HYPOTHESIS TESTS ABOUT THE MEAN AND PROPORTION

HYPOTHESIS TESTS AN INTRODUCTION

- A soft drink company claims that its cans of soft drinks , on an average, contain 12 ounces of soda.
- **How can the claim be verified?**
  - A first approach may be to collect all the cans of soft drink produced by the company, and determine the mean amount of soda in all the cans.
  - A second approach could be to collect a sample of say 100 cans of the soft drink produced by the company, and determined the mean amount of soda in all the cans, for example to be 11.8 ounces.
- **Which of the approaches should we adopt?**
  - The first approach will yield the true mean amount of soda in all the cans. However, this approach would be unrealistic, un-achievable, and cost prohibitive.
  - The second approach may or may not yield a true mean amount of soda in all cans. It is, however, manageable and produce a point estimate of the population mean amount of soda in all the cans. That is,

$$\textbf{Point estimate for } \mu = \bar{x}$$

Hypotheses Test - Introduction

- We are done with this chapter if we chose the first approach.
- So let us examine the second approach further.
  - Can we conclude that the mean amount of soda in all cans is 11.80 ounces, and that the company's claim is false?
  - Not so fast. If there is no nonsampling error, the difference between 11.80 and 12 ounces could have been due to the selected cans in the sample (chance)
  - A different sample could have produced a mean amount of 12.04 ounces.
- **So what can we do to verify the company's claim?**
  - We conduct **hypothesis test** to determine how large is the difference, for example, between 11.80 and 12.0 ounces, and
  - Whether the difference is due to chance.

General Statement

A test of hypothesis is performed whenever a decision is made about a population based on the value of a sample statistic.

Two Hypotheses

Introduction

- In conducting a hypothesis test for our example, we examine two statements:
    1. The company claim is true.
    2. The company claim is false.
- These statements are examined under two hypotheses, **null hypothesis**, denoted by H_0 and **alternative hypothesis**, denoted by H_1.

Definitions

- Null hypothesis is a claim or statement about a population parameter that is assumed to be true until it is proven to be false. In our example, null hypothesis is represented as,

$$\mathbf{H_0:}\ \mu = 12 \ \textbf{ounces} \ \ \textbf{(company's claim is true)}$$

- Alternative hypothesis is a claim about a population parameter which will be true if the null hypothesis is false. In our example, alternative hypothesis is represented as,

$$\mathbf{H_1:}\ \mu < 12 \ \textbf{ounces} \ \ \textbf{(company's claim is false)}$$

Two Hypotheses

- The information we obtained from a sample will be used to verify the company's claim.
  1. That is, the information will confirm whether the null hypothesis, H_0, should be rejected and the alternative hypothesis, H_1, accepted, or
  2. The H_0 should not be rejected.

Two Types of Error

- Like any decision, there is always a possibility of making a wrong decision. The two possible decisions from our example are:
  1. We do not reject H_0.
  2. We reject H_0, that is, H_1 is true.
- There are, however, four possible outcomes from our example:
  1. We correctly do not reject H_0. (company's claim is true).
  2. We correctly rejected H_0 and accepted H_1. (company's claim is false).
  3. We rejected H_0 when we should have not rejected it.
  4. We do not reject H_0 when we should have rejected it
- The types of errors in (3) and (4) are called **Type I** or α errors and **Type II** or β errors, respectively.

Two Type of Error

Definitions

- Type I or α error occurs when a true null hypothesis is rejected.
  1. The value of α is the probability of committing Type I error, and is denoted as,
  $$\alpha = \mathbf{P(H_0 \text{ is rejected} \mid H_0 \text{ is true})}$$
  α also represents the significance level of the test.
  2. The size of the rejection region depends on α
  3. α can be assigned any value, but usually the value does not exceed 0.10.
  4. Common values for α are 0.01, 0.025, 0.05, and 0.10.

- Type II or β error occurs when a false hypothesis is not rejected.
  1. The value of β is the probability of committing this error, and is denoted as, $\beta = \mathbf{P(H_0 \text{ is not rejected} \mid H_0 \text{ is false})}$
  2. The value of $1-\beta$ is called power of the test and represents the probability of not making a Type II error.

Two Type of Error

- The four possible outcomes of our example are represented in Table 1 below.

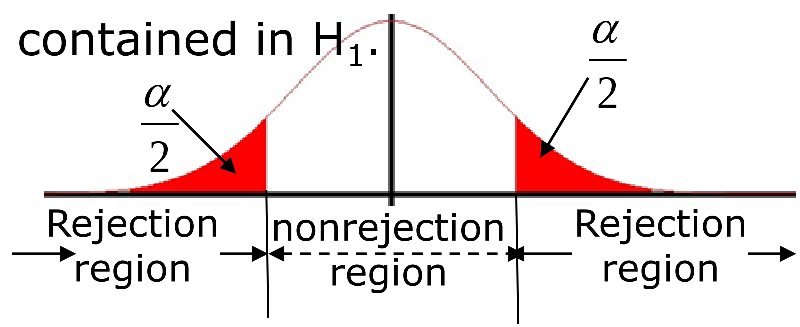
		Actual Solution	
		H_0: Is true	H_0: False
Our Decision	H_0: Do not reject	Correct	Type II error
	H_0: Reject	Type I error	Correct

General Statement

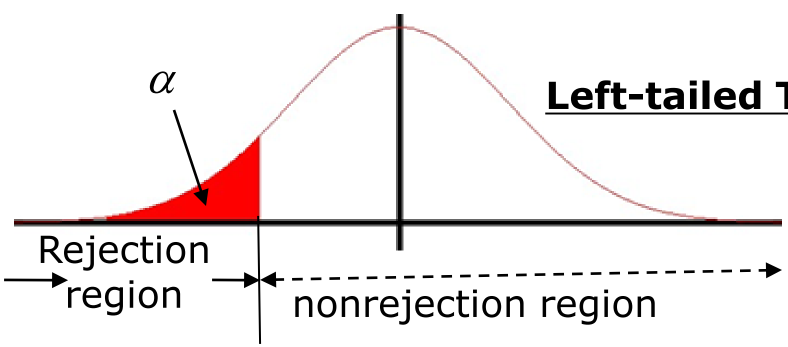
- α and β depend on each other. That is an increase/decrease of one will result in a decrease/increase of the other for a fixed sample size
- Increase in sample size will simultaneously decrease both α and β.

