

* Quartile, Decile, Percentile: (For ungrouped data)

Quartile: Divides data set into 4 equal parts.

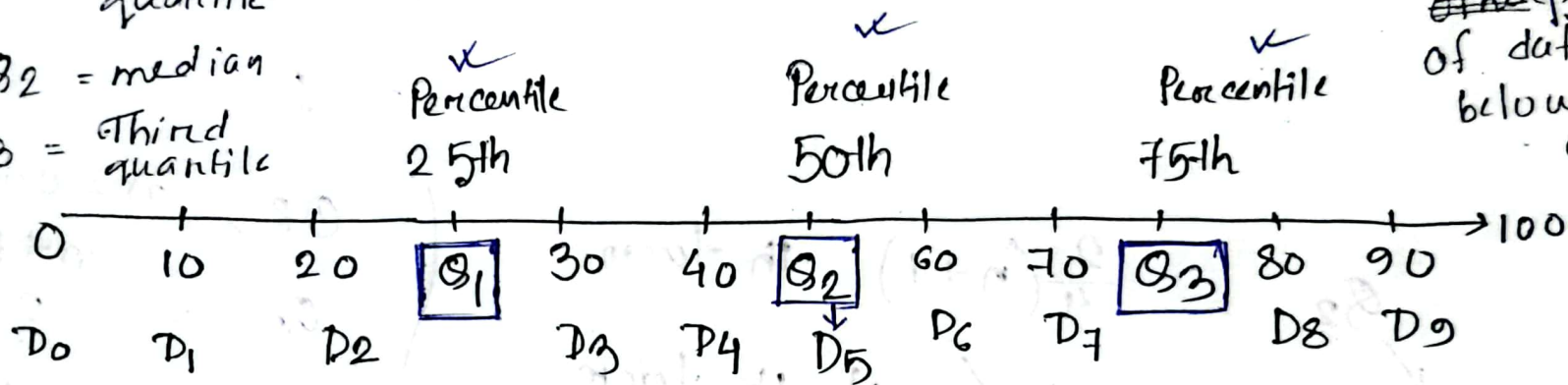
Decile: Divides data set into 10 equal parts.

Percentile: Divides data set into 100 equal parts.

Q_1 = lowest quartile
 Q_2 = median
 Q_3 = Third quartile

$\boxed{Q_1}$ 25% of data falls below Q_1 .

$\boxed{Q_2}$ 50% of data falls below the Q_2 .



1) Q_2 = Median = D_5 = 50th Percentile

2) Q_1 = 25th Percentile

3) Q_3 = 75th Percentile

* 75% of data falls below Q_3 .

Formula:

$$Q_k = \frac{k}{4} (n+1) \text{ -th term}$$

$$D_k = \frac{k}{10} (n+1) \text{ -th term}$$

$$P_k = \frac{k}{100} (n+1) \text{ -th term}$$

($k =$ ~~ex~~ subscript) $Q_3 =$

Math steps: Arrange the data set in ascending order

~~Find~~ Example: 2, 3, 5, 7, 8, 10, 11
13, 15, 16, 19

Find $Q_1, Q_2, Q_3, D_4, P_{75}$ $\frac{k}{4}(n+1); k=1$
 \swarrow \downarrow \swarrow
Lower Median Upper

Soln:

Here, $n = 11$ (odd)

First Quartile, $Q_1 = \frac{1}{4} (n+1) \text{ -th term}$
 $= \frac{1}{4} (11+1) \text{ -th term}$
 $= \frac{12}{4} \text{ -th term}$
 $= 3 \text{rd term}$
 $= 5$

$Q_2 = \frac{2}{4} (n+1) \text{ -th term}$
 \swarrow
Median $= \frac{1}{2} (11+1) \text{ -th term}$
 $= 6 \text{ -th term}$
 $= 10$

OR, $n = \text{odd}$
 $Q_2 = \frac{n+1}{2} \text{ -th term}$
 $= \frac{11+1}{2} \text{ -th}$
 $= 6 \text{ -th}$
 $= 10$

$$\begin{aligned}
 Q_3 &= \frac{3}{4} (n+1) \text{th term} \\
 &= \frac{3}{4} (11+1) \text{th term} \\
 &= 9\text{th term} \\
 &= 15
 \end{aligned}$$

$$\begin{aligned}
 D_4 &= \frac{4}{10} (n+1) \text{th term} \\
 &= \frac{4}{10} (11+1) \text{th term} \\
 &= 4.8 \text{th term}
 \end{aligned}$$

$$\begin{aligned}
 \left. \begin{array}{l} \text{(Interpolation} \\ \text{method)} \end{array} \right\} &= 4\text{th term} + 0.8 (5\text{th} - 4\text{th}) \\
 &= 7 + 0.8 (8 - 7)
 \end{aligned}$$

$$\boxed{D_4 = 7.8}$$

$$P_{75} = Q_3 = 15$$

✓

$$\frac{75}{100} (n+1) \text{th}$$

$$= \frac{75}{100} (11+1) \text{th}$$

$$= 9\text{th}$$

$$= 15$$

Example:

