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Class ⇒ 02

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*Measures of Central Tendency:

Single value that attempts to describe a set of data by identifying the central position with that set of data.

Mean, Median and Mode are all valid measures of central tendency.

1) Mean:

Mean is the arithmatic average and it is probably the measure of central tendency.

Three classical means are

a) AM (Arithmatic Mean)

b) GM (Geometric Mean)

c) HM (Harmonic Mean)

AM for ungrouped data:

$$\frac{\Sigma x}{n}$$

x = sum of no of observation (Individual)

n = Total number

Examp: 7, 10, 4

$$\frac{\Sigma x}{n} = \frac{7+10+4}{3} = 21/3 = 7$$

For Frequency distn A.M. $= \frac{\Sigma fx}{\Sigma f}$;

f. is the freq

For Ungrouped data:

x	Frequency	fx
5	4	20
10	5	50
15	7	105
20	4	80
25	3	75
30	2	60

$$A.M. = \frac{\Sigma fx}{\Sigma f} = 15.6$$

For Grouped data:

Class Interval	Frequency(f_1)	Mid point (x_1)	$f_1 x_1$
0 - 5	4	2.5	10
5 - 10	6	7.5	45
10 - 15	10	12.5	125
15 - 20	16	17.5	280
20 - 25	12	22.5	270
25 - 30	8	27.5	220
30 - 35	4	32.5	130

$$A.M. = \frac{\Sigma f_1 x_1}{\Sigma f_1} = \frac{1080}{60} = 18$$

[Ans.]

Geometric Mean:

Average value or mean which signifies the central tendency of the set of num by taking the root of product of their values.

$$G.M. = \sqrt[n]{x_1 x_2 x_3 \cdots x_n} \quad / (x_1 x_2 \cdots x_n)^{1/n}$$

Example:

Geometric mean of 5, 7, 10

$$\sqrt[3]{5 \times 7 \times 10} = 7.04$$

Harmonic Mean:

defined as the reciprocal of the A.M. of reciprocal of data values.

$$H.M. = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}}$$

Example: H.M. of 4, 6, 8 $\Rightarrow$

$$\frac{3}{\frac{1}{4} + \frac{1}{6} + \frac{1}{8}}$$

Relationship among AM, GM and HM:

$$A.M = \frac{a+b}{2}$$

$$G.M. = \sqrt{a \times b} = (ab)^{1/2}$$

$\therefore ab = (G.M)^2$ G.M. $= \sqrt{ab}$

$(GM)^2 = ab$

$$H.M. = \frac{2}{\frac{1}{a} + \frac{1}{b}} = \frac{2ab}{a+b} = \frac{ab}{\frac{a+b}{2}}$$

$$= \frac{(GM)^2}{AM}$$

$$\therefore HM = \frac{(GM)^2}{AM}$$

Median:

General step to find median =>

a) Arrange the data in ascending order (lowest to larger)

b) Determine whether there is an even or odd num in dataset.

For example:

Ungrouped data => 1, 3, 3, $\boxed{6}$, 7, 8, 9

$n = 7$

So, $\frac{n+1}{2}$

$= \frac{7+1}{2}$

$= 4$th term

$= 6$

Median = 6

1, 2, 3, (4, 5), 6, 8, 9

$$\text{Median} = \frac{4+5}{2}$$

$$= 4.5$$

By Formula,

$n = 8$

$\frac{n}{2} = \frac{8}{2} = 4$th

$\frac{n}{2} + 1 = \frac{8}{2} + 1 = 5$th

Then, $\frac{\frac{n}{2}\text{th} + \left(\frac{n}{2}+1\right)\text{th}}{2}$

$= \frac{4+5}{2}$

$= 9/2 = 4.5$

Num of observation

(Odd)

$\frac{n+1}{2}$ th term

(Even)

median is average of two mid values

1) $\frac{n}{2}$ th term

2) $\left(\frac{n}{2}+1\right)$ th term

then, $\frac{\frac{n}{2}\text{th} + \left(\frac{n}{2}+1\right)}{2}$

Find.

