}$, $\quad P(X = 2) = \frac{2}{6}$, $\quad P(X = 3) = \frac{1}{6}$
Mean, $\mu = E(X) = \sum xf(x) = 1 \times \frac{3}{6} + 2 \times \frac{2}{6} + 3 \times \frac{1}{6} = \frac{5}{3}$
$E(X^2) = \sum x^2 f(x) = 1^2 \times \frac{3}{6} + 2^2 \times \frac{2}{6} + 3^2 \times \frac{1}{6} = \frac{10}{3}$

Variance, $\sigma^2 = E(X^2) - [E(X)]^2 = \frac{10}{3} - \left(\frac{5}{3}\right)^2 = \frac{5}{9}$

2. (a) Given, $f(x) = \frac{3!}{x!(3-x)!}\left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{3-x}$; $x = 0, 1, 2, 3$

$f(x) = {}^3C_x \frac{3^{3-x}}{4^3}$; $\left[\frac{3!}{x!(3-x)!} = {}^3C_x \text{ \& } \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{3-x} = \frac{3^{3-x}}{4^3}\right]$

$P(X = 0) = \frac{27}{64}$

$P(X = 1) = \frac{27}{64}$

$P(X = 2) = \frac{9}{64}$

$P(X = 3) = \frac{1}{64}$

$\mu = E(X) = \sum x f(x) = 0 \times \frac{27}{64} + 1 \times \frac{27}{64} + 2 \times \frac{9}{64} + 3 \times \frac{1}{64} = \frac{48}{64}$

$E(X^2) = \sum x^2 f(x) = 0^2 \times \frac{27}{64} + 1^2 \times \frac{27}{64} + 2^2 \times \frac{9}{64} + 3^2 \times \frac{1}{64} = \frac{72}{64}$

$E[X(X-1)] = E(X^2) - E(X) = \frac{72}{64} - \frac{48}{64} = \frac{24}{64}$

$\sigma^2 = E[X(X-1)] + E(X) - \mu^2 = \frac{24}{64} + \frac{48}{64} - \left(\frac{48}{64}\right)^2 = \frac{9}{16}$

(c) Given, $f(x) = \frac{4!}{x!(4-x)!}\left(\frac{1}{2}\right)^4$, $x = 0, 1, 2, 3, 4$

$f(x) = {}^4C_x \frac{1}{16}$; $\left[\frac{4!}{x!(4-x)!} = {}^4C_x \text{ \& } \left(\frac{1}{2}\right)^4 = \frac{1}{16}\right]$

$P(X = 0) = \frac{1}{16}$

$P(X = 1) = \frac{4}{16}$

$P(X = 2) = \frac{6}{16}$

$P(X = 3) = \frac{4}{16}$

$P(X = 4) = \frac{1}{16}$

$\mu = E(X) = \sum x f(x) = 0 \times \frac{1}{16} + 1 \times \frac{4}{16} + 2 \times \frac{6}{16} + 3 \times \frac{4}{16} + 4 \times \frac{1}{16} = 2$

$E(X^2) = \sum x^2 f(x) = 0^2 \times \frac{1}{16} + 1^2 \times \frac{4}{16} + 2^2 \times \frac{6}{16} + 3^2 \times \frac{4}{16} + 4^2 \times \frac{1}{16} = 5$

$E[X(X-1)] = E(X^2) - E(X) = 5 - 2 = 3$

$\sigma^2 = E[X(X-1)] + E(X) - \mu^2 = 3 + 2 - 2^2 = 1$

3. Given, $E(X + 4) = 10$

$\Rightarrow E(X) + 4 = 10$

$\Rightarrow E(X) = 6$

Again, $E[(X + 4)^2] = 116$

$\Rightarrow E[(X^2 + 8X + 16)] = 116$

$\Rightarrow E(X^2) + 8E(X) + 16 = 116$
$\Rightarrow E(X^2) + 48 + 16 = 116$
$\Rightarrow E(X^2) = 52$

(b) $\mu = E(X) = 6$
(c) $\sigma^2 = Var(X) = E(X^2) - [E(X)]^2 = 52 - (6)^2 = 52 - 36 = 16$
(a) $Var(X + 4) = 1^2 \times Var(X) = 16$; $[Var(ax + b) = a^2 Var(X)]$

