United International **University**

Name: Azizul Islam Nayem ID: 011201262

Section: D Batch: 201 Date: 07.05.2021

Experiment No. 08

Name of the Experiment: Determination of the moment of inertia of the given disc using Torsion pendulum by the method of oscillations (Dynamic Method).

Theory:

A body suspended by a thread or wire which twists first in one direction and then in the reverse direction, in the horizontal plane is called a torsional pendulum. The first torsion pendulum was developed by Robert Leslie in 1793. A simple schematic representation of a torsion pendulum is given here, Fig. 01.

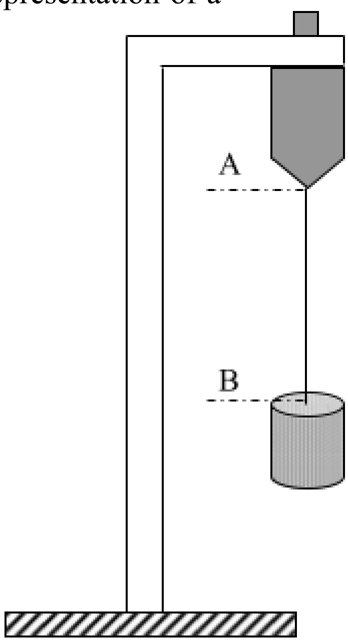


Fig. 01 C

If a heavy body is supported by a vertical wire of length l and radius r so that the axis of the wire passes through its center of gravity, and if the body is turned through an angle and released, it will execute torsional oscillations about a vertical axis. If, at any instant, the angle of twist is θ, the moment of

the torsional couple exerted by the wire will be, $n\pi r^4$

$$\frac{\quad}{2l\theta} \square \; C\theta \; \ldots \ldots \ldots (1)$$

Where, $\frac{\quad}{2l}$ □ $n^{\pi r_4}$ C is a constant and n is the modulus of rigidity of the

the

material of the wire.

Therefore, the motion is simple harmonic and of fixed period.

$$T\square 2\pi\sqrt{\frac{I}{C}} \; \ldots \ldots \ldots (2)$$

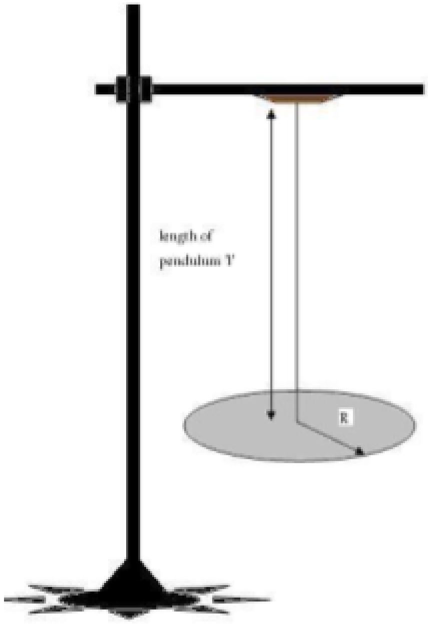
Where, I is the moment of inertia of the body.

From equations (1) and (2), we have,

T_2 □ □ $4\pi C_2 I$ $8n r\pi I_4 l$ Or, n □ $T8_2\pi I r_4 l$ dynes/cm$_2$

Note: For a cylindrical object, having mass *M* and radius *a*, the moment of inertia is given as, $I = \frac{1}{2}Ma^2$

Now, let I_0 be the moment of inertia of the disc alone and I_1 & I_2 be the moment of inertia of the disc with identical masses at distances d_1&d_2 respectively. If I_1 is the moment of inertia of each identical mass about the vertical axis passing through its centre of gravity, then



$$T_0^2 = 4\pi^2 \frac{I_0}{C} \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

$$T_1^2 = 4\pi^2 \frac{I_1}{C} \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

$$T_2^2 = 4\pi^2 \frac{I_2}{C} \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

$$T_2^2 - T_1^2 = \frac{4\pi^2}{C}(I_2 - I_1) \ldots\ldots\ldots (9)$$

$$I_1 = I_0 + 2\ I^1 + 2md_1^2 \ldots\ldots\ldots (3)$$

$$I_2 = I_0 + 2I^1 + 2md_2^2 \ldots\ldots\ldots (4)$$

$$I_2 - I_1 = 2m(d_2^2 - d_1^2) \ldots\ldots\ldots (5)$$

But from equation (2),

Fig.02: A torsional pendulum with disc.

Where T0, T1, T2 are the periods of torsional oscillation without identical mass, with identical pass at position d1, d2 respectively. Dividing equation (6) by (9) and using (5),

$$\frac{T_0^2}{\left(T_2^2 - T_1^2\right)} = \frac{I_0}{\left[I_2 - I_1\right]} = \frac{I_0}{2m\left(d_2^2 - d_1^2\right)} \text{........} (10)$$

Therefore, the moment of inertia of the disc using identical masses,

$$I_0 = 2m\left(d_2^2 - d_1^2\right)\frac{T_0^2}{\left(T_2^2 - T_1^2\right)} \text{..................} (11)$$

Procedure for Simulation

1. The radius of the suspension wire is measured using a screw gauge.
2. The length of the suspension wire is adjusted to suitable values like 0.3m, 0.4m, 0.5m,.....0.9m, 1m etc.
3. The disc is set in oscillation. Find the time for 10 oscillations twice and determine the mean period of oscillation ' T_0 '.
4. The two identical masses are placed symmetrically on either side of the suspension wire as close as possible to the centre of the disc, and measure d_1 which is the distance between the centres of the disc and one of the identical masses.
5. Find the time for 10 oscillations twice and determine the mean period of oscillation ' T_1 '.
6. The two identical masses are placed symmetrically on either side of the suspension wire as far as possible to the centre of the disc, and measure d_2 which is the distance between the centres of the disc and one of the identical masses.
7. Find the time for 10 oscillations twice and determine the mean period of oscillation ' T_2 '.
8. Find the moment of inertia of the disc using the given formulae.

