1.4: INDEPENDENT EVENTS

For certain pairs of events, the occurrence of one of them may or may not change the probability of the occurrence of the other. In the latter case, they are said to be independent events.

If $\boldsymbol{A}$ & $\boldsymbol{B}$ are two independent events, then

$$\boldsymbol{P}(\boldsymbol{A} \cap \boldsymbol{B}) = \boldsymbol{P}(\text{Both } \boldsymbol{A} \,\&\, \boldsymbol{B} \text{ will happen}) = \boldsymbol{P}(\boldsymbol{A}) \times \boldsymbol{P}(\boldsymbol{B})$$

Otherwise, $\boldsymbol{A}$ & $\boldsymbol{B}$ are called dependent events.

Events that are independent are sometimes called **statistically independent**, **stochastically independent**, or **independent in a probabilistic sense**, but in most instances we use independent without a modifier if there is no possibility of misunderstanding.

Problems:

1. A bag contains **8** white and **10** black balls. **Two** balls are drawn in succession with **replacement**. What is the probability that first is white and second is black.
   Solution: Total no. of balls $= \mathbf{8 + 10 = 18}$
   $\boldsymbol{P}(\text{drawing one white ball from } \mathbf{8} \text{ balls}) = \boldsymbol{P}(\boldsymbol{A}) = \frac{\mathbf{8}}{\mathbf{18}}$
   $\boldsymbol{P}(\text{drawing one black ball from } \mathbf{10} \text{ balls}) = \frac{\mathbf{10}}{\mathbf{18}}$

So, $\boldsymbol{P}(\text{drawing first white \& second black}) = \boldsymbol{P}(\boldsymbol{A} \cap \boldsymbol{B}) = \boldsymbol{P}(\boldsymbol{A}) \times \boldsymbol{P}(\boldsymbol{B}) = \frac{\mathbf{8}}{\mathbf{18}} \times \frac{\mathbf{10}}{\mathbf{18}} = \frac{\mathbf{20}}{\mathbf{81}}$

2. Two persons $\boldsymbol{A}$ & $\boldsymbol{B}$ appear in an interview for **2** vacancies for the same post. The probability of $\boldsymbol{A}$'s selection is $\frac{\mathbf{1}}{\mathbf{7}}$ and that of $\boldsymbol{B}$'s selection is $\frac{\mathbf{1}}{\mathbf{5}}$. What is the probability that, **i)** both of them will be selected, **ii)** none of them will be selected.
   Solution: Given that, $\boldsymbol{P}(\boldsymbol{A} \text{ selected}) = \frac{\mathbf{1}}{\mathbf{7}}$ and $\boldsymbol{P}(\boldsymbol{B} \text{ selected}) = \frac{\mathbf{1}}{\mathbf{5}}$
   So, $\boldsymbol{P}(\boldsymbol{A} \text{ will not be selected}) = \boldsymbol{P}(\boldsymbol{A}') = \mathbf{1} - \boldsymbol{P}(\boldsymbol{A}) = \mathbf{1} - \frac{\mathbf{1}}{\mathbf{7}} = \frac{\mathbf{6}}{\mathbf{7}}$ and
   $\boldsymbol{P}(\boldsymbol{B} \text{ will not be selected}) = \boldsymbol{P}(\boldsymbol{B}') = \mathbf{1} - \boldsymbol{P}(\boldsymbol{B}) = \mathbf{1} - \frac{\mathbf{1}}{\mathbf{5}} = \frac{\mathbf{4}}{\mathbf{5}}$
   i) $\boldsymbol{P}(\text{Both of them will be selected}) = \boldsymbol{P}(\boldsymbol{A}) \times \boldsymbol{P}(\boldsymbol{B}) = \frac{\mathbf{1}}{\mathbf{7}} \times \frac{\mathbf{1}}{\mathbf{5}} = \frac{\mathbf{1}}{\mathbf{35}}$
   ii) $\boldsymbol{P}(\text{None of them will be selected}) = \boldsymbol{P}(\boldsymbol{A}') \times \boldsymbol{P}(\boldsymbol{B}') = \frac{\mathbf{6}}{\mathbf{7}} \times \frac{\mathbf{4}}{\mathbf{5}} = \frac{\mathbf{24}}{\mathbf{35}}$

3. A problem in mathematics is given to **3** students $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ whose chances of solving it are $\frac{\mathbf{1}}{\mathbf{2}}, \frac{\mathbf{1}}{\mathbf{3}}$ and $\frac{\mathbf{1}}{\mathbf{4}}$ respectively. What is the probability that the problem will be solved?

