Chapter 3.1

2. Here, $f(x) = \frac{1}{2} = \frac{1}{1-(-1)}$; $-1 \leq x \leq 1$

So, $U(-1,1)$ is a uniform distribution.

Here, $cdf\ F(x) = \begin{cases} 0\ ;\ x < -1 \\ \frac{x+1}{2}\ ;\ -1 \leq x < 1 \\ 1\ ;\ x \geq 1 \end{cases}$

Mean, $\mu = \frac{-1+1}{2} = 0$

Variance, $\sigma^2 = \frac{(1+1)^2}{12} = \frac{1}{3}$

3. Here, $U(0,10)$

So, $a = 0\ ;\ b = 10$

So, $pdf\ \ f(x) = \frac{1}{10-0} = \frac{1}{10}$; $0 \leq x \leq 10$

Here, $cdf\ F(x) = \begin{cases} 0\ ;\ x < 0 \\ \frac{x}{10}\ ;\ 0 \leq x < 10 \\ 1\ ;\ x \geq 10 \end{cases}$

$P(X \geq 8) = 1 - P(X < 8) = 1 - F(8) = 1 - \frac{8}{10} = \frac{2}{10}$

$P(2 \leq X < 8) = F(8) - F(2) = \frac{8}{10} - \frac{2}{10} = \frac{6}{10}$

Mean, $\mu = \frac{0+10}{2} = 5$

Variance, $\sigma^2 = \frac{(10-0)^2}{12} = \frac{25}{3}$

4. Here, $M(t) = \frac{e^{5t} - e^{4t}}{t}$; $t \neq 0$ and $M(0) = 1$; $t = 0$

$\Rightarrow M(t) = \begin{cases} \frac{e^{5t}-e^{4t}}{t(5-4)}\ ;\ t \neq 0 \\ 1\ ;\ t = 0 \end{cases}$

So, $a = 4\,, b = 5$

So, $pdf\ f(x) = \frac{1}{5-4} = 1$; $4 \leq x \leq 5$

Here, $cdf\ F(x) = \begin{cases} 0\ ;\ x < 4 \\ x - 4\ ;\ 4 \leq x < 5 \\ 1\ ;\ x \geq 5 \end{cases}$

Mean, $\mu = \frac{4+5}{2} = 4.5$

Variance, $\sigma^2 = \frac{(5-4)^2}{12} = \frac{1}{12}$

Now, $P(4.2 \le X \le 4.7) = F(4.7\,) - F(4.2) = 0.7 - 0.2 = 0.5$

7.a Here, $f(x) = 4x^c; 0 \le x \le 1$

Now, $\int_0^1 4x^c dx = 1$

$\Rightarrow \frac{4x^{c+1}}{c+1} |_0^1 = 1$

$\Rightarrow \frac{4}{c+1} = 1$

$\Rightarrow c + 1 = 4$

$\Rightarrow c = 3$

Thus, $f(x) = 4x^3\ ;\ 0 \le x \le 1$

So, $cdf\ F(x) = \int_0^x 4w^3\, dw = w^4\, |_0^x = \ x^4\ \ ;\ 0 \le x \le 1$

Mean, $\mu = E(X) = \int_0^1 x\,(4x^3)dx = 4\int_0^1 x^4\, dx = \ \frac{4}{5}\ x^5|_0^1 = \frac{4}{5}$

$E(X^2) = \int_0^1 x^2\,(4x^3)dx = 4\int_0^1 x^5\, dx = \frac{4}{6}x^6|_0^1 = \ \frac{2}{3}$

Variance, $\sigma^2 = \frac{2}{3} - \left(\frac{4}{5}\right)^2 = \ \frac{2}{75}$

7.b Here, $f(x) = c\sqrt{x}\,; 0 \le x \le 4$

Now, $\int_0^4 c\sqrt{x}dx = 1$

$\Rightarrow \frac{c\,x^{\frac{3}{2}}}{\frac{3}{2}} |_0^4 \ = \ 1$

$\Rightarrow 8c = \frac{3}{2}$

$\Rightarrow c = \frac{3}{16}$

Thus, $f(x) = \frac{3}{16}\sqrt{x}\,; 0 \le x \le 4$

So, $cdf\ F(x) = \int_0^x \frac{3}{16}\sqrt{w}\ dw = \frac{3}{16} \times \frac{w^{\frac{3}{2}}}{\frac{3}{2}}\ |_0^x = \frac{1}{8}x^{\frac{3}{2}}\ \ ;\ 0 \le x \le 4$

