Assignment 01

1. During a visit to a doctor, the probability of having neither lab work, nor referral to a specialist is $\mathbf{0.19}$. Of those coming to the doctor, the probability of having lab work is $\mathbf{0.47}$ and the probability of having a referral is $\mathbf{0.51}$. What is the probability of having both lab work and referral?

2. A fair coin is tossed ***three*** times, and the sequence of heads and tails is observed. Let the events $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ be given by, $\boldsymbol{A} = \{\boldsymbol{at\ most\ 2\ tails}\}$, $\boldsymbol{B} = \{\boldsymbol{at\ least\ 2\ heads}\}$ and $\boldsymbol{C} = \{\boldsymbol{1\ head\ and\ 2\ tails}\}$. Find the sample space $\boldsymbol{S}$. Also, find $\boldsymbol{P(A \cap C)}$, $\boldsymbol{P(B \cup C)}$, $\boldsymbol{P(C')}$.

3. A certain test for identifying cancer had tested among $\mathbf{1}$ lac people, we can expect results similar to those given in the following table. If one of $\mathbf{1}$ lac people is selected randomly, find the following probabilities: $\boldsymbol{P(A_1 \backslash B_2)}$, $\boldsymbol{P(A_2 \backslash B_1)}$, $\boldsymbol{P(A_1 \cap B_1)}$ and $\boldsymbol{P(A_2 \cup B_2)}$.

	$\boldsymbol{A_1}$ **Carry cancer**	$\boldsymbol{A_2}$ **Do not carry cancer**	**Totals**
$\boldsymbol{B_1}$ **Test positive**	4500	75500	80000
$\boldsymbol{B_2}$ **Test negative**	500	19500	20000
Totals	5000	95000	100000

4. A shop contains $\mathbf{8}$ Nokia and $\mathbf{7}$ Samsung mobiles. Another shop contains an unknown numbers of Nokia and $\mathbf{11}$ Samsung mobiles. A mobile collected from each shop at random, and the probability of getting ***two*** mobiles of the same company is $\mathbf{\frac{151}{300}}$. How many Nokia mobiles are in the second shop?

5. Each of ***three*** bowlers will attempt to hit the wicket. Let $\boldsymbol{A_i}$ denote the event that the wicket is got by $\boldsymbol{i}$-th player, $\boldsymbol{i = 1, 2, 3}$. Assume that all of the events are mutually independent and that $\boldsymbol{P(A_1) = 0.35}$, $\boldsymbol{P(A_2) = 0.65}$ and $\boldsymbol{P(A_3) = 0.5}$. Find the probability that exactly two players are successful, and probability of no player is successful.

6. At a country fair carnival game there are $\mathbf{25}$ balloons on a board, of which $\mathbf{10}$ balloons are yellow, **8** are red, and **7** are green. A player throws darts at the balloons to win a prize and randomly hits one of them. If the first balloon hit is green, what is the probability of **(i)** next balloon is green, **(ii)** next balloon is not green.

7. A boy has ***three*** red coins and ***five*** white coins in his left hand, ***six*** red coins and ***four*** white coins in his right hand. If he shifts one coin at random from his left to right hand, what is the probability of his then drawing a ***white*** coin from his ***left*** hand?

8. Four inspectors look at a critical component of a product. Their probabilities of detecting an error are different, namely, $\mathbf{0.98, 0.95, 0.92, 0.89}$ respectively. If we in sections are independent, then find the probability of **(i)** no one detecting the error, **(ii)** at least one detecting the error, **(iii)** only one inspector detecting the error.

9. Let $\boldsymbol{P(A) = 0.3}$ and $\boldsymbol{P(B) = 0.6}$. Find $\boldsymbol{P(A \cup B)}$ when $\boldsymbol{A}$ and $\boldsymbol{B}$ are independent. Find $\boldsymbol{P(A|B)}$ when $\boldsymbol{A}$ and $\boldsymbol{B}$ are mutually indepedent.

10. There is a new diagnostic test for a disease that occurs in about $\mathbf{1}\%$ of the population. The test is not perfect, but will detect a person with the disease $\mathbf{99}\%$ of the time. It will, however, say that a person without the disease has the disease about $\mathbf{3}\%$ of the time. A person is selected at random from the population, and the test indicates that this person has the disease. What are the conditional probabilities that **(i)** the person has the disease? **(ii)** the person does not have the disease?

