Note that the number of marks is given in brackets [ ] at the end of each question or part question.
You have to answer all the questions.

Q1 The random variable X has the following probability distribution shown below.

x	1	2	3	4
$P(X = x)$	0.1	a	0.3	b

Given that $E(X) = 3$. Find

(a) The values of a and b. Use the fundamental properties of probability.
(b) Standard deviation of the distribution.
(c) Hence fine $E(2X + 3)$ and $Var(2X + 3)$.

[3+3+2 = 8]

Q2.

(a) Jamia has 75% chance to attend a training session before a football match. If he attends, he is certain to be chosen for the team which plays in the match. If he does not attend, there is 60% chance that he is certain for the team.

(i) Find the probability that Jamia is chosen for the team.

(ii) Find the probability that Jamia attended the training session, given that he was chosen for the team.

(b) A typist makes, on average, 1 error for every 200 keyboard strokes. Assuming the error occur independently and random, find the probability that:

(i) in a document requiring 400 keyboard strokes there is no error.

(ii) in a document requiring 1000 keyboard strokes there is, at most one error.

[2+2+2+2 = 8]

Q3.

Two sisters make contact using an internet messaging service. The length of time for which they are logged on, in minutes is meddled by the random variable T with the probability density function (PDF)given by

$$f(t) = \begin{cases} \frac{1}{k}(40 - t) & 10 \leq t \leq 30 \\ 0 & otherwise \end{cases}, \text{where } k \text{ is a constant.}$$

(a) Show that $k = 400$.
(b) Find the expected time that they logged in.
(c) Find the probability that the time that they logged on for is less than 15 minutes.
(d) Find the corresponding CDF of the above PDF and hence the median time.

[8]

Q4.

(a) Requests come at a voluntary office according to the Poisson process on average 15 in every hour. Let X denote the waiting time in minutes until 2 requests come. Compute $P(X>5)$. What will be the median time if only a single request is considered?

(b) If $X \sim B(n,p)$ and mean and variance of X are 6 and 3.6 respectively. Find

(i) the values of n and p.

(ii) $P(X \geq 2)$.

[4+4=8]

Q5.

(a) In Europe the diameters of women's rings have mean 18.5 mm. Researchers claim that women in Jakarta have smaller fingers than women in Europe. The researchers took a random sample of 20 women in Jakarta and measured the diameters of their rings. The mean diameter was found to be 18.1 mm. Assuming that the diameters of women's rings in Jakarta have a normal distribution with standard deviation 1.1 mm, carry out a hypothesis test at the 5% level of significance to determine whether the researchers' claim is justified.

(b) A company produces an electric tool whose average life time is 300 days and variance 225 days. It is claimed that, in a newly developed process the mean life time can be increased. If the new process has increased the mean life time to 310 days, assuming a sample of 35 bulbs with estimated life time 304 days, find α and β. Also, find the p-value of the claim of the company.

[4+4=8]

Distribution	Pmf/pdf
Hypergeometric	$f(x) = \dfrac{N_1C_x \, N_2C_{n-x}}{NC_n}$; $N = N_1 + N_2$, $x = 1, 2, \ldots \ldots n$
Geometric	$f(x) = q^{x-1}p$; $x = 0, 1, 2, \ldots$
Binomial	$f(x) = nC_x p^x q^{n-x}$; $x = 0, 1, 2, \ldots n$
Poisson	$f(x) = \dfrac{\lambda^x e^{-\lambda}}{x!}$; $x = 0, 1, 2, \ldots$
Uniform	$f(x) = \dfrac{1}{b-a}$; $a \leq x \leq b$
Exponential	$f(x) = \dfrac{1}{\theta} e^{-\frac{x}{\theta}}$; $0 \leq x < \infty$
Gamma	$f(x) = \dfrac{1}{\Gamma(\alpha)\,\theta^\alpha} x^{\alpha-1} e^{-x/\theta}$; $0 \leq x < \infty$
Normal	$f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$; $-\infty < x < \infty$

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997

CamScanner