1 Date: 30.07.2023

Class - 02

HTZ - (1.3, 1.4)

*Permutation and combination:

What exactly permutation it is?

=> Permutation is associated with arranging things from the set of objects with different order.

n = Total number

r = selected things

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

Combination → combiniting things

Basic diff betn Per and comb →

Order matters — Order doesn't matter

$$^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

Multiplication principle: If an operation can be performed in n_1 ways, and if for each of these ways, a 2nd operation can be performed in n_2 ways, → performed together, $n_1 \times n_2$ ways.

Example: A, B ; C C , A , B

Permutation ← P = 2

combination ← C = 1

Example. Say, you've four letters.

A B C D. choose two letters.

How many different ways can we arrange two of the four letters and combine?

Soln:

~~AB~~ ~~BC~~ ~~CD~~ ~~AD~~ ~~BD~~ ~~AC~~

~~BA~~ ~~CB~~ ~~DC~~ ~~DA~~ ~~DB~~ ~~CA~~

AB ↔	BA	CA	DA
AC	~~BC~~	CD	~~DB~~
AD	BD	~~CB~~	DC

P = 12

For combination, C = 6

$^{n}P_{r} = {}^{4}P_{2}$

$= \frac{4!}{(4-2)!}$

$= \frac{4\times3\times\cancel{2\times1}}{\cancel{2\times1}}$

$= 12$

$^{n}C_{r}$

$= {}^{4}C_{2}$

$= \frac{4!}{2!(4-2)!}$

$= \frac{4\times3\times\cancel{2\times1}}{2\times1\times\cancel{2\times1}}$

$= 6$

Consider the three letter a, b, c

Example: 1 In how many different ways can we arrange these 3 letters?

select

abc	bac	cab	6 permutations
acb	bca	cba	

$^3C_3$

$\frac{3!}{3!(0!)} \Rightarrow$ $1! = 1$

$^{3}P_{3} = \frac{3!}{(3-3)!} = \frac{3!}{0!} = 3\times2\times1 = 6$

Example: How many teams of 4 can be produced from pool of 12 engineers?

Soln: Order doesn't matter.

$^{12}C_{4} = \frac{12!}{(12-4)!}$

Problem: In one year, three awards (Research, teaching and service) will be given to a class of 25 graduate students in a stat dep. If each student can receive at most one award, how many possible selections are there?

<u>soln:</u> Total no. of sample points,

$$^{25}P_3 = \frac{25!}{(25-3)!} = 13,800$$

[Ans.]

Problem: How many different letter arrangements can be made from the letters in the word STATISTICS?

<u>soln:</u>

$$\frac{10!}{3!\,3!\,2!\,1!}$$

S = 3

T = 3

I = 2

C = 1

$$= 50,400$$

[Ans]

Problem: A boy asks his mother to select 5 balls from his collection of 10 red and 5 blue balls.

select 5 ball
From

10 red 5 blue

How many ways are there that his mother can $\boxed{\text{select}}$ $\underbrace{\text{3 red}}$ and $\underbrace{\text{2 blue balls}}$?

Solution:

selecting 3 red From 10,

$$^{10}C_3 = \frac{10!}{3!(10-3)!} = 120$$

selecting 2 blue From 5,

$$^{5}C_2 = \frac{5!}{2!(5-2)!} = 10$$

Multiplication Rule = $120 \times 10 = 1200$ ways

[Ans]

$$^{n}C_r = \frac{n!}{r!(n-r)!}$$

$= 1/2$ $\frac{2}{26}$

select

Problem: A bag contains 10 white, 6 red, 4 black and 7 blue. 5 balls are drawn at random. What's the probability that 2 of them are red and one is black?

Soln: Total = 10 + 6 + 7 + 5 = 27

5 balls can be drawn from 27,

$$^{27}C_5 = 80730$$

and 2 red balls can --- from 6 red,

$$= {}^{6}C_2 = 15$$

1 black balls ----- from 4 black

$$= {}^{4}C_1 = 4$$

$$\text{Prob} = \frac{\text{No of favourable cases}}{\text{Total number of cases}}$$

$$= \frac{{}^{6}C_2 \times {}^{4}C_1}{{}^{27}C_5} = \frac{6}{8073}$$

[Ans]

Sort / arrange

Problem: Find the prob. that a hand at bridge will consist of 3 spades, 5 hearts, 2 diamonds. and 3 clubs?

$(3+5+2+3) = 13$

Soln:

$$\text{Prob} = \frac{{}^{13}C_3 \times {}^{13}C_5 \times {}^{13}C_2 \times {}^{13}C_3}{{}^{52}C_{13}}$$

${}^{52}C_{13} \rightarrow$ Total

(select)

Problem: In a committee of 4 persons ~~from~~ form a group of 10 persons, [what's the probability that a particular person is on the committee?

Soln:

$$\text{Prob} = \frac{{}^{9}C_3}{{}^{10}C_4}$$

From a group of 10 persons, we've to select 4 person for a committee.

Conditional Probability:

$A, B \rightarrow$ two events

A → will occur

B → given even

$$P(A/B) = \frac{P(A\cap B)}{P(B)} \quad ✓ \quad P(B) > 0$$

(B → given)

$$P(B/A) = \frac{P(A\cap B)}{P(A)} \quad ✓ \quad P(A) > 0$$

Problems:

1) A bag contain 3 red and 4 white. Two draws are made without replacement. what's the prob that two balls are red?

Condition

given prob

soln: (i) Total = 7

~~Prob~~ P(drawing a red ball on first try)

$= \frac{3}{7} = P(A)$

P(red ball in 2nd drawn that first ball is red)

$= \frac{2}{6} = 1/3 = P(B \mid A)$

$\therefore P(A \cap B) = \frac{3}{7} \times \frac{1}{3} = 1/7$ [Ans.]

