Lecture:

Current, Resistance & Electromotive Force

Ref book: Fundamentals of Physics - D. Halliday, R. Resnick & J. Walker (10^{th} Ed.)

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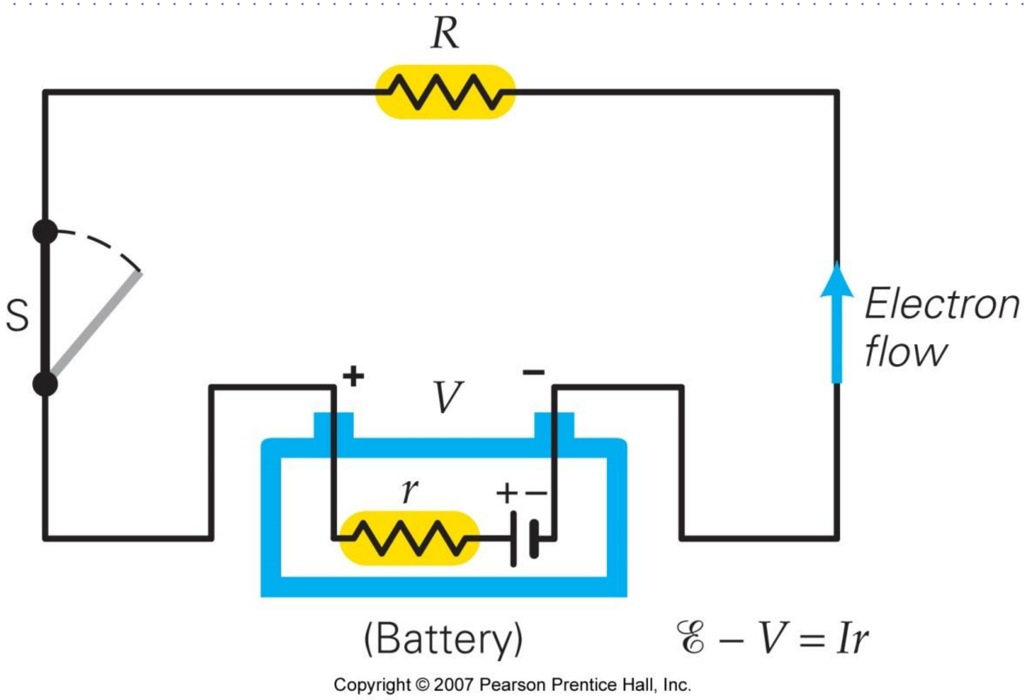
Electric Current, Resistance, and Emf

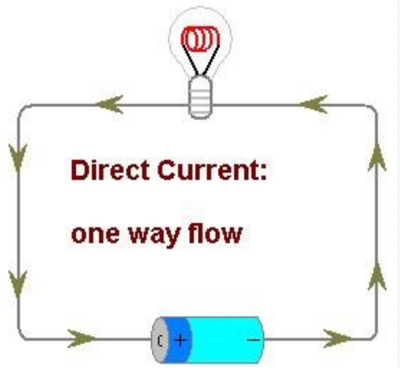


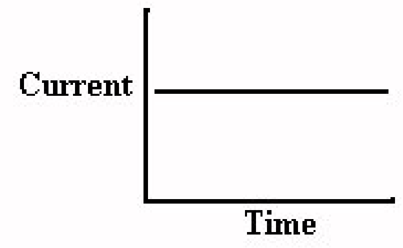
Contents

There are 2 types of Current

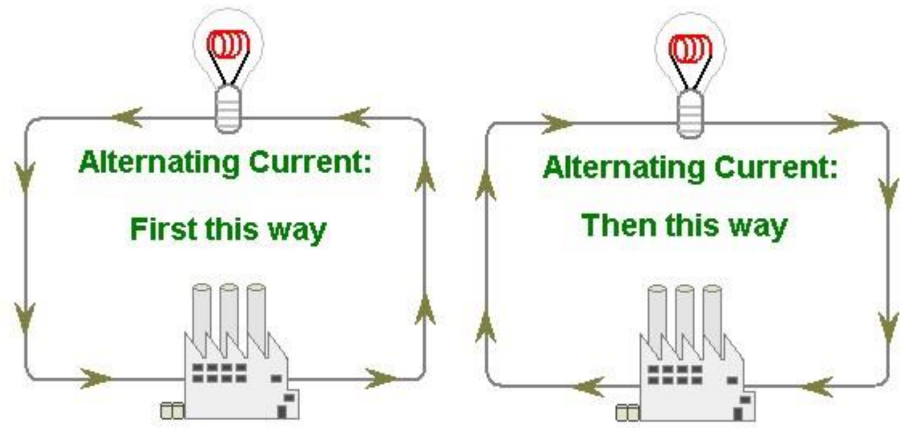
DC = Direct Current - current flows in one direction

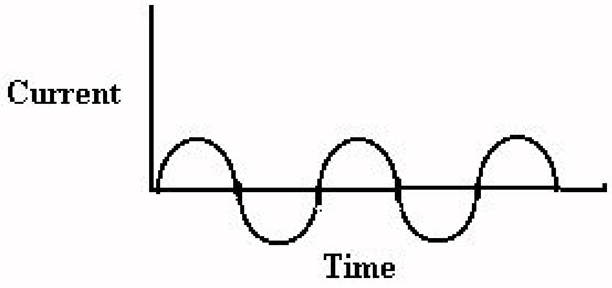
Example: Battery





AC = Alternating Current- current reverses direction many times per second. This suggests that AC devices turn OFF and ON. **Example:** Wall outlet (progress energy)

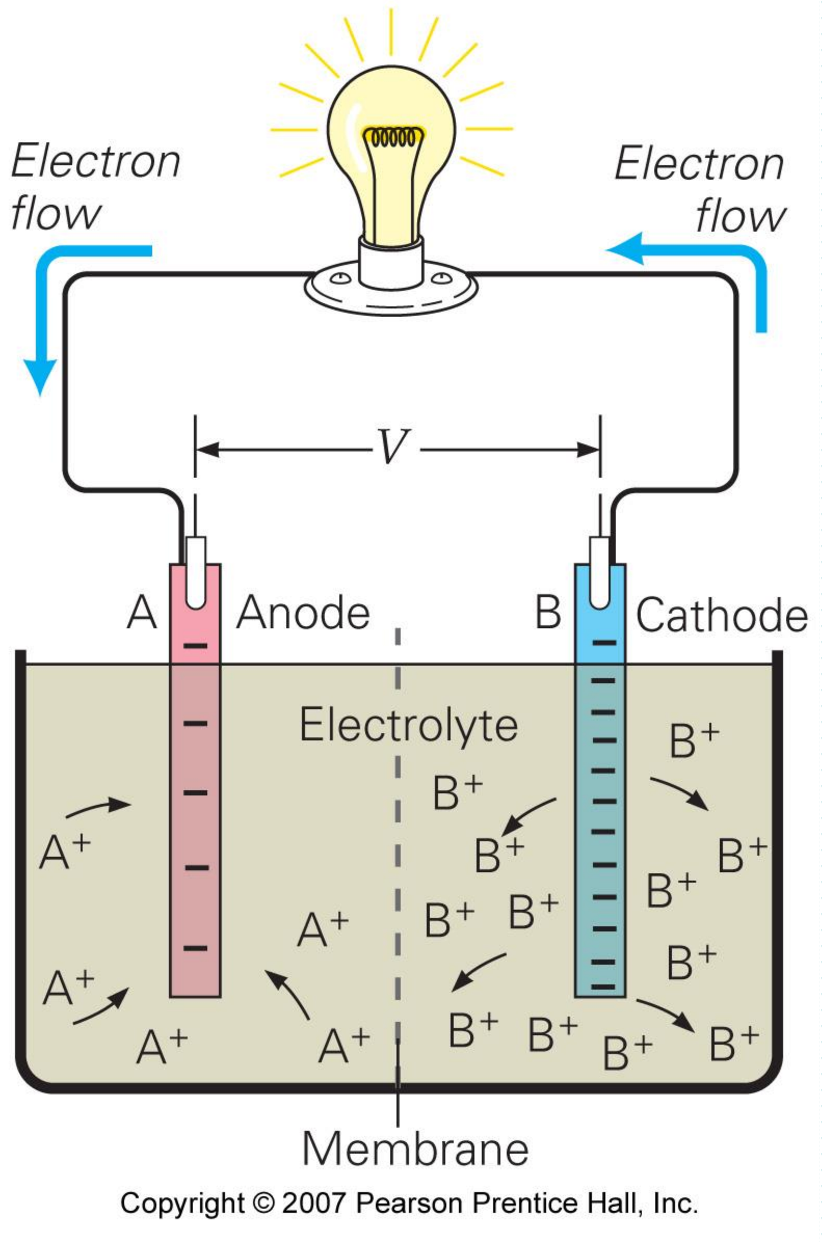




Different types of DC Voltage Sources

- Batteries
  - Electrochemical potential developed between an anode and cathode
- Fuel cells
- Solar cells (Photovoltaics)
- DC generators
  - Electromagnetic induction to produce voltage
- Thermocouples
- Piezoelectric devices, etc.

Batteries and Direct Current



Electric current is the flow of electric charge. A battery is a source of electric energy—it converts chemical energy into electric energy.

Batteries and Direct Current

In a complete circuit, electrons flow from the negative electrode to the positive one.

The positive electrode is called the anode; the negative electrode is the cathode.

A battery provides a constant source of voltage—it maintains a constant potential difference between its terminals.

Batteries and Direct Current :emf

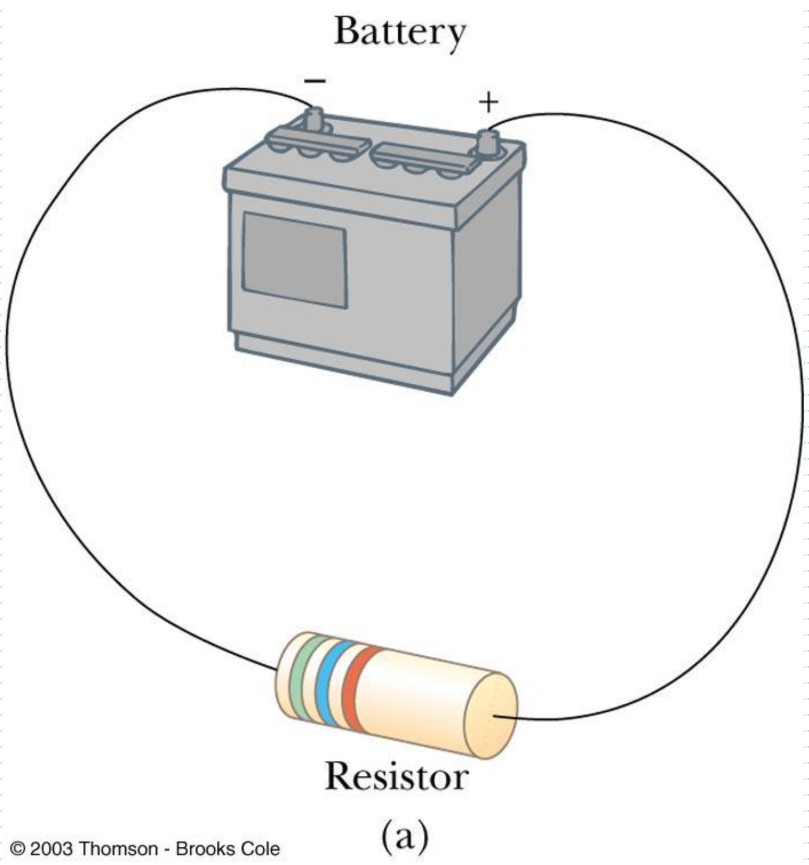
The potential difference between the battery terminals when the battery is not connected to anything is called the electromotive force, emf.

Batteries and Direct Current :emf

- The source that maintains the current in a closed circuit is called a source of *emf (electromotive force)*
  - Any devices that increase the potential energy of charges circulating in circuits are sources of emf
  - Examples include batteries and generators
- SI units are Volts
  - The emf is the work done per unit charge

EMF and Internal Resistance

- A real battery has some internal resistance
- Therefore, the terminal voltage is not equal to the emf



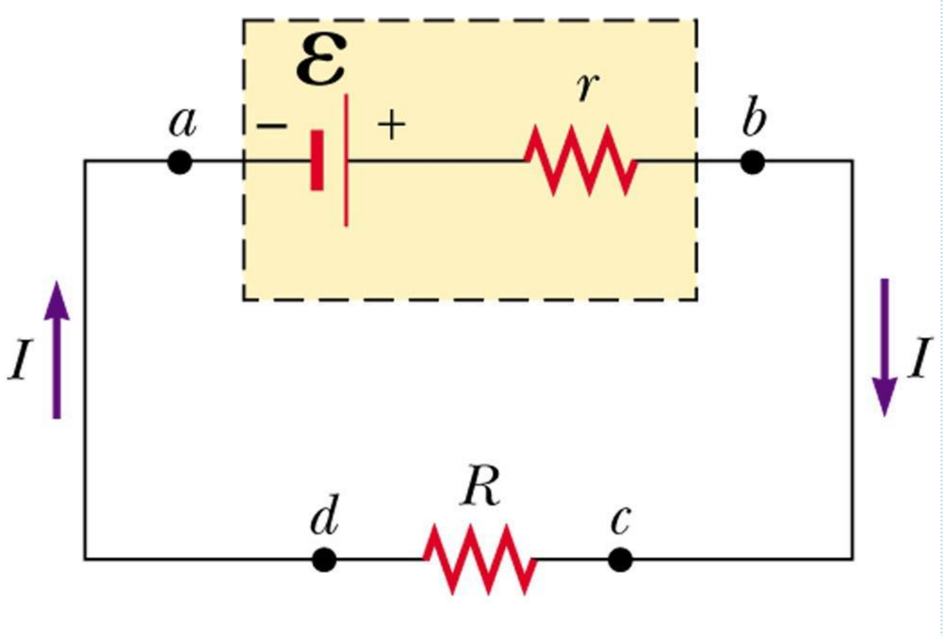
EMF and Internal Resistance

≠ **Show that** $\mathbf{I = \frac{\epsilon}{R+r}}$

- The schematic shows the internal resistance, r
- The terminal voltage is $\Delta V = V_b - V_a$
- $\Delta V = \varepsilon - Ir$
- For the entire circuit, $\varepsilon = IR + Ir$

$$\epsilon = I(R + r)$$

$$\therefore I = \frac{\epsilon}{R + r}$$



(b)

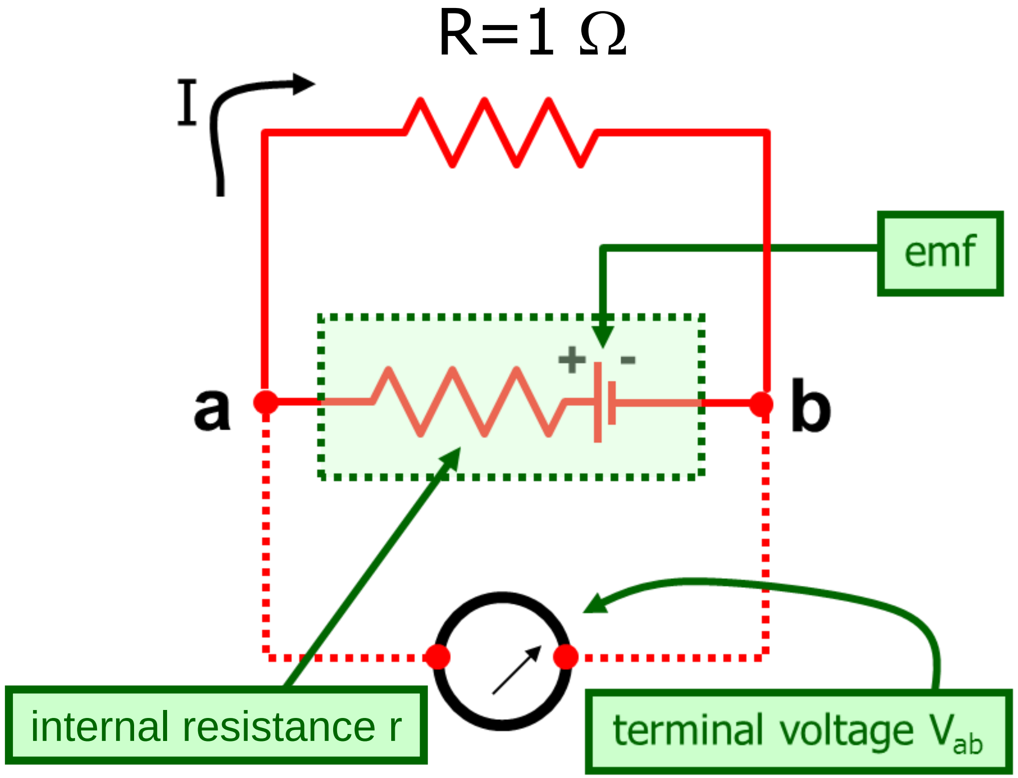
Problem-1: a battery is known to have an emf of 9 volts. If a 1 ohm resistor is connected to the battery, the terminal voltage is measured to be 3 volts. What is the internal resistance of the battery?

Solution: Because the voltmeter draws "no" current, r and R are in series with a current I flowing through both.

$$\varepsilon - Ir - IR = 0$$

IR, the potential drop across the resistor R, is also the potential difference V_{ab}.

$$V_{ab} = IR$$



the voltmeter's resistance is so large that approximately zero current flows through the voltmeter

$$\varepsilon - Ir - IR = 0$$

$$V_{ab} = IR$$

$$Ir = \varepsilon - IR$$

$$I = \frac{V_{ab}}{R}$$

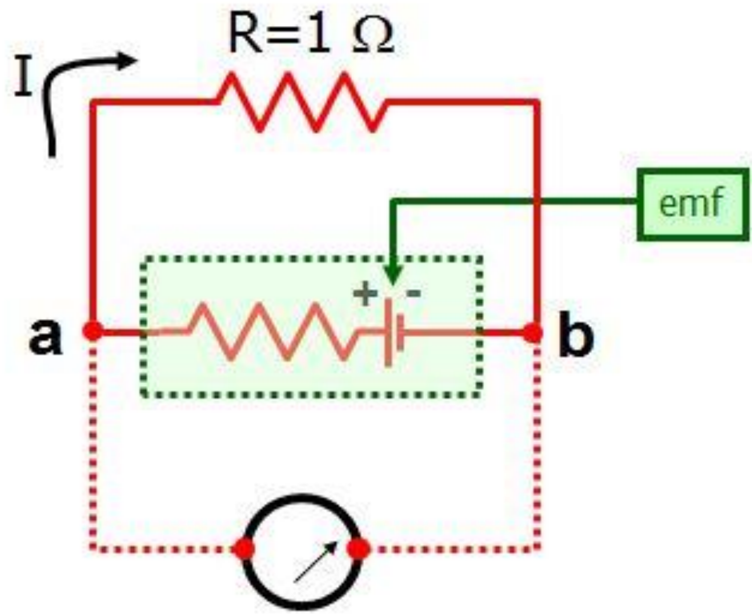
$$r = \frac{\varepsilon - IR}{I}$$

$$r = \frac{\varepsilon}{I} - R$$

$$r = \frac{\varepsilon R}{V_{ab}} - R$$

$$r = R\left(\frac{\varepsilon}{V_{ab}} - 1\right)$$

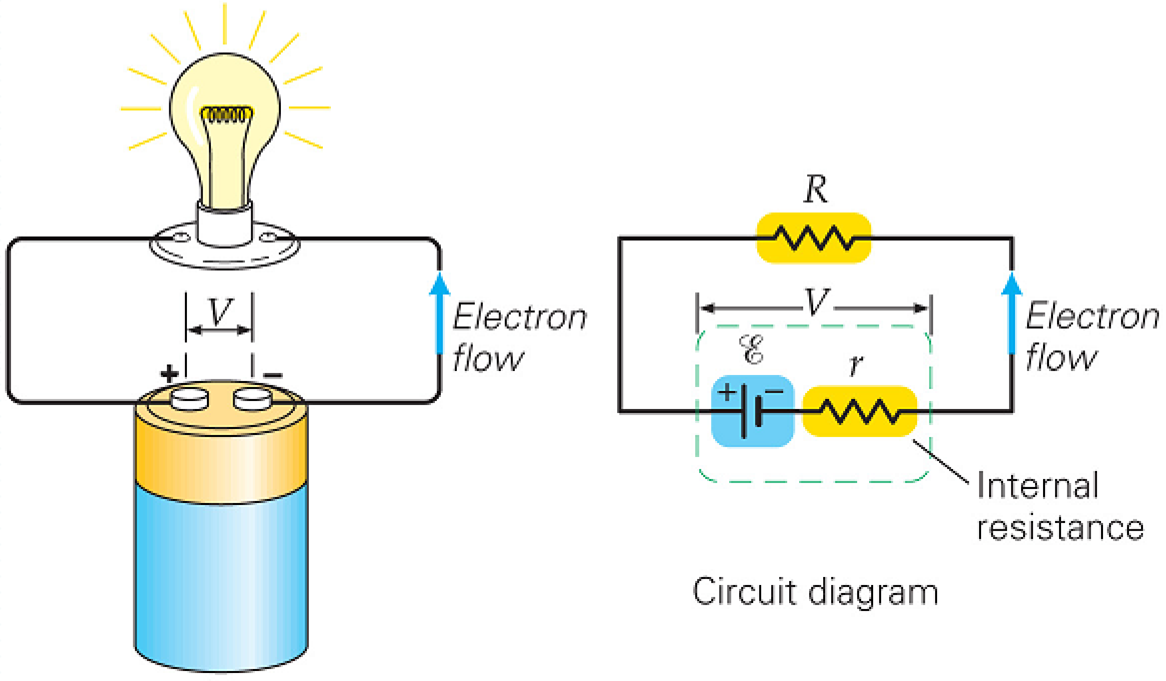
$$r = 1\left(\frac{9}{3} - 1\right) = (3 - 1) = 2\Omega$$



A rather unrealistically large value for the internal resistance of a 9V battery.

Batteries and Direct Current: Terminal Voltage

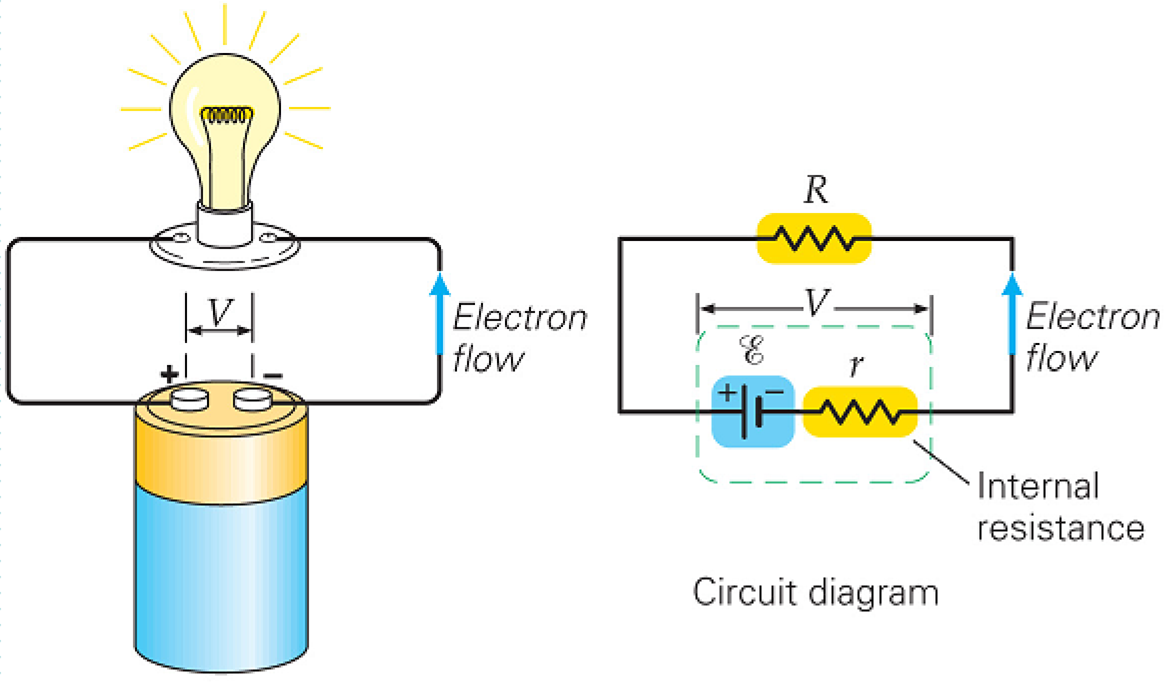
The actual terminal voltage of the battery is always less than the emf, due to internal resistance. Usually the difference is very small.



Terminal voltage

Batteries and Direct Current: Terminal Voltage

Terminal voltage is the **voltage** / potential difference across the **terminals** of a cell or battery. If the cell/battery is not connected to circuit then the **terminal voltage**/p.d is equal to the e.m.f of the cell or battery.



Terminal voltage

Batteries and Direct Current: Terminal Voltage

The actual terminal voltage of the battery is always less than the emf, due to internal resistance. Usually the difference is very small.

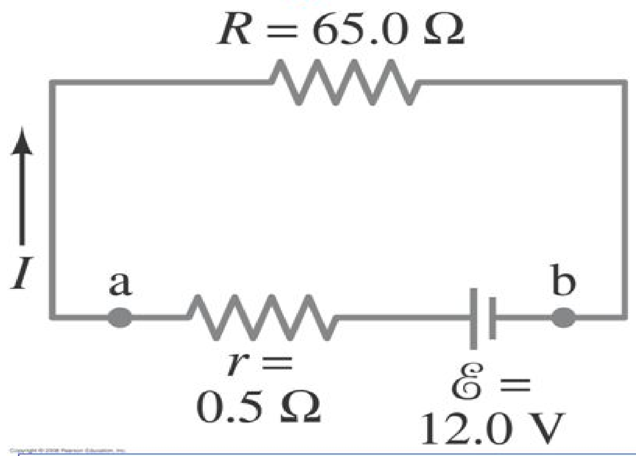
$$V_{term} = \varepsilon - (Ir)$$

Where $\mathcal{E}$ is the emf,
V_{term} is the terminal voltage,
& IR is the internal voltage drop

More on this later in the class….

Problem-2: What is the terminal voltage across the circuit?

Battery Circuit



$$\boxed{V_{ab} = \Delta V = \xi - I \cdot r}$$

$$where \quad I = \frac{\xi}{R + r}$$

- The terminal voltage is always smaller than the *EMF*

Ans: 11.9085 Volt.

Practice problem

Problem-3 : Battery with internal resistance.

Q: A 65.0-Ω resistor is connected to the terminals of a battery whose emf is 12.0 V and whose internal resistance is 0.5 Ω. Calculate (a) the current in the circuit, (b) the terminal voltage of the battery, V_{ab}, and (c) the power dissipated in the resistor R and in the battery's internal resistance r.

