



United International University

Name
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ID No.

Section

Invigilator's
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Course Code

Trimester / Semester : Spring / Summer / Fall, 20.....

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Spring $\Rightarrow 24$

Q1 (b) $3 \rightarrow$ workers (A, B, C)

$$P(A) = 0.7$$

$$P(B) = 0.8$$

$$P(C) = 0.6$$

- i) First worker : $1 - 0.7 = 0.3$
2nd worker : $1 - 0.8 = 0.2$
3rd worker : $1 - 0.6 = 0.4$

$$P(\text{task won't complete}) = 0.3 \times 0.2 \times 0.4 = 0.024$$

$$\text{ii) } P(\text{every worker will complete}) = 0.7 \times 0.8 \times 0.6 = 0.336$$

$$\text{iii) } P(\text{Only 2nd worker will complete the work})$$

$$= 0.3 \times 0.8 \times 0.4 = 0.096$$

\downarrow 1st worker fails to do.
 \downarrow 3rd worker fails to do

[Ans.]

$$c) \quad \begin{array}{l} P(A) = \frac{35}{100} \\ P(B) = \frac{40}{100} \\ P(C) = \frac{25}{100} \end{array} \quad \left| \quad \begin{array}{l} \text{Efficiency} = E \\ P(E|A) = \frac{92}{100} \\ P(E|B) = \frac{97}{100} \\ P(E|C) = \frac{95}{100} \end{array} \right.$$

$$P(B|E) = \frac{P(E|B) P(B)}{P(E|A) P(A) + P(E|B) P(B) + P(E|C) P(C)}$$

2nd programmer

= Do it yourself
(Ans)

Q2(a) hypergeometric distribution
(since sampling done without replacement)

Formula

$$\frac{{}^D C_K \cdot {}^{N-D} C_{n-K}}{{}^N C_n}$$

N = total no of item in population

$$\therefore N = 50$$

D = Total no of defective items = 5

n = sample size = 50

X = no of defective items

$$P(X \leq 2)$$

$$= P(X=0) + P(X=1) + P(X=2)$$

$$= \frac{{}^5C_0 \cdot {}^{450-5}C_{5-0}}{{}^{50}C_5} + \frac{{}^5C_1 \cdot {}^{450-4}C_4}{{}^{50}C_5} + \frac{{}^5C_2 \cdot {}^{450-3}C_3}{{}^{50}C_5}$$

= Do it yourself

(Ans)

Q(9) \Rightarrow (c) For normal distribution,

$$M_X(t) = \exp\left(\mu t + \frac{\sigma^2 t^2}{2}\right)$$

$$\text{Mean } \mu = 24$$

$$\text{Variance, } \frac{1}{2}\sigma^2 = 50 \quad \therefore \sigma = 10$$

$$\text{S.D., } \sigma = \sqrt{50} = 5\sqrt{2}$$

$$i) P(Z \geq k) = 0.025$$

$$\Rightarrow 1 - P(Z \leq k) = 0.025$$

$$\Rightarrow P(Z \leq k) = 0.975$$

$$\Rightarrow \Phi\left(Z < \frac{k-24}{\frac{5\sqrt{2}}{10}}\right) = 0.975$$

$$\Rightarrow \frac{k-24}{\frac{5\sqrt{2}}{10}} = \Phi^{-1}(0.975)$$

$$\Rightarrow \frac{k-24}{\frac{5\sqrt{2}}{10}} = -1.96$$

$$\Rightarrow k-24 = \frac{-1.96 \times 5\sqrt{2}}{10}$$

$$\therefore k = 10.15$$

(Ans.)

$$* \text{ ii) } P(X < 40.2)$$

$$= 1 - P(X > 40.2)$$

$$= 1 - P\left(Z > \frac{40.2 - 24}{10}\right)$$

$$= 1 - \Phi(1.62)$$

$$= 1 - 0.947$$

$$= 0.053$$

$$= 0.053 \text{ (Ans)}$$

$$\checkmark \text{ ii) } P(X < 40.2)$$

$$= P\left(Z < \frac{40.2 - 24}{10}\right)$$

$$= P(Z < 1.62)$$

$$= \Phi(1.62)$$

$$= 0.947$$

$$\text{(Ans)}$$

Q4)