$\underbrace{P(A) \times P(B/A)}$

(ii) What's the prob that both balls are different color?

Total = 7

$P(\text{1st red}) = \frac{3}{7} = P(A)$

P(~~1st red~~ & 2nd white given 1st red)

$= \frac{4}{6} = \frac{2}{3} = P(B/A)$

$P(\text{1st white}) = \frac{4}{7} = P(C)$

$P(\text{2nd red given 1st white}) = \frac{3}{6} = \frac{1}{2}$

$= P(D/C)$

$P(A \cap B) = P(A) \cdot P(B|A) = \frac{3}{7} \times \frac{2}{3} = \frac{2}{7}$ ✓

$P(C \cap D) = P(C) \cdot P(D/C) = \frac{4}{7} \times \frac{1}{2} = \frac{2}{7}$

$\therefore$ Total Prob $= \frac{2}{7} + \frac{2}{7} = \frac{4}{7}$ (Ans)

* [3] Find Prob of drawing a queen and king from a pack of cards in two consecutive draws, the [cards drawn not being replaced.]

condition

successive order / uninterrupted

six consc. numbers

5, 6, 7, 8, 9, 10 ...

Soln: $P(\text{draw queen}) = \frac{4}{52} = P(A)$

$P(\text{draw King}) = \frac{4}{51} = P(B|A)$

$P(A \cap B) = P(A) \, P(B|A) = \frac{4}{52} \times \frac{4}{51} = \frac{4}{663}$

[Ans.]

Problem: In a box, there are 100 registor having resistance and tolerance as sh in following table.

Resistance Ω	5%	10%	Total
22Ω	10	+ 14	= 24
47Ω	28	+ 16	= 44
100Ω	24	+ 8	= 32
	= 62	= 38	= 100

Tolerance

Define 3 events.

A $\longrightarrow$ 47Ω registor

B $\longrightarrow$ 5% tolerance with registor

C $\longrightarrow$ 100Ω registor.

$P(A \backslash B)$, $P(A \backslash C)$, $P(B \backslash C)$

Soln: $P(A) = P(47\Omega) = \frac{44}{100}$ → (Favourable Outcome) [Total = 100Ω]

$P(5\%\ \text{tolerance}) = \frac{62}{100} = P(B)$

$P(100\Omega) = \frac{32}{100} = P(C)$.

$P(A \cap B) = P(47\Omega \cap 5\% \text{ tol}) = \frac{28}{100}$

$P(A \cap C) = P(47\Omega \cap 100\Omega) = 0$

$P(B \cap C) = P(5\% \cap 100\Omega) = \frac{24}{400}$

$\therefore P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{28}{100}}{\frac{62}{100}} = \frac{28}{62}$

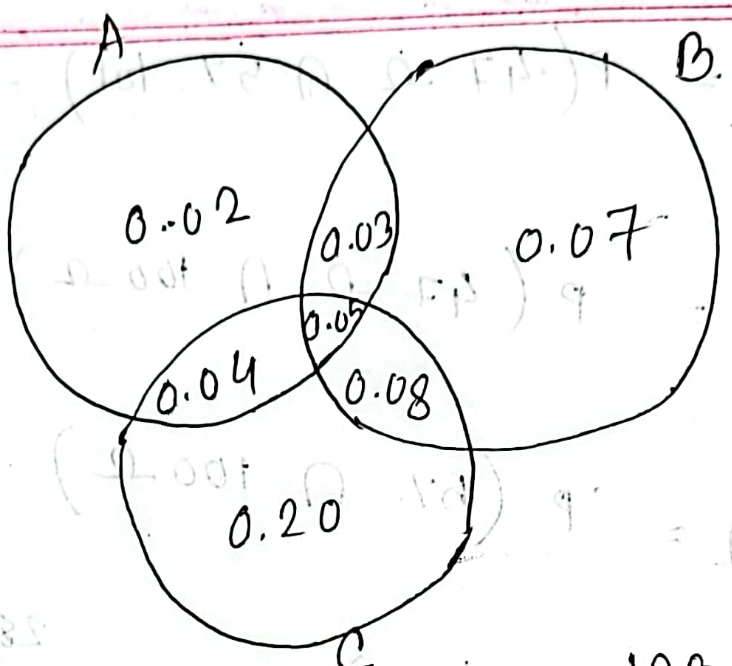
$P(A|C) = \frac{P(A \cap C)}{P(C)} = 0$

$P(B|C) = \frac{P(B \cap C)}{P(C)} = \frac{\frac{24}{100}}{\frac{32}{100}} = \frac{24}{32}$

[Ans]

Prob: The Hindu newspaper publishes three columns entitle politics (A), book (B) cinema (C). Reading habits of a randomly selected reader w.r.t three colums.

