Chapter 1.1

1. Let, A = {Patients visit physical therapist}

B = {Patients visit chiropractor}

Consider, P (A) = x

So, P (B) = x – 16% = x – 0.16

Here, P (A ∩ B) = 28% = 0.28 and P $(A \cup B)'$ = 8% = 0.08

So, P (A ∪ B) = 1 – P$(A \cup B)'$ = 1 – 0.8 = 0.92

We know, P (A ∪ B) = P (A) + P (B) – P (A ∩ B)

⟹ 0.92 = x + x – 0.16 – 0.28

⟹ 2x = 1.36

⟹ x = 0.68

So, P (A) = 0.68 = 68%

2. Let, A = {Customers insure more than one car}

B = {Customers insure a sports car}

Given, P(A) = 85% = 0.85, P(B) = 23% = 0.23, and P (A ∩ B) = 17% = 0.17

Now, P $(A' \cap B')$ = P$(A \cup B)'$ = 1 – P (A ∪ B)

= 1 – [P (A) + P (B) – P (A ∩ B)]

= 1 – (0.85 + 0.23 – 0.17) = 0.09 = 9%

3. Here, n (S) = 52

a) n (A) = 12 and n (B) = 6

So, P (A) = $\frac{12}{52}$ and P (B) = $\frac{6}{52}$

b) n (A ∩ B) = 2

So, P (A ∩ B) = $\frac{n\ (A \cap B)}{n\ (S)} = \frac{2}{52}$

c) P (A ∪ B) = P (A) + P (B) – P (A ∩ B) = $\frac{12}{52} + \frac{6}{52} - \frac{2}{52} = \frac{16}{52}$

d) n (C) = 13 and n (D) = 39

So, P (C) = $\frac{13}{52}$ and P (D) = $\frac{39}{52}$

n (C ∩ D) = 0

So, $P(C \cap D) = \frac{n(C \cap D)}{n(S)} = 0$

e) $P(C \cup D) = P(C) + P(D) - P(C \cap D) = \frac{13}{52} + \frac{39}{52} - 0 = 1$

4. a) The sample space, $S = \begin{Bmatrix} \text{HHHH}, & \text{HHHT}, & \text{HHTH}, & \text{HHTT}, \\ \text{HTHH}, & \text{HTHT}, & \text{HTTH}, & \text{HTTT}, \\ \text{THHH}, & \text{THHT}, & \text{THTH}, & \text{THTT}, \\ \text{TTHH}, & \text{TTHT}, & \text{TTTH}, & \text{TTTT} \end{Bmatrix}$

Here, n(S) = 16.

b) Let, A = {At least 3 heads} = {HHHH, HHHT, HHTH, HTHH, THHH}

B = {At most 2 heads} = {HHTT, TTHH, HTHT, HTTH, HTTT,

THHT, THTH, THTT, TTHT, TTTH, TTTT}

C = {Heads on the third toss} = {HHHH, HTHH, THHH, TTHH, HHHT, HTHT, THHT, TTHT}

D = {1 head and 3 tails} = {HTTT, THTT, TTHT, TTTH}

Now, n(A) = 5, n(B) = 11, n(C) = 8, n(D) = 4

(i) $P(A) = \frac{5}{16}$

(ii) $n(A \cap B) = 0$;

So, $P(A \cap B) = 0$

(iii) $P(B) = \frac{11}{16}$

(iv) $n(A \cap C) = 4$;

So, $P(A \cap C) = \frac{4}{16}$

(v) $P(C) = \frac{n(C)}{n(S)} = \frac{8}{16}$

$P(D) = \frac{n(D)}{n(S)} = \frac{4}{16}$

(vi) $P(A \cup C) = P(A) + P(C) - P(A \cap C) = \frac{5}{16} + \frac{8}{16} - \frac{4}{16} = \frac{9}{16}$

(vii) $n(B \cap D) = 4$;

So, $P(B \cap D) = \frac{4}{16}$

5. Given, $P(A) = \frac{1}{6}$.

So, $P(B) = 1 - \frac{1}{6} = \frac{5}{6}$ $[\because B = A']$

Now, $P(A \cap B) = 0$

So, $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{6} + \frac{5}{6} - 0 = 1$