Mean, $\mu = E(X) = \int_0^4 x\left(\frac{3}{16}\sqrt{x}\right)dx = \frac{3}{16}\int_0^4 x^{\frac{3}{2}}\ dx = \frac{3}{16}\frac{x^{\frac{5}{2}}}{\frac{5}{2}}|_0^4 = \ \frac{12}{5}$

$E(X^2) = \int_0^4 x^2 \left(\frac{3}{16}\sqrt{x}\right) dx = \frac{3}{16}\int_0^4 x^{\frac{5}{2}}\, dx = \frac{3}{16}\frac{x^{\frac{7}{2}}}{\frac{7}{2}}\Big|_0^4 = \frac{48}{7}$

Variance, $\sigma^2 = \frac{48}{7} - \left(\frac{12}{5}\right)^2 = \frac{192}{175}$

7.c Here, $f(x) = \frac{c}{x^{\frac{3}{4}}}$; $0 < x < 1$

Now, $\int_0^1 \frac{c}{x^{\frac{3}{4}}}\, dx = 1$

$\Rightarrow \frac{cx^{\frac{1}{4}}}{\frac{1}{4}}\Big|_0^1 = 1$

$\Rightarrow c = \frac{1}{4}$

Thus, $f(x) = \frac{1}{4x^{\frac{3}{4}}}$; $0 < x < 1$

So, $cdf\ F(x) = \int_0^x \frac{1}{4w^{\frac{3}{4}}}\, dw = \frac{1}{4}\frac{w^{\frac{1}{4}}}{\frac{1}{4}}\Big|_0^x = x^{\frac{1}{4}}; 0 < x < 1$

Mean, $\mu = E(X) = \int_0^1 x\left(\frac{1}{4x^{\frac{3}{4}}}\right) dx = \frac{1}{4}\int_0^1 x^{\frac{1}{4}}\, dx = \frac{1}{4}\frac{x^{\frac{5}{4}}}{\frac{5}{4}}\Big|_0^1 = \frac{1}{5}$

$E(X^2) = \int_0^1 x^2\left(\frac{1}{4x^{\frac{3}{4}}}\right) dx = \frac{1}{4}\int_0^1 x^{\frac{5}{4}}\, dx = \frac{1}{4}\frac{x^{\frac{9}{4}}}{\frac{9}{4}}\Big|_0^1 = \frac{1}{9}$

Variance, $\sigma^2 = \frac{1}{9} - \frac{1}{25} = \frac{16}{225}$

8.a Here, $f(x) = \frac{x^3}{4}; 0 < x < c$

Now, $\int_0^c \frac{x^3}{4} dx = 1$

$\Rightarrow \frac{x^4}{16}\Big|_0^c = 1$

$\Rightarrow c^4 = 16$

$\Rightarrow c = 2$

Thus, $f(x) = \frac{x^3}{4}; 0 < x < 2$

So, $cdf\ F(x) = \int_0^x \frac{w^3}{4} dw = \frac{w^4}{16}\Big|_0^x = \frac{x^4}{16}$; $0 < x < 2$

Mean, $\mu = E(X) = \int_0^2 x\left(\frac{x^3}{4}\right)\,dx = \frac{1}{4}\int_0^2 x^4\,dx = \frac{x^5}{20}\Big|_0^2 = \frac{8}{5}$

$E(X^2) = \int_0^2 x^2\left(\frac{x^3}{4}\right)dx = \frac{1}{4}\int_0^2 x^5\,dx = \frac{x^6}{24}\Big|_0^2 = \frac{8}{3}$

Variance, $\sigma^2 = \frac{8}{3} - \left(\frac{8}{5}\right)^2 = \frac{8}{75}$

8.b Here, $f(x) = \frac{3x^2}{16}\ \ ; -c < x < c$

Now, $\int_{-c}^{c}\frac{3x^2}{16}dx = 1$

$\Rightarrow \frac{x^3}{16}\Big|_{-c}^{c} = 1$

$\Rightarrow 2c^3 = 16$

$\Rightarrow c = 2$

Thus, $f(x) = \frac{3x^2}{16}; -2 < x < 2$

So, $cdf\ F(x) = \int_{-2}^{x}\frac{3w^2}{16}dw = \frac{w^3}{16}\Big|_{-2}^{x} = \frac{x^3+8}{16}\ \ ; -2 < x < 2$

