(1) Date: 20.08.2019

Class → 09

Date :........../........../..........

Normal Distribution Math:

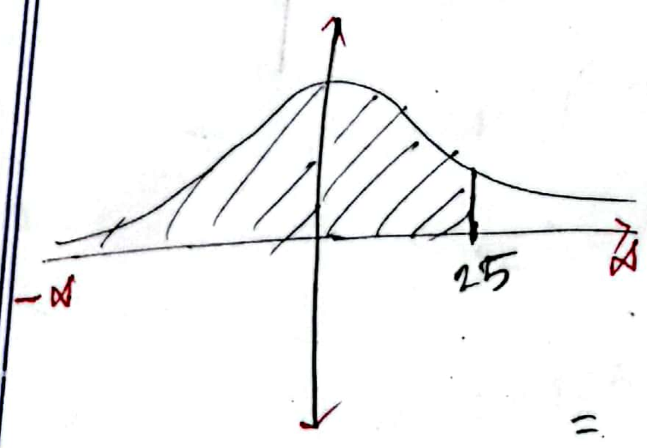
Problem: The weights of bags of red gravel may be modeled by a normally distribution with mean 25.8 kg and S.D = 0.5 kg.

(a) Determine the prob^y that a randomly selected bag of red gravel will weigh:

(i) less than 25 kg

(ii) betn 25.5 kg and 26.5 kg

Soln: (i) $P(X<25) = P\left(Z < \frac{25-25.8}{0.5}\right)$



$= P\left(Z < \frac{-0.8}{0.5}\right)$

$= P(Z < -1.6)$

$= P(Z > 1.6)$

$= 1 - P(Z>1.6)$

$= 1 - 0.94520$

≈ 0.0548

[Ans.]

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(ii) $P(25.5 < X < 26.5)$

$= P(Z < 26.5) - P(Z < 25.5)$

$= P\left(Z < \frac{26.5 - 25.8}{0.5}\right) - P\left(Z < \frac{25.5 - 25.8}{0.5}\right)$

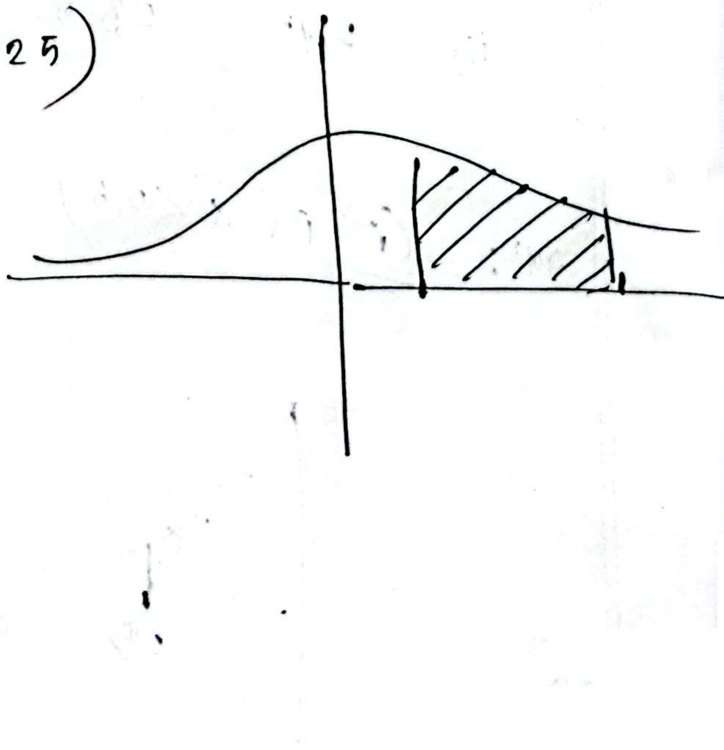
$= P(Z < 1.4) - P(Z < -0.6)$

$= P(Z < 1.4) - P(Z > 0.6)$

$= 0.91924 - (1 - 0.725)$

$= 0.91924 - 0.275$

≈ 0.64424



[Ans.]

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Problem: The volume, L, Litres, of emulsion paint in a plastic tub may be assumed to be normally distributed with mean 10.25 and variance σ^2.

(a) Assuming that $\sigma^2 = 0.04$, determine $P(L<10)$

(b) Find the value of σ so that [98% of tubs contain more than 10 litres] of emulsion paint.

