Lecture on Electricity

Introduction to Electricity: Coulomb's Law

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Reference Books:

1. Fundamentals of Physics

By Halliday-Resnick-Walker (10th edition)

2. Physics for Engineers (Par-II)

By Dr. Giasuddin Ahmed

Concept of charge

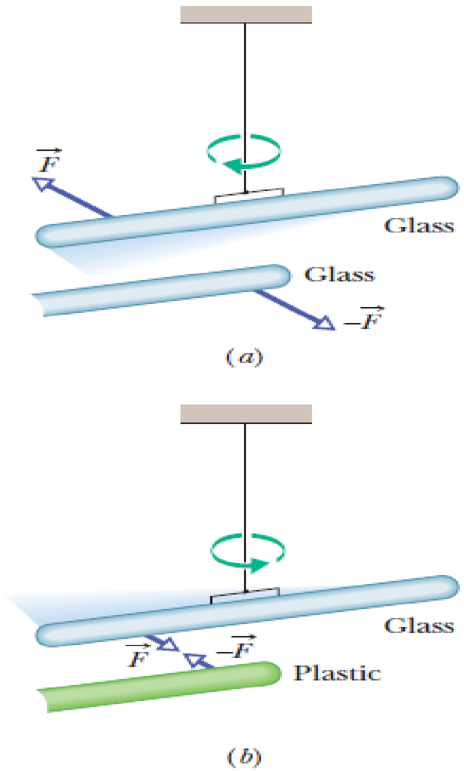


Figure 21-1 (*a*) The two glass rods were each rubbed with a silk cloth and one was suspended by thread. When they are close to each other, they repel each other. (*b*) The plastic rod was rubbed with fur. When brought close to the glass rod, the rods attract each other.

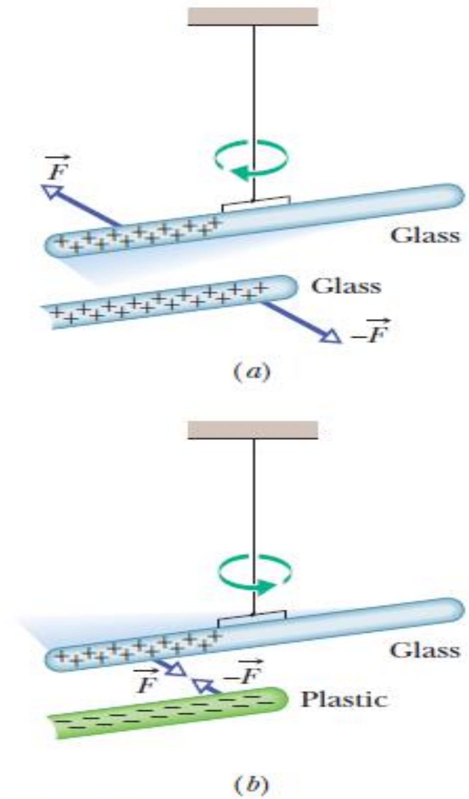


Figure 21-2 (*a*) Two charged rods of the same sign repel each other. (*b*) Two charged rods of opposite signs attract each other. Plus signs indicate a positive net charge, and minus signs indicate a negative net charge.

Introduction Continued

- What is charge?
- How do we visualize it.
- What is the model.
- We only know charge exists because in experiments electric forces cause objects to move.

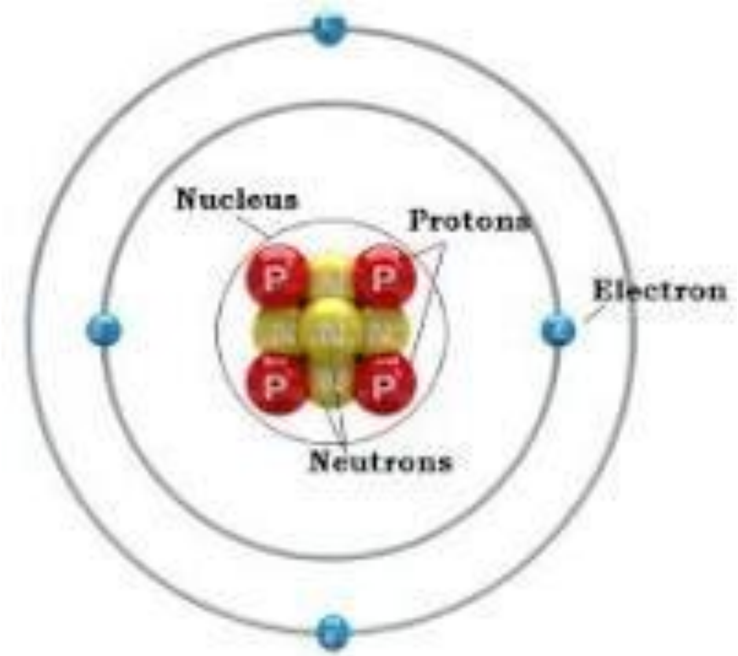
- Electrostatics: study of electricity when the charges are not in motion. Good place to start studying E&M because there are lots of demonstrations.

Some preliminaries

- **Electron**: Considered a point object with radius less than 10^{-18} meters with electric charge e= -1.6×10^{-19} Coulombs (SI units) and mass m_e= 9.11×10^{-31} kg

- **Proton**: It has a finite size with charge +e, mass m_p= 1.67×10^{-27} kg and with radius
  - $0.805 \pm 0.011 \times 10^{-15}$ m scattering experiment
  - $0.890 \pm 0.014 \times 10^{-15}$ m Lamb shift experiment

- **Neutron**: Similar size as proton, but with total charge = 0 and mass m_n=
  - Positive and negative charges exists inside the neutron



Methods of Charging Objects: Friction, Contact, and Induction

- Normally atoms are in the lowest energy state. This means that the material is electrically neutral. You have the same number of electrons as protons in the material.

- How do we change this?

- How do we add more electrons than protons or remove electrons?

Summary Comments

- Silk(+) on teflon(-)
- Silk (-) on acrylic (+)
- Wood doesn’t charge
- Charged objects always attract neutral objects

- Show Triboelectric series
- Not only chemical composition important, structure of surface is important - monolayer of molecules involved, quantum effect.
(nanotechnology)

Triboelectric series

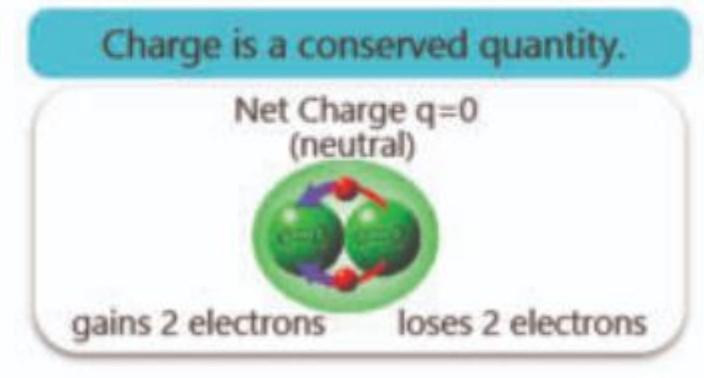
http://www.sciencejoywagon.com/physicszone/lesson/07elecst/static/triboele.htm

Positive (Lose electrons easily)
Air
Human Hands
Asbestos
Rabbit Fur
Glass
Mica
Acrylic
Human Hair
Nylon
Wool
Fur
Lead
Silk
Aluminum
Paper
Cotton
Steel
Wood
Amber
Sealing Wax
Hard Rubber
Nickel, Copper
Brass, Silver
Gold, Platinum
Sulfur
Acetate, Rayon
Polyester
Styrene
Orlon
Saran
Balloon
Polyurethane
Polypropylene
Vinyl (PVC)
Silicon
Teflon
Negative (Gains electrons easily)

Summary:
Electrostatics is based on 4 four empirical facts

- Conservation of charge
- Quantization of charge
- Coulomb’s Law
- The principle of superposition

Conservation of charge



 # <u>Conservation of Charge</u>: Charge conservation is the principle that the total electric charge in an isolated system never changes.

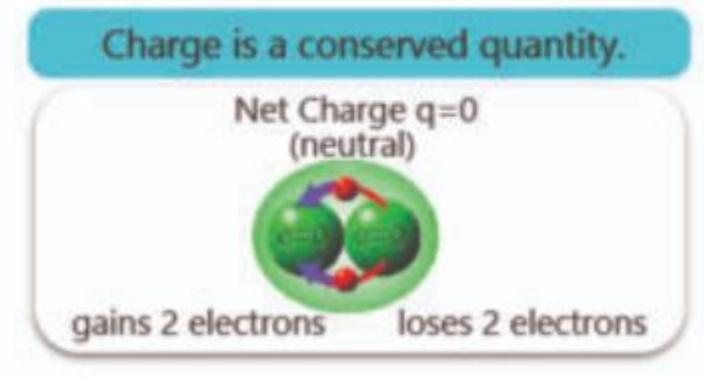
The net quantity of electric charge, the amount of positive charge minus the amount of negative charge in the universe, is always conserved.

Conservation of charges implies that the change in the amount of electric charge in any volume of space is exactly equal to the amount of charge flowing into the volume minus the amount of charge flowing out of the volume.

Conservation of charge requires that the total quantity of charge in the universe is always constant.

Most evidence indicates that the net charge in the universe is zero; that is, there are equal quantities of positive and negative charge.

Conservation of charge



 # Conservation of Charge: Charge conservation is the principle that the total electric charge in an isolated system never changes.

Explanation: This does not mean that individual positive and negative charges cannot be created or destroyed. Electric charge is carried by **subatomic particles** such as **electrons** and **protons**. Charged particles can be created and destroyed in elementary particle reactions. In such reactions, charge conservation means that in reactions that create charged particles, equal numbers of positive and negative particles are always created, keeping the net amount of charge unchanged. Similarly, when particles are destroyed, equal numbers of positive and negative charges are destroyed. This property is supported without exception by all empirical observations so far.

Conservation of charge requires that the total quantity of charge in the universe is always constant.

Conservation of charge: Application

Mathematically, we can state the law of charge conservation as a **continuity equation:**

$$\frac{\partial Q}{\partial t} = \dot{Q}_{\rm IN}(t) - \dot{Q}_{\rm OUT}(t)$$

where $\partial Q/\partial t$ is the electric charge accumulation rate in a specific volume at time *t*, $\dot{Q}_{\rm IN}$ is the amount of charge flowing into the volume and $\dot{Q}_{\rm OUT}$ is the amount of charge flowing out of the volume; both amounts are regarded as generic functions of time.

In electromagnetic theory, The charge density continuity equation **(charge density and current density equation)** can also be expressed as

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0$$

where *ρ is the* charge density (in coulombs per cubic meter) and **J** is the electric current density (in amperes per square meter).
In electromagnetic field theory, vector calculus can be used to express the law in terms of charge density and current density.

The equation equates these two factors, which says that the only way for the charge density at a point to change is for a current of charge to flow into or out of the point. This statement is equivalent to a conservation of Four-current.

Conservation of charge requires that the total quantity of charge in the universe is always constant.

Conservation of charge: More Application

Poisson's equation is an elliptical partial differential equation of broad utility in Engineering, Physics, Aerodynamics, Fluid dynamics, Newtonian Gravity, Electrostatics, **Gaussian Charge density**, Surface reconstruction, etc. **It is used for Charge conservation equation.** For example, the solution to Poisson's equation the potential field caused by a given electric charge or mass density distribution; with the potential field known, one can then calculate electrostatic (charge) or gravitational (force) field. It is a generalization of Laplace's equation, which is named after French mathematician and physicist Simeon Denis Poisson.

Poisson's equation is $\Delta\varphi = f$

where Δ is the Laplace operator, and f and φ are real or complex-valued functions on a manifold. Usually, f is given and φ is sought.

The Laplace operator is often denoted as ∇^2 and so Poisson's equation is frequently written as $\nabla^2\varphi = f$

In three-dimensional Cartesian coordinates, it takes the form

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\varphi(x, y, z) = f(x, y, z)$$

Conservation of charge : More Application

Poisson's equation for electrostatics, can be written as $\nabla^2 \varphi = -\frac{\rho}{\varepsilon}.$

where ε = permittivity of the medium and ρ is a total volume charge density.

