Date : 26.07.2023. Class ⇒ 01

Probability

Reference Book: HTZ (Hogg | Tanis | Zimmerman [Prob. and sta. Inference 9th edition)

ch → 1, 2, 3, 7, 8

ch → 1.1

Probability: → Mathematical defn
→ Statistical defn

Mathematical defn

Let a Random experiment (event) have (n) possible outcomes. and m is favourable outcomes.

$$P = \frac{\text{Number of desired/favourable outcomes}}{\text{Number of Total possible outcomes}}$$

$$= \frac{m}{n}$$

(5,3), (5,5)

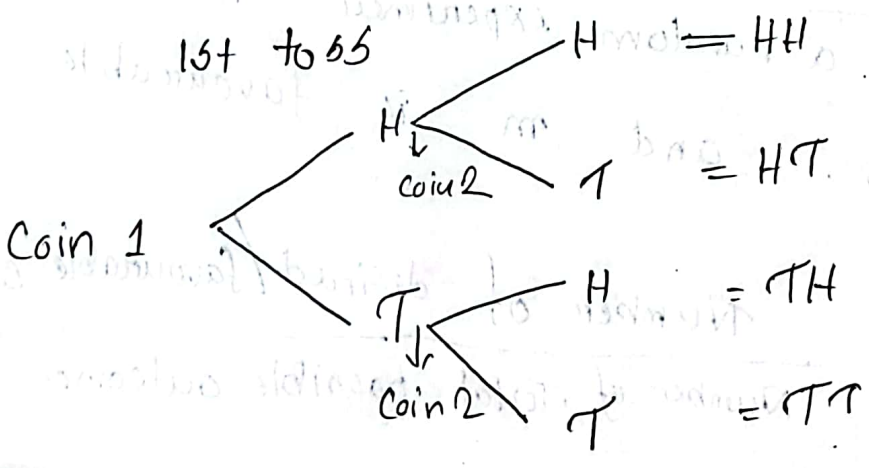
Some important terms

[*] Random experiment:

Any type experiment from where the outcome can't be predicted.

[*] Sample space: set of all possible outcomes from an event / experiment

Example: 2 coins throwing at a time



Total sample space = 4

$2^n = 2^1 = 2$

$2^n = 2^2 =$

$2^3 = 2^3 =$

[*] Trial and Event:

Tossing a coin is a trial.

and getting a ~~Hand~~ Head / Tail is an event.

Null event:

having no sample space is called null event.

denoted by ϕ.

$P(\phi) = 0$

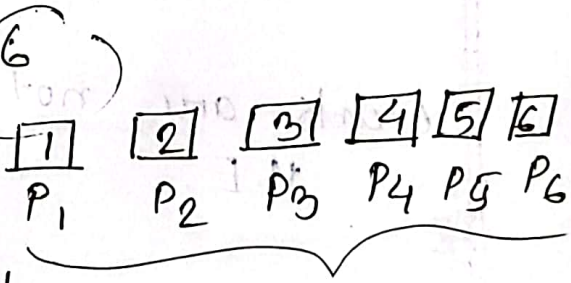
Exhaustive events:

Total num. of possible outcomes in any trail. / union is equal to sample space.

Example: Throwing a dice.

possible outcome = 6

$1 \cup 2 \cup 3 \cup 4 \cup 5 \cup 6 = 6$



6 exhaustive event.

Mutually Exclusive event:

A and B

if A happens then B will not happen and vice-versa.

Example: when tossing a coin, both H and T can't appear in same time.

Example: 6 sided die roll =>

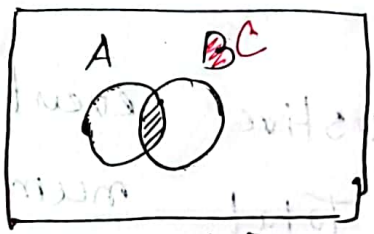
$S = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 2, 3\}$ $B = \{5, 6\}$ $C = \{3, 4, 5\}$

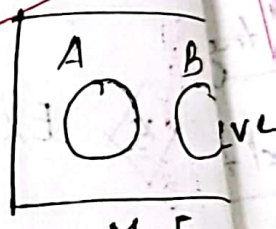
Is A and B are mutually exclusive event? yes

$A \cap B = \emptyset$

$P(A \cap B) = 0$



Not M.E.
Not disjoint



M.E.
disjoint

Events are not M.E. $\Leftarrow$ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Events are M.E. $\Leftarrow$ $P(A \cup B) = P(A) + P(B) - 0$

$P(A \cup B) = P(A) + P(B)$

Example: (Mutually Exclusive Event) (two or more events can't occ at same time)

Tossing a coin. you don't get Head or tail at the same time.

