Take Home Assignment 01

1. During a visit to a doctor, the probability of having neither lab work nor referral to a specialist is 0.19. Of those coming to the doctor, the probability of having lab work is 0.47 and the probability of having a referral is 0.51. What is the probability of having both lab work and referral?

2. A fair coin is tossed three times, and the sequence of heads and tails is observed. Let the events A, B, C be given by, $A = \{at\ most\ 2\ tails\}$, $B = \{at\ least\ 2\ heads\}$ and $C = \{1\ head\ and\ 2\ tails\}$. Find the sample space S. Also, find $P(A \cap C), P(B \cup C),\ P(C')$.

3. A certain test for identifying cancer had tested among 1 lac people, we can expect results similar to those given in the following table. If one of 1 lac people is selected randomly, find the following probabilities: $P(A_1/B_2)$, $P(A_2/B_1)$, $P(A_1 \cap B_1)$ and $P(A_2 \cup B_2)$.

	A_1 Carry cancer	A_2 Do not carry cancer	Totals
B_1 (Test positive)	4500	75500	80000
B_2 (Test negative)	500	19500	20000
Totals	5000	95000	100000

4. A shop contains 8 Nokia and 7 Samsung mobiles. Another shop contains an unknown number of Nokia and 11 Samsung mobiles. A mobile collected from each shop at random, and the probability of getting two mobiles of the same company is $\frac{149}{300}$. How many Nokia mobiles are in the second shop?

5. Each of three bowlers will attempt to hit the wicket. Let A_i denote the event that the wicket is got by player i; $i = 1,2,3$. Assume that all of the events are mutually independent and that $P(A_1) = 0.35,\ P(A_2) = 0.65$ and $P(A_3) = 0.5$. Find the probability that exactly two players are successful, and probability of no player is successful.

6. At a country fair carnival game there are 25 balloons on a board, of which 10 balloons are yellow, 8 are red, and 7 are green. A player throws darts at the balloons to win a prize and randomly hits one of them. If the first balloon hit is green, what is the probability of (i) next balloon is green (ii) next balloon is not green.

7. A boy has three red coins and five white coins in his left hand, six red coins and four white coins in his right hand. If he shifts one coin at random from his left to right hand, what is the probability of his then drawing a white coin from his left hand?

8. A boy has three red coins and five white coins in his left hand, six red coins and four white coins in his right hand. If he shifts one coin at random from his left to right hand, what is the probability of his then drawing a coin of same/different color from his right hand?

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9. Four inspectors look at a critical component of a product. Their probabilities of detecting an error by those inspectors are different, namely, 0.98, 0.95, 0.92, 0.89 respectively. If inspections are independent, then find the probability of (i) no one detecting the error (ii) at least one detecting the error (iii) only one inspector detecting the error.

10. Let $P(A) = 0.3$ and $P(B) = 0.6$. Find $P(A \cup B)$ when A and B are independent. Find $P(A|B)$ when A and B are mutually exclusive.

11. There is a new diagnostic test for a disease that occurs in about 1% of the population. The test is not perfect, but will detect a person with the disease 99% of the time. It will, however, say that a person without the disease has the disease about 3% of the time. A person is selected at random from the population, and the test indicates that this person has the disease. What are the conditional probabilities that (i) the person has the disease? (ii) the person does not have the disease?

12. A test indicates the presence of a particular disease 95% of the time when the disease is present and the presence of the disease 2% of the time when the disease is not present. If 0.2% of the population has the disease, calculate the conditional probability that a person selected at random has the disease if the test indicates the presence of the disease.

13. Die A has orange on one face and blue on five faces, Die B has orange on two faces and blue on four faces, Die C has orange on three faces and blue on three faces. If all are fair dice, picture the sample space.
    i. If the three dice are rolled, find the probability that at least two of the three dice come up orange.
    ii. If you are given, at least two of the three dice come up orange, find the probability that exactly two of the three dice come up orange.