 # Potential of a Gaussian charge density or the potential $\varphi(r)$ approaches the point charge potential

$$\varphi(r) \approx \quad \varphi \approx \frac{1}{4\pi\varepsilon} \frac{Q}{r}$$

where Q is the total charge and r is the radius of Gaussian surface or the potential at distance r from a central point charge Q.

Conservation of charge

- Rubbing does not create charge, it is transferred from one object to another

- Teflon negative - silk positive

- Acrylic positive - silk negative

- Nuclear reactions $\gamma^0 = e^+ + e^-$

- Radioactive decay $^{238}U_{92} = {}^{234}Th_{90} + {}^{4}He_{2}$

- High energy particle reactions $e^- + p^+ = e^- + \pi^+ + n^0$

What is meant by quantization of charge?

- Discovered in 1911 by Robert A. Millikan in the oil drop experiment
- The unit of charge is so tiny that we will never notice it comes in indivisible lumps.

Electric charge is quantized.

$$Q = n \times e$$

What is meant by quantization of charge?

- Discovered in 1911 by Robert A. Millikan in the oil drop experiment
- The unit of charge is so tiny that we will never notice it comes in indivisible lumps.

Example-1: Suppose in a typical experiment we charge an object up with a nano Coulomb of charge (10^{-9} C). How many elementary units of charge is this?

Solution: We know,

$$Q = N \times e \quad \textbf{so} \quad N = \frac{Q}{e} = \frac{10^{-9}\,C}{1.6 \times 10^{-19}\,C} = 6 \times 10^{9}$$

= six billion units of charge or 6 billion electrons.

Ans: 6×10^{9} **unit charge**

Electric Charge (Q)

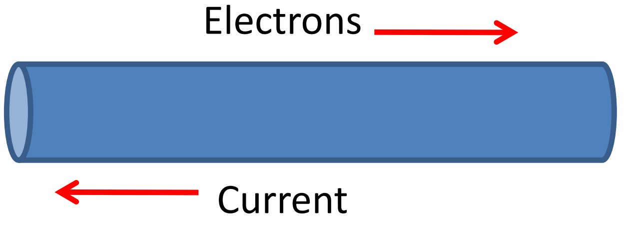
- Characteristic of subatomic particles that determines their electromagnetic interactions

- An electron has a $-1.602 \cdot 10^{-19}$ Coulomb charge

- The rate of flow of charged particles is called current

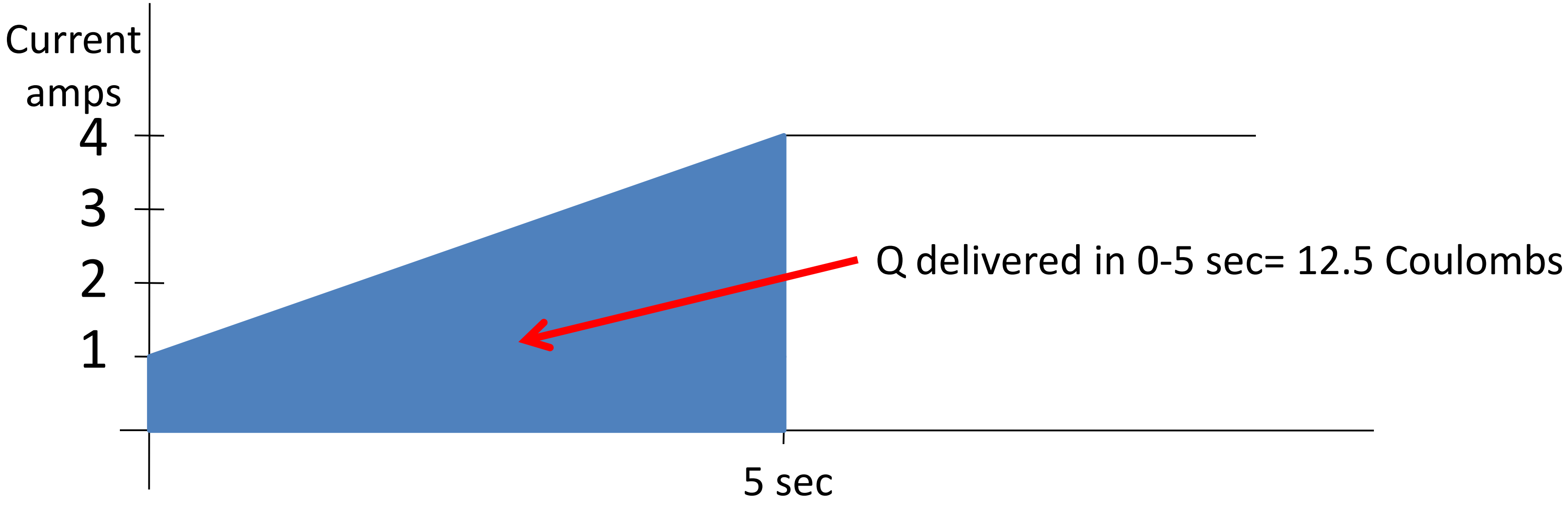
Current (I)

- Current = (Number of electrons that pass in one second) · (charge/electron)
  - -1 ampere = $(6.242 \cdot 10^{18}$ e/sec$) \cdot (-1.602\ 10^{-19}$Coulomb/e)
  - Notice that an ampere = Coulomb/second
- The negative sign indicates that the current inside is actually flowing in the opposite direction of the electron flow



Current

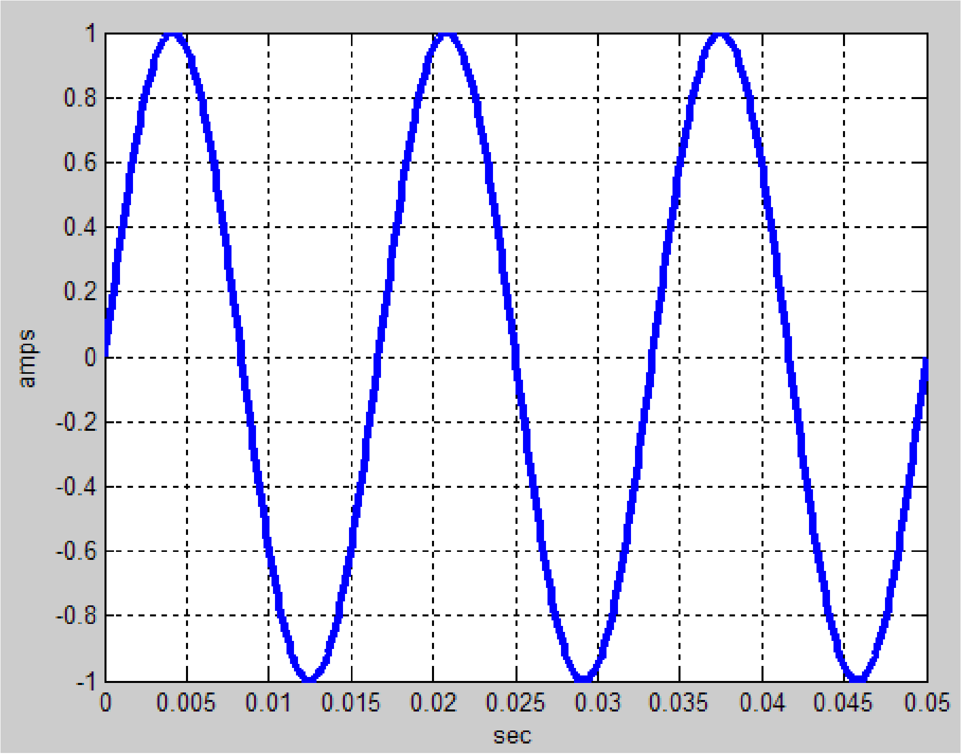
- $i = dq/dt$ – the derivitive or slope of the charge when plotted against time in seconds
- $Q = \int i \cdot dt$ – the integral or area under the current when plotted against time in seconds



AC and DC Current

- DC Current has a constant value

- AC Current has a value that changes sinusoidally



- Notice that AC current changes in value and direction

- No net charge is transferred

Why Does Current Flow?

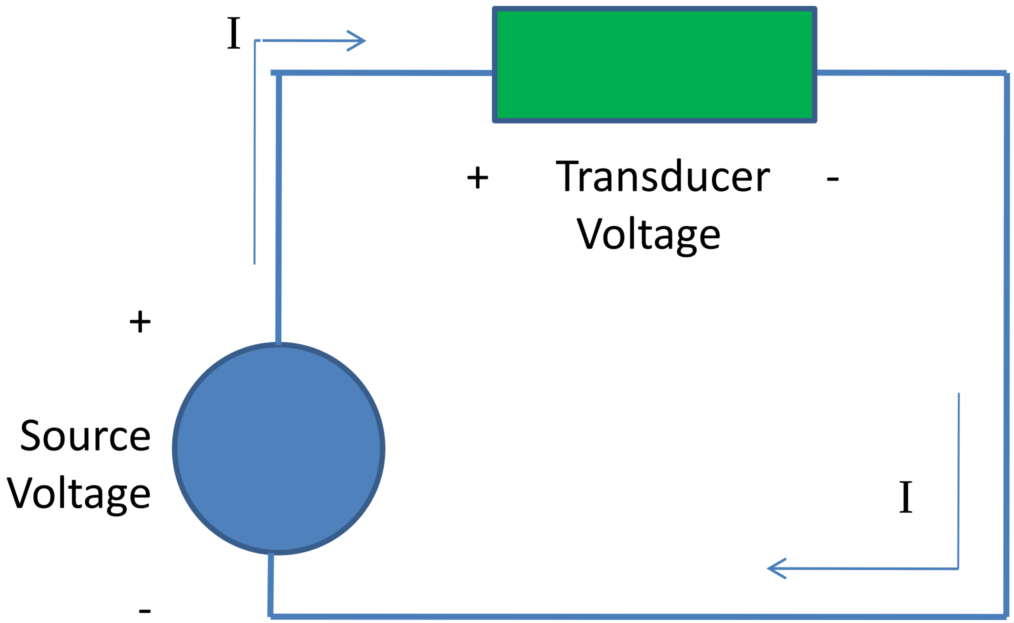
- A voltage source provides the energy (or work) required to produce a current
  - Volts = joules/Coulomb = dW/dQ
- A source takes charged particles (usually electrons) and raises their potential so they flow out of one terminal into and through a transducer (light bulb or motor) on their way back to the source's other terminal

Voltage

- Voltage is a measure of the potential energy that causes a current to flow through a transducer in a circuit
- Voltage is always measured as a difference with respect to an arbitrary common point called ground
- Voltage is also known as electromotive force or EMF outside engineering

A Circuit

- Current flows from the higher voltage terminal of the source into the higher voltage terminal of the transducer before returning to the source



- ➢ The source expends energy & the transducer converts it into something useful

Power

- The rate at which energy is transferred from an active source or used by a passive device
- P in watts = dW/dt = joules/second
- $P = V \cdot I = dW/dQ \cdot dQ/dt$ = volts · amps = watts
- $W = \int P \cdot dt$ – so the energy (work in joules) is equal to the area under the power in watts plotted against time in seconds

