Math-2205 Class Test 04 Section C

1. Identify the value of k for which $f(x) = 4 - kx\,; 0 \le x \le \frac{1}{2}$ could be a *pdf* of a random variable X. Find the *cdf* and hence the median of the distribution of X. Also, find $P(X < 0)$ and $P\left(X \ge \frac{1}{4}\right)$. [6]

2. Consider the distribution $U(5,10)$ of a random variable X. Find the graph of *pdf* and *cdf* of X. Also, estimate $P(6 < X < 9)$. [4]

1.

$$\int_0^{1/2} (4 - kx)\,dx = 1$$

$$\Rightarrow 4x - \frac{1}{2}kx^2 \Big|_0^{1/2} = 1$$

$$\Rightarrow 2 - \frac{1}{8}k = 1$$

$$\Rightarrow \frac{1}{8}k = 1$$

$$\Rightarrow k = 8$$

$$\therefore f(x) = 4 - 8x;\ 0 \le x \le \frac{1}{2}$$

$$F(x) = \int_0^x (4 - 8w)\,dw$$

$$= 4w - 4w^2 \Big|_0^x$$

$$= 4x - 4x^2;\ 0 \le x \le \frac{1}{2}$$

$$F(m) = 0.5$$

$$\Rightarrow 4m - 4m^2 = 0.5$$

$$\Rightarrow 8m^2 - 8m + 1 = 0$$

$$\Rightarrow m = 0.15,\ 0.85$$

$$P(X < 0) = 0$$

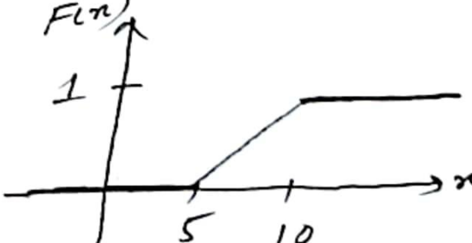
$$P\left(X \ge \frac{1}{4}\right) = 1 - F\left(\frac{1}{4}\right) = 1 - \left[1 - \frac{1}{4}\right] = \frac{1}{4}$$

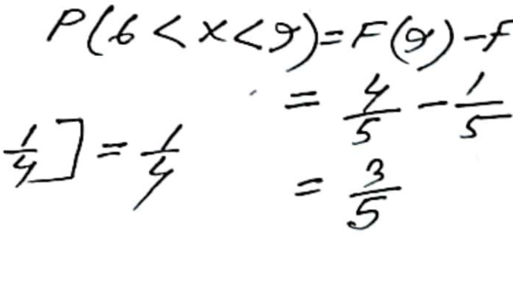
2. $U(5,10)$

$$f(x) = \frac{1}{10-5}$$

$$= \frac{1}{5};\ 5 \le x \le 10$$

$$F(x) = \begin{cases} 0 & ; x < 5 \\ \frac{x-5}{5} & ; 5 \le x < 10 \\ 1 & ; x \ge 10 \end{cases}$$





$$P(6 < X < 9) = F(9) - F(6)$$

$$= \frac{4}{5} - \frac{1}{5}$$

$$= \frac{3}{5}$$

Math-2205 Class Test 04 Section E

1. Identify the value of k for which $f(x) = ke^{-3x};\ 0 \leq x < \infty$ could be a *pdf* of a random variable X. Find the *cdf* to find $P(X \geq 5)$. Also, find the variance of the distribution. [6]

2. Let the *cdf* of the random variable X is $F(x) = \begin{cases} 0; & x < -2 \\ \frac{x+2}{6}; & -2 \leq x < 4 \\ 1; & x \geq 4 \end{cases}$. Identify the distribution and find the corresponding *pdf*. Find the graph of *cdf* of X, and $P(1 < X < 5)$. [4]