4. $E\left(\frac{X-\mu}{\sigma}\right) = \frac{1}{\sigma}E(X - \mu)$

$= \frac{1}{\sigma}[E(X) - \mu]$

$= \frac{1}{\sigma}[\mu - \mu]$; [Since, $\mu = E(X)$]

$= 0$

$E\left[\left(\frac{X-\mu}{\sigma}\right)^2\right] = \frac{1}{\sigma^2}E(X^2 - 2\mu X + \mu^2)$

$= \frac{1}{\sigma^2}[E(X^2) - 2\mu E(X) + \mu^2]$

$= \frac{1}{\sigma^2}[E(X^2) - 2\{E(X)\}^2 + \{E(X)\}^2]$; [Since, $\mu = E(X)$]

$= \frac{1}{\sigma^2}[E(X^2) - \{E(X)^2\}]$

$= \frac{1}{\sigma^2} \times \sigma^2$; [Since, $\sigma^2 = E(X^2) - \{E(X)^2\}$]

$= 1$

$\mu_{\frac{X-\mu}{\sigma}} = E\left(\frac{X-\mu}{\sigma}\right) = 0$

$\sigma^2_{\frac{X-\mu}{\sigma}} = E\left[\left(\frac{X-\mu}{\sigma}\right)^2\right] - \left(E\left(\frac{X-\mu}{\sigma}\right)\right)^2 = 1$

8. Given, $f(x) = \frac{2x-1}{16}$, $x = 1, 2, 3, 4$

$P(X = 1) = \frac{1}{16}$

$P(X = 2) = \frac{3}{16}$

$P(X = 3) = \frac{5}{16}$

$P(X = 4) = \frac{7}{16}$

Mean, $\mu = E(X) = \sum_{x=1}^{4} xf(x)$

$= 1 \times \frac{1}{16} + 2 \times \frac{3}{16} + 3 \times \frac{5}{16} + 4 \times \frac{7}{16}$

$= \frac{50}{16}$

$E(X^2) = \sum_{x=1}^{4} x^2 f(x)$

$= 1^2 \times \frac{1}{16} + 2^2 \times \frac{3}{16} + 3^2 \times \frac{5}{16} + 4^2 \times \frac{7}{16}$

$$= \frac{170}{16}$$

Variance, $\sigma^2 = E(X^2) - [E(X)]^2$

$$= \frac{170}{16} - \left(\frac{50}{16}\right)^2$$

$$= \frac{55}{64}$$

Standard deviation, $\sigma = \sqrt{\frac{55}{64}} = 0.927$

11. Given, $M(t) = \frac{2}{5}e^t + \frac{1}{5}e^{2t} + \frac{2}{5}e^{3t}$

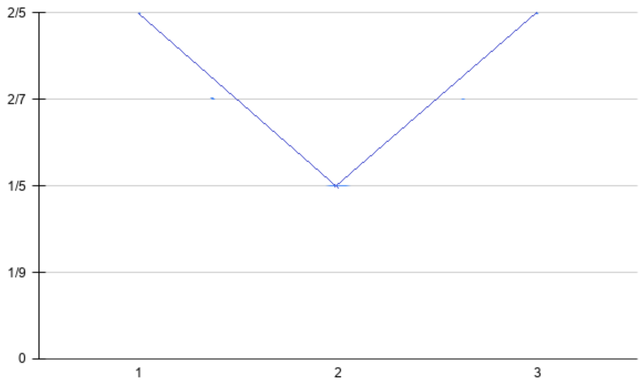
$M(0) = \frac{2}{5} + \frac{1}{5} + \frac{2}{5} = 1$

So, $M(t)$ is mgf.

$P(X = 1) = \frac{2}{5}$

$P(X = 2) = \frac{1}{5}$

$P(X = 3) = \frac{2}{5}$



$M(t) = \sum_{x=1}^{3} e^{tx}\left(\frac{|x-2|+1}{5}\right)$; $\left[\text{Since, } f(x) = \frac{y_{ex} \pm |x - X_{ex}|}{m}\right]$

$= \sum_{x=1}^{3} e^{tx} f(x)$

So, $f(x) = \left(\frac{|x-2|+1}{5}\right)$ is pmf.