Solution: a. The total resistance is 65.5 Ω, so the current is 0.183 A.

b. The voltage is the emf less the voltage drop across the internal resistance: V = 11.9 V.

c. The power is I^2R; in the resistor it is 2.18 W and in the internal resistance it is 0.02 W.

Practice problem

Problem-4: •1 During the 4.0 min a 5.0 A current is set up in a wire, how many (a) coulombs and (b) electrons pass through any cross section across the wire's width?

Ans: (a)1.2×10^3 C, (b) 7.5×10^{21} Hints: $q = it$ and $q = Ne$.

Problem-5: ••2 An isolated conducting sphere has a 10 cm radius. One wire carries a current of 1.000 002 0 A into it. Another wire carries a current of 1.000 000 0 A out of it. How long would it take for the sphere to increase in potential by 1000 V?

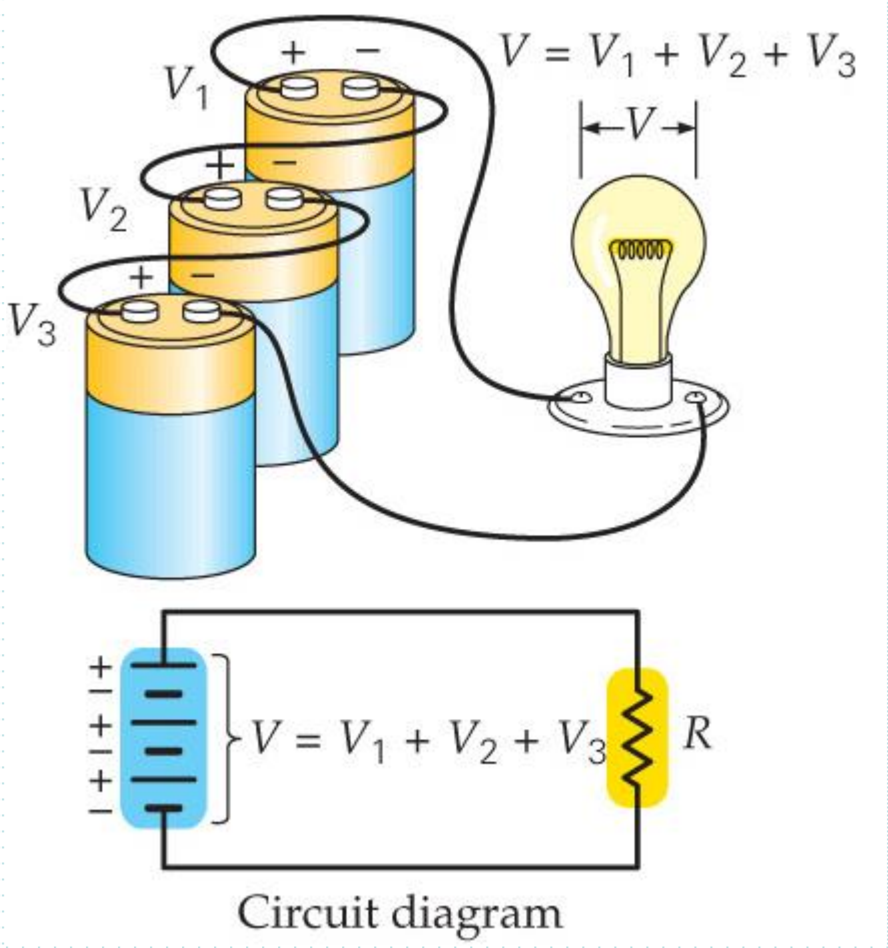
Ans: 5.6 $\text{x}10^{-3}$ s. Hints: $\Delta V = \dfrac{\Delta q}{4\pi\varepsilon_0 r}$, $\Delta q = (i_{\text{in}} - i_{\text{out}})\,\Delta t$

Practice problem

Problem-6: ••3 A charged belt, 50 cm wide, travels at 30 m/s between a source of charge and a sphere. The belt carries charge into the sphere at a rate corresponding to 100 μA. Compute the surface charge density on the belt.

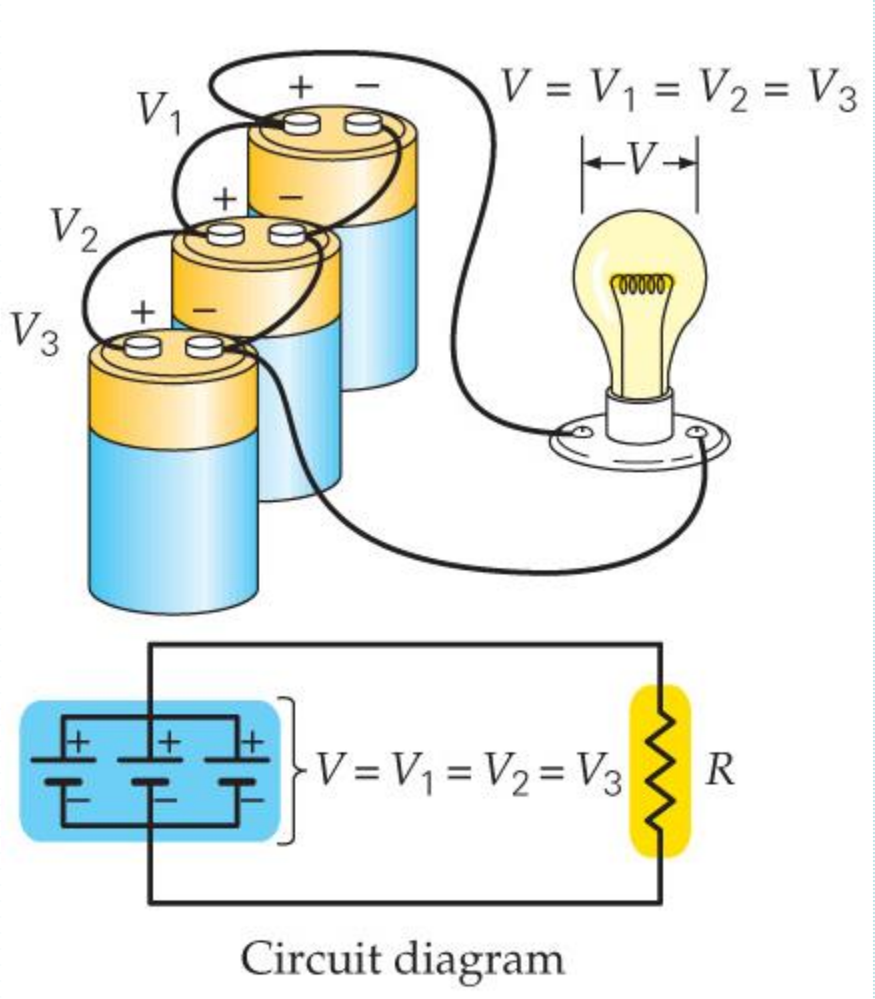
$$\sigma = \frac{i}{vw} = \frac{100 \times 10^{-6}\ \text{A}}{(30\ \text{m/s})(50 \times 10^{-2}\ \text{m})} = 6.7 \times 10^{-6}\ \text{C/m}^2.$$

Batteries and Direct Current : series emf



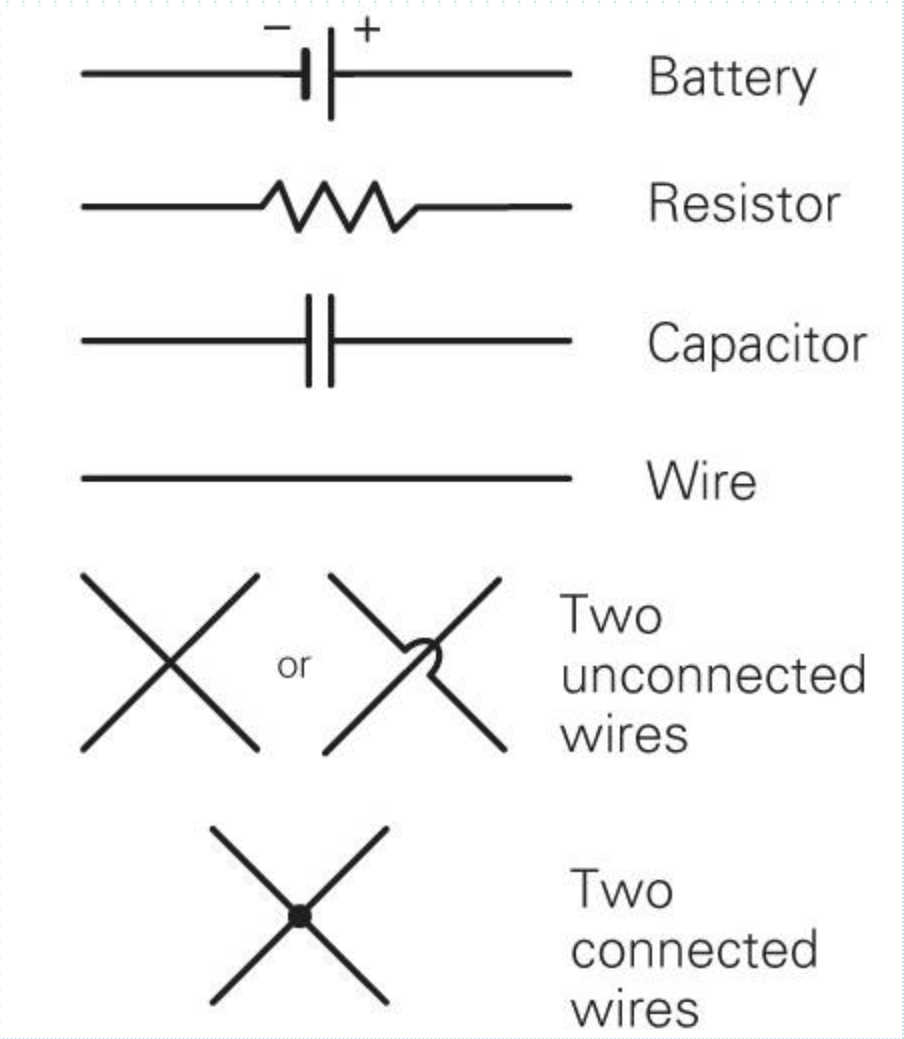
When batteries are connected in series, the total voltage is the sum of the individual voltages. The current is same.

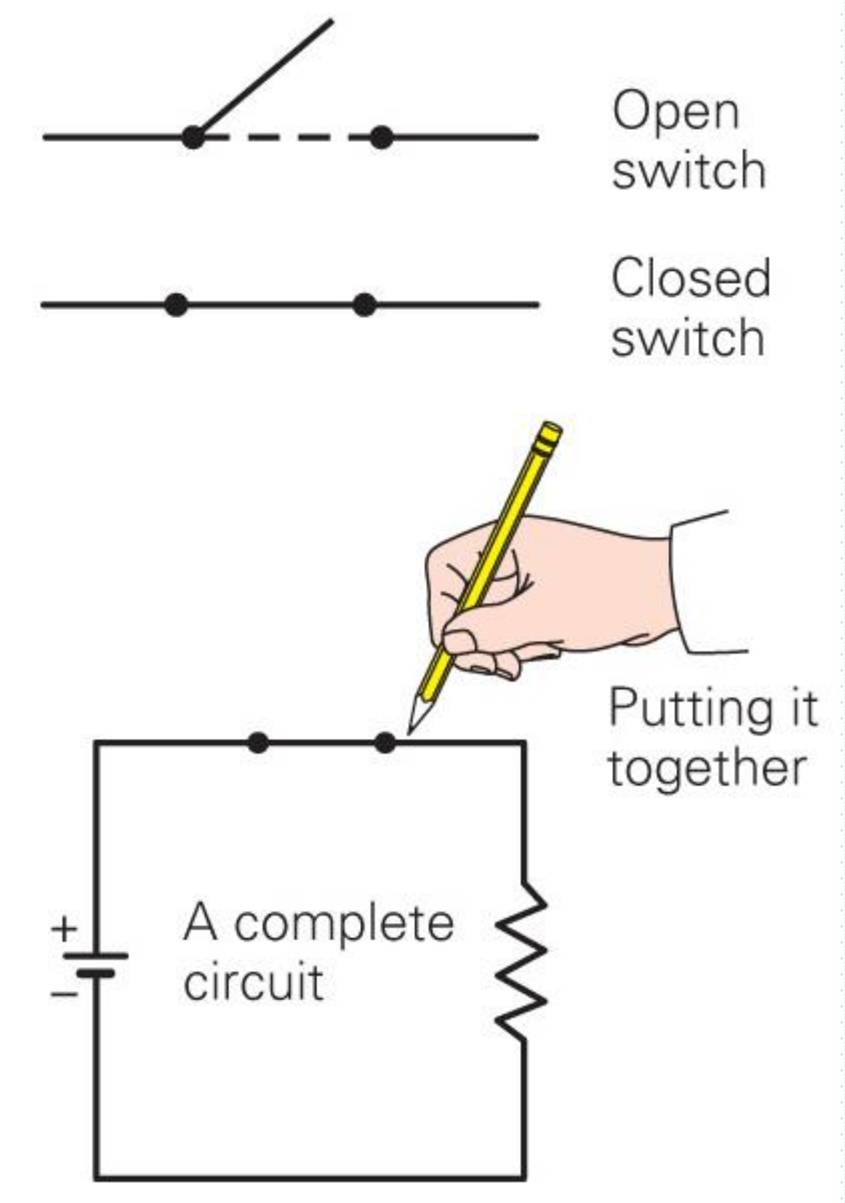
Batteries and Direct Current: parallel emf



When batteries of equal voltage are connected in parallel, the total voltage does not change; each battery supplies part of the total current.

Batteries and Direct Current





Current and Drift Velocity

Current is the time rate of flow of charge.

$$I = \frac{q}{t}$$

SI unit of current: the ampere, A

If charge dq passes through a hypothetical plane (such as $a\acute{a}$) in time dt, then the current i through that plane is defined as

$$i = \frac{dq}{dt} \quad \text{(definition of current).}$$

The current is the same in any cross section.

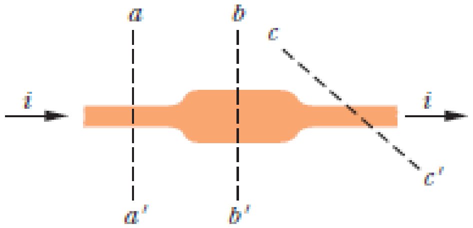


Figure 26-2 The current i through the conductor has the same value at planes aa', bb', and cc'.

We can find the charge that passes through the plane in a time interval extending from 0 to t by integration:

$$q = \int dq = \int_0^t i\, dt,$$

in which the current i may vary with time.

Under steady-state conditions, the current is the same for planes aa', bb', and cc' and indeed for all planes that pass completely through the conductor, no matter what their location or orientation. This follows from the fact that charge is conserved.

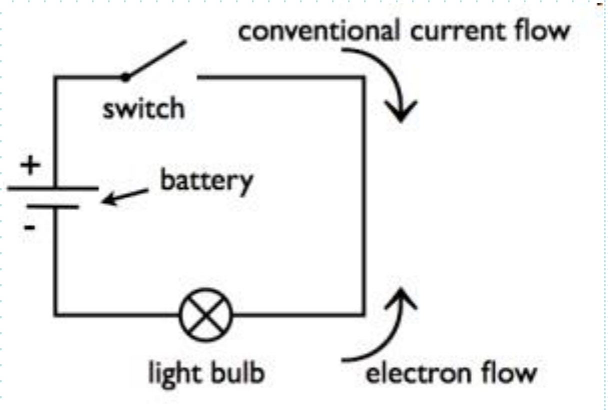
Current and Drift Velocity

Current is defined as the rate at which charge flows through a surface.

$$I = \frac{q}{t} = \frac{dq}{qt} = \frac{\text{Coulombs}}{\text{seconds}} = Ampere = Amp$$

The current is in the same direction as the flow of ***positive*** charge (for this course)

Note: The "I" stands for *intensity*



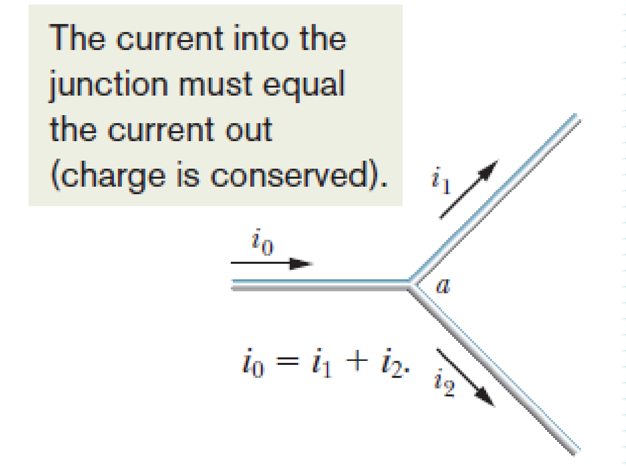
Current and Drift Velocity

- The unit of current is the *ampere* (A), or *amp* for short. It is named for the French physicist André-Marie Ampère (1775–1836).
- **A current of 1 amp is defined as the flow of 1 coulomb of charge in 1 second:**

  $$\mathbf{1\ A = 1\ C/s}$$

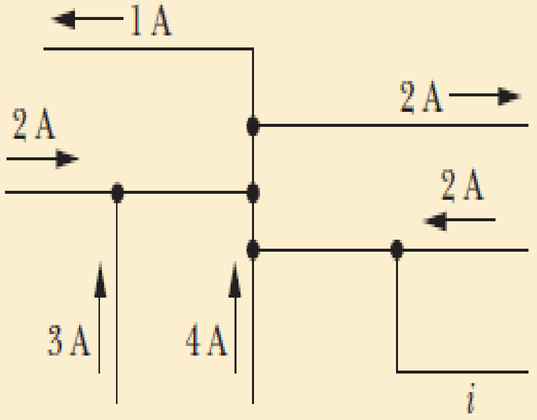
- A 1-amp current is fairly strong. Many electronic devices, like cell phones and digital music players, operate on currents that are a fraction of an amp.
- **NOTE:** 1 C = the charge of 6,240,000,000,000,000,000 electrons

Current and Drift Velocity



Problem-7:

The figure here shows a portion of a circuit. What are the magnitude and direction of the current i in the lower right-hand wire?



Ans: 8A and direction outward

Current and Drift Velocity

Problem-8: 3.8×10^{21} electrons pass through a point in a wire in 4 minutes. What was the average current?

$$I_{av} = \frac{\Delta Q}{\Delta t} = \frac{Ne}{\Delta t}$$

$$I_{av} = \frac{\left(3.8 \times 10^{21}\right)\left(1.6 \times 10^{-19}\right)}{\left(4 \times 60\right)}$$

$$I_{av} = 2.53\text{A}$$

Current and Drift Velocity

Problem-9: the 12-gauge copper wire in a home has a cross-sectional area of 3.31×10^{-6} m^2 and carries a current of 10 A. The conduction electron density in copper is 8.49×10^{28} electrons/m^3. Calculate the drift speed of the electrons.

$$v_d = \frac{I}{nqA}$$

$$|v_d| = \frac{I}{neA}$$

$$|v_d| = \frac{10\,\mathrm{C/s}}{(8.49\times10^{28}\ \mathrm{m}^{-3})(1.60\times10^{-19}\ \mathrm{C})(3.31\times10^{-6}\ \mathrm{m}^2)}$$

$$|v_d| = 2.22\times10^{-4}\ \mathrm{m/s}$$

Current and Drift Velocity

Problem-10: Water flows through a garden hose at a volume flow rate dV/dt of 450 cm^3/s. What is the current of negative charge?

Calculations: We can write the current in terms of the number of molecules that pass through such a plane per second as

$$i = \begin{pmatrix} \text{charge} \\ \text{per} \\ \text{electron} \end{pmatrix} \begin{pmatrix} \text{electrons} \\ \text{per} \\ \text{molecule} \end{pmatrix} \begin{pmatrix} \text{molecules} \\ \text{per} \\ \text{second} \end{pmatrix}$$

or

$$i = (e)(10)\frac{dN}{dt}.$$

Current and Drift Velocity

We substitute 10 electrons per molecule because a water (H_2O) molecule contains 8 electrons in the single oxygen atom and 1 electron in each of the two hydrogen atoms.

We can express the rate dN/dt in terms of the given volume flow rate dV/dt by first writing

$$\begin{pmatrix}\text{molecules}\\ \text{per}\\ \text{second}\end{pmatrix} = \begin{pmatrix}\text{molecules}\\ \text{per}\\ \text{mole}\end{pmatrix}\begin{pmatrix}\text{moles}\\ \text{per unit}\\ \text{mass}\end{pmatrix} \times \begin{pmatrix}\text{mass}\\ \text{per unit}\\ \text{volume}\end{pmatrix}\begin{pmatrix}\text{volume}\\ \text{per}\\ \text{second}\end{pmatrix}.$$

"Molecules per mole" is Avogadro's number N_A. "Moles per unit mass" is the inverse of the mass per mole, which is the molar mass M of water. "Mass per unit volume" is the (mass) density ρ_{mass} of water. The volume per second is the volume flow rate dV/dt. Thus, we have

$$\frac{dN}{dt} = N_A\left(\frac{1}{M}\right)\rho_{\text{mass}}\left(\frac{dV}{dt}\right) = \frac{N_A\rho_{\text{mass}}}{M}\frac{dV}{dt}.$$

Substituting this into the equation for i, we find

$$i = 10eN_AM^{-1}\rho_{\text{mass}}\frac{dV}{dt}.$$

We know that Avogadro's number N_A is 6.02×10^{23} molecules/mol, or $6.02 \times 10^{23}\ \text{mol}^{-1}$, and from Table 15-1 we know that the density of water ρ_{mass} under normal conditions is $1000\ \text{kg/m}^3$. We can get the molar mass of water from the molar masses listed in Appendix F (in grams per mole): We add the molar mass of oxygen (16 g/mol) to twice the molar mass of hydrogen (1 g/mol), obtaining 18 g/mol = 0.018 kg/mol. So, the current of negative charge due to the electrons in the water is

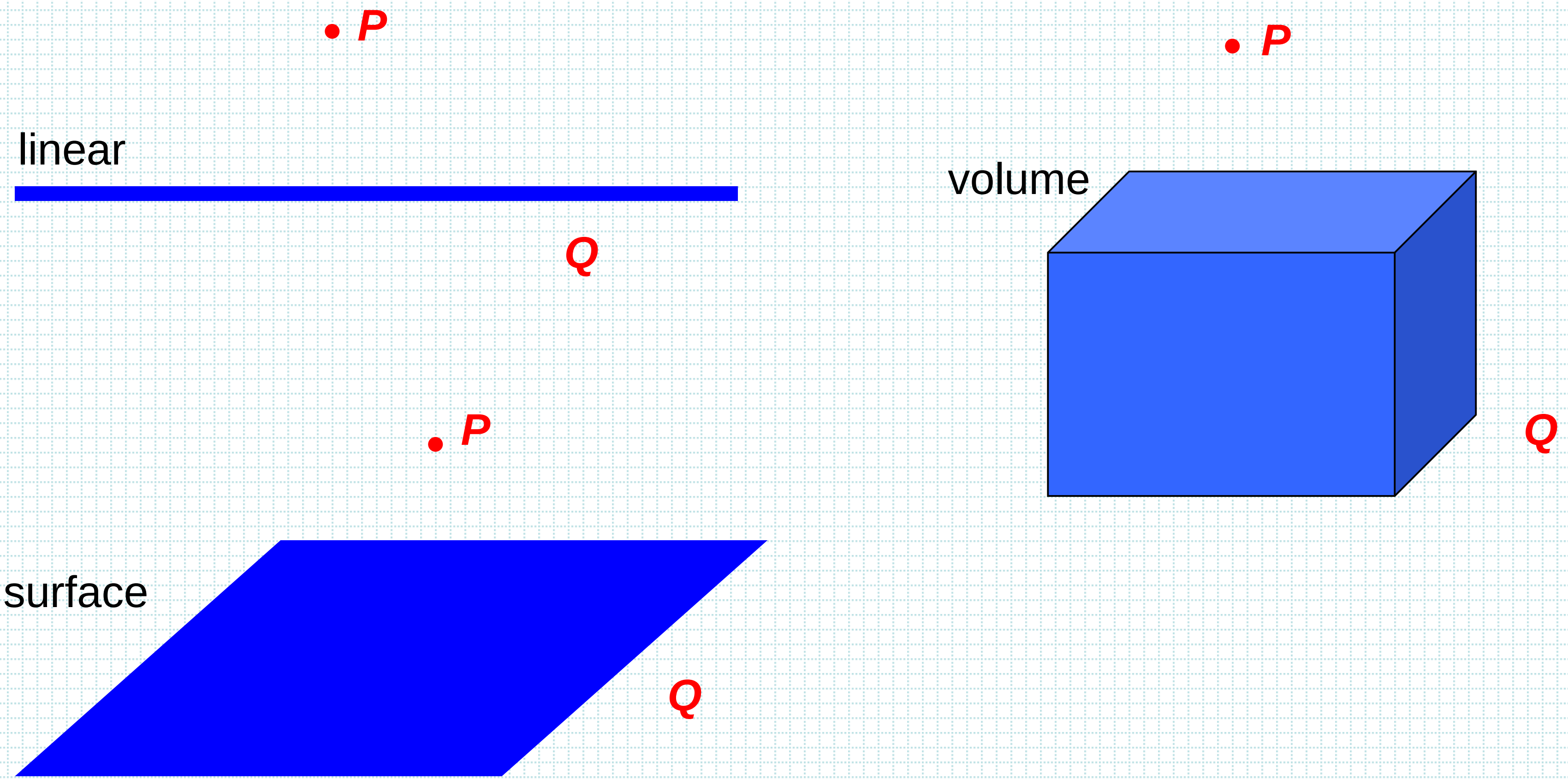
$$\begin{aligned} i &= (10)(1.6 \times 10^{-19}\ \text{C})(6.02 \times 10^{23}\ \text{mol}^{-1}) \\ &\quad \times (0.018\ \text{kg/mol})^{-1}(1000\ \text{kg/m}^3)(450 \times 10^{-6}\ \text{m}^3/\text{s}) \\ &= 2.41 \times 10^7\ \text{C/s} = 2.41 \times 10^7\ \text{A} \\ &= 24.1\ \text{MA}. \qquad \text{(Answer)} \end{aligned}$$

This current of negative charge is exactly compensated by a current of positive charge associated with the nuclei of the three atoms that make up the water molecule. Thus, there is no net flow of charge through the hose.

Current and Drift Velocity: Charge Density

Continuous Charge Distribution

The total electric charge is ***Q*.** What is the electric field at point ***P*** ?



Current and Drift Velocity: Charge Density

Continuous Charge Distribution: Charge Density

The total electric charge is **Q**.