1. $f(x) = ke^{-3x};\ 0 \leq x < \infty$

$$\int_0^\infty ke^{-3x}\,dx = 1$$

$$\Rightarrow \left.\frac{ke^{-3x}}{-3}\right|_0^\infty = 1$$

$$\Rightarrow -0 + \frac{k}{3} = 1$$

$$\Rightarrow k = 3$$

$$\therefore f(x) = 3e^{-3x};\ 0 \leq x < \infty$$

$$F(x) = \int_0^x 3e^{-3w}\,dw$$

$$= -e^{-3w}\Big|_0^x$$

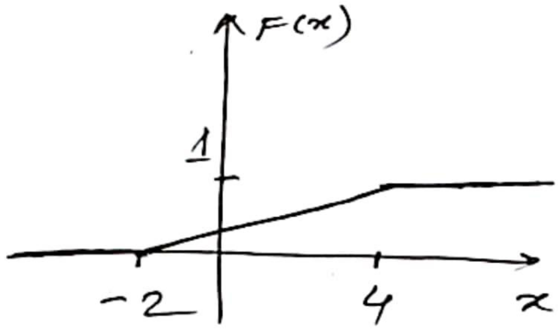
$$= 1 - e^{-3x};\ 0 \leq x < \infty$$

$$P(X \geq 5) = 1 - F(5)$$
$$= 1 - [1 - e^{-15}] = e^{-15}$$

2. $F(x) = \begin{cases} 0; & x < -2 \\ \frac{x-(-2)}{4-(-2)}; & -2 \leq x < 4 \\ 1; & x \geq 4 \end{cases}$

$U(-2, 4)$

$$f(x) = \frac{1}{4-(-2)}$$
$$= \frac{1}{6};\ -2 \leq x \leq 4$$



$$P(1 < X < 5)$$
$$= F(5) - F(1)$$
$$= 1 - \frac{3}{6} = \frac{1}{2}$$

$$E(X) = \int_0^\infty xe^{-3x}\,dx = \left.-\frac{1}{3}xe^{-3x} - \frac{1}{9}e^{-3x}\right|_0^\infty = \frac{1}{9}$$

$$E(X^2) = \int_0^\infty x^2e^{-3x}\,dx = \left.-\frac{1}{3}x^2e^{-3x} - \frac{2}{9}xe^{-3x} - \frac{2}{27}e^{-3x}\right|_0^\infty = \frac{2}{27}$$

$$\sigma^2 = \frac{2}{27} - \left(\frac{1}{9}\right)^2 = 5/81$$

Math-2205 Class Test 04 Section K

1. Identify the value of c for which $f(x) = \frac{3x^2}{8}; 0 \le x \le c$ could be a *pdf* of a random variable X. Find the *cdf* and hence the median of the distribution of X. Also, find the standard deviation of the distribution. [6]

2. Let the random variable X have the *pdf* $f(x) = e^{1-x}; x \ge 1$. Estimate $P(3 < X < 5)$ and the *mgf* of X. [4]

1. $f(x) = \frac{3x^2}{8};\ 0 \le x \le c$

$$\int_0^c \frac{3x^2}{8} = 1$$

$$\Rightarrow \left.\frac{x^3}{8}\right|_0^c = 1$$

$$\Rightarrow c^3 = 8$$

$$\Rightarrow c = 2$$

$$\therefore f(x) = \frac{3x^2}{8};\ 0 \le x \le 2$$

$$F(x) = \int_0^x \frac{3w^2}{8} dw$$

$$= \left.\frac{w^3}{8}\right|_0^x$$

$$= \frac{x^3}{8};\ 0 \le x \le 2$$

$$F(m) = \frac{1}{2}$$

$$\Rightarrow \frac{m^3}{8} = \frac{1}{2}$$

$$\Rightarrow m^3 = 4$$

$$\Rightarrow m = \sqrt[3]{4}$$

$$E(x) = \int_0^2 x\left(\frac{3x^2}{8}\right)dx = \left.\frac{3x^4}{32}\right|_0^2 = \frac{3}{2}$$

$$E(x^2) = \int_0^2 x^2\left(\frac{3x^2}{8}\right)dx = \left.\frac{3x^5}{40}\right|_0^2 = \frac{12}{5}$$

$$\sigma = \sqrt{\frac{12}{5} - \left(\frac{3}{2}\right)^2} = \sqrt{\frac{3}{20}} = 0.3873$$

2. $f(x) = e^{1-x};\ x \ge 1$

$$F(x) = \int_1^x e^{1-w} dw$$

$$= \left.-e^{1-w}\right|_1^x$$

$$= 1 - e^{1-x};\ x \ge 1$$

$$P(3 < x < 5)$$

$$= F(5) - F(3)$$

$$= (1 - e^{-4}) - (1 - e^{-2})$$

$$= e^{-2} - e^{-4}$$

$$M(t) = \int_1^\infty e^{tx} e^{1-x} dx$$

$$= \int_1^\infty e\left(e^{-x(1-t)}\right) dx$$

$$= e \left.\frac{e^{-x(1-t)}}{-(1-t)}\right|_1^\infty \quad \boxed{t \le 1}$$

$$= e\left[\frac{e^{-(1-t)}}{1-t} - 0\right]$$

$$= \frac{e^t}{1-t}$$