Read Reg:	A	B	C	$A \cap B$	$A \cap C$	$B \cap C$	$A \cap B \cap C$
Prob.	0.14	0.23	0.37	0.08	0.09	0.13	0.05



Find

1) $P(A|B)$

2) $P(A|B\cup C)$

3) $P(A|$ atleast one $)$

4) $P(\underbrace{A\cup B}_{A}|\underbrace{C}_{B})$

$A\cap B = \{0.03, 0.05\}$

$P(A\cap B) = 0.03 + 0.05$

$= 0.08$

Soln: 1) $P(A|B) = \dfrac{P(A\cap B)}{P(B)} = \left(\dfrac{0.08}{0.23}\right)$

$P(A|B) = \dfrac{P(A\cap B)}{P(B)}$

2) $P(A|B\cup C) = \dfrac{P(A\cap(B\cup C))}{P(B\cup C)}$

$A = \{0.02, 0.03, 0.04, 0.05\}$

$B = \{0.03, 0.07, 0.05, 0.08\}$

$C = \{0.04, 0.05, 0.08, 0.20\}$

$A\cap(B\cup C) = A\cap\{0.03, 0.04, 0.05, 0.08, 0.07, 0.20\}$

$= \{0.03, 0.04, 0.05\}$

$P(B \cup C) = 0.03 + 0.04 + 0.05 + 0.07 + 0.08 + 0.2 = 0.47$

$P(A \cap (B \cup C)) = 0.03 + 0.04 + 0.05 = 0.12$

$A(A \mid B \cup C) = \frac{12}{47}$

3) $P(A \text{ } \overbrace{\text{read at least one}}^{\text{more than one}}) = \frac{P(A \cap (\overbrace{A \cup B \cup C}^{A \text{ or } B \text{ or both}}))}{P(A \cup B \cup C)}$

$A \cap (A \cup B \cup C) = \{0.02, 0.03, 0.04, 0.05\}$

$P(A \cap (A \cup B \cup C)) = 0.02 + 0.04 + 0.05 + 0.03$
$= 0.14 = P(A)$

$P(A \cup B \cup C) = 0.02 + 0.03 + 0.04 + 0.05 + 0.07 + 0.08 + 0.20$

$= \frac{0.14}{0.49} = \frac{14}{49}$

0.17
0.20
0.37

4) $P(A \cup B / C) = \frac{P((A \cup B) \cap C)}{P(C)} = \frac{17}{37}$

$A \cup B = \{0.02, 0.03, 0.04, 0.05, 0.07, 0.08\}$

$(A \cup B) \cap C = \{0.04, 0.05, 0.08\}$

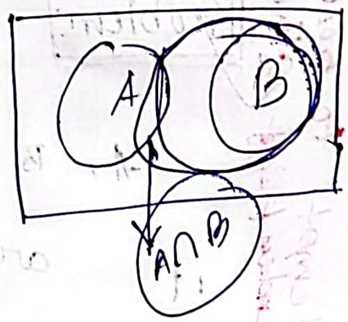
$P((A \cup B) \cap C) = 0.04 + 0.05 + 0.08 = 0.17$

$P(C) = 0.37$

(Ans)

Problems of Independent events:

$$P(A \cap B) = \underbrace{P(A) \times P(B)}_{\text{(Independent event)}}$$



*(Complement Rule)

$\boxed{\text{Problems: (1)}}$ Two person A, B appear for 2 vacacies for the same spot. The prob. A's selection is $\frac{1}{7}$ and that of B's selection is $1/5$.

What's the prob that i) both will be selected
ii) none of them will be selected.

soln:

$P(A \text{ will not be selected}) = P(A') = 1 - P(A)$
$= 1 - 1/7 = \frac{6}{7}$

$P(A) = 1/7$
$P(B) = 1/5$

$P(B \text{ not } '') = P(B') = 1 - P(B) = 1 - 1/5 = \frac{4}{5}$

i) $P(\text{both of them will be selected}) = P(A) \times P(B)$
$= 1/7 \times 1/5$

ii) $P(\text{none will be selected}) = P(A') \times P(B')$
$= \frac{6}{7} \times 4/5 = \frac{24}{35}$

[Ans]

Black (26)

144 15 = (3,1), (3,3), (5,5)
(5,1), (5,3), (5,5)

=> all three students will solve the problem? $(1/2 \times 1/3 \times 1/4) = \frac{1}{24}$.

(each student's attempt to solve the problem is independent of others.)

Problem: A prob of Math → 3 students A, B and C whose chances of solving it are 1/2, 1/3, 1/4 respectively. What's the probability that problem will be solved? (at least one student solves it)

soln: P(A will not ~~be~~ ~~selected~~ solve) $= 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$

P(B not) $= 1 - P(B) = 1 - \frac{1}{3} = \frac{2}{3}$

P(C not) $= 1 - P(C) = 1 - \frac{1}{4} = \frac{3}{4}$

P(all three will not solve) $= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$
$= \frac{1}{4}$

P(all will solve) $= 1 - \frac{1}{4} = \frac{3}{4}$ (Ans)

Problem: What's the chance of getting two sixes in two rolling of a single die?

soln:

One roll $= 1/6$

2nd " $= 1/6$

P(—) $= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$ (Ans)

Problem: A company (An article manufactured by company consists of two parts) Produce

A company

A   B

In the process of manufacture of part A, 9 out of 100 are to be defective. 5 out of 100 are to def in B. Calculate prob the assemble article will not be def.
both

Soln: $P(A \text{ def}) = \frac{9}{100}$

$P(B \text{ def}) = \frac{5}{100}$

$P(A \text{ not}) = 1 - \frac{9}{100} = \frac{91}{100}$

$P(B \text{ not}) = 1 - \frac{5}{100} = \frac{95}{100}$

$P(\text{a.a. will not def}) = \frac{91}{100} \times \frac{95}{100} = 0.86$

[Ans]

16 **Problem:** Prob. of at least one 'H' in four tosses of coin?

For a coin,

Soln: $P(H) = 1/2$ $\longrightarrow 1 - \frac{1}{2} = \frac{2-1}{2} = 1/2$

$P(\text{No } H) = P(T) = 1/2$ (Independent)

$P(\text{total } 1 H) = (1/2)^4 = 1/16$
Num of No Head

$\therefore P(\text{at least } H) = 1 - 1/16 = \frac{15}{16}$

Problem: A person is known to hit target in 3 out of 4 shots. 1)

another $\longrightarrow$ 2 out of 3 shots.

