Assignment 02

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Section A

Problem-01

Here given;

$z = 3x_1 + 5x_2$

Subject to,

$x_1 \leq 4$

$2x_2 \leq 12$

$3x_1 + 2x_2 \leq 18$

And

$x_1 \geq 0;\ x_2 \geq 0$

Turning the inequalities to equalities;

$z - 3x_1 + 5x_2 = 0$ (0)

$x_1 + x_3 = 4$ (1)

$2x_2 + x_4 = 12$ (2)

$3x_1 + 2x_2 + x_5 = 18$ (3)

Here,

$x_1 \leq 4$

$\Rightarrow x_1 < 4$ or $x_1 = 4$

$\Rightarrow x_1 + x_3 = 4$

then, $2x_2 \leq 12$

$\Rightarrow 2x_2 < 12$ or $2x_2 = 12$

$\Rightarrow 2x_2 + x_4 = 12$

And, $3x_1 + 2x_2 \leq 18$

$\Rightarrow 3x_1 + 2x_2 < 18$

or

$3x_1 + 2x_2 = 18$

$\Rightarrow 3x_1 + 2x_2 + x_5 = 18.$

P.T.O

Here, we find 5 different variable and 3 equation, so the degree of freedom is (5-3) or 2.

Let, the non basic variables x_1 and x_2 equal to zero. $x_1 = 0$, $x_2 = 0$.

<u>calculation</u>

$$
\begin{array}{rrrrrrr}
 & -3 & -5 & 0 & 0 & 0 & 0 \\
(+) & 0 & 5 & 0 & 5/2 & 0 & 30 \\
\hline
 & -3 & 0 & 0 & 5/2 & 0 & 30
\end{array}
$$

$$
\begin{array}{rrrrrrr}
 & 3 & 2 & 0 & 0 & 1 & 18 \\
(+) & 0 & -2 & 0 & -1 & 0 & -12 \\
\hline
 & 3 & 0 & 0 & -1 & 1 & 6
\end{array}
$$

$$
\begin{array}{rrrrrrr}
 & 0 & 1 & 0 & 1 & 0 & 0 & 4 \\
(+) & 0 & -1 & 0 & 0 & 1/3 & -1/3 & -2 \\
\hline
 & 0 & 0 & 0 & 1 & 1/3 & -1/3 & 2
\end{array}
$$

$$
\begin{array}{rrrrrrr}
 & -3 & 0 & 0 & 5/2 & 0 & 30 \\
(+) & 3 & 0 & 0 & -1 & 1 & 6 \\
\hline
 & 0 & 0 & 0 & 3/2 & 1 & 36
\end{array}
$$

Basic variable	Eq.	Coefficient of: Z	x_1	x_2	x_3	x_4	x_5	Right side	Ratio
⓪ Z	(0)	1	-3	-5	0	0	0	0	
x_3	(1)	0	1	0	1	0	0	4	$\rightarrow 4/0 = \infty$
x_4	(2)	0	0	2	0	1	0	12	$\rightarrow 12/2 = 6$ (+min)
x_5	(3)	0	3	2	0	0	1	18	$\rightarrow 18/2 = 9$
① Z	(0)	1	-3	0	0	5/2	0	30	
x_3	(1)	0	1	0	1	0	0	4	$\rightarrow 4/1 = 4$
x_2	(2)	0	0	1	0	1/2	0	6	$\rightarrow 6/0 = \infty$
x_5	(3)	0	3	0	0	-1	1	6	$\rightarrow 6/3 = 2$ (+min)
② Z	(0)	1	0	0	0	3/2	1	36 $\rightarrow$ Z	
x_3	(1)	0	0	0	1	1/3	-1/3	2	
x_2	(2)	0	0	1	0	1/2	0	6 $\rightarrow x_2$	
x_1	(3)	0	1	0	0	-1/3	1/3	2 $\rightarrow x_1$	

$\therefore$ Maximize, $Z = 36$.