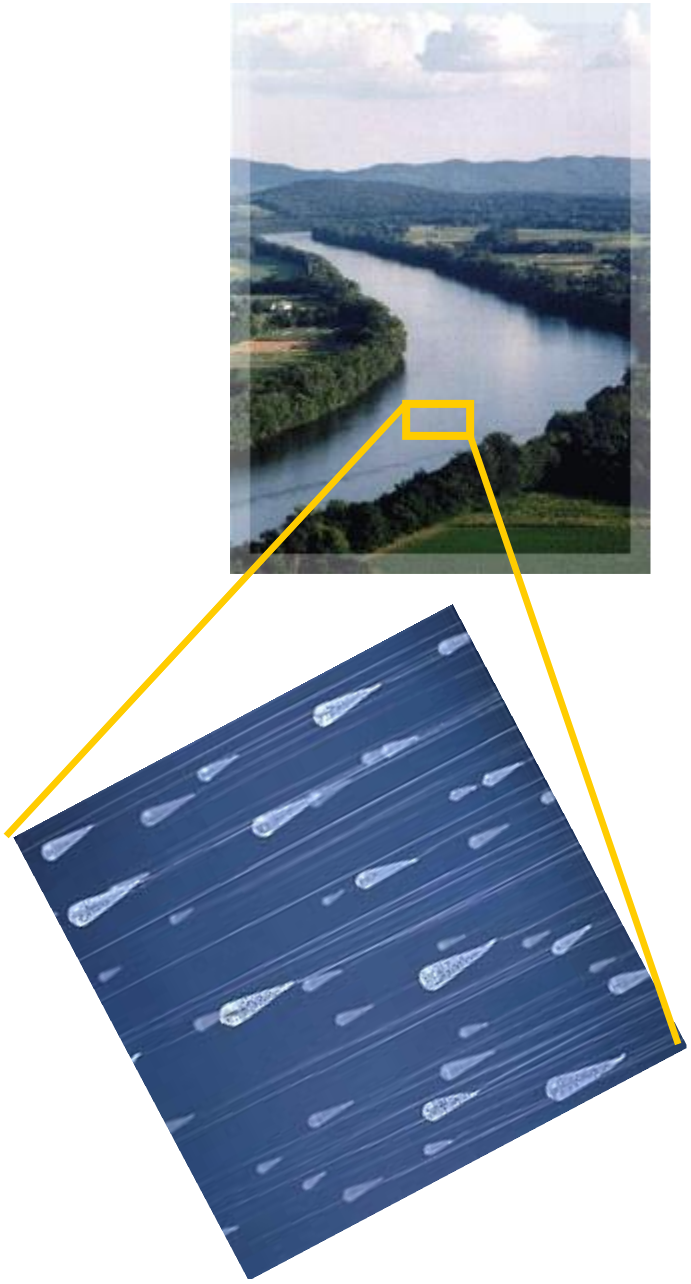
How you should be thinking about electric circuits:

<u>Voltage</u>: a force that pushes the current through the circuit (in this picture it would be equivalent to gravity)

How you should be thinking about electric circuits:

<u>Resistance</u>: friction that impedes flow of current through the circuit (rocks in the river)

How you should be thinking about electric circuits:



Current: the actual "substance" that is flowing through the wires of the circuit (electrons!)

Would This Work?

Would This Work?

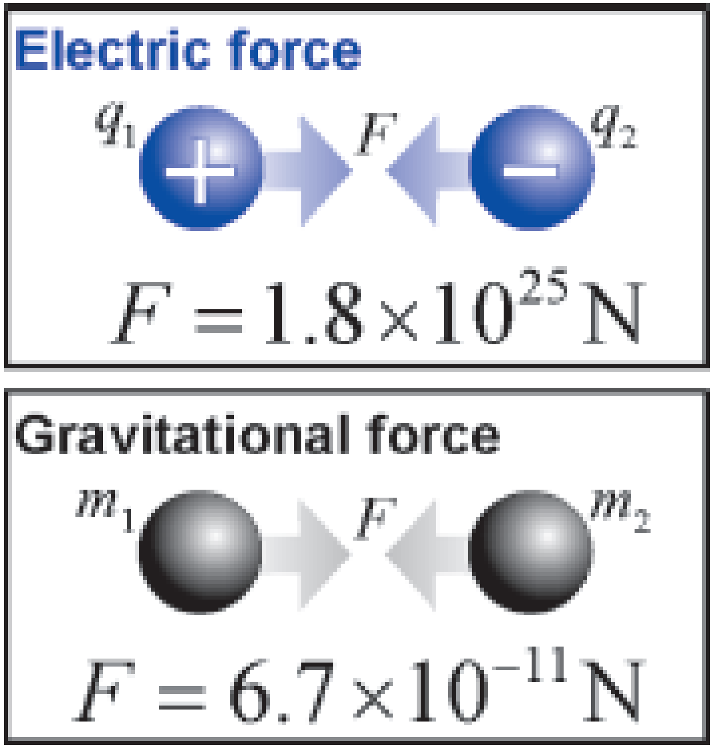
Would This Work?

The Central Concept: Closed Circuit

Electric Charge

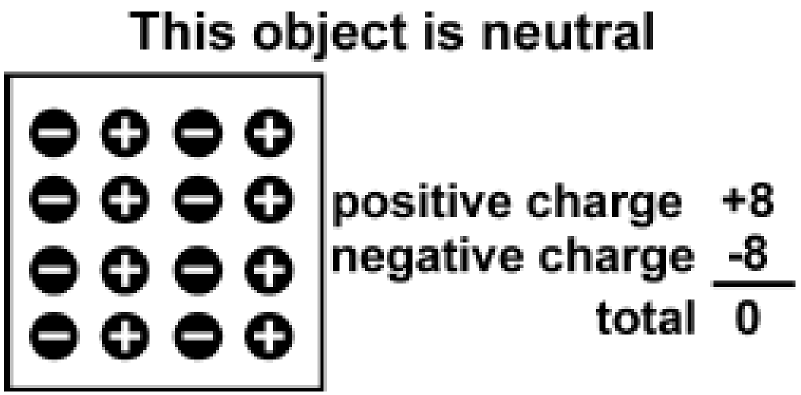
Key Question:

How do electric charges interact?



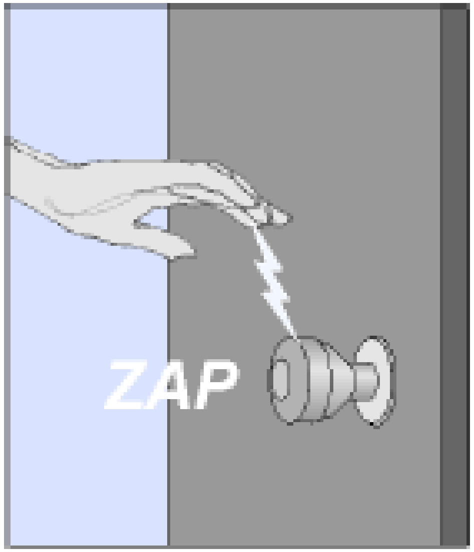
Electric Charge

- All ordinary matter contains both positive and negative charge.
- You do not usually notice the charge because most matter contains the exact same number of positive and negative charges.
- An object is electrically neutral when it has equal amounts of both types of charge.



Electric Charge

- Objects can lose or gain electric charges.
- The net charge is also sometimes called excess charge because a charged object has an excess of either positive or negative charges.
- A tiny imbalance in either positive or negative charge on an object is the cause of static electricity.

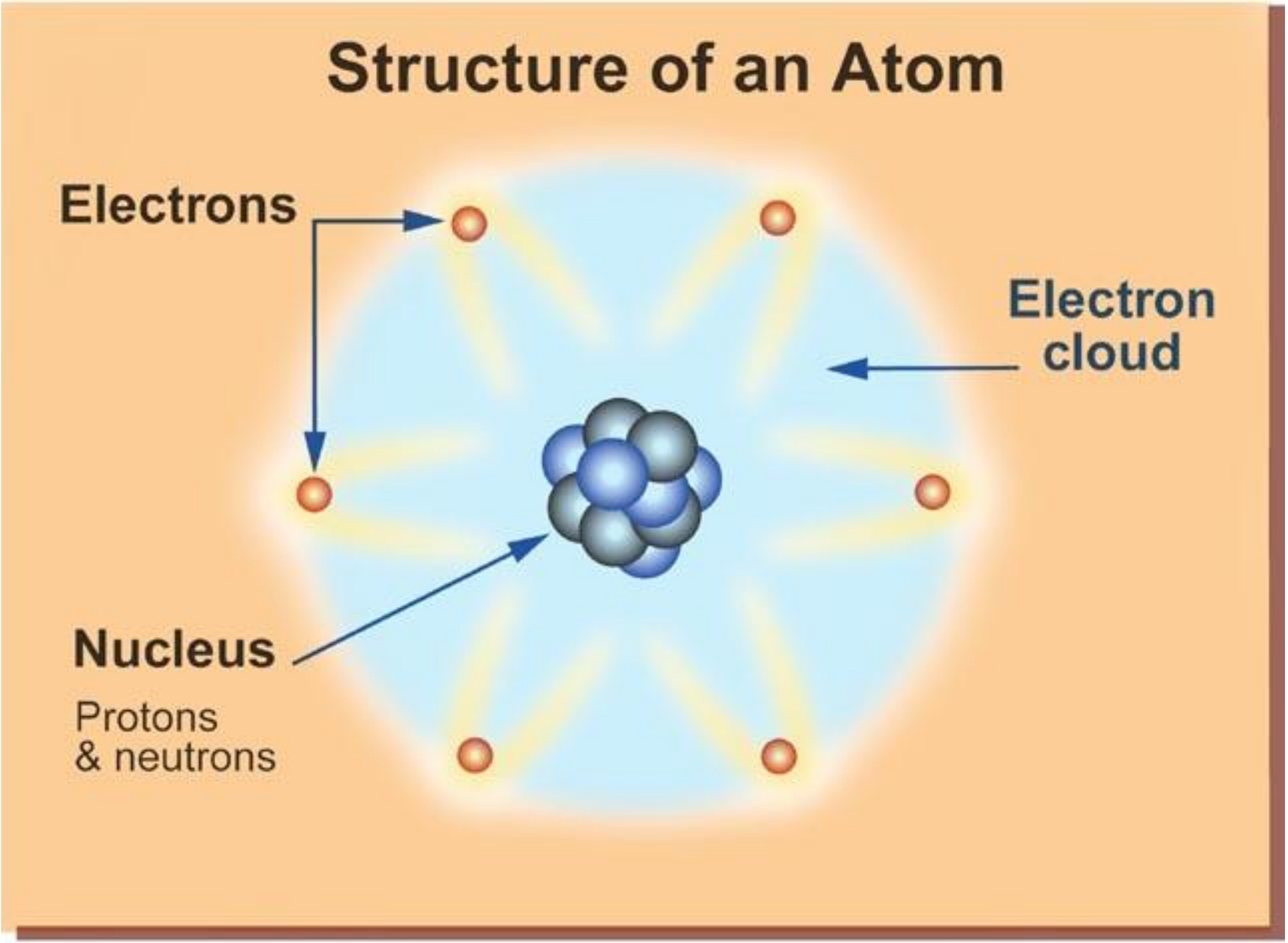


Static electricity

Electric Charge

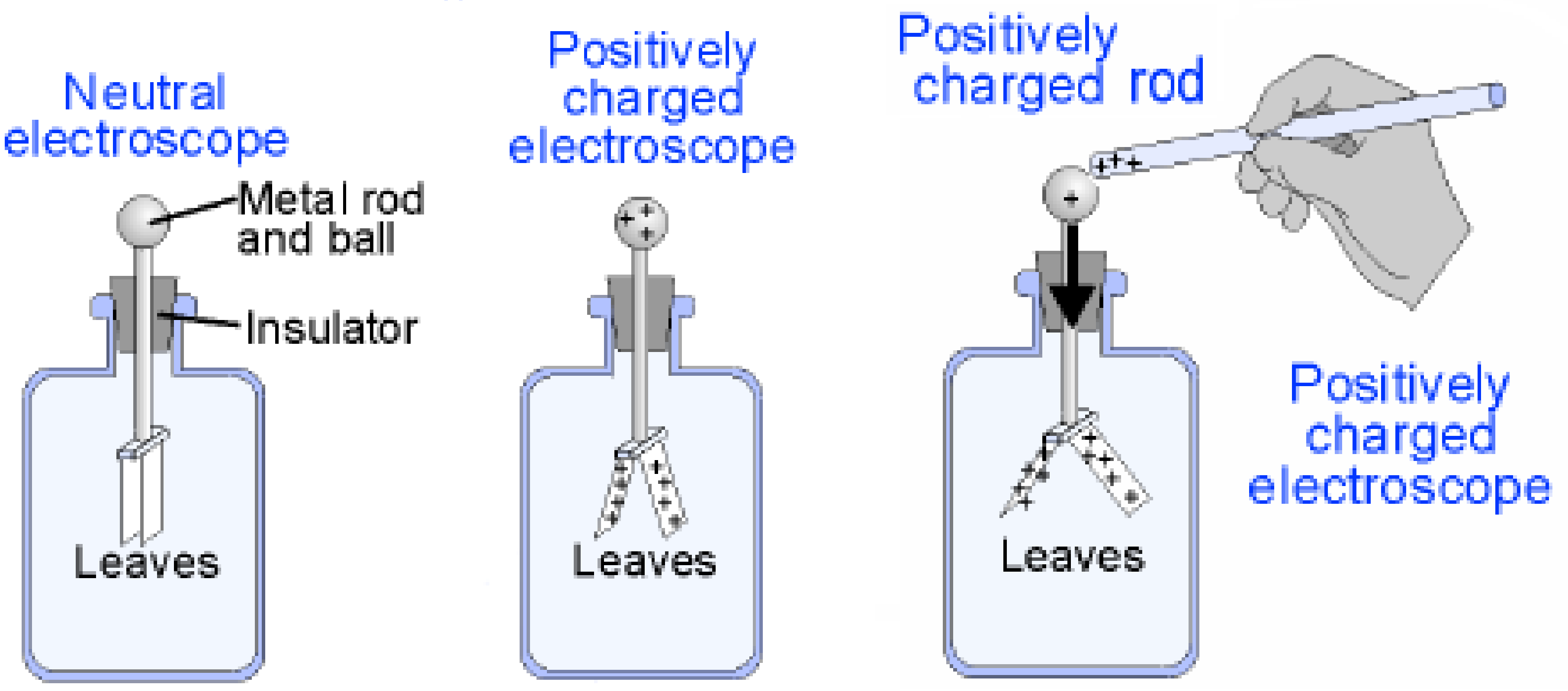
- Electric charge is a property of tiny particles in atoms.
- The unit of electric charge is the coulomb (C).
- A quantity of charge should always be identified with a positive or a negative sign.

