Final Examination

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Section → f

probability & Statistics.

Ans No. 1

(a)

$$0.03 + 2a + 0.32 + a + 0.05 = 1$$

$$3a = 1 - 0.03 - 0.32 - 0.05$$

$$a = 0.2$$

(b)

$$E(x) = \Sigma . x . P(x)$$

$$= 0.03 \times 0 + 2 \times 0.2 \times 1 + 2 \times 0.32 + 3 \times 0.2 + 4 \times 0.05$$

$$= 0 + 0.4 + 0.64 + 0.6 + 0.2$$

$$= 1.84$$

$$\text{Var}(x) = 0^2 \cdot 0.03 + 1^2 \times 2 \times 2 + 2^2 \times 0.32 + 3^2 \times 0.2 + 4^2 \times 0.05 - (1.84)^2$$

$$= 0 + 0.4 + 1.28 + 1.8 + 0.8 - (1.84)^2$$

$$= 0.8944$$

(c) $E(4x-1)$

$= 4 \cdot E(x) - 1$

$= 4 \cdot 1.84 - 1$

$= 6.36$

$Var(4x-1)$

$= 4^2 \, Var(x)$

$= 16 \times 0.8944 = 14.3104$

Ans no. 3

(a) $\int_3^6 k \cdot (6t - t^2) \, dt = 1$

$\Rightarrow k \int_3^6 (6t - t^2) \, dt = 1$

$\Rightarrow k \left[6 \cdot \frac{t^2}{2} - \frac{t^3}{3} \right]_3^6 = 1$

$\Rightarrow k \left[\left(6 \cdot \frac{36}{2} - \frac{216}{3} \right) - \left(6 \cdot \frac{9}{2} - \frac{27}{3} \right) \right] = 1$

$\Rightarrow k \cdot (36 - 18) = 1$

$\Rightarrow 18k = 1$

$\therefore k = \frac{1}{18}$ showed

(b) $E(t) = \int_3^6 \frac{1}{18} \cdot t\,(6t - t^2)\,dt$

$= \frac{1}{18} \cdot \int_3^6 (6t^2 - t^3)\,dt$

$= \frac{1}{18} \left[6 \cdot \frac{t^3}{3} - \frac{t^4}{4}\right]_3^6$

$= \frac{1}{18} \left(6 \cdot \frac{216}{3} - \frac{1296}{4}\right) - \left(6 \cdot \frac{27}{3} - \frac{81}{4}\right)$

$= \frac{1}{18} \left(108 - 33.75\right)$

$= \frac{1}{18} \times 74.25$

$= 4.125$

(c)

$P(X > 5) = \int_5^6 \frac{1}{18} (6t - t^2)\,dt$

$= \frac{1}{18} \int_5^6 (6t - t^2)\,dt$

$= \frac{1}{18} \left[6 \cdot \frac{t^2}{2} - \frac{t^3}{3}\right]_5^6$

$= \frac{1}{18} \left\{\left(6 \cdot \frac{6^2}{2} - \frac{216}{3}\right) - \left(6 \cdot \frac{25}{2} - \frac{125}{3}\right)\right\}$

$= \frac{1}{18} \left(36 - 33.33\right)$

$= \frac{2.67}{18} = 0.148333$

(d)

$$F(x) = \frac{1}{18}\int (6t - t^2)\, dt$$

$$= \frac{1}{18}\left(6\frac{t^2}{2} - \frac{t^3}{3}\right) + c$$

$$= \frac{1}{18}\left(3t^2 - \frac{t^3}{3}\right) + c$$

$t = 3 \qquad F(x) = 0$

$$\Rightarrow \frac{1}{18}\left(3\cdot 3^2 - \frac{3^3}{3}\right) + c = 0$$

$$\Rightarrow \frac{1}{18}(27 - 9) + c = 0$$

$$\Rightarrow \frac{1}{18}\cdot 18 + c = 0$$

$$c = -1$$

$$\therefore F(x) = \frac{1}{18}\left(3t^2 - \frac{t^3}{3}\right) - 1 \quad (\text{cdf})$$

Ans No. 2

$p = 0.6$

(a)

(i) $P(X=12) = {}^{20}C_{12} (0.6)^{12} (1-0.6)^{20-12}$

$= 0.1797$

(ii) $P(X=5) = {}^{20}C_5 (0.6)^5 (1-0.6)^{20-5}$

$= 0.001294$

Exactly 5 don't have any internet connection $= 1 - P(X=5)$

$= 1 - 0.001294$

$= .998706$

(b)

