Students with odd identity numbers are required to answer questions with odd serial numbers, while those with even identity numbers should answer questions with even serial numbers.

1. A company looks at its employee's residence and finds that, all have at least one flat, 80% have a more than one flat, 30% have a duplex flat, and 15% have more than one flat including a duplex flat. Find the probability that an employee selected at random has a flat that is not duplex.

2. A fair dice rolled twice. The event R is such that the sum of the two outcomes is 7. The event S is such that the product of the two outcomes is 12. Find the probability of R and S. Are events R and S independent? Justify your answer.

3. Two dice are rolled. Consider the events are occurred as $A = \{sum\ of\ the\ dice\ is\ 6\}$ and $B = \{both\ of\ the\ dice\ are\ odd\}$. Determine whether the events are independent or not.

4. Two dice are rolled. Consider the events are occurred as $A = \{odd\ sides\ on\ the\ first\ roll\}$ and $B = \{sum\ of\ the\ dice\ is\ odd\}$. Determine whether the events are independent or not.

5. In a bus out of 48 passengers, 34 are men and in another bus out of 56 passengers, 24 are women. Say a passenger is transferred from the first bus to the second bus. For the capacity problem, one of the passengers of the second bus has to come down later, find the probability that this passenger is a man.

6. A boy has three red coins and five white coins in his left hand, six red coins and four white coins in his right hand. If he shifts one coin at random from his left to right hand, what is the probability of his then drawing a coin of same/different color from his right hand?

7. If 3 men try work with the probability of success 0.5, 0.3, 0.6, respectively. What is the probability of the work will be done?

8. In a quality checking line, 4 experts are working with the rate of success 95%, 98%, 96%, and 92%, respectively. If the experts can work independently to find a fault, find the probability that (i) the fault can't be found (ii) one of the experts will fail to find the fault.

9. Four inspectors look at a critical component of a product. Their probabilities of detecting an error by those inspectors are different, namely, 0.98, 0.95, 0.92, 0.89 respectively. If inspections are independent, then find the probability of (i) no one detecting the error (ii) at least one detecting the error (iii) only one inspector detecting the error.

10. Consider $P(A) = 0.54$ and $P(B) = 0.48$ to find $P(A \cup B)$ and $P(A' \cup B')$ such that A and B are of the (i) independent events (ii) mutually exclusive events.

11. In a shopping mall, 3 customer executives sold 30%, 45%, and 25% of the total selling product in a financial year. The rates of efficiency of these three executives are 98%, 99%, and 96% respectively. If a fault is found find the probability that, it may occur by the second executive.

12. Rapid testing is a screening procedure to test Covid-19. The people appearing in the test, 23% of them false-positive while 17% of them false-negative. If the Covid-19 spreads among 7% people in Bangladesh, find the probability of a person who is suffering in Covid-19, when he/she tested negative in the test.

13. At an office, officials are classified and 30% of them efficient, 50% are moderate worker, and 20% are unfit for the work. Of efficient ones, 15% left the job; of the moderate workers, 20% left the job, and of unfit workers, 5% left the job. Given that an employee left the job, what is the probability that the employee is unfit one? Consider independence for the employee classes.

14. A hospital receives 40% of its flu vaccine from Company A and 60% from Company B. Each shipment contains a large number of vials of vaccine. From Company A, 3% of the vials are ineffective; from Company B, 2% are ineffective. A hospital is randomly selected vials from one shipment and finds that is ineffective. What is the conditional probability that this shipment came from Company A?

15. In a company, two managers are working with a workload ratio of $2:3$. The managers do some errors in their work with 3% and 4% of their works. An investigating team found an error in the yearly work summary of the company, which manager will be responsible for this incidence?

16. Suppose there are 5 defective items in a lot of 100 items. A sample of size 15 is taken at random without replacement. Let X denote the number of defective items in the sample. Find the probability that the sample contains (i) at most one defective item (ii) exactly three defective items.

17. Consider the *mgf* $M(t) = \frac{0.3e^t}{1-0.7e^t}$ of random variable X. How X is distributed? Find the mean and variance of X.

18. In a bet, the betting person wins \$1, \$2 and \$3 with probabilities 0.3, 0.2 and 0.1, and loses \$1 with probability 0.4 for each \$1 bet. Find $E[3X^2 - 2X + 4]$.

19. In the gambling game craps, the player wins \$1, \$2 and \$3 with probabilities 0.3, 0.2 and 0.1, and loses \$1 with probability 0.4 for each \$1 bet. What is the expected profit of the game for the player? Also, find the variance of the profit.

