1.0 0.80 / 20

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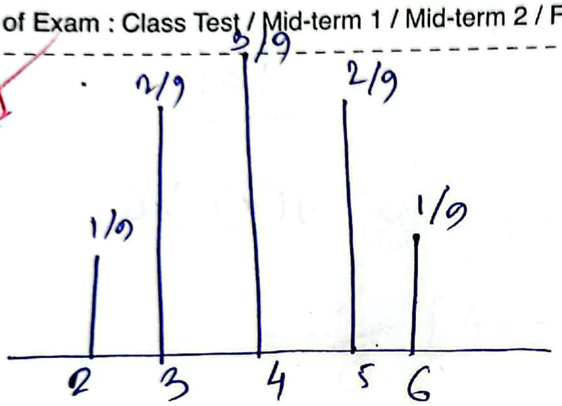


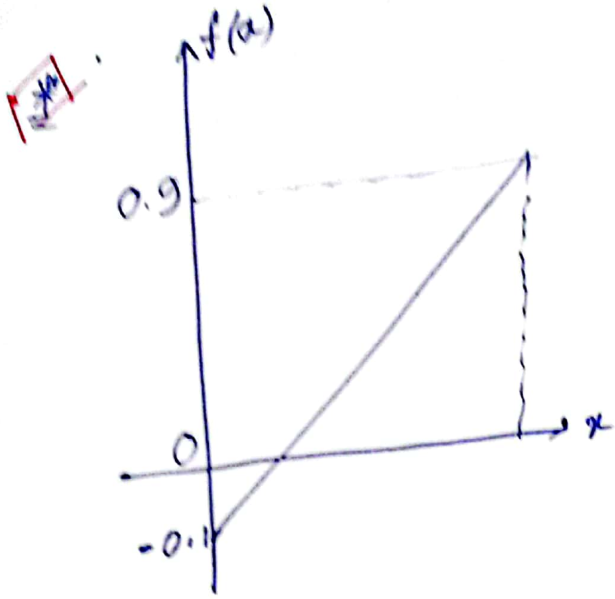
pmf :

$x = 2$	$P(X = x) = 1/9$
$x = 3$	$P(X = x) = 2/9$
$x = 4$	$P(X = x) = 3/9$
$x = 5$	$P(X = x) = 2/9$
$x = 6$	$P(X = x) = 1/9$

~~P(X =~~

$$\left.\begin{array}{ll} f(x) = \dfrac{x-1}{9} ; & x = 2, 3, 4 \\ f(x) = \dfrac{7-x}{9} ; & x = 5, 6 \end{array}\right\} \text{p.m.f}$$

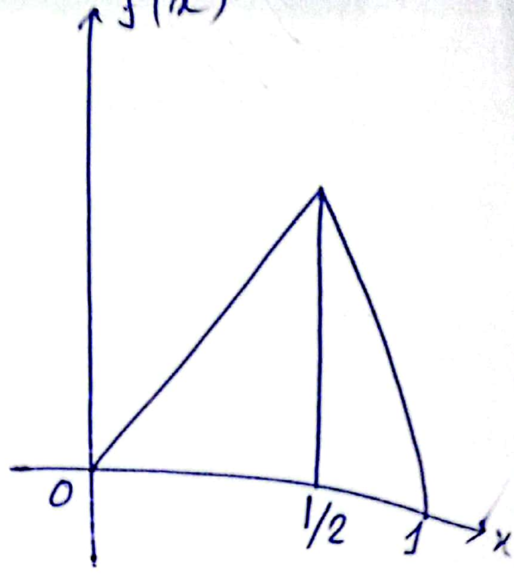
[Ans.]



$\Downarrow$

Not PDF

Here, $f(x) < 0$



Pdf

$f(x) > 0$

$$f(x) = k(4-x) \quad ; \quad 0 \le x \le 4$$

i) $$\int_0^4 k(4-x)\,dx = 1$$

$$\Rightarrow [4x]_0^4 - \left[\frac{x^2}{2}\right]_0^4 = 1/k$$

$$\Rightarrow (4 \times 4) - \left(\frac{1}{2} \times 16\right) = 1/k$$

$$\Rightarrow 16 - 8 = 1/k$$

$$\Rightarrow 8 = \frac{1}{k}$$

$$\therefore k = 1/8$$

iii)