6. Given, $P(A) = 0.4$, $P(B) = 0.5$ and $P(A \cap B) = 0.3$

a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5 + 0.4 - 0.3 = 0.6$

b) $P(A \cap B') = P(A) - P(A \cap B) = 0.4 - 0.3 = 0.1$

c) $P(A' \cup B') = P(A \cap B)' = 1 - P(A \cap B) = 1 - 0.3 = 0.7$

7. Here, $P(A \cup B) = 0.76$ and $P(A \cup B') = 0.87$

We know, $P(A \cup B') = P(A) + P(A \cup B)'$

So, $P(A) = P(A \cup B') - P(A \cup B)'$

$= P(A \cup B') - [1 - P(A \cup B)]$

$= 0.87 - (1 - 0.76) = 0.63$

8. Let, A = {Having lab work}

B = {Having a referral}

Given, $P(A) = 0.41$ and $P(B) = 0.53$

Here, $P(A \cup B)' = 0.21$

Now, $P(A \cup B)' = 0.21$

$\Longrightarrow 1 - P(A \cup B) = 0.21$

$\Longrightarrow P(A \cup B) = 0.79$

So, $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.41 + 0.53 - 0.79 = 0.15$

Chapter 1.3

1. a) $P(B_1) = \frac{5{,}000}{1{,}000{,}000}$

b) $P(A_1) = \frac{78{,}515}{1{,}000{,}000}$

c) $P(A_1 \mid B_2) = \frac{n(A1 \cap B2)}{n(B2)} = \frac{73{,}630}{995{,}000}$

d) $P(B_1 \mid A_1) = \frac{n(A1 \cap B1)}{n(A1)} = \frac{4{,}885}{78{,}515}$

2. a) $P(A_1) = \frac{1041}{1456}$

b) $P(A_1 | S_1) = \frac{n(A1 \cap S1)}{n(S1)} = \frac{392}{633}$

c) $P(A_1 | S_2) = \frac{n(A1 \cap S2)}{n(S2)} = \frac{649}{823}$

3. a) $P(A_1 \cap B_1) = \frac{n(A1 \cap B1)}{n(S)} = \frac{5}{35}$

b) $P(A_1 \cup B_1) = P(A_1) + P(B_1) - P(A_1 \cap B_1) = \frac{n(A1)}{n(S)} + \frac{n(B1)}{n(S)} - \frac{5}{35} = \frac{12}{35} + \frac{19}{35} - \frac{5}{35} = \frac{26}{35}$

c) $P(A_1 | B_1) = \frac{n(A1 \cap B1)}{n(B1)} = \frac{5}{19}$

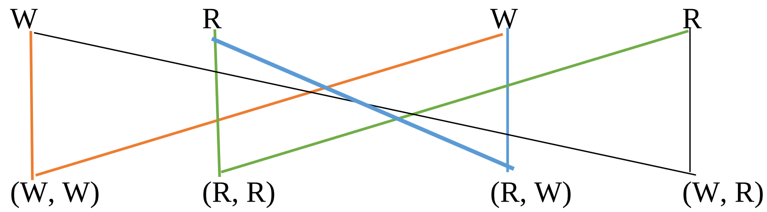
d) $P(B_2 | A_2) = \frac{n(A2 \cap B2)}{n(A2)} = \frac{9}{23}$

4. a) P (two hearts) $= \frac{13}{52} \times \frac{12}{51} = \frac{1}{17}$

b) P (A heart on the first and club on second) $= \frac{13}{52} \times \frac{13}{51} = \frac{13}{204}$

c) P (Non-Ace heart, Ace) + P (Ace of heart, non-heart Ace) $= \frac{12}{52} \times \frac{4}{51} + \frac{1}{52} \times \frac{3}{51} = \frac{1}{52}$

5. <u>Mother</u> <u>Father</u>



a) Sample space, S= {(W, W), (W, R), (R, W), (R, R)}

b) P (WW | White) $= \frac{1}{3}$

6.

	Heart disease	Non-heart disease	Total
Parental	111	223	334
Non-parental	110	538	648
Total	221	761	982

P (Heart disease | Non-parental) $= \frac{n(\text{Heart disease} \cap \text{Non-parental})}{n(\text{Non-parental})} = \frac{110}{648}$

7. P (At least one orange) = $P(O_1 \cap O_2) + P(O_1 \cap B_2) + P(B_1 \cap O_2)$

$$= \frac{2}{4} \times \frac{1}{3} + \frac{2}{4} \times \frac{2}{3} + \frac{2}{4} \times \frac{2}{3} = \frac{5}{6}$$

P (Both orange | At least one orange) = $\frac{\text{P (Both orange} \cap \text{At least one orange)}}{\text{P (At least one orange)}} = \frac{1/6}{5/6} = \frac{1}{5}$