Mean, $\mu = E(X) = \int_{-2}^{2} x\left(\frac{3x^2}{16}\right)dx = \frac{3}{16}\int_{-2}^{2} x^3\,dx = \frac{3}{64}x^4\Big|_{-2}^{2} = 0$

$E(X^2) = \int_{-2}^{2} x^2\left(\frac{3x^2}{16}\right)dx = \frac{3}{16}\int_{-2}^{2} x^5\,dx = \frac{3}{80}x^5\Big|_{-2}^{2} = \frac{12}{5}$

Variance, $\sigma^2 = \frac{12}{5} - 0 = \frac{12}{5}$

8.c Here, $f(x) = \frac{c}{\sqrt{x}}\ ;\ 0 < x < 1$

Now, $\int_0^1 \frac{c}{\sqrt{x}}dx = 1$

$\Rightarrow \frac{cx^{\frac{1}{2}}}{\frac{1}{2}}\Big|_0^1 = 1$

$\Rightarrow c = \frac{1}{2}$

Thus, $f(x) = \frac{1}{2\sqrt{x}}\ ; 0 < x < 1$

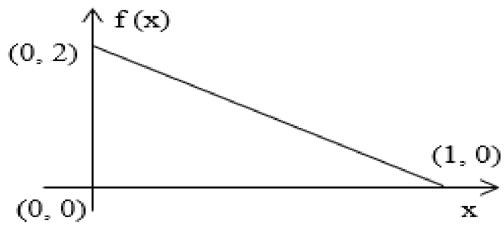
So, $cdf\ F(x) = \int_0^x \frac{1}{2\sqrt{w}}dw = \frac{1}{2}\,\frac{w^{\frac{1}{2}}}{\frac{1}{2}}\Big|_0^x = \sqrt{x}\ ;\ 0 < x < 1$

Mean, $\mu = E(X) = \int_0^1 x\left(\frac{1}{2\sqrt{x}}\right)dx = \frac{1}{2}\int_0^1 \sqrt{x}\,dx = \frac{1}{2}\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\Big|_0^1 = \frac{1}{3}$

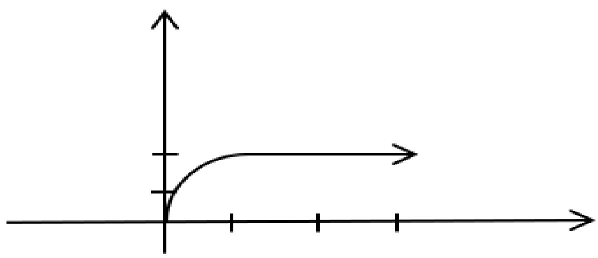
$E(X^2) = \int_0^1 x^2\left(\frac{1}{2\sqrt{x}}\right)dx = \frac{1}{2}\int_0^1 x^{\frac{3}{2}}\,dx = \frac{1}{2}\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\Big|_0^1 = \frac{1}{5}$

Variance, $\sigma^2 = \frac{1}{5} - \frac{1}{9} = \frac{4}{45}$

9. Here, $f(x) = \begin{cases} 2(1-x)\,;\ 0 \le x \le 1 \\ 0\,;\ elsewhere \end{cases}$



So, $cdf, F(x) = \int_0^x 2(1-w)dw = [2w - w^2]_0^x = 2x - x^2; 0 \le x \le 1$



$P\left(0 \le X \le \frac{1}{2}\right) = F\left(\frac{1}{2}\right) - F(0) = \left(1 - \frac{1}{4}\right) - 0 = \frac{3}{4}$

$P\left(\frac{1}{4} \le X \le \frac{3}{4}\right) = F\left(\frac{3}{4}\right) - F\left(\frac{1}{4}\right) = \left(\frac{3}{2} - \frac{9}{16}\right) - \left(\frac{1}{2} - \frac{1}{16}\right) = \frac{1}{2}$

$P(X = \frac{3}{4})$ is not defined or zero

$P\left(X \ge \frac{3}{4}\right) = 1 - P\left(X < \frac{3}{4}\right) = 1 - F\left(\frac{3}{4}\right) = 1 - \left(\frac{3}{2} - \frac{9}{16}\right) = \frac{1}{16}$

