United International University

School of Science and Engineering

Final Examination Trimester: Spring-2024

Course Title: Probability and Statistics

Course Code: Math 2205/Stat 205 Marks: 40 Time: 2 Hours

Answer <u>all</u> the questions. Answer all parts of a question together.

Q1. **(a)** Complete the following table and compute $P(A_1|B_2)$, $P(B_2|A_1)$, and $P(A_1 \cup B_1)$. **[4]**

	$n(A_1)$	$n(A_2)$	Total
$n(B_1)$	25		65
$n(B_2)$			
Total	80		150

(b) If 3 worker independently try to complete a task with the probability of success 0.7, 0.8, and 0.6, respectively. What is the probability of the (i) task will not be completed (ii) every worker will complete the task (iii) only the second worker will complete the work? **[3]**

(c) In a software development team, three programmers contribute to 35%, 40%, and 25% of the total code developed in a project cycle. Their respective coding efficiencies are 92%, 97%, and 95%. If a bug is detected in the code, what is the probability that it originated from the work of the second programmer? **[3]**

Q2. **(a)** Suppose there are 5 defective items in a lot of 50 items. A sample of size 5 is taken at random without replacement. Let X denote the number of defective items in the sample. Find the probability that the sample contains (i) at most two defective item (ii) all defective items. **[3]**

(b) Let the random variable X have the *probability mass function* $f(x) = \frac{x}{10}; x = 1, 2, 3, 4.$ Compute $P(X < 3)$, $E(X^2 + 5)$, and $V(1 - 3X)$. Also, sketch the line graph of X. **[5]**

(c) Suppose that 80% of employees in a certain company are proficient in using a particular software tool. In a random sample of 15 employees, let X be the number of proficient employees. Assuming independence, how is X distributed? Find the *moment generating function* of X. Also, compute $P(X < 3)$ and $P(X \geq 4)$. **[4]**

(d) Let X have a Poisson distribution with $P(X = 1) = \frac{1}{2}P(X = 2)$. Find standard deviation of X and $P(X \geq 1)$. **[3]**

Q3. **(a)** Let $f(y) = \frac{3}{2}y^2$; $-1 < y < 1$ be the *probability density function* of a continuous random variable Y. Find and sketch the *cumulative distribution function of* Y. Find $P(Y \geq 0.5)$ and P_{65} of Y. **[4]**

(b) A continuous random variable X has the *cumulative distribution function* as **[3]**

$$F(x) = \begin{cases} 0; & x < 5 \\ \frac{x-5}{5}; & 5 \le x < 10 \\ 1; & x \ge 10 \end{cases}$$

(i) Identify the distribution and write the *probability density function* of X.

(ii) Estimate $P(3 < X < 12)$.

(iii) Find the mean and the variance of X.

(c) Assume $M(t) = e^{24t+50t^2}$ is the *moment generating function* of a normal variable X, then (i) find a constant k such that $P(Z \ge k) = 0.025$ (ii) evaluate $P(X < 40.2)$. Also, find $-Z_{0.05}$. **[4]**

Q4. Design a decision rule to test the hypothesis that a deck of playing cards is fair if we take a sample of 120 trials of the cards to get black/red and use 10% as the level of significance. Predict the acceptance and critical region. **[4]**

Distribution	***pmf* or *pdf***
Uniform (discrete)	$\boldsymbol{f(x) = \frac{1}{m}\ ;\ x = 1, 2, \dots\dots, m}$
Hypergeometric	$\boldsymbol{f(x) = \frac{N_1 C_x \, N_2 C_{n-x}}{N C_n}\ ;\ \ N = N_1 + N_2,\qquad x = 0, 1, 2, \dots\dots, n}$
Geometric	$\boldsymbol{f(x) = q^{x-1}p\ ;\ x = 1, 2, \dots\dots}$
Binomial	$\boldsymbol{f(x) = n C_x p^x q^{n-x}\ ;\ x = 0, 1, 2, \dots\dots, n}$
Poisson	$\boldsymbol{f(x) = \frac{\lambda^x e^{-\lambda}}{x!}\ ;\ x = 0, 1, 2, \dots\dots}$
Uniform (continuous)	$\boldsymbol{f(x) = \frac{1}{b-a}\ ;\ a \le x \le b}$
Exponential	$\boldsymbol{f(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}\ ;\ 0 \le x < \infty}$
Gamma	$\boldsymbol{f(x) = \frac{1}{\Gamma(\alpha)\, \theta^\alpha} x^{\alpha-1} e^{-x/\theta}\ ;\ 0 \le x < \infty}$
Normal	$\boldsymbol{f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\ ;\ -\infty < x < \infty}$

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997