Stat-205-CT-02 (Section-A)

1. The question was wrong.

Correct question $M(t) = \dfrac{0.25e^t}{1-0.75e^t}$

$$M(0) = \frac{0.25}{1-0.75} = 1$$

Here $M(t) = \dfrac{0.25e^t}{1-0.75e^t} = \dfrac{pe^t}{1-qe^t}$

$\therefore p = 0.25,\ q = 0.75$

The pmf of geometric distribution

$$f(x) = (0.75)^{x-1}(0.25)\ ;\ x = 1, 2, 3, \ldots$$

$$\mu = \frac{1}{p} = \frac{1}{0.25} = 4$$

$$\sigma = \sqrt{\frac{p}{q^2}} = \sqrt{\frac{0.25}{(0.75)^2}} = 0.67$$

2.

$P(X=1) = \frac{11}{36}$

$P(X=2) = \frac{9}{36}$

$P(X=3) = \frac{7}{36}$

$P(X=4) = \frac{5}{36}$

$P(X=5) = \frac{3}{36}$

$P(X=6) = \frac{1}{36}$

Now,

$$\frac{f(x) - \frac{9}{36}}{\frac{11}{36} - \frac{9}{36}} = \frac{x-2}{1-2}$$

$$\Rightarrow \frac{36f(x) - 9}{2} = -x + 2$$

$$\Rightarrow f(x) = \frac{13-2x}{36};$$

$x = 1, 2, 3, 4, 5, 6$

$$M(t) = \sum_{x=1}^{6} e^{tx}\left(\frac{13-2x}{36}\right)$$

$$E(X) = \sum_{x=1}^{6} x f(x) = \frac{91}{36}$$

$$E(X^2) = \sum_{x=1}^{6} x^2 f(x) = \frac{301}{36}$$

or

$$M'(0) = \frac{91}{36}$$

$$M''(0) = \frac{301}{36}$$

$$\text{variance} = \frac{301}{36} - \left(\frac{91}{36}\right)^2 = \frac{2555}{1296} = 1.97$$

3. Here, $n = 10$, $p = 0.4$, $q = 0.6$

$$f(x) = {}^{10}C_x (0.4)^x (0.6)^{10-x}$$

$$P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$$

$$= {}^{10}C_0 (0.4)^0 (0.6)^{10} + {}^{10}C_1 (0.4)^1 (0.6)^9 + {}^{10}C_2 (0.4)^2 (0.6)^8$$

$$= 0.167$$

$$\sigma^2 = npq = 10 \times 0.4 \times 0.6 = 2.4$$

4. Here, $\lambda = \frac{48}{24}$ / hour $= 2$ / hour

$$f(x) = \frac{e^{-2} 2^x}{x!} \; ; \; x = 0, 1, 2, \ldots$$

$$P(X \geq 3) = 1 - P(X \le 2) = 1 - [P(X=0) + P(X=1) + P(X=2)]$$

$$= 1 - \left[\frac{e^{-2} 2^0}{0!} + \frac{e^{-2} 2^1}{1!} + \frac{e^{-2} 2^2}{2!}\right] = 0.323$$

$$SD = \sqrt{\lambda} = \sqrt{2}$$