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Course code: MATH 2205.

Section: A.

Course Title: Probability and Statistics.

Home Assignment 01.

(01)

Let,

A = {having lab work}

B = {having a referral}

From the Question;

$P(A' \cap B') = 0.19$

$P(A) = 0.47$

$P(B) = 0.51$

Neither lab work nor referral occuring;

$P(A \cup B)' = P(A' \cap B') = 0.19$

Now we know;

$P(A \cup B) + P(A \cup B)' = 1$

$\therefore P(A \cup B) = 1 - 0.19 = 0.81$

We have to find both having lab work and referral. So,

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

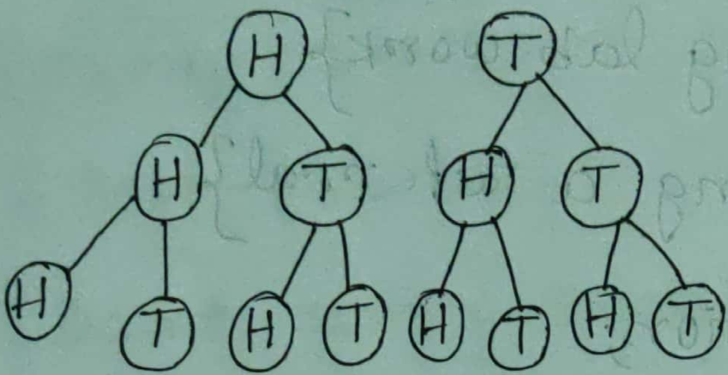
$= 0.47 + 0.51 - 0.81$

$= 0.17$

(Result)

02

Toss



The sample space;

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

Total outcomes 8.

$A = \{$at most 2 tails$\}$

$= \{HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$= 7$. $\therefore P(A) = \frac{7}{8}$.

$B = \{$at least 2 heads$\}$

$= \{HHH, HHT, HTH, THH\}$

$= 4$ $\therefore P(B) = \frac{4}{8}$.

$C = \{$1 head and 2 tails$\}$

$= \{HTT, THT, TTH\}$

$= 3$. $\therefore P(C) = \frac{3}{8}$.

$P(A \cap C) = ?$

$A = \{HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$C = \{HTT, THT, TTH\}$

$\therefore A \cap C = \{HTT, THT, TTH\} = 3.$

$\therefore P(A \cap C) = \frac{3}{8}$,

$P(B \cup C) = ?$

$B = \{HHH, HHT, HTH, THH\}$

$C = \{HTT, THT, TTH\}$

$\therefore B \cup C = \{HHH, HHT, HTH, THH, HTT, THT, TTH\}$

$= 7.$

$\therefore P(B \cup C) = \frac{7}{8}$,

$P(C') = ?$

We know, $P(C) + P(C') = 1$

$\Rightarrow P(C') = 1 - P(C)$

$= 1 - \frac{3}{8} = \frac{8-3}{8}$

$= \frac{5}{8}$,

(Result)

(03)

From the table;

$A_1 = \{\text{carry cancer}\}$

$B_1 = \{\text{Test positive}\}$

$A_2 = \{\text{Don't carry cancer}\}$

$B_2 = \{\text{Test negative}\}$

Total, $A_1 = 5000$, $A_2 = 95000$

$B_1 = 80000$, $B_2 = 20000$

$(A_1 \cap B_1) = 4500$, $(A_1 \cap B_2) = 500$

$(A_2 \cap B_1) = 75500$, $(A_2 \cap B_2) = 19500$

and overall people 100000

$\therefore P(A_1) = \frac{5000}{100000} = \frac{1}{20}$

$\therefore P(A_2) = \frac{95000}{100000} = \frac{19}{20}$

$\therefore P(B_1) = \frac{80000}{100000} = \frac{4}{5}$

$\therefore P(B_2) = \frac{20000}{100000} = \frac{1}{5}$

$\therefore P(A_1 \cap B_1) = \frac{4500}{100000} = \frac{9}{200}$

$\therefore P(A_1 \cap B_2) = \frac{500}{100000} = \frac{1}{200}$

$\therefore P(A_2 \cap B_1) = \frac{75500}{100000} = \frac{151}{200}$

$\therefore P(A_2 \cap B_2) = \frac{19500}{100000} = \frac{39}{200}$

$P(A_1 | B_2) = ?$

We know, $P(A_1/B_2) = \frac{P(A_1 \cap B_2)}{P(B_2)}$

$= \frac{\frac{1}{200}}{\frac{1}{5}}$

$= \frac{1}{40}$

$P(A_2 | B_1) = ?$

We know, $P(A_2 | B_1) = \frac{P(A_2 \cap B_1)}{P(B_1)}$

$= \frac{\frac{151}{200}}{\frac{4}{5}}$

$= \frac{151}{160}$

$P(A_2 \cup B_2) = ?$

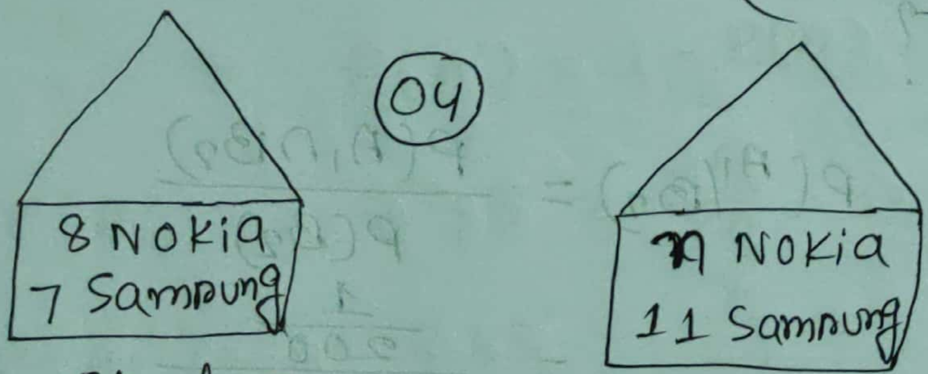
We know,

$$P(A_2 \cup B_2) = P(A_2) + P(B_2) - P(A_2 \cap B_2)$$

$$= \frac{19}{20} + \frac{1}{5} - \frac{39}{200}$$

$$= \frac{191}{200} \quad \text{(Result)}$$

04



1^{st} shop

2^{nd} shop

Total Mobile of 1^{st} shop $= (8+7) = 15$

Total mobile of 2^{nd} shop $= (n+11)$

From statement,

$$P_{(\text{two mobile same company})} = \frac{149}{300}$$

If two phones are from NOKia; the probability will; $\left(\frac{8}{15} \times \frac{n}{n+11}\right)$ or if two phones are samsung. the it will be; $\left(\frac{7}{15} \times \frac{11}{n+11}\right)$,