$$\text{Null: } H_0 = \frac{26}{52} = \frac{1}{2}$$

$$\text{Alternative: } H_a \neq \frac{1}{2}$$

$$\alpha = 0.1$$

$$\text{Sample size, } n = 120$$

$$\Phi(Z_{\alpha/2}) = 1 - \alpha/2$$

$$= 1 - \frac{0.1}{2}$$

$$= 0.95$$

$$Z_{\alpha/2} = \Phi^{-1}(0.95) = 1.65 \text{ (From table)}$$

$$\mu = np = 120 \times \frac{1}{2}$$

$$\sigma = \sqrt{npq}$$

$$= \sqrt{120 \times \frac{1}{2} \times \frac{1}{2}}$$

you have to find

$$-Z_{\alpha/2} \leq Z \leq Z_{\alpha/2}$$

Do it yourself



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Q(1) a) $S_1 = \{1, 2, 3, 4\}$

$S_2 = \{H, T\}$

$P(A) = 2/4 = 1/2$

$P(B) = 1/2$

$P(A \cap B) = 2/4 = 1/2$

$P(A) \times P(B) = 1/2 \times 1/2 = 1/4$

\therefore so independent.

b) $P(\text{algorithm will not be built}) = (1 - 0.48) \times (1 - 0.55) \times (1 - 0.4) \times (1 - 0.48) = 0.0848$

$\therefore P(\text{alg will be built}) = 1 - 0.0848 = 0.9152$

(Ans.)

	1	2	3	4
H	H1	2H	3H	4H
T	T1	T2	T3	T4

82(b)

$$\begin{aligned} E(1 - 3x^2) \\ = E(1) - 3E(x^2) \\ = 1 - 3E(x^2) \end{aligned}$$

$$\begin{aligned} E(x^2) &= \sum x^2 f(x) \\ &= \{(1)^2 \times 0.35\} + \{2^2 \times 0.15\} \\ &\quad + \{3^2 \times 0.05\} + \{4^2 \times 0.45\} \\ &= 0.35 + 0.60 + 0.45 + 0.45 \\ &= ~~0.85~~ \quad 1.85 \end{aligned}$$

$$\begin{aligned} \therefore E(1 - 3x^2) &= 1 - (1.85 \times 3) = 1 - 5.55 \\ &= -4.55 \\ &\quad (\text{Ans}) \end{aligned}$$

c) Five sided dice: $X = \{1, 2, 3, 4, 5\}$

$$P(x=1) = \frac{9}{25}$$

$$P(x=2) = \frac{7}{25}$$

$$P(x=3) = \frac{5}{25}$$

$$P(x=4) = \frac{3}{25}$$

$$P(x=5) = \frac{1}{25}$$

5	(<u>5,1</u>)	(<u>5,2</u>)	(<u>5,3</u>)	(<u>5,4</u>)	(<u>5,5</u>)
4	(<u>4,1</u>)	(<u>4,2</u>)	(<u>4,3</u>)	(<u>4,4</u>)	(<u>4,5</u>)
3	(<u>3,1</u>)	(<u>3,2</u>)	(<u>3,3</u>)	(<u>3,4</u>)	(<u>3,5</u>)
2	(<u>2,1</u>)	(<u>2,2</u>)	(<u>2,3</u>)	(<u>2,4</u>)	(<u>2,5</u>)
1	(<u>1,1</u>)	(<u>1,2</u>)	(<u>1,3</u>)	(<u>1,4</u>)	(<u>1,5</u>)
	1	2	3	4	5

$$p.m.f = \frac{11-2x}{25} ; x = 1, 2, 3, 4, 5$$

$$e) P(x=1) = P(x=2)$$

$$\Rightarrow \frac{\lambda e^{-\lambda}}{1!} = \frac{\lambda^2 e^{-\lambda}}{2!}$$

$$\Rightarrow 1! = \frac{\lambda^2}{2}$$

$$\lambda = 2$$

$$P(x < 1) = P(x=0)$$

$$= \frac{\lambda^x e^{-\lambda}}{x!}$$

$$= \frac{2^0 e^{-2}}{0!}$$

$$S.D. = \sqrt{1} = \sqrt{2} \quad (\text{Ans})$$

$$9, 7, 5, 3, 1$$

$$9 + (x-1)(-2)$$

$$= 29 - 2 + 9$$

$$= 27 + 7$$

$$= 27 + 7 - 2$$

$$= 27 + 7$$

$$9 - 2x + 2$$

$$= 2x + 11$$

83(b)