Find Prob. of $\underbrace{\text{targets being hit}}$ at al when they <u>both person try</u>.

Soln: $P(A) = 3/4$

$P(B) = 2/3$

$$P(\overset{A \cup B}{\cancel{A \cap B}}) = P(A) + P(B) - \underbrace{P(A \cap B)}^{\text{independent}}$$

$$= 3/4 + 2/3 - (3/4 \times 2/3) = 11/12$$

Not M.E.

(M.E ~~event~~ not independent events) [Ans]

Problem: From a bag $\to$ 4 white, $\to$ 6 black

two balls are drawn at random. if the balls are 1 after the other without replacement Find Prob.

1) both are white.

$$P(WW) : \frac{4}{10} \times \frac{3}{9} = 2/15$$

2) both black

$$P(BB) = \frac{6}{10} \times \frac{5}{9} = 1/3$$

3) P(1st white and 2nd black)

independent event $P(A \cap B) = P(A) \times P(B)$

$$= \frac{4}{10} \times \frac{6}{9} = 4/15$$

4) P(~~1st~~ one white and other is black)

$$= \frac{4}{10} \times \frac{6}{9} = 4/15 = P(A)$$

P (1st black and white)

$$= \frac{6}{10} \times \frac{4}{9} = 4/15 = P(B)$$

Mutually exclusive (dependent events)

$$P(A \cup B) = P(A) + P(B)$$

$$= 8/15$$

[If we get white on 1st then also get white then we won't.

if both dice are odd, their sum can be 6.

United International University

Name (Optional)

ID No.

Section

Invigilator's Signature with date

Course Code

Trimester / Semester : Spring / Summer / Fall, 20........

$6^n = 6^2 = 36$

Name of Exam : Class Test / Mid-term / Final

Date: ....

Problem: Two dice are rolled.

*// $A = \{$ sum of the dice is 6 $\}$

$B = \{$ both of the dice are odd $\}$

Determine whether the events are dependent/independent.

$A = (1,5), (2,4), (4,2), (5,1), (3,3)$

6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
	1	2	3	4	5	6

Solⁿ:

$P(A) = \frac{5}{36}$

$P(B) = \frac{9}{36} = \frac{1}{4}$

$A \cap B = 3$

$P(A \cap B) = \frac{3}{36} = \frac{1}{12}$

$P(A) \times P(B) = \frac{5}{36} \times \frac{1}{4}$

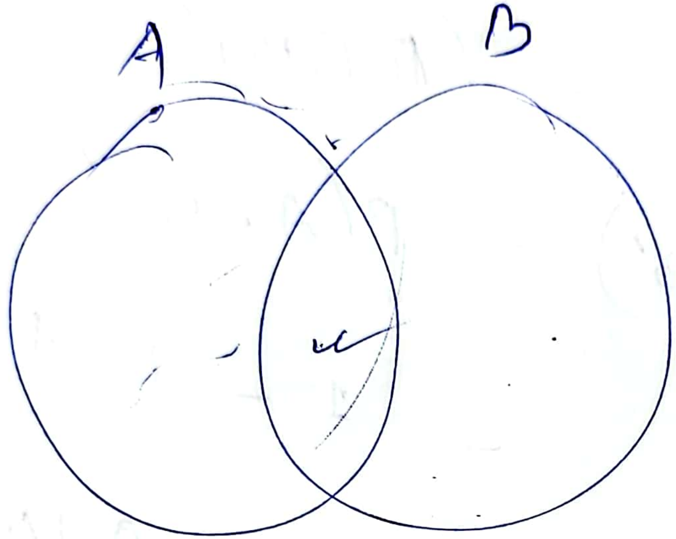
$= \frac{5}{144}$

1 row → 3

$B = (1,1), (1,3), (1,5)$
$(3,1), (3,3), (3,5)$
$(5,1), (5,3), (5,5)$

As, $P(A \cap B) \neq P(A) \times P(B)$

So, not indepenent.



(visual) Representation of A and B)

* must have a common secton.

* $P(A \cap B) = P(A) \times P(B)$

* doesn't affect the probt of one another.

1

Date: 31.07.2023 Class-03

BAYE'S Theorem:

based on conditional probabilities:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A \cap B) = P(B) \cdot P(A|B) = P(A)\, P(B|A)$$

Then, $P(B)\, P(A|B) = P(A)\, P(B|A)$

$$\therefore P(A|B) = \frac{P(B|A)\, P(A)}{P(B)}$$

$$P(A_i|B) = \frac{P(B|A_i) \times P(A_i)}{P(B|A_1)\, P(A_1) + \cdots + P(B|A_n)\, P(A_n)}$$

many events will be occured.

Problem $*_{11}$

University
(6 feet tall)

4% of men, 1% of women

Total student popn is divided in the ration 3:2 in favor of women. If a student is selected at random from among all those over 6 feet tall, what is the prob that the student is a woman?