In total;

$$\left(\frac{8}{15} \times \frac{n}{(n+11)}\right)+\left(\frac{7}{15} \times \frac{11}{(n+11)}\right)=\frac{149}{300}$$

$$\Rightarrow \frac{8n}{15(n+11)}+\frac{77}{15(n+11)}=\frac{149}{300}$$

$$\Rightarrow \frac{8n+77}{15(n+11)}=\frac{149}{300}$$

$$\Rightarrow 2400n+23100=2235n+24585$$

$$\Rightarrow 165n=1485$$

$$\Rightarrow n=9.$$

$\therefore$ There are 9 NOkia mobile in the 2nd shop. (Result)

(5)

Here,

A_i denote the event that the wicket is got by player i. $i = 1, 2, 3$.

$\therefore P(A_1) = 0.35$, $P(A_2) = 0.65$, $P(A_3) = 0.5$.

We know the formula to find complement;

$$P(A) + P(A') = 1$$

$$\Rightarrow P(A') = 1 - P(A).$$

$\therefore P(A_1') = 1 - 0.35 = \frac{13}{20} = 0.65$

$\therefore P(A_2') = 1 - 0.65 = \frac{7}{20} = 0.35$

$\therefore P(A_3') = 1 - 0.5 = \frac{1}{2} = 0.5$

$P_{(\text{exactly two successful})} = ?$

$P_{(\text{No player successful})} = ?$

Let A_1 and A_2 successful, A_3 not.

$\therefore P(A_1 \cap A_2 \cap A_3') = P(A_1)\,P(A_2)\,P(A_3')$

$= (0.35)(0.65)(0.5)$

$= \frac{91}{800} = 0.11375$

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Let, A_1 and A_3 successful, A_2 not.

$$\therefore P(A_1 \cap A_2' \cap A_3) = P(A_1)\,P(A_2')\,P(A_3)$$

$$= (0.35)(0.35)(0.5)$$

$$= \frac{49}{800} = 0.06125$$

Let A_2 and A_3 successful and A_1 not.

$$\therefore P(A_1' \cap A_2 \cap A_3) = P(A_1')\,P(A_2)\,P(A_3)$$

$$= (0.65)(0.65)(0.5)$$

$$= \frac{169}{800} = 0.21125$$

$$\therefore P_{(\text{exactly two successful})} = P(A_1 \cap A_2 \cap A_3') + P(A_1 \cap A_2' \cap A_3) + P(A_1' \cap A_2 \cap A_3)$$

$$= 0.11375 + 0.06125 + 0.21125$$

$$= \frac{309}{800} = 0.38625 \text{ (Result)}$$

$\therefore P_{(\text{No player successful})} = P(A_1' \cap A_2' \cap A_3')$

$\therefore P(A_1' \cap A_2' \cap A_3') = P(A_1') \, P(A_2') \, P(A_3')$

$= (0.65)(0.35)(0.5)$

$= \frac{91}{800} = 0.11375$

(Result).

(6)

10 Yellow
8 red
7 green

Total ballons on the board $= 25$.

$P_{(\text{1st ballon green})} = \frac{7}{25}$

$P(\text{2nd ballon green} / \text{1st ballon green}) = \frac{6}{24}$

(i)

$\therefore P_{(\text{next ballon green})} = \frac{7}{25} \times \frac{6}{24} = \frac{7}{100}$

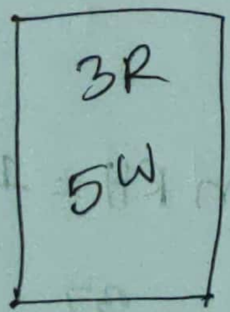
(ii) P(next ballon is not green) = ?

$$\therefore \left(\frac{7}{25} \times \frac{10}{24}\right) + \left(\frac{7}{25} \times \frac{8}{24}\right)$$

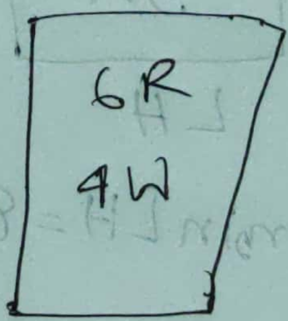
$$= \frac{7}{60} + \frac{7}{25}$$

$$= \frac{21}{100}$$ (Result)

(7)



LH



RH

$$P(LW) = P(LW \cap LR) + P(LW \cap LW)$$

$$= \left(\frac{4}{7} \times \frac{3}{7}\right) + \left(\frac{4}{7} \times \frac{4}{7}\right)$$

$$= \frac{4}{7}.$$

(Result)

(8)

LH	RH
3R	6R
5W	4W

P(same from right hand (RH))

$= P(LW \cap RW) + P(LR \cap RR)$

$= \left(\frac{4}{7} \times \frac{5}{11}\right) + \left(\frac{1}{7} \times \frac{7}{11}\right)$

$= \frac{34}{77}$.

P(Different from RH)

$= P(LW \cap RR) + P(LR \cap RW)$

$= \left(\frac{4}{7} \times \frac{6}{11}\right) + \left(\frac{2}{7} \times \frac{4}{11}\right)$

$= \frac{32}{77}$

(Result)

(09)

Given,

$P(A) = 0.98 \qquad P(A)' = 1 - 0.98 = 0.02$

$P(B) = 0.95 \qquad P(B)' = 1 - 0.95 = 0.05$

$P(C) = 0.92 \qquad P(C)' = 1 - 0.92 = 0.08$

$P(D) = 0.89 \qquad P(D)' = 1 - 0.89 = 0.11.$

(i) $P_{(\text{no one detecting error})} = P(A)' \, P(B)' \, P(C)' \, P(D)'$

$= (0.02)(0.05)(0.08)(0.11)$

$= 8.8 \times 10^{-6}.$

(ii) ~~No or~~

$P_{(\text{at least one detecting})}$

$= 1 - P_{(\text{no one detecting error})}$

$= 1 - (8.8 \times 10^{-6})$

$= 0.9999912.$

(iii) only one defecting error;

$= (P(A)\ P(B)'\ P(C)'\ P(D)') + (P(A)'\ P(B)\ P(C)'\ P(D)')$
$+ (P(A)'\ P(B)'\ P(C)\ P(D)') + (P(A)'\ P(B)'\ P(C)'\ P(D))$

$= (0.98 \times 0.05 \times 0.08 \times 0.11) + (0.02 \times 0.95 \times 0.08 \times 0.11)$
$+ (0.02 \times 0.05 \times 0.92 \times 0.11) + (0.02 \times 0.05 \times 0.08 \times 0.89)$

$= 7.708 \times 10^{-4}$. (Result)

10

$P(A) = 0.3 \quad P(B) = 0.6$

(i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.3 + 0.6 - (0.3 \times 0.6)$ [A and B independent]
$= \frac{18}{25} = 0.72$.

