Level of significance, α	0.10	0.05	0.01	0.005	0.002
Critical values of z for one-tailed tests	-1.28 *or* 1.28	-1.645 *or* 1.645	-2.33 *or* 2.33	-2.58 *or* 2.58	-2.88 *or* 2.88
Critical values of z for two-tailed tests	-1.645 *and* 1.645	-1.96 *and* 1.96	-2.58 *and* 2.58	-2.81 *and* 2.81	-3.08 *and* 3.08

Design a decision rule to test the hypothesis that a coin is fair if we take a sample of 64 tosses of the coin and use significance levels of (*a*) 0.05 and (*b*) 0.01.

SOLUTION

(*a*) **First method**

If the significance level is 0.05, each shaded area in Fig. 10-3 is 0.025 by symmetry. Then the area between 0 and z_1 is $0.5000 - 0.0250 = 0.4750$, and $z_1 = 1.96$; the critical values -1.96 and 1.96 can also be read from Table 10.1. Thus a possible decision rule is:

Accept the hypothesis that the coin is fair if z is between -1.96 and 1.96.
Reject the hypothesis otherwise.

To express this decision rule in terms of the number of heads to be obtained in 64 tosses of the coin, note that the mean and standard deviation of the distribution of heads are given by:

$$\mu = Np = 64(0.5) = 32 \qquad \text{and} \qquad \sigma = \sqrt{Npq} = \sqrt{64(0.5)(0.5)} = 4$$

under the hypothesis that the coin is fair. Then $z = (X - \mu)/\sigma = (X - 32)/4$. If $z = 1.96$, then $(X - 32)/4 = 1.96$ and $X = 39.84$; if $z = -1.96$, then $(X - 32)/4 = -1.96$ and $X = 24.16$. Thus the decision rule becomes:

Accept the hypothesis that the coin is fair if the number of heads is between 24.16 and 39.84 (i.e., between 25 and 39 inclusive).
Reject the hypothesis otherwise.

The manufacturer of a patent medicine claims that it is 90% effective in relieving an allergy for a period of 8 hours. In a sample of 200 people who had the allergy, the medicine provided relief for 160 people. Determine whether the manufacturer's claim is legitimate.

SOLUTION

Let p denote the probability of obtaining relief from the allergy by using the medicine. Then we must decide between two hypotheses:

$H_0 : p = 0.9$, and the claim is correct.
$H_1 : p < 0.9$, and the claim is false.

Since we are interested in determining whether the proportion of people relieved by the medicine is too low, we choose a one-tailed test. If the significance level is taken to be 0.01 (i.e., if the shaded area in Fig. 10-5 is 0.01), then $z_1 = -2.33$, as can be seen either from Problem 10.5(*b*) by using the symmetry of the curve or from Table 10.1. We thus take as our decision rule:

The claim is not legitimate if z is less than -2.33 (in which case we reject H_0).
Otherwise, the claim is legitimate and the observed results are due to chance (in which case we accept H_0).

If H_0 is true, then $\mu = Np = 200(0.9) = 180$ and $\sigma = \sqrt{Npq} = \sqrt{(200)(0.9)(0.1)} = 4.24$. Now 160 in standard units is $(160 - 180)/4.24 = -4.72$, which is much less than -2.33. Thus, according to our decision rule, we conclude that the claim is not legitimate and that the sample results are *highly significant* (see the end of Problem 10.5).

The breaking strengths of cables produced by a manufacturer have a mean of 1800 pounds (lb) and a standard deviation of 100 lb. By a new technique in the manufacturing process, it is claimed that the breaking strength can be increased. To test this claim, a sample of 50 cables is tested and it is found that the mean breaking strength is 1850 lb. Can we support the claim at the 0.01 significance level?

SOLUTION

We have to decide between the two hypotheses:

$H_0 : \mu = 1800$ lb, and there is really no change in breaking strength

$H_1 : \mu > 1800$ lb, and there is a change in breaking strength.

A one-tailed test should be used here; the diagram associated with this test is identical with Fig. 10-4 of Problem 10.5(*a*). At the 0.01 significance level, the decision rule is:

If the z score observed is greater than 2.33, the results are significant at the 0.01 level and H_0 is rejected. Otherwise, H_0 is accepted (or the decision is withheld).

Under the hypothesis that H_0 is true, we find that

$$z = \frac{\bar{X} - \mu}{\sigma/\sqrt{N}} = \frac{1850 - 1800}{100/\sqrt{50}} = 3.55$$

which is greater than 2.33. Hence we conclude that the results are *highly significant* and that the claim should thus be supported.

A company manufactures rope whose breaking strengths have a mean of 300 lb and a standard deviation of 24 lb. It is believed that by a newly developed process the mean breaking strength can be increased.

(*a*) Design a decision rule for rejecting the old process at the 0.01 significance level if it is agreed to test 64 ropes.

(*b*) Under the decision rule adopted in part (*a*), what is the probability of accepting the old process when in fact the new process has increased the mean breaking strength to 310 lb? Assume that the standard deviation is still 24 lb.

SOLUTION

(*a*) If μ is the mean breaking strength, we wish to decide between two hypotheses:

$H_0 : \mu = 300$ lb, and the new process is the same as the old one.

$H_1 : \mu > 300$ lb, and the new process is better than the old one.

For a one-tailed test at the 0.01 significance level, we have the following decision rule [refer to Fig. 10-8(*a*)]:

Reject H_0 if the z score of the sample mean breaking strength is greater than 2.33.
Accept H_0 otherwise.

Since
$$z = \frac{\bar{X} - \mu}{\sigma/\sqrt{N}} = \frac{\bar{X} - 300}{24/\sqrt{64}}$$

we have $\bar{X} = 300 + 3z$. Then if $z > 2.33$, we have $\bar{X} > 300 + 3(2.33) = 307.0$ lb. Thus the above decision rule becomes:

Reject H_0 if the mean breaking strength of 64 ropes exceeds 307.0 lb.
Accept H_0 otherwise.

(*b*) Consider the two hypotheses $H_0 : \mu = 300$ lb and $H_1 : \mu = 310$ lb. The distributions of mean breaking strengths corresponding to these two hypotheses are represented, respectively, by the left and right normal distributions of Fig. 10-8(*b*). The probability of accepting the old process when the new mean breaking strength is actually 310 lb is represented by the region of area β in Fig. 10-8(*b*). To find this, note that 307.0 lb in standard units is $(307.0 - 310)/3 = -1.00$, hence

$$\beta = \text{(area under right-hand normal curve to left of } z = -1.00) = 0.1587$$

This is the probability of accepting $H_0 : \mu = 300$ lb when actually $H_1 : \mu = 310$ lb is true (i.e., it is the probability of making a Type II error).