20. Let the random variable X be the number of days that a certain patient needs to be in the hospital. Suppose X has the *pmf* $f(x) = 0.1(5 - x); x = 1,2,3,4$. If the patient is to receive \$100 for the first day, \$50 for the second day, \$25 for the third day and have to return \$25 for the fourth day, what is the expected payment for the hospitalization? What is the standard deviation of that payment?

21. Let a random experiment be the casting of a pair of fair *five-sided* dice and let X equal the maximum of two outcomes. With reasonable assumptions, find *pmf* of X. Also, find the mean and variance of X.

22. For $f(x) = c(x + 1)^3; x = 0,1,2, \ldots \ldots \ldots 10$, determine the constant c so that $f(x)$ satisfies the conditions of being *pmf* for a random variable X, and then depict *pmf* as line graph and histogram.

23. Let the random variable X have the *pmf* $f(x) = \frac{(|x|-1)^2}{21}; x = -4, -2, 0, 2, 4$. Compute the mean, variance, $E(X^2 - 3X + 4)$ and $V(1 - 2X)$.

24. If the *mgf* of X is $M(t) = \frac{4}{10}e^t + \frac{3}{10}e^{2t} + \frac{2}{10}e^{3t} + \frac{1}{10}e^{4t}$, find the corresponding *pmf*, mean and variance.

25. Consider $E[X] = -0.05$ to find unknown values from the following table.

x	-2	-1	0	1	2
$P(X = x)$	0.2	a	0.1	0.3	b

26. Consider $E[X] = -0.05$ to find unknown values from the following table.

x	-2	-1	a	1	2
$P(X = x)$	0.2	0.25	0.1	b	0.15

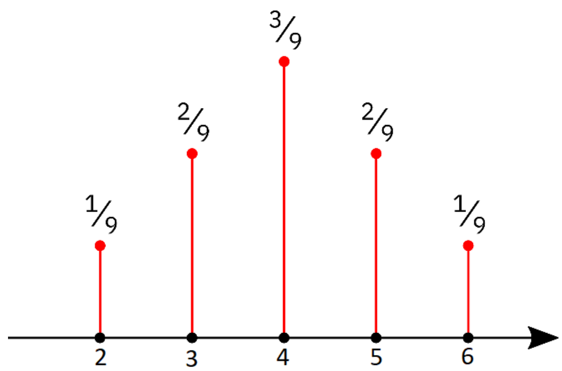
27. Consider $E[X] = -0.05$ and $E[X^2] = 1.95$ to find unknown values from the following table.

x	a	-1	0	b	2
$P(X = x)$	0.2	0.25	0.1	0.3	0.15

28. Consider $E[X] = -0.05$ and $E[X^2] = 1.95$ to find unknown values from the following table.

x	-2	a	0	1	b
$P(X = x)$	0.2	0.25	c	0.3	0.15

29. Line graph of a discrete random variable X is given in the figure below.



(a). Find the *pmf* of the probability distribution of X.
(b). Find the mean of X.
(c). By inspection find the median and mode of X.
(d). What is the *mgf* of X?

30. A boiler has five relief valves. The probability that each does not work is 0.05. Find the probability that (i) none of them work (ii) at least four of them work.

31. It is claimed that 20% of the birds in a particular region have a severe disease. Suppose that 15 birds are selected at random. Let X is the number of birds that are have the disease. Assuming independence, how X is distributed? Find $P(X \geq 2)$ and $P(X \leq 14)$.

32. It is claimed that for a particular lottery, $\frac{1}{10}$ of the 50 million tickets will win a prize. What is the probability of winning at most three prizes if you independently purchase 15 tickets?

33. A random variable X has a binomial distribution with mean 10.5 and variance 3.15. How X is distributed and find $P(X \geq 1)$.

34. Suppose that in a region the probability of arresting an innocent person is 15%. If 500 people are arrested, assuming Bernoulli experiment find the probability of arresting 35 innocent persons. Find the probability by Poisson process as well.

35. Suppose that 90% of UIU students are multi-taskers. In a random sample of 10 students are taken and let X is the number of multi-taskers. Assuming independence, how X is distributed? Find the standard deviation of X. Also, compute $P(X \geq 2)$ and $P(X > 8)$.