8. a) P (WWW) = $\frac{3}{20} \times \frac{2}{19} \times \frac{1}{18} = \frac{1}{1140}$

b) P (WLWW) + P (LWWW) + P (WWLW)

$= \frac{3}{20} \times \frac{17}{19} \times \frac{2}{18} \times \frac{1}{17} + \frac{17}{20} \times \frac{3}{19} \times \frac{2}{18} \times \frac{1}{17} + \frac{3}{20} \times \frac{2}{19} \times \frac{17}{18} \times \frac{1}{17} = \frac{1}{380}$

14. a) $P(A_1) = \frac{n(A1)}{n(S)} = \frac{30}{100}$

b) $P(A_3) = \frac{n(A1)}{n(S)} = \frac{29}{100}$

$P(B_2) = \frac{n(B2)}{n(S)} = \frac{41}{100}$

$P(A_3 \cap B_2) = \frac{n\,(A3 \cap B2)}{n\,(S)} = \frac{9}{100}$

c) $P(A_2 \cup B_3) = P(A_2) + P(B_3) - P(A_2 \cap B_3) = \frac{41}{100} + \frac{28}{100} - \frac{9}{100} = \frac{60}{100}$

d) Probability of A_1 if it is B_2, $P(A_1 | B_2) = \frac{P\,(A1 \cap B2)}{P\,(B2)} = \frac{n\,(A1 \cap B2)/n\,(S)}{n\,(B2)/n\,(S)} = \frac{11/100}{41/100} = \frac{11}{41}$

e) Probability of B_1 if it is A_3, $P(B_1 | A_3) = \frac{P\,(B1 \cap A3)}{P\,(A3)} = \frac{n\,(B1 \cap A3)/n\,(S)}{n\,(A3)/n\,(S)} = \frac{13/100}{29/100} = \frac{13}{29}$

15.

A	B
Red ball = 8	Red ball = n
Blue ball = 7	Blue ball = 9

P (Two balls of same color) = P (RR) + P (BB)

$\Longrightarrow \frac{151}{300} = \frac{8}{15} \times \frac{n}{(n+9)} + \frac{7}{15} \times \frac{9}{(9+n)}$

$\Longrightarrow \frac{151}{300} = \frac{8n+63}{15(n+9)}$

$\Longrightarrow 300\,(8n+63) = 151\,(15n+135)$

$\Longrightarrow$ 2400n + 18900 = 2265n +20385

$\Longrightarrow$135n = 1485

$\therefore$ n = 11

So, there are 11 red balls. (answer)

16.

A	B
Red ball = 4 White ball = 2	Red ball = 4 White ball = 3

P (RB) = P (RA ∩ RB) + P (WA ∩ RB)

= P (RA) P (RB | RA) + P (WA) P (RB | WA)

$= \frac{3}{5} \times \frac{5}{8} + \frac{2}{5} \times \frac{4}{8} = \frac{23}{40}$

Chapter 1.4

1. Given, P (A) = 0.7, P (B) = 0.2 and both A and B are independent.

a) P (A∩B) = P(A)×P(B) = (0.7) × (0.2) = 0.14

b) P (A ∪ B) = P (A) + P (B) – P (A ∩ B) =0.7 + 0.2 – 0.14 = 0.76

c) P (A′ ∪ B′) = P (A ∩ B)′ =1 – P (A∩ B) = 1 – 0.14 = 0.86

2. Given, P(A) = 0.3 & P(B) = 0.6

a) P (A ∪ B) = P (A) + P (B) – P (A ∩ B)

= P(A)+ P(B) – P(A) × P(B) [as A and B are independent]

= 0.3 + 0.6 – 0.3 × 0.6 = 0.72

b) P (A | B) = $\frac{P(A \cap B)}{P(B)}$ = 0 [as A and B are mutually exclusive $P(A \cap B) = 0$]

3. Given, P (A) = $\frac{1}{4}$; P (A)′ = 1 – P (A) = = 1 – $\frac{1}{4}$ = $\frac{3}{4}$

P (B) = $\frac{2}{3}$; P (B)′ = 1 – P (B) = 1 – $\frac{2}{3}$ = $\frac{1}{3}$

a) P (A ∩ B) = P(A) × P(B) = $\frac{1}{4} \times \frac{2}{3} = \frac{1}{6}$

b) P (A ∩ B′)= P (A)×P (B′) = $\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$

c) P (A′ ∩ B′) = P (A′)×P (B′) = = $\frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$

d) $P\,[(A \cup B)'] = P\,(A' \cap B') = \frac{1}{4}$

e) $P\,(A' \cap B) = P\,(A') \times P\,(B) = \frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$