14. Here, $f(x) = \frac{1}{2};\ 0 < x < 1$ or $2 < x < 3$

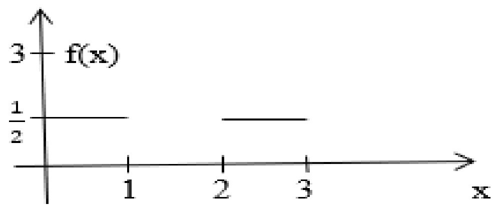
For, $x \le 0,\ F(x) = 0$

$0 < x < 1,\ F(x) = \int_0^x \frac{1}{2}dw = \frac{w}{2}\Big|_0^x = \frac{x}{2}$

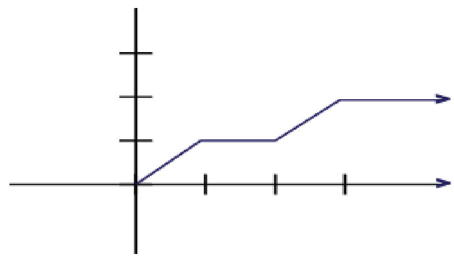
For, $1 \le x \le 2,\ F(x) = \frac{1}{2}$

$2 < x < 3,\ F(x) = \frac{1}{2} + \int_2^x \frac{1}{2} dw = \frac{1}{2} + \frac{w}{2}|_2^x = \frac{1}{2} + \frac{x}{2} - 1 = \frac{x}{2} - \frac{1}{2}$

For, $x \geq 3,\ F(x) = 1$



$$\therefore F(x) = \begin{cases} 0\ ; x \leq 0 \\ \frac{x}{2}\ ; 0 < x < 1 \\ \frac{1}{2}\ ; 1 \leq x \leq 2 \\ \frac{x}{2} - \frac{1}{2}\ ; 2 < x < 3 \\ 1\ ; x \ \geq 3 \end{cases}$$



$F(\pi_{0.25}) = 0.25 \Rightarrow \frac{\pi_{0.25}}{2} = 0.25 \Rightarrow \pi_{0.25} = 0.5$

$F(\pi_{0.5}) = 0.5 \Rightarrow \pi_{0.5} \in [1,2]$, it's not unique

$F(\pi_{0.75}) = 0.75 \Rightarrow \frac{\pi_{0.75}}{2} - \frac{1}{2} = 0.75 \Rightarrow \pi_{0.75} = 2.5$

15. Here, $f(x) = \frac{3x^2}{7^3} e^{-\left(\frac{x}{7}\right)^3}\ ; 0 < x < \infty$

So, $cdf\ F(x) = \int_0^x \frac{3w^2}{7^3} e^{-\left(\frac{w}{7}\right)^3} dw$ Let, $\left(\frac{w}{7}\right)^3 = z \Rightarrow \frac{3w^2}{7^3} dw = dz$

$= \int_0^{\left(\frac{x}{7}\right)^3} e^{-z}\, dz$ $w = 0,\ x \ \rightarrow\ z\ =\ 0,\ \left(\frac{x}{7}\right)^3$

$= -e^{-z}|_0^{\left(\frac{x}{7}\right)^3}$

$= 1 - e^{-\left(\frac{x}{7}\right)^3}\ ;\ 0 < x < \infty$

$P(X \geq 7) = 1 - P(X < 7) = 1\text{–}[1 - e^{-1}] = e^{-1}$

$P(X \geq 10.5) = 1 - P(X < 10.5) = 1\text{–}[1 - e^{-3.375}] = e^{-3.375}$

$P(X \geq 10.5/X \geq 7) = \frac{P\ (X \geq 10.5)}{P(X \geq 7)} = \frac{e^{-3.375}}{e^{-1}} = e^{-2.375}$

16. Here, $f(x) = \begin{cases} \frac{x+1}{2}\ ;\ -1 \leq x \leq 1 \\ 0\ ;\ elsewhere \end{cases}$

So, $cdf\ F(x) = \int_{-1}^{x} \left(\frac{w+1}{2}\right) dw = \left[\frac{(w+1)^2}{4}\right]_{-1}^{x} = \frac{(x+1)^2}{4}; -1 \leq x \leq 1$

$F(\pi_{0.64}) = 0.64 \Rightarrow \frac{(\pi_{0.64}+1)^2}{4} = 0.64 \Rightarrow \frac{\pi_{0.64}+1}{2} = 0.8 \Rightarrow \pi_{0.64} = 0.6$