$M'(t) = \frac{2}{5}e^t + \frac{2}{5}e^{2t} + \frac{6}{5}e^{3t}$

$M''(t) = \frac{2}{5}e^t + \frac{4}{5}e^{2t} + \frac{18}{5}e^{3t}$

Mean, $\mu = M'(0) = 2$

$M''(0) = \frac{24}{5}$

Variance, $\sigma^2 = M''(0) - [M'(0)]^2 = \frac{24}{5} - 2^2 = \frac{4}{5}$

Standard deviation, $\sigma = \sqrt{\frac{4}{5}}$

17. (b) $M(t) = \frac{e^{2t}}{(e^t - 2)^2}$

$M(0) = \frac{e^0}{(e^0 - 2)^2} = 1$;

So, $M(t)$ is mgf.

(c) $M'(t) = \frac{4e^{2t}}{(2 - e^t)^3}$

$M''(t) = \frac{4e^{3t}+16e^{2t}}{(2-e^t)^4}$

Mean, $\mu = M'(0) = \frac{4e^0}{(2-e^0)^3} = \frac{4}{1^3} = 4$

$M''(0) = \frac{4e^0+16e^0}{(2-e^0)^4} = \frac{4+16}{1^4} = 20$

Variance, $\sigma^2 = M''(0) - [M'(0)]^2 = 20 - 4^2 = 4$

19. (a) $M(t) = \frac{44}{120}e^t + \frac{45}{120}e^{2t} + \frac{20}{120}e^{3t} + \frac{10}{120}e^{4t} + \frac{1}{120}e^{5t}$

$M(0) = 1$; So, $M(t)$ is mgf.

$M'(t) = \frac{44}{120}e^t + \frac{90}{120}e^{2t} + \frac{60}{120}e^{3t} + \frac{40}{120}e^{4t} + \frac{5}{120}e^{5t}$

$M'(0) = \frac{239}{120}$

$M''(t) = \frac{44}{120}e^t + \frac{180}{120}e^{2t} + \frac{180}{120}e^{3t} + \frac{160}{120}e^{4t} + \frac{25}{120}e^{5t}$

$M''(0) = \frac{589}{120}$

Mean, $\mu = E(X) = M'(0) = \frac{239}{120}$

Variance, $\sigma^2 = M''(0) - [M'(0)]^2 = \frac{589}{120} - \left(\frac{239}{120}\right)^2 = \frac{13559}{14400}$

(b) $f(x) = \begin{cases} \frac{44}{120} & ; x = 1 \\ \frac{45}{120} & ; x = 2 \\ \frac{120}{120} & ; x = 3 \\ \frac{10}{120} & ; x = 4 \\ \frac{1}{120} & ; x = 5 \end{cases}$

Chapter: 2.4

4. Here, $n = 7$,

$p = 0.15$,

$q = 1 - p = 0.85$,

(a) $f(x) = {}^7C_x(0.15)^x\,(0.85)^{7-x}$. So, X is $b(7, 0.15)$

$$P(X = 0) = {}^7C_0(0.15)^0(0.85)^7 = 0.3205$$

$$P(X = 1) = {}^7C_1(0.15)^1(0.85)^6 = 0.3960$$

$$P(X=2) = {}^{7}C_2(0.15)^2(0.85)^5 = 0.2096$$
$$P(X=3) = {}^{7}C_3(0.15)^3(0.85)^4 = 0.0616$$

(b) (i) $P(X \geq 2) = 1 - P(X \leq 1) = 1 - [P(X=0) + P(X=1)]$
$= 1 - [0.3205 + 0.3960] = 0.2835$

(ii) $P(X = 1) = 0.3960$

(iii) $P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$
$= 0.3205 + 0.3960 + 0.2096 + 0.0616 = 0.9877$