11. A test indicates the presence of a particular disease $\mathbf{95}\%$ of the time when the disease is present and the presence of the disease $\mathbf{2}\%$ of the time when the disease is not present. If $\mathbf{0.2}\%$ of the population has the disease, calculate the conditional probability that a person selected at random has the disease if the test indicates the presence of the disease.

12. ***Die A*** has orange on ***one*** face and blue on ***five*** faces, ***Die B*** has orange on ***two*** faces and blue on ***four*** faces, ***Die C*** has orange on ***three*** faces and blue on ***three*** faces. If all are fair dice, picture the sample space.
    - **a.** If the ***three*** dice are rolled, find the probability that ***at least two*** of the three dice come up orange.
    - **b.** If you are given, at least ***two*** of the three dice come up orange, find the probability that ***exactly two*** of the three dice come up orange.

13. Bowl $\boldsymbol{B_1}$ contains ***two*** white chips and ***one*** red chip, bowl $\boldsymbol{B_2}$ contains ***one*** white chip and ***two*** red chips, bowl $\boldsymbol{B_3}$ contains ***three*** white and ***two*** red chips and bowl $\boldsymbol{B_4}$ contains ***two*** white and ***three*** red chips. The probabilities of selecting bowl and $\boldsymbol{B_1, B_2, B_3}$ and $\boldsymbol{B_4}$ are $\boldsymbol{\frac{3}{8}, \frac{1}{4}, \frac{1}{4}}$ and $\boldsymbol{\frac{1}{8}}$ respectively. A bowl is selected using these probabilities and a chip is drawn at random. Find $\boldsymbol{P(B_3/R)}$, the conditional probability that bowl $\boldsymbol{B_3}$ had been selected, given that a **red** chip was drawn.

14. At an office, officials are classified and $\mathbf{30}\%$ of them efficient, $\mathbf{50}\%$ are moderate worker, and $\mathbf{20}\%$ are unfit for the work. Of efficient ones, $\mathbf{15}\%$ left the job; of the moderate workers, $\mathbf{20}\%$ left the job, and of unfit workers, $\mathbf{5}\%$ left the job. Given that an employee left the job, what is the probability that the employee is ***unfit one***? Consider independence for the employee classes.

15. Suppose there are $\mathbf{5}$ defective items in a lot of $\mathbf{100}$ items. A sample of size $\mathbf{15}$ is taken at random without replacement. Let $\boldsymbol{X}$ denote the number of defective items in the sample. Find the probability that the sample contains **(i)** at most one defective item, **(ii)** exactly three defective items.

16. If the ***mgf*** of $\boldsymbol{X}$ is $\boldsymbol{M(t) = \frac{4}{10}e^{t} + \frac{3}{10}e^{2t} + \frac{2}{10}e^{3t} + \frac{1}{10}e^{4t}}$, find the corresponding ***pmf***, mean and variance.

17. Flaws in a certain type of drapery material appear on the average of ***one*** in $\mathbf{120}$ square feet. If we assume a Poisson distribution, find the probability of no more than one flaw appearing in $\mathbf{60}$ square feet.

18. In the gambling game craps, the player wins $\$\mathbf{1}$, $\$\mathbf{2}$ and $\$\mathbf{3}$ with probabilities $\mathbf{0.3}$, $\mathbf{0.2}$ and $\mathbf{0.1}$, and loses $\$\mathbf{1}$ with probability $\mathbf{0.4}$ for each $\$\mathbf{1}$ bet. What is the expected profit of the game for the player? Also, find the variance of the profit.

19. Let the random variable $\boldsymbol{X}$ be the number of days that a certain patient needs to be in the hospital. Suppose $\boldsymbol{X}$ has the pmf $\boldsymbol{f(x) = \frac{5-x}{10}; x = 1, 2, 3, 4.}$
If the patient is to receive $\$\mathbf{200}$ from an insurance company for each of the first ***two*** days in the hospital and $\$\mathbf{100}$ for each day after the first ***two*** days, what is the expected payment for the hospitalization?

20. A boiler has five relief valves. The probability that each does not work is $\mathbf{0.05}$. Find the probability that **(i)** none of them work, **(ii)** at least four work.

21. If $\boldsymbol{X}$ has a Poisson distribution such that $\boldsymbol{P(X = 1) = 2P(X = 2)}$, evaluate $\boldsymbol{P(X = 5)}$. Also, find the standard deviation of the distribution.

