United International **University**

School of Science and Engineering

Final Examination Trimester: Fall-2023

Course Title: Probability and Statistics

Course Code: Math 2205/Stat 205 Marks: 40 Time: 2 Hours

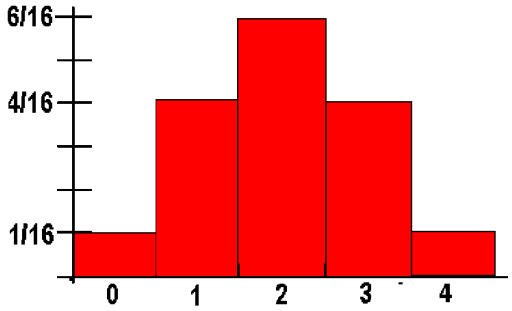
Answer <u>all</u> the questions. Answer all parts of a question together.

Q1. **(a)** A fair coin and a four-sided die are launched at a time. If $A = \{odd\ sides\ in\ the\ die\}$ and $B = \{head\ in\ the\ coin\}$, determine whether they are conditional or independent? **[3]**

(b) If 3 programmers independently trying to build an algorithm with the probability of success 0.55, 0.64, 0.48, respectively. What is the probability of the algorithm will be built? **[2]**

(c) The glucometer is a tool to rapidly test diabetes. Of the people appearing in the test, 12% of them false-positive while 9% of them false-negative. If the people in Bangladesh 5% have diabetes, find the probability of a person suffering from diabetes, when he/she tests negative in the test. **[3]**

Q2. **(a)** Histogram of a discrete random variable X is given in the figure below.



(a). Find the *pmf* of the probability distribution of X. **[1]**

(b). What is the *mgf* of X? **[1]**

(b) In a lottery, a person wins \$1, \$2 and \$3 with probabilities 0.35, 0.15 and 0.05, and loses \$1 with probability 0.45 for each \$1 bet. Find $E[1 - 3X^2]$. **[2.5]**

(c) Let a random experiment be the casting of a pair of fair *five-sided* dice and let X equal the minimum of two outcomes. With reasonable assumptions, find *pmf* of X. **[3]**

(d) Let engaging in research is independently distributed. A group of 10 students are selected at random with probability of engaging in research 0.4. Find the probability that more than 2 students engaged in research. Also, evaluate the variance of the corresponding distribution? **[3]**

(e) If X has a Poisson distribution such that $8P(X = 1) = P(X = 2)$, evaluate $P(X < 1)$. Also, find the standard deviation of the distribution. **[2.5]**

Q3. **(a)** A random variable T that is representing the time in hour for the duration daily traffic jam in the Dhaka metropolitan is distributed $f(t) = k(3t - t^2); 0 \leq t \leq 3$.

(i) Find the value of the constant k. **[1]**
(ii) Find the standard deviation of the duration of the traffic jam. **[2.5]**
(iii) Find the probability that the duration of the traffic jam more than 1 hour. **[1.5]**
(iv) Find the mode time of duration of the traffic jam. **[1]**

(b) Let $M(t) = \begin{cases} \frac{e^{4t}-1}{4t} ; t \neq 0 \\ 1 ; t = 0 \end{cases}$ is the *mgf* of the random variable X satisfies uniform distribution. Find the *cdf* and hence the median of the distribution. **[2]**

(c) The average weight of the cement bags satisfied by a normal distribution with mean 50.5 Kg and standard deviation 1.25 Kg.

(i) Determine the probability that a randomly selected bag of cement will weigh: **[2]**
(ia) Less than 49 Kg. **(ib)** More than 51 Kg.
(ii) Determine, to two decimal places, exceeded by 60% of cement bags. **[2]**

Q4. **(a)** In a certain machine-learning conference, an author delivered a talk to 400 people and half of them responded at the end of the session. If 120 responses were positive. Find an approximate 95% confidence interval for the fraction p of the people who are being encouraged by the speaker. **[3]**

(b) Design a decision rule to test the hypothesis that a six-sided die is fair (based on appearing even/odd sides in the trials) if a sample of 60 trials of the die is taken to test the die as fair with a significance of 0.01. Predict the acceptance and critical region. **[4]**

Abbreviations

pmf for *probability mass function*

pdf for *probability density function*

cdf for *cumulative distribution function*

mgf for *moment generating function*

Confidence interval for ***mean*** $\bar{X} - z\alpha_{/2}\left(\frac{\sigma}{\sqrt{n}}\right) \leq \mu \leq \bar{X} + z\alpha_{/2}\left(\frac{\sigma}{\sqrt{n}}\right)$

Confidence interval for ***proportion*** $\frac{Y}{n} - z\alpha_{/2}\sqrt{\frac{\frac{Y}{n}\left(1-\frac{Y}{n}\right)}{n}} \leq p \leq \frac{Y}{n} + z\alpha_{/2}\sqrt{\frac{\frac{Y}{n}\left(1-\frac{Y}{n}\right)}{n}}$

Distribution	***pmf* or *pdf***
Uniform (discrete)	$\boldsymbol{f(x) = \frac{1}{m}\ ;\ x = 0, 1, 2, \ldots . m}$
Hypergeometric	$\boldsymbol{f(x) = \frac{N_{1C_x} N_{2C_{n-x}}}{N_{C_n}}\ ;\quad N = N_1 + N_2,\qquad x = 1, 2, \ldots \ldots n}$
Geometric	$\boldsymbol{f(x) = q^{x-1}p\ ;\ x = 0, 1, 2, \ldots .}$
Binomial	$\boldsymbol{f(x) = n_{C_x} p^x q^{n-x}\ ;\ x = 0, 1, 2, \ldots . n}$
Poisson	$\boldsymbol{f(x) = \frac{\lambda^x e^{-\lambda}}{x!}\ ;\ x = 0, 1, 2, \ldots ..}$
Uniform (continuous)	$\boldsymbol{f(x) = \frac{1}{b-a}\ ;\ a \leq x \leq b}$
Exponential	$\boldsymbol{f(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}\ ;\ 0 \leq x < \infty}$
Gamma	$\boldsymbol{f(x) = \frac{1}{\Gamma(\alpha)\, \theta^\alpha} x^{\alpha-1} e^{-x/\theta}\ ;\ 0 \leq x < \infty}$
Normal	$\boldsymbol{f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\ ;\ -\infty < x < \infty}$

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997