Linear, length ***L***

Q

Amount of charge in a small volume ***dl***:

$$dq = \frac{Q}{L}dl = \lambda dl \qquad \lambda = \frac{Q}{L}$$

Linear charge density

Volume ***V***

Q

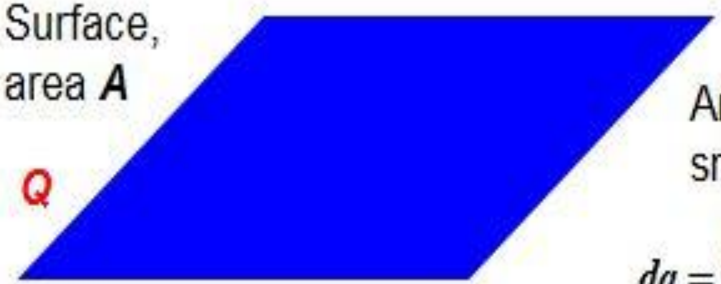
Amount of charge in a small volume ***dV***:

$$dq = \frac{Q}{V}dV = \rho dV \qquad \rho = \frac{Q}{V}$$

Volume charge density

Surface, area ***A***

Q



Amount of charge in a small volume ***dA***:

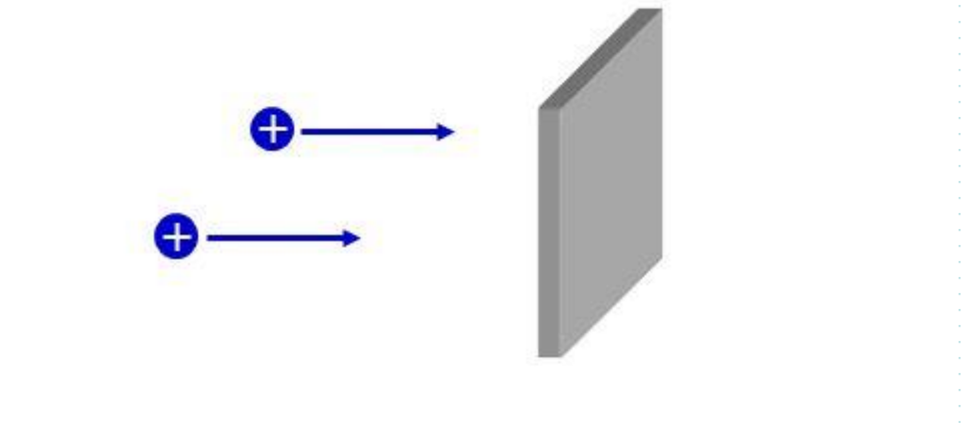
$$dq = \frac{Q}{A}dA = \sigma dA \qquad \sigma = \frac{Q}{A}$$

Surface charge density

Current and Drift Velocity: Current Density

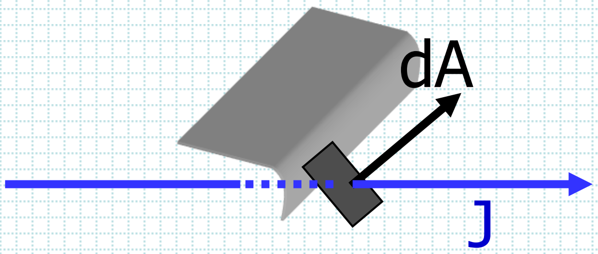
When we study details of charge transport, we use the concept of current density.

Current density is the amount of charge that flows across a unit of area in a unit of time.



Current density: charge per area per time.

A current density J flowing through an infinitesimal area dA produces an infinitesimal current dI.



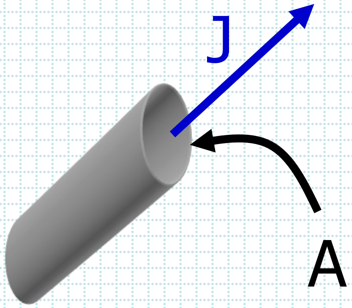
$$dI = \vec{J} \cdot d\vec{A}$$

Current density **is a vector**. Its direction is the direction of the velocity of positive charge carriers.

The total current passing through A is just

$$I = \int_{\text{surface}} \vec{J} \cdot d\vec{A}$$

Current and Drift Velocity: Current Density



If J is constant and parallel to dA (like in a wire), then

$$I = \int_{\text{surface}} \vec{J} \cdot d\vec{A} = J \int_{\text{surface}} dA = JA \quad \Rightarrow \quad J = \frac{I}{A}$$

Unit: A/m^2

Simply, Current per unit area is called current density.

This is a convenient concept for relating the microscopic motions of electrons to the macroscopic current.

Current and Drift Velocity: Current Density

Current density:

$$i = \int \vec{J} \cdot d\vec{A}. \qquad (26\text{-}4)$$

If the current is uniform across the surface and parallel to $d\vec{A}$, then $\vec{J}$ is also uniform and parallel to $d\vec{A}$. Then Eq. 26-4 becomes

$$i = \int J\,dA = J \int dA = JA,$$

so

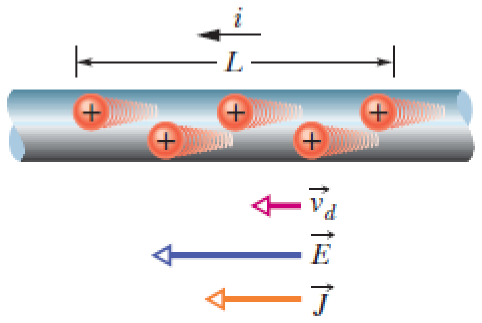
$$J = \frac{i}{A}, \qquad (26\text{-}5)$$

where A is the total area of the surface. From Eq. 26-4 or 26-5 we see that the SI unit for current density is the ampere per square meter (A/m^2).

Current and Drift Velocity

#Establish a relation for drift velocity/Derive a relation for Current density.

Current is said to be due to positive charges that are propelled by the electric field.



convenience, Fig. 26-5 shows the equivalent drift of *positive* charge carriers in the direction of the applied electric field $\vec{E}$. Let us assume that these charge carriers all move with the same drift speed v_d and that the current density J is uniform across the wire's cross-sectional area A. The number of charge carriers in a length L of the wire is nAL, where n is the number of carriers per unit volume. The total charge of the carriers in the length L, each with charge e, is then

$$q = (nAL)e.$$

Because the carriers all move along the wire with speed v_d, this total charge moves through any cross section of the wire in the time interval

$$t = \frac{L}{v_d}.$$

Equation 26-1 tells us that the current i is the time rate of transfer of charge across a cross section, so here we have

$$i = \frac{q}{t} = \frac{nALe}{L/v_d} = nAev_d. \qquad (26\text{-}6)$$

Solving for v_d and recalling Eq. 26-5 ($J = i/A$), we obtain

$$v_d = \frac{i}{nAe} = \frac{J}{ne}$$

or, extended to vector form,

$$\vec{J} = (ne)\vec{v}_d. \qquad (26\text{-}7)$$

Current and Drift Velocity

Problem-11: (a) The current density in a cylindrical wire of radius $R = 2.0$ mm is uniform across a cross section of the wire and is $J = 2.0 \times 10^5$ A/m^2. What is the current through the outer portion of the wire between radial distances $R/2$ and R (Fig. 26-6*a*)?

KEY IDEA

Because the current density is uniform across the cross section, the current density J, the current i, and the cross-sectional area A are related by Eq. 26-5 ($J = i/A$).

Calculations: We want only the current through a reduced cross-sectional area A' of the wire (rather than the entire area), where

$$A' = \pi R^2 - \pi\left(\frac{R}{2}\right)^2 = \pi\left(\frac{3R^2}{4}\right)$$

$$= \frac{3\pi}{4}(0.0020\text{ m})^2 = 9.424 \times 10^{-6}\text{ m}^2.$$

So, we rewrite Eq. 26-5 as

$$i = JA'$$

and then substitute the data to find

$$i = (2.0 \times 10^5\text{ A/m}^2)(9.424 \times 10^{-6}\text{ m}^2)$$

$$= 1.9\text{ A}. \qquad \text{(Answer)}$$

Current and Drift Velocity

(b) Suppose, instead, that the current density through a cross section varies with radial distance r as $J = ar^2$, in which $a = 3.0 \times 10^{11}$ A/m^4 and r is in meters. What now is the current through the same outer portion of the wire?

KEY IDEA

Because the current density is not uniform across a cross section of the wire, we must resort to Eq. 26-4 ($i = \int \vec{J} \cdot d\vec{A}$) and integrate the current density over the portion of the wire from $r = R/2$ to $r = R$.

Calculations: The current density vector $\vec{J}$ (along the wire's length) and the differential area vector $d\vec{A}$ (perpendicular to a cross section of the wire) have the same direction. Thus,

$$\vec{J} \cdot d\vec{A} = J\,dA \cos 0 = J\,dA.$$

We need to replace the differential area dA with something we can actually integrate between the limits $r = R/2$ and $r = R$. The simplest replacement (because J is given as a function of r) is the area $2\pi r\,dr$ of a thin ring of circumference $2\pi r$ and width dr (Fig. 26-6b). We can then integrate with r as the variable of integration. Equation 26-4 then gives us

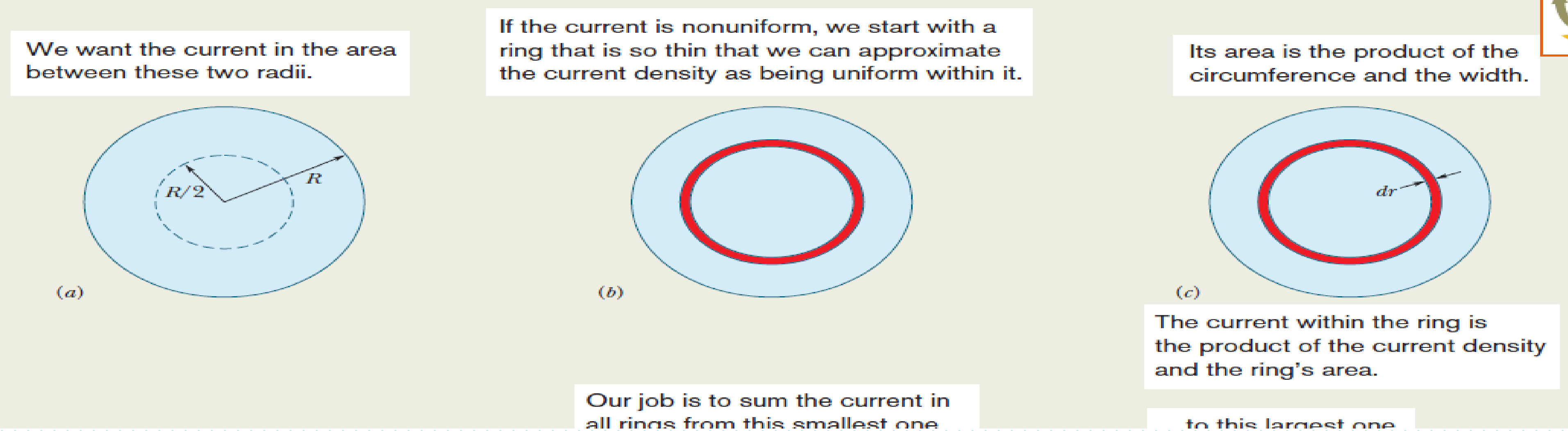
$$i = \int \vec{J} \cdot d\vec{A} = \int J\,dA$$

$$= \int_{R/2}^{R} ar^2\, 2\pi r\,dr = 2\pi a \int_{R/2}^{R} r^3\,dr$$

$$= 2\pi a \left[\frac{r^4}{4}\right]_{R/2}^{R} = \frac{\pi a}{2}\left[R^4 - \frac{R^4}{16}\right] = \frac{15}{32}\pi a R^4$$

$$= \frac{15}{32}\pi(3.0 \times 10^{11}\ \text{A/m}^4)(0.0020\ \text{m})^4 = 7.1\ \text{A}.$$

(Answer)



Current and Drift Velocity

Problem-12:

What is the drift speed of the conduction electrons in a copper wire with radius $r = 900\ \mu m$ when it has a uniform current $i = 17$ mA? Assume that each copper atom contributes one conduction electron to the current and that the current density is uniform across the wire's cross section.

KEY IDEAS

1. The drift speed v_d is related to the current density $\vec{J}$ and the number n of conduction electrons per unit volume according to Eq. 26-7, which we can write as $J = nev_d$.
2. Because the current density is uniform, its magnitude J is related to the given current i and wire size by Eq. 26-5 ($J = i/A$, where A is the cross-sectional area of the wire).
3. Because we assume one conduction electron per atom, the number n of conduction electrons per unit volume is the same as the number of atoms per unit volume.

Calculations: Let us start with the third idea by writing

$$n = \begin{pmatrix}\text{atoms} \\ \text{per unit} \\ \text{volume}\end{pmatrix} = \begin{pmatrix}\text{atoms} \\ \text{per} \\ \text{mole}\end{pmatrix}\begin{pmatrix}\text{moles} \\ \text{per unit} \\ \text{mass}\end{pmatrix}\begin{pmatrix}\text{mass} \\ \text{per unit} \\ \text{volume}\end{pmatrix}.$$

The number of atoms per mole is just Avogadro's number N_A ($= 6.02 \times 10^{23}\ \text{mol}^{-1}$). Moles per unit mass is the inverse of the mass per mole, which here is the molar mass M of copper. The mass per unit volume is the (mass) density ρ_{mass} of copper. Thus,

$$n = N_A\left(\frac{1}{M}\right)\rho_{\text{mass}} = \frac{N_A \rho_{\text{mass}}}{M}.$$

Taking copper's molar mass M and density ρ_{mass} from Appendix F, we then have (with some conversions of units)

$$n = \frac{(6.02 \times 10^{23}\ \text{mol}^{-1})(8.96 \times 10^{3}\ \text{kg/m}^3)}{63.54 \times 10^{-3}\ \text{kg/mol}}$$
$$= 8.49 \times 10^{28}\ \text{electrons/m}^3$$

or

$$n = 8.49 \times 10^{28}\ \text{m}^{-3}.$$

Next let us combine the first two key ideas by writing

$$\frac{i}{A} = nev_d.$$

Substituting for A with πr^2 ($= 2.54 \times 10^{-6}\ \text{m}^2$) and solving for v_d, we then find

$$v_d = \frac{i}{ne(\pi r^2)}$$
$$= \frac{17 \times 10^{-3}\ \text{A}}{(8.49 \times 10^{28}\ \text{m}^{-3})(1.6 \times 10^{-19}\ \text{C})(2.54 \times 10^{-6}\ \text{m}^2)}$$
$$= 4.9 \times 10^{-7}\ \text{m/s}, \qquad \text{(Answer)}$$

which is only 1.8 mm/h, slower than a sluggish snail.

Lights are fast: You may well ask: "If the electrons drift so slowly, why do the room lights turn on so quickly when I throw the switch?" Confusion on this point results from not distinguishing between the drift speed of the electrons and the speed at which *changes* in the electric field configuration travel along wires. This latter speed is nearly that of light; electrons everywhere in the wire begin drifting almost at once, including into the lightbulbs. Similarly, when you open the valve on your garden hose with the hose full of water, a pressure wave travels along the hose at the speed of sound in water. The speed at which the water itself moves through the hose—measured perhaps with a dye marker—is much slower.

Practice Problem

Problem-13 : Electron speeds in a wire.

Q: A copper wire 3.2 mm in diameter carries a 5.0-A current. Determine (a) the current density in the wire, and (b) the drift velocity of the free electrons. (c) Estimate the rms speed of electrons assuming they behave like an ideal gas at 20°C. Assume that one electron per Cu atom is free to move (the others remain bound to the atom).

Ans: (a) 6.2×10^5 A/m^2 [j = I/A =]
(b) 4.6 x 10-5 m/s. [j/ne =]
(c) 1.2×10^5 m/s [$\sqrt{3KT/m}$]

Practice Problem

Problem-14: Electric field inside a wire.

Q: What is the electric field inside the wire of Example 25–14 (copper wire)? (The current density was found to be 6.2 x 10^5 A/m^2.) The resistivity of copper wire=1.69x10^{-8} Ωm.

Ans: 0.01 V/m.[E = ρj =]

Current and Drift Velocity

 # Derive a relation for resistivity or Establish a relation among conductivity, current density, and electric field.

$$\rho = \frac{E}{J} \quad \text{(definition of } \rho\text{)}. \tag{26-10}$$

(Compare this equation with Eq. 26-8.)

If we combine the SI units of E and J according to Eq. 26-10, we get, for the unit of ρ, the ohm-meter ($\Omega \cdot m$):

$$\frac{\text{unit }(E)}{\text{unit }(J)} = \frac{V/m}{A/m^2} = \frac{V}{A}\,m = \Omega \cdot m.$$

(Do not confuse the *ohm-meter*, the unit of resistivity, with the *ohmmeter*, which is an instrument that measures resistance.) Table 26-1 lists the resistivities of some materials.

We can write Eq. 26-10 in vector form as

$$\vec{E} = \rho\vec{J}. \tag{26-11}$$

Equations 26-10 and 26-11 hold only for *isotropic* materials—materials whose electrical properties are the same in all directions.

We often speak of the **conductivity** σ of a material. This is simply the reciprocal of its resistivity, so

$$\sigma = \frac{1}{\rho} \quad \text{(definition of } \sigma\text{)}. \tag{26-12}$$

The SI unit of conductivity is the reciprocal ohm-meter, $(\Omega \cdot m)^{-1}$. The unit name mhos per meter is sometimes used (mho is ohm backwards). The definition of σ allows us to write Eq. 26-11 in the alternative form

$$\vec{J} = \sigma\vec{E}. \tag{26-13}$$

Current and Drift Velocity

$$E = V/L \quad \text{and} \quad J = i/A. \tag{26-14}$$

We can then combine Eqs. 26-10 and 26-14 to write

$$\rho = \frac{E}{J} = \frac{V/L}{i/A}. \tag{26-15}$$

However, V/i is the resistance R, which allows us to recast Eq. 26-15 as

$$R = \rho\frac{L}{A}. \tag{26-16}$$

Current and Drift Velocity : Resistivity

What is Resistivity?

Resistivity is just a property of the conductor.

Every material has a **resistivity**.

It is actually the **resistance** of a **1m long** piece of wire with a cross-sectional **area of 1m²**.

As you can imagine this is always a very low number.

For Copper $\rho = 1.72 \times 10^{-8}\ \Omega m$

$$\boxed{\rho = \frac{RA}{L}}$$

$$\boxed{\text{Units} = \Omega m}$$

Current and Drift Velocity : Conductance & Conductivity

Conductance is the reciprocal of resistance. Mathematically,

$$G = R^{-1} = i/v$$

Unit for conductance is S (siemens) or (mhos) or (ohm^{-1})

$$G = A\sigma/L$$

where σ is conductivity,

which is the inverse of resistivity, ρ

The inverse of resistivity is conductivity.

Unit for conductivity is $(\Omega m)^{-1}$

Current and Drift Velocity

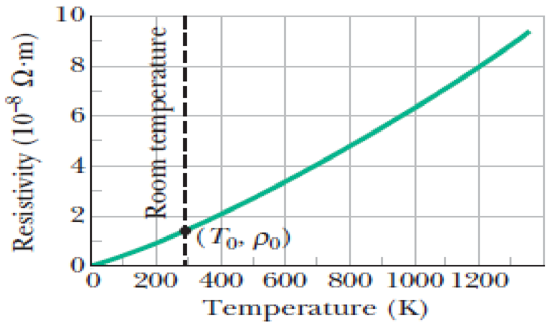
Variation with Temperature

The values of most physical properties vary with temperature, and resistivity is no exception. Figure 26-10, for example, shows the variation of this property for copper over a wide temperature range. The relation between temperature and resistivity for copper—and for metals in general—is fairly linear over a rather broad temperature range. For such linear relations we can write an empirical approximation that is good enough for most engineering purposes:

$$\rho - \rho_0 = \rho_0\alpha(T - T_0). \tag{26-17}$$

Here T_0 is a selected reference temperature and ρ_0 is the resistivity at that temperature. Usually $T_0 = 293$ K (room temperature), for which $\rho_0 = 1.69 \times 10^{-8}\ \Omega\cdot\text{m}$ for copper.

Because temperature enters Eq. 26-17 only as a difference, it does not matter whether you use the Celsius or Kelvin scale in that equation because the sizes of degrees on these scales are identical. The quantity α in Eq. 26-17, called the *temperature coefficient of resistivity,* is chosen so that the equation gives good agreement with experiment for temperatures in the chosen range. Some values of α for metals are listed in Table 26-1.



Resistivity can depend on temperature.

Figure 26-10 The resistivity of copper as a function of temperature. The dot on the curve marks a convenient reference point at temperature $T_0 = 293$ K and resistivity $\rho_0 = 1.69 \times 10^{-8}\ \Omega\cdot\text{m}$.

Current and Drift Velocity : resistivity Chart

Table 26-1 Resistivities of Some Materials at Room Temperature (20°C)

Material	Resistivity, ρ ($\Omega \cdot m$)	Temperature Coefficient of Resistivity, α (K^{-1})
	Typical Metals	
Silver	1.62×10^{-8}	4.1×10^{-3}
Copper	1.69×10^{-8}	4.3×10^{-3}
Gold	2.35×10^{-8}	4.0×10^{-3}
Aluminum	2.75×10^{-8}	4.4×10^{-3}
Manganin[a]	4.82×10^{-8}	0.002×10^{-3}
Tungsten	5.25×10^{-8}	4.5×10^{-3}
Iron	9.68×10^{-8}	6.5×10^{-3}
Platinum	10.6×10^{-8}	3.9×10^{-3}
	Typical Semiconductors	
Silicon, pure	2.5×10^{3}	-70×10^{-3}
Silicon, *n*-type[b]	8.7×10^{-4}	
Silicon, *p*-type[c]	2.8×10^{-3}	
	Typical Insulators	
Glass	$10^{10} - 10^{14}$	
Fused quartz	$\sim 10^{16}$	

Current and Drift Velocity

Problem-15:

A rectangular block of iron has dimensions 1.2 cm × 1.2 cm × 15 cm. A potential difference is to be applied to the block between parallel sides and in such a way that those sides are equipotential surfaces (as in Fig. 26-8*b*). What is the resistance of the block if the two parallel sides are (1) the square ends (with dimensions 1.2 cm × 1.2 cm) and (2) two rectangular sides (with dimensions 1.2 cm × 15 cm)?

KEY IDEA

The resistance R of an object depends on how the electric potential is applied to the object. In particular, it depends on the ratio L/A, according to Eq. 26-16 ($R = \rho L/A$), where A is the area of the surfaces to which the potential difference is applied and L is the distance between those surfaces.

Calculations: For arrangement 1, we have $L = 15$ cm $= 0.15$ m and

$$A = (1.2\text{ cm})^2 = 1.44 \times 10^{-4}\text{ m}^2.$$

Substituting into Eq. 26-16 with the resistivity ρ from Table 26-1, we then find that for arrangement 1,

$$R = \frac{\rho L}{A} = \frac{(9.68 \times 10^{-8}\,\Omega\cdot\text{m})(0.15\text{ m})}{1.44 \times 10^{-4}\text{ m}^2}$$
$$= 1.0 \times 10^{-4}\,\Omega = 100\,\mu\Omega. \qquad \text{(Answer)}$$

Similarly, for arrangement 2, with distance $L = 1.2$ cm and area $A = (1.2\text{ cm})(15\text{ cm})$, we obtain

$$R = \frac{\rho L}{A} = \frac{(9.68 \times 10^{-8}\,\Omega\cdot\text{m})(1.2 \times 10^{-2}\text{ m})}{1.80 \times 10^{-3}\text{ m}^2}$$
$$= 6.5 \times 10^{-7}\,\Omega = 0.65\,\mu\Omega. \qquad \text{(Answer)}$$

Current and Drift Velocity

Problem-16: the 12-gauge copper wire in a home has a cross-sectional area of 3.31×10^{-6} m^2 and carries a current of 10 A. Calculate the magnitude of the electric field in the wire.

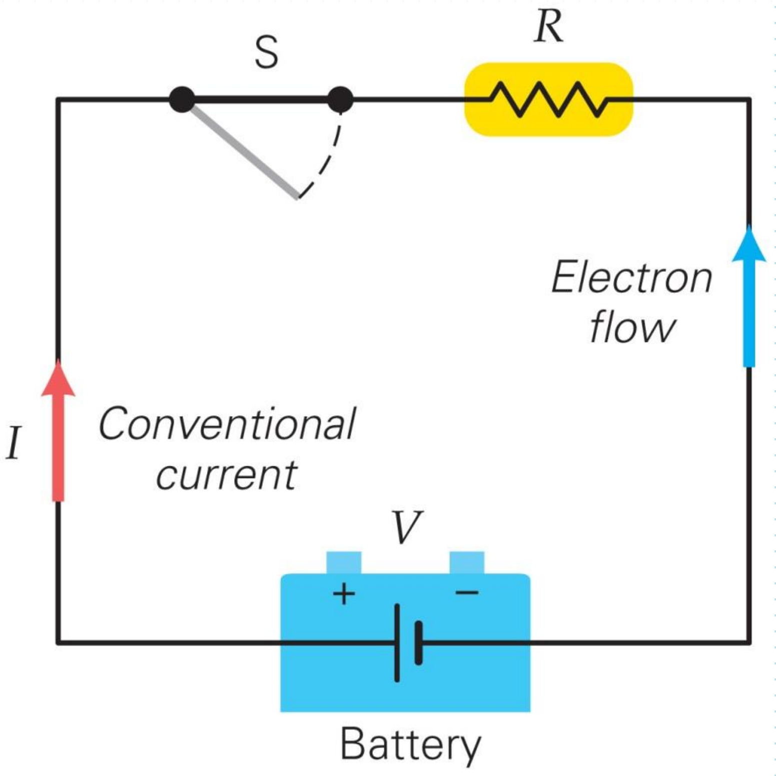
$$E = \rho J = \rho \frac{I}{A}$$

$$E = \frac{(1.72 \times 10^{-8}\ \Omega \cdot \mathrm{m})\left(10\,\mathrm{C/s}\right)}{(3.31 \times 10^{-6}\ \mathrm{m}^2)}$$

$$E = 5.20 \times 10^{-2}\ \mathrm{V/m}$$

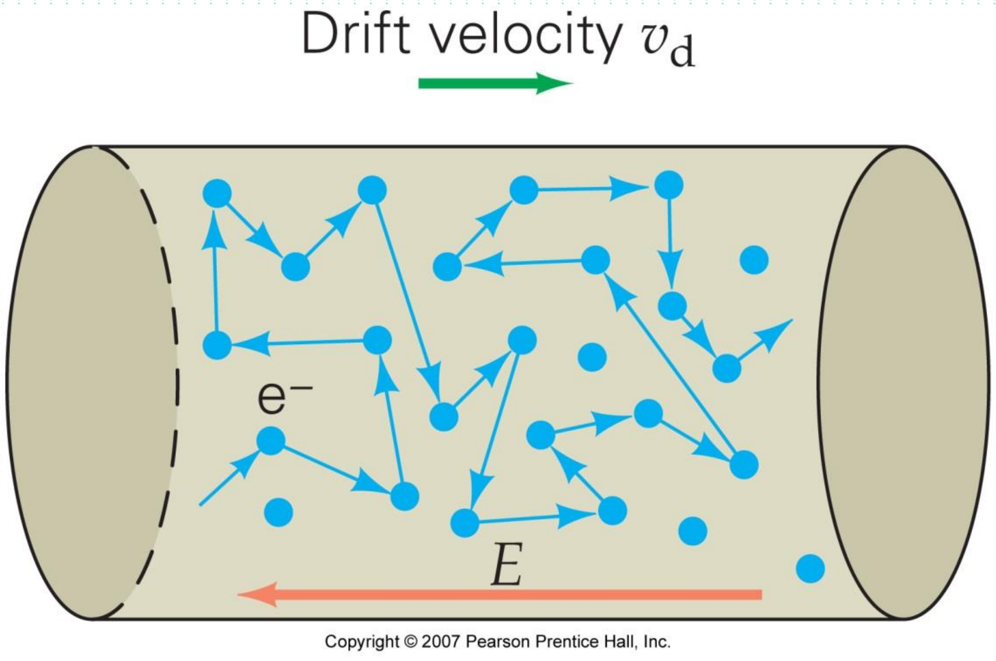
Current and Drift Velocity

Historically, the direction of current has been taken to be from positive to negative; this is opposite to the way electrons flow. However, this seldom matters.