5. Given, $P\,(A) = 0.8$, $P\,(B) = 0.5$ & $P\,(A \cup B) = 0.9$

$P\,(A \cap B) = P\,(A) \times P\,(B) = 0.8 \times 0.5 = 0.4$

$P\,(A \cap B) = P\,(A) + P\,(B) - P\,(A \cup B) = 0.8 + 0.5 - 0.9 = 0.4$

As they are same, so A and B are independent.

7. Given, $P\,(A_1) = 0.5$, $P\,(A_2) = 0.7$, $P(A_3) = 0.6$

a) P (Exactly one player is successful)

$= P\,(A_1)\,P\,(A_2)'\,P(A_3)' + P\,(A_1)'\,P\,(A_2)\,P(A_3)' + P\,(A_1)'\,P\,(A_2)'\,P(A_3)$

$= 0.5 \times (1 - 0.7) \times (1 - 0.6) + (1 - 0.5) \times 0.7 \times (1 - 0.6) + (1 - 0.5) \times (1 - 0.7) \times 0.6 = 0.29$

b) P (Exactly two players make a goal)

$= P\,(A_1)\,P\,(A_2)\,P(A_3)' + P\,(A_1)\,P\,(A_2)'\,P(A_3) + P\,(A_1)'\,P\,(A_2)\,P(A_3)$

$= 0.5 \times 0.7 \times (1 - 0.6) + 0.5 \times (1 - 0.7) \times 0.6 + (1 - 0.5) \times 0.7 \times 0.6 = 0.47$

8. Let, A = {Orange comes up die on A}

B = {Orange comes up die on B}

C = {Orange comes up die on C}

$P(A) = \frac{1}{6}$; $P\,(A') = \frac{5}{6}$

$P(B) = \frac{2}{6}$; $P\,(B') = \frac{4}{6}$

$P(C) = \frac{3}{6}$; $P\,(C') = \frac{3}{6}$

P (exactly two players make a goal)

$= P(A)\,P(B)\,P(C') + P\,(A)\,P\,(B)'\,P\,(C) + P\,(A')\,P\,(B)\,P(C)$

$= \frac{1}{6} \times \frac{2}{6} \times \frac{3}{6} + \frac{1}{6} \times \frac{4}{6} \times \frac{3}{6} + \frac{5}{6} \times \frac{2}{6} \times \frac{3}{6} = \frac{2}{9}$

9. Given, $P(A) = 0.5$; $P\,(A') = 0.5$

$P(B) = 0.8$; $P\,(B') = 0.2$

$P(C) = 0.9$; $P\,(C') = 0.1$

a) P (All three events occur) $= P(A) \times P(B) \times P(C) = 0.5 \times 0.8 \times 0.9 = 0.36$

b) P (Exactly two events occur) = $P(A)\,P(B)\,P(C)' + P(A)\,P(B)'\,P(C) + P(A)'\,P(B)\,P(C)$

$= 0.5 \times 0.8 \times 0.1 + 0.5 \times 0.2 \times 0.9 + 0.5 \times 0.8 \times 0.9 = 0.49$

c) P (None of the events occur) = $P(A)'\,P(B)'\,P(C)' = 0.5 \times 0.2 \times 0.1 = 0.01$

Chapter 1.5

1.

B_1	B_2	B_3	B_4
Red ball = 0	Red ball = 2	Red ball = 2	Red ball = 1
White ball = 4	White ball = 0	White ball = 2	White ball = 3

Given, $P(B_1) = \frac{1}{2}$, $P(B_2) = \frac{1}{4}$, $P(B_3) = \frac{1}{8}$, $P(B_4) = \frac{1}{8}$

a) $P(W) = P(W \cap B_1) + P(W \cap B_2) + P(W \cap B_3) + P(W \cap B_4)$

$= P(B_1)\,P(W\,|\,B_1) + P(B_2)\,P(W\,|\,B_2) + P(B_3)\,P(W\,|\,B_3) + P(B_4)\,P(W\,|\,B_4)$

$= \frac{1}{2} \times 1 + \frac{1}{4} \times 0 + \frac{1}{8} \times \frac{2}{4} + \frac{1}{8} \times \frac{3}{4} = \frac{21}{32}$

b) $P(B_1\,|\,W) = \frac{P(W \cap B1)}{P(W)} = \frac{1/2}{21/32} = \frac{16}{21}$