And $(x_1, x_2) = (2, 6)$ (Result)

Problem 02

Maximize,

$Z = 3x_1 + 2x_2$

Subject to,

$2x_1 + x_2 \leq 6$

$x_1 + 2x_2 \leq 6$

And,

$x_1 \geq 0, x_2 \geq 0$

Turning the inequalities to equalities;

$Z - 3x_1 - 2x_2 = 0$ (0)

$2x_1 + x_2 + x_3 = 6$ (1)

$x_1 + 2x_2 + x_4 = 6$ (2)

Here,

$2x_1 + x_2 \leq 6$

$\Rightarrow 2x_1 + x_2 < 6$ or

$2x_1 + x_2 = 6$

$\Rightarrow 2x_1 + x_2 + x_3 = 6$

then; $x_1 + 2x_2 \leq 6$

$\Rightarrow x_1 + 2x_2 < 6$ or

$x_1 + 2x_2 = 6$

$\Rightarrow x_1 + 2x_2 + x_4 = 6$.

Here, we find 4 different variables and 2 equations. So the degree of freedom is (4-2) or 2.

Let, the non Basic variables x_1, x_2 equal to zero. $x_1 = 0$, $x_2 = 0$.

calculation

$$
\begin{array}{rrrrrrr}
 & 1 & -3 & -2 & 0 & 0 & 0 \\
(+) & 0 & 3 & 3/2 & 3/2 & 0 & 9 \\
\hline
 & 1 & 0 & -1/2 & 3/2 & 0 & 9
\end{array}
$$

$$
\begin{array}{rrrrrrr}
 & 0 & 1 & 2 & 0 & 1 & 6 \\
(+) & 0 & -1 & -1/2 & -1/2 & 0 & -3 \\
\hline
 & 0 & 0 & 3/2 & -1/2 & 1 & 3
\end{array}
$$

$$
\begin{array}{rrrrrr}
1 & 0 & -1/2 & 3/2 & 0 & 9 \\
0 & 0 & 1/2 & -1/6 & 1/3 & 1 \\
\hline
1 & 0 & 0 & 4/3 & 1/3 & 10
\end{array}
$$

$$
\begin{array}{rrrrrr}
0 & 1 & 1/2 & 1/2 & 0 & 3 \\
0 & 0 & -1/2 & 1/6 & -1/3 & -1 \\
\hline
0 & 1 & 0 & 2/3 & -1/3 & 2
\end{array}
$$

$$
\begin{array}{rrrrrr|r}
 & 1 & 0 & -1 & 3/2 & 0 & 9 \\
(+) & 0 & 0 & 1 & -1/3 & 2/3 & 2 \\
\hline
 & 1 & 0 & 0 & 7/6 & 2/3 & 11
\end{array}
$$

Basic Variable	Eq.	Z	x_1	x_2	x_3	x_4	Right Side	Ratio
Z	(0)	1	-3	-2	0	0	0	
x_3	(1)	0	$\boxed{2}$	1	1	0	6	$\rightarrow 6/2 = 3$ (+min)
x_4	(2)	0	$\boxed{1}$	2	0	1	6	$\rightarrow 6/1 = 6$
Z	(0)	1	0	$-1/2$	$3/2$	0	9	
x_1	(1)	0	1	$\boxed{1/2}$	$1/2$	0	3	$\rightarrow 3/(1/2) = 6$
x_4	(2)	0	0	$\boxed{3/2}$	$-1/2$	1	3	$\rightarrow 3/(3/2) = 2$ (+min)
Z	(0)	1	0	0	$1/3$	$1/3$	10 $\rightarrow Z$	
x_1	(1)	0	1	0	$2/3$	$-1/3$	2 $\rightarrow x_1$	
x_2	(2)	0	0	1	$-1/3$	$2/3$	2 $\rightarrow x_2$	

$\therefore$ maximize, $Z = 10$

And $(x_1, x_2) = (2, 2)$ (Result)

Problem 03

Maximize, $z = 3x_1 + 2x_2$

Subject to,

$x_1 \leq 4$

$x_1 + 3x_2 \leq 15$

$2x_1 + x_2 \leq 10$

And

$x_1 \geq 0$, $x_2 \geq 0$

Here, $x_1 \leq 4$

$\Rightarrow x_1 < 4$ or $x_1 = 4$

$\therefore x_1 + x_3 = 4$.

then, $x_1 + 3x_2 \leq 15$

$\Rightarrow x_1 + 3x_2 < 15$ or $x_1 + 3x_2 = 15$

$\therefore x_1 + 3x_2 + x_4 = 15$

And, $2x_1 + x_2 \leq 10$

$\Rightarrow 2x_1 + x_2 < 10$ or

$2x_1 + x_2 = 10$

$\therefore 2x_1 + x_2 + x_5 = 10$.