Current and Drift Velocity

Electrons do not flow like water in a pipe. In the absence of voltage, they move randomly at high speeds, due to their temperature.



When a voltage is applied, a very small drift velocity is added to the thermal motion, typically around 1 mm/s; this is enough to yield the observed current.

Resistance and Ohm's Law

If there is a potential difference across a conductor, how much current flows?

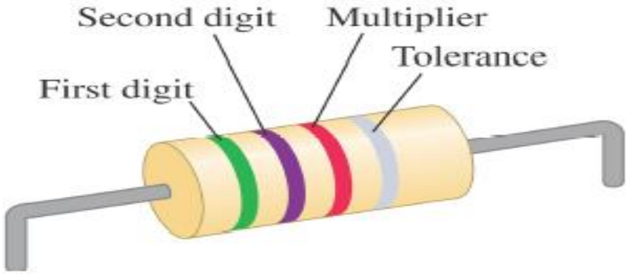
The ratio between the voltage and the current is called the resistance.

$$R = \frac{V}{I}$$

SI unit of resistance: the ohm, Ω

Resistance and Color code

Resistor: circuit device with a fixed R between its ends.



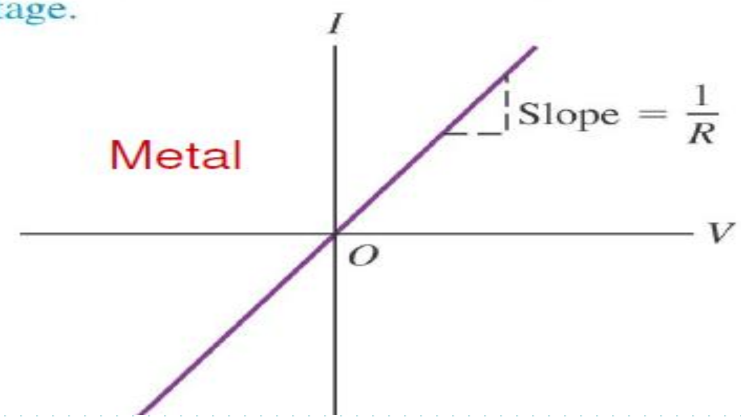
Ex: 5.7 kΩ = green (5) violet (7) red multiplier (100)

Table 25.3 Color Codes for Resistors

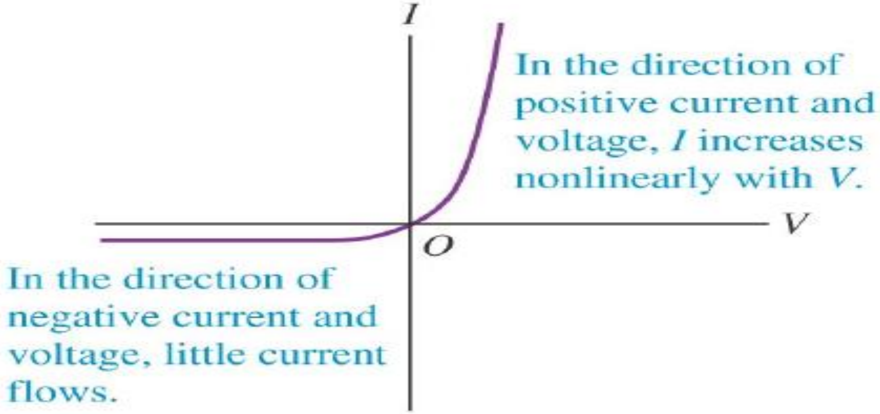
Color	Value as Digit	Value as Multiplier
Black	0	1
Brown	1	10
Red	2	10^2
Orange	3	10^3
Yellow	4	10^4
Green	5	10^5
Blue	6	10^6
Violet	7	10^7
Gray	8	10^8
White	9	10^9

Current-voltage curves

Ohmic resistor (e.g., typical metal wire): At a given temperature, current is proportional to voltage.



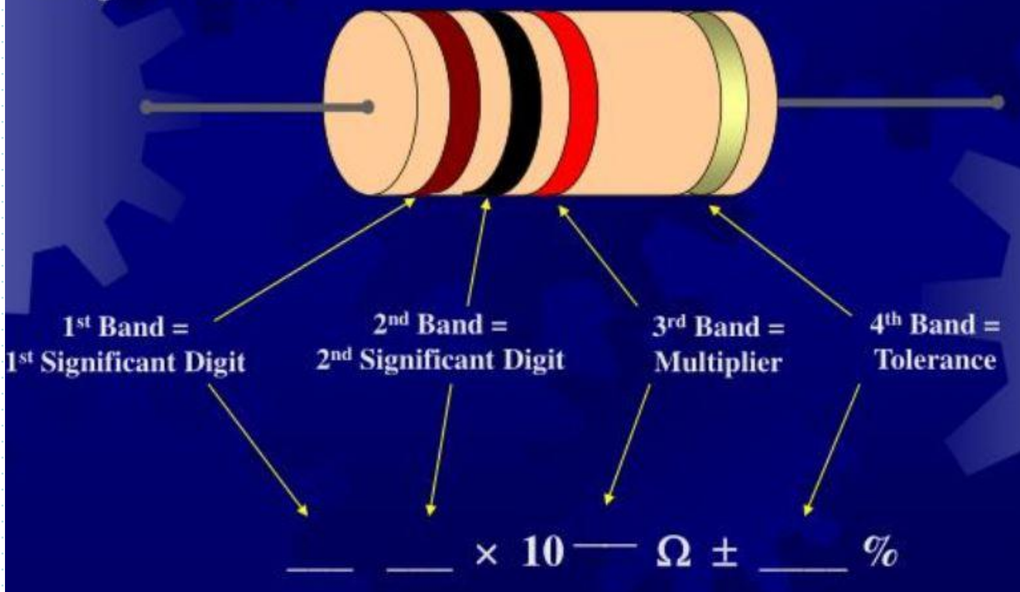
Semiconductor diode: a nonohmic resistor



Resistance and Color code

4-Band Resistors

The resistor nominal value is encoded in the color code in Powers of Ten Notation. The template for determining the nominal value and tolerance of a resistor with 4 color bands is given below:



How do we know which color corresponds to which number?

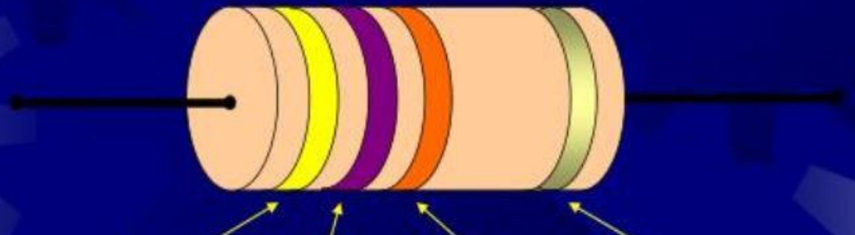
Answer: Using the Resistor Color Code Table

Color	Digit	Multiplier	Tolerance
Black	0	$10^0 = 1$	
Brown	1	$10^1 = 10$	±1%
Red	2	$10^2 = 100$	±2%
Orange	3	$10^3 = 1,000$	
Yellow	4	$10^4 = 10,000$	
Green	5	$10^5 = 100,000$	
Blue	6	$10^6 = 1,000,000$	
Violet	7	$10^7 = 10,000,000$	
Gray	8	$10^8 = 100,000,000$	
White	9	$10^9 = 1,000,000,000$	
Silver		$10^{-2} = 0.01$	±10%
Gold		$10^{-1} = 0.1$	±5%
No band	---	---------	±20%

Resistance and Color code

Example 1. Determine the nominal resistance value and the tolerance for the resistor shown below.

Example 2. a) Determine the nominal value and tolerance for the resistor below.
b) What is the minimum resistance value this resistor can actually have?
c) What is the maximum resistance value this resistor can actually have?



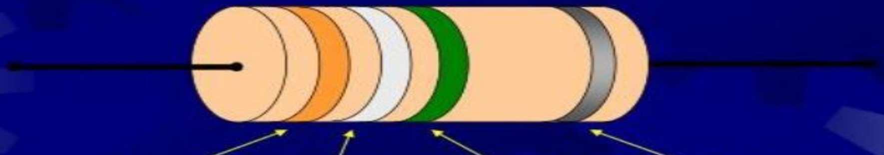
Solution:

$$\underline{4}\ \underline{7} \times 10^{\underline{3}}\ \Omega \pm \underline{5}\ \%$$

Resistor nominal value $= 47\times10^3\Omega$
$= 47{,}000\Omega$
$= 47k\Omega$.

Tolerance = ±5%

Example 3. a) Determine the nominal value and tolerance for the resistor below.
b) What is the minimum resistance value this resistor can actually have?
c) What is the maximum resistance value this resistor can actually have?



Solution: continued

- Minimum resistance value:
  Multiply the nominal value by the tolerance and then *subtract* this from the nominal value:

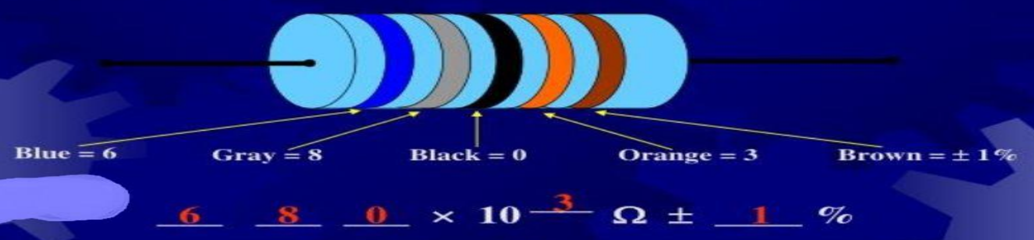
$$= 47k\Omega - 47k\Omega * 0.05$$
$$= 47k\Omega - 2.35k\Omega$$
$$= 44.65k\Omega$$

- Maximum resistance value:
  Multiply the nominal value by the tolerance and then *add* this to the nominal value:

$$= 47k\Omega + 47k\Omega * 0.05$$
$$= 47k\Omega + 2.35k\Omega$$
$$= 49.35k\Omega$$

Resistance and Color code

Example 5. Determine the nominal resistance and tolerance for the resistor shown below.



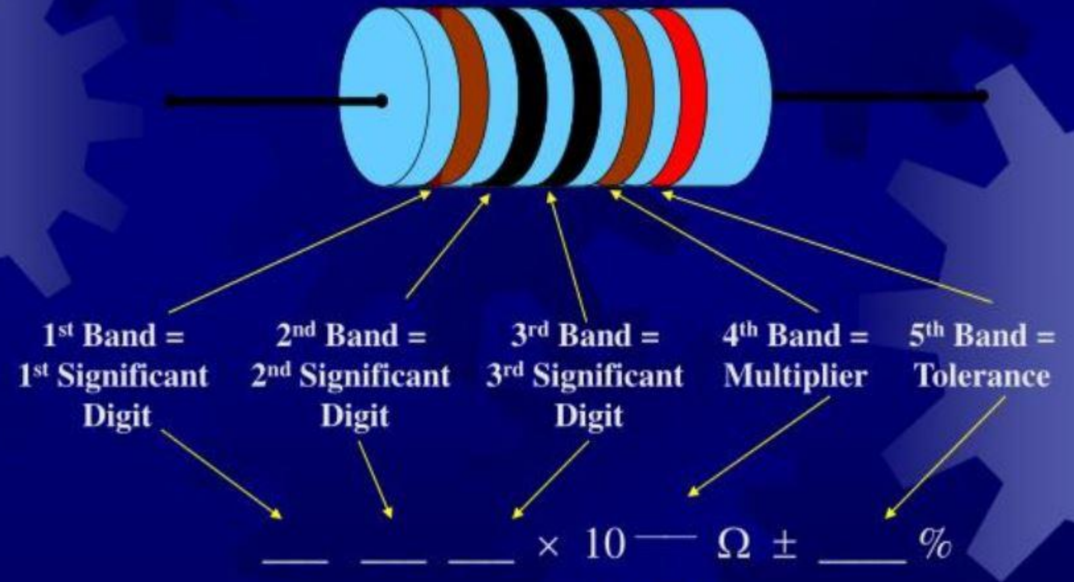
Example 6. Specify the color code of a resistor with nominal value of 27kΩ and a tolerance of ±10%.

Example 7. Specify the color code of a resistor with nominal value of 1.5kΩ and a tolerance of ±5%.

Example 8. Specify the color code of a resistor with nominal value of 2.5MΩ and a tolerance of ±1%.

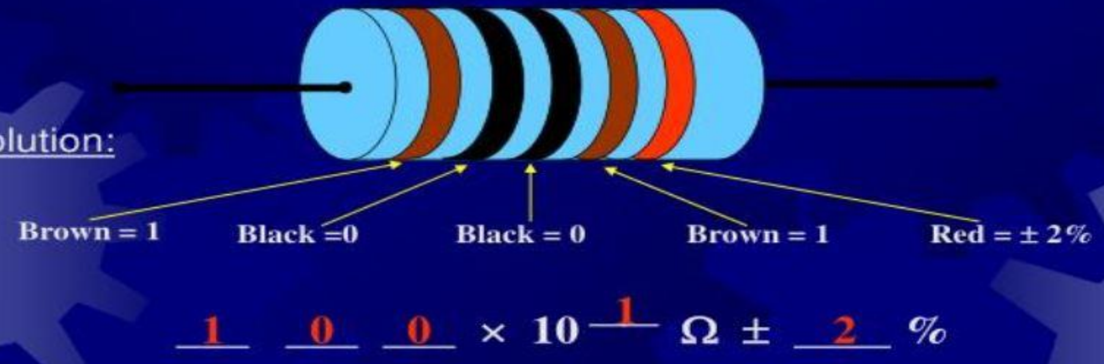
5-Band Resistors

- For resistors with ±1% or ±2% tolerance, the color code consists of 5 bands.
- The template for 5-band resistors is:



Example 4. Determine the nominal resistance and tolerance for the resistor shown below.

Solution:



Resistor nominal value $= 100 \times 10^1 \Omega$

$= 1{,}000\Omega$

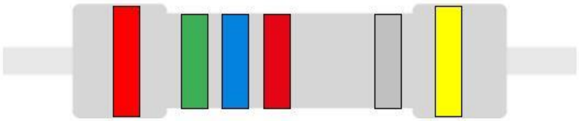
$= 1k\Omega.$

Tolerance = ±2%

Resistance and Color code

Example-10 : Determine the nominal resistance, maximum-minimum value of resistance and tolerance for the resistor shown below. Given temperature coefficient of resistor is Yellow.

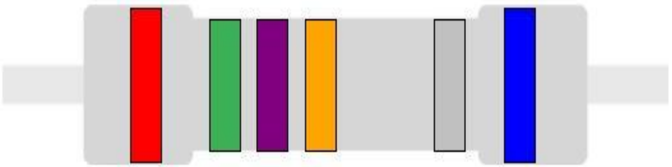
6 strip Resistor



1st Digit	2nd Digit	3rd Digit	Multiplier	Tolerance	Tempco
2 RED	5 Greer	6 Blue	x100 Re	± 10% !	25 Yello

Example-9: Determine the nominal resistance, maximum-minimum value of resistance and tolerance for the resistor shown below. Given temperature coefficient of resistor is Blue.

6 strip Resistor



1st Digit	2nd Digit	3rd Digit	Multiplier	Tolerance	Tempco
2 RED	5 Greer	7 Violet	x1k Ora	± 10% !	10 Blue

Soln:

Resistance:	257.00k	ohms
Tolerance:	± 10%	
Minimum:	231.300000k	ohms
Maximum:	282.700000k	ohms
Tempco:	10	ppm/°C

Color	Value
Black	N/A
Brown	100 ppm/°C
Red	50 ppm/°C
Orange	15 ppm/°C
Yellow	25 ppm/°C
Green	N/A
Blue	10 ppm/°C
Violet	5 ppm/°C
Grey	N/A
White	N/A

Fig: Temperature co-efficient of resistor chart

Resistance and Color code

Color	Value
Black	N/A
Brown	±1%
Red	±2%
Orange	±3%
Yellow	±4%
Green	±0.5%
Blue	±0.25%
Violet	±0.10%
Grey	±0.05%
White	N/A
Gold	±5%
Silver	±10%

Fig: Tolerance chart of resistor

At a Glance

± 1% Brown
± 2% Red
± 3% Orange
± 4% Yellow
± 0.5% Green
± 0.25% Blue
± 0.10% Violet
± 0.05% Gray
± 5% Gold
± 10% Silver

Fig: Tolerance chart

100 Brown
50 Red
15 Orange
25 Yellow
10 Blue
5 Violet

Fig: Tempco chart

Resistance and Ohm's Law

An ohmic material is one whose resistance is constant.

Ohm's law:"The voltage (potential difference, emf) is directly related (proportional) to the current, when the resistance is constant"

$$\Delta V \alpha I$$

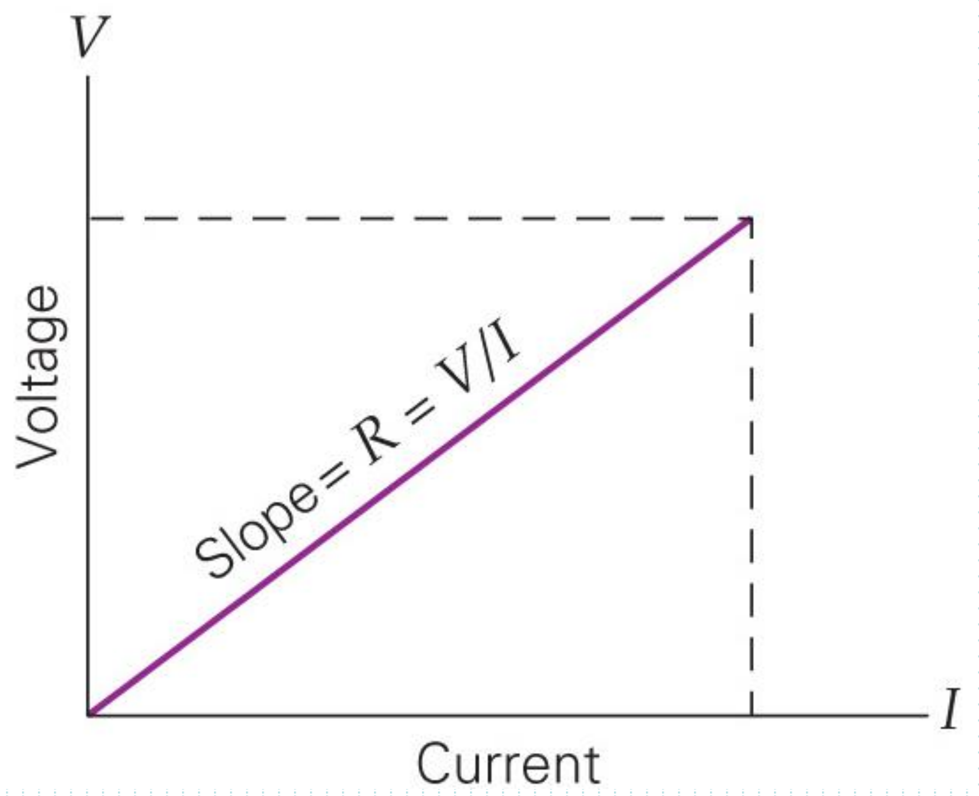
$$R = \text{constant of proportionality}$$

$$R = \text{Resistance}$$

$$\Delta V = IR$$

$$\varepsilon = IR$$

Since R=ΔV/I, the resistance is the SLOPE of a ΔV vs. I graph



Resistance and Ohm's Law

Derive Ohm's law.

Solution:

$$I = J \cdot A$$

$$V = E \cdot L$$

R = resistance

$$E = \frac{V}{L} = \rho \cdot J = \rho \frac{I}{A} \quad \rightarrow \quad V = \boxed{\frac{\rho \cdot L}{A}} I$$

Resistance: $R = \frac{V}{I} = \frac{\rho \cdot L}{A}$

$\boxed{V = I \cdot R}$ Ohm's law (conductors)

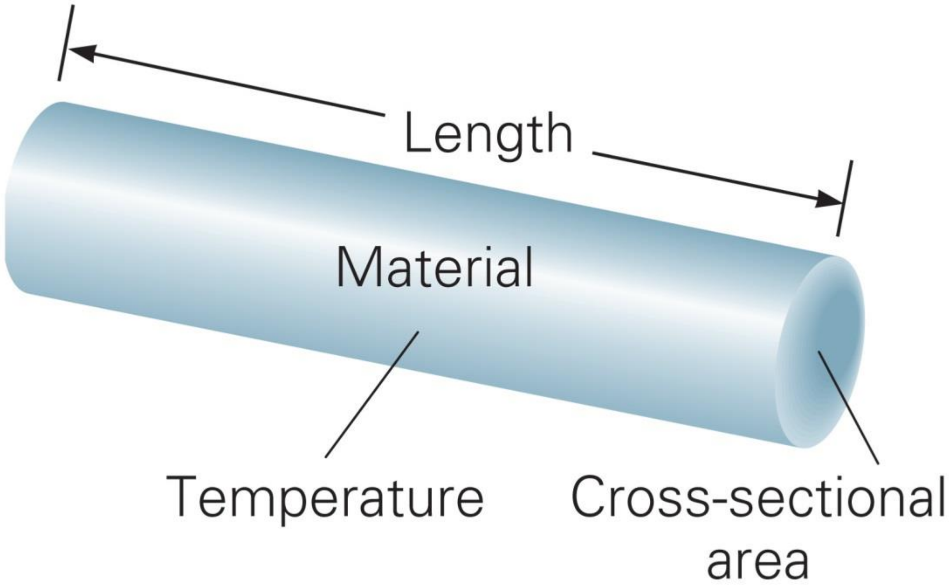
Units: Ohm = Ω = 1 V/A

$$\boxed{R(T) = R_0[1 + \alpha(T - T_0)]}$$

Resistance and Ohm's Law

Ohm's law is valid only for ohmic materials:

$$V = IR$$



The resistance of a particular object depends on its length, cross-sectional area, material, and temperature.

Resistance and Ohm's Law: Resistivity

As expected, the resistance is proportional to the length and inversely proportional to the cross-sectional area (why?):

$$R = \rho\left(\frac{L}{A}\right)$$

The constant ρ is called the resistivity, and is characteristic of the material.

Resistance and Ohm's Law: Resistivity

Temperature Dependence of Resistivity

Many materials have resistivities that depend on temperature. We can **model*** this temperature dependence by an equation of the form

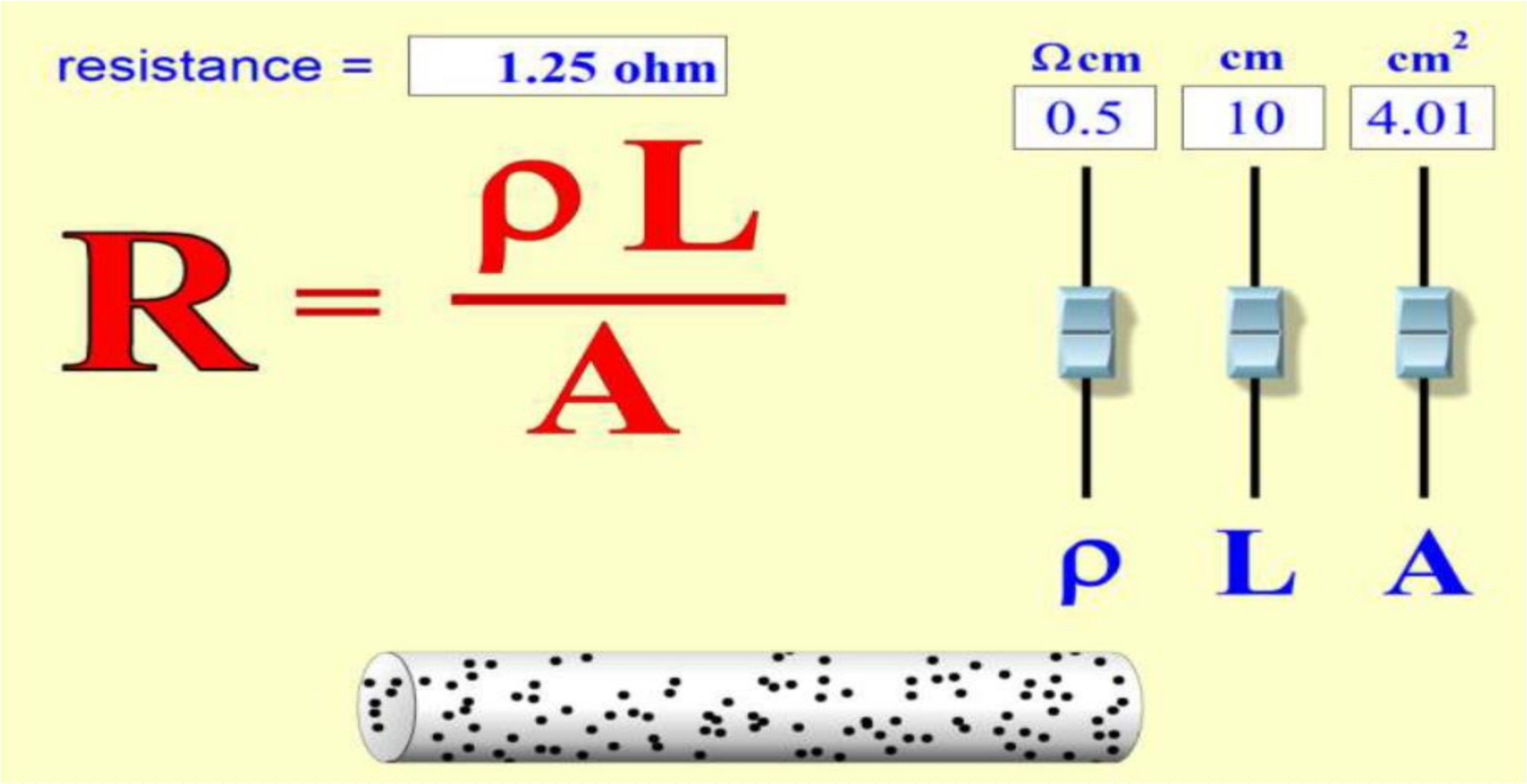
$$\rho = \rho_0\left[1+\alpha\left(T-T_0\right)\right],$$

where ρ_0 is the resistivity at temperature T_0, and α is the *temperature coefficient of resistivity*.

