Math-2205 Class Test 03 Section C

1. In a factory, workers A and B are working with ratio of $3:2$ of the total tasks. They used to do 15% and 10% fault of their products. A concerning team found a fault in an investigation; find the probability that A will be responsible for this fault? [3]

2. Let a random experiment be the casting of a pair of fair *four-sided* dice and let X equal the minimum of two outcomes. With reasonable assumptions, find *pmf* of X. What is the corresponding *mgf*. [3]

3. Let a random variable X be the number of days that an employee needs to be in leave satisfying the *pmf* $f(x) = \frac{1}{15}(6-x) ; x = 1,2,3,4,5$. If the employee is to receive \$100 for the first two day, \$50 for the next two day, and have to return \$25 for the final day, what are the expected payment for the hospitalization and the corresponding standard deviation? [4]

1. $P(A) = 0.6$
   $P(B) = 0.4$
   $P(F/A) = 0.15$
   $P(F/B) = 0.10$

   $P(A/F) = \dfrac{0.6 \times 0.15}{0.6 \times 0.15 + 0.4 \times 0.10} = 0.69$

2. $P(x=1) = \frac{7}{16}$
   $P(x=2) = \frac{5}{16}$
   $P(x=3) = \frac{3}{16}$
   $P(x=4) = \frac{1}{16}$

   $(1,1)\ (1,2)\ (1,3)\ (1,4)$
   $(2,1)\ (2,2)\ (2,3)\ (2,4)$
   $(3,1)\ (3,2)\ (3,3)\ (3,4)$
   $(4,1)\ (4,2)\ (4,3)\ (4,4)$

   $\dfrac{f(x) - \frac{9}{16}}{\frac{1}{16} - \frac{7}{16}} = \dfrac{x-1}{4-1}$. $\Rightarrow f(x) = \dfrac{9-2x}{16} ; x = 1,2,3,4$

   $\therefore M(t) = \sum_{x=1}^{4} e^{tx}\left(\dfrac{9-2x}{16}\right)$

3. $P(x=1) = \frac{5}{15}$
   $P(x=2) = \frac{4}{15}$
   $P(x=3) = \frac{3}{15}$
   $P(x=4) = \frac{2}{15}$
   $P(x=5) = \frac{1}{15}$

   $E(x) = 100 \times \frac{5}{15} + 200 \times \frac{4}{15} + 250 \times \frac{3}{15} + 300 \times \frac{2}{15}$
   $+ 275 \times \frac{1}{15} = 195$

   $E(x^2) = 100^2 \times \frac{5}{15} + 200^2 \times \frac{4}{15} + 250^2 \times \frac{3}{15}$
   $+ 300^2 \times \frac{2}{15} + 275^2 \times \frac{1}{15} = \frac{653125}{15}$

   $\therefore$ payment $= \$195$

   $\sigma = \sqrt{\dfrac{653125}{15} - (195)^2} = \$74.27$.

Math-2205 Class Test 03 Section E

1. The glucometer is a tool to rapidly test diabetes. Of the people appearing in the test, 10% of them false-positive while 5% of them false-negative. If the people in Bangladesh 1% have diabetes, find the probability of a person suffering from diabetes, when he/she tests negative in the test. [3]

2. Let a random experiment be the casting of a pair of fair *four-sided* dice and let X equal the maximum of two outcomes. With reasonable assumptions, find *pmf* of X. What is the corresponding *mgf*. [3]

3. In a bet, the betting person wins \$1, \$2 and \$3 with probabilities 0.3, 0.2 and 0.1, and loses \$1 with probability 0.4 for each \$1 bet. Find μ, $V(x)$, $E[10 - X^2]$ and $V[10 - 2X]$. [4]

1.
$$P(T^-/D+) = 0.05$$
$$P(T^+/D+) = 0.95$$
$$P(T^+/D^-) = 0.10$$
$$P(T^-/D^-) = 0.90$$
$$P(D+) = 0.01$$
$$P(D^-) = 0.99$$

$$P(D^+/T^-) = \frac{0.01 \times 0.05}{0.01 \times 0.05 + 0.99 \times 0.90}$$
$$= 5.61 \times 10^{-4}$$

2.
$$P(x=1) = \frac{1}{16}$$
$$P(x=2) = \frac{3}{16}$$
$$P(x=3) = \frac{5}{16}$$
$$P(x=4) = \frac{7}{16}$$