	Mass (kg)	Charge (coulombs)
Electron	9.109×10^{-31}	-1.602×10^{-19}
Proton	1.673×10^{-27}	$+1.602 \times 10^{-19}$
Neutron	1.675×10^{-27}	0


Structure of an Atom
Electrons
Electron cloud
Nucleus
Protons & neutrons

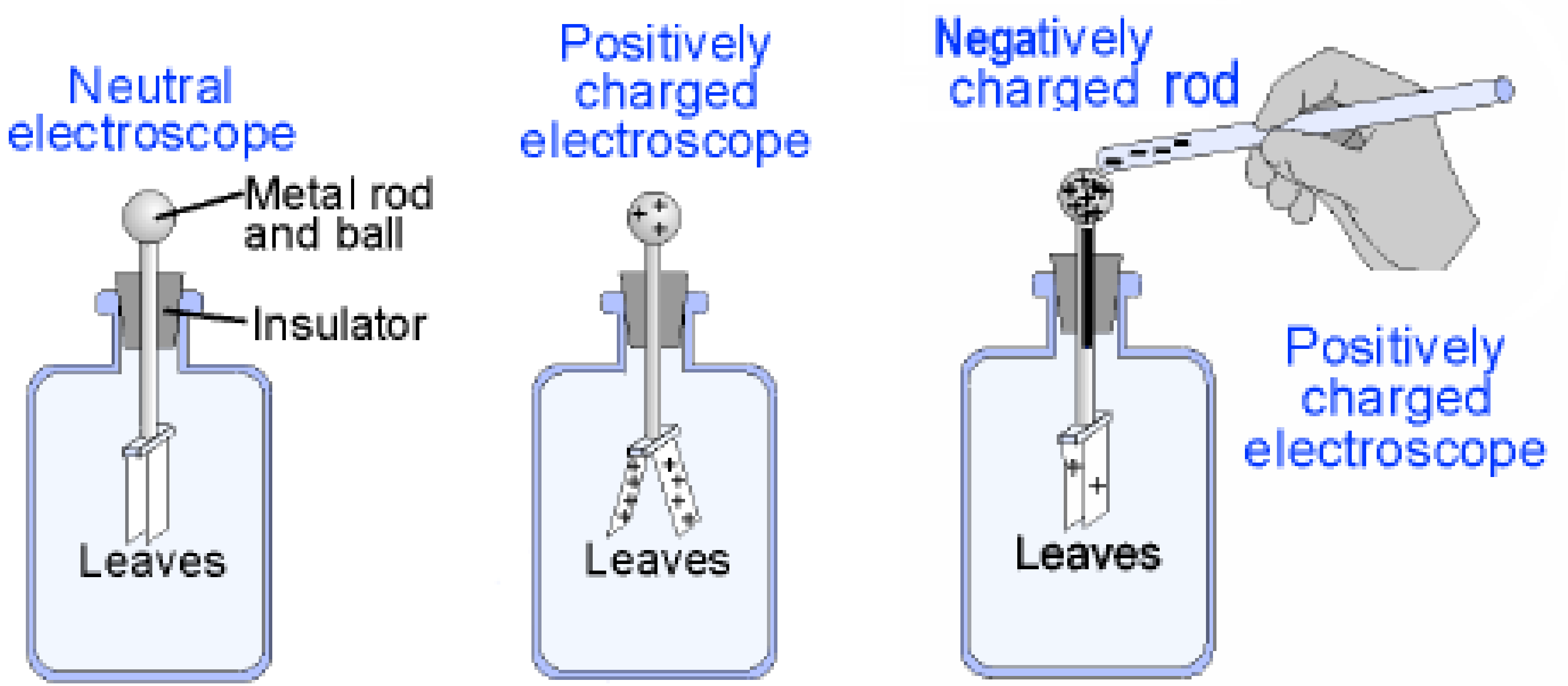
Electric forces

- The forces between the two kinds of charge can be observed with an electroscope.



Electric forces

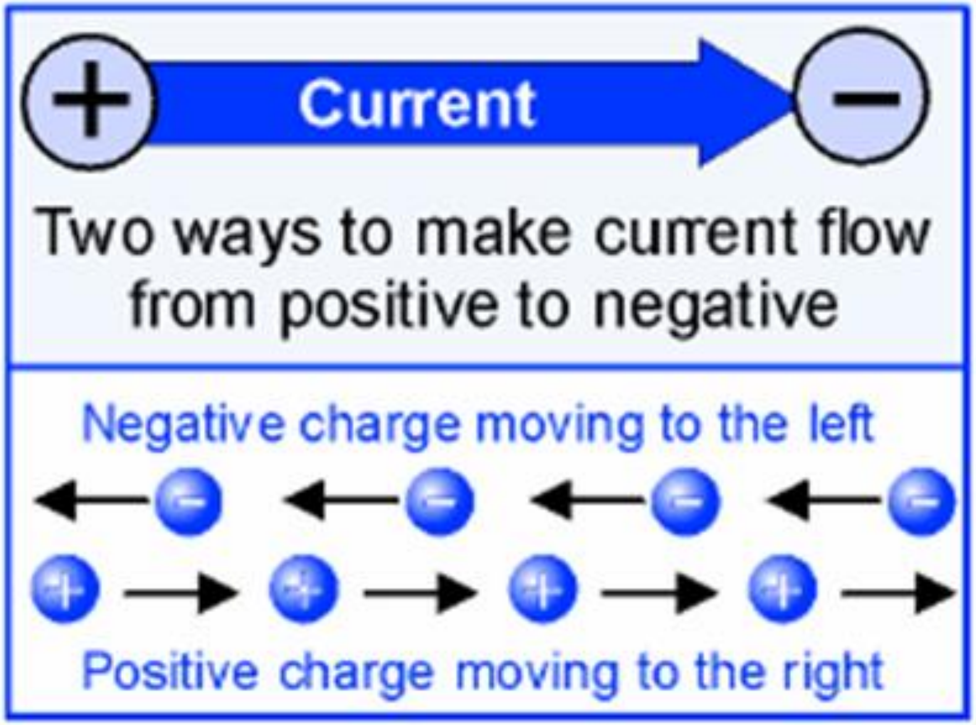
- Charge can be transferred by conduction.



Electric current

- **The direction of current was historically defined as the direction that <u>positive</u> <u>charges</u> move.**
- **Both positive and negative charges can carry current.**

- In conductive liquids (salt water) <u>both</u> positive and negative charges carry current
- In solid metal conductors, only the <u>electrons</u> can move, so current is carried by the flow of negative electrons.



Electric current

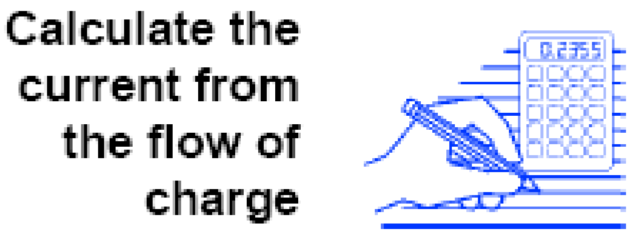
- Current is the movement of electric charge through a substance.

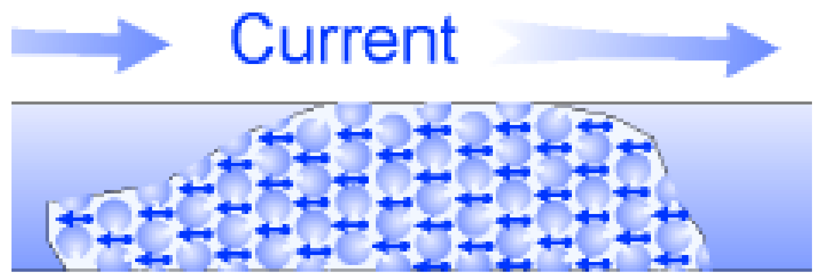
Current (amps) → $I = \frac{q}{t}$

Charge that flows (coulombs)

Time (sec)

Calculate current

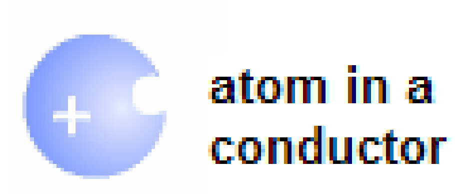


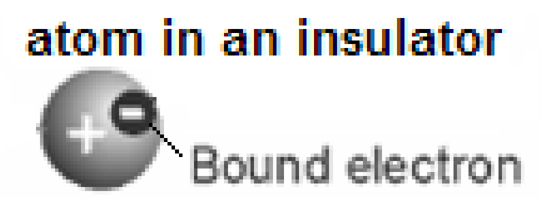


- Two coulombs of charge pass through a wire in five seconds.
- Calculate the current in the wire.

Conductors and insulators

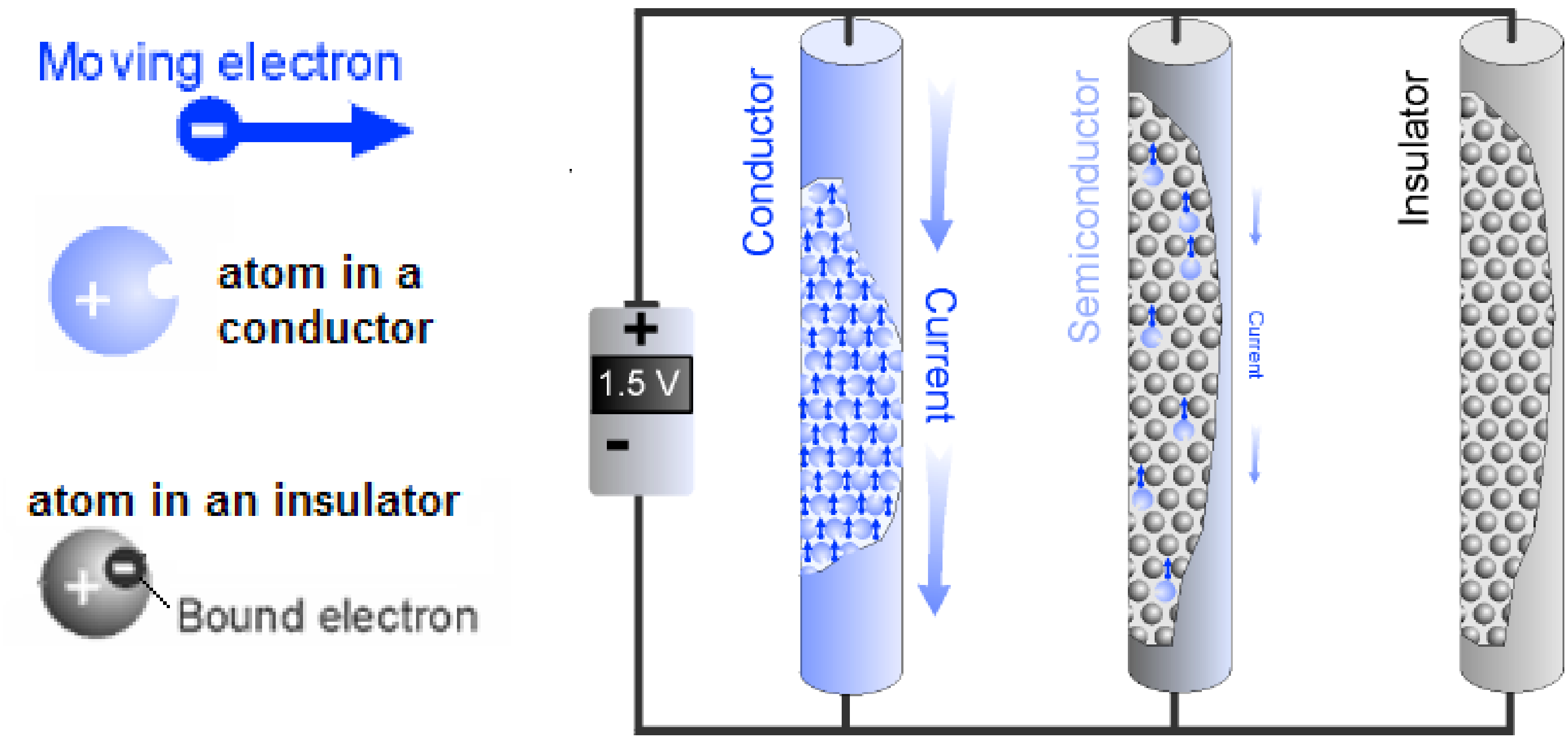
- All materials contain electrons.
- The electrons are what carry the current in a conductor.
- The electrons in insulators are not free to move—they are tightly bound inside atoms.





