United International **University**

School of Science and Engineering

Final Examination Trimester: Summer-2024

Course Title: Probability and Statistics

Course Code: Math 2205/Stat 205 Marks: 50 Time: 2 Hours

Answer <u>all</u> the questions. Answer all parts of a question together.

Q1. **(a)** Two ordinary fair dices are rolled. Using a possibility diagram or otherwise, find the probability of obtaining **[4]**

(i). at least one 6.

(ii). two numbers whose product is 6.

(b) A coin is biased such that the probability that three successive tosses all result in heads in $\frac{125}{512}$. Find the probability of tail to be appeared when tossed. **[2]**

(c) In a survey, 50% of the participants own a desktop (D), 60% own a laptop (L) and 15% own both. By using Venn diagram or otherwise, find what percentage of participants owns neither a desktop nor a laptop? **[2]**

(d) The following table gives information about all the animals on a farm. Find the probability that a randomly selected animal is **[4]**

(i). male or a goat.

(ii). a sheep or female.

	Male	Female
Goats	5	25
Sheep	3	22

(e) Complete the following table for a total of 250 patients who are taking treatment. **[2]**

Treatment	Recovered	Suffered	Total
Surgery	110		155
Medication		25	
Total	180		

Q2. **(a)** A factory produces half-liters tins of oil. The volume of oil in a tin is normally distributed with a mean 506 ml and standard deviation 2.9 ml. A tin is randomly selected what is the probability that it will be **[6]**

(i). less than half a liter (500 ml) of oil.

(ii). within one standard deviation.

(b) A footballer has a 95% chance of scoring each penalty kick that she takes. Find the probability that she **[4]**

(i). scores from all of her next 10 penalty kicks.

(ii). fails to score exactly 2 of her next 10 penalty kicks.

(c) The probability distribution for random variables Y is given in the following table. **[4]**

y	0	1	2	3	4
$P\ (Y\ =\ y)$	0.03	$2p$	0.32	p	0.05

(i). Find the value of p.

(ii). Find the standard deviation of Y.

Q3. The weekly petrol consumption, in hundreds of liters of a sales representative, may be modeled by the random variable X with *pdf* $f(x) = ax(4 - 3x)$ for $0 \leq x \leq \frac{4}{3}$.

(i). Find the value of a. **[2]**

(ii). Find the mean of the weekly consumption. **[3]**

(iii). Find the probability that the weekly consumption will be less than 1 hundred liters. **[3]**

(iv). Find the mode of the weekly consumption. **[3]**

(v). Construct the corresponding *cdf* for the above *pdf*. **[3]**

Q4. **(a)** A machine is supposed to produce metal rods which are 5.7 cm long. A random sample of 100 rods are produced by the machine are measured with mean 5.71 cm and standard deviation 0.048 cm. Calculate a 95% confidence interval for the mean length of a rod produced by the machine. **[3]**

(b) An engineer designs a novel jet engine and claimed that it will reduce the fuel cost remarkably with 90% accuracy. Now design decision rule for the process with significance 0.1 by testing 20 jet engines. **[5]**

Distribution	*pmf* or *pdf*	*cdf*	*mgf*	*Mean*	*Variance*
Binomial	$f(x) = n_{C_x} p^x q^{n-x}$; $x = 0,1,2, \dots \dots, n$	$F(x) = \sum_{x=0}^{x_t} n_{C_x} p^x q^{n-x}$	$M(t) = (q + pe^t)^n$	$\mu = np$	$\sigma^2 = npq$
Normal	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$; $-\infty < x < \infty$	$F(x) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{w^2}{2}} dw$; $z = \frac{x-\mu}{\sigma}$	$M(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$	$\mu = \mu$	$\sigma^2 = \sigma^2$

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997