Turning the inequalities to equalities.

$z - 3x_1 - 2x_2 = 0$ (0)

$x_1 + x_3 = 4$ (1)

$x_1 + 3x_2 + x_4 = 15$ (2)

$2x_1 + x_2 + x_5 = 10$ (3)

Here, we find 5 different variable and 3 equation, so the degree of freedom is (5-3) or 2.

	Basic variable	Eq.	Z	x_1	x_2	x_3	x_4	x_5	Right side	Ratio
0	Z	(0)	1	-3	-2	0	0	0	0	
	x_3	(1)	0	1	0	1	0	0	4	$\rightarrow 4/1 = 4$ (+min)
	x_4	(2)	0	1	3	0	1	0	15	$\rightarrow 15/1 = 15$
	x_5	(3)	0	2	1	0	0	1	10	$\rightarrow 10/2 = 5$
1	Z	(0)	1	0	-2	3	0	0	12	
	x_1	(1)	0	1	0	1	0	0	4	$\rightarrow 4/0 = \propto$
	x_4	(2)	0	0	3	-1	1	0	11	$\rightarrow 11/3 = 3.67$
	x_5	(3)	0	0	1	-2	0	1	2	$\rightarrow 2/1 = 2$ min
2	Z	(0)	1	0	0	-1	0	1	16	
	x_1	(1)	0	1	0	1	0	0	4	$\rightarrow 4/1 = 4$
	x_4	(2)	0	0	0	5	1	-3	5	$\rightarrow 5/5 = 1$ (+min)
	x_2	(3)	0	0	1	-2	0	1	2	$\rightarrow 2/-2 = -1$
3	Z	(0)	1	0	0	0	1/5	2/5	17	$\rightarrow Z$
	x_1	(1)	0	1	0	0	-1/5	3/5	3	$\rightarrow x_1$
	x_3	(2)	0	0	0	1	1/5	-3/5	1	
	x_2	(3)	0	0	1	0	2/5	-1/5	4	$\rightarrow x_2$

Calculations

Let, the non Basic variables x_1, x_2 equal to zero. $x_1 = 0, x_2 = 0$.

$$\begin{array}{lcccccccc} & 1 & -3 & -2 & 0 & 0 & 0 & 0 \\ (+); & 0 & 3 & 0 & 3 & 0 & 0 & 12 \\ \hline & 1 & 0 & -2 & 3 & 0 & 0 & 12 \end{array}$$

$$\begin{array}{lcccccccc} & 0 & 1 & 3 & 0 & 1 & 0 & 15 \\ (+); & 0 & -1 & 0 & -1 & 0 & 0 & -4 \\ \hline & 0 & 0 & 3 & -1 & 1 & 0 & 11 \end{array}$$

$$\begin{array}{lcccccccc} & 0 & 2 & 1 & 0 & 0 & 1 & 10 \\ (+) & 0 & -2 & 0 & -2 & 0 & 0 & -8 \\ \hline & 0 & 0 & 1 & -2 & 0 & 1 & 2 \end{array}$$

$$\begin{array}{lcccccccc} & 1 & 0 & -2 & 3 & 0 & 0 & 12 \\ (+) & 0 & 0 & 2 & -4 & 0 & 1 & 4 \\ \hline & 1 & 0 & 0 & -1 & 0 & 1 & 16 \end{array}$$

$$\begin{array}{ccccccc} 0 & 0 & 3 & -1 & 1 & 0 & 11 \\ 0 & 0 & -3 & 6 & 0 & -3 & -6 \\ \hline 0 & 0 & 0 & 5 & 1 & -3 & 5 \end{array}$$