2. Here, $P(A) = 40\% = 0.4$; $P(G\,|\,A) = 85\% = 0.85$

$P(B) = 60\% = 0.6$; $P(G\,|\,B) = 75\% = 0.75$

a) $P(G) = P(A)\,P(G\,|\,A) + P(B)\,P(G\,|\,B) = 0.4 \times 0.85 + 0.6 \times 0.75 = 0.79 = 79\%$

b) $P(A\,|\,G) = \frac{P(A)\,P(G|A)}{P(G)} = \frac{0.4 \times 0.85}{0.79} = 0.43 = 43\%$

5. Let, A = {Patients are critical}

B = {Patients are serious}

C = {Patients are stable}

$P(A) = 20\% = 0.2$; $P(D\,|\,A) = 30\% = 0.3$

$P(B) = 30\% = 0.3$; $P(D\,|\,B) = 10\% = 0.1$

$P(C) = 50\% = 0.5$; $P(D\,|\,C) = 1\% = 0.01$

Now, $P(D) = P(D \cap A) + P(D \cap B) + P(D \cap C)$

$= P(A)\,P(D\,|\,A) + P(B)\,P(D\,|\,B) + P(C)\,P(D\,|\,C)$

$= 0.2 \times 0.3 + 0.3 \times 0.1 + 0.5 \times 0.01 = 0.095$

Then, $P(A\,|\,D) = \frac{P(A \cap D)}{P(D)} = \frac{P(A)P(D\,|\,A)}{P(D)} = \frac{0.2 \times 0.3}{0.095} = 63\%.$

7. Given, $P(I^+) = 20\% = 0.2$; $P(I^-) = 80\% = 0.8$

$P(D^+ \mid I^+) = 0.9$; $P(D^- \mid I^+) = 0.1$

$P(D^+ \mid I^-) = 0.05$; $P(D^- \mid I^-) = 0.95$

Now, $P(D^+) = P(D^+ \cap I^+) + P(D^+ \cap I^-)$

$= P(I^+)\,P(D^+ \mid I^+) + P(I^-)\,P(D^+ \mid I^-)$

$= 0.2 \times 0.9 + 0.8 \times 0.05 = 0.22$

Then, $P(I^+ \mid D^+) = \frac{P(I^+ \cap D^+)}{P(D^+)} = \frac{P(I^+)P(D^+|I^+)}{P(D^+)} = \frac{0.2\times0.9}{0.22} = 81\%$.

9. Here, P (disease) = 0.05% = 0.0005; P (non-disease) = 0.9995

P (detect | disease) = 99% = 0.99; P (not detect | disease) = 0.01

P (detect | non-disease) = 3% = 0.03; P (not detect | non-disease) = 0.97

Now, $P(\text{disease} \mid \text{detect}) = \frac{P(\text{disease} \cap \text{detect})}{P(\text{detect})}$

$$= \frac{P(\text{disease})\,P(\text{detect} \mid \text{disease})}{P(\text{disease})\,P(\text{detect} \mid \text{disease}) + P(\text{non-disease})\,P(\text{detect} \mid \text{non-disease})}$$

$$= \frac{0.0005\times0.99}{0.0005\times0.99 + 0.9995\times0.03} = 0.016$$

Then, P (non-disease | detect) = 1 – P (disease | detect) = 1 – 0.016 = 0.984

10. Given, $P(A^+) = 0.02$; $P(A^-) = 0.98$

$P(D^- \mid A^+) = 0.08$; $P(D^+ \mid A^+) = 0.92$

$P(D^- \mid A^-) = 0.95$; $P(D^+ \mid A^-) = 0.05$

a) $P(D^+) = P(D^+ \cap A^+) + P(D^+ \cap A^-)$

$= P(A^+)\,P(D^+ \mid A^+) + P(A^-)\,P(D^+ \mid A^-)$

$= 0.02 \times 0.92 + 0.98 \times 0.05 = 0.0674$

b) $P(A^- \mid D^+) = \frac{P(D^+ \cap A^-)}{P(D^+)} = \frac{P(A^-)P(D^+|A^-)}{P(D^+)} = \frac{0.98\times0.05}{0.0674} = 0.727$

So, $P(A^+ \mid D^+) = 1 - P(A^- \mid D^+) = 1 - 0.727 = 0.273$