# Assignment - 2

1. Let $X$ equals the weight of soap in a "500-gram" bottle. A random sample of $X$ yielded with weights $509, 499, 513, 498, 501, 495, 511, 505, 493$ & $503$ grams respectively. Order them increasingly and find the median, $Q_1$, $D_9$, and $P_{43}$. Is there any mode exists? Determine the semi-range of the given weights.

2. Let $Y_1 < Y_2 < Y_3 < Y_4 < Y_5 < Y_6$ be the order statistics of six independent observations $X_1, X_2, X_3, X_4, X_5, X_6$ each from the distribution with the *pdf* $f(x) = 3x^2$ defined in the interval $0 < x < 1$. Find $P(Y_4 \leq \frac{2}{3})$ and $P(Y_5 > \frac{3}{5})$. Also, using $g_4(y)$ and $g_5(y)$ find the corresponding means $\mu_4$ and $\mu_5$, respectively.

3. Let $Y_1 < Y_2 < Y_3 < Y_4 < Y_5$ be the order statistics of five independent observations $X_1, X_2, X_3, X_4, X_5$ each from the distribution with the *pdf* $f(x) = \frac{1}{5}e^{-\frac{x}{5}}$ defined in the interval $0 \leq x < \infty$. Find $P(Y_3 \leq 3)$ and $P(Y_2 > 2)$. Also, find the *pdf* of sample median $Y_3$.

4. Let $Y_1 < Y_2 < Y_3 < Y_4$ be the order statistics of four independent observations $X_1, X_2, X_3, X_4$ each from the distribution with the *pdf* $f(x) = \frac{1}{5}$ defined in the interval $5 \leq x \leq 10$. Find $P(Y_3 \leq 8)$ and $P(Y_4 > 9)$. Also, using $g_2(y)$ find the corresponding means $\mu_2$.

5. Let $X_1, X_2, \dots \dots \dots, X_n$ be a random sample from the distributions with the *pmf* given below. Find the maximum likelihood estimator(s) of the corresponding parameter(s).
   (a) Bernoulli distribution $f(x{:}\, p) = p^x(1-p)^{1-x}$ ; $x = 0,1$ for estimator $\hat{p}$ of $p$.
   (b) Exponential distribution $f(x{:}\, \theta) = \frac{1}{\theta}e^{-\frac{x}{\theta}}$ ; $0 \leq x < \infty$ for the estimator $\hat{\theta}$ of $\theta$.
   (c) Geometric distribution $f(x{:}\, p) = (1-p)^{x-1}p$ ; $0 < p < 1$ for the estimator $\hat{p}$ of $p$.
   (d) Normal distribution $f(x{:}\, \theta_1, \theta_2) = \frac{1}{\sqrt{2\pi\theta_2}}\, exp[-\frac{(x-\theta_1)^2}{2\theta_2}]$; $x = 1,2, \dots \dots$ for the estimators $\widehat{\theta_1}$ & $\widehat{\theta_2}$ of $\theta_1$ & $\theta_2$ respectively.
   (e) Poisson distribution $\frac{\lambda^x e^{-\lambda}}{x!}$ ; $x = 1,2, \dots \dots$ for the estimator $\hat{\lambda}$ of $\lambda$.

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6. Let $X$ equal the daily sell of foods in kg by the super shops. Suppose the variance of $X$ is $50$ kg. To estimate the mean $\mu$ of $X$, an agency took a random sample of $15$ super shops and found they sold in a total of $675$ kg of foods in a day. Find an approximate $80\%$ confidence interval for $\mu$.

7. Let $X$ equal to the amount of drinks in milliliter per day consumed by a student. Suppose the variance of $X$ is $36$. To estimate the mean $\mu$ of $X$, a survey team took a random sample of $50$ students and found they consumed $10$ litter drinks per day. Find an approximate $95\%$ confidence interval for $\mu$.

8. In a forest there are $1200$ animal under severe virus infection, $65\%$ of the animals are rescued from the forest. If $\frac{2}{3}$ of the rescued animals survived after the attempt, find the confidence interval of the proportion with $8\%$ significance level. Is the rescue process effective? Why?

9. In a certain political campaign, one candidate has a poll taken at random among $1500$ people with $65\%$ of them are voter. If the candidate secured $50\%$ of the total votes. Find an approximate $10\%$ significance interval for the fraction $p$ of the voting population who favor the candidate.

10. The breaking strengths of cables produced by a manufacturer have a mean of $2000$ pounds and standard deviation of $100$ pounds. By a new technique in the manufacturing process, it is claimed that the breaking strength can be increased. To test this claim, a sample of $200$ cables is tested and it is found that the mean breaking strength is $2050$ pounds. Construct a hypothesis test. Can we support the claim at the $0.1$ significance level?

11. A company produces electric bulbs whose average life time is 180 days and average variation 10 days. It is claimed that, in a newly developed process the mean life time can be increased.
    (a) Design a decision rule for the process at the 0.1 significance to test 100 bulbs.
    (b) What about the decision if the average life time of a bulb (i) 184 days (ii) 187 days.

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(c) If the new process has increase the mean life time to 185 days. Find $\alpha$ and $\beta$ for the estimated mean 183 days for 80 samples.

(d) If the estimated average life time for 55 samples is 184 days, find the $p$-value of the claim of the manufacturer.

12. Design a decision rule to test the hypothesis that a die is fair if we take a sample of 150 trials of the die to get even/odd faces and use 0.95 as the confidence level. Predict the acceptance and critical region.

13. Design a decision rule to test the hypothesis that a coin is fair if we take a sample of 120 trials of the die to get head/tail and use 0.01 as the significance level. Predict the acceptance and critical region.

14. A company produces an electric tool whose average life time is $260$ days and variance $169$ days. It is claimed that, in a newly developed process the mean life time can be increased. If the new process has increase the mean life time to $276$ days, assuming a sample of $80$ bulbs with estimated life time $269$ days, find $\alpha$ and $\beta$.

15. A pharmaceutical company produces a new medicine and they claimed that it will reduce the migraine pain very fast with $85\%$ accuracy. Design a decision rule for the process considering the level of significance $0.01$ by apply the medicine to $150$ people.