Solution: Here, $\boldsymbol{P}(\boldsymbol{A} \text{ will not solve the problem}) = \boldsymbol{P}(\boldsymbol{A}') = \mathbf{1} - \boldsymbol{P}(\boldsymbol{A}) = \mathbf{1} - \frac{\mathbf{1}}{\mathbf{2}} = \frac{\mathbf{1}}{\mathbf{2}}$,

$\boldsymbol{P}(\boldsymbol{B} \text{ will not solve the problem}) = \boldsymbol{P}(\boldsymbol{B}') = \mathbf{1} - \boldsymbol{P}(\boldsymbol{B}) = \mathbf{1} - \frac{\mathbf{1}}{\mathbf{3}} = \frac{\mathbf{2}}{\mathbf{3}}$ and

$\boldsymbol{P}(\boldsymbol{C} \text{ will not solve the problem}) = \boldsymbol{P}(\boldsymbol{C}') = \mathbf{1} - \boldsymbol{P}(\boldsymbol{C}) = \mathbf{1} - \frac{\mathbf{1}}{\mathbf{4}} = \frac{\mathbf{3}}{\mathbf{4}}$.

So, $\boldsymbol{P}(\text{all three will not solve the problem}) = \boldsymbol{P}(\boldsymbol{A}') \times \boldsymbol{P}(\boldsymbol{B}') \times \boldsymbol{P}(\boldsymbol{C}') = \frac{\mathbf{1}}{\mathbf{2}} \times \frac{\mathbf{2}}{\mathbf{3}} \times \frac{\mathbf{3}}{\mathbf{4}} = \frac{\mathbf{1}}{\mathbf{4}}$

$\therefore\ \boldsymbol{P}(\text{the problem will be solved}) = \mathbf{1} - \frac{\mathbf{1}}{\mathbf{4}} = \frac{\mathbf{3}}{\mathbf{4}}$

4. What is the chance of getting two sixes in two rolling of a single die?
   Sol: $\boldsymbol{P}(\text{getting a six in first rolling}) = \frac{\mathbf{1}}{\mathbf{6}}$ and $\boldsymbol{P}(\text{getting a six in second rolling}) = \frac{\mathbf{1}}{\mathbf{6}}$
   Since two rolling are independent, so $\boldsymbol{P}(\text{getting two sixes in } \mathbf{2} \text{ rolls}) = \frac{\mathbf{1}}{\mathbf{6}} \times \frac{\mathbf{1}}{\mathbf{6}} = \frac{\mathbf{1}}{\mathbf{36}}$

5. An article manufactured by a company consists of two parts $\boldsymbol{A}$ & $\boldsymbol{B}$. In the process of manufacture of part $\boldsymbol{A}$, $\mathbf{9}$ out of $\mathbf{100}$ are likely to be defective. Similarly, $\mathbf{5}$ 0ut of $\mathbf{100}$ are likely to be defective in the manufacture of part $\boldsymbol{B}$. Calculate the probability that the assembled article will not be defective.

Solution: Here, $\boldsymbol{P}(\boldsymbol{A} \text{ will be defective}) = \frac{\mathbf{9}}{\mathbf{100}}$ and $\boldsymbol{P}(\boldsymbol{B} \text{ will be defective}) = \frac{\mathbf{5}}{\mathbf{100}}$

So, $\boldsymbol{P}(\boldsymbol{A} \text{ will not be defective}) = \mathbf{1} - \frac{\mathbf{9}}{\mathbf{100}} = \frac{\mathbf{91}}{\mathbf{100}}$ and

$\boldsymbol{P}(\boldsymbol{B} \text{ not will be defective}) = \mathbf{1} - \frac{\mathbf{5}}{\mathbf{100}} = \frac{\mathbf{95}}{\mathbf{100}}$

$\therefore$ $\boldsymbol{P}(\text{the assembled article will not be defective}) = \boldsymbol{P}(\boldsymbol{A} \text{ will not be defective}) \times$

$$\boldsymbol{P}(\boldsymbol{B} \text{ will not be defective}) = \frac{\mathbf{91}}{\mathbf{100}} \times \frac{\mathbf{95}}{\mathbf{100}} = \mathbf{0.86}$$

6. What is the probability of at least one "**H**" in four tosses of a coin?

Sol: For a coin, $\boldsymbol{P}(\boldsymbol{H}) = \frac{\mathbf{1}}{\mathbf{2}}$ and $\boldsymbol{P}(\text{no } \boldsymbol{H}) = \boldsymbol{P}(\boldsymbol{T}) = \mathbf{1} - \boldsymbol{P}(\boldsymbol{H}) = \mathbf{1} - \frac{\mathbf{1}}{\mathbf{2}} = \frac{\mathbf{1}}{\mathbf{2}}$

Now, $\boldsymbol{P}(\text{Total No. no } \boldsymbol{H}) = (\frac{\mathbf{1}}{\mathbf{2}})^{\mathbf{4}} = \frac{\mathbf{1}}{\mathbf{16}}$

$\boldsymbol{P}(\text{at least one } \boldsymbol{H}) = \mathbf{1} - \boldsymbol{P}(\text{Total No. no } \boldsymbol{H}) = \mathbf{1} - \frac{\mathbf{1}}{\mathbf{16}} = \frac{\mathbf{15}}{\mathbf{16}}$

7. A person is known to hit the target in $\mathbf{3}$ out of $\mathbf{4}$ shots, whereas another person is known to hit the target in $\mathbf{2}$ out of $\mathbf{3}$ shots. Find the probability of the targets being hit at all when they both person try.

