United International University

School of Science and Engineering

Final Examination Trimester- Summer 2022

Course: Probability and Statistics, Course code: Math -2205

Total marks - 40; Duration – 2 hours **Date: 26.09.22**

[*Note that the number of marks is given in brackets at the end of each question or part question. You have to answer all the questions in order.*]

Q1

(a) 1% of a population have a certain disease and the remaining 99% are free from this disease. A test is used to detect this disease. This test is positive in 95% of the people with the disease and is also (falsely) positive in 2% of the people free from the disease. If a person, selected at random from this population, has tested positive, what is the probability that she/he has the disease? D | P

(b) The time taken in minutes by candidates to answer a question in an examination has probability density function (PDF)given by

$$f(t) = \begin{cases} k(6t - t^2) & 3 \le t \le 6 \\ 0 & otherwise \end{cases}$$, where k is a constant.

-18k = 1   18k = 1 $k = \frac{1}{18}$

(i) Show that $k = \frac{1}{18}$

(ii) Find the standard deviation of the time that is taken by candidates

(iii) Find the probability that a candidate chosen a random, takes longer than 5 minutes to answer the questions **[3+7=10]**

Q2

(a) The lifetime of the Powerhouse battery has a normal distribution with mean 210 hours. It is found that 4% of these batteries operate for more than 222 hours. Find the variance of the distribution, correct to 2 decimal places. $X \sim N(210, \sigma^2)$ $P(X>222) = 0.04$

(b) A typist makes, on average, 1 error for every 200 keyboard strokes. Assuming the error occur independently and random, find the probability that

(i) in a document requiring 400 keyboard strokes there is no error $P(X=r)$ $X \sim Po(\lambda)$

(ii) in a document requiring 1000 keyboard strokes there is, at most one error $\frac{e^{-\lambda} \times \lambda^r}{r!}$

(c) In a large college, 28% of the students do not play any musical instrument, 52% play exactly one musical instrument and the remainder play two or more musical instruments. A random sample of 12 students from the college is chosen, find the probability that more than 9 of these students play one or more musical instruments.

A+B − (A∩B) **[4+3+3 = 10]**

Q3.

(a) The random variable X takes the values -2, 1, 2 and 3. It is given that $P(X = x) = kx^2$, where k is a constant.

(i) Draw up the probability distribution table for x, giving the probabilities in numerical fraction.

(ii) Find $p(x \ge 2)$

(iii) Find $E(x)$ and $Var(x)$ of the random variable X.

(b) A javelin thrower noted the length of a random of 50 of her throws. The sample mean was 72.3 m and an unbiased estimate of the population variance was 64.3 m². Calculate a 92% confidence interval for the population mean length of throws by this athlete.

$c = z \times \frac{\sigma}{\sqrt{n}}$

[7+3=10]

Q4

(a) In the past, the mean time for Jenny's morning run was 28.2 minutes. She does some extra training, and she wishes to test whether her mean time has been reduced. After the training Jenny takes a random sample of 40 morning runs. She decides that if the sample mean run is less then 27 minutes, she will conclude that the training has been effective. After the training, Jenny's run time has a standard deviation of 4.0 minutes.

$R < 27$

Reject Typ I

Ans

True

False Type 2

(i) State the null and alternative hypothesis for Jenny's test.

(ii) Find the probability of $P(\bar{x} < 27)$ and hence the find the probability that she will make a type I error.

(b) In a game a ball is thrown and lands in one of 4 slots, labelled A, B, C and D. Raju wishes to test whether the probability that the ball land in slot A is $\frac{1}{4}$.

hyp

(i) State suitable null and alternative hypothesis for Raju's test.

(ii) The ball is thrown 100 times and it lands in slot A 15 times. Use a suitable approximating distribution to carry out the test at the 2% significance level.

[5+5= 10]

Required Formulae

Distribution	pmf/pdf
Discrete Pro. distribution	$E(x) = \sum x\, p(x)$ Var $(x) = \sum x^2\, p(x) - (E(x))^2$
Binomial	$f(x) = n_{C_x} p^x (1-p)^{n-x}$; $x = 0, 1, 2, \dots n$
Poisson	$f(x) = \dfrac{\lambda^x e^{-\lambda}}{x!}$; $x = 0, 1, 2, \dots$
Uniform	$f(x) = \dfrac{1}{b-a}$; $a \le x \le b$
Normal	$f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$; $-\infty < x < \infty$
Continues random variable	$E(x) = \int_{-\infty}^{\infty} x\, f(x)dx$ Var $(x) = \int_{-\infty}^{\infty} x^2\, f(x)dx - (E(x))^2$

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