Soln:

$$P(F) = \frac{3}{5}$$

$$P(M) = \frac{2}{5}$$

$$\underbrace{P(T|M)} = \frac{4}{100} \qquad \underbrace{P(T|F)} = \frac{1}{100}$$

Using thm,

$$\left[P(\underset{\text{A event}}{F}|\underset{\text{B event}}{T})\right. = \frac{\overbrace{P(T|F)}^{P(B|A)}\ \overbrace{P(F)}^{P(A)}}{\underbrace{P(T|M)P(M) + P(T|F)\ P(F)}_{P(B)}}$$

$$= \frac{3}{11} \text{ [Ans]}$$

Problem:

*

[A Factory is (manufacturing) bolts using 3 machines]

100

of the Total output, A is

A → 25%

B → 35%

C → Rest

From Prev experience with machines, 5% of output from A is defective, 4% from B and 2% from C. A bolt is chosen from production and found to be defective. Find Prob that it came from i) A
ii) B
iii) C

$D = \{\text{bolt is defective}\}$

Soln:

$P(A) = \frac{25}{100} = 0.25$

$P(B) = 0.35$; $P(C) = 40\% = \frac{40}{100} = 0.4$

$P(D|A) = 0.05$

$P(D|C) = 0.02$

$P(D|B) = 0.04$

Both is defective

$$P(A|D) = \frac{P(D|A)\,P(A)}{P(D)} = .11$$

$$P(B|D) = \frac{P(D|B)\,P(B)}{P(D)} = .40$$

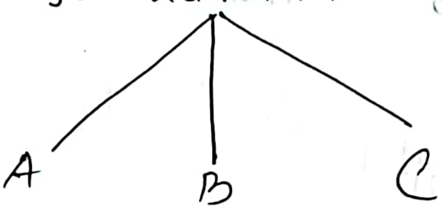
$$P(C|D) = \frac{P(D|C)\,P(C)}{P(D)} = .15$$

$$P(D) = P(D|A)\,P(A) + P(D|B)\,P(B) + P(D|C)\,P(C)$$

$= 0.0345$ [Ans]

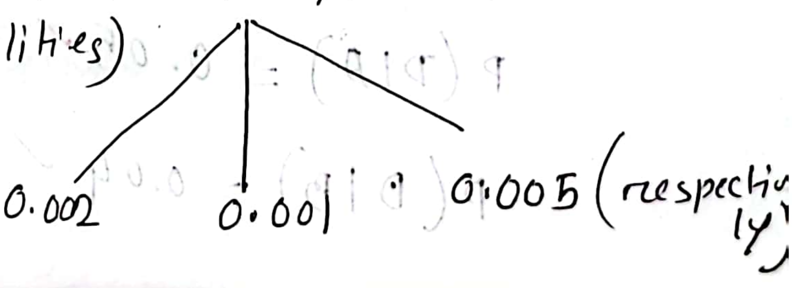
$\boxed{\text{Prob.}}$ An engineering company

* Job advertisement



These papers attract undergrad readership in proportion 2:3:1.

Reader sees and applies for job (Probabilities)



Assume under g. students see only one Job ad.

a) If comp. receives one reply to this ad, then Find Prob. that the applicant has seen the job in place i) A ii) B, iii) C.

b) If company receives two replies, what is the prob. that both applicants saw the job advertised in A?

Sol^n:

$$P(A) = \frac{2}{6} = 1/3$$

$$P(B) = 3/6 = 1/2$$

$$P(C) = 1/6$$

$P(R|A) = 0.002$; $P(R|B) = 0.001$

and $P(R|C) = 0.005$

a) $$P(A|R) = \frac{P(R|A)P(A)}{P(R)}$$

$$P(B/R) = \frac{P(R|B)P(B)}{P(R)}$$

$$P(C|R) = \frac{P(R|C)P(C)}{P(R)}$$

$P(R)$

$= P(R|A) \cdot P(A)$

$+ P(R|B) \cdot P(B)$

$+ P(R|C) \cdot P(C)$

b) P(both appli. read A)

$$= P(A|R) \times P(A|R) = 1/3 \times 1/3 = 1/9$$

[Ans]

(replies and readership independent)

Prob (1.5 - 4): See the Follow. table:

An ins. comp knows the following probabilities relating to automobiles accid. 2nd ~~out~~ column refers to prob. policyholder has at least one accident during policy period:

Age of driver	Prob of Accident	Company's insured driver
16 - 25	0.05	0.10
26 - 50	0.02	0.55
51 - 65	0.03	0.20
66 - 90	0.04	0.15

select a random driver from 2nd column what's the prob. that driver is in cond.

16–25 age?

Soln:

$$P(A_i \mid B) \qquad = P(A_1 \mid B)$$

Age (from A_i), accident (from B)

$i = 4$

B = Accident

A = Age

$$= \frac{P(A_1/B) \cdot P(B)}{P(A_1/B) \cdot P(B) + P(A_2 \mid B) \cdot P(B) + P(A_3 \mid B) \cdot P(B) + P(A_4 \mid B) \cdot P(B)}$$

$$= \frac{0.05 \times 0.10}{(0.05 \times 0.10) + (0.02 \times 0.55) + (0.03 \times 0.20) + (0.04 \times 0.15)}$$

$= 0.179$ [Ans]

Date : ..../........./...................

Random variable:

A random process => Outcomes ↓ numbers

Draw a coin / flip coin,

Define R.V. 'X' = $\begin{cases} 1 & \text{if head} \\ 0 & \text{if Tail} \end{cases}$

variable whose value is determined by any Random event / experiment.

=> Discrete R.V. [Distinct values / value which is countable]

=> Continuous R.V. [Any value in interval]

$$X = \begin{cases} 1 & ; \text{Head} \\ 0 & ; \text{Tail} \end{cases} \Rightarrow \text{Discrete R.V.}$$

Y = Mass of a random animal / ~~Height~~ / weight

It may take value within any interval.

Probability Distn:

For a random variable x of discrete type, the probab $P(X=x)$ is ~~frequently~~ defined as ~~denoted by $f(x)$~~, $\cancel{f(x)}$

Probab ; distribution.

Example: Number of 'Heads' after 3 flip of a fair coin.

Soln: X = Num of Heads

$P(X=x) = ?$

x	0	1	2	3
$P(X=x)$	$1/8$	$3/8$	$3/8$	$1/8$

⇓

Probab distribution

HHH, HHT, HTH, HTT, THH, THT, TTH, $\boxed{TTT}$

↓

$x = \{0, 1, 2, 3\}$

⇓

Outcome / (Sample Space)

Date : ..../......../........