14. Bowl B_1 contains two white chips and one red chip, bowl B_2 contains one white chip and two red chips, bowl B_3 contains three white and two red chips and bowl B_4 contains two white and three red chips. The probabilities of selecting bowl B_1, B_2, B_3 and B_4 are $\frac{3}{8}$, $\frac{1}{4}$, $\frac{1}{4}$ and $\frac{1}{8}$ respectively. A bowl is selected using these probabilities and a chip is drawn at random. Find $P(B_3/R)$, the conditional probability that bowl B_3 had been selected, given that a red chip was drawn.

15. At an office, officials are classified and 30% of them efficient, 50% are moderate worker, and 20% are unfit for the work. Of efficient ones, 15% left the job; of the moderate workers, 20% left the job, and of unfit workers, 5% left the job. Given that an employee left the job, what is the probability that the employee is unfit one? Consider independence for the employee classes.

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16. Suppose there are 5 defective items in a lot of 100 items. A sample of size 15 is taken at random without replacement. Let X denote the number of defective items in the sample. Find the probability that the sample contains (i) at most one defective item (ii) exactly three defective items.

17. If the *mgf* of X is $M(t) = \frac{4}{10}e^{t} + \frac{3}{10}e^{2t} + \frac{2}{10}e^{3t} + \frac{1}{10}e^{4t}$, find the corresponding *pmf*, mean and variance.

18. Flaws in a certain type of drapery material appear on the average of one in 120 square feet. If we assume a Poisson distribution, find the probability of no more than one flaw appearing in 60 square feet.

19. In the gambling game craps, the player wins \$1, \$2 and \$3 with probabilities 0.3, 0.2 and 0.1, and loses \$1 with probability 0.4 for each \$1 bet. What is the expected profit of the game for the player? Also, find the variance of the profit.

20. A boiler has five relief valves. The probability that each does not work is 0.05. Find the probability that (i) none of them work (ii) at least four of them work.

21. If X has a Poisson distribution such that $P(X = 1) = 2P(X = 2)$, evaluate $P(X = 5)$. Also, find the standard deviation of the distribution.

22. Let X have a Poisson distribution with a standard deviation of 2. Find $P(X \geq 1)$.

23. For $f(x) = c(x + 1)^3; x = 0,1,2, \dots \dots \dots 10$, determine the constant c so that $f(x)$ satisfies the conditions of being *pmf* for a random variable X, and then depict *pmf* as line graph and histogram.

24. Given that $E[X + c] = 10$ and $E[(X + c)^2] = 116$. Find the mean and variance of X.

25. It is claimed that 20% of the birds in a particular region have a severe disease. Suppose that 15 birds are selected at random. Let X is the number of birds that are have the disease. Assuming independence, how X is distributed? Find $P(X \geq 2)$ and $P(X \leq 14)$.

26. Let a random experiment be casting of pair of fair six-sided dice and let X equal the maximum of two outcomes. With reasonable assumptions, find the *pmf* of X and draw a probability histogram of the *pmf* of X.

27. A random variable X has a binomial distribution with mean 10.5 and variance 3.15. How X is distributed and find $P(X \geq 1)$.

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28. Let the random variable X be the number of days that a certain patient needs to be in the hospital. Suppose X has the *pmf* $f(x) = 0.1(5-x); x = 1,2,3,4$. If the patient is to receive \$100 for the first day, \$50 for the second day, \$25 for the third day and have to return \$25 for the fourth day, what is the expected payment for the hospitalization?

29. Suppose that in a region the probability of arresting an innocent person is 15%. If 500 people are arrested, assuming Bernoulli experiment find the probability of arresting 35 innocent persons. Find the probability by Poisson process as well.

30. Suppose that 90% of UIU students are multi-taskers. In a random sample of 10 students are taken and let X is the number of multi-taskers. Assuming independence, how X is distributed? Find the standard deviation of X. Also, compute $P(X \geq 2)$ and $P(X > 8)$.