(ii) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0}{0.6}$ [Since A and B are mutually exclusive. So, $(A \cap B) = 0$]

$= 0$.

(Result)

(11)

Here given;

$P(D^+) = 0.01$; So, $P(D^-) = 0.99$

$P(T^+/D^+) = 0.99$, $P(T^-/D^+) = 0.01$

$P(T^+/D^-) = 0.03$, $P(T^-/D^-) = 0.97$.

So,

$\therefore P(T^+) = P(T^+ \cap D^+) + P(T^+ \cap D^-)$

$= P(T^+/D^+)\, P(D^+) + P(T^+/D^-)\, P(D^-)$

$= (0.99 \times 0.01) + (0.03 \times 0.99)$

$= 0.0396$.

Now; $P(D^+/T^+) = \dfrac{P(D^+ \cap T^+)}{P(T^+)}$

$= \dfrac{P(D^+)\, P(T^+/D^+)}{P(T^+)}$

$= \dfrac{0.01 \times 0.99}{0.0396} = 0.25$

$\therefore P(D^-/T^+) = 1 - P(D^+/T^+) = 1 - 0.25 = 0.75$.
(Result)

Here given,

$P(D^+) = 0.002$, So, $P(D^-) = 0.998$

$P(T^+/D^+) = 0.95$, So, $P(T^-/D^+) = 0.05$

$P(T^+/D^-) = 0.02$, $P(T^-/D^-) = 0.98$

Now;

$$P(T^+) = P(T^+ \cap D^+) + P(T^+ \cap D^-)$$

$$= P(T^+/D^+)P(D^+) + P(T^+/D^-)\ P(D^-)$$

$$= (0.95 \times 0.002) + (0.02 \times 0.998)$$

$$= 0.02186$$

So,

$$\therefore P(D^+/T^+) = \frac{P(T^+/D^+)\ P(D^+)}{P(T^+)}$$

$$= \frac{0.95 \times 0.002}{0.02186}$$

$$= 0.0869167 \text{ (Result)}$$

Die A :- 1 orange , 5 Blue

Die B :- 2 orange, 4 Blue.

Die C :- 3 orange, 3 Blue.

Let q = { Number of come up orange }

(i) $P(q \geq 2) = 1 - P(q < 2)$

$= 1 - \{P(q=0) + P(q=1)\}$ -- (i)

$P(q=0) = \left(\frac{5}{6}\right)\left(\frac{4}{6}\right)\left(\frac{3}{6}\right) = \frac{5}{18}$

$P(q=1) = \left(\frac{1}{6} \times \frac{4}{6} \times \frac{3}{6}\right) + \left(\frac{5}{6} \times \frac{2}{6} \times \frac{3}{6}\right) + \left(\frac{5}{6} \times \frac{4}{6} \times \frac{3}{6}\right) = \frac{17}{36}$

Now, From (i),

$P(q \geq 2) = 1 - \frac{5}{18} - \frac{17}{36} = \frac{1}{4}$.

(ii) $P(g=2) = \left(\frac{1}{6} \times \frac{2}{6} \times \frac{3}{6}\right) + \left(\frac{5}{6} \times \frac{2}{6} \times \frac{3}{6}\right) + \left(\frac{1}{6} \times \frac{4}{6} \times \frac{3}{6}\right) = \frac{2}{6}$

(Result)

(14)

B_1	B_2	B_3	B_4
1R	2R	2R	3R
2W	1W	3W	2W

From statement;

$P(B_1) = \frac{3}{8}$, $P(B_2) = \frac{1}{4}$, $P(B_3) = \frac{1}{4}$, $P(B_4) = \frac{1}{8}$

Now,

$P(R) = \left(\frac{1}{3} \times \frac{3}{8}\right) + \left(\frac{2}{3} \times \frac{1}{4}\right) + \left(\frac{2}{5} \times \frac{1}{4}\right) + \left(\frac{3}{5} \times \frac{1}{8}\right)$

$= \frac{7}{15}$

$\therefore P(B_3/R) = \frac{P(R/B_3)\ P(B_3)}{P(R)}$

$= \frac{\frac{2}{5} \times \frac{1}{4}}{\frac{7}{15}} = \frac{3}{14}$

(Result)

Let,

$A = \{$Efficient worker$\}$

$B = \{$Moderate worker$\}$

$C = \{$unfit worker$\}$

$\therefore P(A) = 0.3 \quad P(B) = 0.5, \quad P(C) = 0.2$

$P(L/A) = 0.15, \quad P(L/B) = 0.2, \quad P(L/C) = 0.05$

$\therefore P(L) = (0.15 \times 0.3) + (0.2 \times 0.5) + (0.05 \times 0.2)$

$= 0.155.$

$\therefore P(C/L) = \dfrac{P(L/C)\,P(C)}{P(L)} = \dfrac{0.05 \times 0.2}{0.155}$

$= \dfrac{2}{31}$

(Result)

(16)

From statement;

$N = 100, \quad N_1 = 5, \quad N_2 = 95, \quad n = 15$

(i) $P(X \le 1) = P(X=0) + P(X=1)$

$$= \frac{5c_0 \times 95c_{15}}{100c_{15}} + \frac{5c_1 \times 95c_{14}}{100c_{15}}$$

$$= 0.8390.$$

(ii) $P(X=3) = \frac{5c_3 \times 95c_{12}}{100c_{15}} = 0.0216.$

(Result)

(17)

$$M(t) = \frac{4}{10} e^{t} + \frac{3}{10} e^{2t} + \frac{2}{10} e^{3t} + \frac{1}{10} e^{4t}$$