Soln: (a) $L \sim N(10.25, \sigma^2)$

Assume, $\sigma^2 = 0.04$

Now, $P(L<10) = P\left(Z < \frac{10-10.25}{\sqrt{0.04}}\right)$

$= P(Z< -1.25)$

$= P(Z > 1.25)$

$= 1 - P(Z > 1.25)$

≈ 0.105650

(b) Given, $P(L>10) = 0.98$

$\Rightarrow \cancel{P\left(Z > \frac{10-\mu}{\sigma}\right) = 0.98}$

$\Rightarrow \cancel{\phi\left(\frac{10-\mu}{\sigma}\right) = 0.98}$

$\Rightarrow \cancel{\frac{10-\mu}{\sigma} = \phi^{-1}(0}$

$\Rightarrow \cancel{\frac{10-10.25}{\sigma} =}$

(b) $P(L > 10) = 0.98$

$\Rightarrow P\left(Z > \frac{10-\mu}{\sigma}\right) = 0.98$

$\Rightarrow 1 - P\left(Z < \frac{10-\mu}{\sigma}\right) = 0.98$

$\Rightarrow P\left(Z < \frac{10-\mu}{\sigma}\right) = 0.02$

$\Rightarrow \phi\left(\frac{10-\mu}{\sigma}\right) = 0.02$

$\Rightarrow \frac{10-\mu}{\sigma} = \phi^{-1}(0.02)$

$\Rightarrow \phi\left(\frac{10-\mu}{\sigma}\right) = 0.98$

$\Rightarrow \frac{10-10.25}{\sigma} = -\phi^{-1}(0.98)$

$\Rightarrow \frac{10-10.25}{\sigma} = -2.053749$

$\therefore \sigma = 0.1217$

{ 0.5 এর কম হলে '+' sign
0.5 " বেশি হলে '-' sign

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Problem: The weights, W grams, of shaving foam canisters are normally distributed with a mean of 125 and S.D. = 4.

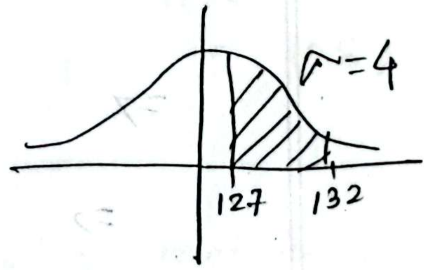
a) Determine prob^y the weight of one such canister will be betn 127 and 132 grams.

b) Find the value of b, so that $P(b < W < 128) = 0.7672$

c) Determine $P(W < 125 \mid W < 128)$

Soln:

a) $P(127 < X < 132)$



$= P(127 < Z < 132)$

$= P\left(Z < \frac{132-125}{4}\right) - P\left(Z < \frac{127-125}{4}\right)$

$= P(Z < 1.75) - P(Z < 0.5)$

$= \phi(1.75) - \phi(0.5) = 0.959 - 0.695$

$\underbrace{\qquad\qquad}_{\text{(From table)}}$

$= 0.2684$

[Ans.]

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(b) $P(b < W < 128) = 0.7672$