Tails of a Test

- The outcomes of every test of hypothesis are classified into a **rejection** region or **nonrejection** region.
- The partition of the total region depends on the value of α
- The rejection region can be on:
  1. Both sides of the nonrejection region (two-tailed test).
  2. The left side of the nonrejection region (left-tailed test).
  3. The right side of the nonrejection region (right-tailed test)
- The type of tail test is based on the sign contained in H_1.
  - "$\neq$" signifies two-tailed test.
  - "<" signifies left-tailed test.
  - ">" signifies right-tailed test.

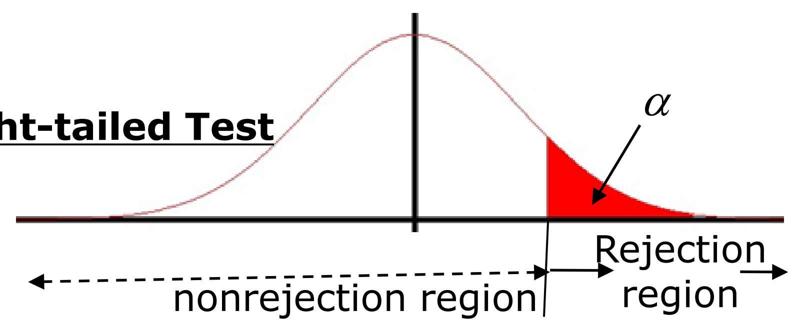


Two-tailed Test



Left-tailed Test

Right-tailed Test



Signs in H_0 and H_1 and Tails of a Test

	Two-Tailed Test	Left-Tailed Test	Right-Tailed Test
Sign in the null hypothesis H_0	$=$	$=$ or $\geq$	$=$ or $\leq$
Sign in the alternative hypothesis H_1	$\neq$	$<$	$>$
Rejection region	In both tails	In the left tail	In the right tail

Tails of a Test – A two-tailed test

Definition

A two-tailed test of hypothesis has two rejection regions, one in each tail of the distribution curve and a nonrejection region in the middle.

Issue

In 1980, a sample survey of 3000 newly born children in U.S. was conducted to determine their mean weight. The survey showed that the mean weight was 7 pounds. A group of STA120 students has decided to verify whether the mean weight of newly born children has changed since 1980.

Analysis

- Let μ = the mean weight of the newly born children in U.S. since 1980. That is, $\mu = 7$ **pounds.**
- The two possible decisions from this analyzes are:
  - The mean weight of the children has not changed. That is, $\mu = 7$ **pounds.**
  - The mean weight of the children has changed. That is, $\mu \neq 7$ **pounds.**
- So, the null and alternative hypotheses can be written as,

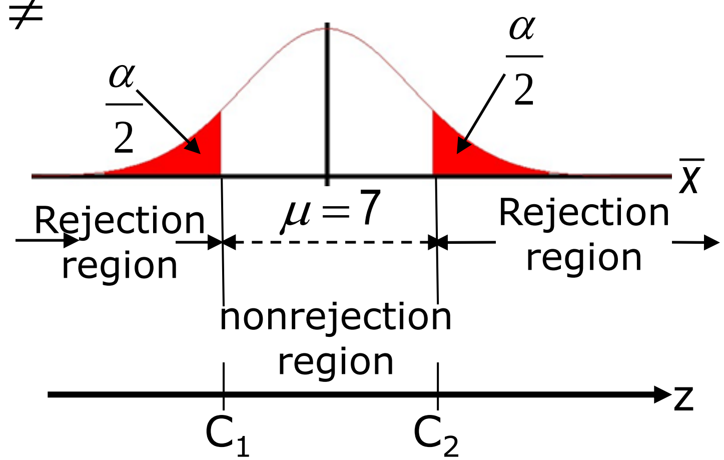
H_0: $\mu = 7$ **pounds (mean weight has not changed)**

H_1: $\mu \neq 7$ **pounds (mean weight has changed)**

Tails of a Test – A two-tailed test

Analysis

- This test is two-tailed because H_1 contains "$\neq$"
- The figure to the right shows the sampling distribution of $\bar{x}$ assuming that it is normally distributed.
- The figure also assumes that H_0 is true and the mean weight is equal to 7 pounds.
- The area in each tail under the curve is $\frac{\alpha}{2}$
- The sum of the areas is α.
- As shown in the figure, a two-tailed test has two critical values, C_1 and C_2, which separate the rejection regions from nonrejection region



- **Decision**
  - Reject H_0 if the value of $\bar{x}$ falls within one of the rejection regions. $(\mu - \bar{x})$ is too large to have been caused only be sampling errors.
  - Do not reject H_0 if the value of $\bar{x}$ falls within the nonrejection region. $(\mu - \bar{x})$ is too small to assume that the difference is caused by sampling errors.

Tails of a Test – A left-tailed test

Definition

A left-tailed test applies when the rejection region is to the left of the nonrejection region.

Issue

In the case of the company claiming that its cans of soft drinks, on an average, contain 12 ounces of soda.

Analysis

- The two possible decisions from this analyzes are:
  - The mean amount of soda in the cans of soft drinks is equal to 12 ounces. That is, $\mu \geq 12$ oz.
  - The mean amount of soda in the cans of soft drinks is less than 12 ounces. That is, $\mu < 12$ **oz.**
- Then, the null and alternative hypotheses can be written as,

$$\mathbf{H_0}\text{: } \mu = 12 \textbf{ oz } (\mu \geq 12 \textbf{ oz})$$

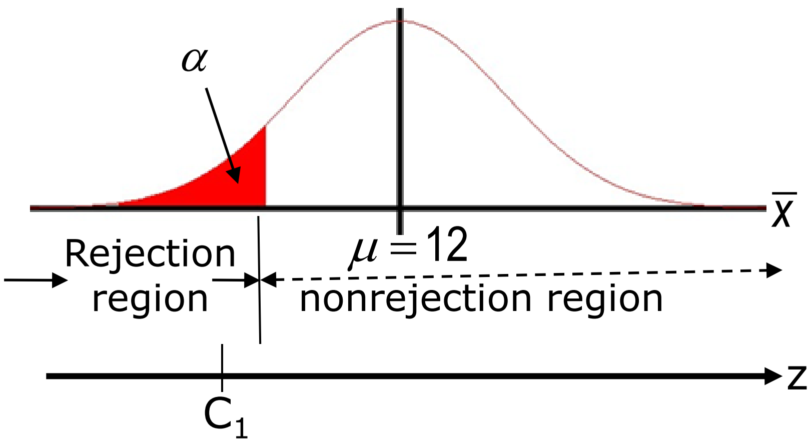
$$\mathbf{H_1}\text{: } \mu < 12 \textbf{ oz}$$

- Since the sign in H_1 is "<", this test is left-tailed.