1 3 7 7

16
= 82

2.1 27 30 31

Find Q_3 and Q_1

Soln:

$(n = 9)$

Lower quartile
 Q_1

$$Q_1 = \frac{1}{4}(n+1) \text{th term}$$

$$= \frac{1}{4}(9+1) \text{th term}$$

$$= 2.5 \text{th term}$$

$$= \text{2nd term} + 0.5(\text{3rd} - \text{2nd})$$

$$= 3 + 0.5(7 - 3)$$

$$= 5$$

Upper quartile
 Q_3

$$Q_3 = \frac{3}{4}(n+1) \text{th term}$$

$$= \frac{3}{4}(9+1) \text{th term}$$

$$= 7.5 \text{th term}$$

$$= 7 \text{th term} + 0.5(8 \text{th} - 7 \text{th})$$

$$= 27 + 0.5(30 - 27)$$

$$= 28.5$$

(Ans.)

OR,
for Q_1 ,

$$\frac{4}{2} \text{th} = \text{2nd} = 3$$

$$\frac{4}{2} + 1 = \text{3rd} = 7$$

$$\frac{3+7}{2} = 5$$

$$\frac{4}{2} \text{th} = \text{2nd}$$

$$\left(\frac{4}{2} + 1\right) \text{th} = \text{3rd}$$

$$\frac{27+30}{2}$$

$$= \frac{57}{2}$$

$$= 28.5$$

Course Code

Trimester / Semester : Spring / Summer / Fall, 20.....

Signature with date

Name of Exam : Class Test / Mid-term / Final

Date:

Problem:

$$\begin{array}{c} \overbrace{(45, 46, 46, 49, 52, 55)} \\ \underbrace{(68, 69, 73)} \end{array} \quad \begin{array}{c} \overbrace{(57, 59, 63)} \\ \downarrow 0.2 \end{array}$$

Find D_6 .

7.8

soln:

$$n = 12.$$

$$\begin{aligned} D_6 &= \frac{6}{10} (n+1) \text{th term} \\ &= \frac{6}{10} (12+1) \text{th term} \\ &= 7.8 \text{th term} \\ &= 7\text{th term} + 0.8 (8\text{th} - 7\text{th}) \\ &= 57 + 0.8 (59 - 57) \\ &= 58.6 \end{aligned}$$

[Ans.]

Problem:

1, 2, 2, 2, 3, 3, 4, 4, 5, 6, 6, 8, 8, 8, 9, 9.

Lower Quartile $Q_2 = \text{median}$ Upper Quartile

Find Q_1, Q_2, Q_3 .

Soln:

$$n = 16$$

$$\text{Median} = Q_2 = \frac{2}{4}(n+1) + h$$

$$= \frac{1}{2}(16+1) + h$$

$$= 8.5 + h$$

$$= 8.5 + h$$

$$= 8 + h + 0.5(9 + h - 8 + h)$$

$$= 4 + 0.5(n - 4)$$

$$Q_2 = 4.5$$

$$\frac{8}{2} + h = 4 + h = 8$$

$$\left(\frac{8}{2} + 1\right) + h = 9 + h = 8$$

$$\frac{8+8}{2} = \frac{16}{2} = 8$$

$$\text{Upper quartile} = Q_3 = \frac{3}{4}(n+1) + h \text{ term}$$

$$Q_1 =$$

$$\frac{8}{2} + h = 4 + h = 2$$

$$\left(\frac{8}{2} + 1\right) + h = 5 + h = 3$$

$$\frac{2+3}{2} = \frac{5}{2}$$

$$= 2.5$$

$$= \frac{3}{4}(16+1) + h$$

$$= \frac{51}{4} + h = 12.75 + h$$

$$= 12 + h + 0.75(13 + h - 12 + h)$$

$$= 8 + 0.75(8 - 8) = 8$$

similarly,

$$\text{Lower quartile} = Q_1 =$$

$$\frac{1}{4}(16+1) + h$$

$$= \frac{17}{4} + h$$

$$= 4.25 + h$$

$$= 4 + h + 0.25(5 + h - 4 + h)$$

$$= 2 + 0.25(3 + h)$$

$$= 2.25$$

Quartile, Decile, Percentile (For Grouped data):

$$Q_k = L + \frac{h}{f} \left(\frac{kN}{4} - C_f \right) \quad [\text{Quartile}]$$

$$D_k = L + \frac{h}{f} \left(\frac{kN}{10} - C_f \right) \quad [\text{Decile}]$$

$$P_k = L + \frac{h}{f} \left(\frac{kN}{100} - C_f \right) \quad [\text{Percentile}]$$

Problem:

Class boundary

Frequency

5

10 - 12

8

13 - 15

5

16 - 18

10

19 - 21

2

22 - 24

find Q_3 , 4th decile (D_4), 19th Percentile (P_{19}).