$F(\pi_{0.25}) = 0.25 \Rightarrow \frac{(\pi_{0.25}+1)^2}{4} = 0.25 \Rightarrow \frac{\pi_{0.25}+1}{2} = 0.5 \Rightarrow \pi_{0.25} = 0$

$F(\pi_{0.81}) = 0.81 \Rightarrow \frac{(\pi_{0.81}+1)^2}{4} = 0.81 \Rightarrow \frac{\pi_{0.81}+1}{2} = 0.9 \Rightarrow \pi_{0.81} = 0.8$

Chapter - 3.2

1.a Here, $M(t) = \frac{1}{1-3t} = \frac{1}{1-\theta t}$. So, $\theta = 3$

$M(0) = \frac{1}{1-0} = 1$. So, $M(t)$ is an mgf.

Mean, $\mu = \theta = 3$

Variance, $\sigma^2 = \theta^2 = 9$

Thus, $pdf\ f(x) = \frac{1}{3} e^{-\frac{x}{3}};\ 0 \leq x < \infty$

1.b Here, $M(t) = \frac{3}{3-t}$

$M(0) = \frac{3}{3-0} = 1$. So, $M(t)$ is an mgf.

$M(t) = \frac{1}{1-\frac{t}{3}} = \frac{1}{1-\theta t}$. So, $\theta = \frac{1}{3}$

Mean, $\mu = \theta = \frac{1}{3}$

Variance $\sigma^2 = \theta^2 = \frac{1}{9}$

Thus, $pdf\ f(x) = \frac{1}{1/3} e^{-\frac{x}{1/3}} = 3e^{-3x};\ 0 \leq x < \infty$

2. Given by, $\lambda = \frac{2}{3} \Longrightarrow \theta = \frac{1}{\lambda} = \frac{3}{2}$

So, $f(x) = \frac{1}{3/2} e^{-\frac{x}{3/2}} = \frac{2}{3} e^{-\frac{2x}{3}}\,; 0 \le x < \infty$

Thus, $cdf\ F(x) = 1 - e^{-\frac{2x}{3}}\,; 0 \le x < \infty$

Mean, $\mu = \theta = \frac{3}{2}$

Variance. $\sigma^2 = \theta^2 = \frac{9}{4}$

Now, $P(X > 2) = 1 - P(X \le 2) = 1 - F(2) = 1 - \left(1 - e^{-\frac{4}{3}}\right) = e^{-\frac{4}{3}}$

Median, $Me = \theta\, ln2 = \frac{3}{2} ln2$

6. Given by, $\lambda = \frac{3}{100} \Longrightarrow \theta = \frac{1}{\lambda} = \frac{100}{3}$

So, $f(x) = \frac{1}{100/3} e^{-\frac{x}{100/3}} = \frac{3}{100} e^{-\frac{3x}{100}}\,; 0 \le x < \infty$

Thus, $cdf\ F(x) = 1 - e^{-\frac{3x}{100}}\,; 0 \le x < \infty$

Now, $P(X > 40) = 1 - P(X \le 40) = 1 - F(40) = 1 - \left(1 - e^{-\frac{120}{100}}\right) = e^{-\frac{6}{5}}$

Also, $M(t) = \frac{1}{1-\theta t} = \frac{1}{1-\frac{100t}{3}} = \frac{3}{3-100t}$

Median, $Me = \theta\, ln2 = \frac{100}{3} ln2$

8. Given by, $\alpha = 2, \theta = 4$

So, $f(x) = \frac{1}{\Gamma(2) \times 4^2} x^{2-1} e^{-\frac{x}{4}} = \frac{1}{16} x e^{-\frac{x}{4}}\ ;\ 0 \le x < \infty$

Now, $P(X < 5) = \frac{1}{16} \int_0^5 x e^{-\frac{x}{4}}\, dx = \frac{1}{16}\left[-4xe^{-\frac{x}{4}} - 16^{-\frac{x}{4}}\right]_0^5$

$$= \frac{1}{16}\left[16 - 36e^{-\frac{5}{4}}\right] = 1 - \frac{9}{4} e^{-\frac{5}{4}}$$