Tails of a Test – A left-tailed test

Analysis

- The figure below shows the sampling distribution of $\bar{x}$ assuming that it is normally distributed.
- The figure also assumes that H_0 is true and the sampling distribution has a mean of 12 ounces.
- The area in the left tail under the curve is α.
- As shown below, a left-tailed test has one critical values, C_1, which separate the rejection region from nonrejection region.
- **Decision**
  - Reject H_0 if the value of $\bar{x}$ falls within the rejection region. $(\mu - \bar{x})$ is too large to have been caused only be sampling errors.
  - Do not reject H_0 if the value of $\bar{x}$ falls within the nonrejection region. $(\mu - \bar{x})$ is too small to assume that the difference is caused by sampling errors.



Tails of a Test – A right-tailed test

Definition

A right-tailed test applies when the rejection region is to the right of the nonrejection region.

Issue

In 2002, the average income of entry level engineers is \$63,000. Suppose we want to verify whether the current mean income of entry level engineers is greater than \$63,000.

Analysis

- The two possible decisions from the test of hypothesis are:
  - The mean income of entry level engineers is not higher than \$63,000. That is, $\mu = \$63,000$.
  - The mean income of entry level engineers is higher than \$63,000. That is, $\mu > \$63,000$.
- Then, the null and alternative hypotheses can be written as,

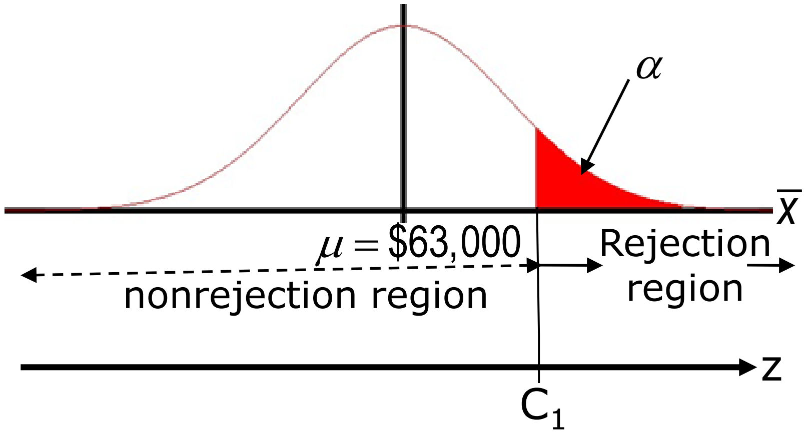
$$\mathbf{H_0}\mathbf{:}\ \mu = \$63,000$$
$$\mathbf{H_1}\mathbf{:}\ \mu > \$63,000$$

- Since the sign in H_1 is ">", this test is right-tailed.

Tails of a Test – A right-tailed test

Analysis

- The figure below shows the sampling distribution of $\bar{x}$ assuming that it is normally distributed.
- The figure also assumes that H_0 is true and the sampling distribution has a mean of \$63,000.
- The area in the right tail under the curve is α.
- As shown below, a right-tailed test has one critical values, C_1, which separate the rejection region from the nonrejection region.
- **Decision**
  - Reject H_0 if the value of $\bar{x}$ falls within the rejection region. $(\mu - \bar{x})$ is too large to have been caused only be sampling errors.
  - Do not reject H_0 if the value of $\bar{x}$ falls within the nonrejection region. $(\mu - \bar{x})$ is too small to assume that the difference is caused by sampling errors.



Hypothesis Tests - Introduction

Example #1

Explain which of the following is a two-tailed test, a left-tailed test, or a right-tailed test.

a. $H_0: \mu = 12,\ \ H_1: \mu < 12;$ b. $H_0: \mu \le 85,\ \ H_1: \mu > 85;$ c. $H_0: \mu = 33,\ \ H_1: \mu \ne 33;$

Show the rejection and nonrejection regions for each of these cases by drawing a sampling distribution curve for the sample mean, assuming that it is normally distributed

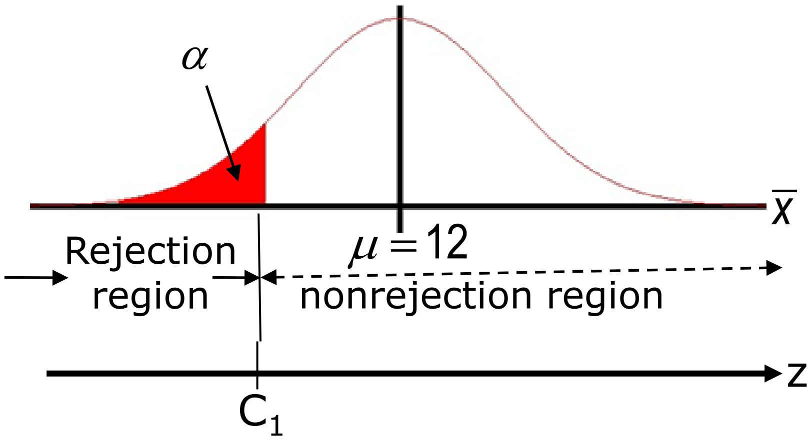
Example #1 – Solution

a. Given:

$$H_0: \mu = 12$$

$$H_1: \mu < 12$$

This is a left-tailed test because the sign "<" is contained in H_1.