OR, Rolling a 2 and 3 on a dice.

* Equally Likely Events:

Example: Coin 1 → H
Coin 1 → T

A → what is the probability of getting a head?

$$= \frac{1}{2}$$

B → Probability (Tail) = 1/2

both events have same proba. So, it gives us likelihood of events.

(Equally likely events)

[*] Independent event : [*] Dependent event

Example: A bag consists 8 red marbles, 7 blue, 6 green and 4 yellow. what's the prob of selecting a) a red marble?

b) a blue in the first try and green on the 2nd with replacement?

(Red, Blue, Green, Yellow.)

8, 7, 6, 4

(We are going to put that Blue marble back in back, we still have total 25)

c) a yellow on the 1st try and then red on 2nd with replacement?

d) 2 blue marble with replacement?

e) 2 green without replacement

Soln:

a) Total = $8 + 4 + 6 + 7 = 25$ marbles

$$P(\text{Red}) = \frac{8}{25} = 0.32$$

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B

Soln:

a) Total = $8+4+6+7 = 25$ marbles

$$P(\text{Red}) = \frac{8}{25} = 0.32$$

(c) $P(BG) = \frac{7}{25} \cdot \frac{6}{25} = \frac{42}{625}$

Independent event

(Total outcomes remains unchanged)

$= 0.0672 = 6.72\%$.

(b) $P(YR) = \frac{4}{25} \cdot \frac{8}{24} = \frac{4}{75}$

Dependent event

$= 0.053$

$= 5.3\%$.

(Total outcome changes)

(a) $P(GG) = \left(\frac{6}{25}\right) \cdot \frac{5}{24}$ ← Dependent event

$= \frac{30}{600} = \frac{1}{20}$

(d) $P(BB) = \frac{7}{25} \cdot \frac{7}{25} = \frac{49}{625}$

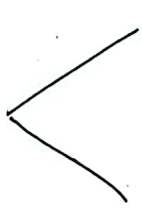
↙ Independent event

[Ans]

Math Problems:

Problem: In tossing a coin, what's the probability of getting a head?

Soln: Total outcome = 2



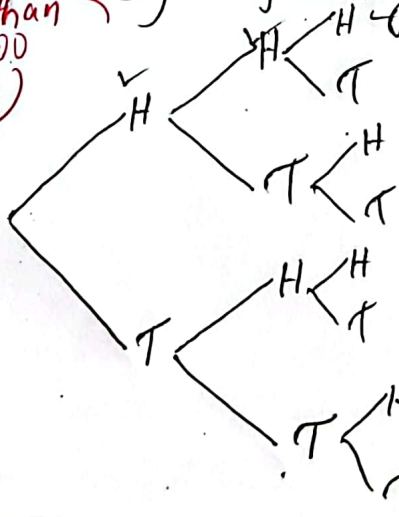
$P(H) = 1/2$

Problem: In tossing a coin 3 times,

a) what's the prob of getting 2 He

b) getting at least 2 Head (2 or more)

c) getting at most 2 Head. (two or less than two (0, 1, 2))



= HHH
= HHT
= HTH
= HTT
= THH
= THT
= TTH
= TTT

H	0 (no head)	1	2	3
	1	3	3	1

exactly (2 Head)

a) getting 2 Heads

$= P(2H) = \frac{3}{8}$

b) at least 2 Head

$= P(2H/3H) = \frac{4}{8} \rightarrow 3+1$

$= 1/2$

c) at most 2 Head

$= P(\text{0H}/1H/2H) = \frac{7}{8}$

Problem: In throwing a dice, what's the probability of getting 2?