Thus, *mgf* $M(t) = \frac{1}{(1-4t)^2}$

Mean, $\mu = \alpha\theta = 8$

Variance, $\sigma^2 = \alpha\theta^2 = 32$

9. Here, $M(t) = (1-7t)^{-20} = \frac{1}{(1-7t)^{20}} = \frac{1}{(1-\theta t)^{\alpha}}$

Given by, $\alpha = 20, \theta = 7$

So, $f(x) = \frac{x^{20-1}e^{-\frac{x}{7}}}{\Gamma(20)\times 7^{20}} = \frac{x^{19}e^{-\frac{x}{7}}}{7^{20}(19)!}$

Mean, $\mu = \alpha\theta = 140$

Variance, $\sigma^2 = \alpha\theta^2 = 980$

Chapter - 3.3

1. If Z is $N(0,1)$, then

a. $P(0.53 < Z \leq 2.06) = \varphi(2.06) - \varphi(0.53) = 0.9803 - 0.7019 = 0.2784$

b. $P(-0.79 \leq Z < 1.52) = \varphi(1.52) - \varphi(-0.79) = \varphi(1.52) - \big(1 - \varphi(0.79)\big)$

$= 0.9357 - (1 - 0.7852) = 0.7209$

c. $P(Z > -1.77) = P(Z < 1.77) = \varphi(1.77) = 0.9616$

d. $P(Z > 2.89) = 1 - P(Z \leq 2.89) = 1 - \varphi(2.89) = 1 - 0.9981 = 0.0019$

e. $P(|Z| < 1.96) = P(-1.96 < Z < 1.96) = \varphi(1.96) - \varphi(-1.96)$

$= \varphi(1.96) - (1 - \varphi(1.96)) = 2\varphi(1.96) - 1 = 2 \times 0.9750 - 1 = 0.95$

f. $P(|Z| < 1) = P(-1 < Z < 1) = \varphi(1) - \varphi(-1) = \varphi(1) - (1 - \varphi(1))$

$= 2\varphi(1.96) - 1 = 2 \times 0.8413 - 1 = 0.6826$

g. $P(|Z| < 2) = P(-2 < Z < 2) = \varphi(2) - \varphi(-2) = \varphi(2) - (1 - \varphi(2))$

$= 2\varphi(2) - 1 = 2 \times 0.9772 - 1 = 0.9544$

h. $P(|Z| < 3) = P(-3 < Z < 3) = \varphi(3) - \varphi(-3) = \varphi(3) - (1 - \varphi(3))$

$= 2\varphi(3) - 1 = 2 \times 0.9987 - 1 = 0.9974$

2. If Z is $N(0,1)$, then

a. $P(0 < Z \leq 0.87) = \varphi(0.87) - \varphi(0) = 0.8106 - 0.5 = 0.3106$

b. $P(-2.64 \leq Z < 0) = \varphi(0) - \varphi(-2.64) = \varphi(0) - \big(1 - \varphi(2.64)\big)$

$= 0.5 - (1 - 0.9959) = 0.4959$

c. $P(-2.13 \leq Z < -0.56) = P(0.56 \leq Z < 2.13) = \varphi(2.13) - \varphi(0.56)$