Hypothesis Tests - Introduction

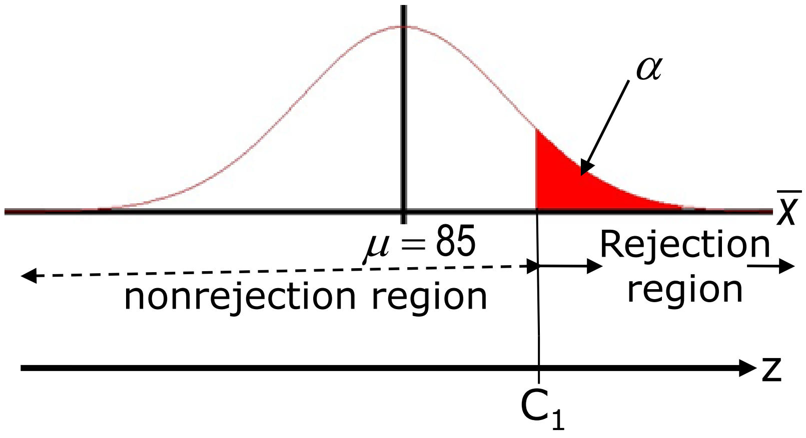
Example #1 – Solution

b. Given:

$H_0 : \mu \leq 85$

$H_1 : \mu > 85$

This is a right-tailed test because the sign ">" is contained in H_1.

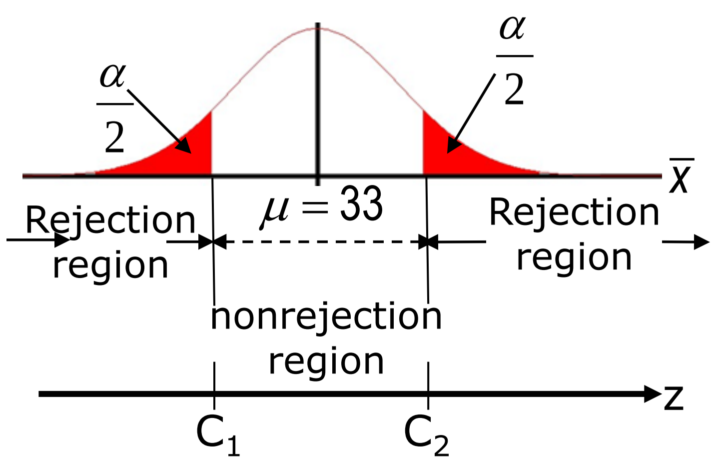


c. Given:

$H_0 : \mu = 33$

$H_1 : \mu \neq 33$

This is a two-tailed test because the sign "$\neq$" is contained in H_1.



Hypothesis Tests - Introduction

Example #2

Write the null and alternative hypotheses for each of the following examples. Determine if each is a case of a two-tailed test, a left-tailed test, or a right-tailed test.

a. To test if the mean number of hours spent working per week by college students who hold jobs is different from 20 hours.
b. To test whether or not a bank's ATM is out of service for an average of more than 10 hours per month.

Example #2 – Solution

a. Let $\mu =$ mean number of hours spent working per week by college students who hold jobs.

The two possible decisions are:

(1) The mean number of hour spent working per week by college students who hold jobs is not different from 20 hours. That is, $H_0 : \mu = 20$ hr

(2) The mean number of hour spent working per week by college students who hold jobs is different from 20 hours. That is, $H_0 : \mu \neq 20$ hr

Hypotheis: $H_0 : \mu \neq 20$ hr

$H_1 : \mu \neq 20$ hr

Thus, this is a case of a two-tailed test because the sign "$\neq$" is contained in H_1.

Hypothesis Tests - Introduction

Example #2

Write the null and alternative hypotheses for each of the following examples. Determine if each is a case of a two-tailed test, a left-tailed test, or a right-tailed test.

b. To test whether or not a bank's ATM is out of service for an average of more than 10 hours per month.

Example #2 – Solution

b. Let $\mu =$ mean number of hours per month a bank's ATM was out of service.

The two possible decisions are:

(1) The mean number of hours per month a bank's ATM was out of service is not more than 10 hours per month. That is, $H_0 : \mu \leq 10$ hr

(2) The mean number of hours per month a bank's ATM was out of service is more than 10 hours per month. That is, $H_0 : \mu > 10$ hr

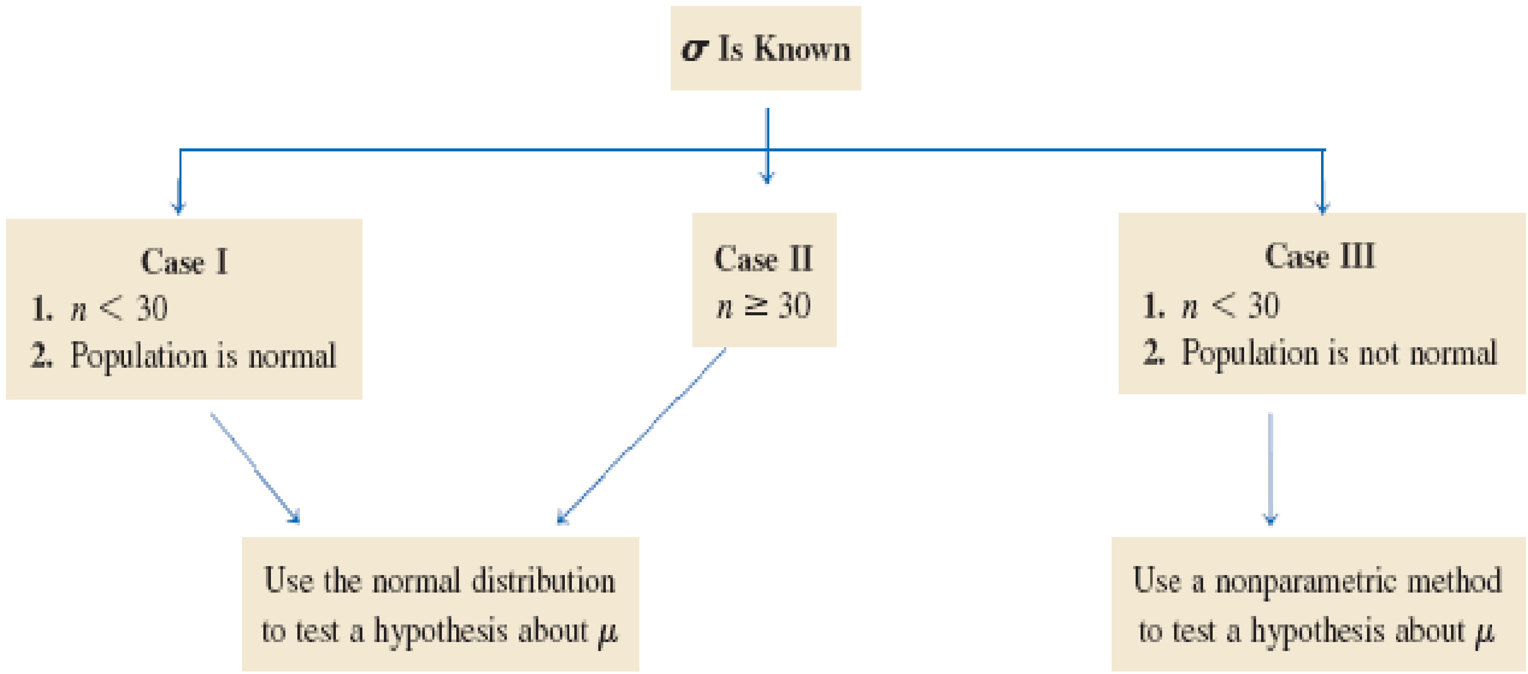
Hypotheis: $H_0 : \mu \leq 10$ hours per month