31. Let the random variable X have the *pmf* $f(x) = \frac{(|x|-1)^2}{21}; x = -4, -2, 0, 2, 4$. Compute the mean, variance, $E(X^2 - 3X + 4)$ and $V(1 - 2X)$.

32. Verify that $M(t) = (0.4 + 0.6e^t)^{15}$ is a *mgf* of a binomial distribution and find the *pmf* of it. Evaluate the mean and variance of the binomial distribution?

33. Consider the *mgf* $M(t) = \frac{0.3e^t}{1-0.7e^t}$ of random variable X. How X is distributed? Find the mean and variance of X.

34. Let the *mgf* of the random variable X satisfies uniform distribution is $M(t) = \begin{cases} \frac{e^{4t}-1}{4t} ; t \neq 0 \\ 1 ; t = 0 \end{cases}$. Find the *pdf*, mean and variance of X. Also, find $P(X > 3.5)$.

35. Let the random variable X have the *pdf* $f(x) = 2e^{1-2x}; x \geq \frac{1}{2}$, find the *cdf* and hence the 3rd decile of the distribution.

36. For $f(x) = 4x^c ;\ 0 \leq x \leq 1$, find the constant c so that $f(x)$ is a *pdf* of a random variable X. Find μ, σ^2 and cdf of X. Also, sketch the graph of *pdf* and *cdf*.

37. Assume the *pdf* of X be $f(x) = 3e^{-3x} ;\ 0 \leq x < \infty$. Then, (i) Estimate the *cdf* of X (ii) Calculate the mean and variance of X (iii) Find $P(X \geq 2)$.

38. The life X (in years) of a voltage regulator of a car has the *pdf* $f(x) = \frac{3x^2}{7^3} e^{-(\frac{x}{7})^3}$ defined in $0 \leq x < \infty$. What is the probability that it will last at least 10.5 years? If it has lasted for 10.5 years, find the conditional probability that it will last at least 7 years more. Also, find the median (position) of X.

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39. Telephone calls arrive at a physician's office according to the Poisson process on average 2 every 5 minute. Let X denote the waiting time in minutes until 3 calls arrive. Find the *pdf* and compute $P(X > 2)$.

40. Accident occurs at a countryside road at a mean rate of 4 per day. Assuming that the number of accidents per hour has a Poisson process, find the probability that at most 2 customers arrive in a particular day.

41. Customers arrive at a travel agency at a mean rate of 3 per 2 hours. Assuming that the number of arrivals per hour has a Poisson process, find the probability of waiting 2 hours for the first customers.

42. If X has a gamma distribution with $\theta = 5$ and $\alpha = 2$. Find $P(X \geq 6)$. What are the mean and variance of the gamma distribution?

43. If the *mgf* of a gamma distribution of a random variable X is $M(t) = (1 - 5t)^{-3}$, find the pdf, mean and variance of X. Also, find $P(X > 4)$.

44. Complains come to a police station according to a Poisson process on the average of 5 in every hour. Let X denote the waiting time in minutes until the first complain comes at a certain office hour. What is the *pdf* of X? Find $P(X \geq 10)$. Also, find the *median* and *mgf* of X.

45. If the *mgf* of the normal variable X is $M(t) = e^{25t+18t^2}$, find *pdf* of X. Also, find a constant c such that $P(|X - 25| \leq c) = 0.9332$.

46. If the *mgf* of the normal variable X is $M(t) = e^{30t+18t^2}$, then (i) Find a constant k such that $P(|z| \leq k) = 0.9544$ (ii) Evaluate $P(42.6 \leq x \leq 55.8)$. Also, find $-Z_{09656}$.

47. If X is a random variable satisfying $N(650, 625)$, find $P(631 \leq X \leq 676)$. Also, find a constant $c > 0$ such that $P(|X - 650| \leq c) = 0.6826$.