$= 0.9834 - 0.7123 = 0.2711$

d. $P(|Z| > 1.39) = P(Z > 1.39) + P(Z < -1.39)$

$= 1 - P(Z < 1.39) + 1 - P(Z < 1.39) = 2 - 2P(Z < 1.39) = 2 - 2\varphi(1.39)$

$= 2 - 2 \times 0.9177 = 0.1646$

e. $P(Z < -1.62) = 1 - P(Z < 1.62) = 1 - \varphi(1.62) = 1 - 0.9474 = 0.0526$

f. $P(|Z| > 1) = P(Z > 1) + P(Z < -1) = 1 - P(Z < 1) + 1 - P(Z < 1)$

$= 2 - 2P(Z < 1) = 2 - 2\varphi(1) = 2 - 2 \times 0.8413 = 0.3174$

g. $P(|Z| > 2) = P(Z > 2) + P(Z < -2) = 1 - P(Z < 2) + 1 - P(Z < 2)$

$= 2 - 2P(Z < 2) = 2 - 2\varphi(2) = 2 - 2 \times 0.9772 = 0.0456$

h. $P(|Z| > 3) = P(Z > 3) + P(Z < -3) = 1 - P(Z < 3) + 1 - P(Z < 3)$

$= 2 - 2P(Z < 3) = 2 - 2\varphi(3) = 2 - 2 \times 0.9987 = 0.0026$

3. If Z is $N(0,1)$ Find value of c such that,

a. $P(Z \geq c) = 0.025$

$\Rightarrow 1 - P(Z < c) = 0.025$

$\Rightarrow P(Z < c) = 0.975$

$\Rightarrow \varphi(c) = 0.975$

So, $c = 1.96$

b. $P(|Z| \leq c) = 0.95$

$\Rightarrow P(-c \leq Z \leq c) = 0.95$

$\Rightarrow \varphi(c) - \varphi(-c) = 0.95$

$\Rightarrow \varphi(c) - (1 - \varphi(c)) = 0.95$

$\Rightarrow 2\varphi(c) = 1.95$

$\Rightarrow \varphi(c) = 0.975$

So, $c = 1.96$

c. $P(Z > c) = 0.05$

$\Rightarrow 1 - P(Z \leq c) = 0.05$

$\Rightarrow P(Z \leq c) = 0.95$

$\Rightarrow \varphi(c) = 0.95$

So, $c = 1.64$ or 1.645

d. $P(|Z| \leq c) = 0.9$

$\Rightarrow P(-c \leq Z \leq c) = 0.9$

$\Rightarrow \varphi(c) - \varphi(-c) = 0.9$

$\Rightarrow \varphi(c) - (1 - \varphi(c)) = 0.9$

$\Rightarrow 2\varphi(c) = 1.9$

$\Rightarrow \varphi(c) = 0.95$

So, $c = 1.64$ or 1.645

4. Find the value of Z, such that

a. $Z_{0.10}$

$\Rightarrow \varphi(Z_0) = 0.10$

$\Rightarrow Z_0 = -1.28$

So, $Z_0 = -1.28$

Alternative

$Z_{0.10}$

$\Rightarrow \varphi(Z_0) = 0.10$

$\Rightarrow 1 - \varphi(Z_0) = 0.90$

$\Rightarrow \varphi(-Z_0) = 0.90$

$\Rightarrow -Z_0 = 1.28$

So, $Z_0 = -1.28$

b. $-Z_{0.05}$

$\Rightarrow \varphi(-Z_0) = 0.05$

$\Rightarrow -Z_0 = -1.64$ or -1.645

So, $Z_0 = 1.64$ or 1.645

Alternative

$-Z_{0.05}$

$\Rightarrow \varphi(-Z_0) = 0.05$

$\Rightarrow 1 - \varphi(Z_0) = 0.05$

$\Rightarrow \varphi(Z_0) = 0.95$

$\Rightarrow Z_0 = 1.64$ or 1.645

So, $Z_0 = 1.64$ or 1.645

c. $-Z_{0.0485}$

$\Rightarrow \varphi(-Z_0) = 0.0485$

$\Rightarrow -Z_0 = -1.66$

So, $Z_0 = 1.66$

Alternative

$-Z_{0.0485}$

$\Rightarrow \varphi(-Z_0) = 0.0485$

$\Rightarrow 1 - \varphi(Z_0) = 0.0485$

$\Rightarrow \varphi(Z_0) = 0.9515$

$\Rightarrow Z_0 = 1.66$

So, $Z_0 = 1.66$

d. $Z_{0.9656}$

$\Rightarrow \varphi(Z_0) = 0.9656$

$\Rightarrow Z_0 = 1.82$

So, $Z_0 = 1.82$

5. If X is normally distributed with mean $\mu = 5$ and variance $\sigma^2 = 25$. So, $\sigma = 5$.

a. $P(6 \leq X \leq 12) = P\left(\frac{6-6}{5} \leq \frac{X-6}{5} \leq \frac{12-6}{5}\right) = P(0 \leq Z \leq 1.2)$

$= \varphi(1.2) - \varphi(0) = 0.8849 - 0.5 = 0.3849$

b. $P(0 \leq X \leq 8) = P\left(\frac{0-6}{5} \leq \frac{X-6}{5} \leq \frac{8-6}{5}\right) = P(-1.2 \leq Z \leq 0.4)$

$= \varphi(0.4) - \varphi(-1.2) = \varphi(0.4) - (1 - \varphi(1.2)) = 0.6554 - (1 - 0.8849) = 0.5403$

c. $P(-2 < X \leq 0) = P\left(\frac{-2-6}{5} \leq \frac{X-6}{5} \leq \frac{0-6}{5}\right) = P(-1.6 < Z \leq -1.2)$