$M(0) = \frac{4}{10} + \frac{3}{10} + \frac{2}{10} + \frac{1}{10} = 1$, So, $M(t)$

is mgf.

pmf, $f(x) = \frac{5-x}{10}$

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$\therefore$ Mean, for finding mean we have to find first $M'(t)$.

$$\therefore M'(t) = \frac{4}{10}e^{t} + \frac{6}{10}e^{2t} + \frac{6}{10}e^{3t} + \frac{4}{10}e^{4t}$$

$$\therefore M'(0) = \frac{4}{10} + \frac{6}{10} + \frac{6}{10} + \frac{4}{10} = 2$$

$$\therefore \text{mean}, \mu = M'(0) = 2.$$

Now;

$$M''(t) = \frac{4}{10}e^{t} + \frac{12}{10}e^{2t} + \frac{18}{10}e^{3t} + \frac{16}{10}e^{4t}$$

$$\therefore M''(0) = \frac{4}{10} + \frac{12}{10} + \frac{18}{10} + \frac{16}{10} = 5$$

Varience, $\sigma^2 = M''(0) - \{M(0)\}^2$

$$= 5 - 2^2$$

$$= 5 - 4 = 1$$

(Result)

(18)

From statement,

$n = \frac{1}{120}$, $P = 60$ $\therefore \lambda = np = \frac{1}{120} \times 60 = \frac{1}{2}$

$\therefore P(X \leq 1) = P(X=0) + P(X=1)$

$= \frac{e^{-1/2} \times (\frac{1}{2})^0}{0!} + \frac{e^{-1/2} (\frac{1}{2})^1}{1!}$

$= 0.91$ (Result)

(19)

Here,

$E(X) = (1 \times 0.3) + (2 \times 0.2) + (3 \times 0.1) + (-1 \times 0.4)$

$= \frac{3}{5} = 0.6$

The profit is 0.6.

$E(X^2) = (1^2 \times 0.3) + (2^2 \times 0.2) + (3^2 \times 0.1) + ((-1)^2 \times 0.4)$

$= 2.4.$

$\therefore$ varience, $\sigma^2 = E(X^2) - \{E(X)\}^2$

$= 2.4 - (0.6)^2 = 2.04.$

(Result)

Here, From statement;

$n=5$, $p=0.95$, $q=0.05$

$f(x)=5_{C_x}(0.95)^x(0.05)^{5-x}$, $X=b(5,0.95)$

$P(X=0)=3.125\times10^{-7}$

$P(X=1)=2.96\times10^{-5}$

$P(X=2)=1.128\times10^{-3}$

$P(X=3)=0.0214$

(i) $P(X=0)=3.125\times10^{-7}$

(ii) $P(X\geq4)=1-P(X\leq3)$

$=1-\{P(X=0)+P(X=1)+P(X=2)+P(X=3)\}$

$=0.0226$

(Result)

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(21)

Here, $2P(X=2) = P(X=1)$

$$\Rightarrow 2 \times \frac{e^{-\lambda}\lambda^2}{2!} = \frac{e^{-\lambda}\lambda}{1!}$$

$$\Rightarrow \lambda^2 = \frac{e^{-\lambda}\lambda}{e^{-\lambda}}$$

$$\Rightarrow \lambda^2 = \lambda \quad [e^{-\lambda} \neq 0]$$

$$\Rightarrow \lambda = 1$$

$$\therefore P(X=5) = \frac{e^{-1}.1^5}{5!} = 3.066 \times 10^{-3}$$

(Result)

(22)

Here,

$$\lambda = 6^2 = (2)^2 = 4.$$

$$P(X \geq 1) = 1 - P(X=0)$$

$$= 1 - \frac{e^{-4}.4^0}{0!}$$

$$= 0.98$$

(Result).

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(23)

$f(x) = c(x+1)^3, \quad x = 0, 1, 2, \cdots 10,$

It is known to us that;

$$\sum f(x) = 1$$

$$\Rightarrow c\left(\frac{n(n+1)}{2}\right)^2 = 1$$

$$\Rightarrow c \times \left(\frac{11(11+1)}{2}\right)^2 = 1 \quad [\text{Here, } n = 11]$$