$x < 0, \quad f(x) = 0$

$0 \le x \le 4, \quad F(X \le x)$

$$= \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{x} f(x)\,dx$$

$$= \frac{1}{8} \int_{0}^{x} (4 - x)\,dx$$

$$= \frac{1}{8} \left[4x - \frac{x^2}{2} \right]_0^x$$

$$= \frac{1}{8} \left[4x - \frac{x^2}{2} \right]$$

$$= \frac{x}{2} - \frac{x^2}{16}$$

$x > 0$;

$$F(x) = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{4} f(x)\,dx + \int_{4}^{x} f(x)\,dx$$

$$= \left[\frac{x}{2} - \frac{x^2}{16} \right]_0^4$$

$$= \cancel{\frac{4}{3} - \frac{16}{24}}$$

$$= \cancel{\frac{32 - 16}{24}}$$

$$= \cancel{\frac{16}{24}}$$

$$= \frac{4}{2} - \frac{16}{16}$$

$$2 - 1 = 1$$

$$\text{CDF } F(x) = \begin{cases} 0 & ; \; x < 0 \\ \dfrac{x}{3} - \dfrac{x^2}{24} & ; \; 0 \le x \le 4 \\ 1 & ; \; x > 0 \end{cases}$$

* Median time:

C.D.F. = x

$$F(x) = \frac{1}{8}\left[4x - \frac{x^2}{2}\right] = 0.5$$

$$\Rightarrow 4x - \frac{x^2}{2} = 4$$

$$\Rightarrow 8x - x^2 = 8$$

$$\Rightarrow x^2 - 8x + 8 = 0$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(8)}}{2(1)}$$

$$= 4 \pm 2\sqrt{2}$$

$4 + 2\sqrt{2} = 6.82$

$4 - 2\sqrt{2} = 1.17$ → $[0 < x < 4]$

$\therefore$ Median time: 1.171 (Ans.)

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*1) Discrete Uniform distribution:

Every possible outcome has the same proby.

pmf, $P(X = x) = 1/x$ for each x

[Example: Rolling a six-sided dice]

2) Conts Uniform distn:

=> Proby is spread evenly over an interval $[a, b]$.

Defined PDF $f(x) = \begin{cases} \frac{1}{b-a} & ; a \le x \le b \\ 0 & ; \text{else} \end{cases}$

Problem: A cont's R.V. x has cdf as

(Final Spring-24) Q(m) (b)

$$F(x) = \begin{cases} 0 & ; \quad x < 5 \\ \dfrac{x-5}{5} & ; \quad 5 \le x < 10 \\ 1 & ; \quad x \geq 10 \end{cases}$$

i) Identify the distribution and write the pdf.

ii) Estimate $P(3 < x < 12)$

iii) Find the mean and variance of x.

Soln: i) Uniform distn.

$$PDF = \begin{cases} 0 & ; \quad x < 5 \\ \frac{1}{5} & ; \quad 5 \le x < 10 \\ 0 & ; \quad x \geq 10 \end{cases}$$

ii) Estimate : $P(3 < x < 12)$

$= P(x = 12) - P(x = 3)$

$= ~~0 - 1~~ \quad 1 - 0$

$= 1$.

iii) Mean, $E(x) = \int_{-\infty}^{\infty} x f(x)$ for PDF

$$= \int_{5}^{10} x f(x) \, dx$$

$$= \frac{1}{5} \int_{5}^{10} x \, dx$$

$$= \frac{1}{5} \left[\frac{x^2}{2} \right]_{5}^{10}$$

$$= \frac{1}{5} \left[\frac{100}{2} - \frac{25}{2} \right]$$

$$= 7.5$$

Variance, $E(x^2) - \{E(x)\}^2$

$$= \int_{5}^{10} x^2 f(x) \, dx - (7.5)^2$$

$$= \frac{1}{5} \left[\frac{x^3}{3} \right]_{5}^{10} - (7.5)^2$$

$$= 1/5 \left[\frac{1000}{3} - \frac{125}{3} \right] - (7.5)^2$$

$$= 58.33 - 56.25$$

$$= 2.08$$

[An-]

Final spring 24(Q(3)a)

Problem: Let, $f(y) = \frac{3}{2} y^2 \; ; \; -1 < y < 1$ be the pdf of cont^s R.V. Y. Find and sketch the cumulative distribution of Y. Find $P(Y \geq 0.5)$ and P_{65} of Y.

Soln:

$y < -1$, $\displaystyle\int_{-\infty}^{0} f(y)\,dy = 0$

$-1 < y < 1$; $\displaystyle\int_{-\infty}^{0} f(y)\,dy + \int_{-1}^{y} f(y)\,dy$

$$= \int_{-1}^{y} \frac{3}{2} y^2 \, dy$$

$$= \frac{3}{2} \left[\frac{y^3}{3} \right]_{-1}^{y}$$

$$= \frac{1}{2} \left[y^3 + 1 \right]$$

$y > 1$; $\displaystyle\int_{-\infty}^{0} f(y)\,dy + \int_{-1}^{1} f(y)\,dy + \int_{1}^{y} f(y)\,dy$

$$= 1$$

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$$CDF = \begin{cases} 0 & ; y < -1 \\ \frac{1}{2}(y^3+1) & ; -1 < y < 1 \\ 1 & ; y > 1 \end{cases}$$