$H_1 : \mu > 10$ hous per month

Thus, this is a case of a right-tailed test because the sign ">" is contained in H_1.

HYPOTHESIS TESTS ABOUT μ: σ KNOWN

Three Possible Cases



HYPOTHESIS TESTS ABOUT μ: σ KNOWN

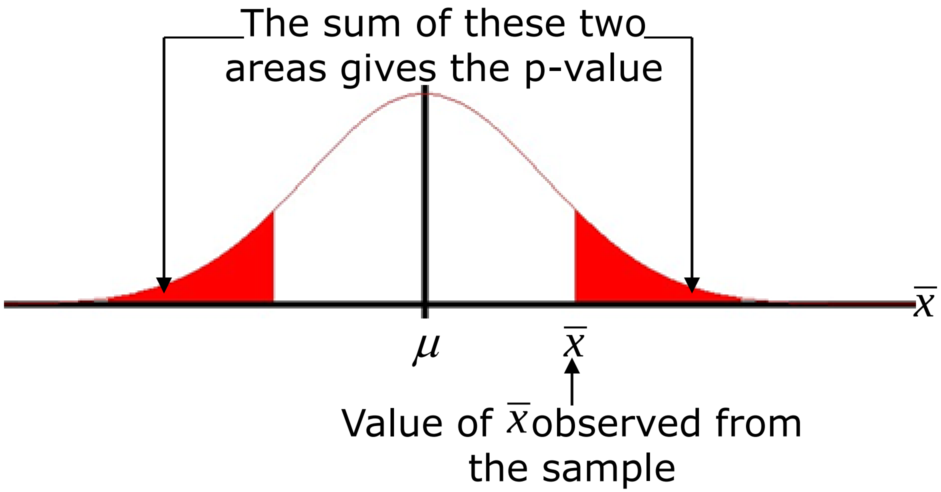
There are two techniques for performing the tests of hypothesis about μ. These techniques are the:

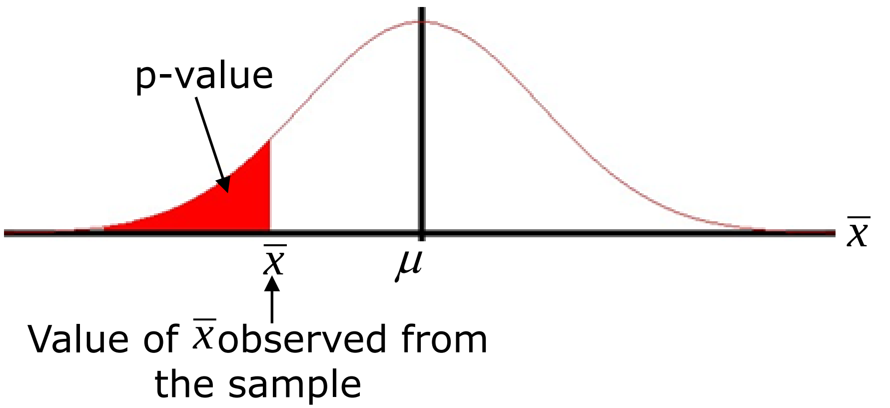
- p-Value approach
- Critical-Value approach

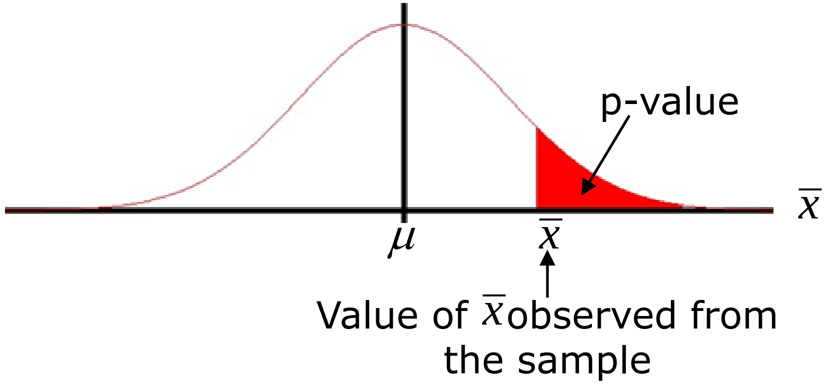
<u>The p-Value Approach</u>

- The p-value approach determines a probability value such that the null hypothesis, H_0, is:
  - Rejected for any α greater than the value.
  - Not rejected for any α less than the value.
- Thus, p-value is the smallest significance value at which the null hypothesis, H_0, is rejected.
- If α is unknown, the p-value is used to make/state a decision.
- If α is known, the p-value is compared to the α and a decision is made.
- Our decision could be:
  1. Reject H_0 if $\text{p-value} < \alpha$ or $\alpha > \text{p-value}$
  2. Do not reject H_0 if $\text{p-value} \geq \alpha$ or $\alpha \leq \text{p-value}$

The *p*–value for a left-tailed, two-tailed and right-tailed tests







The p-value Approach

- To find the p-value, we first have to find the z-value for $\overline{x}$ by using the formula,

$$z = \frac{\overline{x} - \mu}{\sigma_{\overline{x}}} \quad \text{where} \quad \sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$$

- The calculated z-value is also called the observed value of z
- We can find the area in the tail of the curve beyond the value of z by using the standard normal distribution table. This area is the p-value (one-tailed) or ½ of the p-value (2-tailed).