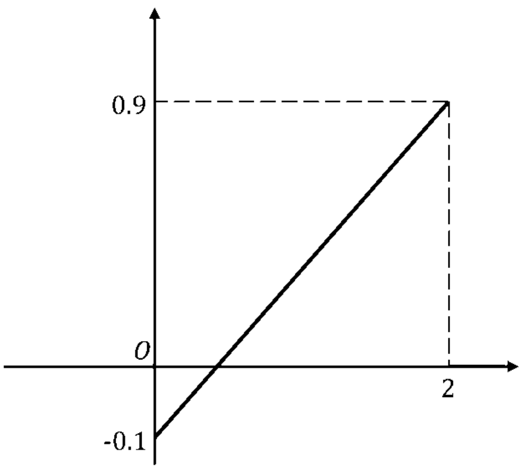
36. Verify that $M(t) = (0.4 + 0.6e^t)^{15}$ is a *mgf* of a binomial distribution and find the *pmf* of it. Evaluate the mean and variance of the binomial distribution?

37. Let X have a Poisson distribution with a standard deviation of 2. Find $P(X \geq 1)$.

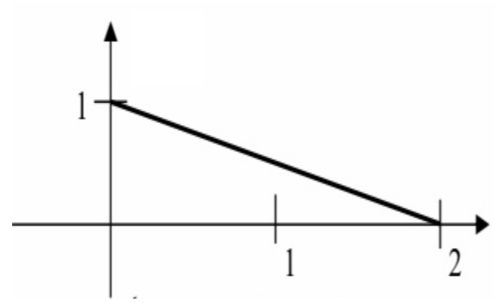
38. If X has a Poisson distribution such that $P(X = 1) = 2P(X = 2)$, evaluate $P(X = 5)$. Also, find the standard deviation of the distribution.

39. Flaws in a certain type of drapery material appear on the average of one in 120 square feet. If we assume a Poisson distribution, find the probability of no more than one flaw appearing in 60 square feet.

40. The following functions $f(x)$ are proposed as probability density functions. In each case state whether or not they could provide a suitable probability density function.



(a)



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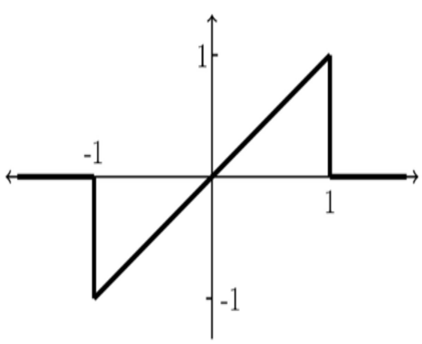
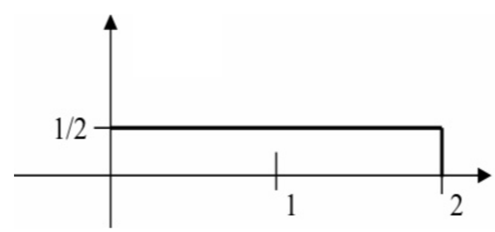


(c)



(d)

41. Identify which of the following could represent a probability density function (*pdf*). If it could not be probability density function state why, and if it could then give the value of k.

$$f(x) = \begin{cases} kx(x-2); 1 \le x \le 3 \\ 0; \quad \text{otherwise} \end{cases}$$

$$f(x) = \begin{cases} kx(4-x); 2 \le x \le 4 \\ 0; \quad \text{otherwise} \end{cases}$$

42. For $f(x) = 4x^c$; $0 \le x \le 1$, find the constant c so that $f(x)$ is a *pdf* of a random variable X. Find μ, σ^2 and cdf of X. Also, sketch the graph of *pdf* and *cdf*.

43. The time for which Lucy has to wait at a certain traffic light each day is T minutes, where T has probability density function (*pdf*) given by

$$f(t) = \begin{cases} k\left(t - \frac{t^2}{2}\right); & 0 \leq t \leq 2 \\ 0; & otherwise \end{cases}$$

(a). Find the value of the constant k.
(b). Find the expected time that Lucy has to wait and $\mathrm{Var}(T)$.
(c). Construct the cumulative density function (*cdf*) for T.
(d). Find the probability that Lucy has to wait less than 0.5 minutes.
(e). Find the mode time.
(f). Find the median time that Lucy has to wait.

44. A continuous random variable X has cumulative distribution function

$$F(x) = \begin{cases} 0; & x < -2 \\ \frac{x+2}{6}; & -2 \leq x \leq 4 \\ 1; & x > 4 \end{cases}$$

(a). Find the probability density function $f(x)$ of X.
(b). Find the mean and the variance of X and $P(X < 0)$.
(c). Write down the value of $P(X = 1)$.