Problem: Show that $P(X=x) = f(x) = x/6$; $x = 1, 2, 3$ defines a prob^y distribution.

Soln: $\sum f(x) = 1/6 + 2/6 + 3/6 = 1$

$f(x) \geq 0$. So, it's prob^y dist^n (Proved)

1) Probability mass function. (Pmf)

2) Probability density function. (Pdf)

3) Cumulative distri^n function (cdf)

→ for discrete r.v.

→ for cont^s r.v.

→ for both r.v.

$$\begin{array}{ccc} \text{PMF} & \text{PDF} & \text{CDF} \\ \downarrow & \downarrow & \downarrow \\ \text{Discrete R.V.} & \text{Cont}^s \text{ R.V.} & \text{Discr and co (Both R.V)} \end{array}$$

→ Frequency function.

$\boxed{\text{Pmf:}}$ For r.v. X, Pmf is defined ~~by~~ as $P(X = x)$ / as $f(x)$; this $\underbrace{f(x) = P(X = x)}$ is defined as Pmf.

Basically, Prob^y $P(X = x)$ is frequently denoted by $f(x)$.

Properties:

1) $P(X = x) = f(x) > 0$; $x \in S$ → Sample space

2) $\Sigma f(x) = 1$

3) $f(x) = 0$; $x \notin S$

$\boxed{4)}$ $P(X \in A) = \sum_{x \in A} f(x)$; $A \subset S$

Problem: A college statistics has 20 students. The ages of these students are as follows:

total

One student is 16 y.o.

Four are 18 y. ; nine are 19 y

three are 20 y

two are 21 y

one is 30 y

Let, x = age of student (random selec)

Find Pmf : Mass Function

Soln: $f(x) = P(X = x)$

x	16	18	19	20	21	30
$P(X=x)$	$1/20$	$4/20$	$9/20$	$3/20$	$2/20$	$1/20$

1) $\sum f(x) = 1$

2) $f(x) > 0$

2.1-3

Example: Roll a fair four-sided dice. Let X be the maximum of two outcomes. Find Pmf of X.

Soln: $S = \underbrace{\{1, 2, 3, 4\}}$

$\underset{\text{R.V.}}{X} = \{\text{max of two outcomes}\}$

4	(4,1)	(4,2)	(4,3)	(4,4)
3	(3,1)	(3,2)	(3,3)	(3,4)
2	(2,1)	(2,2)	(2,3)	(2,4)
1	(1,1)	(1,2)	(1,3)	(1,4)
	1	2	3	4

$P(X = x)$

x = 1 must come

$P(X = 1) = P[\underset{\text{possible outcome}}{(1,1)}] = 1/16$

$P(X = 2) = P[(1,2), (2,1), (2,2)] = \boxed{3/16}$

$P(X = 3) = P[(1,3), (3,1), (3,3), (3,2), (2,3)] = 5/16$

$P(X = 4) = P[(1,4), (4,1), (4,4), (4,3), (2,4), (3,4), (4,2)]$

$= \dfrac{7}{16}$

P.m.f, $f(x) = \dfrac{2x-1}{16}$; $x = 1, 2, 3, 4$ [Ans]

Class-04

Date: 01.08.2023

Ch ⇒ (2.1, 2.2, 2.3, 3.1)

(2.1-7)

Problem: Roll a fair four sided dice. Let X be ~~the outcome~~ of sum of two equal outcomes. Find pmf and draw histogram of pmf.

Soln:

$S = \{\underbrace{2, 3, 4, 5, 6, 7, 8}_{x}\}$; $x \in S$

(a)