Sol: The probability that the first person hit the target = $\boldsymbol{P}(\boldsymbol{A}) = \frac{\mathbf{3}}{\mathbf{4}}$

The probability that the second person hit the target = $\boldsymbol{P}(\boldsymbol{B}) = \frac{\mathbf{2}}{\mathbf{3}}$

The two events are not mutually exclusive, since both persons hit the same target.

So, $\boldsymbol{P}(\boldsymbol{A}\ \boldsymbol{or}, \boldsymbol{B}) = \boldsymbol{P}(\boldsymbol{A} \cup \boldsymbol{B}) = \boldsymbol{P}(\boldsymbol{A}) + \boldsymbol{P}(\boldsymbol{B}) - \boldsymbol{P}(\boldsymbol{A} \cap \boldsymbol{B}) = \frac{\mathbf{3}}{\mathbf{4}} + \frac{\mathbf{2}}{\mathbf{3}} - \frac{\mathbf{3}}{\mathbf{4}} \times \frac{\mathbf{2}}{\mathbf{3}} = \frac{\mathbf{11}}{\mathbf{12}}$

8. From a bag containing $\mathbf{4}$ white and $\mathbf{6}$ black balls, two balls are drawn at random. If the balls are drawn one after the other **without replacement**, find the probability that

i) both balls are white.
ii) both balls are black.
iii) the first ball is white and the second ball is black.
iv) one ball is white and the other is black.

Solution: Total no. of balls $= \mathbf{4} + \mathbf{6} = \mathbf{10}$

i) $\boldsymbol{P}(\text{first ball is white}) = \frac{\mathbf{4}}{\mathbf{10}}$ and $\boldsymbol{P}(\text{second ball is white}) = \frac{\mathbf{3}}{\mathbf{9}}$

$\therefore$ $\boldsymbol{P}(\text{both balls are white}) = \frac{\mathbf{4}}{\mathbf{10}} \times \frac{\mathbf{3}}{\mathbf{9}} = \frac{\mathbf{2}}{\mathbf{15}}$

ii) $\boldsymbol{P}(\text{first ball is black}) = \frac{\mathbf{6}}{\mathbf{10}}$ and $\boldsymbol{P}(\text{second ball is black}) = \frac{\mathbf{5}}{\mathbf{9}}$

$\therefore$ $\boldsymbol{P}(\text{both balls are black}) = \frac{\mathbf{6}}{\mathbf{10}} \times \frac{\mathbf{5}}{\mathbf{9}} = \frac{\mathbf{1}}{\mathbf{3}}$

iii) $\boldsymbol{P}(\text{first ball is white}) = \frac{\mathbf{4}}{\mathbf{10}}$ and $\boldsymbol{P}(\text{second ball is black}) = \frac{\mathbf{6}}{\mathbf{9}}$

$\therefore$ $\boldsymbol{P}(\text{first ball is white \& second ball is black}) = \frac{\mathbf{4}}{\mathbf{10}} \times \frac{\mathbf{6}}{\mathbf{9}} = \frac{\mathbf{4}}{\mathbf{15}}$

iv) $\boldsymbol{P}(\text{first ball is white \& second ball is black}) = \frac{4}{10} \times \frac{6}{9} = \frac{4}{15}$ and

$\boldsymbol{P}(\text{first ball is black \& second ball is white}) = \frac{6}{10} \times \frac{4}{9} = \frac{4}{15}$

Hence both events are mutually exclusive,

$\therefore\ \boldsymbol{P}(\text{one ball is white \& the other is black}) = \frac{4}{15} + \frac{4}{15} = \frac{8}{15}$

9. Find the probability in each of the below ***four*** cases, if the balls are drawn one after the other with **replacement**. A bag containing **4** white & **6** black balls, **2** balls are drawn at random.

i) both balls are white.
ii) both balls are black.
iii) the first ball is white and the second ball is black.
iv) one ball is white and the other is black.

Answer: i) $\frac{4}{25}$, **ii)** $\frac{9}{25}$, **iii)** $\frac{6}{25}$ and **iv)** $\frac{12}{25}$ **(Try yourself)**

Examples: 1.4-1 to 1.4-9 (See yourself)
Exercises: 1.4-1 to 1.4-9.& **1.4-12 to 1.4-16 (Try yourself)**