45. Let the *mgf* of the random variable X satisfies uniform distribution is $M(t) = \begin{cases} \frac{e^{4t}-1}{4t}\ ; t \neq 0 \\ 1\ ; t = 0 \end{cases}$.

Find the *pdf*, mean and variance of X. Also, find $P(X > 3.5)$.

46. Let the random variable X have the *pdf* $f(x) = 2e^{1-2x}; x \geq \frac{1}{2}$, find the *cdf* and hence the 3rd decile of the distribution.

47. Assume the *pdf* of X be $f(x) = 3e^{-3x}$; $0 \leq x < \infty$. Then, (i) Estimate the *cdf* of X (ii) Calculate the mean and variance of X (iii) Find $P(X \geq 2)$.

48. Let X have the *pdf* $f(x) = 4x^3 e^{-x^4}; 0 \leq x < \infty$. Find the *cdf* and hence median of X. Also, find $P(X > -2)$.

49. Consider $f(x) = \frac{3x^2}{8}; 0 \leq x \leq 2$ as the *pdf* of X. Sketch the graphs of *pdf* and *cdf* of X.

50. Let X have the *pdf* $f(x) = e^{-x-1}; x > -1$. Find mean, median, variance, standard deviation and $P(X \geq 1)$.

51. The life X (in years) of a voltage regulator of a car has the *pdf* $f(x) = \frac{3x^2}{7^3} e^{-\left(\frac{x}{7}\right)^3}$ defined in $0 \leq x < \infty$. What is the probability that it will last at least 10.5 years? If it has lasted for 10.5 years, find the conditional probability that it will last at least 7 years more. Also, find the median (position) of X. Find the 30th percentile (in years) of voltage regulator.

52. Complains come to a police station according to a Poisson process on the average of 5 in every hour. Let X denote the waiting time in minutes until the first complain comes at a certain office hour. What is the *pdf* of X? Find $P(X \geq 10)$.

53. Customers arrive at a travel agency at a mean rate of 3 per 2 hours. Assuming that the number of arrivals per hour has a Poisson process, find the probability of waiting 2 hours for the first customers.

54. Investigators visit at a company at a mean rate of 5 per 2 hours. Assuming that the number of visits per hour has a Poisson process, find the probability of waiting at most 3 hours for the first investigator. What are the median and *mgf* of the distribution?

55. Telephone calls arrive at a physician's office according to the Poisson process on average 2 every 5 minutes. Let X denote the waiting time in minutes until 3 calls arrive. Find the *pdf* and compute $P(X > 2)$.

56. Accident occurs at a factory at a mean rate of 2 every 5 hours. Assuming that the number of accidents per hour has a Poisson process and X denotes the waiting time in hours for the accidents to occur. Find the probability that it is required to wait more than 4 hours to occur 3 accidents. What is the standard deviation of the distribution?

57. If the *mgf* of a gamma distribution of a random variable X is $M(t) = \left(1 - \frac{t}{2}\right)^{-3}$, find the *pdf*, mean and variance of X. Also, find $P(X > 4)$.

58. If the *mgf* of the normal variable X is $M(t) = e^{25t+18t^2}$, find *pdf* of X. Also, find a constant c such that $P(|X - 25| \leq c) = 0.9332$.

59. If the *mgf* of the normal variable X is $M(t) = e^{30t+18t^2}$, then (i) Find a constant k such that $P(|Z| \leq k) = 0.9544$ (ii) Evaluate $P(42.6 \leq X \leq 55.8)$. Also, find $-Z_{09656}$.

60. If X is a random variable satisfying $N(650, 625)$, find $P(631 \leq X \leq 676)$. Also, find a constant $c > 0$ such that $P(|X - 650| \leq c) = 0.6826$.

61. Consider the *mgf* of a Normal variate X is defined as $M(t) = e^{-8t+2t^2}$. Find $P(X < -5)$ and $P(X \geq -10)$. Also, find the value of c such that $P(X \leq c) = 0.725$.

62. The lengths of pine needles, in cm, are normally distributed. It is further given that 11.51% of these pine needles are shorter than 6.2 cm and 3.59% are longer than 9.5 cm. Find the mean and the standard deviation of the length of these pine needles.

63. The time, X minutes, taken by Fred Fast to install a satellite dish may be assumed to be a normal random variable with mean 134 and standard deviation 16.

    (a). Determine $P(X < 150)$.
    (b). Determine, to one decimal place, the time exceeded by 10% of installations.