$P(X=2) = \frac{1}{16} = 0.0625$

$P(X=3) = \frac{2}{16} = 0.125$

$P(X=4) = \frac{3}{16} = 0.1875$

$P(X=5) = \frac{4}{16} = 0.25$ — max

$P(X=6) = \frac{3}{16} = 0.1875$

$P(X=7) = 2/16 = 0.125$

$P(X=8) = 1/16 = 0.0625$

For $x = 2, 3, 4$

PMF, $f(x) = \frac{x-1}{16}$

For $x = 5, 6, 7, 8$

PMF, $f(x) = \frac{9-x}{16}$

CDF

$P(X \le x) = F(x)$

$P(X \le 2) = P(X=0) + P(X=2)$
$= 0 + \frac{1}{16} = 1/16$

$P(X \le 3) = \frac{1}{\ldots}$

$P(X=1) = \frac{1}{16}$ $\quad$ $P(X=4) = 7/16$

$P(X=2) = \frac{3}{16}$

$P(X=3 = 5/16$

CDF

$P(X \le 1) = \frac{1}{16}$

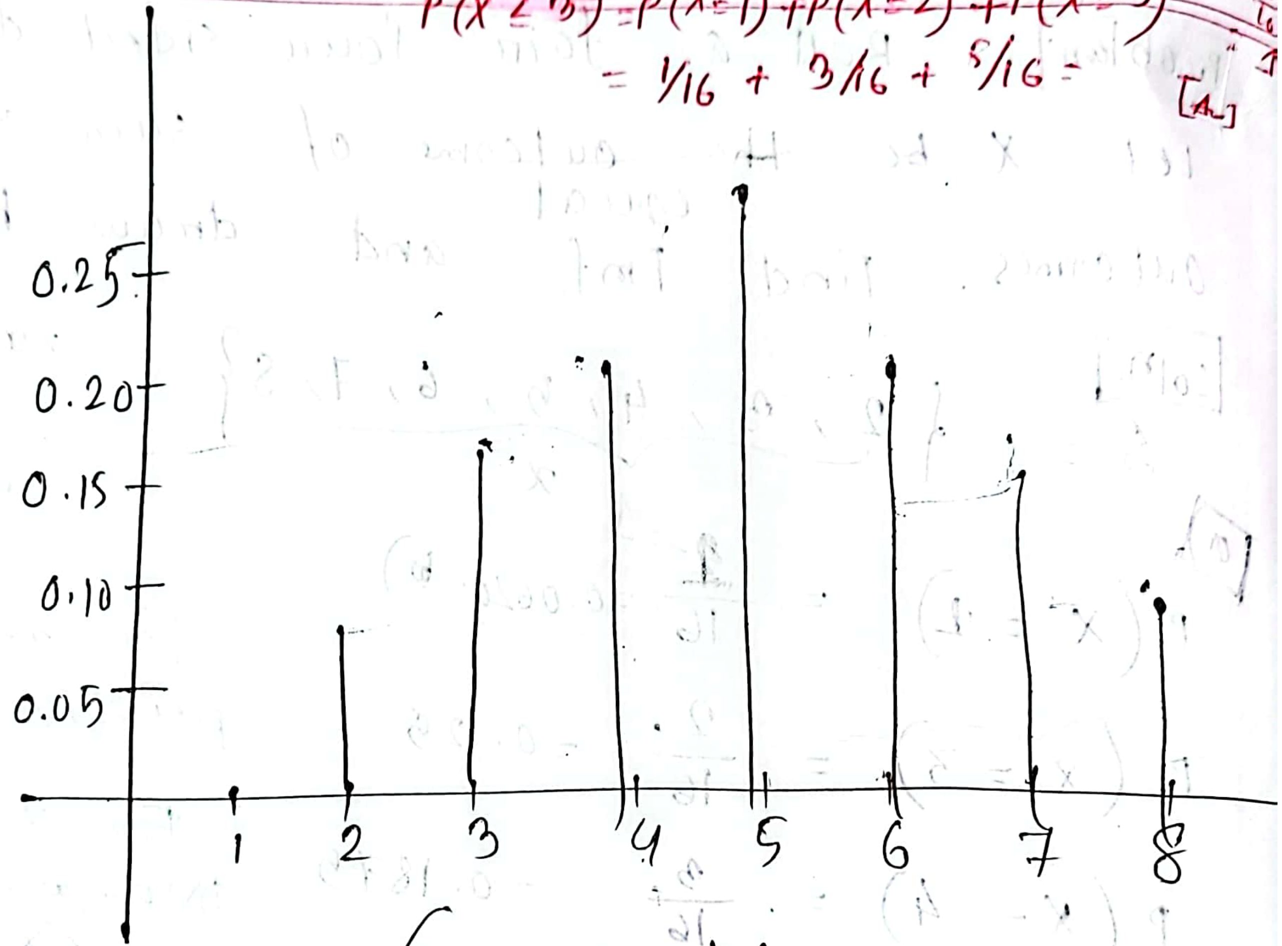
$P(X \le 2) = P(X=1) + P(X=2) = \frac{1}{16} + \frac{3}{16}$