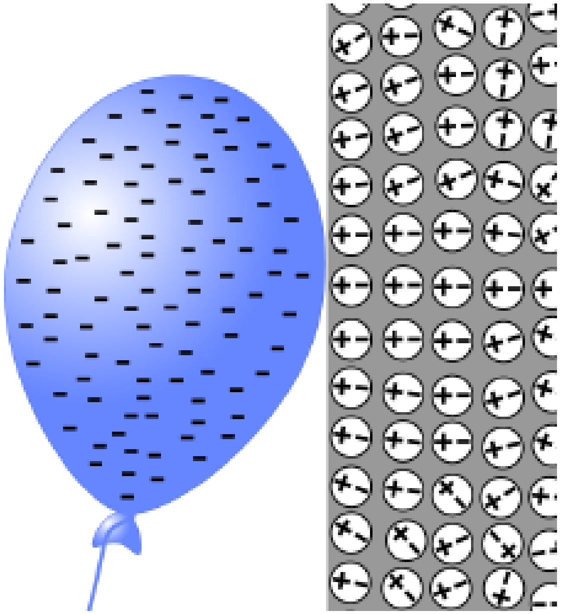
Conductors and insulators

- A **semiconductor** has a few free electrons and atoms with bound electrons that act as insulators.



Conductors and insulators

- When two neutral objects are rubbed together, charge is transferred from one to the other and the objects become oppositely charged.
- This is called charging by friction.
- Objects charged by this method will attract each other.



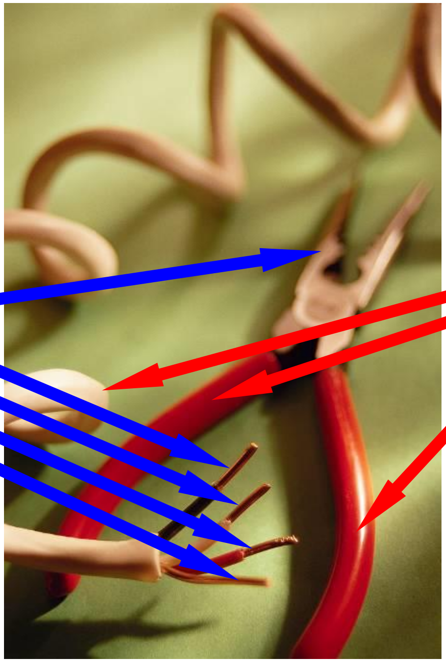
Conductors and Insulators

Conductors	Insulators
Electrons flow easily between atoms.	Electron flow is difficult between atoms.
1-3 valence electrons in outer orbit.	5-8 valence electrons in outer orbit.
Examples: Silver, Copper, Gold, Aluminum, etc.	Examples: Mica, Glass, Quartz, etc.

Conductors and Insulators

Identify conductors and insulators

Conductors



Insulators

Coulombs Law

Lab Experiment

In 1785 Charles Augustin Coulomb reported in the Royal Academy Memoires using a torsion balance two charged mulberry pith balls repelled each other with a force that is inversely proportional to the distance.

$$F=\frac{kq_1q_2}{r^2}$$ where $k=8.99*10^9\ Nm^2/C^2$ in SI unit

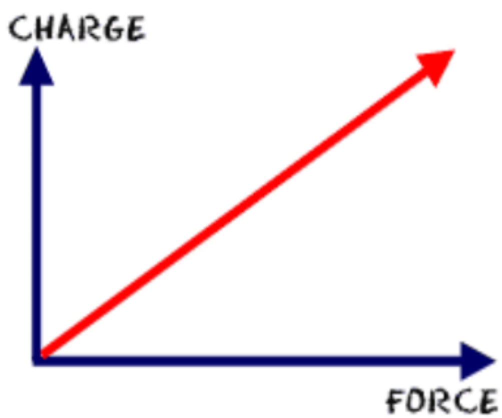
$k \sim 10^{10}\ Nm^2/C^2$

F = electric force

k = Coulomb constant

q_1, q_2 = charges

r = distance of separation



When you have two **charged particles**, an **electric force** is created. If you have larger charges, the forces will be larger.

Coulomb's law, or Coulomb's inverse-square law, is an experimental law of physics that quantifies the amount of force between two stationary, electrically charged particles. The electric force between charged bodies at rest is conventionally called electrostatic force or Coulomb force.

Coulombs Law : Coulomb's Constant

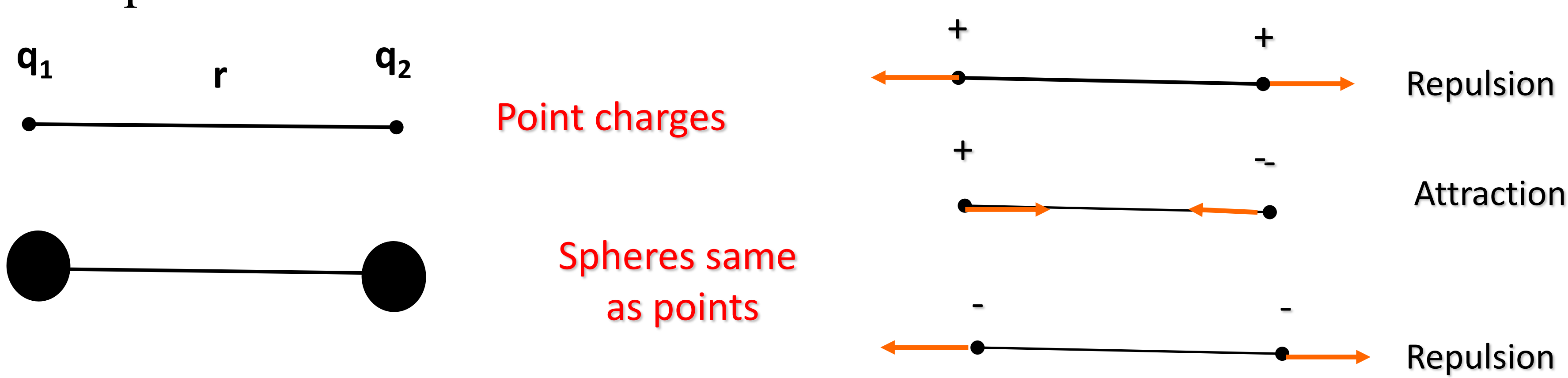
 # Calculation of Coulomb's constant:

$$k_e = \frac{1}{4\pi\varepsilon_0} = \frac{c^2\mu_0}{4\pi} = c^2 \times (10^{-7}\ \mathrm{H{\cdot}m^{-1}})$$

$$= 8.987\ 551\ 787\ 368\ 1764 \times 10^9\ \mathrm{N{\cdot}m^2{\cdot}C^{-2}} = 9 \times 10^9\ \mathrm{N\ m^2C^{-2}}$$

where vacuum permeability was defined as $\mu_0 = 4\pi\times10^{-7}\ \mathrm{Hm^{-1}}$, speed of light in vacuum c =2.99792458 x10^8 m/s =3x10^8 $\mathrm{ms^{-1}}$ and ϵ_0= vacuum permittivity or permittivity of free space = $8.8541878128\times10^{-12}\ \mathrm{F{\cdot}m^{-1}} = 8.854\times10^{-12}\ \mathrm{Fm^{-1}} = 8.854\times10^{-12}\ \mathrm{C^2{\cdot}N^{-1}{\cdot}m^{-2}}$

$$F = \frac{kq_1q_2}{r^2}$$



Coulombs Law: Vector Form

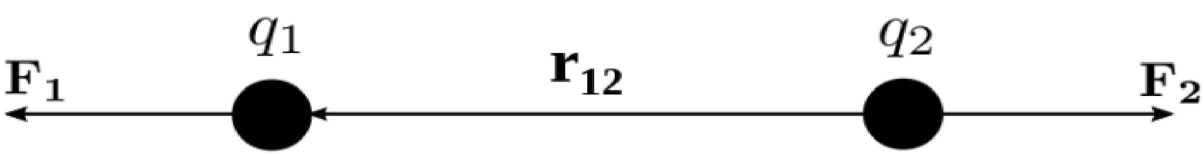
 # Coulomb's law in Vector form:

$$\mathbf{F} = k_e \frac{Qq}{r^2} \hat{\mathbf{e}}_r$$

where $\hat{\mathbf{e}}_r$ is a unit vector in the **r**-direction.

$$k_e = \frac{1}{4\pi\varepsilon_0}$$

and ϵ_0 is the vacuum permittivity.



The scalar and vector forms of the mathematical equations are

$$|\mathbf{F}| = k_e \frac{|q_1 q_2|}{r^2} \quad \text{and} \quad \mathbf{F}_1 = k_e \frac{q_1 q_2}{|\mathbf{r}_{12}|^2} \hat{\mathbf{r}}_{12}$$

q_1 and q_2 are the signed magnitudes of the charges, the scalar r is the distance between the charges, the vector $\mathbf{r}_{12} = \mathbf{r}_1 - \mathbf{r}_2$ is the vectorial distance between the charges and $\hat{\mathbf{r}}_{12} = \frac{\mathbf{r}_{12}}{|\mathbf{r}_{12}|}$ (a unit vector pointing from q_2 to q_1).

The vector form of the equation calculates the force $\mathbf{F}_1$ applied on q_1 by q_2.

If $\mathbf{r}_{21}$ is used instead, then the effect on q_2 can be found, i.e., $\mathbf{r}_{21} = \mathbf{r}_2 - \mathbf{r}_1$ and $\hat{\mathbf{r}}_{21} = \frac{\mathbf{r}_{21}}{|\mathbf{r}_{21}|}$ (a unit vector pointing from q_1 to q_2).

Coulombs Law : Vector Form

Coulomb's law in Vector form:

Coulomb's law obeys Newton's third law: $\mathbf{F}_2 = -\mathbf{F}_1$.

Explanation: Let there be two charges q_1 and q_2, with position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ respectively. Now, since both the charges are of the same sign, there will be a repulsive force between them. Let the force on the q_1 charge due to q_2 be $\mathbf{F}_{12}$ and force on q_2 charge due to q_1 charge be $\mathbf{F}_{21}$. The corresponding vector from q_1 to q_2 is $\mathbf{r}_{21}$ vector.