***T_0 is a reference temperature, often taken to be 0 °C or 20 °C. This approximation can be used if the temperature range is "not too great;" i.e. 100 °C or so.**

Resistance and Ohm’s Law: Resistivity

#Factors affecting the resistance of a wire: There are mainly 4 factors for which the resistance of a wire can vary.



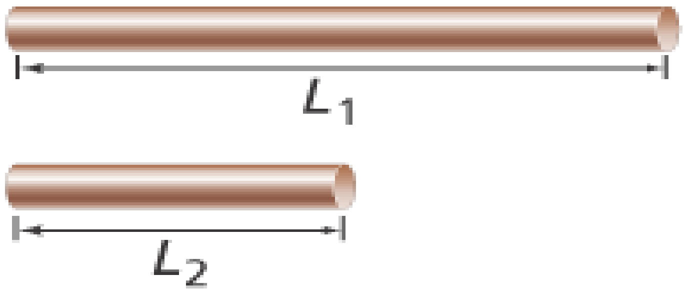
Resistance and Ohm's Law: Law of resistance

Factors affecting the resistance:

1. The length of a wire, L. [Rα L]
2. The cross-sectional area, A. [R$\alpha \frac{1}{A}$]
3. The resistivity of a wire, ρ (i.e., type of the element of a wire). [Rα ρ]
4. The temperature effect, T. [Rα T]

Factors that affect the resistance of a wire:

1. Increasing the length (L) of the wire will increase the resistance of the wire.



$R_{L1} > R_{L2}$

Long/thin wires = high resistance

This is because the current (electrons) will now have further to travel and will encounter and collide with an increasing number of atoms.

Factors that affect the resistance of a wire:

2. Increasing the cross-sectional area (A) of a wire will decrease the wires resistance.

$R_{A1} > R_{A2}$

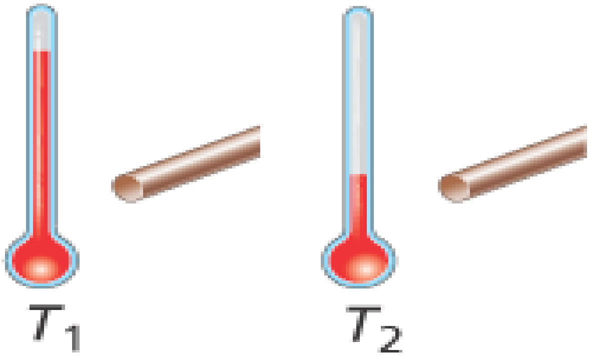
$A = \pi r^2$

Short/Thick wires = low resistance

This occurs because in making the wire thicker there is now more spaces between atoms through which the electrons can travel and thus flow easier. The wire isn't resisting the flow as much.

Factors that affect the resistance of a wire:

3. RESISTIVITY (*p*) – this is a characteristic of a material that depends on its electronic structure and temperature. If a wire is made of a material that has a high resistivity then it will have a high resistance.

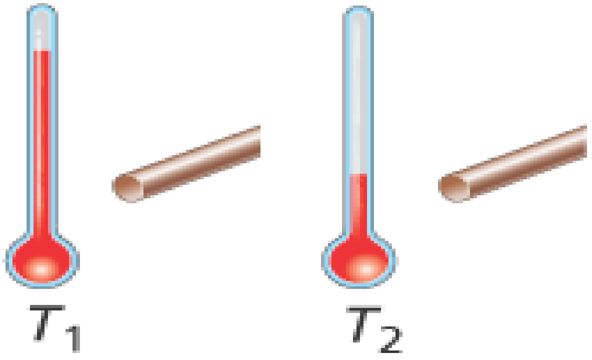


$R_{T1} > R_{T2}$

Resistivities at 20°C	
Material	**Resistivity (Ω•m)**
Aluminum	2.82×10^{-8}
Copper	1.72×10^{-8}
Gold	2.44×10^{-8}
Nichrome	$150. \times 10^{-8}$
Silver	1.59×10^{-8}
Tungsten	5.60×10^{-8}

Factors that affect the resistance of a wire:

4. Temperature (T) – it is a characteristic of a material that any material has high resistance if it is in high temperature. A cooler wire has less resistance than a warmer wire. Cooler particles have less kinetic energy, so they move more slowly. Therefore, they are less likely to collide with moving electrons in current. Hotter particles have high kinetic energy, so they have high resistance. Some elements show anomaly like semiconductors which does not behave like metal.

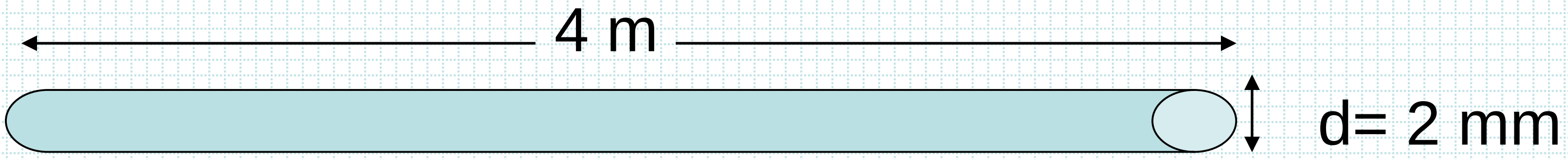


$R_{T1} > R_{T2}$

$T_1 > T_2$

Resistance and Ohm's Law: Problem

Problem-17: Determine the resistance of a 4.00 m length of copper wire having a diameter of 2 mm. Assume the temperature of 20°C.



$p_{\text{copper}} = 1.72 \times 10^{-8}\ \Omega\bullet\text{m}$

$$R = \frac{pL}{A} = \frac{(1.72 \times 10^{-8}\ \Omega\bullet\text{m})(4\text{m})}{\pi(.001\text{m})^2} = .0219\ \Omega$$

Resistance and Ohm's Law: Problem

Problem-18: Suppose you want to connect your stereo to remote speakers.
(a) If each wire must be 20 m long, what diameter copper wire should you use to make the resistance 0.10 Ω per wire.

Solution: $R = \rho L / A$

$\Rightarrow A = \rho L / R$

$\Rightarrow A = \pi (d/2)^2$

$\Rightarrow \pi (d/2)^2 = \rho L / R$

$\Rightarrow (d/2)^2 = \rho L / \pi R$

$\Rightarrow d/2 = (\rho L / \pi R)^{1/2}$

$\Rightarrow \boxed{d = 2 (\rho L / \pi R)^{1/2}}$

$\Rightarrow d = 2 [(1.68 \times 10^{-8}) (20) / \pi (0.1)]^{1/2}$

$\boxed{d = 0.0021 \text{ m} = 2.1 \text{ mm}}$

(b) If the current to each speaker is 4.0 A, what is the voltage drop across each wire?

$V = I R$ $\Rightarrow V = (4.0) (0.10)$ $\Rightarrow V = 0.4 \text{ V}$

Resistance and Ohm's Law: Problem

Problem-19: a carbon resistance thermometer in the shape of a cylinder 1 cm long and 4 mm in diameter is attached to a sample. The thermometer has a resistance of 0.030 Ω. What is the temperature of the sample?

Soln: Look up resistivity of carbon, use it to calculate resistance.

$$\rho_0 = 3.519 \times 10^{-5}\ \Omega \cdot \mathrm{m}$$

This is the resistivity at 20 °C.

$$T_0 = 20°\mathrm{C} \qquad L = 0.01\ \mathrm{m} \qquad r = 0.002\ \mathrm{m}$$

$$R_0 = \frac{\rho_0 L}{\pi r^2} = 0.028\ \Omega$$

This is the resistance at 20 °C.

Resistance and Ohm's Law: Problem

$$\alpha = -0.0005\ ^\circ C^{-1}$$

$$\rho(R) = \frac{RA}{L}$$

$$T(R) = T_0 + \frac{1}{\alpha}\left[\frac{\rho}{\rho_0} - 1\right]$$

$$T(0.030) = -122.6\ ^\circ C$$

The result is very sensitive to significant figures in resistivity and α.

Resistance and Ohm's Law

Derive a relation for mean free time and resistivity using drift velocity of electron.

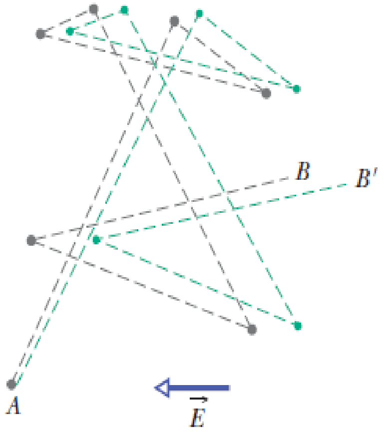
If an electron of mass m is placed in an electric field of magnitude E, the electron will experience an acceleration given by Newton's second law:

$$a = \frac{F}{m} = \frac{eE}{m}. \qquad (26\text{-}18)$$

After a typical collision, each electron will—so to speak—completely lose its memory of its previous drift velocity, starting fresh and moving off in a random direction. In the average time τ between collisions, the average electron will acquire a drift speed of $v_d = a\tau$. Moreover, if we measure the drift speeds of all the electrons at any instant, we will find that their average drift speed is also $a\tau$. Thus, at any instant, on average, the electrons will have drift speed $v_d = a\tau$. Then Eq. 26-18 gives us

$$v_d = a\tau = \frac{eE\tau}{m}. \qquad (26\text{-}19)$$

Figure 26-12 The gray lines show an electron moving from A to B, making six collisions en route. The green lines show what the electron's path might be in the presence of an applied electric field $\vec{E}$. Note the steady drift in the direction of $-\vec{E}$. (Actually, the green lines should be slightly curved, to represent the parabolic paths followed by the electrons between collisions, under the influence of an electric field.)



Combining this result with Eq. 26-7 ($\vec{J} = ne\vec{v}_d$), in magnitude form, yields

$$v_d = \frac{J}{ne} = \frac{eE\tau}{m}, \qquad (26\text{-}20$$

which we can write as

$$E = \left(\frac{m}{e^2 n\tau}\right) J. \qquad (26\text{-}21$$

Comparing this with Eq. 26-11 ($\vec{E} = \rho\vec{J}$), in magnitude form, leads to

$$\rho = \frac{m}{e^2 n\tau}. \qquad (26\text{-}22$$

Equation 26-22 may be taken as a statement that metals obey Ohm's law if w can show that, for metals, their resistivity ρ is a constant, independent of th strength of the applied electric field $\vec{E}$. Let's consider the quantities in Eq. 26-2: We can reasonably assume that n, the number of conduction electrons per vol ume, is independent of the field, and m and e are constants. Thus, we only need t convince ourselves that τ, the average time (or *mean free time*) between colli sions, is a constant, independent of the strength of the applied electric fielc Indeed, τ can be considered to be a constant because the drift speed v_d caused b the field is so much smaller than the effective speed v_{eff} that the electron speed– and thus τ—is hardly affected by the field. Thus, because the right side o Eq. 26-22 is independent of the field magnitude, metals obey Ohm's law.

Resistance and Ohm's Law

Problem-20:

(a) What is the mean free time τ between collisions for the conduction electrons in copper?

KEY IDEAS

The mean free time τ of copper is approximately constant, and in particular does not depend on any electric field that might be applied to a sample of the copper. Thus, we need not consider any particular value of applied electric field. However, because the resistivity ρ displayed by copper under an electric field depends on τ, we can find the mean free time τ from Eq. 26-22 ($\rho = m/e^2 n\tau$).

Calculations: That equation gives us

$$\tau = \frac{m}{ne^2\rho}. \tag{26-23}$$

The number of conduction electrons per unit volume in copper is $8.49 \times 10^{28}\ \text{m}^{-3}$. We take the value of ρ from Table 26-1. The denominator then becomes

$$\begin{aligned}(8.49 \times 10^{28}\ \text{m}^{-3})&(1.6 \times 10^{-19}\ \text{C})^2(1.69 \times 10^{-8}\ \Omega\cdot\text{m}) \\ &= 3.67 \times 10^{-17}\ \text{C}^2\cdot\Omega/\text{m}^2 = 3.67 \times 10^{-17}\ \text{kg/s},\end{aligned}$$

where we converted units as

$$\frac{\text{C}^2\cdot\Omega}{\text{m}^2} = \frac{\text{C}^2\cdot\text{V}}{\text{m}^2\cdot\text{A}} = \frac{\text{C}^2\cdot\text{J/C}}{\text{m}^2\cdot\text{C/s}} = \frac{\text{kg}\cdot\text{m}^2/\text{s}^2}{\text{m}^2/\text{s}} = \frac{\text{kg}}{\text{s}}.$$

Using these results and substituting for the electron mass m, we then have

$$\tau = \frac{9.1 \times 10^{-31}\ \text{kg}}{3.67 \times 10^{-17}\ \text{kg/s}} = 2.5 \times 10^{-14}\ \text{s}. \qquad \text{(Answer)}$$

(b) The mean free path λ of the conduction electrons in a conductor is the average distance traveled by an electron between collisions. (This definition parallels that in Module 19-5 for the mean free path of molecules in a gas.) What is λ for the conduction electrons in copper, assuming that their effective speed v_{eff} is 1.6×10^6 m/s?

KEY IDEA

The distance d any particle travels in a certain time t at a constant speed v is $d = vt$.

Calculation: For the electrons in copper, this gives us

$$\begin{aligned}\lambda &= v_{\text{eff}}\tau &\text{(26-24)}\\ &= (1.6 \times 10^6\ \text{m/s})(2.5 \times 10^{-14}\ \text{s}) \\ &= 4.0 \times 10^{-8}\ \text{m} = 40\ \text{nm}. &\text{(Answer)}\end{aligned}$$

This is about 150 times the distance between nearest-neighbor atoms in a copper lattice. Thus, on the average, each conduction electron passes many copper atoms before finally hitting one.

Resistance and Ohm's Law

Problem-21: You are given a length of uniform heating wire made of a nickel–chromium–iron alloy called Nichrome; it has a resistance R of 72 Ω. At what rate is energy dissipated in each of the following situations? (1) A potential difference of 120 V is applied across the full length of the wire. (2) The wire is cut in half, and a potential difference of 120 V is applied across the length of each half.

KEY IDEA

Current in a resistive material produces a transfer of mechanical energy to thermal energy; the rate of transfer (dissipation) is given by Eqs. 26-26 to 26-28.

Calculations: Because we know the potential V and resistance R, we use Eq. 26-28, which yields, for situation 1,

$$P = \frac{V^2}{R} = \frac{(120\text{ V})^2}{72\ \Omega} = 200\text{ W.} \qquad \text{(Answer)}$$

In situation 2, the resistance of each half of the wire is (72 Ω)/2, or 36 Ω. Thus, the dissipation rate for each half is

$$P' = \frac{(120\text{ V})^2}{36\ \Omega} = 400\text{ W,}$$

and that for the two halves is

$$P = 2P' = 800\text{ W.} \qquad \text{(Answer)}$$

This is four times the dissipation rate of the full length of wire. Thus, you might conclude that you could buy a heating coil, cut it in half, and reconnect it to obtain four times the heat output. Why is this unwise? (What would happen to the amount of current in the coil?)

Current and Drift velocity

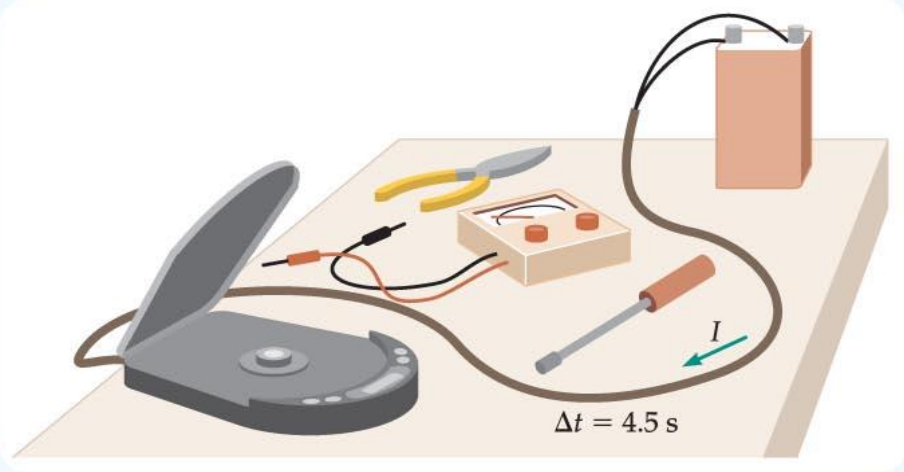
Problem-22:

GUIDED Example 21.2 | Mega Blaster — Electric Current

The disk drive in a portable CD player is connected to a battery that supplies it with a current of 0.22 A. How many electrons pass through the disk drive in 4.5 s? (Recall that the charge of an electron has the magnitude $e = 1.60 \times 10^{-19}$ C.)

Picture the Problem

Our sketch shows the CD player with a current $I = 0.22$ A flowing through it. Also indicated is the time, $\Delta t = 4.5$ s, during which the current flows.



Strategy

Since we know both the current, I, and the amount of time, Δt, we can use the definition of current, $I = \Delta Q/\Delta t$, to find the amount of charge, ΔQ, that flows through the player. Once we know the amount of charge, the number of electrons, N, is that amount divided by the magnitude of the electron's charge. That is, $N = \Delta Q/e$, where $e = 1.60 \times 10^{-19}$ C.

Known	Unknown
$I = 0.22$ A	$\Delta Q = ?$
$\Delta t = 4.5$ s	$N = ?$
$e = 1.60 \times 10^{-19}$ C	

Current and Drift velocity

GUIDED Example 21.2 | Mega Blaster *(Continued)* **Electric Current**

Solution

1 Rearrange $I = \Delta Q/\Delta t$ to solve for the amount of charge, ΔQ, that flows through the drive:

$$\Delta Q = I\,\Delta t = (0.22\ \text{A})(4.5\ \text{s}) = 0.99\ \text{C}$$

Math HELP Significant Figures See Lesson 1.4

2 Divide the amount of charge, ΔQ, by the magnitude of the electron's charge, e, to find the number of electrons, N:

$$N = \frac{\Delta Q}{e} = \frac{0.99\ \cancel{\text{C}}}{1.60 \times 10^{-19}\ \cancel{\text{C}}/\text{electron}} = \boxed{6.2 \times 10^{18}\ \text{electrons}}$$

Insight

Thus, even a modest current flowing for a brief time corresponds to the transport of an extremely large number of electrons.

Resistance and Ohm's Law

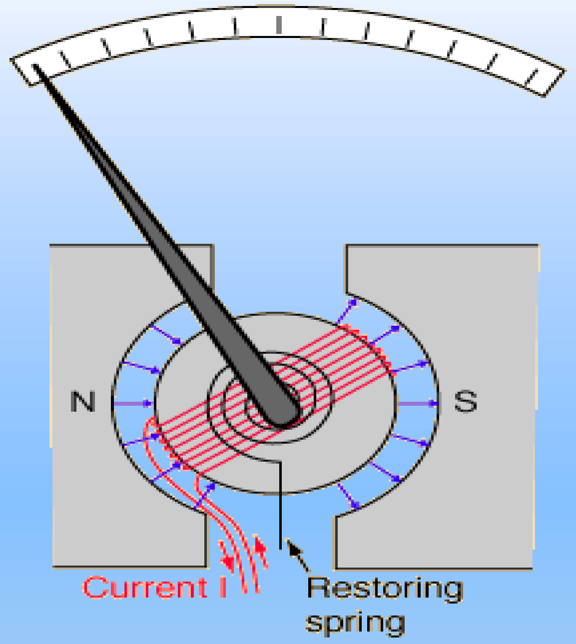
In this table, you can easily see the differences between the resistivities of conductors, semiconductors, and insulators.

TABLE 17.1 Resistivities (at 20°C) and Temperature Coefficients of Resistivity for Various Materials*

	$\rho\ (\Omega \cdot m)$	$\alpha\ (1/C°)$		$\rho\ (\Omega \cdot m)$	$\alpha\ (1/C°)$
Conductors			*Semiconductors*		
Aluminum	2.82×10^{-8}	4.29×10^{-3}	Carbon	3.6×10^{-5}	-5.0×10^{-4}
Copper	1.70×10^{-8}	6.80×10^{-3}	Germanium	4.6×10^{-1}	-5.0×10^{-2}
Iron	10×10^{-8}	6.51×10^{-3}	Silicon	2.5×10^{2}	-7.0×10^{-2}
Mercury	98.4×10^{-8}	0.89×10^{-3}			
Nichrome (alloy of nickel and chromium)	100×10^{-8}	0.40×10^{-3}	*Insulators*		
Nickel	7.8×10^{-8}	6.0×10^{-3}	Glass	10^{12}	
Platinum	10×10^{-8}	3.93×10^{-3}	Rubber	10^{15}	
Silver	1.59×10^{-8}	4.1×10^{-3}	Wood	10^{10}	
Tungsten	5.6×10^{-8}	4.5×10^{-3}			

*Values for semiconductors are general ones, and resistivities for insulators are typical orders of magnitude.

Galvanometer



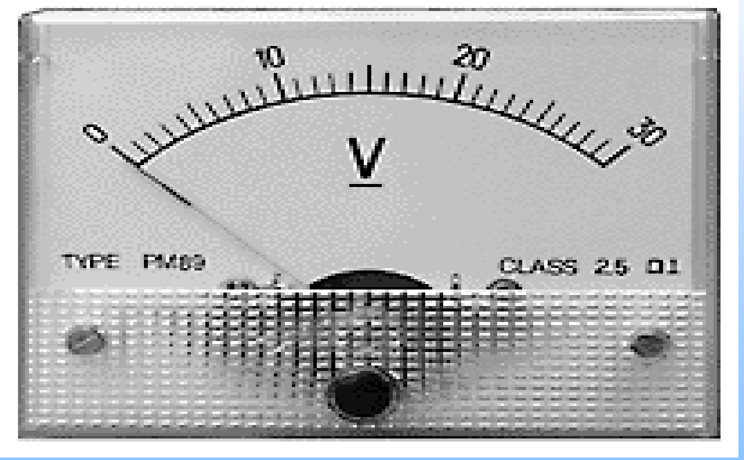
- Electric current detector.

- It has a coil in a magnetic field. When a current is passed through the coil, the coil experiences a torque proportional to the current.

Ammeter

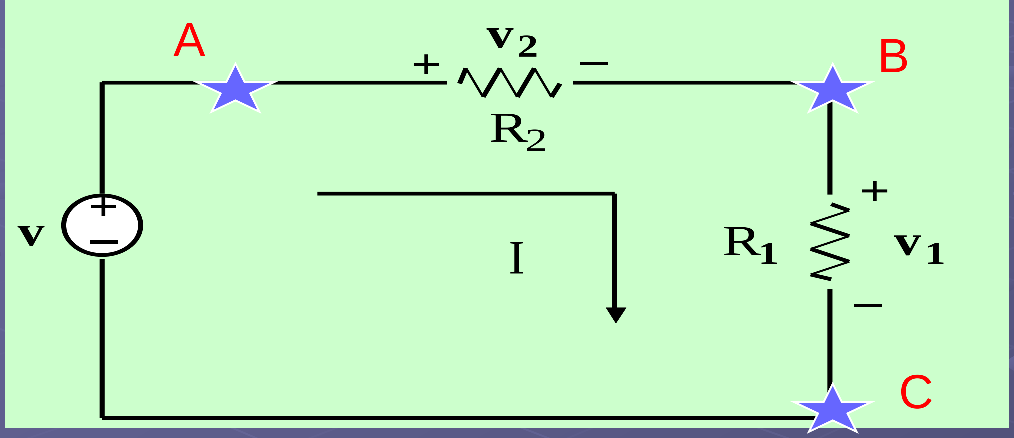
- Measures electric current.
- It must be placed in series with the measured branch.
- It must have very low resistance to avoid significant alteration of the current it is measuring.

Voltmeter



- Measures the change in voltage between two points in an electric circuit.
- It must be connected in parallel with the portion of the circuit on which the measurement is made.
- Must have a very high resistance so that it does not have an appreciable affect on the current or voltage associated with the measured circuit.

