

0.042



United International University
School of Science and Engineering
Department of Computer Science and Engineering
Final Examination Trimester- Fall – 2022
Course: Math -2205
Total marks - 40; Duration – 2 hours

[Note that the number of marks is given in brackets | | at the end of each question or part question. You are requested to answer all the questions in order.]

Q1

- (a) Two fair six-sided dice each with faces marked 1, 2, 3, 4, 5, 6, are thrown at the same time. For a single throw of the two dice, the score is the sum of the numbers on the top faces.
- Find the probability that the score is 6 on a single throw of the two dice.
 - Find the probability that the score is more than 9 on a single throw of the two dice.
 - Find the probability that the score is not greater than 4 on a single throw of the two dice.
- (b) A teacher calculates that 70% of the students do their homework regularly. If a student regularly completes their homework the probability that they will pass the examination is 0.8 and that if the students do not do the homework the probability of passing is only 0.4. Calculate the probability that a randomly selected student:
- will pass the examination
 - does not do the homework regularly given that passes the examination.

[5+5=10]

Q2

- (a) Batteries for a radio have a mean life of 160 hours and a standard deviation of 24 hours. Assuming the battery life follows a normal distribution, calculate the probability that a randomly selected battery has a life of between 150 hours to 180 hours. p
- (b) A balloon manufacturer claims that 95% of his balloons will not burst when blown up for a birthday part. In a party 16 balloons were blown up. What is the probability that none of them burst when blown up? $P(X=0)$
- (c) The random variable X has the following probability distribution shown below.
- | | | | | |
|----------|-----|-----|-----|-----|
| x | 1 | 2 | 3 | 4 |
| $P(X=x)$ | 0.2 | p | 0.1 | q |

Given that $E(X) = 2.4$. Find The values of p and q .

[4+4+2=10]

Q3.

- (a) In a particular coastal area, on average 35% of the rocks contain fossils. Jamila selects 12 rocks at random from this area and breaks them open. She finds fossils in two of the rocks. Test at the 10% level of significance if there is evidence to show the average is correct.
 $H_0: \mu = 0.35$
- (b) A small shop sells, on average, six laptops per week. Following a price hike, the number of laptops sold reduced to four per week. Test at 5% level of significance whether the sales of laptops have significantly reduced.
 $H_0: \mu \geq 6$ $H_1: \mu < 6$ $P(X \leq 4) \approx 5\%$
- (c) A nutritionist wishes to investigate the mean sugar content in some cereal bars. He takes a random sample of 10 of the bars and measures the mass, in gram and found the average 11.8 gm. Assume that variance of mass of the bar is 0.001. Calculate a 99% confidence interval for μ . [4+4+2=10]

Q4

The time for which Lucy has to wait at a certain traffic light each day is T minutes, where T has probability density function (PDF) given by

$$f(t) = \begin{cases} k \left(t - \frac{t^2}{2} \right) & 0 \leq t \leq 2 \\ 0 & \text{otherwise} \end{cases}, \text{ where } k \text{ is a constant.}$$

- Find the value of k 1.5
- Find the expected time that Lucy has to wait
- Find the probability that Lucy has to wait less than 0.5 minutes.
- Find the mode time.
- Construct the CDF for T

[10]

Related Formulae

Distribution

pmf/pdf

Binomial

$$f(x) = n_c p^x (1-p)^{n-x}; x = 0, 1, 2, \dots, n$$

Poisson

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}; x = 0, 1, 2, \dots$$

Uniform

$$f(x) = \frac{1}{b-a}; a \leq x \leq b$$

Normal

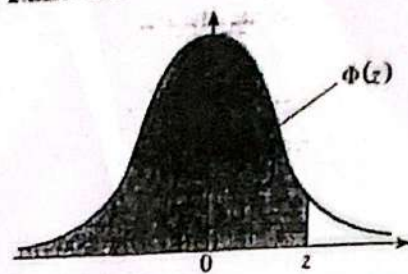
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}; -\infty < x < \infty$$

The Normal Distribution Function

If Z has a normal distribution with mean 0 and variance 1 then, for each value of z , the table gives the value of $\Phi(z)$, where

$$\Phi(z) = P(Z \leq z).$$

For negative values of z use $\Phi(-z) = 1 - \Phi(z)$.



z	0	1	2	3	4	5	6	7	8	9	ADD								
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	7	11	14	18	22	25	29	32
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	16	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	2	5	7	9	12	14	16	19	21
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	2	4	6	8	10	12	14	16	18
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	2	4	6	7	9	11	13	15	17
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	2	3	5	6	8	10	11	13	14
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	1	3	4	6	7	8	10	11	13
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	1	2	4	5	6	7	8	10	11
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	1	2	3	4	5	6	7	8	9
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	1	2	3	4	4	5	6	7	8
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	1	1	2	3	4	4	5	6	6
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	1	1	2	2	3	4	4	5	5
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	0	1	1	2	2	3	3	4	4
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	0	1	1	2	2	2	3	3	4
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	0	1	1	1	2	2	2	3	3
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	0	1	1	1	1	2	2	2	2
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	0	0	1	1	1	1	1	2	2
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0	0	0	1	1	1	1	1	1
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	0	0	0	0	1	1	1	1	1
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	0	0	0	0	0	1	1	1	1
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0	0	0	0	0	0	0	1	1
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0	0	0	0	0	0	0	0	0

Critical values for the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p , the table gives the value of z such that $P(Z \leq z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

$$P(Z=3)^2$$