~~$P(X \le 3) = P(X=1) + P(X=2) + P(X=3)$~~

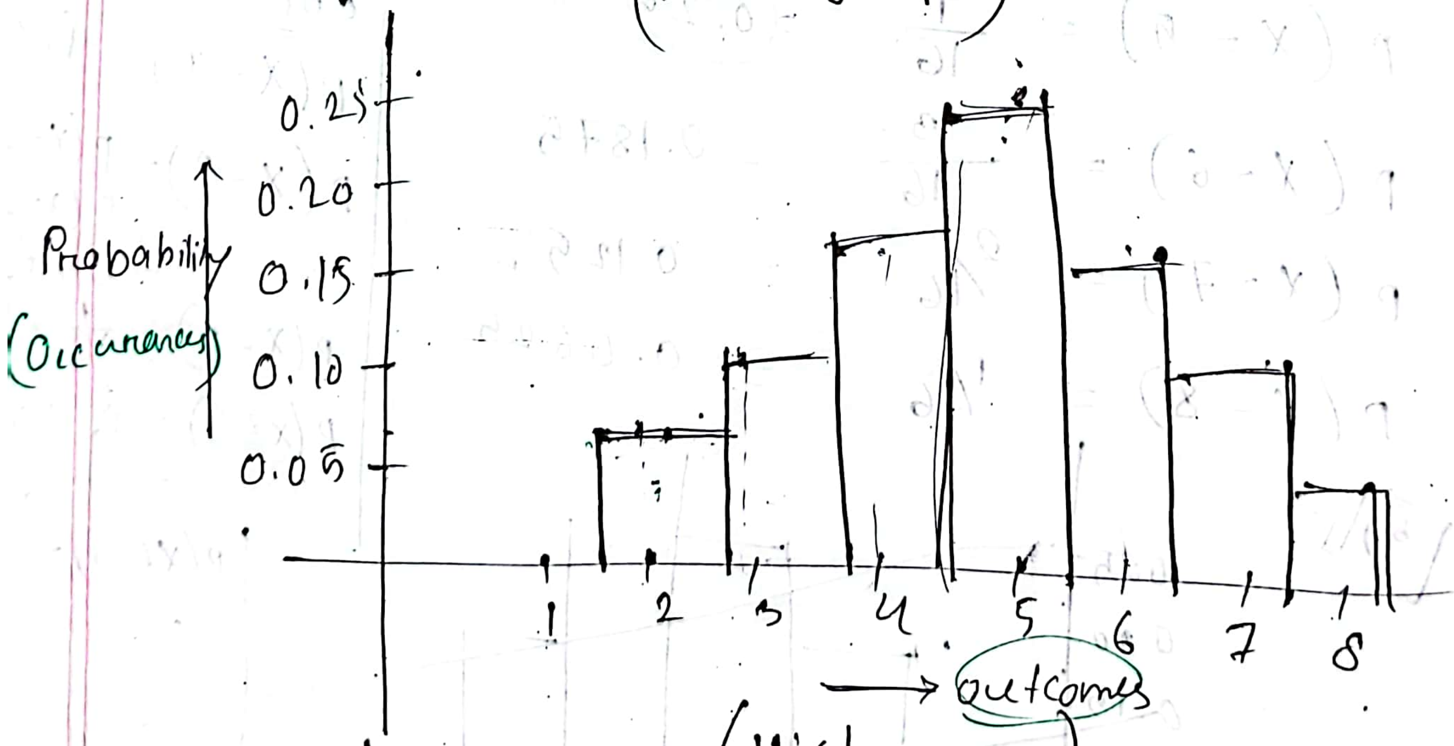
$= 1/16 + 3/16 + 5/16 =$

$P(X \le 4) =$ $1/16 + 3/16$ $5/16$ $= \frac{16}{16}$ $= 1$

[Ans]



(Line graph)



(Histogram)

Outcomes

Probability Density Function: [For Continuous Random Variable]

A Function $f(x)$ is PDF if

(i) $f(x) \geq 0$; $-\infty < x < \infty$

(ii) $\int_{-\infty}^{\infty} f(x)\, dx = 1$

It is also known as density function.

Example: If X is a cont's Random Variable with the following function

$$f(x) = \begin{cases} \alpha(2x - x^2) & ; \; 0 \leq x \leq 2 \\ & \quad x \geq 0 \; ; \; x \leq 2 \\ 0 & ; \; \text{otherwise} \end{cases}$$

Find
1) α
11) $P(X > 1)$

Soln: (i) By defn of p.d.f,

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \Rightarrow \underbrace{\int_{-\infty}^{0} f\,dx}_{} + \int_{0}^{2} f\,dx \quad \underbrace{\int_{2}^{\infty} f}_{=1}$$

$$\Rightarrow \int_{0}^{2} f(x)\,dx = 1$$

$$\Rightarrow \int_{0}^{2} \alpha(2x - x^2) = 1$$

$$\Rightarrow \alpha\left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^2 = 1$$

$\frac{12-8}{3}$

$= \frac{4}{3}$

$$\Rightarrow \alpha\left[4 - \frac{8}{3}\right] = 1$$

$$\therefore \alpha = \frac{3}{4}$$

(ii) $$P(x > 1) = \int_{1}^{\infty} f(x)\,dx$$

$$= \int_{1}^{2} f(x)\,dx$$

$$= \int_{1}^{2} \frac{3}{4}(2x - x^2)\,dx$$

$$= \frac{3}{4}\left[x^2 - \frac{x^3}{3}\right]_1^2$$

$$= \frac{3}{4}\left[4 - \frac{8}{3}\right] - \frac{3}{4}\left[1 - \frac{1}{3}\right]$$

$= 1/2$ [Ans]

Problem: A Random Variable X has density function,

$$f(x) = \begin{cases} Kx^2 & ; -3 < x < 3 \\ 0 & ; \text{otherwise} \end{cases}$$

1) Find K

2) Find $P(1 \le X \le 2)$, $P(X \le 2)$ and $P(X > 1)$

Soln: 1) By def^n of p.d.f

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow \int_{-\infty}^{-3} f\,dx + \int_{-3}^{3} f\,dx + \int_{3}^{\infty} f\,dx = 1$$

$$\Rightarrow \int_{-3}^{3} Kx^2\,dx = 1$$

$$\Rightarrow K\left[\frac{x^3}{3}\right]_{-3}^{3} = 1$$

$$\Rightarrow \frac{K}{3}\left[3^3 - (-3)^3\right] = 1$$

$$\Rightarrow K/3\,[27 + 27] = 1$$

$$= 2K\int_{0}^{3} x^2\,dx = 1$$

$$\Rightarrow 2K\left[\frac{x^3}{3}\right]_0^3 = 1$$

$$\Rightarrow \frac{2K}{\cancel{3}}\left[3^{\cancel{3}} - 0\right] = 1$$

$$\Rightarrow 2K \times 9 = 1$$

$$\therefore K = 1/18$$

[Ans]

2) $p(1 \le x \le 2)$

$$= \int_1^2 f(x)\, dx$$

$$= \int_1^2 \frac{1}{18} x^2\, dx$$

$$= \frac{1}{18} \left[\frac{x^3}{3}\right]_1^2$$

$$= \frac{1}{18} \left[\cancel{\frac{27}{3}}\ \frac{8}{3} - \frac{1}{3}\right]$$

$$= \frac{7}{54}$$

$p(x \le 2)$

$$= \int_{\infty}^{2} f(x)\, dx$$

$$= \int_{-3}^{2} f(x)\, dx$$

$$= \frac{1}{18} \left[\frac{x^3}{3}\right]_{-3}^{2}$$

$$= \frac{1}{18} \left[\frac{8}{3} + \frac{27}{3}\right]$$

$$= \frac{35}{54}$$

$$p(x > 1) = \int_1^{\infty} f(x)\, dx$$

$$= \int_1^3 \frac{1}{18} x^2\, dx$$

$$= \frac{1}{18} \left[\frac{x^3}{3}\right]_1^3$$

$$= \frac{1}{18} \left[\frac{27}{3} - \frac{1}{3}\right]$$

$$= \frac{26}{54}$$ [Ans]