Stat-205-CT-04 (Section A)

1. Here, $\sigma = 2kg$

$n = 100$

$\bar{x} = \frac{1 \times 1000}{2 \times 100} = 5\ kg$

$1-\alpha = 0.95$

$\therefore \alpha = 0.05$

$\phi(z_{\alpha/2}) = \frac{2-\alpha}{2} = \frac{2-0.05}{2} = 0.975$

$\Rightarrow z_{\alpha/2} = 1.96$

Now, $\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + z_{\alpha/2}\,\sigma/\sqrt{n}$

$\Rightarrow 5 - 1.96 \times 2/\sqrt{100} \le \mu \le 5 + 1.96 \times 2/\sqrt{100}$

$\Rightarrow 5 - 0.392 \le \mu \le 5 + 0.392$

$\Rightarrow 4.608 \le \mu \le 5.392$

2. Given by $H_0 : \mu = 360$ days

$H_1 : \mu > 360$ days

For the one-tailed (right) test,

$n = 100$

$\alpha = 0.1$

$\sigma = 60$ days

so, $P(Z > z_\alpha) = \alpha$

$\Rightarrow P(Z > z_\alpha) = 0.1$

$\Rightarrow 1 - \phi(z_\alpha) = 0.1$

$\Rightarrow \phi(z_\alpha) = 0.1$

$\Rightarrow z_\alpha = 1.28$

so, H_1 may accept for

$z > z_\alpha$

$\Rightarrow \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}} > z_\alpha$

$\Rightarrow \bar{x} > \mu + z_\alpha \sigma/\sqrt{n}$

$\Rightarrow \bar{x} > 360 + 1.28 \times 60/\sqrt{100}$

$\Rightarrow \bar{x} > 367.68$

Again, $H_1 : \mu = 375$ days

$n = 120$

$\bar{x} = 370$ days

$\alpha = P(E_1)$

$= P(\bar{X} \geq 370 ; \mu = 360)$

$= P\left(\dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}} \geq \dfrac{370 - 360}{60/\sqrt{120}}\right)$

$= P(Z \geq 1.83)$

$= 1 - \phi(1.83)$

$= 1 - 0.96638$

$= 0.03362$

$\beta = P(E_2)$

$= P(\bar{x} < 370 ; \mu = 375)$

$= P\left(\frac{\bar{x} - \mu}{\sigma/\sqrt{n}} < \frac{370 - 375}{60/\sqrt{120}} \right)$

$= P(z < -0.91)$

$= 1 - \phi(0.91)$

$= 1 - 0.81859$

$= 0.18141$

$p\text{-value} = P(\bar{x} > 368 ; \mu = 360)$

$= P\left(\frac{\bar{x} - \mu}{\sigma/\sqrt{n}} > \frac{368 - 360}{60/\sqrt{80}} \right)$

$= P(z > 1.19)$

$= 1 - \phi(1.19)$

$= 1 - 0.88298$

$= 0.11702$

$\mu = 360$

$n = 80$

$\bar{x} = 368$ days