Soln:

<u>Original class boundary</u>		<u>Frequency</u>	<u>Cum-Freq</u>
9.5 - 12.5		5	5
12.5 - 15.5		8	13
15.5 - 18.5		5	18
18.5 - 21.5		10	28
21.5 - 24.5		2	30
		$n = 30$	

Pentile class / Decile class

Quantile class

Here, h = lower value of quantile class

c_f = Cum. Frequency of pre quantile class

h = size of quantile class

n = Total Frequency

k = n -th term which we want to find.

Quantile class

$$= \frac{Kn}{4}$$

$$= \frac{3 \times 30}{4}$$

$$= 22.5$$

Quantile class = $\frac{KN}{4}$ is slightly greater than c_f .

$$Q_3 = l + \frac{h}{f} \left(\frac{KN}{4} - C_f \right)$$

$$\frac{KN}{4} = 22.5$$

$$= 18.5 + \frac{3}{10} (22.5 - 18)$$

$$\approx 19.85$$

$$D_4 = l + \frac{h}{f} \left(\frac{KN}{10} - C_f \right)$$

$$\frac{KN}{10}$$

$$= 12.5 + \frac{3}{8} (12 - 5)$$

$$= \frac{.4 \times 30}{.10}$$

$$= 12$$

$$\approx 15.125$$

$$P_{19} = l + \frac{h}{f} \left(\frac{KN}{100} - C_f \right)$$

$$\frac{KN}{100} = \frac{19 \times 30}{100}$$

$$= 5.7$$

$$= 12.5 + \frac{3}{8} (5.7 - 5)$$

$$\approx 12.76 \quad (\text{Ans.})$$

* Quantile Deviation:

Lower upper quantile
 Difference between first quantile and third quantile. This is also known as interquantile Range.

Upper quantile
 Lower quantile

* Step: Arrange the data in ascending order.

Formula :

$$\frac{Q_3 - Q_1}{2}$$

co-efficient of Quantile deviation:

$$\frac{Q_3 - Q_1}{Q_3 + Q_1}$$

Problem:

2, 3, 4, 5, 6, 6, 7, 7, 8, 9, 10, 12
 Q_1 Q_2 Q_3

Find Q.D.

$$Q.D. = \frac{Q_3 - Q_1}{2} ; n = 12$$

Soln:

$$Q_1 = \frac{1}{4}(n+1) \cdot th$$

$$= \frac{1}{4}(12+1) \cdot th$$

$$= 3.25 \cdot th$$

$$= 3rd + 0.25(4th - 3rd)$$

$$= 4 + 0.25(5 - 4) = 4.25$$

$$Q.D. = \frac{8.75 - 4.25}{2}$$

$$= 2.25$$

(Ans.)

$$Q_3 = \frac{3}{4}(n+1) \cdot th$$

$$= \frac{3}{4}(12+1) \cdot th$$

$$= \left(\frac{3}{4} \times 13\right) \cdot th$$

$$= 9.75 \cdot th$$

$$= 9th + 0.75(10th - 9th)$$

$$= 8 + 0.75(9 - 8)$$

$$= 8.75$$

H.W. Find ~~mean and~~ Quartile deviation and 1 for the following data set.

<u>class interval</u>	<u>Frequency</u>
29.5 - 39.5	8
39.5 - 49.5	87
49.5 - 59.5	190
59.5 - 69.5	304
69.5 - 79.5	211
79.5 - 89.5	85
89.5 - 99.5	20

<u>Soln:</u>	<u>C.I.</u>	<u>Frequency</u>	<u>Cumulative Frequency</u>
		8	8
	29.5 - 39.5		95
	39.5 - 49.5	87	285 ✓
	49.5 - 59.5	190	
	59.5 - 69.5	304	589
	69.5 - 79.5	211	800 ✓
	79.5 - 89.5	85	885
	89.5 - 99.5	20	905
		<u>905</u>	

$$Q_1 = l + \frac{h}{f} \left(\frac{KN}{4} - C_f \right)$$

$$= 49.5 + \frac{10}{190} (226.25 - 95)$$

$$\approx 56.40$$

$$\frac{KN}{4}$$

$$= \frac{1}{4} \times 905$$

$$= 226.25$$

$$Q_3 = l + \frac{h}{f} \left(\frac{KN}{4} - C_f \right)$$

$$= 69.5 + \frac{10}{211} (678.75 - 589)$$

$$= 73.753$$

$$\frac{KN}{4}$$

$$\frac{3 \times 905}{4}$$

$$= 678.75$$

Quartile deviation = $\frac{Q_3 - Q_1}{2}$

$$= \frac{73.753 - 56.40}{2}$$

$$= 8.6765$$

co-eff. of Q.D. = $\frac{Q_3 - Q_1}{Q_3 + Q_1}$

$$= 0.133$$

[Ans.]

[Ans.]