Steps for a test of hypothesis using the p-Value Approach

1. State the null and alternative hypothesis
2. Select the distribution to use (z or t distribution)
3. Calculate the p-value
4. Make a decision

The p-value Approach

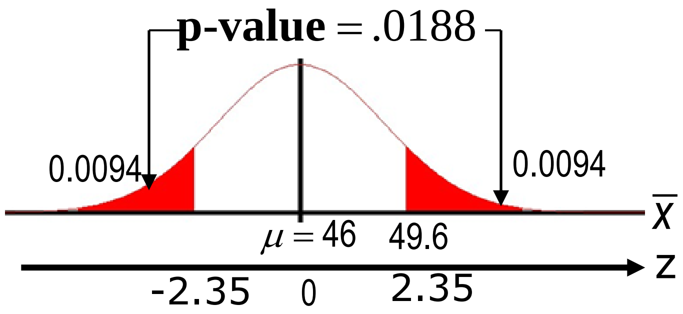
Example #3

Find the p-value for each of the following hypothesis tests.

a. $H_0: \mu = 46, \quad H_1: \mu \neq 46, \quad n = 40, \quad \bar{x} = 49.60, \quad \sigma = 9.7$

Example #3 – Solution

a. $H_0: \mu = 46, \quad H_1: \mu \neq 46, \quad n = 40, \quad \bar{x} = 49.60, \quad \sigma = 9.7$



(i) Null and alternative hypothesis

$H_0: \mu = 46$

$H_1: \mu \neq 46$

(ii) Select distribution to use

Since $n > 30$, sampling distribution of $\bar{x}$ is normal (CLT). Thus, we can use normal distribution to find p-value.

(iii) Calculate the p-value

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{9.7}{\sqrt{40}} = 1.5337 \quad \text{and} \quad z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{49.60 - 46}{1.5337} = 2.35$$

Thus, the area to the right of $x = 49.60$ is equal to the area under the normal curve to the right of $z = 2.35$.

From Table IV, the area to the right of $z = 2.35$ is $= 1 - .9906 = .0094$.

Since this test is two-tailed, then the p-value $= 2(.0094) = 0.0188$

The p-value Approach

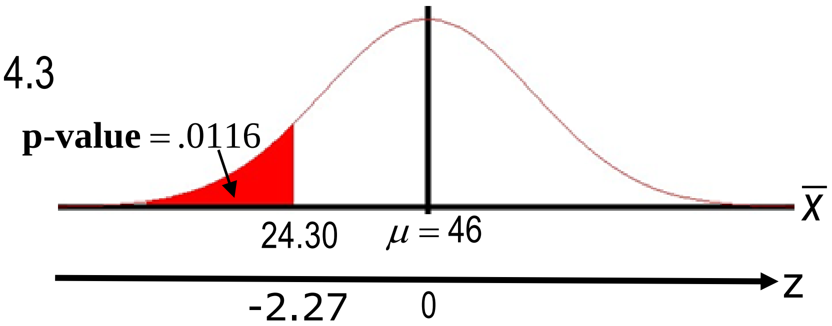
Example #3

Find the p-value for each of the following hypothesis tests.

b. $H_0 : \mu = 26, \quad H_1 : \mu < 26, \quad n = 33, \quad \overline{x} = 24.30, \quad \sigma = 4.3$

Example #3 – Solution

b. $H_0 : \mu = 26, \quad H_1 : \mu < 26, \quad n = 33, \quad \overline{x} = 24.30, \quad \sigma = 4.3$



(i) Null and alternative hypothesis

$H_0 : \mu = 26$

$H_1 : \mu < 26$

(ii) Select distribution to use

Since $n > 30$, sampling distribution of $\overline{x}$ is normal (CLT). Thus, we can use normal distribution to find p-value.

(iii) Calculate the p-value

$$\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}} = \frac{4.3}{\sqrt{33}} = .7485 \text{ and } z = \frac{\overline{x} - \mu}{\sigma_{\overline{x}}} = \frac{24.30 - 26}{.7485} = -2.27$$

From Table IV, the area to the left of $z = -2.27$ is $= 0.0116$.

Since this test is left-tailed, then the p-value $= .0116$.

The p-value Approach

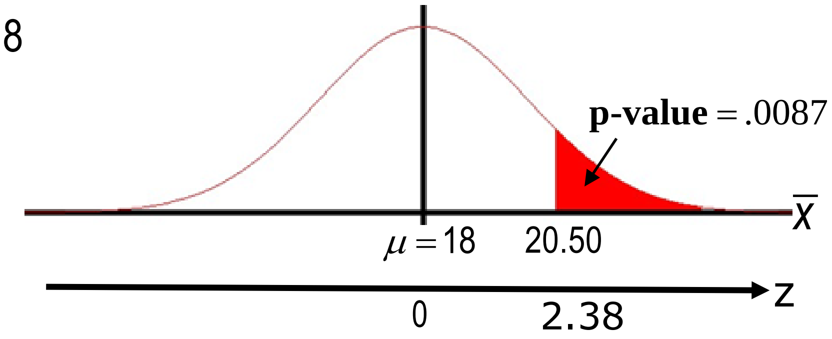
Example #3

Find the p-value for each of the following hypothesis tests.

c. $H_0: \mu = 18, \quad H_1: \mu > 18, \quad n = 55, \quad \bar{x} = 20.50, \quad \sigma = 7.8$

Example #3 – Solution

c. $H_0: \mu = 18, \quad H_1: \mu > 18, \quad n = 55, \quad \bar{x} = 20.50, \quad \sigma = 7.8$



(i) Null and alternative hypothesis

$H_0: \mu = 18$

$H_1: \mu > 18$

(ii) Select distribution to use

Since $n > 30$, sampling distribution of $\bar{x}$ is normal (CLT). Thus, we can use normal distribution to find p-value.

(iii) Calculate the p-value

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{7.8}{\sqrt{55}} = 1.0518 \text{ and } z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{20.50 - 18}{1.0518} = 2.38$$

From Table IV, the area to the right of $z = 2.38$ is $= 1 - .9913 = .0087$

Since this test is right-tailed, then the p-value $= .0087$.