$n = 550$

$np = 550 \times .60$

$= 330 > 5$

$nq = 550 \times (1-.60)$

$= 220 > 5$

we have to convert it into normal distribution

$X \sim N(330, 220)$ $\mu = 330$

$\sigma^2 = npq = 330 \times (1-.60)$

$= 132$

$P(\bar{x} > 360.5)$ (by half Continuity Correction)

$= P\left(z > \frac{360.5 - 330}{22/\sqrt{132}}\right)$

$= P(z > 2.65)$

$= 1 - \phi(2.65)$

$= 1 - .99598$

$= .00402$

2 (c) $\lambda = \frac{4500 \times 4}{3600} = 5$

$\therefore P(X<2) = P(0) + P(1)$

$= \frac{e^{-5} \cdot (5)^0}{0!} + \frac{e^{-5} \cdot (5)^1}{1!}$

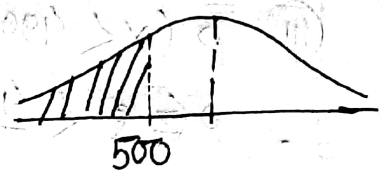
$= 0.04042 \times 6$

Ans No. 4

(a)

$X \sim N(506.18, 8.7616)$ $\quad 2.96^2 = 8.7616$

(i) $P(X < 500)$



$= P\left(Z < \frac{500 - 506.18}{2.96}\right)$

$= P(Z < -2.08)$

$= 1 - P(Z < 2.08)$

$= 1 - \Phi(2.08)$

$= 1 - .98124$

$= 0.01876$

(ii)

$$P(x > 512.24)$$

$$= P\left(Z > \frac{512.24 - 506.18}{2.96}\right)$$

$$= P(Z > 2.04)$$

$$= 1 - \Phi(2.04)$$

$$= 1 - .97932$$

$$= 0.0268$$

* Expected tin $= .0268 \times 40$

$$= 1.072$$

(iii) $P(x < 490)$

$$= P\left(Z < \frac{490 - 506.18}{2.96}\right)$$

$$= P(Z < -5.46)$$

$$= 1 - \Phi(5.46)$$

$$=$$

(4b)

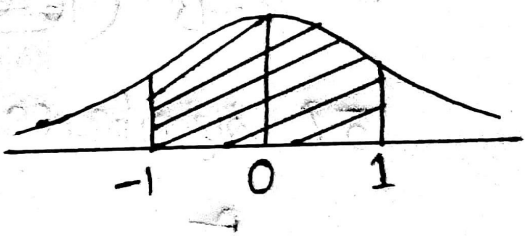
$X \sim N(506.18, 2.96^2)$

$X \sim N(\mu, \sigma^2)$

$= P(\mu - \sigma < x < \mu + \sigma)$

$= P\left(\frac{\mu - \sigma - \mu}{\sigma} < z < \frac{\mu + \sigma - \mu}{\sigma}\right)$

$= P(-1 < z < 1)$



$= \Phi(1) - \Phi(-1)$

$= \Phi(1) - [1 - \Phi(1)]$

$= 2\Phi(1) - 1$

$= (2 \times 0.8413) - 1$

$= 0.6826$

$= 68.26\%$

Ans No. 5

(a) $X \sim B(16, .5)$

(i) $H_0 : \mu = 13$

$H_1 : \mu > 13$

(ii) $X \sim B(16, 0.5)$

5% significance level

$p = .95$

$= .65$

$P(X>13) = P(14) + P(15) + P(16)$

$= {}^{16}C_{14}(.5)^{14}(.5)^2 + {}^{16}C_{15}(.5)^{15}(.5)^1 + {}^{16}C_{16}(.5)^{16}(.5)^0$

$= .00299$

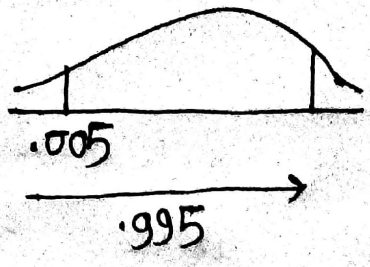
$= .0$

$P(X=13) = {}^{16}C_{13}$

5b $X \sim N(500, 30)$

$\mu = 1500$

$\sigma = 30$



~~$z = 2.58$~~

$$\bar{x} - z\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z \cdot \frac{\sigma}{\sqrt{n}}$$

$P(X > 1468)$

$= P\left(z > \frac{1468 - 1500}{\sqrt{30}}\right)$

$= P\left(z > \left(-\frac{32}{\sqrt{30}}\right)\right)$

$= P(z > 5.84)$

$= 1 - P(5.84)$