$$M_x(t) = \frac{e^{tb} - e^{ta}}{(b-a)t}$$

$$= \frac{e^{4t} - e^{0t}}{4t}$$

$$\text{Hence, } b = 4$$

$$a = 0$$

C.D.F

$$= \begin{cases} 0 & ; x < 0 \\ \frac{x-0}{4} & ; 0 \leq x \leq 4 \\ 1 & ; x > 4 \end{cases}$$

$$= \begin{cases} 0 & ; x < 0 \\ \frac{x}{4} & ; 0 \leq x \leq 4 \\ 1 & ; x > 4 \end{cases}$$

$p^2 a, n-x$

$$\begin{aligned}\text{Median : } m &= \frac{a+b}{2} \\ &= \frac{0+4}{2} \\ &= 2 \quad (\text{Ans.})\end{aligned}$$

** Summer 2024

$$\text{mean} = 160 = \mu$$

$$\text{SD} = 24 = \sigma$$

$$\begin{aligned}P(150 < X < 180) &= P\left(\frac{150-160}{24} < Z < \frac{180-160}{24}\right) \\ &= P(-0.416 < Z < 0.83)\end{aligned}$$

$$\begin{aligned}&= P(Z < 0.83) - P(Z < -0.416) \\ &= P(Z < 0.83) - P(Z > 0.416) \\ &= P(Z < 0.83) - [1 - P(Z < 0.416)] \\ &= P(Z < 0.83) + P(Z < 0.416) - 1 \\ &= 0.79673 + 0.66276 - 1 \\ &= 0.45949 \quad (\text{Ans.})\end{aligned}$$



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☒ $P(A) = \checkmark$ Given
 $P(B) = \checkmark$ Given
 $P(A \cap C) = \checkmark$ Given

$$P(A \cap B) = ?$$

$$P(C) = ?$$

$$P(B \cap C) = ?$$

A, B, C
Independent

$$1) P(A \cap B) = \frac{P(A) \times P(B)}{1} = 1$$

$$2) P(B \cap C) = P(B) \times P(C)$$

$$3) P(A \cap C) = \frac{P(A) \times P(C)}{P(A)} = P(C) = 1$$

☒ For binomial distⁿ \Rightarrow
What's the prob^y of them will work \Rightarrow
$$P(X=0) = {}^n C_x p^x q^{n-x}$$

$$\Rightarrow \mu = 6$$

$$\alpha = 0.05$$

$$z > -z_{\alpha} = 1 - 0.05$$

$$z < z_{\alpha} = 0.95$$

$$\Phi(z_{\alpha}) = 0.95 = 1.65$$

From table

$$z_{\alpha} = 1.65$$

$$z > -z_{\alpha}$$

$$\Rightarrow \frac{\bar{x} - \mu}{\sigma} > -z_{\alpha}$$

$$\Rightarrow \bar{x} - \mu > -z_{\alpha} \times \sigma$$

$$\Rightarrow \bar{x} > \mu - (z_{\alpha} \times \sigma)$$

$$\bar{x} > 6 - (1.65 \times 0.5)$$

$$\Rightarrow Q_3^{(b)} > Q_2^{(a)}$$

$$H_0 = 4$$

$$H_a < 4$$

$$\mu = 6 \times 4 = 24$$

$$\sigma = \sqrt{6 \times 4 \times 3}$$

$$p + q = 1$$

$$4 + q = 1$$

$$q = -3$$

$$\left\{ \begin{array}{l} \Rightarrow x \quad 1 \quad 2 \quad 3 \quad 4 \\ P(X=x) \quad 0.2 \quad p \quad 0.5 \quad q \end{array} \right\}$$

Given $E(X) = 2.5$; ~~which~~

Find a and b

$$E(X) = 2.5$$

$$\Rightarrow \sum x P(x) = 2.5$$

$$\Rightarrow (1 \times 0.2) + 2p + (3 \times 0.5) + 4q = 2.5 \longrightarrow (1)$$

sum of \Rightarrow probabilities will be 1.

$$0.2 + p + q + 0.5 = 1 \longrightarrow (2)$$

\Downarrow

solve $\begin{cases} p = \\ q = \end{cases}$
 (1) and (2)

(Ans.)