PDF

$$P(Y \geq 0.5) = \int_{0.5}^{\infty} f(y)\, dy$$

$$= \int_{0.5}^{1} \frac{3}{2} y^2 \, dy$$

$$= \frac{3}{2} \left[\frac{y^3}{3} \right]_{0.5}^{1}$$

$$= \frac{1}{2} [1 - (0.5)^3]$$

$$= 0.4375$$

To find P_{65} of Y,

We set, $F(y) = 0.65$

that, C.D.F $\Rightarrow \frac{1}{2}(y^3+1) = 0.65$

$$\Rightarrow \frac{1}{2}y^3 = 0.65 - \frac{1}{2}$$

$$\Rightarrow y^3 = 0.30$$

$$\therefore y = \sqrt[3]{0.30}$$

$$= 0.669$$

$P_{65} \approx 0.669$

[Ans.]

* Binomial and Poisson Distribution:

Binomial

Deals with a fixed number of $\boxed{\text{independent}}$ trials, where each trial has two possible outcomes

- Success
- Failure

Poisson

Number of events ~~can~~ occur within a fixed interval of time, space on ~~others~~ at a average rate.

Binomial range 0 to n

Poisson Range 0 to ∞.

* Formula (Binomial) $P(X=x) = {}^{n}C_{x}\, p^{x}\, q^{(n-x)}$

$p+q=1$

$\boxed{\text{Poisson}}$ $P(X=x) = \dfrac{\lambda^{x} e^{-\lambda}}{x!}$

* Moment generating function (Mgf) of Bionomial:

$(pe^{t}+q)^{n}$

CamScanner

Final spring 24 Q2(c)

Problem: Let, x have a poisson distribution with $P(x=1) = \frac{1}{2} P(x=2)$. Find SD and $P(x \geq 1)$.

Soln: We know, $P(x=k) = \dfrac{\lambda^x e^{-\lambda}}{k!}$; $k = 0, 1, 2, \ldots \infty$

Now, $P(x=1) = \frac{1}{2} P(x=2)$

$$\Rightarrow \frac{\lambda e^{-\lambda}}{1!} = \frac{1}{2} \frac{\lambda^2 \cancel{e^{-\lambda}}}{2!}$$

$$\Rightarrow \lambda = \frac{\lambda^2}{4}$$

$$\Rightarrow 1 = \frac{\lambda}{4}$$

$$\therefore \lambda = 4$$

Now, S.D. $= \sqrt{4} = 2$

Now, $P(x \geq 1)$

$= 1 - P(x < 1)$

$= 1 - \dfrac{\lambda^0 e^{-\lambda}}{0!}$

$= 1 - e^{-\lambda}$

$= 1 - e^{-4}$

$\approx 1 - 0.0183$

≈ 0.9817

(Ans)

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$V(x) = E(x^2) - \{E(x)\}^2$

$V(1-3x) = E(1-3x)^2 - \{E(1-3x)\}^2$

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<u>Problem:</u> Let the R.V. X have the prob mass function $f(x) = \frac{x}{10}$; $x = 1,2,3,4$. compute $P(x<3)$; $E(x^2+5)$ and $V(1-3x)$. Also, sketch the line graph of x.

<u>Soln:</u>

$x=1$; $f(x=1) = 1/10$

$x=2$; $f(x=2) = 2/10 = 1/5$

$x=3$; $f(x=3) = 3/10$

$x=4$; $f(x=4) = 4/10$

$P(x<3) = P(x=1) + P(x=2)$

$= 1/10 + \frac{2}{10}$

$= \frac{3}{10}$

OR,

$1 - P(x \geq 3)$

$= 1 - \left(\frac{4}{10} + \frac{3}{10}\right)$

$= \frac{10-7}{10}$

$= \frac{3}{10}$

$1 - \frac{2}{5} = 3/5$

$E(x^2+5)$

$= E(x^2)+5$

$E(x^2) = \sum x^2 P(x)$

$= \left\{(1)^2 \times \frac{1}{10}\right\} + \left(4\times\frac{2}{10}\right) + \left(9\times\frac{3}{10}\right) + \left(16\times\frac{4}{10}\right)$

$= \frac{1}{10} + \frac{8}{10} + \frac{27}{10} + \frac{64}{10} = \frac{100}{10}$

$= 10$

$\therefore E(x^2+5) = 10+5 = 15$

(Ans.)

Problem: Suppose that 80% employees in a certain company are proficient in using a particular software tool. In a random sample of 15 employees, Let X be the no of Proficient employees. Assuming independence, how is X distributed? Find the m.g.f of X. Also, compute $P(X<3)$ and $P(x\geq 4)$.

Soln: Binomial distribution.