$\vec{\mathbf{r}}_{21} = \vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1$ $\quad \hat{\mathbf{r}}_{21} = \dfrac{\vec{\mathbf{r}}_{21}}{|\vec{\mathbf{r}}_{21}|}$ $\quad$ Now, the force on charge q_2 due to q_1, in vector form is: $\vec{\mathbf{F}}_{21} = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1 q_2}{r_{21}^2}\hat{\mathbf{r}}_{21}$

$\vec{\mathbf{r}}_{12} = \vec{\mathbf{r}}_1 - \mathbf{r}_2$ $\quad \hat{\mathbf{r}}_{12} = \dfrac{\vec{\mathbf{r}}_{12}}{|\vec{\mathbf{r}}_{12}|}$ $\quad$ Now, the force on charge q_1 due to q_2, in vector form is: $\vec{\mathbf{F}}_{12} = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1 q_2}{\mathrm{r}^2{}_{12}}\hat{\mathbf{r}}_{12}$

The above equations are the vector form of Coulomb's Law.

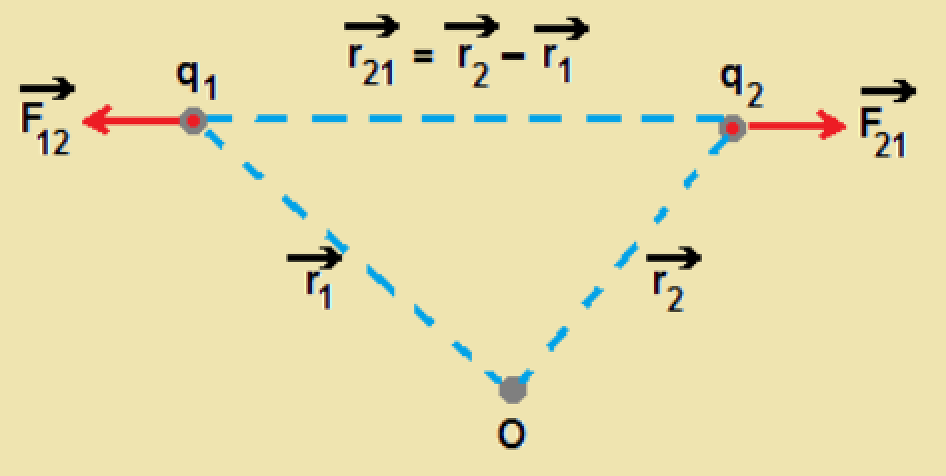
The magnitude of vector $\mathbf{r}_{21}$ is denoted by $|\mathbf{r}_{21}|$.

 # Attraction and Repulsion: The above equation is valid for any sign of q_1 and q_2. If q_1 and q_2 are of same sign, $\mathbf{F}_{21}$ is along $\mathbf{r}_{21}$, which denotes repulsion. If they are opposite sign, $\mathbf{F}_{21}$ is along $-\mathbf{r}_{21}$ that denotes attraction. No need to write separate equation for like and unlike charges.

Coulombs Law: Vector Form

 # Coulomb's law in Vector form: Coulomb's law obeys Newton's third law: $\mathbf{F}_2 = -\mathbf{F}_1$.

Since force is vector, we need to write Coulombs law in vector notation.



$\vec{\mathbf{r}}_{21} = \vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1$ $\quad \hat{\mathbf{r}}_{21} = \dfrac{\vec{\mathbf{r}}_{21}}{|\vec{\mathbf{r}}_{21}|}$ Now, the force on charge q_2 due to q_1, in vector form is: $\vec{\mathbf{F}}_{21} = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1 q_2}{r_{21}^2}\hat{\mathbf{r}}_{21}$

$\vec{\mathbf{r}}_{12} = \vec{\mathbf{r}}_1 - \mathbf{r}_2$ $\quad \hat{\mathbf{r}}_{12} = \dfrac{\vec{\mathbf{r}}_{12}}{|\vec{\mathbf{r}}_{12}|}$ Now, the force on charge q_1 due to q_2, in vector form is: $\vec{\mathbf{F}}_{12} = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1 q_2}{\mathrm{r}^2{}_{12}}\hat{\mathbf{r}}_{12}$

The above equations are the vector form of Coulomb's Law.

Coulombs Law: Application

Coulomb's law is Used in Different form: Foundation of Electrostatics

- Used in measuring the electrical forces between two objects.
- Used in Coulomb's constant or vacuum permittivity constant calculation. $$k_e = \frac{1}{4\pi\varepsilon_0}.$$
- Used in distance (atomic, radius) measurement in Coulomb's law. $$\mathbf{F} = k_e \frac{Qq}{r^2}\hat{\mathbf{e}}_r.$$
- Used in Electric Potential Energy calculation. $$U_E(r) = k_e \frac{Qq}{r}.$$
- Used in Electric Field calculation. $$\mathbf{E} = k_e \sum_{i=1}^{N} \frac{Q_i}{r_i^2}\hat{\mathbf{r}}_i.$$
- Used in vacuum permittivity or permittivity of free space calculation. $$k_e = \frac{1}{4\pi\varepsilon_0}.$$
- Coulomb's law can be used to derive Gauss's law, and vice versa.
- Used in Quantum field theory origin (Coulomb potential derives from the QED, Quantum Electrodynamics).

Coulombs Law: Limitations

Coulomb’s law has some limitations:

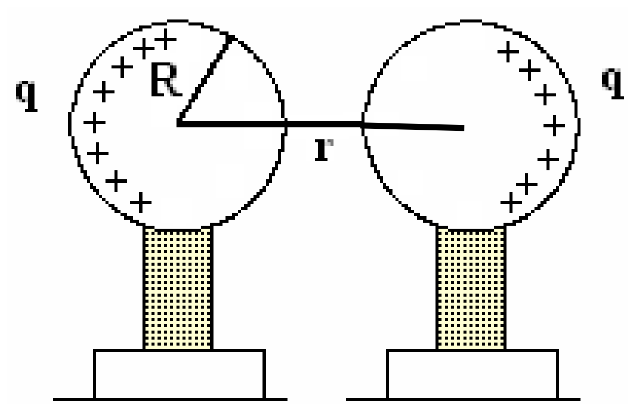
There are mainly three conditions to be fulfilled for the validity of Coulomb's inverse square law:

1. The charges must have a spherically symmetric distribution (e.g. be point charges, or a charged metal sphere).
2. The charges must not overlap (e.g. they must be distinct point charges).
3. The charges must be stationary with respect to each other.

The last of these is known as the **electrostatic approximation**. When movement takes place, **Einstein’s theory of relativity** must be taken into consideration, and a result, an extra factor is introduced, which alters the force produced on the two objects. This extra part of the force is called the **magnetic force**, and is described by **magnetic fields.** For slow movement, the magnetic force is minimal and Coulomb's law can still be considered approximately correct, but when the charges are moving more quickly in relation to each other, the full **electrodynamics** rules (incorporating the magnetic force) must be considered.

Uniformly charged metal spheres of Radius R

$$F=\frac{kq^2}{(r)^2}$$



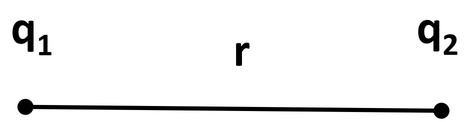
$$F- \frac{kq^2}{(r+2R)^2}$$

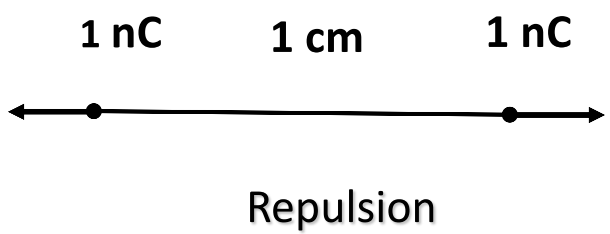
Demo: Show uniformity of charge around sphere using electrometer.
Demo: Show charging spheres by induction using electrometer

Coulombs Law
Two Positive Charges

Example-2: What is the force between two positive charges each 1 nano Coulomb, 1cm apart in a typical demo? Why is the force so weak here?

Solution:





$$F = \frac{kq_1q_2}{r^2}$$

$$F = \frac{\left(10^{10}\,\frac{Nm^2}{C^2}\right)\left(10^{-9}\,c\right)^2}{\left(10^{-2}\,m\right)^2} = 10^{-4}\,N$$

(equivalent to a weight of something with a mass of 10^{-5} kg = 10^{-2} gm or 10 mg - long strand of hair)

Coulombs Law
Two Pennies without electrons

Example-3: (i) What is the force between two 3 gm pennies one meter apart if we remove all the electrons from the copper atoms? [Modeling] (ii) What is their acceleration as they separate?

Solution: (i) We know,

$$F=\frac{kq_1q_2}{r^2}=\frac{\left(10^{10}\ \frac{Nm^2}{C^2}\right)q^2}{(1m)^2}$$

The force is $$F=\frac{\left(10^{10}\ \frac{Nm^2}{C^2}\right)\left(1.4\times 10^5 C\right)^2}{1m^2}=2\times 10^{20}N$$

The atom Cu has 29 protons and a 3 gm penny has

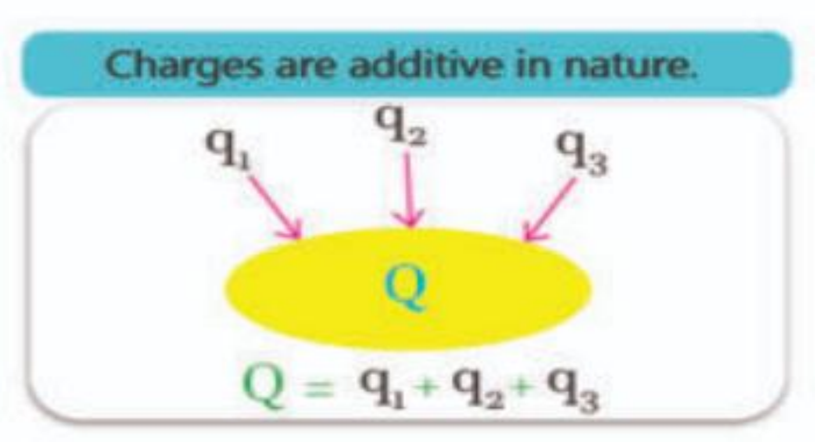
$$=\left(\frac{3gm}{63.5gm}\right)\times 6\times 10^{23}\,atoms=3\times 10^{22}\,atoms$$

The total charge is $q=29\times 3\times 10^{22}\,atoms\times 1.6\times 10^{-19}C=1.4\times 10^5 C$

(ii) What is their acceleration as they separate?

$$a=\frac{F}{m}=\frac{2x10^{20}N}{3x10^{-3}kg}=7x10^{22}\ \frac{m}{s^2}$$

Principle of Superposition: Superposition of forces Three charges In a line



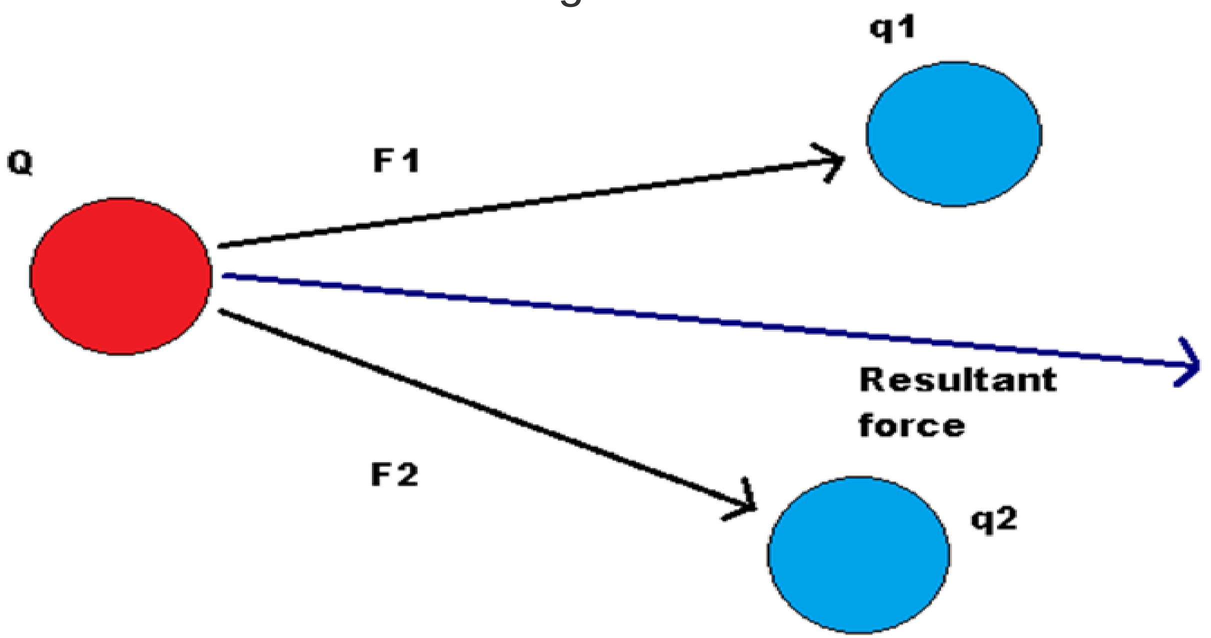
The **principle of superposition of forces** in electrostatics or **principle of superposition in electrostatics** states that when a number of **charges** are interacting, the electrostatic force between two **charges** is not affected by the existence of the other charges and the total electrostatic force on a charge is the vector sum of all the forces due to other **charges**.