$$\Rightarrow c \times 4356 = 1$$

$$\Rightarrow c = \frac{1}{4356}$$

$\therefore f(x) = \frac{(x+1)^2}{4356}; \quad x = 0, 1, 2, \cdots 10$

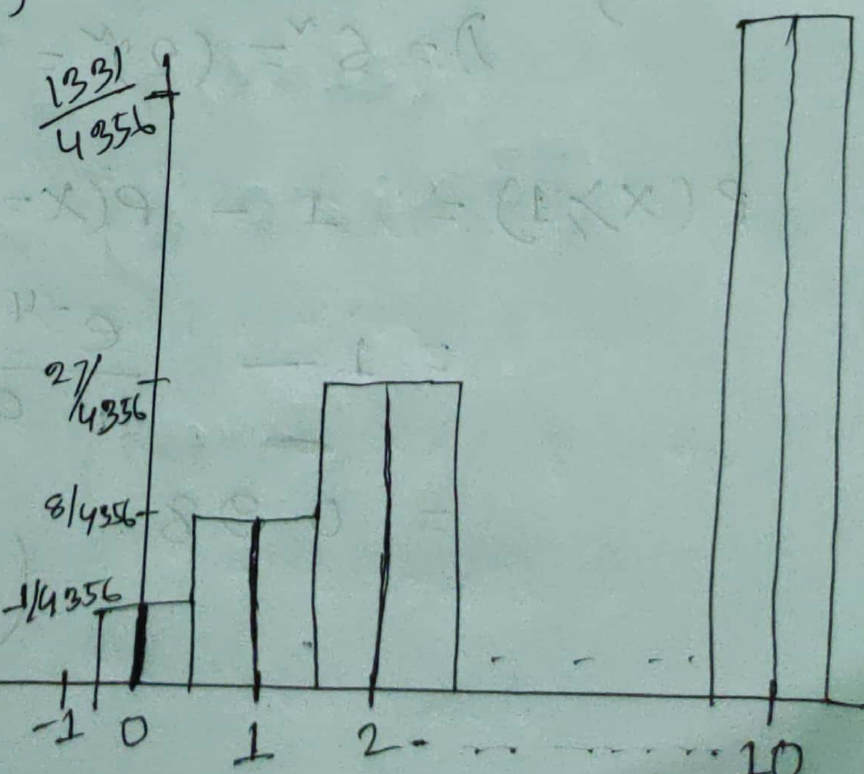
$\therefore P(X=0) = \frac{1}{4356}$

$\therefore P(X=1) = \frac{8}{4356}$

$\therefore P(X=2) = \frac{27}{4356}$

$\vdots$

$\therefore P(X=10) = \frac{1331}{4356}$



Here, $E[X+c]=10$

$\Rightarrow E(X)+c=10$

$\Rightarrow E(X)=10-c$

$\mu=10-c$

Again, $E[(X+c)^2]=116$

$\Rightarrow E[X^2+2Xc+c^2]=116$

$\Rightarrow E(X^2)+2cE(X)+c^2=116$

$\Rightarrow E(X^2)+2c(10-c)+c^2=116$

$\Rightarrow E(X^2)+20c-2c^2+c^2=116$

$\Rightarrow E(X^2)+20c-c^2=116$

$\Rightarrow E(X^2)=116-20c+c^2$

$\sigma^2=E(X^2)-\{E(X)\}^2$

$=116-20c+c^2-(10-c)^2$

$=116-20c+c^2-100+20c-c^2$

$=16$ (Result)

(25)

Here,

$$\left.\begin{aligned} p &= 0.2 \\ q &= 0.8 \\ n &= 15 \end{aligned}\right\} \begin{aligned} &f(x) = {}^{15}C_x (0.2)^x (0.8)^{15-x} \\ &\text{So, } x \text{ is } b(5, 0.2) \end{aligned}$$

$$\therefore P(X \geq 2) = 1 - P(X \leq 1)$$

$$= 1 - P(X=0) - P(X=1)$$

$$= 1 - 0.0352 - 0.1319$$

$$= 0.8329$$

$$\therefore P(X \leq 14) = 1 - P(X > 14)$$

$$= 1 - P(X = 15)$$

$$= 1 - (3.2768 \times 10^{-11})$$

$$= 1.$$

(Result)

(26)

Here,

$P(X=1) = \frac{1}{36}$

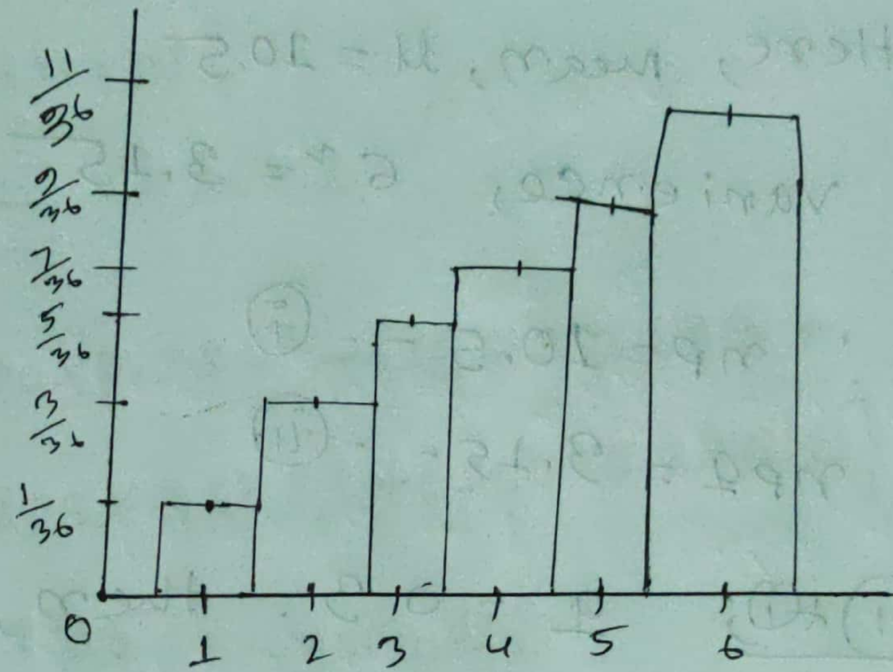
$P(X=2) = \frac{3}{36}$

$P(X=3) = \frac{5}{36}$

$P(X=4) = \frac{7}{36}$

$P(X=5) = \frac{9}{36}$

$P(X=6) = \frac{11}{36}$



Let, $P(X=x_1) = \frac{3}{36}$, $P(X=x_0) = \frac{1}{36}$, $x_1 = 2$, $x_0 = 1$.

Then, $$\frac{f(x) - \frac{3}{36}}{\frac{1}{36} - \frac{3}{36}} = \frac{x-2}{1-2}$$

$$\Rightarrow 36f(x) - 3 = -(x-2) \times -2$$

$$\Rightarrow f(x) = \frac{2x-1}{36} \; ; \quad x = 1, 2, 3, 4, 5, 6.$$

(Result)

Here, mean, $\mu = 10.5$

varience, $\sigma^2 = 3.15$

$\therefore np = 10.5$ -- (i)

$npq = 3.15$ -- (ii)

(ii) $\div$ (i); $q = 0.3$. then, $p = 0.7$.

from (i); $n = \frac{10.5}{0.7} = 15$

$\therefore f(x) = {}^{15}C_x \, (0.7)^x \, (0.3)^{15-x}$

$\therefore P(X \geq 1) = 1 - P(X=0)$

$= 1 - 1.435 \times 10^{-8}$

$= 0.999999985\,7$.

(Result)

(28)

Expected payment,

$$E(X) = \sum_{n=1}^{4} n f(n)$$

$$= \{(100 \times 0.1 \times 4) + (150 \times 0.1 \times 3) + (175 \times 0.1 \times 2) + (150 \times 0.1 \times 0.1)\}$$

$$= \$135 \text{ (Result)}$$

(29)

Here, $n = 500$, $p = 0.15$, $q = 0.85$

$$\therefore f(x) = 500_{C_x} (0.15)^x (0.85)^{500-x}$$

$$\therefore P(X=35) = 1.8348 \times 10^{-8}$$

Again, $\lambda = np = 500 \times 0.15 = 75$

$$f(x) = \frac{e^{-75} \cdot 75^x}{x!}$$

$$\therefore P(X=35) = \frac{e^{-75} \times 75^{35}}{35!}$$

$$= 1.0986 \times 10^{-7} \text{ (Result)}$$

Here, $p = 0.9$

$q = 0.1$

$n = 10$

$f(x) = {}^{10}C_x (0.9)^x (0.1)^{10-x}$

$P(x \geq 2) = 1 - P(X < 2)$

$= 1 - P(X = 1) - P(X = 0)$

$= 1 - 9 \times 10^{-9} - 1 \times 10^{-10}$

$\doteq 0.9999$ (Result)

