United International University
School of Science and Engineering
Final Exam Trimester: Spring 2023
Course Title: Probability and Statistics
Course Code: Math 2205/Stat 205 Marks: 40 Time: 2 Hours

[All questions are of equal marks. Answer all the questions in order]

Q1. (a) The probability distribution table for a random variable X is shown below:

x	-2	-1	0.5	1	2
$P(X=x)$	0.12	p	q	0.16	0.3

Given that $E(x) = 0.28$, find the value p and q. **[4]**.

(b) Drop of water fall randomly from a leaking tap at a constant average rate of 5.2 per minutes. Find the probability that at least 3 drops fall during a randomly chosen 30-second period. **[3]**

(c) A company produces bags of sugar. An inspector finds that on average 10% of the bags are underweight. 9 bags are chosen at random, find the probability that fewer than 3 of these bags are underweight. **[3]**

Q2

(a) In a game a ball is rolled down a slope and along a track until it stops. The distance, in meters travelled by the ball is modelled by the random variable X with the probability density function (*pdf*)

$$f(x) = \begin{cases} -k(x-1)(x-3) & 1 \le x \le 3 \\ 0 & otherwise \end{cases}$$

where k is a constant.

(i) Find the value of k **[3]**
(ii) With or without calculation, Explain or show that $E(x) = 2$ **[1]**
(iii) Find the standard deviations of the distance and the mode **[4]**

(b) In a large population, the systolic blood pressure (SBP) of adults is normally distributed with mean 125.4 and standard deviation 18.6. Find the probability that the SBP of a randomly chosen adult is less than 132. **[2]**

Q3 (a) A spinner has five sectors, each printed with a different color. Sushma and Sanjay both wish to test whether the spinner is biased so that it lands on red on fewer spins than it would if it were fair. Sushma spins the spinner 40 times. She finds that it lands on red exactly 4 times.
Use a binomial distribution to carry out the test at the 5% significance level. **[4]**

(b) In the past, the mean length of a particular variety of worm has been 10.3 cm, with standard deviation 2.6 cm. Following a change in the climate, it is thought that the mean length of this variety of worm has decreased. The length of a random sample of 100 worms of this variety are found and the mean of this sample is found to be 9.8cm.
Assuming that the standards deviation remains at 2.6 cm, carry out a test at 2% significance level whether the mean length has decreased. **[4]**

(c) A builders' merchant sells stones of different sizes. The masses of size A stones, have standard deviation of 6 grams. The mean mass of a random sample of 200 size A stones is 45 grams. Find a 95% confidence interval for the population mean mass of size A stones. **[2]**

P.T.O.

Q4 (a) Two six-sided dice of red and blue colors are rolled. The event A is such that the sum of the two outcomes is 7. The event B is such that the product of the two outcomes is 12. Find the probability of $A \cap B$ and $A \cup B$. Determine whether the events A and B independent or not? [4]

(b) In an apartment out of 12 dwellers, 7 are men,and in another apartment out of 13 dwellers, 7 are women., Say a dweller is shifted from the first apartment to the second apartment. For the capacity problem, one of the dwellers of the second apartment has to leave later, find the probability that this dweller is a man. [3]

(c) There is a new diagnostic test for a disease that occurs in about 1% of the population. The test is not perfect but will detect a person with the disease 99% of the time. It will, however, say that a person without the disease has the disease about 3% of the time. A person is selected at random from the population, and the test indicates that this person has the disease. What are the conditional probabilities that the person does not have the disease? [3]

Important Formulae

Distribution	pmf /pdf
Binomial	$f(x) = n_{c_x} p^x (1-p)^{n-x}\ ;\ x = 0, 1, 2, \ldots .n$
Poisson	$f(x) = \dfrac{\lambda^x e^{-\lambda}}{x!}\ ;\ x = 0, 1, 2, \ldots$
Uniform	$f(x) = \dfrac{1}{b-a}\ ;\ a \le x \le b$
Normal	$f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\ ;\ -\infty < x < \infty$

For continuous random variables $E(x) = \int_{-\infty}^{\infty} x f(x)dx$

$$\mathrm{Var}(x) = \int_{-\infty}^{\infty} x^2 f(x)dx - (E(x))^2$$

$$CI = \bar{x} \pm z\frac{s}{\sqrt{n}}$$