The p-value Approach

Example #4

Consider $H_0 : \mu = 72$ versus $H_1 : \mu > 72$. A random sample of 16 observations taken from this population produced a sample mean of 75.2. The population is normally distributed with $\sigma = 6$.

a) Calculate p-value.

Example #4 – Solution

$\text{Given} : n = 16,\ \bar{x} = 75.2, \sigma = 6.0$

(i) Null and alternative hypotheses

$$H_0 : \mu = 72$$

$$H_1 : \mu > 72$$

(ii) Distribution

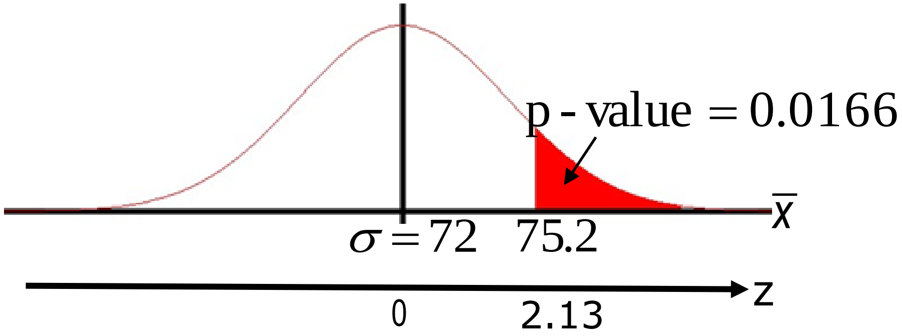
Since population is normally distributed, then the sampling distibution of $\bar{x}$ is normally distributed with a mean of μ and a standard deviation of $\bar{x}$ is $\sigma_{\bar{x}} = \sigma / \sqrt{n}$. Thus, we can use normal distribution to find p-value and perform the test.

The p-value Approach

Example #4

Consider $H_0: \mu = 72$ versus $H_1: \mu > 72$. A random sample of 16 observations taken from this population produced a sample mean of 75.2. The population is normally distributed with $\sigma = 6$.

a) Calculate p-value.



Example #4 – Solution

$\text{Given}: n = 16,\ \bar{x} = 75.2, \sigma = 6.0$

(iii) Calculate p - value

This is a right - tailed test because ">" is contained in H_1.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{6}{\sqrt{16}} = 1.5 \text{ and } z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{75.2 - 72}{1.5} = 2.1333 \approx 2.13$$

The area in the right tail of the curve to the right of $z = 2.13$ is equal to

$$1.0 - .9834 = 0.0166$$

Since this test is right - tailed, then p - value $= 0.0166$

The p-value Approach

Example #4

Consider $H_0: \mu = 72$ versus $H_1: \mu > 72$. A random sample of 16 observations taken from this population produced a sample mean of 75.2. The population is normally distributed with $\sigma = 6$.

b) Considering the p-value of part a, would you reject the null hypothesis if the test were made at the significance level of .01?

c) Considering the p-value of part a, would you reject the null hypothesis if the test were made at the significance level of .025?

Example #4 – Solution

b. Given: $n = 16$, $\bar{x} = 75.2$, $\sigma = 6.0$, $\alpha = 0.01$ and from part a p-value $= 0.0166$

Since $\alpha = 0.01 <$ p-value $= 0.0166$, then we do not reject H_0, and conclude that the mean is not different from 72.

c. Given: $n = 16$, $\bar{x} = 75.2$, $\sigma = 6.0$, $\alpha = 0.025$ and from part a p-value $= 0.0166$

Since $\alpha = 0.025 >$ p-value $= 0.0166$, then we reject H_0, and conclude that the mean is greater than 72.

HYPOTHESIS TESTS ABOUT μ: σ KNOWN

The Critical-Value Approach

- The critical-value approach is also called a classical or traditional approach.
- This approach relies on a predetermined significance level, α which represents the rejection region under the normal distribution curve.
- It involves:

1. Finding the critical value of the test statistic, z_1, based on a given significance level α .
2. Calculating the value of the test statistic, z_2, for the observed value of sample statistic, $\bar{x}$.

$$z_2 = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} \text{ where } \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

3. Compare (1) and (2) and make decision as follows:
   a. For left-tailed test, **reject H_0** if $z_1 > z_2$ in (2) and **do not reject H_0** if $z_1 < z$ in (2),
   b. For right-tailed test, **reject H_0** if $z_1 < z_2$, and **do not reject H_0** if $z_1 > z_2$, and
   c. For two-tailed test, **reject H_0** if z_2 is not between the z_1's and **do not reject H_0** if z_2 is between the z_1's.

The Critical-Value Approach

Steps for a test of hypothesis using the critical-Value Approach

1. State the null and alternative hypothesis
2. Select the distribution to use (z or t distribution)
3. Determine rejection and nonrejection regions
4. Calculate the value of test statistic, z-value
5. Make a decision

General Statements

1. Some authors used the phrase "statistically significantly different" or "statistically not significantly different" to make conclusion on hypothesis test.
2. The phrase "statistically significantly different" means that the difference between the observed value of $\bar{x}$ and the hypothesized value of the μ is so large that it is not could not only be caused by sampling errors. A such the null hypothesis is rejected.
3. The phrase "statistically not significantly different" means that the difference between $\bar{x} - \mu$ is so small that it may have occurred only because of sampling error. A such the null hypothesis is not rejected.

Example

The TIV Telephone Company provides long-distance telephone service in an area. According to the company's records, the average length of all long-distance calls placed through this company in 2009 was 12.44 minutes. The company's management wanted to check if the mean length of the current long-distance calls is different from 12.44 minutes. A sample of 150 such calls placed through this company produced a mean length of 13.71 minutes with a standard deviation of 2.65 minutes. Using the 2% significance level, can you conclude that the mean length of all current long-distance calls is different from 12.44 minutes?