Statement: Superposition Principle: The principle of superposition states that every charge in space creates an electric field at point independent of the presence of other charges in that medium. The resultant electric field is a vector sum of the electric field due to individual charges.

Every charged particle in the universe creates an electric field in the space surrounding it. This field can be calculated with the help of Coulomb's law.

Principle of Superposition : Superposition of forces

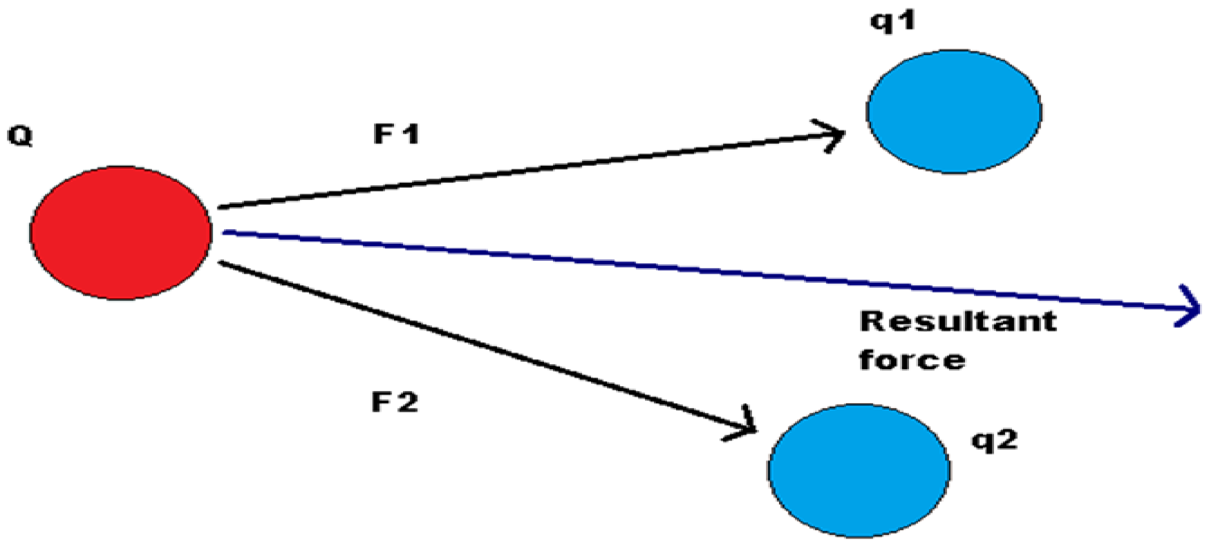
The superposition principle is helpful when there are large number of charges in a system. Let's consider the following case:



For our convenience let us consider one positive charge, and two negative charges exerting a force on it, from the superposition theorem we know that the resultant force is the vector sum of all the forces acting on the body, therefore the force $\mathbf{F}_r$, the resultant force can be given as follows,

$$\overrightarrow{F_r} = \frac{1}{4\pi\in}\left[\frac{Qq_1}{r_{12}^2}\hat{r}_{12} + \frac{Qq_2}{r_{13}^2}\hat{r}_{13}\right]$$

Principle of Superposition : Superposition of forces



Where,

$\hat{\mathbf{r}}_{12}$ and $\hat{\mathbf{r}}_{13}$ are the unit vectors along the direction of q_1 and q_2.

$\in$ is the permittivity constant for the medium in which the charges are placed in.

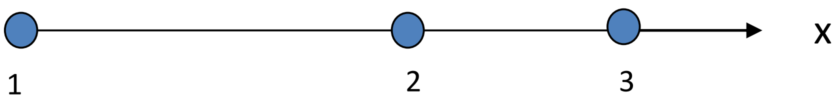
Q, q_1 and q_2 are the magnitude of the charges respectively.

r_{12} and r_{13} are the distances between the charges Q and q_1 & Q and q_2 respectively.

Principle of Superposition
Three charges In a line

- In the previous example we tacitly assumed that the forces between nuclei simply added and did not interfere with each other. That is the force between two nuclei in each penny is the same as if all the others were not there. This idea is correct and is referred to as the Principle of Superposition.

Example-4: Example of charges in a line.



Three charges lie on the x axis: q_1=+25 nC at the origin, q_2= -12 nC at x =2m, q_3=+18 nC at x=3 m. What is the net force on q_1?

Solution: We simply add the two forces keeping track of their directions. Let a positive force be one in the + x direction.

$$F=-kq_1\left(\frac{q_2}{(2m)^2}+\frac{q_3}{(3m)^2}\right)$$

$$=-\left(10^{10}\,\frac{Nm^2}{C^2}\right)(25\times10^{-9}\,C)\left(\frac{-12\times10^{-9}\,C}{(2m)^2}+\frac{18\times10^{-9}\,C}{(3m)^2}\right)$$

$$=2.5\times10^{-7}\,N$$

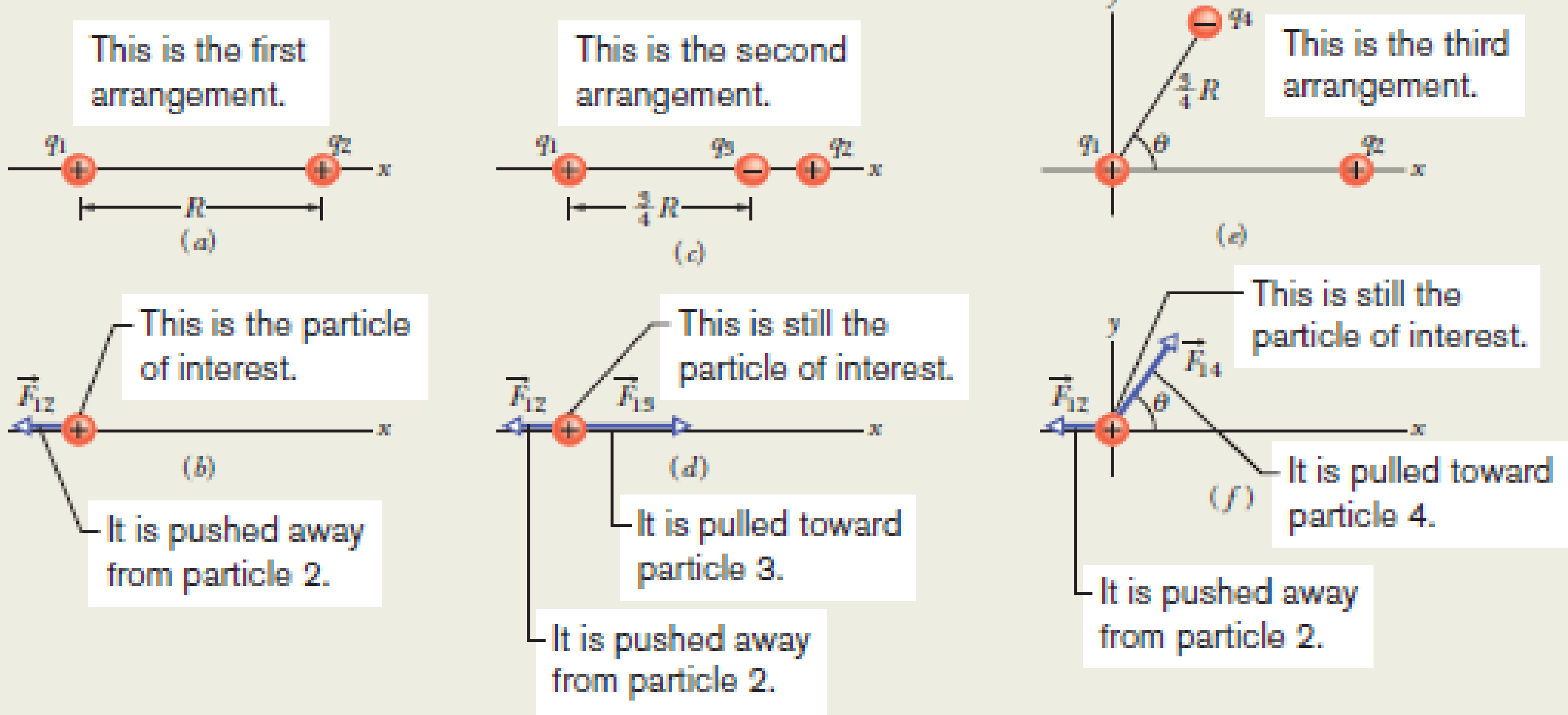


Figure 21-7 (*a*) Two charged particles of charges q_1 and q_2 are fixed in place on an x axis. (*b*) The free-body diagram for particle 1, showing the electrostatic force on it from particle 2. (*c*) Particle 3 included. (*d*) Free-body diagram for particle 1. (*e*) Particle 4 included. (*f*) Free-body diagram for particle 1.

This sample problem actually contains three examples, to build from basic stuff to harder stuff. In each we have the same charged particle 1. First there is a single force acting on it (easy stuff). Then there are two forces, but they are just in opposite directions (not too bad). Then there are again two forces but they are in very different directions (ah, now we have to get serious about the fact that they are vectors). The key to all three examples is to draw the forces correctly *before* you reach for a calculator, otherwise you may be calculating nonsense on the calculator. (Figure 21-7 is available in *WileyPLUS* as an animation with voiceover.)

$$F_{12} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{R^2}$$
$$= (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)$$
$$\times \frac{(1.60 \times 10^{-19}\ \text{C})(3.20 \times 10^{-19}\ \text{C})}{(0.0200\ \text{m})^2}$$
$$= 1.15 \times 10^{-24}\ \text{N}.$$

Thus, force $\vec{F}_{12}$ has the following magnitude and direction (relative to the positive direction of the x axis):

$$1.15 \times 10^{-24}\ \text{N} \quad \text{and} \quad 180°. \qquad \text{(Answer)}$$

We can also write $\vec{F}_{12}$ in unit-vector notation as

$$\vec{F}_{12} = -(1.15 \times 10^{-24}\ \text{N})\hat{\text{i}}. \qquad \text{(Answer)}$$

Three particles: To find the magnitude of $\vec{F}_{13}$, we can rewrite Eq. 21-4 as

$$F_{13} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_3|}{(\frac{3}{4}R)^2}$$
$$= (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)$$
$$\times \frac{(1.60 \times 10^{-19}\ \text{C})(3.20 \times 10^{-19}\ \text{C})}{(\frac{3}{4})^2(0.0200\ \text{m})^2}$$
$$= 2.05 \times 10^{-24}\ \text{N}.$$

We can also write $\vec{F}_{13}$ in unit-vector notation:

$$\vec{F}_{13} = (2.05 \times 10^{-24}\ \text{N})\hat{\text{i}}.$$

The net force $\vec{F}_{1,\text{net}}$ on particle 1 is the vector sum of $\vec{F}_{12}$ and $\vec{F}_{13}$; that is, from Eq. 21-7, we can write the net force $\vec{F}_{1,\text{net}}$ on particle 1 in unit-vector notation as

$$\vec{F}_{1,\text{net}} = \vec{F}_{12} + \vec{F}_{13}$$
$$= -(1.15 \times 10^{-24}\ \text{N})\hat{\text{i}} + (2.05 \times 10^{-24}\ \text{N})\hat{\text{i}}$$
$$= (9.00 \times 10^{-25}\ \text{N})\hat{\text{i}}. \qquad \text{(Answer)}$$

Thus, $\vec{F}_{1,\text{net}}$ has the following magnitude and direction (relative to the positive direction of the x axis):

$$9.00 \times 10^{-25}\ \text{N} \quad \text{and} \quad 0°. \qquad \text{(Answer)}$$

Four particles: We can rewrite Eq. 21-4 as

$$F_{14} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_4|}{(\frac{3}{4}R)^2}$$
$$= (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)$$
$$\times \frac{(1.60 \times 10^{-19}\ \text{C})(3.20 \times 10^{-19}\ \text{C})}{(\frac{3}{4})^2(0.0200\ \text{m})^2}$$
$$= 2.05 \times 10^{-24}\ \text{N}.$$

Then from Eq. 21-7, we can write the net force $\vec{F}_{1,\text{net}}$ on particle 1 as

$$\vec{F}_{1,\text{net}} = \vec{F}_{12} + \vec{F}_{14}.$$

Because the forces $\vec{F}_{12}$ and $\vec{F}_{14}$ are not directed along the same axis, we *cannot* sum simply by combining their magnitudes. Instead, we must add them as vectors, using one of the following methods.