5. Here, $n = 25$,
$p = 0.2$,
$q = 1 - p = 0.8$,

$f(x) = {}^{25}C_x(0.2)^x\,(0.8)^{25-x}$. So, $X\ is\ b(25, 0.2)$

$$P(X=0) = {}^{25}C_0(0.2)^0\,(0.8)^{25} = 0.0038$$
$$P(X=1) = {}^{25}C_1(0.2)^1\,(0.8)^{24} = 0.0236$$
$$P(X=2) = {}^{25}C_2(0.2)^2\,(0.8)^{23} = 0.0708$$
$$P(X=3) = {}^{25}C_3(0.2)^3\,(0.8)^{22} = 0.1358$$
$$P(X=4) = {}^{25}C_4(0.2)^4\,(0.8)^{21} = 0.1867$$

(a) $P(X \leq 4) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$
$= 0.0038 + 0.0236 + 0.0708 + 0.1358 + 0.1867 = 0.4207$

(b) $P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0.4207 = 0.5793$

(c) $P(X = 6) = {}^{25}C_6(0.2)^6\,(0.8)^{19} = 0.1633$

(d) Mean, $\mu = np = 25 \times 0.2 = 5$
Variance, $\sigma^2 = npq = 25 \times 0.2 \times 0.8 = 4$
Standard deviation, $\sigma = \sqrt{4} = 2$

6. Here, $n = 15$,
$p = 0.75$,
$q = 1 - p = 0.25$,

(a) $f(x) = {}^{15}C_x(0.75)^x\,(0.25)^{15-x}$. So, $X\ is\ b(15, 0.75)$

$$P(X=10) = {}^{15}C_{10}(0.75)^{10}\,(0.25)^5 = 0.1651$$
$$P(X=11) = {}^{15}C_{11}(0.75)^{11}\,(0.25)^4 = 0.2252$$

$$P(X=12)= {}^{15}C_{12}(0.75)^{12}\ (0.25)^{3}\ = 0.2252$$
$$P(X=13)= {}^{15}C_{13}(0.75)^{13}\ (0.25)^{2}\ = 0.1559$$
$$P(X=14)= {}^{15}C_{14}(0.75)^{14}\ (0.25)^{1}\ = 0.0668$$
$$P(X=15)= {}^{15}C_{15}(0.75)^{15}\ (0.25)^{0}\ = 0.0134$$

(b) $P(X \geq 10) = P(X=10) + P(X=11) + P(X=12) + P(X=13)$
$+P(X=14) + P(X=15)$

$= 0.1651 + 0.2252 + 0.2252 + 0.1559 + 0.0668 + 0.0134$
$= 0.8516$

(c) $P(X \leq 10) = 1 - [P(X=11) + P(X=12) + P(X=13)$
$+P(X=14) + P(X=15)]$
$= 1 - (0.2252 + 0.2252 + 0.1559 + 0.0668 + 0.0134) = 0.3135$

(d) $P(X=10)\ = 0.1651$

(e) Mean, $\mu = np = 15 \times 0.75\ = 11.25$
Variance, $\sigma^2 = npq\ = 15 \times 0.75 \times 0.25\ = 2.8125$
Standard deviation, $\sigma = \sqrt{2.8125}\ = 1.6771$

8. Here, $n\ = 4,$
$p\ = 0.99,$
$q\ = 1 - p = 0.01,$

$f(x)\ = {}^{4}C_x(0.99)^x\ (0.01)^{4-x}$. So, $X\ is\ b(4, 0.99)$

(a) $P(X \geq 1) = 1 - P(X=0) = 1 - {}^{4}C_0(0.99)^0\ (0.01)^4 = 0.999999999$

(b) $P(X=4)\ = {}^{4}C_4(0.99)^4\ (0.01)^0\ = 0.9606$

9. Here, $n\ = 20,$
$p\ = 0.8,$
$q\ = 1 - p = 0.2,$

(a) $f(x) = {}^{20}C_x(0.8)^x\ (0.2)^{20-x}$. So, $X\ is\ b(20, 0.8)$

(b) Mean, $\mu = np = 20 \times 0.8\ = 16$
Variance, $\sigma^2 = npq\ = 20 \times 0.8 \times 0.2\ = 3.2$
Standard deviation, $\sigma = \sqrt{3.2}\ = 1.7889$