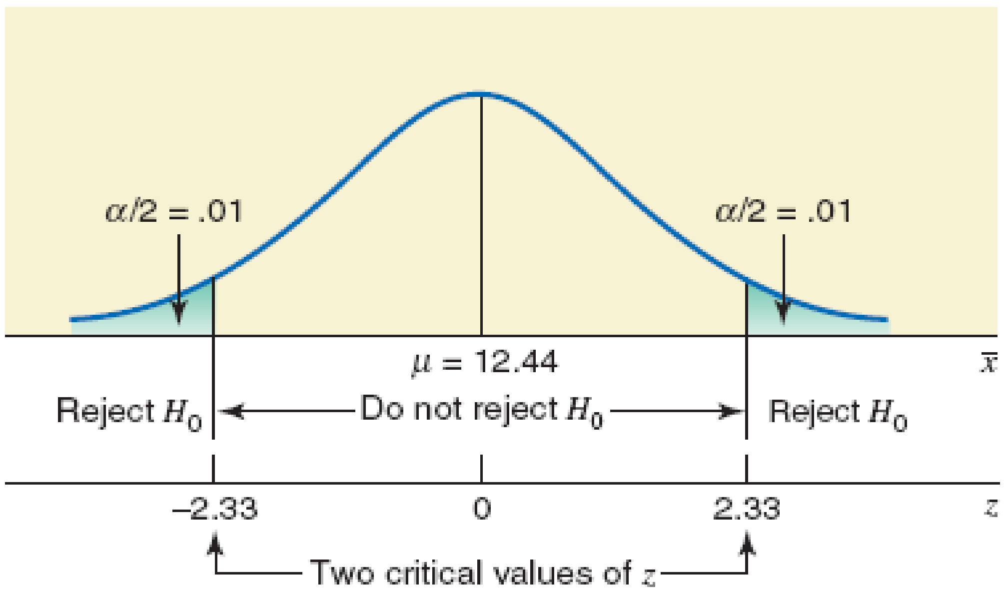
Solution

- Step 1: $H_0 : \mu = 12.44 \quad H_1 : \mu \neq 12.44$
- Step 2: The population standard deviation σ is known, and the sample size is large (more than 30). Due to the Central Limit Theorem, we will use the normal distribution to perform the test.

Solution

- Step 3: $\alpha = .02$
- The $\neq$ sign in the alternative hypothesis indicates that the test is two-tailed
- Area in each tail $= \alpha / 2 = .02 / 2 = .01$
- The z values for the two critical points are -2.33 and 2.33

Figure



$\alpha/2 = .01$
$\alpha/2 = .01$
$\mu = 12.44$
$\bar{x}$
Reject H_0
Do not reject H_0
Reject H_0
–2.33
0
2.33
z
Two critical values of z

Calculating the Value of the Test Statistic

When using the normal distribution, the ***value of the test statistic z*** for $\bar{x}$ for a test of hypothesis about μ is computed as follows:

$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} \quad \textbf{where} \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

The value of z for x is also called the ***observed value of z***.

Solution

- Step 4:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2.65}{\sqrt{150}} = .21637159$$

$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{13.71 - 12.44}{.21637159} = 5.87$$

Solution

- Step 5: This value of $z = 5.87$ is greater than the critical value of $z = 2.33$, and it falls in the rejection region in the right tail in Figure 9.9. Hence, we reject H_0 and conclude that based on the sample information, it appears that the mean length of all such calls is not equal to 12.44 minutes.

Example

The mayor of a large city claims that the average net worth of families living in this city is at least \$300,000. A random sample of 25 families selected from this city produced a mean net worth of \$288,000. Assume that the net worths of all families in this city have a normal distribution with the population standard deviation of \$80,000. Using the 2.5% significance level, can you conclude that the mayor’s claim is false?

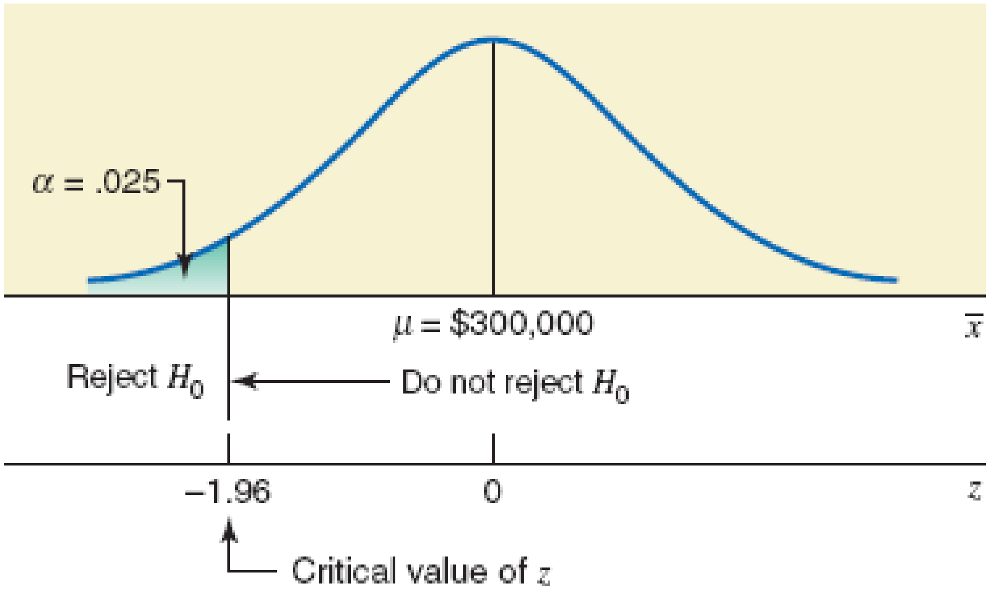
Solution

- Step 1: $H_0 : \mu \geq \$300,000$
  $H_1 : \mu < \$300,000$
- Step 2: The population standard deviation σ is known, the sample size is small (n < 30), but the population distribution is normal. Consequently, we will use the normal distribution to perform the test.

Solution

- Step 3: α = .025
- The < sign in the alternative hypothesis indicates that the test is left-tailed
- Area in the left tail = α = .025
- The critical value of z is -1.96

Figure


α = .025
μ = $300,000
$\bar{x}$
Reject H_0
Do not reject H_0
−1.96
0
z
Critical value of z

Solution

- Step 4:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{80,000}{\sqrt{25}} = \$16,000$$

$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{288,000 - 300,000}{16,000} = -.75$$

Solution

- Step 5: This value of z = -.75 is greater than the critical value of z = -1.96, and it falls in the nonrejection region. As a result, we fail to reject H_0. Therefore, we can state that based on the sample information, it appears that the mean net worth of families in this city is not less than $300,000.