1 dice.

1	2	3	4	5	6

$P(2) = \frac{1}{6}$

Problem: Find the probability of getting 7 with 2 dice.

(3,1), (5,3), (5,5)

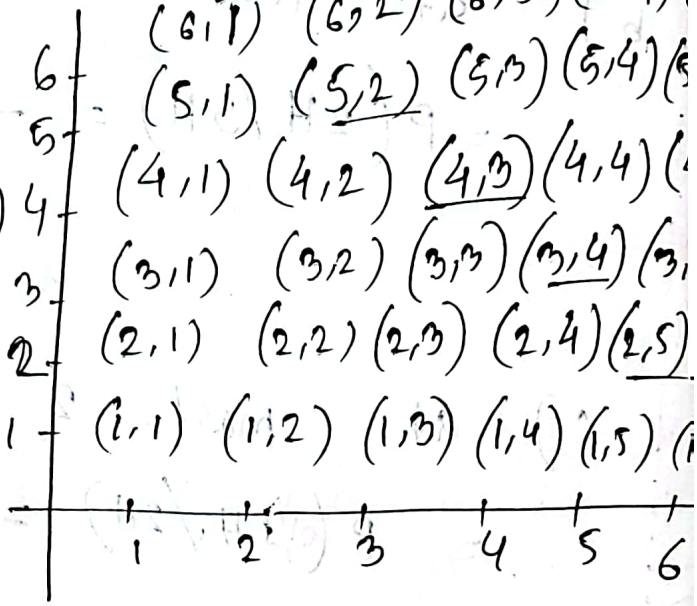
1 Dice $\longrightarrow 6$

2 Dice $\longrightarrow 6^2 = 36$

3 dice $\longrightarrow 6^3 = 216$

7 $\longrightarrow$ P. outcomes $\rightarrow$

(6,1), (5,2), (4,3)
(3,4), (2,5), (1,6)



$$P(7) = \frac{6}{36}$$

$$= 1/6$$

Problem: A bag $\rightarrow$ 6 red ball
$\rightarrow$ 7 black ball.

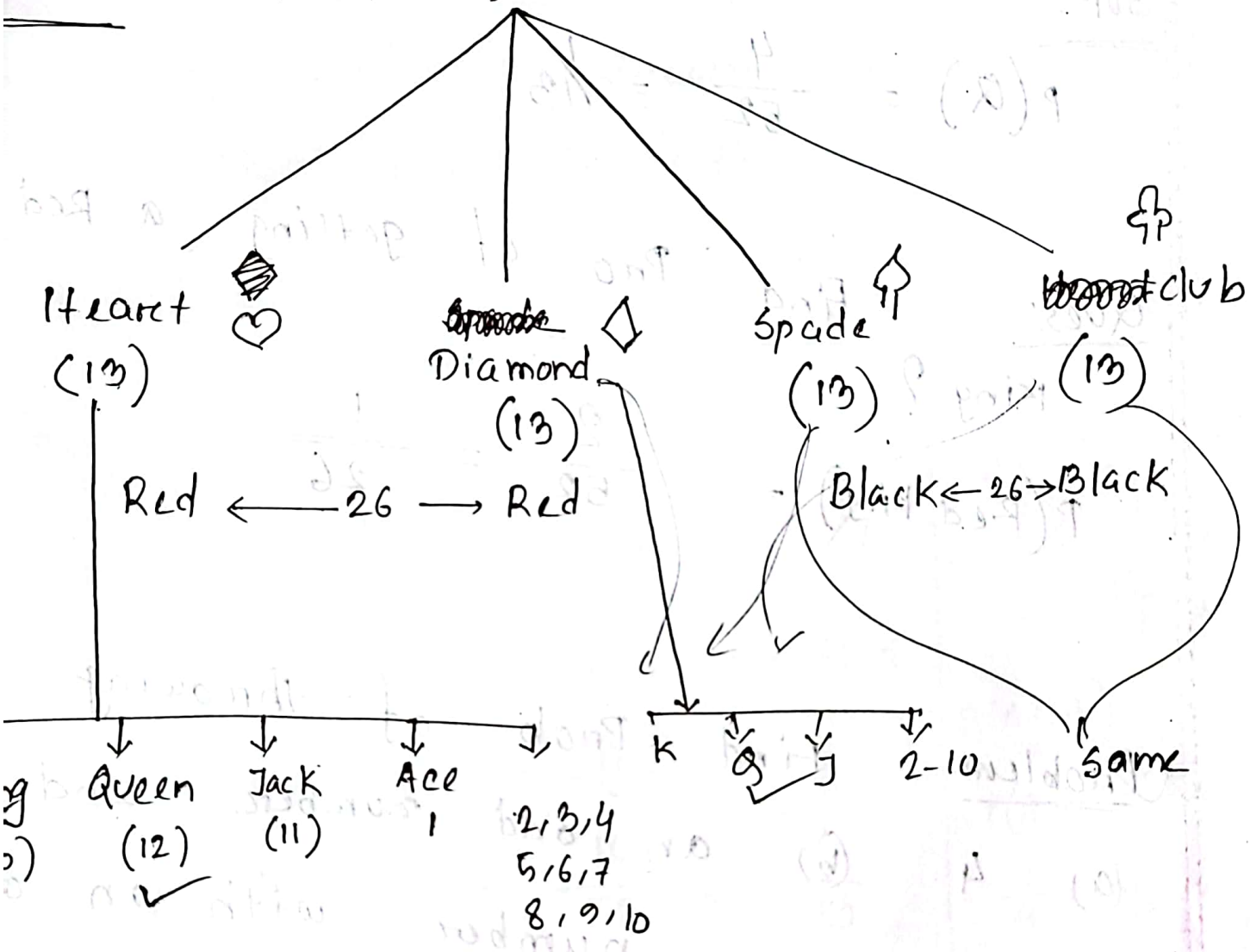
Find Prob of drawing a red ball.

