Stat- 205 - CT-03 (Section - A)

1. $M(t) = \dfrac{e^{2t} - 1}{2t}$

$= \dfrac{e^{2t} - e^{0 \cdot t}}{t(2-0)}$; $t \neq 0$

$M(0) = 1$

$\therefore \; a = 0, \; b = 2$

$\therefore f(x) = \dfrac{1}{b-a} = \dfrac{1}{2}$

$$F(x) = \begin{cases} 0 & ; \; x < 0 \\ \frac{x}{2} & ; \; 0 \leq x < 2 \\ 1 & ; \; x \geq 2 \end{cases}$$

$P(0.5 < x < 1.2) = F(1.2) - F(0.5)$

$= \dfrac{1.2}{2} - \dfrac{0.5}{2} = 0.35$

2.

$f(x) = \dfrac{x^2}{K}$; $-1 < x < 1$

so, $\displaystyle\int_{-1}^{1} \frac{x^2}{K} dx = 1$

$\Rightarrow \dfrac{1}{K} \left. \dfrac{x^3}{3} \right|_{-1}^{1} = 1$

$\Rightarrow \dfrac{2}{3K} = 1$

$\Rightarrow K = \dfrac{2}{3}$

$\therefore f(x) = \dfrac{3x^2}{2}$; $-1 < x < 1$

$$M(t) = \int_{-1}^{1} e^{tx}\, \frac{3x^2}{2}\, dx$$

$$\begin{array}{ccc} x^2 & (+) & e^{tx} \\ 2x & (-) & \frac{e^{tx}}{t} \\ 2 & (+) & \frac{e^{tx}}{t^2} \\ 0 & & \frac{e^{tx}}{t^3} \end{array}$$

$$= \frac{3}{2} \int_{-1}^{1} x^2 e^{tx}\, dx$$

$$= \frac{3}{2} \left[\frac{x^2 e^{tx}}{t} - \frac{2x e^{tx}}{t^2} + \frac{2 e^{tx}}{t^3} \right]_{-1}^{1}$$

$$= \frac{3}{2} \left[\frac{e^{t}}{t} - \frac{2e^{t}}{t^2} + \frac{2e^{t}}{t^3} - \frac{e^{-t}}{t} - \frac{2e^{-t}}{t^2} - \frac{2e^{-t}}{t^3} \right]$$

3. $\lambda = \frac{120}{24} /\text{hour} = 5/\text{hour}$

$\theta = \frac{1}{\lambda} = \frac{1}{5}\ \text{hour}$

$\alpha = 4$

$$f(x) = \frac{x^{4-1} e^{-x/1/5}}{\Gamma 4 \left(\frac{1}{5}\right)^4} = \frac{625}{6} x^3 e^{-5x}$$

$$P(0 \le X \le 1) = \int_{0}^{1} \frac{625}{6} x^3 e^{-5x}\, dx$$

$$= \frac{625}{6} \left[-\frac{x^3 e^{-5x}}{5} - \frac{3x^2 e^{-5x}}{25} - \frac{6x e^{-5x}}{125} - \frac{6e^{-5x}}{625} \right]_{0}^{1}$$

$$= \frac{625}{6} \left[-\frac{1}{5} e^{-5} - \frac{3}{25} e^{-5} - \frac{6}{125} e^{-5} - \frac{6e^{-5}}{625} + \frac{6}{625} \right]$$

$= 0.735$

Here, 2 PM to 3 PM is 1 hour, so $P(0 \le X \le 1)$.

$$\sigma^2 = \alpha\theta^2 = 4 \times \frac{1}{25} = 0.16$$

4.

$$f(x) = \frac{1}{9}x^2 \; ; 0 < x < 3$$

$$f(y) = \frac{1}{9}y^2 \; ; 0 < y < 3$$

$$F(x) = \int_0^x \frac{1}{9} w^2 dw = \frac{x^3}{27} \; ; 0 < x < 3$$

$$F(y) = \frac{y^3}{27} \; ; 0 < y < 3$$

$$G_5(y) = \sum_{k=5}^{6} {}^6C_k \left(\frac{y^3}{27}\right)^k \left(1 - \frac{y^3}{27}\right)^{6-k}$$

$$P(Y_5 \le 2.25) = G_5(2.25)$$

$$= {}^6C_5 \left(\frac{2.25^3}{27}\right)^5 \left(1 - \frac{2.25^3}{27}\right) + \left(\frac{2.25^3}{27}\right)^6$$

$$= 0.052$$

$$g_5(y) = \frac{6!}{4!\,1!} \left(\frac{y^3}{27}\right)^4 \left(1 - \frac{y^3}{27}\right) \frac{y^2}{9}$$

$$= 30 \, \frac{y^{14}(27 - y^3)}{27^5}$$

$$= \frac{30}{27^5 \times 9} \left(27y^{14} - y^{17}\right)$$

$$\mu_3 = \frac{30}{27^5 \times 9} \int_0^3 y\,(27\,y^{14} - y^{17})\,dy$$

$$= \frac{30}{27^5 \times 9} \left[\frac{27}{16} y^{16} - \frac{y^{19}}{19} \right]_0^3$$

$$= 2.665$$