Resistors in series:



1. Because of the charge conservation, all charges going through the resistor R_2 will also go through resistor R_1. Thus, currents in R_1 and R_2 are the same,

$$I_1 = I_2 = I$$

2. Because of the energy conservation, total potential drop (between A and C) equals to the sum of potential drops between A and B and B and C,

$$\Delta V = IR_1 + IR_2$$

By definition,

$$\Delta V = IR_{eq}$$

Thus, R_{eq} would be

$$R_{eq} \equiv \frac{\Delta V}{I} = \frac{IR_1 + IR_2}{I} = R_1 + R_2$$

$$\boxed{R_{eq} = R_1 + R_2}$$

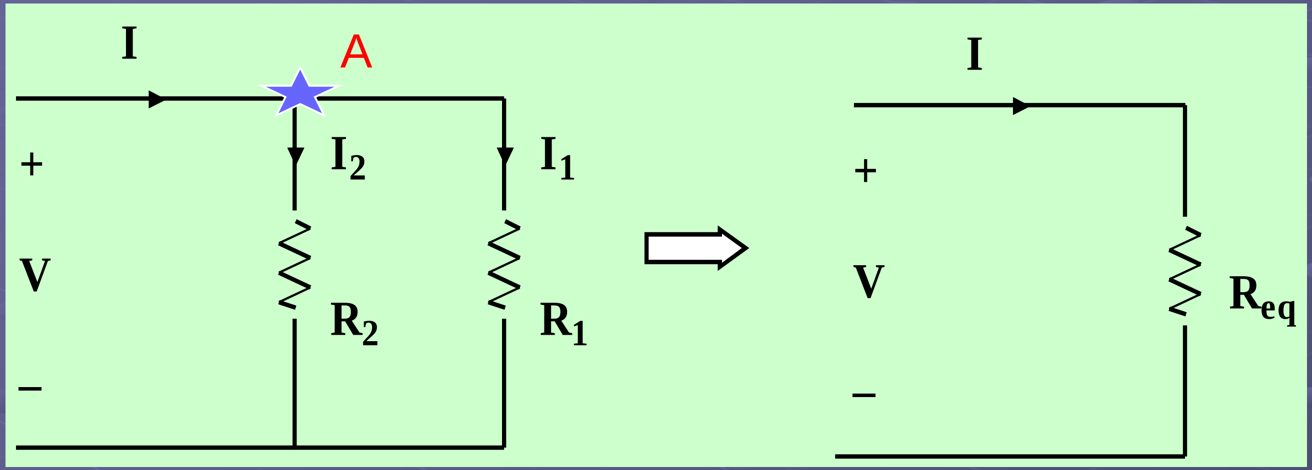
Resistors in series: notes

- Analogous formula is true for any number of resistors,

$$R_{eq} = R_1 + R_2 + R_3 + \ldots \qquad \text{(series combination)}$$

- It follows that the equivalent resistance of a series combination of resistors is greater than any of the individual resistors

Resistors in parallel



1. Since both R_1 and R_2 are connected to the same battery, potential differences across R_1 and R_2 are the same,

$$V_1 = V_2 = V$$

2. Because of the charge conservation, current, entering the junction A, must equal the current leaving this junction,

$$I = I_1 + I_2$$

By definition,

$$I = \frac{V}{R_{eq}}$$

Thus, R_{eq} would be

$$I = \frac{V}{R_{eq}} = \frac{V_1}{R_1} + \frac{V_2}{R_2} = \frac{V}{R_1} + \frac{V}{R_2}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

or

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Resistors in parallel: notes

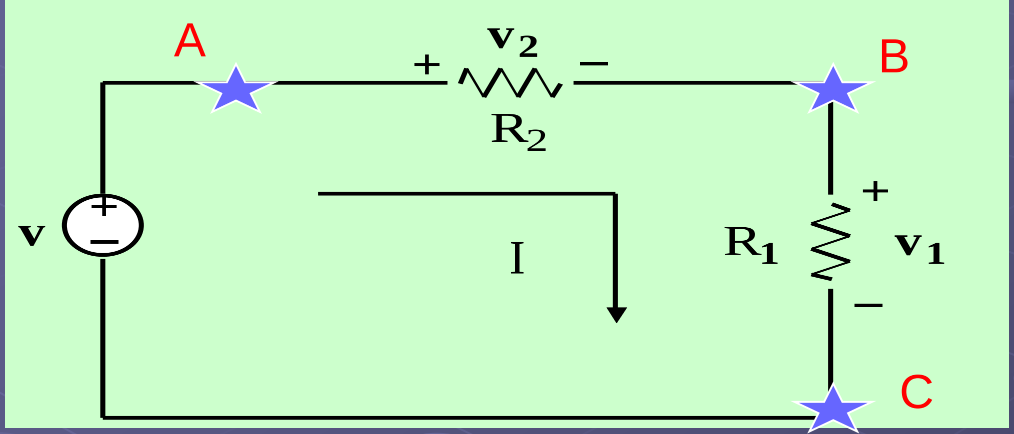
- Analogous formula is true for any number of resistors,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \qquad \text{(parallel combination)}$$

- It follows that the equivalent resistance of a parallel combination of resistors is always less than any of the individual resistors

VDR Rule

In the electrical circuit below, find voltage across the resistor R_1 in terms of the resistances R_1, R_2 and potential difference between the battery's terminals V.



Energy conservation implies:

$$V = V_1 + V_2$$

with $V_1 = IR_1$ and $V_2 = IR_2$

Then, $V = I\left(R_1 + R_2\right)$, so $I = \dfrac{V}{R_1 + R_2}$

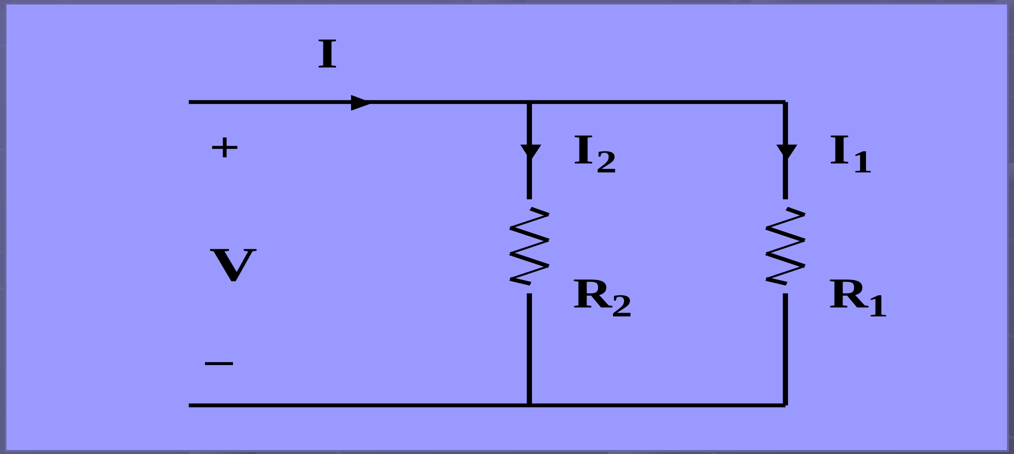
Thus,

$$\boxed{V_1 = V\frac{R_1}{R_1 + R_2}}$$

This circuit is known as **voltage divider**.

CDR Rule

In the electrical circuit below, find current through the resistor R_1 in terms of the resistances R_1, R_2 and total current I induced by the battery.



Charge conservation implies:

$$I = I_1 + I_2$$

with $$I_1 = \frac{V}{R_1},\ \text{and}\ I_2 = \frac{V}{R_2}$$

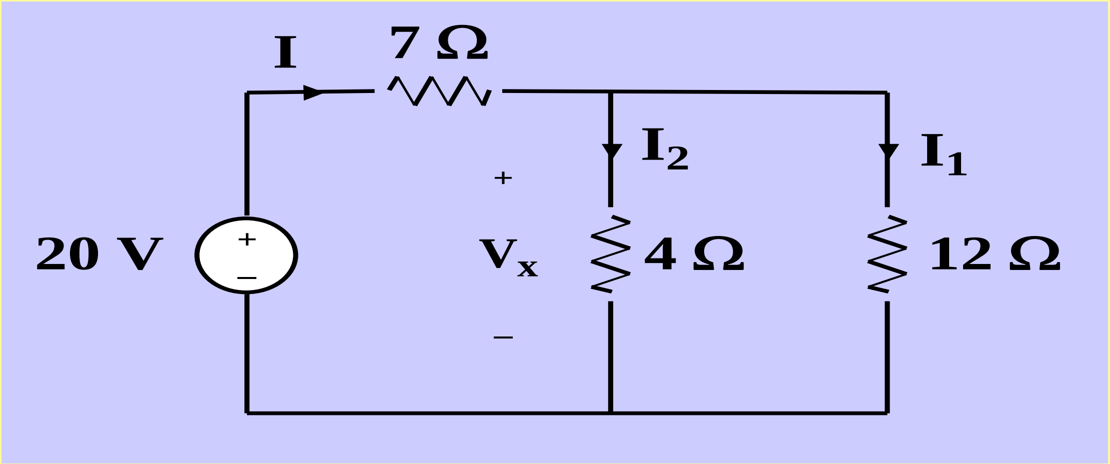
Then, $$I_1 = \frac{IR_{eq}}{R_1},\ \text{with}\ R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Thus, $$\boxed{I_1 = I\,\frac{R_2}{R_1 + R_2}}$$

This circuit is known as **current divider**.

Problem-23: Direct current circuits: example

Find the currents I_1 and I_2 and the voltage V_x in the circuit shown below.

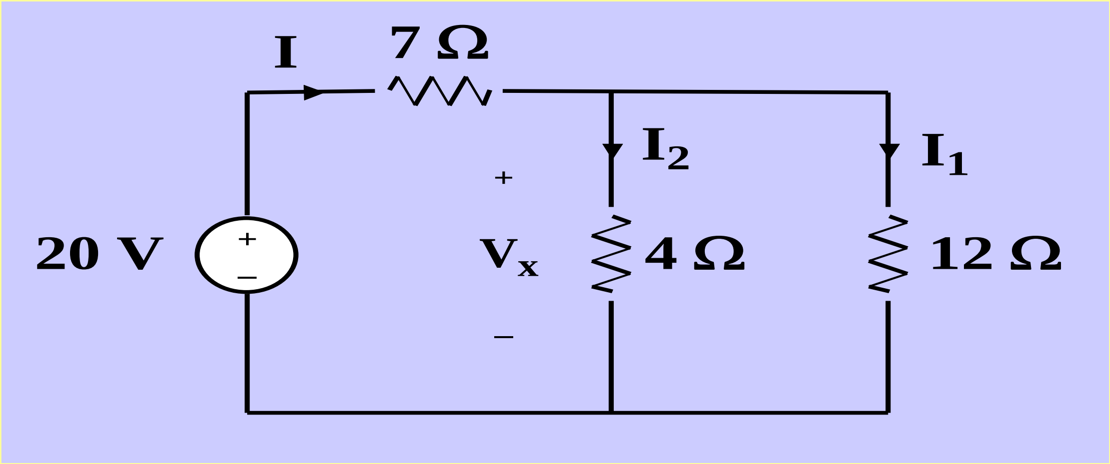


Strategy:

1. Find current I by finding the equivalent resistance of the circuit
2. Use current divider rule to find the currents I_1 and I_2
3. Knowing I_2, find V_x.

Problem-24: Direct current circuits: example

Find the currents I_1 and I_2 and the voltage V_x in the circuit shown below.



First find the equivalent resistance seen by the 20 V source:

$$R_{eq} = 7\Omega + \frac{4\Omega(12\Omega)}{12\Omega + 4\Omega} = 10\ \Omega$$

Then find current I by,

$$I = \frac{20V}{R_{eq}} = \frac{20V}{10\Omega} = 2\,A$$

We now find I_1 and I_2 directly from the current division rule:

$$I_1 = \frac{2A(4\Omega)}{12\Omega + 4\Omega} = 0.5\,A, \text{ and } I_2 = I - I_1 = 1.5A$$

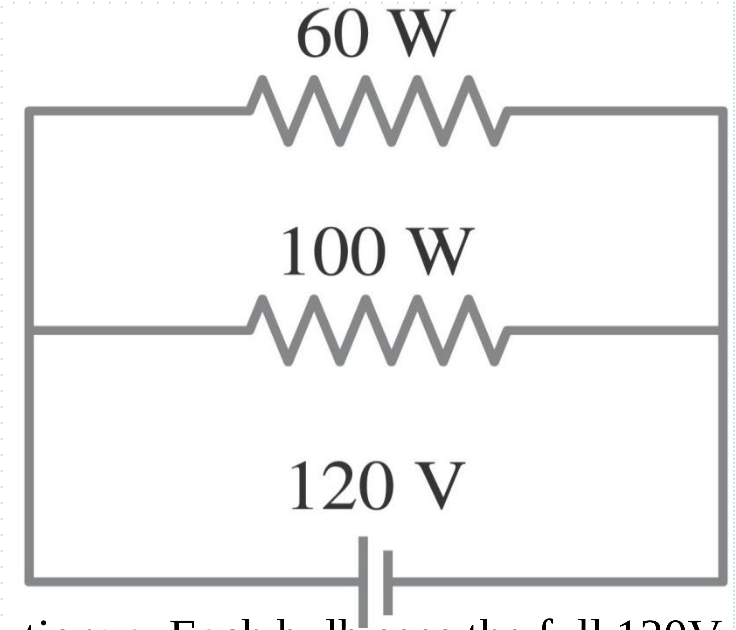
Finally, voltage V_x is

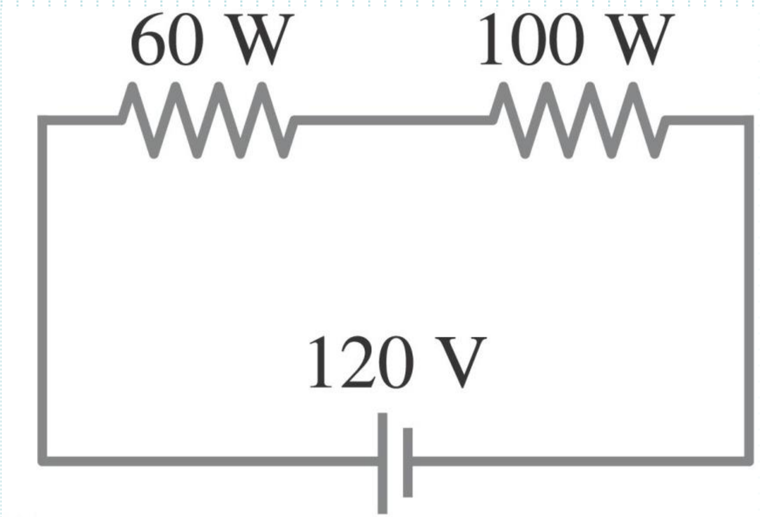
$$V_x = I_2(4\Omega) = 1.5A(4\Omega) = 6V$$

Practice problem

Problem-25: Conceptual Example: An illuminating surprise.

Q: A 100-W, 120-V lightbulb and a 60-W, 120-V lightbulb are connected in two different ways as shown. In each case, which bulb glows more brightly? Ignore change of filament resistance with current (and temperature).



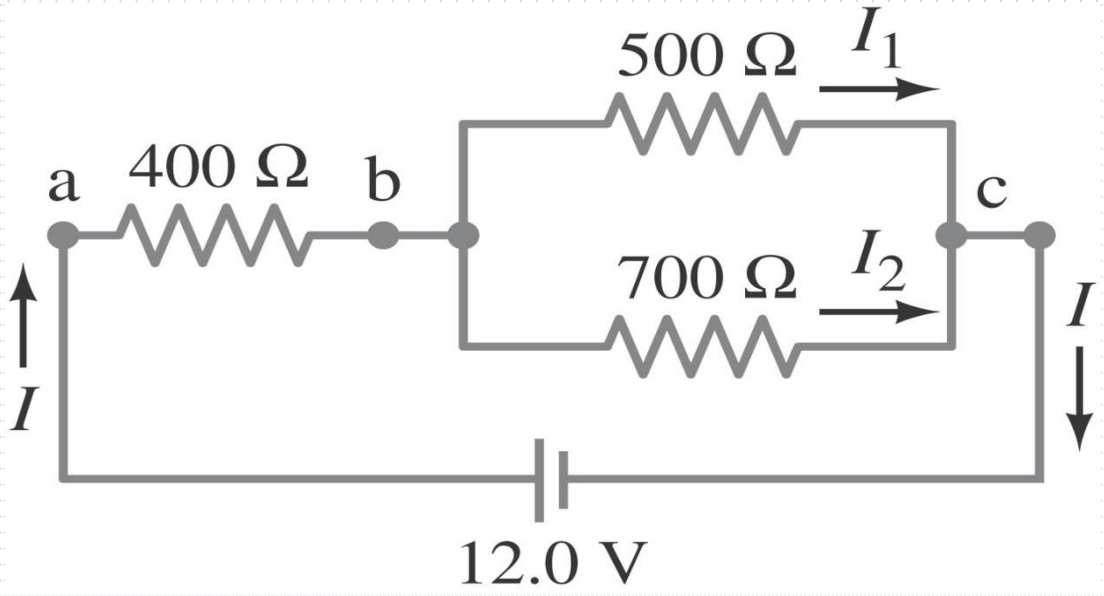


Solution: a. Each bulb sees the full 120V drop, as they are designed to do, so the 100-W bulb is brighter.
b. $P = V^2/R$, so at constant voltage the bulb dissipating more power will have lower resistance. In series, then, the 60-W bulb – whose resistance is higher – will be brighter. (More of the voltage will drop across it than across the 100-W bulb).

Practice problem

Problem-26: Circuit with series and parallel resistors.

Q: How much current is drawn from the battery shown?

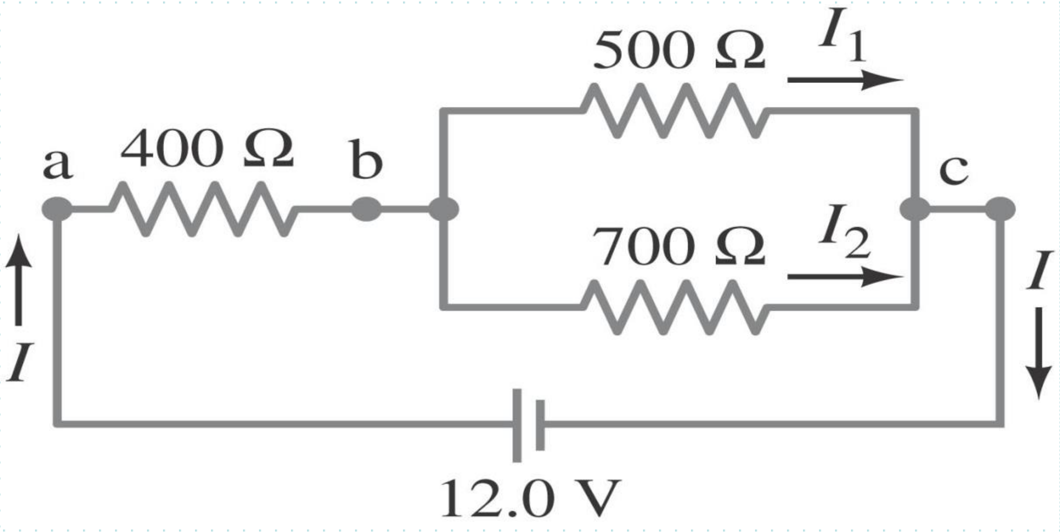


Solution: First we find the equivalent resistance of the two resistors in parallel, then the series combination of that with the third resistance. For the two parallel resistors, R = 290 Ω (remember that the usual equation gives the INVERSE of the resistance – students are often confused by this). The series combination is then 690 Ω, so the current in the battery is V/R = 17 mA.

Practice problem

Problem-27: Current in one branch.

Q: What is the current through the 500-Ω resistor shown? (Note: This is the same circuit as in the previous problem.) The total current in the circuit was found to be 17 mA.

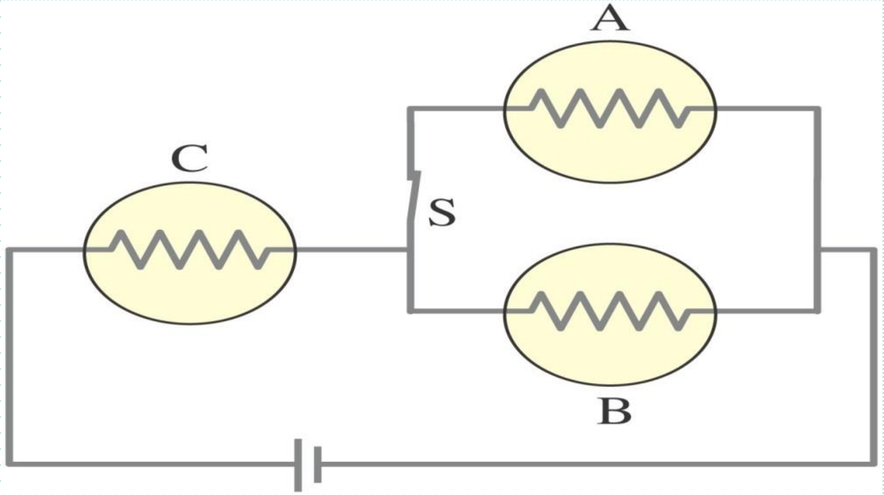


Solution: The total current is 17 mA, so the voltage drop across the 400-Ω resistor is V = IR = 7.0 V. This means that the voltage drop across the 500-Ω resistor is 5.0 V, and the current through it is I = V/R = 10 mA.

Practice Problem

Problem-28: Conceptual : Bulb brightness in a circuit.

Q: The circuit shown has three identical lightbulbs, each of resistance R. (a) When switch S is closed, how will the brightness of bulbs A and B compare with that of bulb C? (b) What happens when switch S is opened? Use a minimum of mathematics in your answers.



Solution: a. When S is closed, the bulbs in parallel have an equivalent resistance equal to half that of the series bulb. Therefore, the voltage drop across them is smaller. In addition, the current splits between the two of them. Bulbs A and B will be equally bright, but much dimmer than C.

b. With switch S open, no current flows through A, so it is dark. B and C are now equally bright, and each has half the voltage across it, so C is somewhat dimmer than it was with the switch closed, and B is brighter.

Practice problem

Problem-29: A two-speed fan.

One way a multiple-speed ventilation fan for a car can be designed is to put resistors in series with the fan motor. The resistors reduce the current through the motor and make it run more slowly. Suppose the current in the motor is 5.0 A when it is connected directly across a 12-V battery. (a) What series resistor should be used to reduce the current to 2.0 A for low-speed operation? (b) What power rating should the resistor have?

Solution: a. The resistance of the motor is V/I = 2.4 Ω. For the current to be 2.0 A, the voltage across the motor must be 4.8 V; the other 7.2 V must appear across the resistor, whose resistance is therefore 7.2V/2.0A = 3.6 Ω.

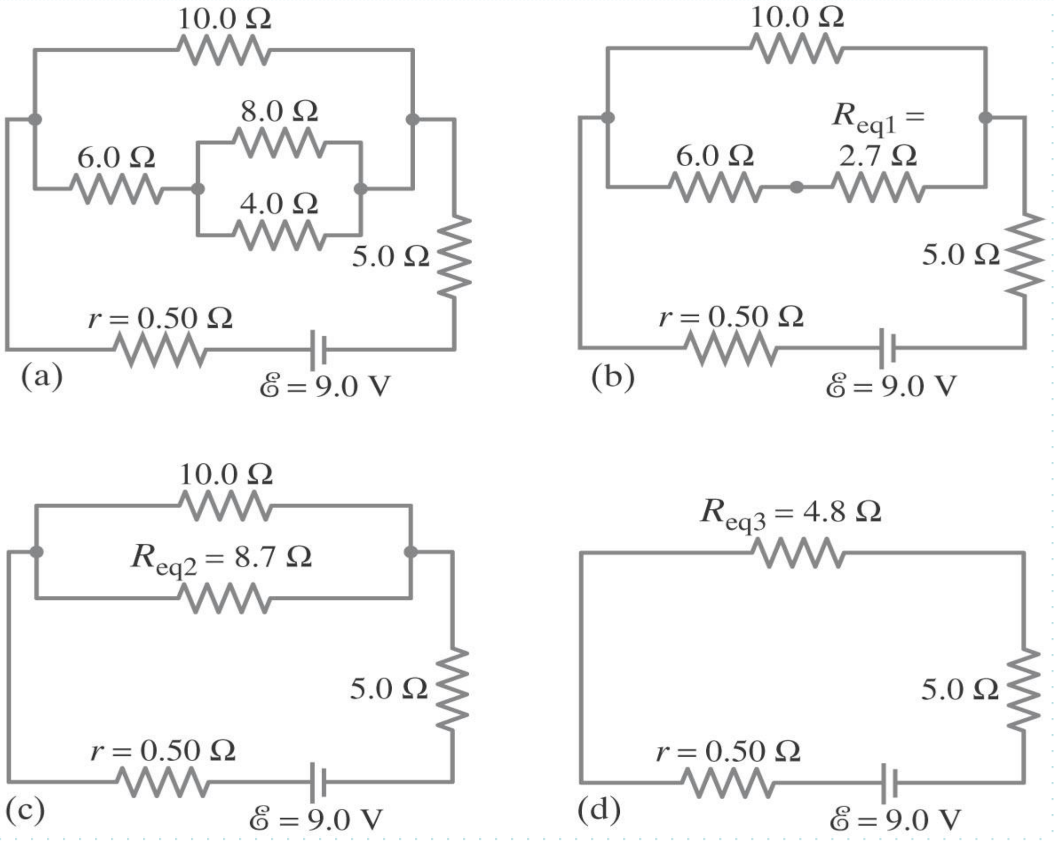
b. The power is IV = 14.4 W; a 20-W rating would be reasonably safe.

Practice Problem

Problem-30: Analyzing a circuit.

Q: A 9.0-V battery whose internal resistance *r* is 0.50 Ω is connected in the circuit shown. (a) How much current is drawn from the battery? (b) What is the terminal voltage of the battery? (c) What is the current in the 6.0-Ω resistor?

Solution: a. First, find the equivalent resistance. The 8-Ω and 4-Ω resistors in parallel have an equivalent resistance of 2.7 Ω; this is in series with the 6.0-Ω resistor, giving an equivalent of 8.7 Ω. This is in parallel with the 10.0-Ω resistor, giving an equivalent of 4.8 Ω; finally, this is in series with the 5.0-Ω resistor and the internal resistance of the battery, for an overall resistance of 10.3 Ω. The current is 9.0 V/10.3 Ω = 0.87 A.
b. The terminal voltage of the battery is the emf less Ir = 8.6 V.
c. The current across the 6.0-Ω resistor and the 2.7-Ω equivalent resistance is the same; the potential drop across the 10.0-Ω resistor and the 8.3-Ω equivalent resistor is also the same. The sum of the potential drop across the 8.3-Ω equivalent resistor plus the drops across the 5.0-Ω resistor and the internal resistance equals the emf of the battery; the current through the 8.3-Ω equivalent resistor is then 0.48 A.



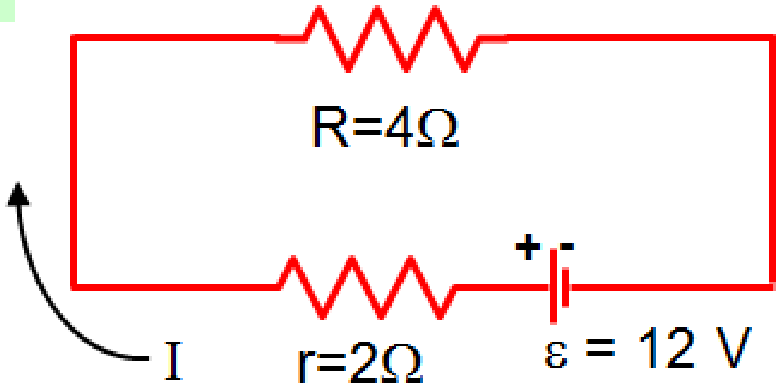
Practice problem

Problem-31: A 12 V battery with 2 Ω internal resistance is connected to a 4 Ω resistor. Calculate (a) the rate at which chemical energy is converted to electrical energy in the battery, (b) the power dissipated internally in the battery, and (c) the power output of the battery.

(a) Rate of energy conversion.

