

$$f(x_i; \theta) = \frac{1}{\theta} e^{-x_i/\theta}$$

$$L(\theta) = \prod_{i=1}^n f(x_i; \theta)$$

$$= \prod_{i=1}^n \frac{1}{\theta} e^{-x_i/\theta}$$

$$= \frac{1}{\theta^n} e^{-\sum x_i/\theta}$$

$$\ln L(\theta) \\ = -n \ln \theta - \frac{\sum x_i}{\theta}$$

$$\frac{d \ln L(\theta)}{d\theta} = -\frac{n}{\theta} + \frac{\sum x_i}{\theta^2} = 0 \quad (\text{let})$$

$$\Rightarrow \frac{\sum x_i}{\theta^2} = \frac{n}{\theta}$$

$$\Rightarrow n\theta = \sum x_i$$

$$\frac{d^2 \ln L(\theta)}{d\theta^2} \\ = \frac{n}{\theta^2} - \frac{2\sum x_i}{\theta^3} < 0$$

for $\theta = \frac{\sum x_i}{n}$

$$\hat{\theta} = \frac{\sum x_i}{n} = \bar{x}$$

$$\text{Ex-2} \quad f(x_i; p) = (1-p)^{x_i-1} p^{n_i-1}$$

$$L(p) = \prod_{i=1}^n f(x_i; p) = \prod_{i=1}^n (1-p)^{\sum x_i - n} p^n$$

$$= (1-p)^{\sum x_i - n} p^n$$

$$\ln L(p) = (\sum x_i - n) \ln(1-p) + n \ln p \quad (\text{let})$$

$$\frac{d \ln L(p)}{dp} = -\frac{\sum x_i - n}{1-p} + \frac{n}{p} = 0$$

$$\Rightarrow \frac{\sum x_i - n}{1-p} = \frac{n}{p}$$

$$\Rightarrow n - np = p \sum x_i - np$$

$$\Rightarrow p = \frac{n}{\sum x_i}$$

$$\frac{d^2 \ln L(p)}{dp^2} = -\frac{\sum x_i - n}{(1-p)^2} - \frac{n}{p^2} < 0$$

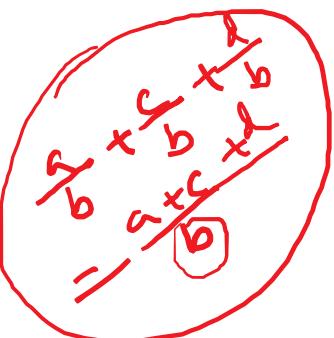
$$\text{for } p = \frac{n}{\sum x_i}$$

$$\hat{p} = \frac{n}{\sum x_i} = \frac{1}{\bar{x}}$$

Ex-3

$$f(x; \theta_1, \theta_2) = \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x-\theta_1)^2}{2\theta_2}}$$

$$\begin{aligned} L(\theta_1, \theta_2) &= \prod_{i=1}^n \frac{(2\pi\theta_2)^{-1/2}}{e^{-\frac{\sum(x_i - \theta_1)^2}{2\theta_2}}} \\ &= (2\pi\theta_2)^{-n/2} e^{-\frac{\sum(x_i - \theta_1)^2}{2\theta_2}} \end{aligned}$$



$$\begin{aligned} \ln L(\theta_1, \theta_2) &= -\frac{n}{2} \ln(2\pi\theta_2) - \frac{1}{2} \ln \theta_2 \\ &= -\frac{n}{2} \ln(2\pi) - \frac{n}{2} \ln \theta_2 - \frac{\sum(x_i - \theta_1)^2}{2\theta_2} \end{aligned}$$

$$\begin{aligned} \frac{\partial \ln L(\theta_1, \theta_2)}{\partial \theta_1} &= -\frac{2 \sum(x_i - \theta_1)(-1)}{2\theta_2} \\ &= \frac{\sum(x_i - \theta_1)}{\theta_2} = 0 \quad (\text{let}) \end{aligned}$$

$$\Rightarrow \frac{\sum x_i - n\theta_1}{\theta_2} = 0$$

$$\Rightarrow \frac{\sum x_i - n\theta_1}{\theta_2} = 0$$

$$\begin{aligned} \Rightarrow \frac{\sum x_i - n\theta_1}{\theta_2} &= 0 \\ \Rightarrow \theta_1 &= \frac{\sum x_i}{n} \end{aligned}$$

$$\frac{\partial^2 L(\theta_1, \theta_2)}{\partial \theta_1^2} = -\frac{n}{\theta_2} < 0 \quad \text{for } \theta_1 = \frac{\sum x_i}{n}$$

$$\hat{\theta}_1 = \frac{\sum x_i}{n} = \mu$$

$$\text{so, } \ln L(\theta_1, \theta_2) = -\frac{n}{2} \ln(2\pi) - \frac{n}{2} \ln \theta_2 - \frac{\sum (x_i - \mu)^2}{2\theta_2}$$

$$\frac{\partial \ln L(\theta_1, \theta_2)}{\partial \theta_2} = -\frac{n}{2\theta_2} + \frac{\sum (x_i - \mu)^2}{2\theta_2^2} = 0 \quad (\text{ok})$$

$$\Rightarrow \frac{n}{2\theta_2} = \frac{\sum (x_i - \mu)^2}{2\theta_2^2}$$

$$\Rightarrow \theta_2 = \frac{\sum (x_i - \mu)^2}{n}$$

$$\Rightarrow \theta_2 = \frac{\sum (x_i - \mu)^2}{n} < 0$$

$$\frac{\partial^2 \ln L(\theta_1, \theta_2)}{\partial \theta_2^2} = \frac{n}{2\theta_2^2} - \frac{\sum (x_i - \mu)^2}{\theta_2^3}$$

$$\text{for } \theta_2 = \frac{\sum (x_i - \mu)^2}{n}$$

$$\hat{\theta}_2 = \frac{\sum (x_i - \mu)^2}{n} = \tilde{s}^2$$