$n = 15$

$p = 80\% = 0.80$

$p + q = 1$

$\therefore q = 1 - 0.80$

$= 0.20$

Now, $m.g.f = (pe^t + q)^n$

$= (0.80e^t + 0.20)^{15}$

Now, $P(x < 3)$

$= P(x=0) + P(x=1) + P(x=2)$

$= {}^{15}C_0\, 0.80^0 (0.20)^{15-0} + {}^{15}C_1 (0.80)^1 (0.20)^{15-1} + {}^{15}C_2 (0.80)^2 (0.20)^{15-2}$

= calculate it

And $P(x \geq 4)$

$= 1 - P(x < 4)$

$= 1 - \{P(x=0) + P(x=1) + P(x=2) + P(x=3)\}$

= Calculate it.

(Ans.)

* To find $v(1-3x)$

$v(x) = E(x^2) - \{E(x)\}^2$

$v(1-3x) = E\{(1-3x)\}^2 - \{E(1-3x)\}^2$

$= E(1-6x+9x^2) - \{E(1-3x)\}^2$

$E(1-6x+9x^2) = 1-(6\times 3) + (9\times 10)$

$= 1 - 18 + 90$

$= 73$

$E(1-3x) = 1-(3\times 3) = 1-9 = -8$

Now, $v(1-3x)$

$= 73 - (-8)^2$

$= 73 - 64$

$= 9$ (Ans.)

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$x = 1$ $f(x) = 1/5$

$x = 1$; $1/5$

$x = 3 = 2/5$

$x = 5 = 3/5$

$x = 7 = 2/5$

$x = 9 = 1/5$

$(xy + y^2)\,dx + x^2 dy = 0$

$\cancel{\frac{\partial M}{\partial y} = x + 2y}$

$y = ux$

$dy = u\,dx + x\,du$

$(x.ux + u^2x^2)\,dx + x^2(u\,dx + x\,du) = 0$

$\Rightarrow (ux^2 + u^2x^2)\,dx + x^2u\,dx + x^3\,du = 0$

$\cancel{\Rightarrow x^2(u + u^2)\,dx +}$

$ux^2dx + u^2x^2\,dx + x^2u\,dx + x^3\,du = 0$

$2x^2u\,dx + u^2x^2\,dx + x^3\,du = 0$

$x^2(2u + u^2)\,dx + x^3\,du = 0$

$$\frac{x^2\,dx}{x^3} = \frac{du}{2u + u^2}$$

$$\Rightarrow \int \frac{dx}{x} = \int \frac{du}{u(u+2)}$$

$$\frac{1}{u(u+2)} = \frac{A}{u} + \frac{B}{u+2}$$

$1 \equiv A(u+2) + Bu$

$u = 0$

$1 = 2A + 0$

$A = 1/2$

$u = -2$

$1 = 0 + 2B$

$2B = -1 \therefore B = -1/2$

$\frac{1}{2u} - \frac{1}{2(u+2)}$

$= \frac{1}{2}\ln(u) - \frac{1}{2}\ln(u+2)$

Fundamental

$\Rightarrow$ $\Rightarrow$

$\Rightarrow$ Probability

$\Rightarrow$ Fall '23 } Question
Spring '24 } solve

Summer 2024 (Question)

Linear $\Rightarrow$ Spring 24
Fall 23

$\Leftarrow$ Question solve

$91 - 6 = 85$

$E(1-3x)$

$= 1 - (3\times 1) = -2$

$E(1-6x+9x^2)$

$= 1 - 6E(x) + 9E(x^2)$

$= 1 - 6 + 90$

$= 91 - 6$

$= 85$

$E(x) = \Sigma x\, f(x)$

$= (1\times \frac{1}{10}) + (2\times \frac{1}{10}) + (3\times \frac{1}{10}) + (4\times \frac{1}{10})$

$= \frac{10}{10} = 1$

$85 - (-2)^2$

$= 85 - 4$

$= 81$

$1 - P(x \leq 1.5)$

$$P(x \leq 1.5) = \int_{-\infty}^{0} f(x)\,dx + \int_0^1 f(x)\,dx + \int_1^x f(x)\,dx$$

$$= \int_0^1 x\,dx + \int_1^x 2 - x\,dx$$

$$= \left[\frac{x^2}{2}\right]_0^1 + [2x]_1^x - \left[\frac{x^2}{2}\right]_1^x$$

$$= \frac{1}{2} + 2(x-1) - \frac{1}{2}[x^2 - 1]$$

$$= \cancel{\frac{1}{2} + 2x + 2 - \frac{1}{2}x^2 + 1/2}$$

$$-2 - \frac{1}{2} = \frac{-4-1}{2} = -5/2$$

$$= \frac{1}{2} + 2x - 2 - \frac{1}{2}x^2 + 1/2$$

$$= -1 + 2x - \frac{1}{2}x^2$$

$$P(x \leq 1.5) = 1 - 0.875$$

$$= 0.125$$