The total resistance in the circuit is 6 Ω, so

$V = I R_{total}$

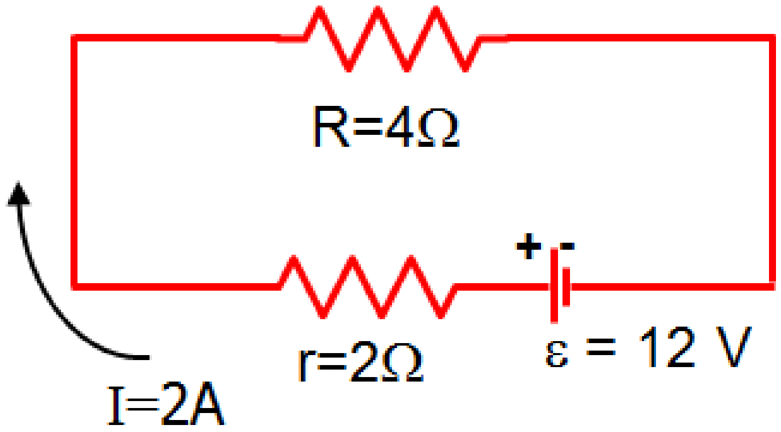


$I = \varepsilon / R_{total} = 12\ V / 6\ \Omega = 2\ A$

Energy is converted at the rate $P_{converted} = I\varepsilon = (2\ A)(12\ V) = 24 W$.

Practice problem

(b) Power dissipated internally in the battery.



$$P_{dissipated} = I^2 r = (2\ A)^2\,(2\ \Omega) = 8\ W.$$

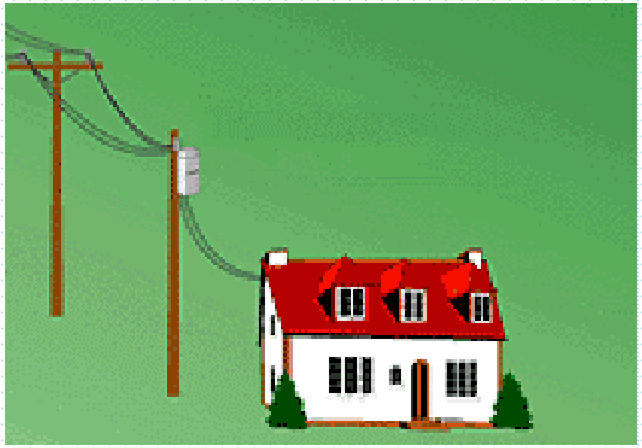
(c) Power output of the battery.

$$P_{output} = P_{converted} - P_{dissipated} = 24\ W - 8\ W = 16W.$$

Practice problem

Problem-32: the electric utility company supplies your house with electricity from the main power lines at 120 V. The wire from the pole to your house has a resistance of 0.03 Ω. Suppose your house is drawing 110 A of current…

(a) Find the voltage at the point where the power wire enters your house.



$$\Delta V_{HT} = IR$$

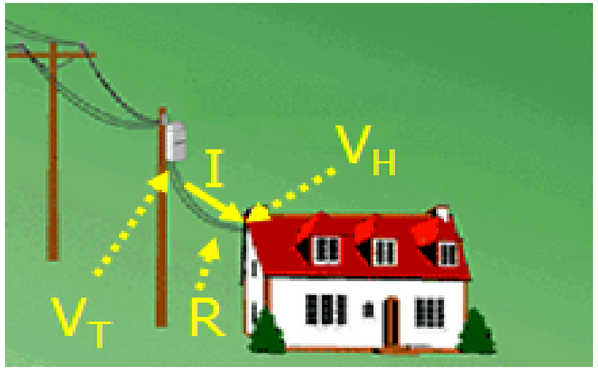
$$V_T - V_H = IR$$

$$V_H = V_T - IR$$

$$V_H = (120\ \text{V}) - (110\ \text{A})\,(0.03\Omega) = 116.7\ \text{V}$$

Practice problem

(b) How much power is being dissipated in the wire from the pole to your house?



Three different ways to solve; all will give the correct answer.

$$P = I\Delta V = I^2R = (\Delta V)^2/R$$

$$P = I(V_T - V_H) = I^2R = (V_T - V_H)^2/R$$

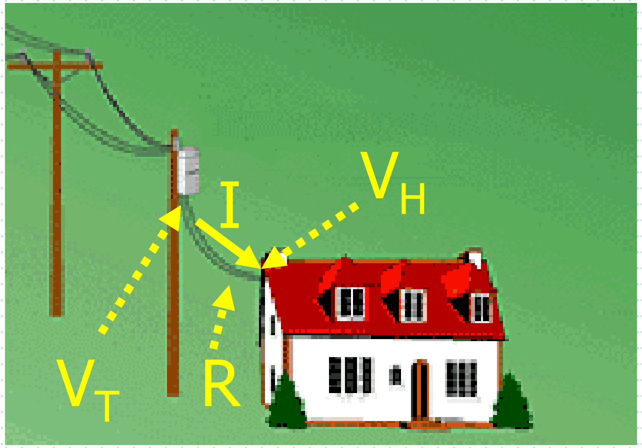
$$P = (110\ A)\ (120\ V - 116.7\ V) = 363\ W$$

$$\text{or } P = (110\ A)^2\ (0.03\Omega) = 363\ W$$

$$\text{or } P = (120\ V - 116.7\ V)^2\ /\ (0.03\Omega) = 363\ W$$

Practice problem

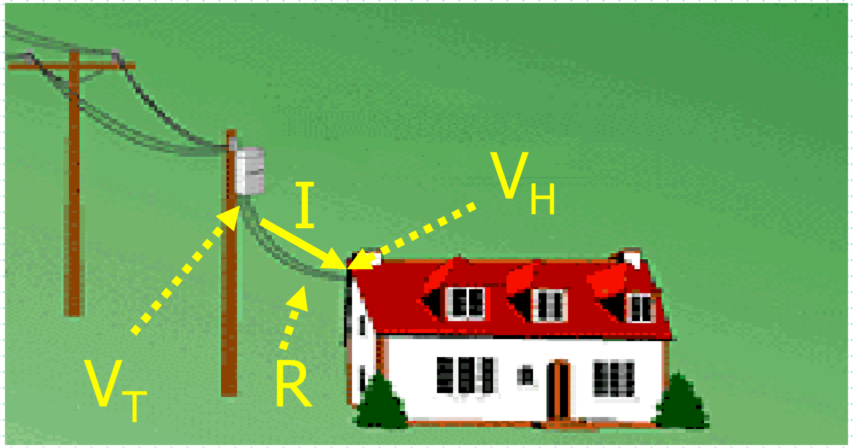
(c) How much power are you using inside your house?



You need to understand that your household voltage represents the potential difference between the "incoming" and "outgoing" power lines, and the "outgoing" is at ground (0 V in this case)…except…

…because the "outgoing" power line is at 0 V, you can "accidentally" get this correct if you simply multiply the current by the voltage at the point where the power wire enters your house.

(c) How much power are you using inside your house?



$$P = I\Delta V$$

$$P = (110\ \text{A})\ (116.7\ \text{V} - 0\ \text{V})$$

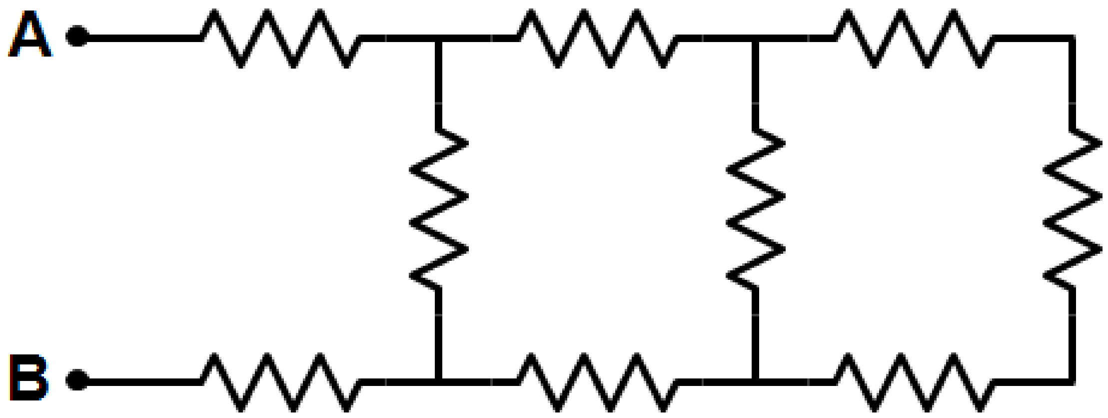
$$P = 12840\ \text{W}$$

You don’t want to use the $P=I^2R=V^2/R$ equations because you don’t know the effective resistance of your house (although you could calculate it).

$P = (110\ \text{A})\ (120\ \text{V}) - (110\ \text{A})(3.3\ \text{V})$ is also a reasonable way to work this part.

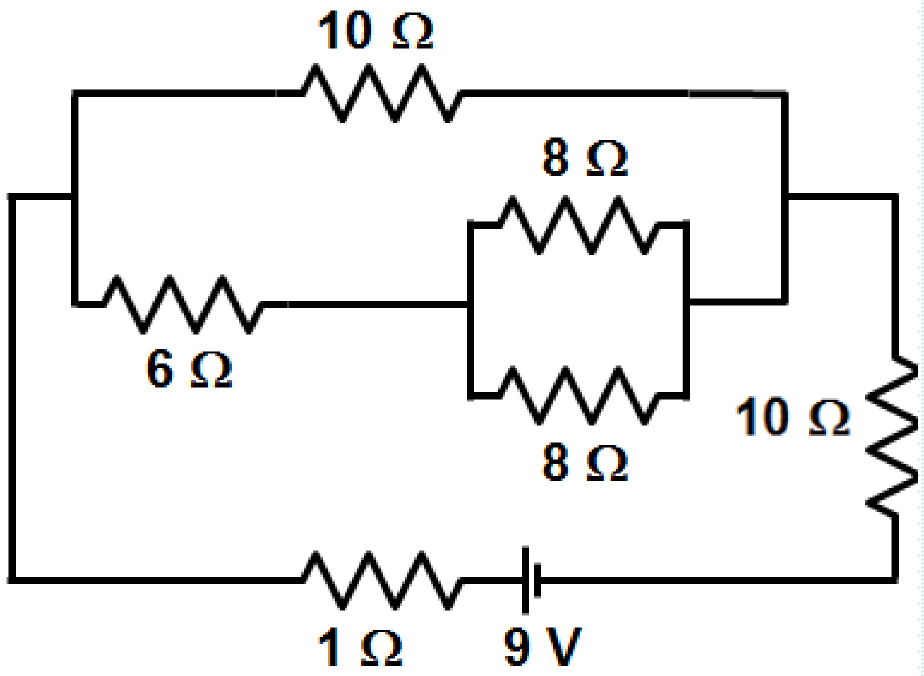
Practice problem

Problem-33: calculate the equivalent resistance of the resistor "ladder" shown. All resistors have the same resistance R.



Practice problem

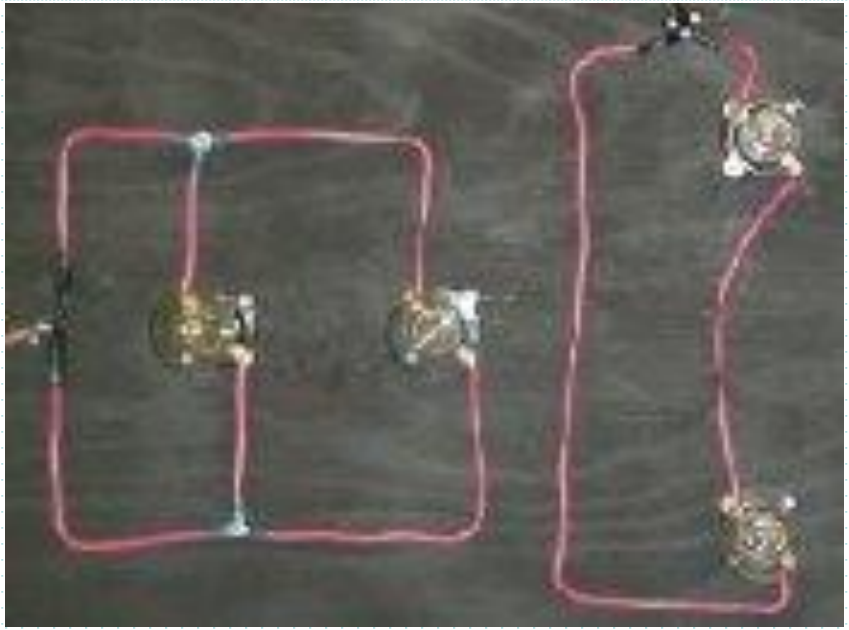
Problem-34: For the circuit below, calculate the current drawn from the battery and the current in the 6 Ω resistor.



Practice problem

Problem-35: two 100 Ω light bulbs are connected (a) in series and (b) in parallel to a 24 V battery. For which circuit will the bulbs be brighter?

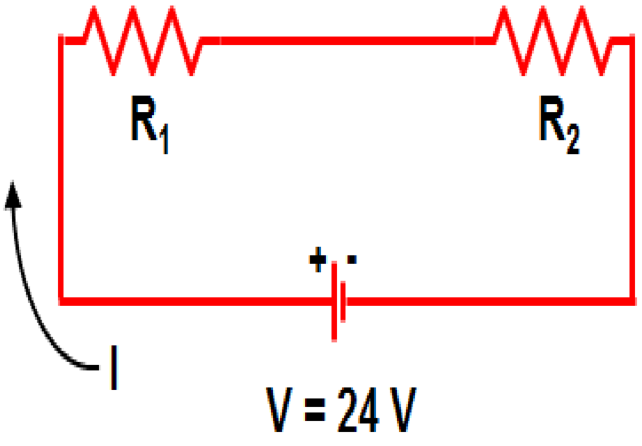
1. parallel (left)
2. series (right)



Practice problem

Problem-36: two 100 Ω light bulbs are connected (a) in series and (b) in parallel to a 24 V battery. What is the current through each bulb and what is the equivalent resistance of each circuit? (c) For which of the two circuits above would the bulb(s) be brighter

(a) Series combination.



$R_{eq} = \Sigma R_i$

$R_{eq} = R_1 + R_2$

$V = I\, R_{eq}$

$V = I\,(R_1 + R_2)$

$I = V / (R_1 + R_2) = 24\text{ V} / (100\ \Omega + 100\ \Omega) = \boxed{0.12\text{ A}}$

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The same current of 0.12 A flows through each bulb.

The equivalent resistance is

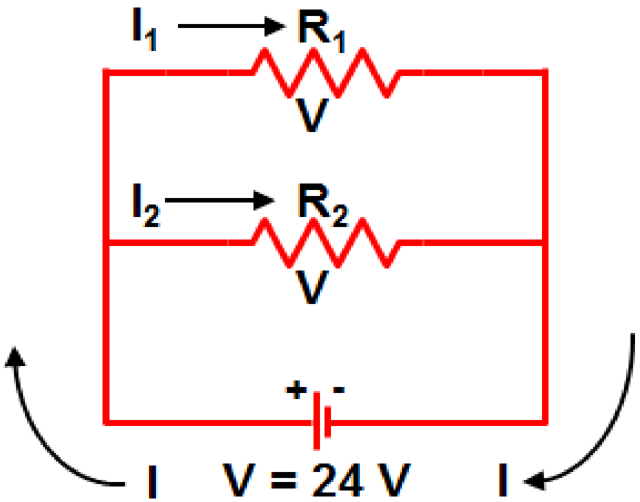
$$R_{eq} = R_1 + R_2$$

$$R_{eq} = 100\ \Omega + 100\ \Omega = \boxed{200\ \Omega}.$$

(b) Parallel combination.

$$\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$



Practice problem

$$V = I\,R_{eq}$$

$$V = \frac{I}{\frac{1}{R_1} + \frac{1}{R_2}}$$

$$\boxed{I = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$

$$I = 24V\left(\frac{1}{100\ \Omega} + \frac{1}{100\ \Omega}\right)$$

$$I = 24\left(\frac{200}{10000}\right) = \boxed{0.48\ \text{A}}$$

The equivalent resistance is

$$\frac{1}{R_{eq}} = \left(\frac{1}{100\ \Omega} + \frac{1}{100\ \Omega}\right) = \left(\frac{200\ \Omega}{10000\ \Omega^2}\right)$$

$$\boxed{R_{eq} = 50\ \Omega}$$

(c) Q: For which of the two circuits above would the bulb(s) be brighter…

To answer the question, we must calculate the power dissipated in the bulbs for each circuit. The more power "consumed," the brighter the bulb.

Practice problem

In both circuits, the bulbs are identical and have identical currents passing through them. We pick either bulb for the calculation.

Series circuit: we know the resistance and current through each bulb, so we use:

$$P = I^2R$$

$$P = (0.12\ \text{A})^2\ (100\ \Omega)$$

$$P = 1.44\ \text{W}$$

Parallel circuit: we know the resistance and voltage drop across each bulb, so we use:

$$P = V^2 / R$$

$$P = (24\ \text{V})^2 / (\ 100\ \Omega)$$

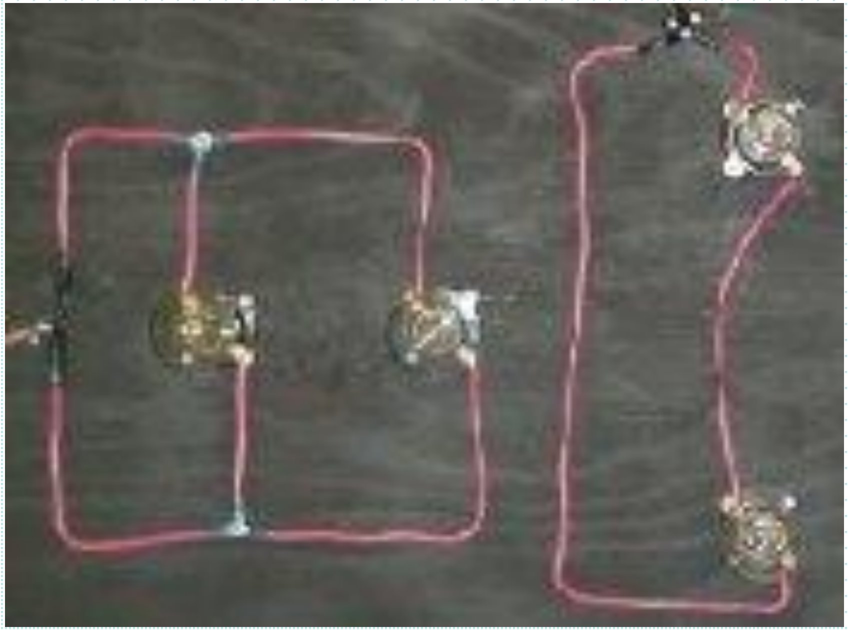
$$P = 5.76\ \text{W}$$

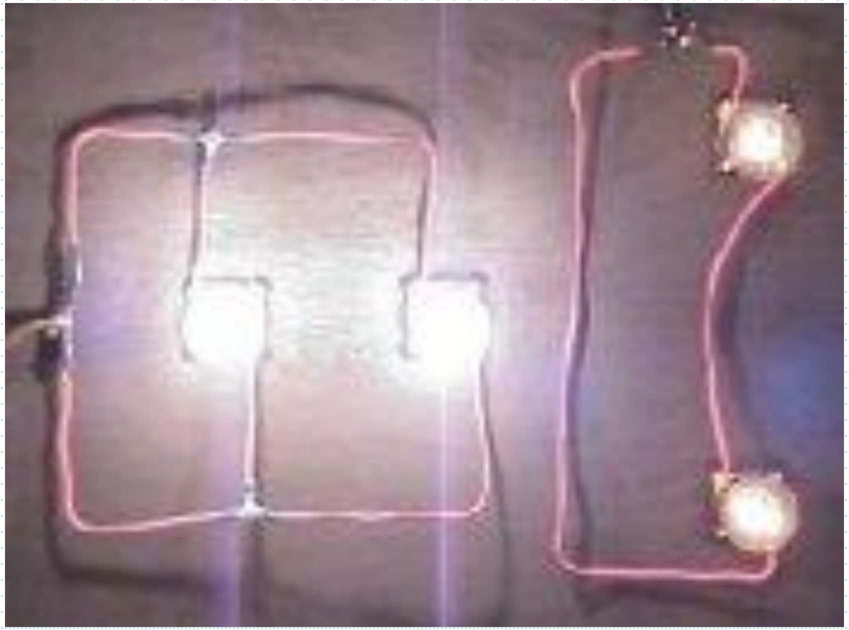
Compare: $P_{series} = 1.44\ \text{W}$

$P_{parallel} = 5.76\ \text{W}$

The bulbs in parallel are brighter.

Practice Problem

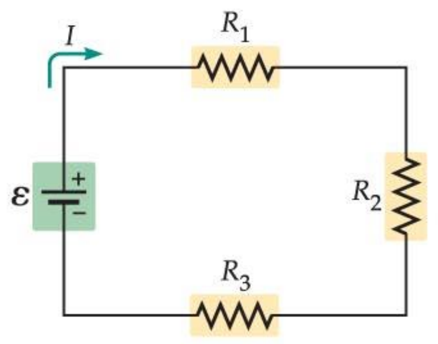




Problem-37

GUIDED Example 21.5 | Three Resistors in Series Series Circuit

A circuit consists of three resistors connected in series to a 24.0-V battery. The current in the circuit is 0.0320 A. Given that $R_1 = 250.0\ \Omega$ and $R_2 = 150.0\ \Omega$, find (**a**) the resistance of R_3 and (**b**) the potential difference across each resistor.



Picture the Problem

The circuit is shown in our sketch. Notice that the same current, $I = 0.0320$ A, flows through each of the three resistors. This is the key characteristic of a series circuit.

Strategy

(**a**) First, we can find the equivalent resistance of the circuit by using Ohm's law, $R_{eq} = \varepsilon/I$. Since the resistors are in series, we also know that $R_{eq} = R_1 + R_2 + R_3$. We can solve this relation for the only unknown, R_3. (**b**) We can then calculate the potential difference across each resistor using Ohm's law, $V = IR$.

Known	Unknown
$\varepsilon = 24.0$ V	(**a**) $R_3 = ?$
$I = 0.0320$ A	(**b**) $V_1 = ?$
$R_1 = 250.0\ \Omega$	$V_2 = ?$
$R_2 = 150.0\ \Omega$	$V_3 = ?$

GUIDED Example 21.5 | Three Resistors in Series *(Continued)* **Series Circuit**

Solution

Part (a)

1 Use Ohm's law to find the equivalent resistance of the circuit:

$$R_{eq} = \frac{\varepsilon}{I} = \frac{24.0\ \text{V}}{0.0320\ \text{A}}$$
$$= 750\ \Omega$$

Math HELP
Solving Simple Equations
See Math Review, Section III

2 Set R_{eq} equal to the sum of the individual resistances, and solve for R_3:

$$R_{eq} = R_1 + R_2 + R_3$$
$$R_3 = R_{eq} - R_1 - R_2$$
$$= 750\ \Omega - 250.0\ \Omega - 150.0\ \Omega$$
$$= \boxed{350\ \Omega}$$

Part (b)

3 Use Ohm's law to calculate the potential difference across R_1:

$$V_1 = IR_1$$
$$= (0.0320\ \text{A})(250.0\ \Omega)$$
$$= \boxed{8.00\ \text{V}}$$

4 Find the potential difference across R_2:

$$V_2 = IR_2$$
$$= (0.0320\ \text{A})(150.0\ \Omega)$$
$$= \boxed{4.80\ \text{V}}$$

5 Find the potential difference across R_3:

$$V_3 = IR_3$$
$$= (0.0320\ \text{A})(350\ \Omega)$$
$$= \boxed{11.2\ \text{V}}$$

Insight

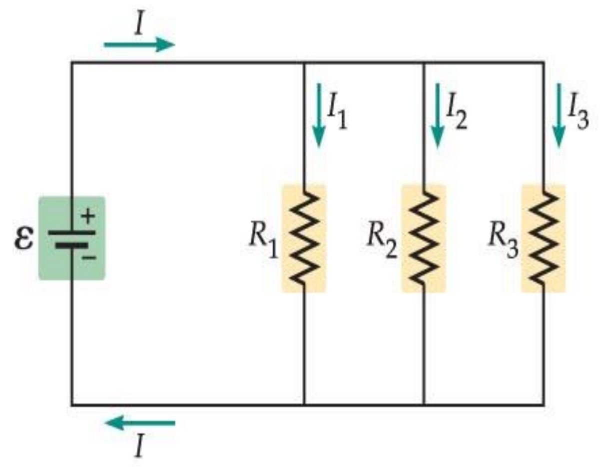
Notice that the greater the resistance, the greater the potential difference. In addition, the sum of the individual potential differences is the voltage of the battery, $8.00\ \text{V} + 4.80\ \text{V} + 11.2\ \text{V} = 24.0\ \text{V}$, as expected.

Problem-38:

GUIDED Example 21.6 | Three Resistors in Parallel

Parallel Circuit

Consider a circuit with three resistors, $R_1 = 250.0\ \Omega$, $R_2 = 150.0\ \Omega$, and $R_3 = 350.0\ \Omega$, connected in parallel with a 24.0-V battery. Find (**a**) the total current supplied by the battery and (**b**) the current through each resistor.



Picture the Problem

Our sketch indicates the parallel connection of the resistors with the battery. Notice that each of the resistors experiences precisely the same potential difference—namely, the 24.0 V produced by the battery. This is the feature that characterizes parallel connections.

Strategy

(**a**) We can find the total current by solving Ohm's law, $V = IR$, for the current. This gives $I = \varepsilon/R_{eq}$, where $\varepsilon = V = 24.0$ V and $1/R_{eq} = 1/R_1 + 1/R_2 + 1/R_3$. (**b**) For each resistor the current is given by Ohm's law, $I = \varepsilon/R$.

