United International **University**

School of Science and Engineering

Final Assessment Trimester: Summer-2020

Course Title: Probability and Statistics

Course Code: Stat 205 Marks: 25 Time: 1 Hour 15 Mins

There are 4 questions. Answer question no.* 1, 2 *and any* <u>one</u> *from 3 and 4.

1. a) Consider the following frequency distribution table for the following questions. **[6]**

Class	42-47	48-53	54-59	60-65	66-71
Frequency	12	9	6	15	8

Find the first four raw moments about $\boldsymbol{A = 51}$; convert them to the central moments. Also, find the co-efficient of skewness and kurtosis. Make comments about the distribution.

$$\boldsymbol{m_2 = m_2' - {m_1'}^2;\quad m_3 = m_3' - 3m_2'm_1' + 2{m_1'}^3;}$$
$$\boldsymbol{m_4 = m_4' - 4m_3'm_1' + 6m_2'{m_1'}^2 - 3{m_1'}^4}$$

b) If the correlation coefficient of two variables is $\boldsymbol{0.76}$ and regression coefficient of $\boldsymbol{x}$ on $\boldsymbol{y}$ is $\boldsymbol{2.39}$. Also, $\boldsymbol{\bar{x} = 29.7}$ and $\boldsymbol{\bar{y} = 47.3}$. **Find** and **sketch** the regression line $\boldsymbol{y}$ on $\boldsymbol{x}$. If possible from the graph predict the value of $\boldsymbol{y}$ when $\boldsymbol{x}$ is $\boldsymbol{37}$. **[4]**

2. a) Let $\boldsymbol{X_1, X_2, \ldots, X_n}$ be a random sample from the Poisson distribution with ***pmf*** $\boldsymbol{f(x;\lambda) = \frac{\lambda^x e^{-\lambda}}{x!}}$; $\boldsymbol{x = 1, 2}$, ... Find the maximum likelihood estimator $\boldsymbol{\hat{\lambda}}$ for $\boldsymbol{\lambda}$. **[4]**

b) Let $\boldsymbol{X}$ equals the weight of a soap. A random sample of size $\boldsymbol{12}$ of $\boldsymbol{X}$ yielded with weights $\boldsymbol{513}$, $\boldsymbol{493}$, $\boldsymbol{498}$, $\boldsymbol{501}$, $\boldsymbol{495}$, $\boldsymbol{480}$, $\boldsymbol{511}$, $\boldsymbol{505}$, $\boldsymbol{501}$, $\boldsymbol{493}$, $\boldsymbol{511}$ & $\boldsymbol{503}$ grams respectively. **Find** the $\boldsymbol{Me}$, $\boldsymbol{Q_1}$, $\boldsymbol{D_7}$ and $\boldsymbol{P_{43}}$. Is there any mode exists? Determine the **semi-range** of the given weights. **[5]**

3. a) In a forest there are $\boldsymbol{1200}$ animal under severe virus infection, $\boldsymbol{65}$% of the animals are rescued from the forest. If $\boldsymbol{\frac{2}{3}}$ of the rescued animals survived after the attempt, find the confidence interval of the proportion with $\boldsymbol{8}$% significance level. Is the rescue process effective? Why? **[3]**

b) Let $\boldsymbol{X}$ equal the daily sell of foods in **kg** by the super shops. Suppose the variance of $\boldsymbol{X}$ is $\boldsymbol{50}$ **kg**. To estimate the mean $\boldsymbol{\mu}$ of $\boldsymbol{X}$, an agency took a random sample of $\boldsymbol{15}$ super shops and found they sold in a total of $\boldsymbol{675}$ kg of foods in a day. Find an approximate $\boldsymbol{80}$% confidence interval for $\boldsymbol{\mu}$. **[3]**

4. A company produces mechanical tools whose average lifetime is $\mathbf{25}$ years and an average variation of $\mathbf{7}$ years. It is claimed that, in a new process the mean life time can be increased. **[6]**

(i) Design a decision rule for the process at the $\mathbf{0.04}$ significance level to test $\mathbf{35}$ tools.

(ii) If the estimated average life time for $\mathbf{29}$ samples is $\mathbf{30.5}$ years, find the **p**-value of the claim of the producer.

(iii) If the new process has increase the mean life time to $\mathbf{31.25}$ years. Find $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ for the estimated mean $\mathbf{28.75}$ years for $\mathbf{32}$ samples.

Probability table for Normal distribution

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990