$\Rightarrow P(W < 128) - P(W < b) = 0.7672$

$\Rightarrow P\left(z < \frac{128-125}{4}\right) - [1 - P(W > b)] = 0.7672$

$\Rightarrow P(z < 0.75) + P\left(W > \frac{b-125}{4}\right) = 1.7672$

$\Rightarrow \phi(0.75) + P\left(W > \frac{b-125}{4}\right) = 1.7672$

$\Rightarrow 0.7734 + P\left(W > \frac{b-125}{4}\right) = 1.7672$

$\Rightarrow \frac{b-125}{4} = -\phi^{-1}(0.9938)$

$\Rightarrow \frac{b-125}{4} = -2.50$

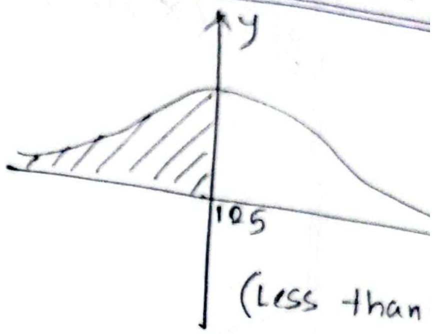
$\Rightarrow b - 125 = -10$

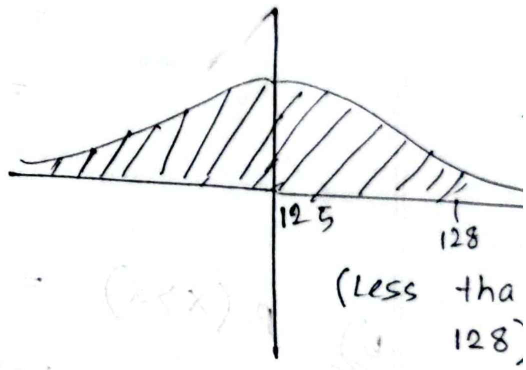
$\therefore b = 125 - 10 = 115$

[Ans.]

c) $P(W < 125 \mid W < 128)$

$= P($





$\boxed{\text{Problem:}}$ The number of miles Mark's motorbike can travel on a full tank of petrol, can be modelled by a Normal distribution with a m[...] of 135 and S.D = 12.

a) Determine the prob^y that Mark can trave[...] at least 165 miles on a full tank of petrol.

b) Find, to the nearest mile, the ~~lon~~ longest journey that Mark can make, if he is to have atleast a 90% chance of completing it on a single full tank of petrol.

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a) $P(X > 165) = 1 - P(X < 165)$

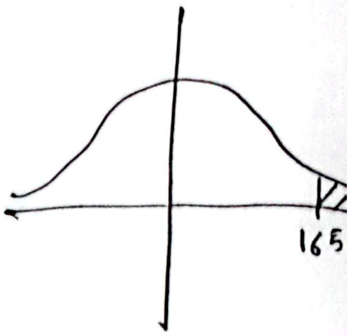
$= 1 - P\left(Z < \frac{165 - 135}{12}\right)$

$= 1 - P(Z < 2.5)$

$= 1 - \phi(2.5)$

$= 1 - 0.9938$

$= 0.0662$



b) $P(X > a) = 90\%$

$\Rightarrow P\left(Z > \frac{a - 135}{12}\right) = 0.9$

$\Rightarrow \frac{a - 135}{12} = -\phi^{-1}(0.9)$

$\Rightarrow \frac{a - 135}{12} = -1.2816$

$\therefore a = 119.62$

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Problem: If the m.g.f of normal variable X is
$M(t) = e^{30t + 18t^2}$.

a) Find a constant K such that $P(|Z| \le K) = 0.954$

b) Evaluate $P(42.6 \le X \le 55.8)$

Soln: a) From given $M_X(t) = e^{30t + 18t^2}$

For normal dist^n, $M_X(t) = e^{\mu t + \frac{\alpha^2 t^2}{2}}$

$M'(t) = e^{\mu t + \frac{\alpha^2 t^2}{2}} \left(\mu + \frac{\alpha^2}{2} 2t\right)$

Here, $\mu = 30$

$\frac{\alpha^2}{2} = 18$

$\Rightarrow \alpha^2 = 36$

$\therefore \alpha = \sqrt{36} = 6$

Then, $P(|Z| \le K) = 0.9544$

$P(Z \le K) = 0.9544$

$\Rightarrow P\left(Z < \frac{K-\mu}{\alpha}\right) = 0.9544$

$\Rightarrow \cancel{1 - P\left(Z > \frac{K-30}{6}\right) = 0.9544}$

$\Rightarrow \phi\left(\frac{K-30}{6}\right) = -0.9544$

$\Rightarrow \frac{K-30}{6} = -\phi^{-1}(0.9544)$

$\Rightarrow K - 30 = -(6 \times 1.69$

$\Rightarrow K = 30 - 10.14$

$\therefore K = 19.86$ [Ans.]

Again, $P(-Z \le K) = 0.95$

$\Rightarrow P(Z \ge K) = 0.954$

$\Rightarrow P\left(Z \ge \frac{K-30}{6}\right) = 0.95$

b) $P(42.6 \leq x \leq 55.8)$

$= P(z < 55.8) - P(z < 42.6)$

$= P\left(z < \frac{55.8-30}{6}\right) - P\left(z < \frac{42.6-30}{6}\right)$

$= P(z < 4.3) - P(z < 2.1)$

$= \phi(4.3) - \phi(2.1)$

$= 0.99996 - 0.98214$

≈ 0.01783 [Ans.]