31

$P(X = -4) = \frac{9}{21}$

$P(X = -2) = \frac{1}{21}$

$P(X = 0) = \frac{1}{21}$

$P(X = 2) = \frac{1}{21}$

$P(X = 4) = \frac{9}{21}$

$\therefore E(X) = \left(-4 \times \frac{9}{21}\right) + \left(-2 \times \frac{1}{21}\right) + \left(0 \times \frac{1}{21}\right) + \left(2 \times \frac{1}{21}\right) + \left(4 \times \frac{9}{21}\right) = 0$

$\therefore E(X^2) = \left((-4)^2 \times \frac{9}{21}\right) + \left((-2)^2 \times \frac{1}{21}\right) + \left(0^2 \times \frac{1}{21}\right) + \left(2^2 \times \frac{1}{21}\right) + \left(4^2 \times \frac{9}{21}\right)$

$\therefore \mu = 0$

$\sigma^2 = E(X^2) - \{E(X)\}^2 = \frac{296}{21} - 0^2 = \frac{296}{21}$

$\therefore E(X^2 - 3X + 4) = E(X^2) - 3E(X) + 4$

$= \frac{296}{21} - (3 \times 0) + 4 = \frac{380}{21}$

$V(1-2x) = (-2)^2 \, Var(x) = (-2)^2 \times \frac{296}{21} \quad [Var(x) = \sigma^2]$

$= \frac{1184}{4}$

$= 9.785$

(Result)

(32)

$M(t) = (0.4 + 0.6e^t)^{15}$

$\therefore M(0) = (0.4 + 0.6)^{15} = 1^{15} = 1$, So, mgf

Here, $n = 15$, $q = 0.4$, $p = 0.6$

$\therefore f(x) = {}^{15}C_x (0.6)^x (0.4)^{15-x}$

Mean, $\mu = np = 15 \times 0.6 = 9$

varience, $\sigma^2 = npq = 15 \times 0.6 \times 0.4 = 3.6$

(33)

Given, $M(t) = \frac{0.3e^t}{1 - 0.7e^t}$ [mgf, geometric distribution]

$\therefore M(0) = \frac{0.3}{1 - 0.7} = 1$. So. mgf.

$f(x) = q^{x-1} p$

$p = 0.3$
$q = 0.7$

$\therefore \mu = \frac{1}{p} = \frac{1}{0.3}$

$= \frac{10}{3} = 3.3333.$

$\therefore \sigma^2 = \frac{q}{p^2} = \frac{0.7}{(0.3)^2} = \frac{70}{9} = 7.777777778.$

(Result)

(34)

Here,

$$V(0,4) = \begin{cases} \frac{e^{4t}-e^{0t}}{(4-0)t} ; & t \neq 0 \\ 1 ; & t = 1 \end{cases}$$

$\therefore$ pdf is $f(x) = \frac{1}{b-a} = \frac{1}{4} ; \quad 0 \le x \le 4$

$\therefore \mu = \frac{4+0}{2} = 2; \quad \sigma^2 = \frac{(4-0)^2}{12} = \frac{4}{3}$

$$\text{cdf}, F(x) = \begin{cases} 0 ; & x < 0 \\ \frac{x-0}{4-0} ; & 0 \le x \le 4 \\ 1 ; & x \geq 4 \end{cases}$$

$$P(X > 3.5) = 1 - P(X \le 3.5)$$

$$= 1 - \frac{3.5}{4} = \frac{7}{8}$$

(Result)

$$F(x) = \int_0^x 2e^{1-2w}\,dw$$

$$= 2\int_0^x e^{1-2w}\,dw$$

$$= 2e\int_0^x e^{-2w}\,dw$$

$$= 2e\left[\frac{e^{-2w}}{-2}\right]_0^x$$

$$= -e\left[e^{-2x}-1\right] = e - e^{1-2x}$$

For 3rd decile = 0.3.

$$F(\pi_{0.3}) = 0.3$$

$$\Rightarrow e^1 - e^{1-2\pi_{0.3}} = 0.3$$

$$\Rightarrow \ln(e) - \ln(e^{1-2\pi_{0.3}}) = 0.3$$

$$\Rightarrow 1 - 1 - 2\pi_{0.3} = \ln(0.3)$$

$$\Rightarrow \pi_{0.3} = \frac{-\ln(0.3)}{2} = 0.601986$$

(Result)

36

$$\int_0^1 f(x)\,dx = \int_0^1 4x^c\,dx$$

$$= 4\left[\frac{x^{c+1}}{c+1}\right]_0^1$$

$$= 4\left[\frac{1^{c+1}}{c+1} - \frac{0^{c+1}}{c+1}\right]$$

$$= \frac{4}{c+1}$$

We know, $\int_0^1 4x^c\,dx = 1$

$$\Rightarrow \frac{4}{c+1} = 1$$

$$\Rightarrow 4 = c+1$$

$$\Rightarrow c = 3$$

$\therefore f(x) = 4x^3;\ 0 < x < 1$

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$$\therefore F(x) = \int_0^x f(\omega)\, d\omega$$

$$= \int_0^x 4\omega^3\, d\omega$$

$$= \left[\omega^4\right]_0^x = x^4$$

$$\therefore \text{cdf}, F(x) = \begin{cases} 0; & x < 0 \\ x^4; & 0 \leq x < 1 \\ 1; & x \geq 1 \end{cases}$$