22. Let $\boldsymbol{X}$ have a Poisson distribution with a standard deviation of $\mathbf{2}$. Find $\boldsymbol{P(X \geq 1)}$.

23. For $\boldsymbol{f(x) = c(x + 1)^3}$; $\boldsymbol{x = 0, 1, 2, \ldots \ldots \ldots 10}$, determine the constant $\boldsymbol{c}$ so that $\boldsymbol{f(x)}$ satisfies the conditions of being ***pmf*** for a random variable $\boldsymbol{X}$, and then depict ***pmf*** as line graph and histogram.

24. Given that $\boldsymbol{E[X + c] = 10}$ and $\boldsymbol{E[(X + c)^2] = 116}$. Find the mean and variance of $\boldsymbol{X}$.

25. It is claimed that $\mathbf{20}\%$ of the birds in a particular region have severe disease. Suppose that $\mathbf{15}$ birds are selected at random. Let $\boldsymbol{X}$ is the number of birds that are have the disease. Assuming independence, how $\boldsymbol{X}$ is distributed? Find $\boldsymbol{P(X \geq 2)}$ and $\boldsymbol{P(X \leq 14)}$.

26. Let a random experiment be casting of pair of fair ***six*** sided dice and let $\boldsymbol{X}$ equal the maximum of ***two*** outcomes. With reasonable assumptions, find the ***pmf*** of $\boldsymbol{X}$ and draw a probability histogram of the ***pmf*** of $\boldsymbol{X}$.

27. A random variable $\boldsymbol{X}$ has a binomial distribution with mean $\mathbf{10.5}$ and variance $\mathbf{3.15}$. How $\boldsymbol{X}$ is distributed and find $\boldsymbol{P(X \geq 1)}$.

28. Let the random variable $\boldsymbol{X}$ be the number of days that a certain patient needs to be in the hospital. Suppose $\boldsymbol{X}$ has the ***pmf*** $\boldsymbol{f(x) = 0.1(5 - x)}$; $\boldsymbol{x = 1, 2, 3, 4}$. If the patient is to receive $\$\mathbf{100}$ for the first day, $\$\mathbf{50}$ for the second day, $\$\mathbf{25}$ for the third day and have to return $\$\mathbf{25}$ for the fourth day, what is the expected payment for the hospitalization?

29. Suppose that in a region the probability of arresting an innocent person is $\mathbf{15}\%$. If $\mathbf{500}$ people are arrested, assuming Bernoulli experiment find the probability of arresting $\mathbf{35}$ innocent persons. Find the probability by Poisson process as well.

30. Suppose that $\mathbf{90\%}$ of UIU students are multi-taskers. In a random sample of $\mathbf{10}$ students are taken and let $\boldsymbol{X}$ is the number of multi-taskers. Assuming independence, how $\boldsymbol{X}$ is distributed? Find the standard deviation of $\boldsymbol{X}$. Also, compute $\boldsymbol{P(X \geq 2)}$ and $\boldsymbol{P(X > 8)}$.

31. Let the random variable $\boldsymbol{X}$ have the ***pmf*** $\boldsymbol{f(x) = \frac{(|x|-1)^2}{21}; x = -4, -2, 0, 2, 4}$. Compute the mean, variance, $\boldsymbol{E(X^2 - 3X + 4)}$ and $\boldsymbol{V(1 - 2X)}$.

32. Verify that $\boldsymbol{M(t) = (0.4 + 0.6e^t)^{15}}$ is a ***mgf*** of a binomial distribution and find the ***pmf*** of it. Evaluate the mean and variance of the binomial distribution?

33. Consider the ***mgf*** $\boldsymbol{M(t) = \frac{0.3e^t}{1-0.7e^t}}$ of random variable $\boldsymbol{X}$. How $\boldsymbol{X}$ is distributed? Find the mean and variance of $\boldsymbol{X}$.

34. The life $\boldsymbol{X}$ (in years) of a voltage regulator of a car has the ***pdf*** $\boldsymbol{f(x) = \frac{3x^2}{7^3} e^{-\left(\frac{x}{7}\right)^2}}$ defined in $\boldsymbol{0 \leq x < \infty}$. What is the probability that it will last at least $\mathbf{10.5}$ years? If it has lasted for $\mathbf{10.5}$ years, find the conditional probability that it will last at least $\mathbf{7}$ years more. Also, find the median (position) of $\boldsymbol{X}$.