$= \varphi(-1.2) - \varphi(-1.6) = \varphi(1.6) - \varphi(1.2) = 0.9452 - 0.8849 = 0.0603$

d. $P(X \geq 21) = 1 - P(X < 21) = 1 - P\left(\frac{X-6}{5} < \frac{21-6}{5}\right) = 1 - P(Z < 3)$

$= 1 - \varphi(3) = 1 - 0.9987 = 0.0013$

e. $P(|X - 6| \leq 5) = P(-5 < X - 6 < 5) = P\left(-\frac{5}{5} < \frac{X-6}{5} < \frac{5}{5}\right) = P(-1 < Z < 1)$
$= \varphi(1) - \varphi(-1) = \varphi(1) - \big(1 - \varphi(1)\big) = 2\varphi(1) - 1 = 2 \times 0.8413 = 0.6826$

f. $P(|X - 6| \leq 10) = P(-10 < X - 6 < 10) = P\left(-\frac{10}{5} < \frac{X-6}{5} < \frac{10}{5}\right)$

$= P(-2 < Z < 2) = \varphi(2) - \varphi(-2) = \varphi(2) - \big(1 - \varphi(2)\big) = 2\varphi(2) - 1$

$= 2 \times 0.9772 - 1 = 0.9544$

g. $P(|X - 6| \leq 15) = P(-5 < X - 6 < 5) = P\left(-\frac{15}{5} < \frac{X-6}{5} < \frac{15}{5}\right)$

$= P(-3 < Z < 3) = \varphi(3) - \varphi(-3) = \varphi(3) - \big(1 - \varphi(3)\big) = 2\varphi(3) - 1$

$= 2 \times 0.9987 - 1 = 0.9974$

h. $P(|X - 6| \leq 12.41) = P(-12.41 < X - 6 < 12.41) = P\left(-\frac{12.41}{5} < \frac{X-6}{5} < \frac{12.41}{5}\right)$
$= P(-2.48 < Z < 2.48) = \varphi(2.48) - \varphi(-2.48) = \varphi(2.48) - (1 - \varphi(2.48))$

$= \varphi(2.48) - 1 = 2 \times 0.9934 - 1 = 0.9868$

6. Here, compare with $M(t) = e^{166t+200t^2} = e^{166t+\frac{1}{2}(400)t^2} = e^{\mu t+\frac{\sigma^2}{2}t^2}$. So, mean $\mu = 166$ and variance $\sigma^2 = 400$. So, $\sigma = 20$.

$P(170 \leq X \leq 200) = P\left(\frac{170-166}{20} \leq \frac{X-166}{20} \leq \frac{200-166}{20}\right) = P(0.2 \leq Z \leq 1.7)$

$= \varphi(1.7) - \varphi(0.2) = 0.9554 - 0.5793 = 0.3761$

$P(148 \leq X \leq 172) = P\left(\frac{148-166}{20} \leq \frac{X-166}{20} \leq \frac{172-166}{20}\right) = P(-0.9 \leq Z \leq 0.3)$

$= \varphi(0.3) - \varphi(-0.9) = \varphi(0.3) - (1 - \varphi(0.9)) = 0.6179 - (1 - 0.8159) = 0.4338$

7. Here, given by $N(650,625)$. So, we get $\mu = 650$, $\sigma^2 = 625 \Rightarrow \sigma = 25$.

$P(600 \leq X \leq 660) = P\left(\frac{600-650}{25} \leq \frac{X-650}{25} \leq \frac{660-650}{25}\right) = P(-2 \leq Z \leq 0.4)$

$= \varphi(0.4) - \varphi(-2) = \varphi(0.4) - (1 - \varphi(2)) = 0.6554 - (1 - 0.9772) = 0.6326$

$P(|X - 650| \leq c) = 0.9544$

$\Rightarrow P(-c \leq X - 650 \leq c) = 0.9544$

$\Rightarrow P\left(-\frac{c}{25} \leq \frac{X-650}{25} \leq \frac{c}{25}\right) = 0.9544$

$\Rightarrow \varphi\left(\frac{c}{25}\right) - \varphi\left(-\frac{c}{25}\right) = 0.9544$

$\Rightarrow 2\varphi\left(\frac{c}{25}\right) - 1 = 0.9544$

$\Rightarrow 2\varphi\left(\frac{c}{25}\right) = 1.9544$

$\Rightarrow \varphi\left(\frac{c}{25}\right) = 0.9772$

$\Rightarrow \frac{c}{25} = 2$

$\Rightarrow c = 50$