Soln:

$$P(\text{Red}) = \frac{6}{6+7}$$

$$= \frac{6}{13}$$

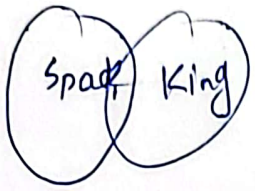
Card Total = 52



Question: Find prob of a card drawn at random from pack, is a diamond.

Soln: $P(\text{Dia}) = \frac{13}{52} = 1/4$

Question: From a pack of 52 cards, 1 card is drawn at random. Find prob of getting a queen?



total = 4 (King) Spade (13) → King Spade (13)

Problem: If from a pack of cards, a single card is drawn, what's the probability that it is either spade or a king?

Soln:

$$P(\text{spade}) = \frac{13}{52} = 1/4$$

$$P(\text{King}) = \frac{4}{52} = 1/13$$

Spade and King taken together

Both (A and B) = (13) + 3 = (16)

$$\therefore P(\text{S or K}) = \frac{16}{52} = \frac{4}{13}$$

Or,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A) \times P(B) = \frac{1}{4} \times \frac{1}{13} = 1/52$$

$$P(A \cup B) = 1/4 + 1/13 - 1/52 = 4/13 \text{ (Ans.)}$$

Problem: If $P(A) = 0.35$; $P(B) = 0.73$, $P(A \cap B) = 0.14$. Find $P(A' \cup B')$

$P(A') = 1 - P(A)$

Soln:

$$P(A' \cup B') = (A \cap B)' \quad \text{[De Morgan's Law]}$$

$$\Rightarrow P(A' \cup B') = \boxed{P(A \cap B)'}$$

$$= 1 - P(A \cap B) = 1 - 0.14 = 0.86 \text{ (Ans)}$$

Problem: A die is loaded in such a way that an even number is twice as likely to occur as an odd number. If E is the event that a number less than 4 occurs on a single toss of the dice. Find P(E)?

Soln: Let, odd = x

even = $2x$

$S = \{1, 2, 2, 3, 4, 4, 5, 6, 6\}$ ⇒ Total sample space

$E = \{1, 2, 2, 3\}$ ⇒ (Less than 4 occurs)

So, $P(E) = \frac{4}{9}$. [Ans.]