$$G_r(y) = P(Y_r \leq y) = \sum_{k=r}^{n} {}^{n}C_k [F(y)]^k [1-F(y)]^{n-k}$$

cdf of continuous order Statistic

Give value

Target order

$$g_r(y) = \frac{n!}{(r-1)!\,(n-r)!} [F(y)]^{r-1} [1-F(y)]^{n-r} f(y)$$

pdf of the cont. order statistic

Ex-2

$$f(x) = 2x \; ; \; 0 \leq x \leq 1$$

$$f(y) = 2y \; ; \; 0 \leq y \leq 1$$

$$F(x) = \int_0^x 2w\,dw = x^2 \; ; \; 0 \leq x \leq 1$$

$$F(y) = y^2 \; ; \; 0 \leq x \leq 1$$

$$P(Y_r < y) = G_r(y) = \sum_{k=r}^{5} {}^{5}C_r (y^2)^k (1-y^2)^{5-k}$$

$$P\left(Y_4 < \frac{1}{2}\right) = \sum_{k=4}^{5} {}^{5}C_k \left(\frac{1}{4}\right)^k \left(1-\frac{1}{4}\right)^{5-k}$$

$Y_1 < Y_2 < Y_3 < Y_4 < \frac{1}{2}$

$$= 5\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)^1 + 1\left(\frac{1}{4}\right)^5\left(\frac{3}{4}\right)^0 = ?$$

$$g_r(y) = \frac{5!}{(r-1)!\,(5-r)!}(y^2)^{r-1}(1-y^2)^{5-r}(2y)$$

$$g_4(y) = \frac{5!}{3!\,1!}(y^2)^3(1-y^2)^1(2y)$$

$$= 40\, y^7(1-y^2)$$

$$= 40\,(y^7 - y^9)$$

$$\mu_4 = \int_0^1 y\, g_4(y)\,dy = \int_0^1 40\,(y^8 - y^{10})\,dy$$

$$= 40\left(\frac{y^9}{9} - \frac{y^{11}}{11}\right)\Big|_0^1$$

$$= 40\left(\frac{1}{9} - \frac{1}{11}\right)$$

$$= \frac{80}{99}$$

$n = 5$

$G_1(y) = ?$ $\quad y = 0.1$

$G_2(y) = ?$ $\quad y = 0.35$

$G_3(y) = ?,$ $\quad y = 0.4$

$G_5(y) = ?$ $\quad y = 0.9$

$g_1(y) = ?$

$g_2(y) = ?$

$\mu_1 = \int_0^1 y \, g_1(y) \, dy$

$\mu_2 = \int_0^1 y \, g_2(y) \, dy$

$\mu_3 = ?$

$\mu_5 = ?$

Exe → 7

$n = 6$

$r = 3$

$$G_3(y) = P(Y_3 < y) = \sum_{K=3}^{6} {}^6C_K (y^2)^K (1-y^2)^{6-K}$$

$G_3(0.6)$

$$g_3(y) = \frac{6!}{2! \, 3!} (y^2)^2 (1-y^2)^3 (2y)$$

$$\mu_3 = \int_0^1 y \, g_3(y) \, dy$$

Exc-3

$$f(x)=\frac{1}{3}e^{-x/3} \;;\quad 0\le x<\infty$$

$$F(x) = 1-e^{-x/3} \;;\quad 0\le x<\infty$$

$$f(y)=\frac{1}{3}e^{-y/3} \;;\quad 0\le y<\infty$$

$$F(y)=1-e^{-y/3} \;;\quad 0\le y<\infty$$

ⓐ $$g_3(y)=\frac{5!}{2!\,2!}\left[F(y)\right]^2\left[1-F(y)\right]^2 f(y)$$

$$\mu_3=\int_0^\infty y\, g_3(y)\,dy$$

ⓑ $$P(Y_4<y)=G_4(y)$$

$$=\sum_{K=4}^{5} 5C_K\left[F(y)\right]^K\left[1-F(y)\right]^{5-K}$$

$$P(Y_4<5)=\,?$$

ⓒ $$P(Y_1>1)=1-P(Y_1\le 1)$$

$$=1-G_1(1)$$

$$G_1(y)=\sum_{K=1}^{5} 5C_K\left[F(y)\right]^K\left[1-F(y)\right]^{5-K}$$

$$G_1(1)=\,?$$

6.4

Ex → ***

$$f(x:p) = p^{x}(1-p)^{1-x}; \quad x = 0, 1 \quad 0 \le p \le 1$$

Sol^n

Let $$L(p) = \prod_{i=1}^{n} f(x_i : p)$$

$$= \prod_{i=1}^{n} p^{x_i} (1-p)^{1-x_i}$$

$$= p^{\sum x_i} (1-p)^{n - \sum x_i}$$

$$\ln L(p) = (\sum x_i) \ln p + (n - \sum x_i) \ln(1-p)$$

$$\frac{d \ln L(p)}{dp} = \frac{\sum x_i}{p} - \frac{n - \sum x_i}{1-p}$$

Let $$\frac{d \ln L(p)}{dp} = 0$$

$$\Rightarrow \frac{\sum x_i}{p} = \frac{n - \sum x_i}{1-p}$$

$$\Rightarrow \sum x_i - p\sum x_i = np - p\sum x_i$$

$$\Rightarrow np = \sum x_i$$

$$\Rightarrow p = \frac{\sum x_i}{n}$$

$$\frac{d^2 \ln L(p)}{d^2 p} = \frac{-\sum x_i}{p^2} - \frac{n - \sum x_i}{(1-p)^2} < 0$$

for $p = \sum x_i / n$

$$\therefore \hat{p} = \frac{\sum x_i}{n} = \bar{x}$$

Exe→3

$$f(x:\lambda) = \frac{e^{-\lambda}\lambda^{x}}{x!} \; ; \; 0 \le x < \infty, \; 0 \le \lambda < \infty$$

$$L(\lambda) = \prod_{i=1}^{n} f(x_i : \lambda)$$

$$= \prod_{i=1}^{n} \frac{e^{-\lambda}\lambda^{x_i}}{x_i!} = \frac{e^{-n\lambda}\lambda^{\sum x_i}}{\prod_{i=1}^{n}(x_i!)}$$

$$= \frac{e^{-n\lambda}\lambda^{\sum x_i}}{K} \text{ (say)}$$

$$\ln L(\lambda) = -n\lambda + \Sigma x_i \ln\lambda - \ln K$$

$$\frac{d \ln L(\lambda)}{d\lambda} = -n + \frac{\Sigma x_i}{\lambda}$$

Let $\frac{d \ln L(\lambda)}{d\lambda} = 0$

$$\Rightarrow \quad -n + \frac{\Sigma x_i}{\lambda} = 0$$

$$\lambda = \frac{\Sigma x_i}{n}$$

$$\Rightarrow \quad \frac{d^2 \ln L(\lambda)}{d\lambda^2} = -\frac{\Sigma x_i}{\lambda^2} < 0 \text{ for } \lambda = \frac{\Sigma x_i}{n}$$

$$\hat{\lambda} = \frac{\Sigma x_i}{n} = \bar{x}$$

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