Mean, $\mu = E(X) = \int_0^1 x f(x)\, dx$

$$= \int_0^1 x \cdot 4x^3\, dx$$

$$= \int_0^1 4x^4\, dx$$

$$= 4\left[\frac{x^5}{5}\right]_0^1$$

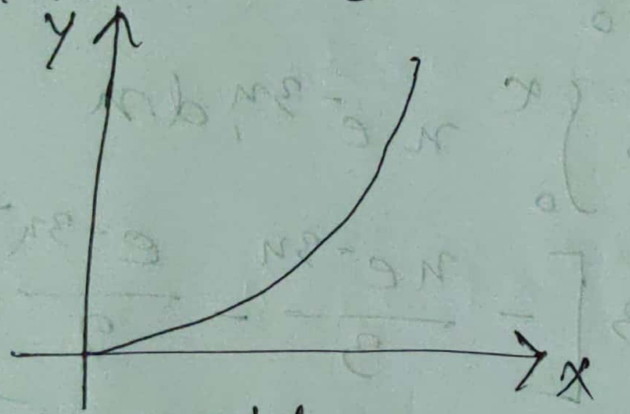
$$= \frac{4}{5}$$

$$E(X^2) = \int_0^1 x^2 f(x)\,dx$$

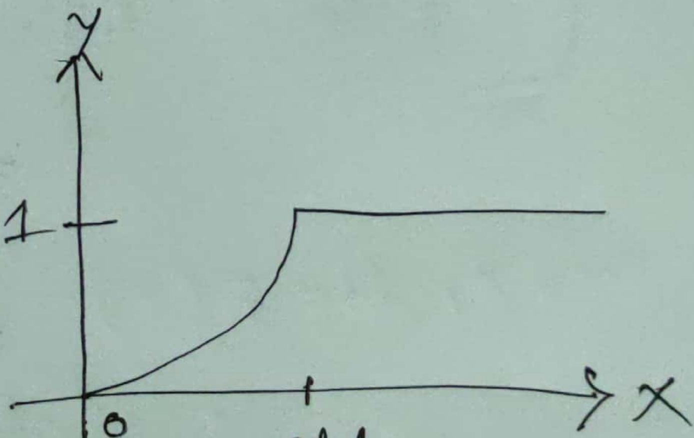
$$= \int_0^1 x^2\, 4x^3\,dx$$

$$= \int_0^1 4x^5\,dx = 4\left[\frac{x^6}{6}\right] = \frac{2}{3}$$

$\therefore$ varience, $\sigma^2 = \frac{2}{3} - \left(\frac{4}{5}\right)^2 = 0.02667.$



pdf



cdf.

(37)

$f(x) = 3e^{-3x}$; $0 \le x < \alpha$

(1) $F(x) = \int_0^x 3e^{-3w}\,dw$

$$= 3\left[\frac{e^{-3w}}{-3}\right]_0^x = -\left[e^{-3w}\right]_0^x$$

$$= 1 - e^{-3x}.$$

$$\therefore \text{cdf}, F(x) = \begin{cases} 0 \; ; & x < 0 \\ 1 - e^{-3x} \; ; & 0 \leq x < 1 \\ 1 \; ; & x > 1 \end{cases}$$

(ii) Mean, $\mu = E(X) = \int_0^\infty x f(x)\, dx$

$$= \int_0^\infty x \cdot 3e^{-3x}\, dx$$

$$= 3\int_0^\infty x e^{-3x}\, dx$$

$$= 3\left[-\frac{xe^{-3x}}{3} - \frac{e^{-3x}}{9} \right]_0^\infty$$

$$= 3\left[(-0-0) - \left(-0 - \frac{1}{9}\right) \right]$$

$$= \frac{1}{3}$$

$$E(x^2) = \int_0^\infty x^2 \cdot 3e^{-3x}\, dx$$

$$= 3\int_0^\infty x^2 e^{-3x}\, d\omega$$

$$= 3\left[\frac{x^2 e^{-3x}}{-3} - \frac{2xe^{-3x}}{9} - \frac{2e^{-3x}}{27} \right]_0^\infty$$

$$= 3\left[(0-0-0)-(0-0-\frac{2}{27})\right] = \frac{2}{9},$$

$\therefore$ varience, $\sigma^2 = E(X^2) - \{E(X)\}^2$

$$= \frac{2}{9} - \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

(iii) $P(X \geq 2) = 1 - P(X < 2)$

$$= 1 - (1 - e^{-3.2})$$

$$= e^{-6} \text{ (Result)}$$

38

cdf, $F(x) = \int_0^x \frac{3w^2}{7^3} e^{(-\frac{w}{7})^3} dw$

| $z = \left(\frac{w}{7}\right)^3$
| $dz = \frac{3w^2}{7^3} dw$
| $\frac{w^2}{7^3} dw = \frac{dz}{3}$

$$= \int_0^{(x/7)^3} e^{-z} dz$$

$$= \left[-e^{-z}\right]_0^{(x/7)^3}$$

$$= 1 - e^{(-x/7)^3}; \quad 0 < x < \alpha.$$

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$\therefore P(X \geq 10.5) = 1 - P(X < 10.5)$

$= 1 - \left[1 - e^{\left(\frac{-10.5}{7}\right)^3}\right]$

$= 0.034.$

$\therefore P(X \geq 17.5) = 1 - P(X < 17.5)$

$= 1 - \left[1 - e^{\left(\frac{-17.5}{7}\right)^3}\right]$

$= 1.637 \times 10^{-7}.$

$\therefore P(X \geq 17.5 \mid X \geq 10.5) = \frac{1.637 \times 10^{-7}}{0.034}$

$= 4.815 \times 10^{-6}.$

median, $\pi_{0.5}$

$F(\pi_{0.5}) = 0.5$

$\Rightarrow 1 - e^{\left(\frac{\pi_{0.5}}{7}\right)^3} = 0.5$

$\Rightarrow \frac{\pi_{0.5}}{7} = \sqrt[3]{\ln(0.5)}$

$\Rightarrow \pi_{0.5} = 7 \times \sqrt[3]{\ln(0.5)}$

$= -6.195$ (Result)

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(39)

Here, $\lambda = \frac{2}{5}$, $\therefore \theta = \frac{5}{2}$

$$f(x) = \frac{1}{5/2} e^{-x/5/2}$$

$$= \frac{2}{5} e^{-2x/5}$$

$$F(x) = 1 - e^{-2x/5}$$

$$\therefore P(X > 2) = 1 - P(X \leq 2) = 1 - F(2)$$

$$= 1 - \left[1 - e^{-4/5}\right] = e^{-4/5}$$

(Result)

(40)

$\lambda = \frac{4}{24} = \frac{1}{6}$, $\theta = \frac{1}{\lambda} = 6$

$$\therefore f(x) = \frac{1}{\theta} e^{-x/\theta} = \frac{1}{6} e^{-x/6}$$

$$F(x) = 1 - e^{-x/6}$$

$$\therefore P(X \leq 2) = F(3) = 1 - e^{-3/6} = 0.3935.$$

(Result)