$\boxed{\text{Hypothesis testing:}}$

Null hypothesis : denoted by H_0
(Possible answer of any event)

Alternative hypothesis : against null hypone:
denoted by H_a.

Some parameters:

$\bar{x}$ = Sample mean

μ = popn mean

σ = popn ~~variance~~ s.d.

S = sample s.d.

n = sample size

Type - I error : denoted by α.

Type - II error: denoted by β.

Mean, $\mu = np$
and ~~v~~s.d, $\sigma = \sqrt{npq}$

Class: 10

Date: 23.08.2023

Hypothesis testing Math:

Problems: Design a decision rule to test the hypothesis that a die is fair if we take a sample of 150 trials of the die to get even/odd faces and use 0.05 as the level of significance. Predict the acceptance and rejection region.

Soln:

Null hypothesis : $H_0 : \mu = 0.5$

Alternative hypothesis: $H_a : \mu \neq 0.5$

Sample size, $n = 150$

Level of significance, $\alpha = 0.05$

As, $n > 30$,

$$Z = \frac{\bar{x} - \mu}{\alpha/\sqrt{n}}$$

This is a two-tailed test, so,

$$P\left(-Z_{\alpha/2} < Z < Z_{\alpha/2}\right) = 1 - \frac{\alpha}{2}$$

$$\Rightarrow \phi\left(Z_{\alpha/2}\right) = 1 - \frac{0.05}{2}$$

$$\Rightarrow \phi\left(Z_{\alpha/2}\right) = 0.975$$

$$\therefore Z_{\alpha/2} = 1.96$$

$\mu = np = 150 \times 0.5 = 75$

and $\sigma = \sqrt{npq} = 6.124$

$p = 0.5 ; q = 0.5$

we will accept the claim if,

$-Z_{\alpha/2} \le Z \le Z_{\alpha/2}$.

$\Rightarrow -Z_{\alpha/2} \le \frac{\bar{x} - \mu}{\sigma} \le Z_{\alpha/2}$

$\Rightarrow \mu - (Z_{\alpha/2} \times \sigma) \le \frac{\bar{x}}{\sigma} \le \mu + (Z_{\alpha/2} \times$

$\Rightarrow 75 - (1.96 \times 6.124) \le \bar{x} \le 75 + (1.96$

$\therefore 62.997 \le \bar{X} \le 87.003$

Problem: 02) A pharmaceutical company produces a new medicine and they claimed that it will reduce the migraine pain ve fast with 95% accuracy. Now, design decision rule for the process with significance 0.01 by applying the medicine to 120 people. Make decision if 115 people get releif from the migraine pain by using medicine.

n

Soln:

Null : $H_0 : \mu = 0.95$

Alternative : $H_a : \mu < 0.95$

Now, $P(Z > -Z\alpha) = 1 - \alpha$

$\Rightarrow \phi(Z\alpha) = 1 - 0.01$

$\Rightarrow Z\alpha = \phi^{-1}(0.99)$

$\therefore Z\alpha = 2.33$

Then, $\mu = np = 120 \times 0.95 = 114$

$p = 0.95$
$q = 0.05$ $\quad \sigma = \sqrt{npq} = 2.38$ $\quad q = 5\% = 0.05$

Now, $\cancel{P(Z > -Z\alpha) =}$

$$\frac{\bar{x} - \mu}{\sigma/\sqrt{n}} > -Z\alpha$$

$$\Rightarrow \bar{x} > \mu - Z\alpha \cdot$$

$$> 0.95 - (2.3$$

$Z > -Z\alpha$

or, $\frac{\bar{x} - \mu}{\sigma} > -Z\alpha$

or, $\bar{x} > \mu - Z\alpha\,\sigma$

or, $\bar{x} > 114 - (2.33 \times 2.38)$

$\bar{x} > 108.4546$

claim will be Accepted if more than 109 people get relief from pain.

Question: Problem 03) A company [illegible] electric bulbs whose average lifetime is 180 days and an average variation of 15 days. It is claimed that by using a developed process the mean lifetime of the bulbs can be increased

(i) Design a de. Rule at 0.01 S.L. to test 100 bulbs. Also, what about the decision if A. lifetime of a bulb 182 and 197 days.

(ii) If the new process has increased the mean lifetime to 192 days and assuming the estimated sample mean 187 days, find α and β for 30 samples.