64. The weights of bags of red gravel may be modeled by a normal distribution with mean 25.8 Kg and standard deviation 0.5 Kg.

    (a). Determine the probability that a randomly selected bag of red gravel will weigh:
        (i) Less than 25 Kg.
        (ii) Between 25.5 Kg and 26.5 Kg.
        (iii) More than 28 Kg.
    (b). Determine, to two decimal places, exceeded by 75% of bags.

65. The volume, L liters, of emulsion paint in a plastic tub may be assumed to be normally distributed with mean 10.25 and variance σ^2.

    (a). Assuming that $\sigma^2 = 0.04$, determine $P(L < 10)$.
    (b). Find the value of σ so that 98% of tubs contain more than 10 liters of emulsion paint.
    (c). Find the probability that within 7% of mean liter.

66. The volume of shower gel bottles, V ml, is normally distributed with a mean of 250 and a variance of 10.

    (a). Find the probability that the volume of one of these shower gel bottles picked at random will be between 249 ml and 254 ml.
    (b). Determine the value of V exceeded by 1% of the shower gel bottles.
    (c). Three shower gel bottles are picked at random. Find the probability that the volume of only one of these three bottles will be between 249 ml and 254 ml.

67. The lifetimes, in hours, of a certain make of light bulbs are assumed to be Normally distributed with a mean of 5500 hours and a standard deviation of 120.

    (a). Find the probability that the lifetime of a light bulb picked at random will exceed 5764 hours.
    (b). Determine the lifetime not achieved by 0.4% of these light bulbs.
    (c). Find the probability that two out of these thirty light bulbs will have a lifetime exceeding 5764 hours.

68. The weights of marmalade jars are normally distributed with a mean of 250 grams.
    (a). Calculate, correct to 1 decimal place, the standard deviation of these jars if 1% of the jars are heavier than 256 grams.
    (b). Using the answer of part (a), determine the probability that the weight of one such marmalade jar is between 249 and 253 grams.
    (c). Given that the weight of a randomly picked marmalade jar is between 249 and 253 grams, find the probability that the jar weighs more than 250 grams.

69. Let X equal the dirt in kg per day produced by a typical family in Dhaka city. Suppose the standard deviation of X is 2 kg. To estimate the mean μ of X, an agency took a random sample of 100 families and found they produced 0.5 metric ton of dirt every day. Find an approximate 95% confidence interval for μ.

70. Let X equal to the amount of food in pound per day consumed by a labor. Suppose the variance σ^2 of X is 0.25 . To estimate the mean μ of X, an agency took a random sample of 50 labors and found they consumed on average 4 found food per day. Find an approximate 85% confidence interval for μ.

71. In a certain motivational conference, a speaker delivered a speech to 300 people and two-third of them responded after the conference. If 75% responses were positive. Find an approximate 90% confidence interval for the fraction p of the people who motivated by the speaker.

72. In a factory there were 300 workers under flu infection, 75% of the workers were hospitalized. If half of the total workers survived after the treatment, find the confidence interval of the proportion with a 3% significance level. Is the treatment effective? Why?

73. Design a decision rule to test the hypothesis that a coin is fair if we take a sample of 250 trials of the die to test the coin as fair and use 0.99 as the confidence level. Predict the acceptance and critical region.

74. An engineer designs a novel jet engine and claimed that it will reduce the fuel cost remarkably with 90% accuracy. Now design decision rule for the process with significance 0.1 by testing 20 jet engines.

75. A company produces mosquito killing bat whose average lifetime is 360. days and average variation 60 days. It is claimed that in a newly developed process the mean lifetime can be increased. Design a decision rule for 100 samples with 0.1 significance. If the new process has increased the mean lifetime to 375 days, assuming a sample of 120 bats with estimated

lifetime 370 days, find α and β. Again, a sample of 80 bats is tested and it is found that the average lifetime is 368 days. Find the p −value of the test.

76. A company produces electric bulbs whose average life time is 180 days and average variation 10 days. It is claimed that, in a newly developed process the mean life time can be increased.

    (a). Design a decision rule for the process at the 0.05 significance to test 100 bulbs.
    (b). What about the decision if the average life time of a bulb (i) 184 days (ii) 187 days?
    (c). If the new process has increased the mean life time to 185 days. Find α and β for the estimated mean 183 days for 80 samples.
    (d). If the estimated average life time for 55 samples is 184 days, find the p −value of the claim of the manufacturer.