35. Let the ***mgf*** of the random variable $\boldsymbol{X}$ satisfies uniform distribution is $\boldsymbol{M(t) = \begin{cases} \frac{e^{4t}-1}{4t}; & t \neq 0 \\ 1; & t = 0 \end{cases}}$.

Find the ***pdf***, mean and variance of $\boldsymbol{X}$. Also, find $\boldsymbol{P(X > 3.5)}$.

36. Telephone calls arrive at a physician's office according to the Poisson process on average $\mathbf{2}$ every $\mathbf{5}$ minute. Let $\boldsymbol{X}$ denote the waiting time in minutes until $\mathbf{3}$ calls arrive. Find the ***pdf*** and compute $\boldsymbol{P(X > 2)}$.

37. Customers arrive at a travel agency at a mean rate of $\mathbf{48}$ per day. Assuming that the number of arrivals per hour has a Poisson process, find the probability that more than $\mathbf{3}$ customers arrive in a given hour.

38. Assume the ***pdf*** of $\boldsymbol{X}$ be $\boldsymbol{f(x) = 3e^{-3x}; 0 \leq x < \infty}$. Then, **(i)** Estimate the ***cdf*** of $\boldsymbol{X}$, **(ii)** Calculate the mean and variance of $\boldsymbol{X}$, $(\boldsymbol{iii})$ Find $\boldsymbol{P(X \geq 2)}$.

39. Customers arrive at a travel agency at a mean rate of $\mathbf{3}$ per $\mathbf{2}$ hour. Assuming that the number of arrival per hour has a Poisson distribution, find the probability of waiting $\mathbf{2}$ hours for the first customers.

40. If $\boldsymbol{X}$ has a gamma distribution with $\boldsymbol{\theta = 5}$ and $\boldsymbol{\alpha = 2}$. Find $\boldsymbol{P(X \geq 6)}$. What are the mean and variance of the gamma distribution?

41. For $\boldsymbol{(x) = 4x^c; 0 \leq x \leq 1}$, find the constant $\boldsymbol{c}$ so that $\boldsymbol{f(x)}$ is a ***pdf*** of a random variable $\boldsymbol{X}$. Find $\boldsymbol{\mu, \sigma^2}$ and ***cdf*** of $\boldsymbol{X}$. Also, ***sketch*** the graph of ***pdf*** and ***cdf***.

42. If the ***mgf*** of a Gamma distribution of a random variable $\boldsymbol{X}$ is $\boldsymbol{M(t) = (1 - 5t)^{-3}}$, find the ***pdf***, mean and variance of $\boldsymbol{X}$. Also, find $\boldsymbol{P(X > 4)}$.

43. Telephone calls arrive at a department according to a Poisson process on the average of three every ***four*** minutes. Let $\boldsymbol{X}$ denote the waiting time until the ***first*** call arrives at a certain time duration. What is the ***pdf*** of $\boldsymbol{X}$? Find $\boldsymbol{P(X \geq 3)}$. Also, find the ***mgf*** of $\boldsymbol{X}$.

44. Let the random variable $\boldsymbol{X}$ have the ***pdf*** $\boldsymbol{f(x) = 2e^{1-2x};\ x \geq \frac{1}{2}}$, find the ***cdf*** and hence the $\mathbf{3^{rd}}$ decile of the distribution.

45. If the ***mgf*** of the normal variable $\boldsymbol{X}$ is $\boldsymbol{M(t) = e^{25t+182t^2}}$, find ***pdf*** of $\boldsymbol{X}$. Also, find a constant $\boldsymbol{c}$ such that $\boldsymbol{P(|X - 25| \leq c) = 0.9332}$.

46. If the ***mgf*** of the normal variable $\boldsymbol{X}$ is $\boldsymbol{M(t) = e^{30t+18t^2}}$, then **(i)** Find a constant $\boldsymbol{k}$ such that $\boldsymbol{P(|z| \leq k) = 0.9544}$ and **(ii)** Evaluate $\boldsymbol{P(42.6 \leq x \leq 55.8)}$.

47. If $\boldsymbol{X}$ is a random variable satisfying $\boldsymbol{N(650, 625)}$, find $\boldsymbol{P(631 \leq X \leq 676)}$. Also, find a constant $\boldsymbol{c > 0}$ such that $\boldsymbol{P(|X - 650| \leq c) = 0.6826}$.