Method 1. Summing directly on a vector-capable calculator. For $\vec{F}_{12}$, we enter the magnitude 1.15×10^{-24} and the angle 180°. For $\vec{F}_{14}$, we enter the magnitude 2.05×10^{-24} and the angle 60°. Then we add the vectors.

Method 2. Summing in unit-vector notation. First we rewrite $\vec{F}_{14}$ as

$$\vec{F}_{14} = (F_{14}\cos\theta)\hat{\text{i}} + (F_{14}\sin\theta)\hat{\text{j}}.$$

Substituting 2.05×10^{-24} N for F_{14} and 60° for θ, this becomes

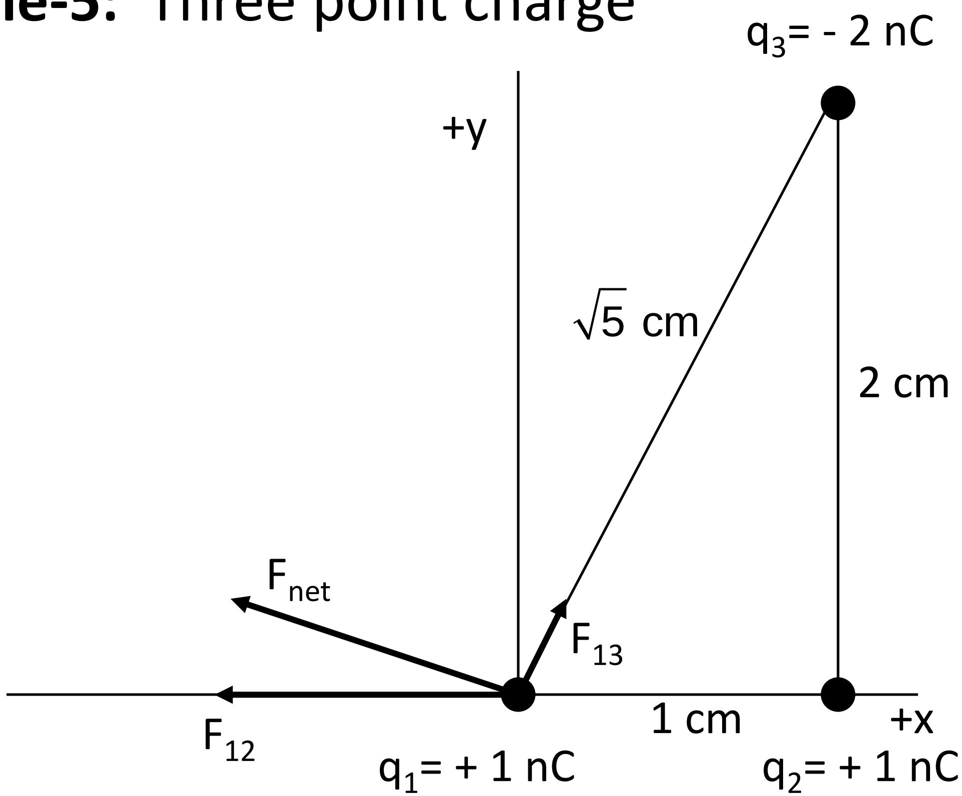
$$\vec{F}_{14} = (1.025 \times 10^{-24}\ \text{N})\hat{\text{i}} + (1.775 \times 10^{-24}\ \text{N})\hat{\text{j}}.$$

Then we sum:

$$\vec{F}_{1,\text{net}} = \vec{F}_{12} + \vec{F}_{14}$$
$$= -(1.15 \times 10^{-24}\ \text{N})\hat{\text{i}}$$
$$+ (1.025 \times 10^{-24}\ \text{N})\hat{\text{i}} + (1.775 \times 10^{-24}\ \text{N})\hat{\text{j}}$$
$$\approx (-1.25 \times 10^{-25}\ \text{N})\hat{\text{i}} + (1.78 \times 10^{-24}\ \text{N})\hat{\text{j}}.$$

(Answer)

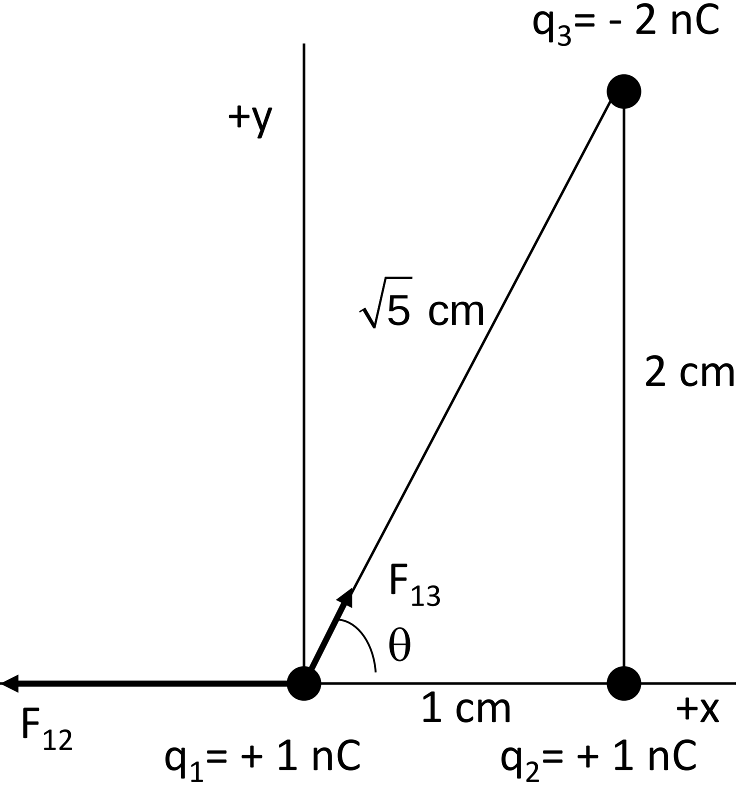
Example-5: Three point charge



Question: What is the net force on q_1 and in what direction?

Hint : Find x and y components of force on q_1 due to q_2 and q_3 and add them up.

Solution:



x and y Components of force due to q_2

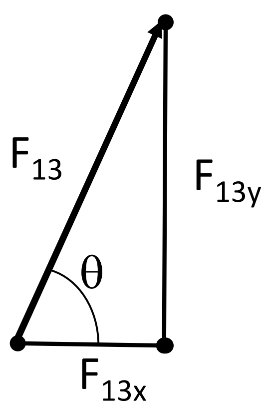
$$F_{12x} = -10^{10}\,\tfrac{Nm^2}{C^2}\,\frac{(10^{-9}C)^2}{(10^{-2}m)^2} = -1\times 10^{-4}N$$

$$F_{12y} = 0$$

x and y Components of force due to q_3

Magnitude of Force due to q_3

$$|F_{13}| = 10^{10}\,\tfrac{Nm^2}{C^2}\,\frac{(2\times 10^{-9}C)(1\times 10^{-9}C)}{(\sqrt{5}\times 10^{-2}m)^2} = 0.40\times 10^{-4}N$$

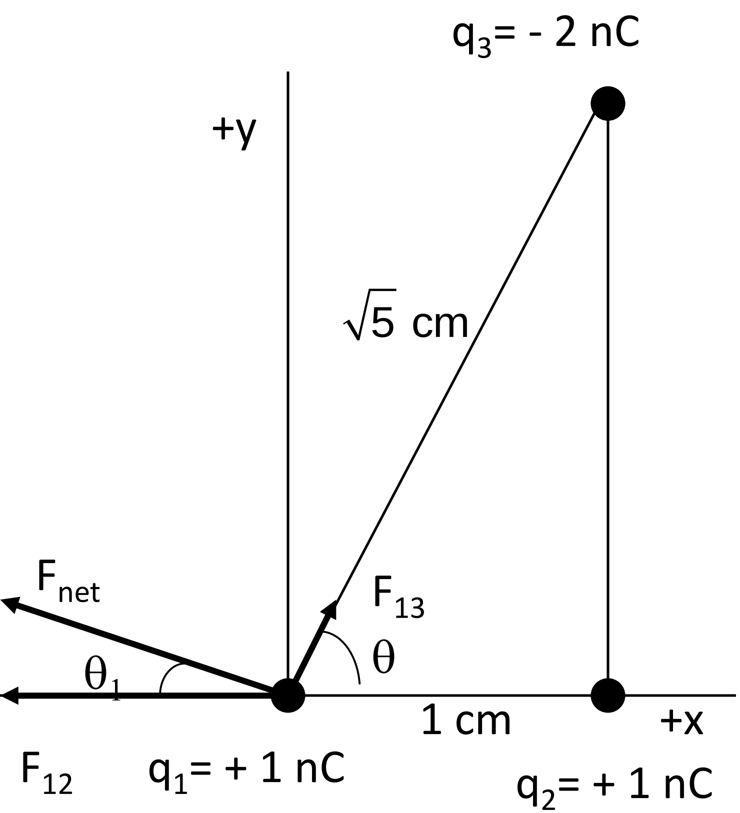


$$\tan\theta = \frac{\text{y - axis value}}{\text{x - axis value}} \quad \Rightarrow \theta = \tan^{-1}(\tfrac{2}{1}) = 63.43 \text{ deg}$$

$$F_{13x} = F\cos\theta = (0.40N)(\cos 63.43) = 0.179\times 10^{-4}N$$

$$F_{13y} = F\sin\theta = (0.40N)(\sin 63.43) = 0.358\times 10^{-4}N$$

Example Cont.



Total force along the x-axis

$$F_{x_{net}} = F_{12x} + F_{13x} = (-1\times10^{-4} + 0.179\times10^{-4})N = -0.821\times10^{-4}N$$

Total force along the y-axis

$$F_{y_{net}} = F_{12y} + F_{13y} = (0 + 0.358\times10^{-4})N = 0.358\times10^{-4}N$$

$$F_{net} = \sqrt{F_{x_{net}}^2 + F_{y_{net}}^2} = \sqrt{((0.821)^2 + (0.358)^2)\times(10^{-4})^2}N$$

$$F_{net} = +0.802\ 10^{-4}N$$

$$\tan\theta_1 = \frac{F_y}{F_x}$$

$$\theta_1 = \tan^{-1}\left(\frac{F_y}{F_x}\right) \qquad \theta_1 = \tan^{-1}\left(\frac{0.358\times10^{-4}N}{-0.821\times10^{-4}N}\right)$$

$$\theta_1 = 23.6^\circ \text{ from the } -x \text{ axis}$$

Example-6: In an atom can we neglect the gravitational force between the electrons and protons? What is the ratio of Coulomb's electric force to Newton's gravity force for 2 electrons separated by a distance r ?

Solution:

q **r** **q**

$$F_c = \frac{kee}{r^2}$$

m **r** **m**

$$F_g = \frac{Gmm}{r^2}$$

$$\frac{F_c}{F_g} = \frac{ke^2}{Gm^2}$$

$$\frac{F_c}{F_g} = \frac{\left(10^{10}\ Nm^2/C^2\right)\left(1.6\times10^{-19}\ C\right)^2}{\left(6.67\times10^{-11}\ Nm^2/kg^2\right)\left(9.1\times10^{-31}\ kg\right)^2}$$

$$= 4.6\times10^{42}$$

Huge number, pure ratio