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(41)

Here, $\gamma = \frac{3}{2}$; $\theta = \frac{2}{3}$

$$f(x) = \frac{1}{\frac{2}{3}} e^{-x/2/3} = 3/2 \, e^{-3x/2}$$

$$F(x) = 1 - e^{-3x/2}$$

$$\therefore P(X < 2) = F(2) = 1 - e^{-3.2/2} = 0.9502.$$

(Result)

(42)

Here, $\theta = 5$, $\alpha = 2$

$$f(x) = \frac{1}{\Gamma 2 \, 5^2} x^1 e^{-x/5}$$

$$= \frac{1}{25} x e^{-x/5}$$

$\therefore \mu = \alpha\theta = 2 \times 5 = 10$

$\therefore \sigma^2 = \alpha\theta^2 = 2 \times 5^2$
$= 50$

$$F(x) = \int_0^x \frac{1}{25} z e^{-z/5} dz$$

$$= \frac{1}{25} \int_0^x z e^{-z/5} dz$$

$$= \frac{1}{25} \left[-5 z e^{-z/5} - 25 e^{-z/5} \right]_0^x$$

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$$= \frac{1}{25}\left[\left(-5xe^{-x/5} - 25e^{-x/5}\right) - (0-25)\right]$$

$$= \frac{1}{25}\left[-5xe^{-x/5} - 25e^{-x/5} + 25\right]$$

$\therefore P(X \geq 6) = 1 - P(X < 6) = 1 - F(6)$

$= 0.6625.$

(Result)

(43)

Here, $M = (1-5t)^{-3} = \frac{1}{(1-5t)^3}$

$\theta = 5, \quad \alpha = 3.$

pdf, ; $f(x) = \frac{1}{\Gamma 3 \, 5^3} x^{3-1} e^{-x/5}$

$= \frac{1}{2 \times 5^3} x^2 e^{-x/5}$

$= \frac{1}{250} x^2 e^{-x/5}$

$F(x) = \int_0^x \frac{1}{250} w^2 e^{-w/5} \, dw$

$= \frac{1}{250}\left[-5w^2 e^{-w/5} - 50we^{-w/5} - 250e^{-w/5}\right]_0^x$

$= \frac{1}{250}\left[-5x^2 e^{-x/5} - 500x e^{-x/5} - 250 e^{-x/5} + 250\right]$

$\therefore P(X>4) = 1 - P(X \leq 4) = 1 - F(4)$

$= 0.9526$

(Result)

44

Here, $\lambda = \frac{5}{60} = \frac{1}{12}$, $\theta = 12$

Pdf, $f(x) = \frac{1}{12} e^{-x/12}$

Median; $m = 12 \ln(2) = 8.3178$.

mgf, $M(t) = \frac{1}{1-12t}$; $t < \frac{1}{12}$

$\therefore F(x) = 1 - e^{-x/12}$

$\therefore P(X \geq 10) = 1 - P(X<10)$

$= 1 - (1 - e^{-10/12})$

$= e^{-10/12}$ (Result)

(45)

Here, $\mu = 25$, $\sigma^2 = 36$, $\sigma = 6$

pdf, $f(x) = \frac{1}{6\sqrt{2\pi}} e^{\frac{-(x-25)^2}{72}}$

$$z = \frac{x-25}{6}$$

$\therefore P(|x-25| \leq c) = 0.9332$

$\Rightarrow P(-c \leq x-25 \leq c) = 0.9332$

$\Rightarrow P\left(\frac{-c}{6} \leq \frac{x-25}{6} \leq \frac{c}{6}\right) = 0.9332$

$\Rightarrow P\left(\frac{-c}{6} \leq z \leq \frac{c}{6}\right) = 0.9332$

$\Rightarrow \phi\left(\frac{c}{6}\right) - \phi\left(-\frac{c}{6}\right) = 0.9332$

$\Rightarrow \phi\left(\frac{c}{6}\right) - 1 + \phi(c/6) = 0.9332$

$\Rightarrow \phi(c/6) = 0.9666$

$\Rightarrow c/6 = 1.83$

$\Rightarrow c = 10.98$

(Result)

(46)

Here, $\mu = 30$, $\sigma^2 = 36$, $\sigma = 6$.

$$z = \frac{x-30}{6}.$$

(i) $P(|z| \leq k) = 0.9544$

$\Rightarrow P(-k \leq z \leq k) = 0.9544$

$\Rightarrow \phi(k) - \phi(-k) = 0.9544$

$\Rightarrow \phi(k) - 1 + \phi(k) = 0.9544$

$\Rightarrow \phi(k) = 0.9722$

$\therefore k = 2.$

(ii) $P(42.6 \leq x \leq 55.8)$

$$\Rightarrow P\left(\frac{42.6-30}{6} \leq \frac{x-30}{6} \leq \frac{55.8-30}{6}\right)$$

$\Rightarrow P(2.1 \leq z \leq 4.3)$

$\Rightarrow \phi(4.3) - \phi(2.1)$

$\Rightarrow 1 - 0.98214 = 0.01786$

$\phi(z) = 0.9656$

$\Rightarrow z = 1.82 \quad \therefore -z = -1.82$ (Result)

(47)

Here given,

$\mu = 650, \quad \sigma^2 = 625, \quad \sigma = 25$

(i) $P(631 \leq X \leq 676)$

$= P\left(\frac{631-650}{25} \leq \frac{X-650}{25} \leq \frac{676-650}{25}\right)$

$= P(-0.76 \leq Z \leq 1.04)$

$= \Phi(1.04) - \Phi(-0.76)$

$= 0.85083 - 0.22363 = 0.6272$

(ii) $P(|X-650| \leq c) = 0.6826$

$\Rightarrow P(-c \leq X-650 \leq c) = 0.6826$

$\Rightarrow P\left(\frac{-c}{25} \leq \frac{X-650}{25} \leq \frac{c}{25}\right) = 0.6826$

$\Rightarrow P\left(-\frac{c}{25} \leq Z \leq \frac{c}{25}\right) = 0.6826$

$\Rightarrow \phi\left(\frac{c}{25}\right) - 1 + \phi\left(\frac{c}{25}\right) = 0.6826$

$\Rightarrow \phi\left(\frac{c}{25}\right) = 0.8413$

$\Rightarrow \frac{c}{25} = 1 \quad \Rightarrow c = 25.$ (Result)