



# United International University

School of Science and Engineering

Final Assessment Trimester: Fall-2020

Course Title: Probability and Statistics

Course Code: Stat 205 Marks: 25 Time: 1 Hour 15 Mins

**There are 3 questions. Answer question no. 1 and any one from 2 and 3.**

1. a) Consider the following frequency distribution table. [10]

Class	61-65	66-70	71-75	76-80
Frequency	9	13	21	7

- (i) **Sketch** the cumulative frequency polygon and find the median from it.  
 (ii) **Find** mode and mean-deviation from mode.  
 (iii) **Find** the first four raw moments about  $A = 75$ ; **convert** them to the central moments. Also, **find** the co-efficient of skewness and kurtosis. **Make comments** about the distribution.

$$m_2 = m'_2 - m'_1{}^2; \quad m_3 = m'_3 - 3m'_2m'_1 + 2m'_1{}^3;$$

$$m_4 = m'_4 - 4m'_3m'_1 + 6m'_2m'_1{}^2 - 3m'_1{}^4$$

- b) If the correlation coefficient of two variables is **0.5** and regression coefficient of  $x$  on  $y$  is **0.8**. Also,  $\bar{x} = 31.5$  and  $\bar{y} = 42.75$ . **Find** and **sketch** the regression line of  $y$  on  $x$ . **Predict** the value of  $y$  when  $x$  is **40** and **verify** it graphically. [5]

2. a) Let  $Y_1 < Y_2 < Y_3 < Y_4$  be the order statistics of four independent observations  $X_1, X_2, X_3, X_4$  each from the distribution with pdf  $f(x) = \frac{1}{4}x^3; 0 < x < 2$ . **Find**  $P\left(Y_3 \leq \frac{3}{2}\right)$  and the pdf of the order statistics. Also, **find**  $\mu_3(y)$ . [4]

$$G_r(y) = \sum_{k=r}^n \binom{n}{k} [F(y)]^k [1 - F(y)]^{n-k}$$

$$g_r(y) = \frac{n!}{(n-r)!(r-1)!} [F(y)]^{r-1} [1 - F(y)]^{n-r} f(y)$$

- b) Let  $X$  equal the daily sell of foods in **kg** by the super shops. Suppose the variance of  $X$  is **25 kg**. To estimate the mean  $\mu$  of  $X$ , an agency took a random sample of **15** super shops and found they sold in a total of **750 kg** of foods in a day. **Find** an approximate **80%** confidence interval for  $\mu$ . [2.5]

- c) The breaking strengths of cables produced by a manufacturer have a mean of **2500** pounds and standard deviation of **100** pounds. By a new technique in the manufacturing process, it is claimed that the breaking strength can be increased. To test the claim, a sample of **100** cables is tested and it is found that the mean breaking strength is **2575** pounds. Construct a hypothesis test. Can we support the claim at the **0.1** significance level? [3.5]
3. a) Consider **1009, 999, 1013, 998, 1001, 995, 1011, 1005, 993 & 1003** grams are the weights of soap in a “**1000-gram**” bottle, defined by the random variable  **$X$** . Order them increasingly and find the **mode, median,  $D_9$ , and  $P_{45}$** . [4]
- b) In a forest **100** crocodiles are infected by viral disease, **half** of the crocodiles are brought to a zoo. If **30** of the crocodiles survived in the zoo, **find** the confidence interval of the proportion with **10%** significance level. Is the transfer process effective? [2.5]
- c) **ABC** company produces machineries with the average lifetime is **30** years and the standard deviation **5** years. It is claimed that, in a new process the mean life time can be increased. [3.5]
- (i) If the estimated average life time for **40** samples is **31.5** years, find the  **$p$**  –value of the claim of the producer.
  - (ii) If the new process has increase the mean life time to **32** years. Find  **$\alpha$**  and  **$\beta$**  for the estimated mean **31.25** years for **35** samples.