Problem: 03)

(i) Null hypothesis : $H_0 : \mu = 180$

Alternative : $H_a : \mu > 180$

Right-tailed test,

$$P(Z > Z_\alpha) = 1 - \alpha$$

$$\Rightarrow P(Z > Z_\alpha) = 1 - 0.01$$

$$\Rightarrow P(Z > Z_\alpha) = 0.99$$

$$\Rightarrow \phi(Z_\alpha) = 0.99$$

$$\therefore Z_\alpha = \phi^{-1}(0.99) = 2.33$$

Then,

$\mu = 180$

$\sigma = 15$

$n = 100$

$$Z > Z_\alpha$$

$$\Rightarrow \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} > Z_\alpha$$

$$\Rightarrow \bar{x} > \mu + Z_\alpha \cdot \frac{\sigma}{\sqrt{n}}$$

$$\Rightarrow \bar{x} > 180 + \left(2.33 \times \frac{15}{\sqrt{100}}\right)$$

$$\therefore \bar{x} > 183.495$$

claim will be accepted if the lifetime increased more than 183.495 days.

(ii) Ha : $\mu > 192$

$n = 30$; sample mean, $\bar{x} = 187$

α = Type I error (Ho reject but true)

$= P(\bar{x} \geq 187 \; ; \mu = 180)$

$= P\left(\dfrac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \geq \dfrac{187 - 180}{\frac{15}{\sqrt{30}}}\right)$

$= P(Z \geq 2.56) = 1 - P(Z < 2.56)$

$= \phi(2.56)$

$= 0.0052$

β = Type II error (Ho accept but false)

$= P(\bar{x} < 18.7 \; ; \mu = 192)$

$= P\left(\dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}} < \dfrac{187 - 192}{15/\sqrt{30}}\right)$

$= P(Z < -\cancel{18.3}\,1.83) = 1 - P(Z > 1.83)$

$= 0.034$

[Ans.]

Question : $\boxed{\text{Problem: 04)}}$

A telecom service provider claims that individual customers pay on an average 400 rs. per month with s.d. of 25 rs. A sample of 50 customers bills during a given month is taken with a mean of 250 and s.d. of 15. What to say with r.t. the claim by the service provider?

$\boxed{\text{Soln:}}$

Problem: 04) Null, $H_0 : \mu = 400$

Alternative, $H_1 : \mu \neq 400$

sample size, $n = 50$

s.d, $\sigma = 25$

sample mean, $\bar{x} = 250$

sample s.d., $s = 15$

Now, $P\left(-Z_{\alpha/2} < Z < Z_{\alpha/2}\right) = 1 - \alpha/2$

$\Rightarrow \phi(Z_{\alpha/2}) = 1 - 0.025$

$= 0.975$

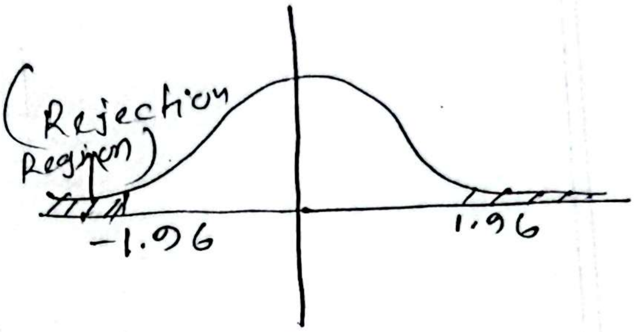
$\therefore Z_{\alpha/2} = 1.96$

As $n \geq 30$,

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$= \frac{250 - 400}{25/\sqrt{50}}$$

$$= -42.42$$



So, H_0 is rejected here.

$X \sim$ Normal dist^n

$\boxed{X}$

(a^2 / a known)

yes

(z - test)

$$z = \frac{\bar{x} - \mu}{a/\sqrt{n}}$$

no

($n \geq 30$)

(yes)

z-test

No

(t- test)

$$\frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

If $\bar{x}$ is not given, then, $z = \frac{\bar{x} - \mu}{a}$ is used.

One- tail test

$1 - \alpha$ If: $H_a > \square$

$H_a < \square$

two - tail test

$1 - \frac{\alpha}{2}$ If $H_a \neq \square$

α = level of significance

standard value of α = 0.05