$$\begin{array}{cccccccc} 0 & 0 & 1 & -2 & 0 & 1 & 2 \\ 0 & 0 & 0 & 2 & 2/5 & -6/5 & 2 \\ \hline 0 & 0 & 1 & 0 & 2/5 & -1/5 & 4 \end{array}$$

$$\begin{array}{lcccccccc} & 1 & 0 & 0 & -1 & 0 & 1 & 16 \\ (+) & 0 & 0 & 0 & 1 & 1/5 & -3/5 & 1 \\ \hline & 1 & 0 & 0 & 0 & 1/5 & 2/5 & 17 \end{array}$$

$$\begin{array}{ccccccc} 0 & 1 & 0 & 1 & 0 & 0 & 4 \\ 0 & 0 & 0 & -1 & -1/5 & 3/5 & -1 \\ \hline 0 & 1 & 0 & 0 & -1/5 & 3/5 & 3 \end{array}$$

So, maximize, $z = 17$

And $(x_1, x_2) = (3, 4)$ (Result)

Problem 04

Maximize,

$z = 6x_1 + x_2 + 4x_3$

Subject to,

$3x_1 + 7x_2 + x_3 \leq 15$

$x_1 - 2x_2 + 3x_3 \leq 20$

$x_1, x_2, x_3 \geq 0$;

Turning the inequalities to equalities.

$z - 6x_1 - x_2 - 4x_3 = 0 \quad (0)$

$3x_1 + 7x_2 + x_3 + x_4 = 15 \quad (1)$

$x_1 - 2x_2 + 3x_3 + x_5 = 20 \quad (2)$

Here we find 5 different variables and 2 equation. So the degree of freedom $(5-2)$ or 3.

Here,

$3x_1 + 7x_2 + x_3 \leq 15$

$\Rightarrow 3x_1 + 7x_2 + x_3 < 15$

or

$3x_1 + 7x_2 + x_3 = 15$

$\Rightarrow 3x_1 + 7x_2 + x_3 + x_4 = 15$

then,

$x_1 - 2x_2 + 3x_3 \leq 20$

$\Rightarrow x_1 - 2x_2 + 3x_3 < 20$

or

$x_1 - 2x_2 + 3x_3 = 20$

$\Rightarrow x_1 - 2x_2 + 3x_3 + x_5 = 20$

Calculation

	1	-6	-1	-4	0	0	0
(+)	0	6	14	2	2	0	30
	1	0	13	-2	2	0	30

	0	1	-2	3	0	1	20
(+)	0	-1	-7/3	-1/3	-1/3	0	-5
	0	0	-13/3	8/3	-1/3	1	15

	1	0	13	-2	2	0	30
(+)	0	0	-13/4	2	-1/4	3/4	11.26
	1	0	39/4	0	7/4	3/4	41.26

	0	1	7/3	1/3	1/3	0	5
(+)	0	0	13/24	-1/3	1/24	-1/8	-1.87
	0	1	23/8	0	3/8	-1/8	3.13

Basic variable	Eq.	Z	x_1	x_2	x_3	x_4	x_5	Right side	Ratio
Z	(0)	1	-6	-1	-4	0	0	0	
x_4	(1)	0	3	7	1	1	0	15	$\to 15/3 = 5$ (min)
x_5	(2)	0	1	-2	3	0	1	20	$\to 20/1 = 20$
Z	(0)	1	0	13	-2	2	0	30	
x_1	(1)	0	1	$7/3$	$1/3$	$1/3$	0	5	$\to 5/\frac{1}{3} = 15$
x_5	(2)	0	0	$-13/3$	$8/3$	$-1/3$	1	15	$\to 15/\frac{8}{3} = 5.63$ (min)
Z	(0)	1	0	$39/4$	0	$7/4$	$3/4$	41.26	$\to Z$
x_1	(1)	0	1	$23/8$	0	$3/8$	$-1/8$	3.13	$\to x_1$
x_3	(2)	0	0	$-13/8$	1	$-1/8$	$3/8$	5.63	$\to x_2$

The value of x_2 is zero here.

As there is no negative value of Z row, so the iteration will stop here. and maximize, $Z = 41.26$

And $(x_1, x_2, x_3) = (3.13, 0, 5.63)$

(Result)