(1,1) (1,2) (1,3) (1,4)
(2,1) (2,2) (2,3) (2,4)
(3,1) (3,2) (3,3) (3,4)
(4,1) (4,2) (4,3) (4,4)

$$\frac{f(x) - \frac{3}{16}}{\frac{5}{16} - \frac{3}{16}} = \frac{x-2}{3-2} \Rightarrow f(x) = \frac{2x-1}{16}; \quad x = 1, 2, 3, 4$$

$$\therefore M(k) = \sum_{x=1}^{4} e^{tx} \left(\frac{2x-1}{16} \right)$$

3.
$$E(x) = 1 \times 0.3 + 2 \times 0.2 + 3 \times 0.1 + (-1) \times 0.4 = 0.6$$
$$E(x^2) = 1^2 \times 0.3 + 2^2 \times 0.2 + 3^2 \times 0.1 + (-1)^2 \times 0.4 = 2.4$$

$$\mu = 0.6$$
$$V(x) = 2.4 - (0.6)^2 = 2.04$$
$$E(10 - x^2) = 10 - E(x^2) = 10 - 2.4 = 7.6$$
$$V(10 - 2x) = 4 V(x) = 4 \times 2.04 = 8.16$$

Math-2205 Class Test 03 Section K

1. D-Test is a screening procedure to test Dengue fever. The people appearing in the test, 7% of them false-positive while 4% of them false-negative. If the Dengue fever occurred among 2% people in Bangladesh, find the probability of a person who is suffering from Dengue fever, when he/she tested negative in the test. [3]

2. For $f(x) = \frac{x^2}{k}$; $x = 1,2,3,4,5$, determine the constant k for which $f(x)$ is the *pmf* for a random variable X, and then write the corresponding *mgf*. Also, sketch the line graph of the distribution. **[3]**

3. Line graph of a discrete random variable X is given in the figure below. Find the *pmf* of the probability distribution of X. Also, find the corresponding mean and standard deviation.[4]
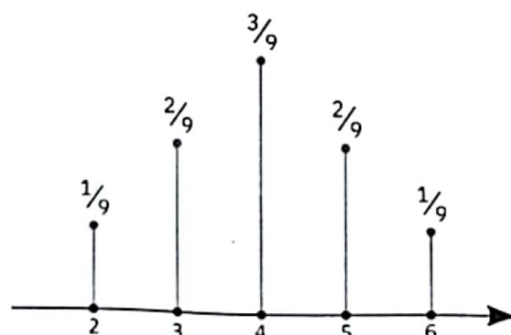
1. $P(T^+/D^-) = 0.07$
   $P(T^-/D^-) = 0.93$
   $P(T^-/D^+) = 0.04$
   $P(T^+/D^+) = 0.96$
   $P(D^+) = 0.02$
   $P(D^-) = 0.98$

   $P(D^+/T^-) = \dfrac{0.02 \times 0.04}{0.02 \times 0.04 + 0.98 \times 0.93}$
   $= 8.77 \times 10^{-4}$

2. $\sum \frac{x^2}{k} = 1 \Rightarrow \frac{1}{k}(1^2 + 2^2 + 3^2 + 4^2 + 5^2) = 1$

   $\Rightarrow k = \frac{5(5+1)(10+1)}{6} = 55$

   $\therefore f(x) = \frac{x^2}{55}$

   $P(x=1) = \frac{1}{55}$, $P(x=2) = \frac{4}{55}$, $P(x=3) = \frac{9}{55}$,
   $P(x=4) = \frac{16}{55}$, $P(x=5) = \frac{25}{55}$

3. $P(x=2) = \frac{1}{9}$
   $P(x=3) = \frac{2}{9}$
   $P(x=4) = \frac{3}{9}$
   $P(x=5) = \frac{2}{9}$
   $P(x=6) = \frac{1}{9}$

   $f(x) = \dfrac{3 - |x-4|}{9}$; $x = 2,3,4,5,6$

   $E(x) = 2 \times \frac{1}{9} + 3 \times \frac{2}{9} + 4 \times \frac{3}{9} + 5 \times \frac{2}{9} + 6 \times \frac{1}{9} = 4 = \mu$

   $E(x^2) = 4 \times \frac{1}{9} + 9 \times \frac{2}{9} + 16 \times \frac{3}{9} + 25 \times \frac{2}{9} + 36 \times \frac{1}{9} = \frac{156}{9}$

   $\sigma = \sqrt{\frac{156}{9} - 4^2} = 1.15$