(c) (i) $P(X=15) = {}^{20}C_{15}(0.8)^{15}\ (0.2)^5 = 0.1746$

(ii) $P(X > 15) = P(X = 16) + P(X = 17) + P(X = 18) + P(X = 19) + P(X = 20)$

$= {}^{20}C_{16}(0.8)^{16}\ (0.2)^4 + {}^{20}C_{17}(0.8)^{17}\ (0.2)^3 + {}^{20}C_{18}(0.8)^{18}\ (0.2)^2$
$+ {}^{20}C_{19}(0.8)^{19}\ (0.2)^1 + {}^{20}C_{20}(0.8)^{20}\ (0.2)^0$
$= 0.2182 + 0.2054 + 0.1369 + 0.0576 + 0.0115 = 0.6296$

(iii) $P(X \le 15) = 1 - P(X > 15) = 1 - 0.6296 = 0.3704$

10. Here, $n = 8$,
$p = 0.9$,
$q = 1 - p = 0.1$,

(a) $f(x) = {}^{8}C_x(0.9)^x\ (0.1)^{8-x}$. So, $X\ is\ b(8, 0.9)$

$P(X = 6) = {}^{8}C_6(0.9)^6\ (0.1)^2 = 0.1489$
$P(X = 7) = {}^{8}C_7(0.9)^7\ (0.1)^1 = 0.3826$
$P(X = 8) = {}^{8}C_8(0.9)^8\ (0.1)^0 = 0.4305$

(b) (i) $P(X = 8) = 0.4305$

(ii) $P(X \le 6) = 1 - (P(X = 7) + P(X = 8))$
$= 1 - (0.3826 + 0.4305) = 0.1869$

(iii) $P(X \ge 6) = P(X = 6) + P(X = 7) + P(X = 8)$
$= 0.1489 + 0.3826 + 0.4305 = 0.962$

11. Given that,
Mean, $\mu = np = 6$ ....(i)
Variance, $\sigma^2 = npq = 3.6$ ....(ii)

(ii) ÷ (i) $\Rightarrow q = 0.6$ and hence $p = 0.4$,
So, $n = \frac{6}{p} = \frac{6}{0.4} = 15$ [from (i)]

$f(x) = {}^{15}C_x(0.4)^x\ (0.6)^{15-x}$

$P(X = 4) = {}^{15}C_4(0.4)^4\ (0.6)^{11} = 0.1267$

13.(a) Here, $n = 10$,
$p = 0.1$,
$q = 1 - p = 0.9$,

$f(x) = {}^{10}C_x(0.1)^x\ (0.9)^{10-x}$. So, $X\ is\ b(10, 0.1)$

$P(X \ge 1) = 1 - P(X = 0) = 1 - {}^{10}C_0(0.1)^0\ (0.9)^{10} = 1 - 0.3487 = 0.6513$

(b) Here, $n = 15,$

$p = 0.1,$

$q = 1 - p = 0.9,$

$f(x) = {}^{15}C_x (0.1)^x\ (0.9)^{10-x}$. So, $X\ is\ b(15, 0.1)$

$P(X \geq 1) = 1 - P(X = 0) = 1 - {}^{15}C_0 (0.1)^0\ (0.9)^{15} = 1 - 0.2059 = 0.7941$

17. Here, $n = 5,$

$p = 0.6,$

$q = 1 - p = 0.4,$

$f(x) = {}^5C_x (0.6)^x\ (0.4)^{4-x}$. So, $X\ is\ b(5, 0.4)$

(a) $P(X = 5) = {}^5C_5 (0.6)^5 (0.4)^0 = 0.0778$

(b) $P(X = 3) = {}^5C_3 (0.6)^3 (0.4)^2 = 0.3456$

(c) $P(X \geq 1) = 1 - P(X = 0) = 1 - {}^5C_0 (0.6)^0 (0.4)^5 = 0.9898$

19. (a) Given that,

$M(t) = \frac{1}{3} + \frac{2}{3} e^t$

$M(0) = \frac{1}{3} + \frac{2}{3} = 1$ So, this is a *mgf*.