~~For~~ grouped data (Find Median):

1) Find the cumulative frequency class and $n/2$.

2) Locate the class whose cf is great than $\frac{n}{2}$. That class is median class.

(nearest to).

nearmost greatest value.

Example:

class	Frequency	c.f.	Mid point
0 - 10	6	6	5
10 - 20	7	13	15
20 - 30	15	28	25
30 - 40	16	44	35
40 - 50	4	48	45
50 - 60	2	50	55
	= 50		

$N = 50 = \Sigma f_i$

Median class $= \left(\frac{N}{2}\right)$th term

$= \left(\frac{50}{2}\right)$th term

$=$ 25th term

cum. frequency of class 20 - 30 which is slightly > than 25.

$\therefore$ Median class = 20 - 30

$$l + \frac{h}{f}\left(\frac{n}{2} - cf\right)$$

Frequency

Cumulative Frequency

$$l_m + \frac{h}{f_m}\left(\frac{n}{2} - F_{(m)-1}\right)$$

$$= 20 + \frac{10}{15}\left(\frac{50}{2} - 13\right)$$

$$\approx 26.6$$

[Ans.]

l_m = Lower Limit of median class

n = Total Frequency

f_m = Frequency of median class

$F_{(m)-1}$ = c.f. of Pre-median class

h = Width of median class

Mode:

The mode is the value that appears most frequently in a data set.

Example:

Data set : 3, 4, 11, 15, 19, 19, 22, 22, 23, 23, 26

19, 22, 23 ⟶ multi - modal

When data set has two modes, we call it bimodal.

When the data set has only one mode, it is uni-modal.

Problem: For the following data,

Age (in years)	Frequency
24.5 – 29.5	3
29.5 – 34.5	9
34.5 – 39.5	15
39.5 – 44.5	12
44.5 – 49.5	7
49.5 – 54.5	4

Find Mode.

Soln: Mode

$$= l_0 + h\left(\frac{\Delta_1}{\Delta_1+\Delta_2}\right)$$

(Highest Frequency

l_0 = Lower Limit of modal class

h = size of modal class

Δ_1 = Difference in the Frequencies of modal and pre-modal classes

Δ_2 = differences in modal and post modal classes.

Here, $l_0 = 34.5$

$h = 5$

$\Delta_1 = 15-9$; $\Delta_2 = 15-12$

Then, Mode $= 34.5 + 5\left(\frac{6}{6+3}\right)$

$= 37.83$

[Ans.]

Problem: Compute Mean, Median and Mode.

Payment	Frequency
9.5 – 12.5	3
12.5 – 15.5	14
15.5 – 18.5	23
18.5 – 21.5	12
21.5 – 24.5	8
24.5 – 27.5	4
27.5 – 30.5	1

Soln:

C. I.	(f_i) Frequency	(x_i) Mid-value	c.f.	$f_i x_i$
9.5 – 12.5	3	11	3	33
12.5 – 15.5	14	14	17	196
15.5 – 18.5	23	17	40	391
18.5 – 21.5	12	20	52	240
21.5 – 24.5	8	23	60	184
24.5 – 27.5	4	26	64	104
27.5 – 30.5	1	29	65	29

$\Sigma f_i = 66$

N

Mean, $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$

$= \frac{1177}{65}$

$= 18.107$

Median, $\tilde{m} = l_m + \frac{h}{f_m}\left(\frac{n}{2} - F_{(m)-1}\right)$

$N = \frac{65}{2}$ th term

$= 32.5$ th term

Median class is $15.5 - 18.5$ which is slightly $>$ than 32.5

Now, $\tilde{m} = 15.5 + \frac{3}{23}\left(\frac{65}{2} - 17\right)$

$= 17.52$

Mode, $= l_0 + h\left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right)$

Here, $l_0 = 15.5$

$h = 3$

$\Delta_1 = 23 - 14$

$= 9$

$\Delta_2 = 23 - 12 = 11$

$\therefore$ Mode $= 16.85$

[Ans.]