Stat-205-CT-05 (Section-A)

1. $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$; $-\infty < x < \infty$

$$I = \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

Let $z = \frac{x-\mu}{\sigma}$

$dz = \frac{dx}{\sigma}$

$x = -\infty, \infty$

$z = -\infty, \infty$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz$$

$$I^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-x^2/2} dx \int_{-\infty}^{\infty} e^{-y^2/2} dy$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\frac{x^2+y^2}{2}} dx\, dy$$

Let $r^2 = x^2 + y^2$

$r\,dr\,d\theta = dA$

$= dx\,dy$

$x, y = -\infty, \infty$

$r = 0, \infty$

$\theta = 0, 2\pi$

$$= \frac{1}{2\pi} \int_0^{2\pi} \int_0^{\infty} e^{-r^2/2}\, r\,dr\,d\theta$$

$$= \frac{1}{2\pi} \int_0^{2\pi} \left[-e^{-r^2/2}\right]_0^{\infty} d\theta$$

$$= \frac{1}{2\pi} [-0+1] \int_0^{2\pi} d\theta$$

$$= \frac{1}{2\pi} [\theta]_0^{2\pi}$$

$$= \frac{1}{2\pi} [2\pi - 0] = 1$$

So, $I = 1$, hence $f(x)$ is a pdf.

2. Here,

$$\bar{x}_1 = 150 \qquad n_1 = 17$$
$$\bar{x}_2 = 170 \qquad n_2 = 12$$
$$\bar{x}_3 = 140 \qquad n_3 = ?$$
$$\bar{\bar{x}} = 153.25$$

$$\bar{\bar{x}} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2 + n_3\bar{x}_3}{n_1 + n_2 + n_3}$$

$$\Rightarrow 153.25 = \frac{17\times150 + 12\times170 + n_3\times140}{17+12+n_3}$$

$$\Rightarrow 153.25 = \frac{4590 + 140n_3}{n_3 + 29}$$

$$\Rightarrow 153.25n_3 + 4444.25 = 4590 + 140n_3$$

$$\Rightarrow 13.25n_3 = 145.75$$

$$\Rightarrow n_3 = 11$$

3. Given

$$M_0 = 65.5$$
$$L = 60.5$$
$$c = 10$$
$$\Delta_1 = 7$$
$$\Delta_2 = f_m - 14, \quad f_m \text{ is required.}$$

$$\therefore 65.5 = 60.5 + \frac{7}{7 + f_m - 14}\times 10$$

$$\Rightarrow 0.5 = \frac{7}{f_m - 7}$$

$\Rightarrow f_m - 7 = 14$

$\Rightarrow f_m = 21$

4. Given, $\sigma = 3.6$

$CSD = 6.55\%$

$\bar{x} = ?$

$CSD = \frac{\sigma}{\bar{x}} \times 100\%$

$\Rightarrow 6.55\% = \frac{3.6 \times 100}{\bar{x}}\%$

$\Rightarrow \bar{x} = \frac{3.6 \times 100}{6.55}$

$\Rightarrow \bar{x} = 54.96$