Again, we know, $M(t) = (q + pe^t)^n$

Here, $M(t) = \left(\frac{1}{3} + \frac{2}{3} e^t\right)^1$

So, here, $q = \frac{1}{3}, p = \frac{2}{3}, n = 1$

$f(x) = {}^1C_x \left(\frac{2}{3}\right)^x \left(\frac{1}{3}\right)^{1-x}$

Mean, $\mu = np = 1 \times \frac{2}{3} = \frac{2}{3}$

Variance, $\sigma^2 = npq = 1 \times \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$

Standard deviation, $\sigma = \frac{\sqrt{2}}{3}$

(b) Given that,

$M(t) = (0.25 + 0.75e^t)^{12}$

$M(0) = (0.25 + 0.75)^{12} = 1$ So, this is *mgf*.

Again, we know, $M(t) = (q + pe^t)^n$

Here, $M(t) = (0.25 + 0.75e^t)^{12}$

So, here, $q = 0.25, p = 0.75, n = 12$

$f(x) = {}^{12}C_x (0.75)^x\ (0.25)^{12-x}$

Mean, $\mu = np = 12 \times 0.75 = 9$
Variance, $\sigma^2 = npq = 12 \times 0.75 \times 0.25 = 2.25$
Standard deviation, $\sigma = \sqrt{2.25} = 1.5$

20. (a) $M(t) = (0.3 + 0.7e^t)^5 = (q + pe^t)^n$
This is a *mgf* of the Binomial distribution.

Here, $p = 0.7, q = 0.3, n = 5$
So, $f(x) = {}^5C_x(0.7)^x\,(0.3)^{5-x}$

Mean, $\mu = np = 5 \times 0.7 = 3.5$
Variance, $\sigma^2 = npq = 5 \times 0.7 \times 0.3 = 1.05$
$P(1 \le X \le 2) = P(X = 1) + P(X = 2) = 0.02835 + 0.1323 = 0.16065$

(b) $M(t) = \frac{0.3e^t}{1-0.7e^t} = \frac{pe^t}{1-qe^t}$
So, this is a *mgf* of Geometric distribution.

Here, $p = 0.3, q = 0.7$
So, $f(x) = (0.7)^{x-1}(0.3)$

Mean, $\mu = \frac{1}{p} = \frac{1}{0.3} = \frac{10}{3}$
Variance, $\sigma^2 = \frac{q}{p^2} = \frac{0.7}{0.3^2} = \frac{70}{9}$
$P(1 \le X \le 2) = P(X = 1) + P(X = 2) = 0.3 + 0.21 = 0.51$

(c) $M(t) = (0.45 + 0.55e^t)^1 = (q + pe^t)^n$
This is a *mgf* of the Binomial distribution.

Here, $p = 0.55, q = 0.45, n = 1$
As $n = 1$, this is a Bernoulli distribution.
So, $f(x) = (0.55)^x\,(0.45)^{1-x}; x = 0,1$

Mean, $\mu = p = 0.55$
Variance, $\sigma^2 = pq = 0.55 \times 0.45 = 0.2475$
$P(1 \le X \le 2) = P(X = 1) + P(X = 2) = 0.55 + 0 = 0.55$

(d) $M(t) = 0.3e^t + 0.4e^{2t} + 0.2e^{3t} + 0.1e^{4t}$
Does not satisfy any distribution.

$M'(t) = 0.3e^t + 0.8e^{2t} + 0.6e^{3t} + 0.4e^{4t};\ \ M'(0) = 2.1$
$M''(t) = 0.3e^t + 1.6e^{2t} + 1.8e^{3t} + 1.6e^{4t};\ \ M''(0) = 5.3$

Mean, $\mu = M'(0) = 2.1$
Variance, $\sigma^2 = M''(0) - \{M'(0)\}^2 = 0.53 - (2.1)^2 = 0.89$
$P(1 \leq X \leq 2) = P(X = 1) + P(X = 2) = 0.3 + 0.4 = 0.7$

(e) $M(t) = \sum_{x=1}^{10}(0.1)e^{tx} = \sum_{x=1}^{10}\frac{e^{tx}}{10} = \sum_{x=1}^{10}\frac{e^{tx}}{m}$
This is a *mgf* of the Uniform distribution.