Apparatus:

- A uniform wire □ Stopwatch □ Vernier scale
- Two identical □ Screw □ gauge cylindrical masses □ Meter scale etc. Given torsional
- Suitable clamp pendulum

Experimental Data:

(A) Mass of each identical masses, m = 5gm

(B) Length of the suspension wire, *l* =

(C) Radius of the suspension wire, r =0.04cm

(D) d_1= 1.5cm and d_2= 4cm

(E) Radius of the disc, a = 5cm

(F) Mass of the disc or cylinder, M= 1kg=1000g

(G) Moment of Inertia of the cylinder (using simulator), I = $\frac{1}{2}Ma^2$ = 12500 gm cm^2

(H) Table for the time period, T

No. of obs.	Length of the suspension wire, l (cm)	Time for 10 oscillations (s)									Period of oscillation (s)			$T_{o2}/(T_2^2-T_1^2)$	Mean $T_{o2}/(T_2^2-T_1^2)$
		Without mass (t_o s)			With mass at d_1 (t_1 s)			With mass at d_2 (t_2 s)							
		1 (s)	2 (s)	Mean (s)	1 (s)	2 (s)	Mean (s)	1 (s)	2 (s)	Mean (s)	T_o (s)	T_1 (s)	T_2 (s)		
1	35	22.514	22.088	22.301	23.978	23.60	23.789	23.938	23.50	23.754	2.2301	2.389	2.3554	52.67	
2	45	26.487	26.250	26.3685	26.609	26.098	26.3535	26.700	26.303	26.5015	2.6368	2.6353	2.65015	88.88	96.248
3	55	29.171	29.308	29.2395	29.339	29.567	29.453	29.403	29.809	29.606	2.9239	2.9453	2.9606	94.61	
4	65	32.006	32.335	32.1705	31.968	32.1005	32.0342	32.145	32.443	32.294	3.21705	3.2342	3.2294	113.75	
5	75	34.55	34.36	34.45	34.515	34.692	34.603	34.634	35.014	34.82	3.445	3.469	3.482	131.33	

Calculation:

$T_0 = 2.88$s $\quad$ $T_1 = 2.93$s $\quad$ $T_2 = 2.933$s

$$I_0 = 2m\left(d_2^2 - d_1^2\right)\frac{T_0^2}{\left(T_2^2 - T_1^2\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

Moment of inertia of the given disc is, $I_o = 2*5(4\verb|^|2 - 1.5\verb|^|2)\,\dfrac{2.88\verb|^|2}{(2.933\verb|^|2-2.93\verb|^|2)} = 18840.5253$

Or,

I_o = 2*5(4^2-1.5^2)*(96.248) = 13234.1 gm cm^2 = (13234.1/1000)*10^-4 = 0.00132 kg m^2

Difference = [(Experimental Result - Theoretical Result)/Theoretical Result] x 100% = [(13234.1-12500)/12500]*100% = 5.87%

Accuracy = 100% - % Difference = 100%-5.87% = 94.13% ‘

Result:

The moment of inertia of the given disc is, $I_0 = 0.00132$ kg m^2

Discussions:

Q: **How do the length and diameter of the wire affect the period of oscillation of a torsional pendulum?**

Answer:
The length and diameter of the string affects the pendulum's period of oscillation such that the longer the length of the string, the longer the pendulum's period. A pendulum with a longer string has a lower frequency, meaning it swings back and forth less times in a given amount of time than a pendulum with a shorter string length and diameter.

Q: **What type of oscillation did you observe in this experiment? Explain.**

Answer:
A torsion pendulum is analogous to a mass-spring oscillator. Instead of a mass at the end of a helical spring, which oscillates back and forth along a straight line, however, it has a mass at the end of a torsion wire, which rotates back and forth. To set the mass-spring in motion, you displace the mass from its equilibrium position by moving it in a straight line and then releasing it.

Q: **On what factors does the time period of oscillation depend?**

Answer:
There are four factors which act to lengthen the period of the pendulum. 1. The increase in the moment of inertia due to the masses of the added weights. 2. The change in dimensions of the suspending wire. 3. The decreased torsional stiffness of this wire. and 4. The energy used in raising and lowering the disk

Q: **Does the period of oscillation depend on the amplitude of oscillation of the cylinder?**

Answer:
The period of oscillation does depend on the amplitude. The pendulum is not quite a simple harmonic oscillator, but provided the angular amplitude is kept small, this is a small effect.

Q: **How will the period of oscillation be affected if the bob of the pendulum be made heavy?**

Answer:
The mass of the bob does not affect the period of oscillation of a pendulum because the mass of the bob is being accelerated toward the ground at a constant rate - the gravitational constant, g(9.81m/s^2).