Here, $m = 10$
So, $f(x) = \frac{1}{10}$; $x = 1,2,3, \ldots\ldots,10$

Mean, $\mu = \frac{m+1}{2} = \frac{10+1}{2} = 5.5$
Variance, $\sigma^2 = \frac{m^2-1}{12} = \frac{10^2-1}{12} = 8.25$
$P(1 \leq X \leq 2) = P(X = 1) + P(X = 2) = \frac{1}{10} + \frac{1}{10} = 0.2$

Chapter: 2.6

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1. Given that, $\lambda = 4$, so $f(x) = \frac{e^{-4}\,4^x}{x!}$

(a) $P(2 \leq X \leq 5) = P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)$

$$= \frac{e^{-4}\,4^2}{2!} + \frac{e^{-4}\,4^3}{3!} + \frac{e^{-4}\,4^4}{4!} + \frac{e^{-4}\,4^5}{5!} = 0.693$$

(b) $P(X \geq 3) = 1 - P\,(X \leq 2) = 1\ - [P(X = 0) + P(X = 1) + P(X = 2)]$

$$= 1 - \left[\frac{e^{-4}\,4^0}{0!} + \frac{e^{-4}\,4^1}{1!} + \frac{e^{-4}\,4^2}{2!}\right] = 0.762$$

(c) $P(X \leq 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$

$$= \frac{e^{-4}\,4^0}{0!} + \frac{e^{-4}\,4^1}{1!} + \frac{e^{-4}\,4^2}{2!} + \frac{e^{-4}\,4^3}{3!} = 0.433$$

2. Given that, $\lambda = 3 = \sigma^2$, so $f(x) = \frac{e^{-3}\,3^x}{x!}$

$P(X = 2) = \frac{e^{-3}\,3^2}{2!} = 0.224$

3. Given that, $\mu = \lambda = 11$, so $f(x) = \frac{e^{-11}\,11^x}{x!}$

$P(X > 10) = 1 – P(X \leq 10) = 1 - \sum_{x=0}^{10}\frac{e^{-11}\,11^x}{x!} = 1 - 0.46 = 0.54$

4. Given that, $3P(X = 1) = P(X = 2)$

$$\Longrightarrow 3\frac{e^{-\lambda}\lambda}{1!} = \frac{e^{-\lambda}\,\lambda^{2}}{2!}$$
$$\Longrightarrow 6\lambda = \lambda^{2};\ e^{-\lambda} \neq 0$$
$$\Longrightarrow \lambda = 6;\ \lambda \neq 0$$

So, $f(x) = \frac{e^{-6}\,6^{x}}{x!}$

$P(X = 4) = \frac{e^{-6}\,6^{4}}{4!} = 0.1338$

5. Here, $p = \frac{1}{150}$ & $n = 225$, so $\lambda = np = 1.5$

So, $f(x) = \frac{1.5^{x}e^{-1.5}}{x!}$

$P(X \leq 1) = P(X = 0) + P(X = 1) = \frac{1.5^{0}e^{-1.5}}{0!} + \frac{1.5^{1}e^{-1.5}}{1!} = 0.558$

6. Here, $p = \frac{1}{100}$ & $n = 50$, so $\lambda = np = 0.5$

So, $f(x) = \frac{0.5^{x}e^{-0.5}}{x!}$

$P(X = 0) = \frac{0.5^{0}e^{-0.5}}{0!} = 0.606$

8. Here, $p = 0.005$ & $n = 1000$, so $\lambda = np = 5$

So, $f(x) = \frac{5^{x}e^{-5}}{x!}$

(a) $P\ (X \leq 1) = P(X = 0) + P(X = 1) = \frac{5^{0}e^{-5}}{0!} + \frac{5^{1}e^{-5}}{1!} = 0.0404$

(b) $P(4 \leq\ X \leq 6) = P(X = 4) + P(X = 5) + P(X = 6)$

$$= \frac{5^{4}e^{-5}}{4!} + \frac{5^{5}e^{-5}}{5!} + \frac{5^{6}e^{-5}}{6!} = 0.4972$$