Known	Unknown
$\varepsilon = 24.0$ V	(**a**) $I = ?$
$R_1 = 250.0\ \Omega$	(**b**) $I_1 = ?$
$R_2 = 150.0\ \Omega$	$I_2 = ?$
$R_3 = 350.0\ \Omega$	$I_3 = ?$

GUIDED Example 21.6 | Three Resistors in Parallel *(Continued)* **Parallel Circuit**

Solution

Part (a)

1 Find the equivalent resistance of the circuit:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$= \frac{1}{250.0\ \Omega} + \frac{1}{150.0\ \Omega} + \frac{1}{350.0\ \Omega}$$

$$= 0.01352\ \Omega^{-1}$$

$$R_{eq} = \frac{1}{0.01352\ \Omega^{-1}} = 73.96\ \Omega$$

2 Use Ohm's law to find the total current:

$$I = \frac{\varepsilon}{R_{eq}} = \frac{24.0\ \text{V}}{73.96\ \Omega} = \boxed{0.325\ \text{A}}$$

Part (b)

3 Calculate I_1 using $I_1 = \varepsilon / R_1$ with $\varepsilon = 24.0\ \text{V}$:

$$I_1 = \frac{\varepsilon}{R_1} = \frac{24.0\ \text{V}}{250.0\ \Omega} = \boxed{0.0960\ \text{A}}$$

4 Repeat the calculation of Step 3 for resistors 2 and 3:

$$I_2 = \frac{\varepsilon}{R_2} = \frac{24.0\ \text{V}}{150.0\ \Omega} = \boxed{0.160\ \text{A}}$$

$$I_3 = \frac{\varepsilon}{R_3} = \frac{24.0\ \text{V}}{350.0\ \Omega} = \boxed{0.0686\ \text{A}}$$

Insight

As expected, the smallest resistor, R_2, carries the greatest current. The three currents combined yield the total current, as they must. That is, $I_1 + I_2 + I_3 = 0.0960\ \text{A} + 0.160\ \text{A} + 0.0686\ \text{A} = 0.325\ \text{A} = I$. Also, notice that the equivalent resistance ($73.96\ \Omega$) is less than that of the smallest resistor in the parallel circuit ($150.0\ \Omega$).

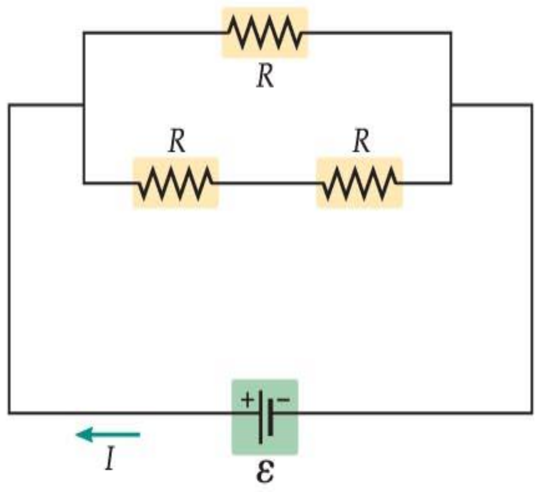
Problem-39:

GUIDED Example 21.7 | Combination Special — Combination Circuit

In the circuit shown in the diagram below, the emf of the battery is 12 V, and each resistor has a resistance of 450 Ω. Find the current supplied by the battery to this circuit.

Picture the Problem

The circuit for this problem has three resistors connected to a battery. The lower two resistors are in series with one another, and they are in parallel with the upper resistor. The battery has an emf of 12 V.



Strategy

The current supplied by the battery is given by Ohm's law, $I = \varepsilon/R_{eq}$, where R_{eq} is the equivalent resistance of the three resistors. To find R_{eq} we first note that the lower two resistors are in series, giving a net resistance of $2R$. Next, the upper resistor, R, is in parallel with the pair of resistors having resistance $2R$. Calculating the equivalent resistance of this combination yields the desired R_{eq}.

Known	Unknown
$\varepsilon = 12\ \text{V}$	$I = ?$
$R = 450\ \Omega$	

GUIDED Example 21.7 | Combination Special *(Continued)* — Combination Circuit

Solution

1 Calculate the equivalent resistance of the two *lower* resistors:

$$R_{\text{eq,lower}} = R + R = 2R$$

2 Use the result of Step 1 to calculate the equivalent resistance of R in parallel with $2R$:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R} + \frac{1}{2R} = \frac{3}{2R}$$

$$R_{\text{eq}} = \tfrac{2}{3}R$$

$$= \tfrac{2}{3}(450\ \Omega)$$

$$= 300\ \Omega$$

Math HELP
Solving Simple Equations
See Math Review, Section III

3 Find the current supplied by the battery, I:

$$I = \frac{\varepsilon}{R_{\text{eq}}}$$

$$= \frac{12\ \text{V}}{300\ \Omega}$$

$$= \boxed{0.040\ \text{A}}$$

Insight

Notice that the total resistance of the three 450-Ω resistors is less than 450 Ω—in fact, it is only 300 Ω.

Problem-40:

QUICK Example 21.8 What's the Current?

A handheld electric fan operates on a 3.0-V battery. If the power generated by the fan is 2.2 W, what is the current supplied by the battery?

Solution

Solving $P = IV$ for the current, we find

$$I = \frac{P}{V}$$

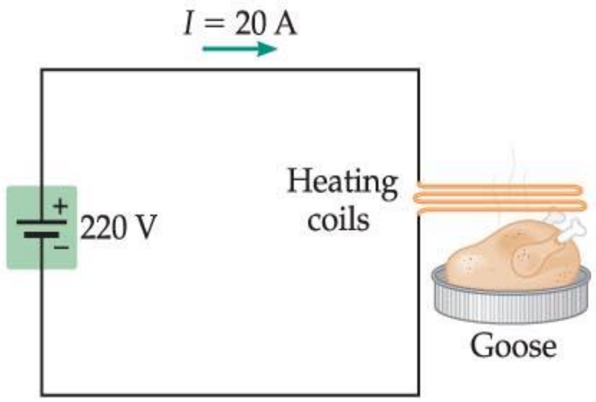
$$= \frac{2.2\ \text{W}}{3.0\ \text{V}}$$

$$= \boxed{0.73\ \text{A}}$$

Problem-41:

GUIDED Example 21.12 | Your Goose Is Cooked — Energy Cost

A holiday goose is cooked in an electric oven for 4.00 h. Assume that the stove draws a current of 20.0 A, operates at a voltage of 220.0 V, and uses electric energy that costs \$0.085 per kilowatt-hour. How much does it cost to cook your goose?



Picture the Problem

We show a schematic representation of the stove cooking the goose in our sketch. The current in the circuit is 20.0 A, and the voltage difference across the heating coils is 220 V.

Strategy

The cost equals the amount of energy used (in kilowatt-hours) times the rate (\$0.085 per kilowatt-hour). To find the energy used, we multiply power by time. The time is given, and the power is calculated using $P = IV$. To summarize, we find the power, multiply by the time, and then multiply by \$0.085/kWh to find the cost.

Known	Unknown
$\Delta t = 4.00\ \mathrm{h}$	cost of cooking $= ?$
$I = 20.0\ \mathrm{A}$	
$V = 220.0\ \mathrm{V}$	
rate $= \$0.085/\mathrm{kWh}$	

GUIDED Example 21.12 | Your Goose Is Cooked *(Continued)* — Energy Cost

Solution

1 Calculate the power delivered to the stove:

$$P = IV$$
$$= (20.0\ \text{A})(220.0\ \text{V})$$
$$= 4.40\ \text{kW}$$

Math HELP
Dimensional Analysis
See Lesson 1.3

2 Multiply the power by the time to determine the total energy supplied to the stove during the 4.00 h of cooking:

$$\Delta PE = P\Delta t$$
$$= (4.40\ \text{kW})(4.00\ \text{h})$$
$$= 17.6\ \text{kWh}$$

3 Multiply by the cost per kilowatt-hour to find the total cost of cooking:

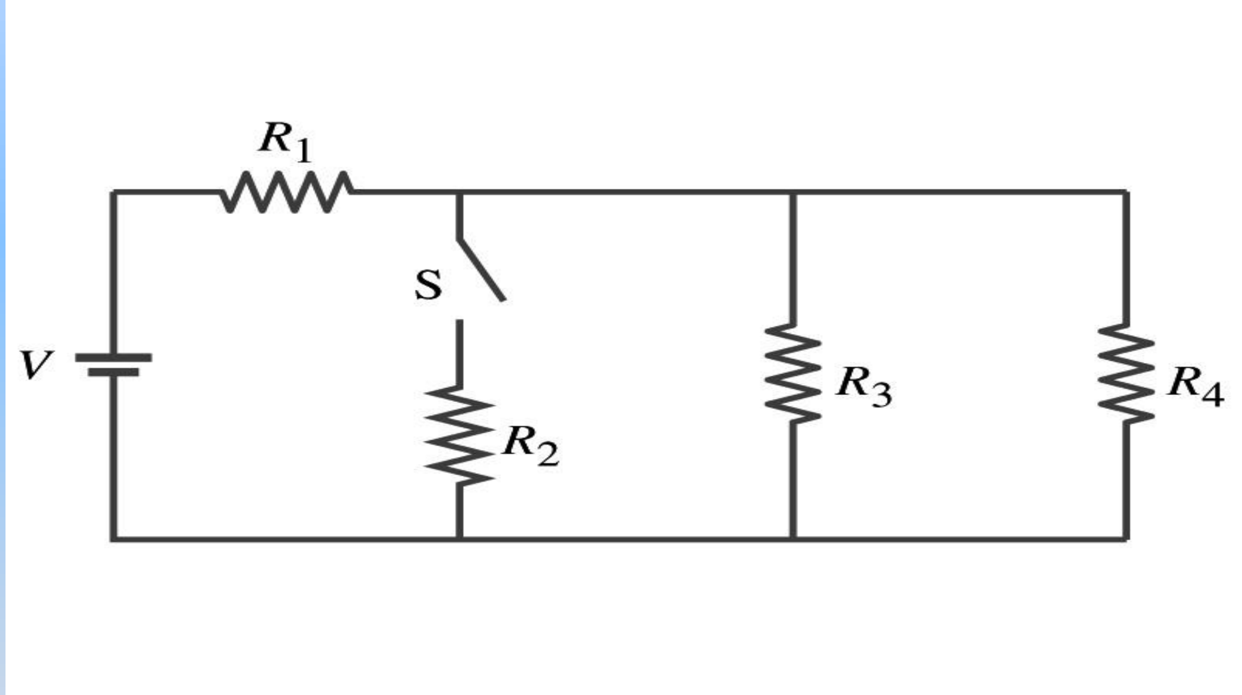
$$\text{cost} = \text{energy used} \times \text{rate}$$
$$= (17.6\ \cancel{\text{kWh}})(\$0.085/\cancel{\text{kWh}})$$
$$= \boxed{\$1.50}$$

Insight

Thus, your goose can be cooked for about a dollar and a half.

Practice Problem

Problem-42:



Given:

$$R_1 = 1500\Omega$$

$$R_3 = 1000\Omega$$

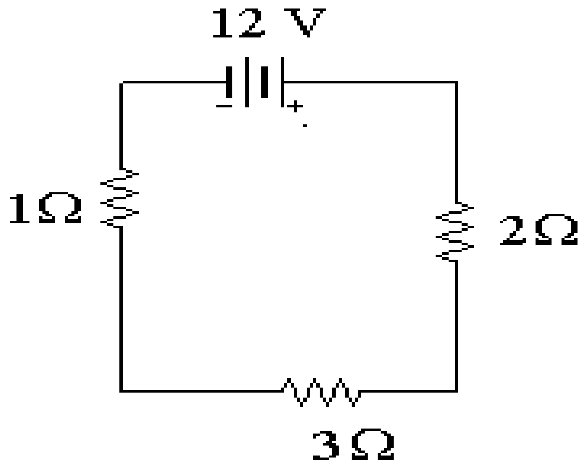
$$R_4 = 3000\Omega$$

V = 6 Volts

Find the current and the voltage across each resistor.

Practice Problem

Problem-43:



A series circuit is shown to the left.

a) What is the total resistance?

R(series) = 1 + 2 + 3 = 6Ω

b) What is the total current?

ΔV=IR   12=I(6)   I = 2A

c) What is the current across EACH resistor?

They EACH get 2 amps!

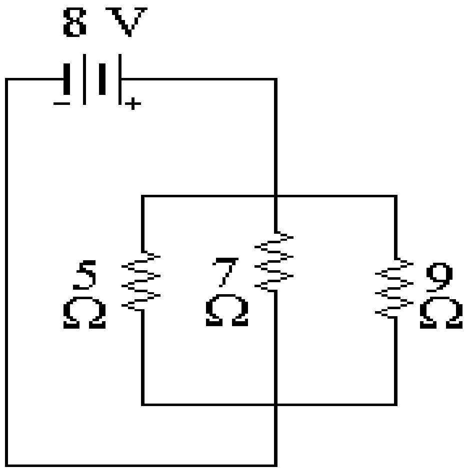
d) What is the voltage drop across each resistor?(Apply Ohm's law to each resistor separately)

$V_{1\Omega}$=(2)(1)= 2 V $V_{3\Omega}$=(2)(3)= 6V $V_{2\Omega}$=(2)(2)= 4V

Notice that the individual VOLTAGE DROPS add up to the TOTAL!!

Practice Problem

Problem-44:



To the left is an example of a parallel circuit.

a) What is the total resistance?

$$\frac{1}{R_P} = \frac{1}{5} + \frac{1}{7} + \frac{1}{9}$$

$$\frac{1}{R_p} = 0.454 \rightarrow R_P = \frac{1}{0.454} =$$ **2.20 Ω**

b) What is the total current?

$$\Delta V = IR$$

$$8 = I(R) =$$ **3.64 A**

c) What is the voltage across EACH resistor?

8 V each!

d) What is the current drop across each resistor? (Apply Ohm's law to each resistor separately)

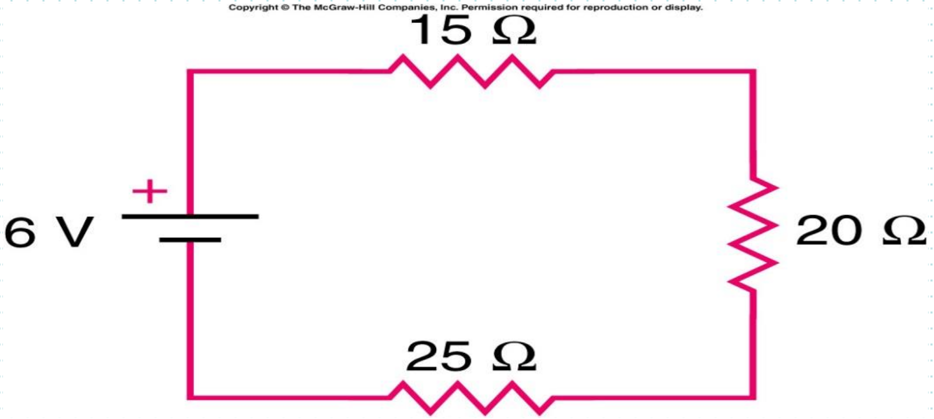
$$\Delta V = IR$$

$I_{5\Omega} = \frac{8}{5} =$ **1.6 A** $I_{7\Omega} = \frac{8}{7} =$ **1.14 A** $I_{9\Omega} = \frac{8}{9} =$ **0.90 A**

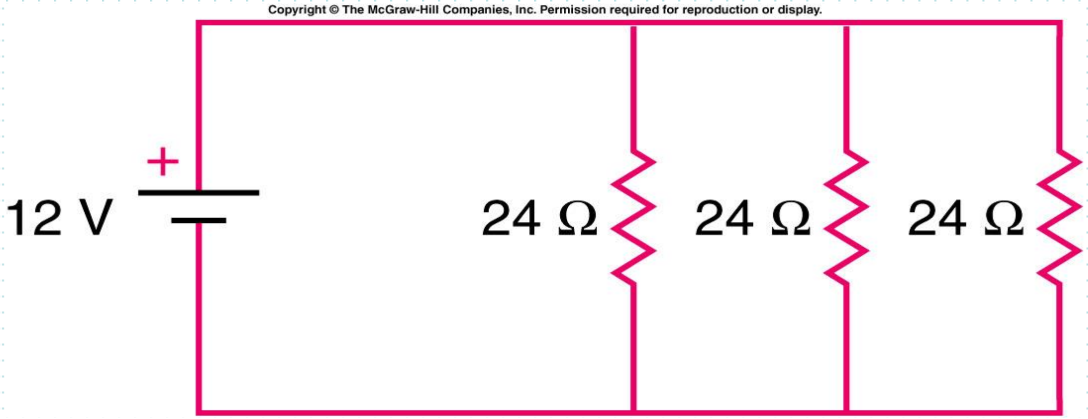
Notice that the individual currents ADD to the total.

Practice problem:

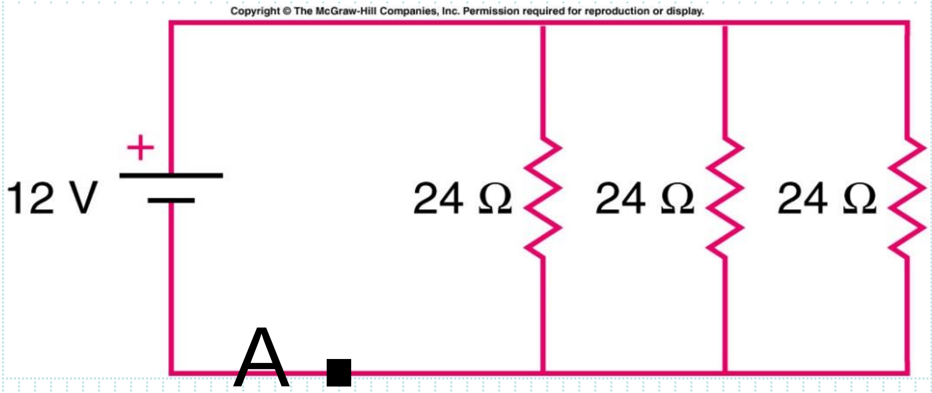
Problem-45: (i)What is the current that flows in this circuit?



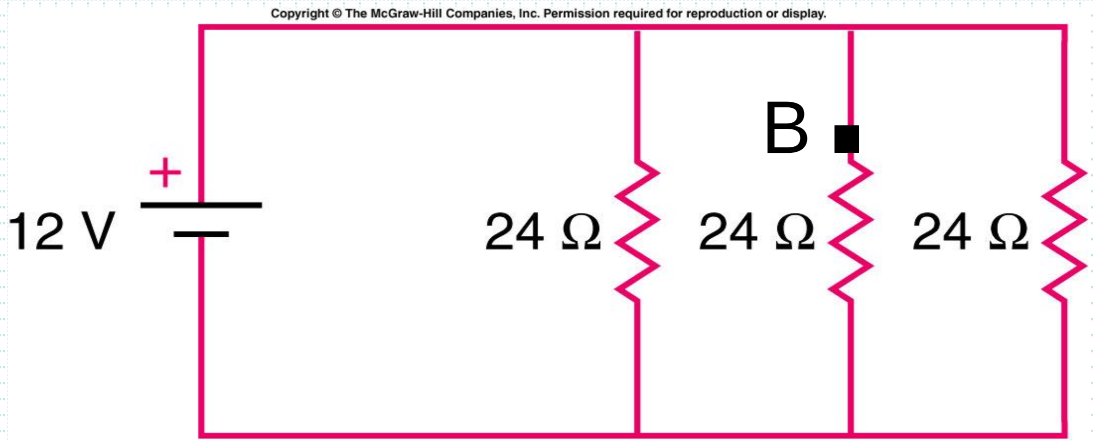
Problem-45:(ii) What is the total resistance of this circuit?



Problem-45:(iii) What is the current that flows in this circuit at point A?

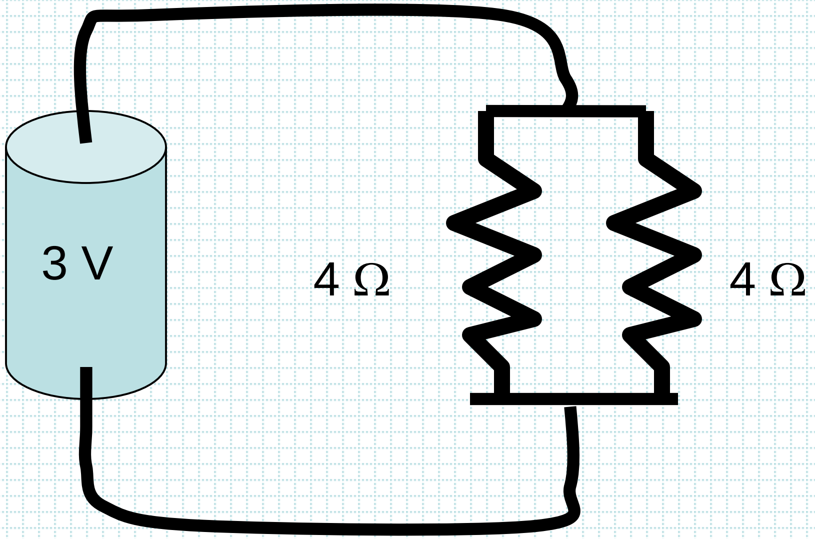


Problem-45:(iv) What is the current that flows in this circuit at point B?

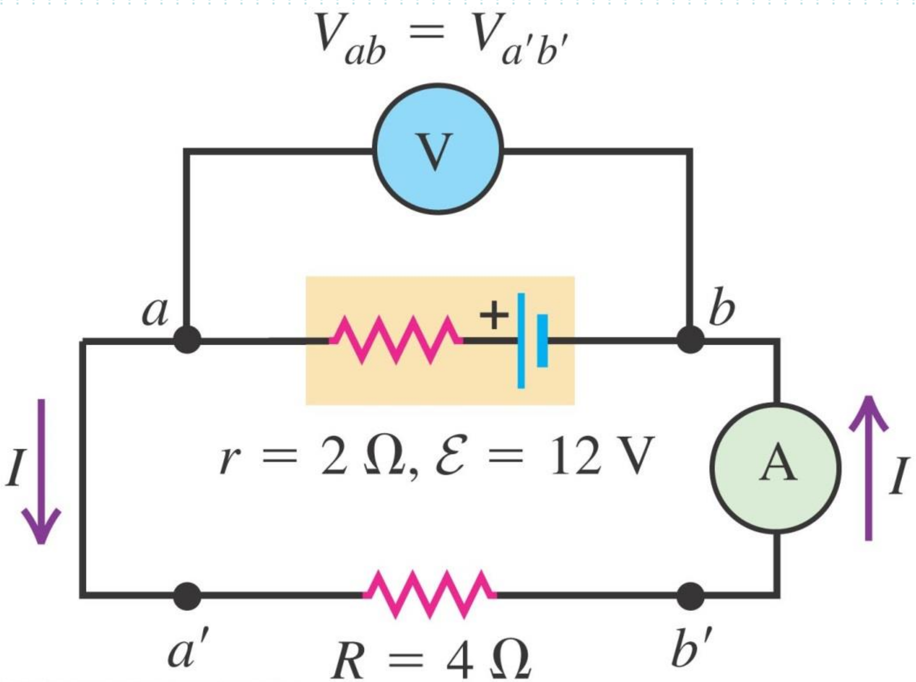


Practice problem:

Problem-45: (v) How much current I runs through this circuit of source 3V battery?



Problem-45: (vi) What are voltmeter and ammeter readings in the following ckt?



Electric Power

Power, as usual, is the rate at which work is done. For work done by electricity:

$$\overline{P} = \frac{W}{t} = \frac{qV}{t}$$

Chemical energy → Electric potential energy → Kinetic energy of charge carriers → Dissipation/Joule heat (heating the resistor through collisions with its atoms)

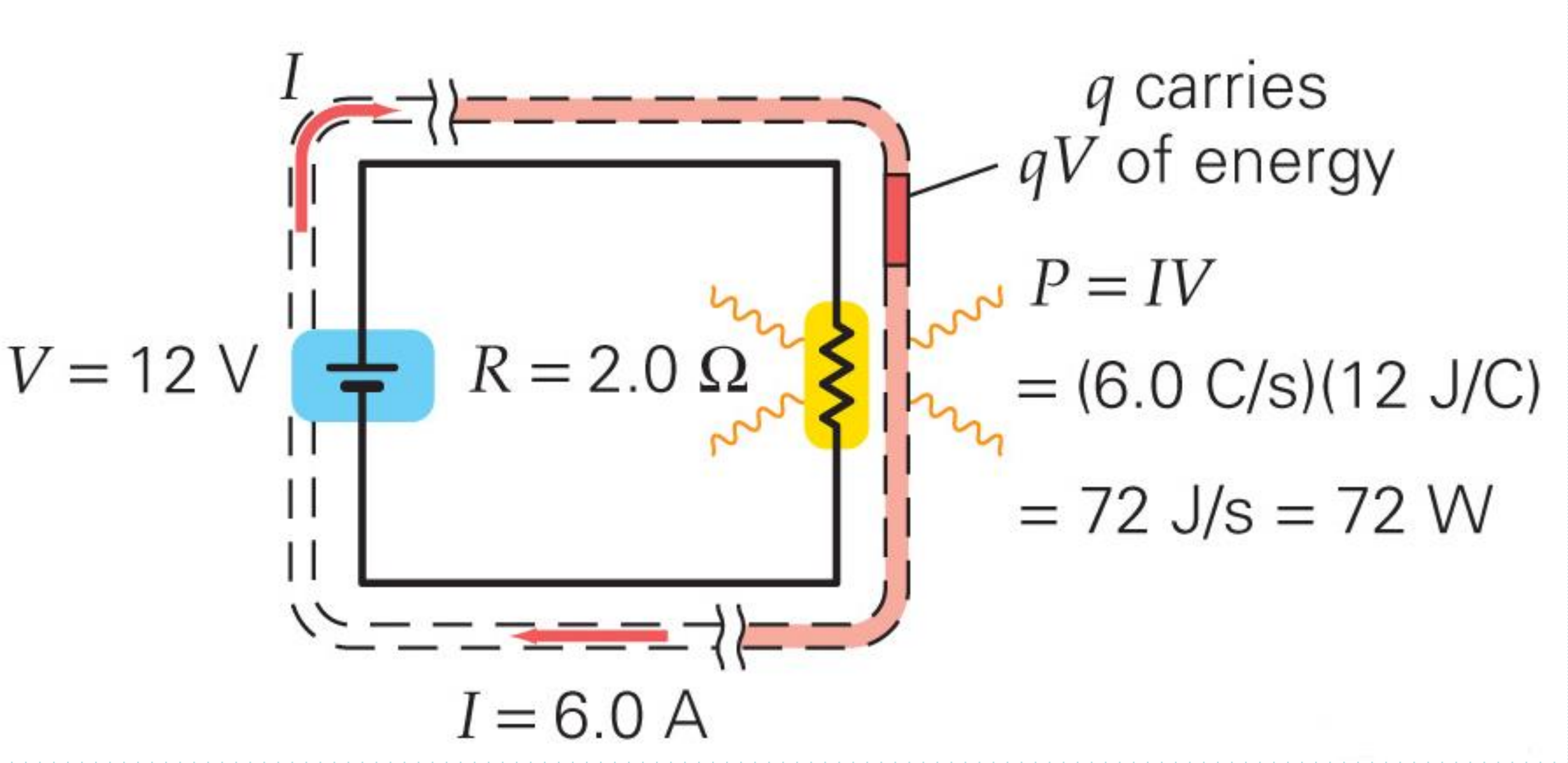
Rewriting, $P = IV$

For ohmic materials, we can write:

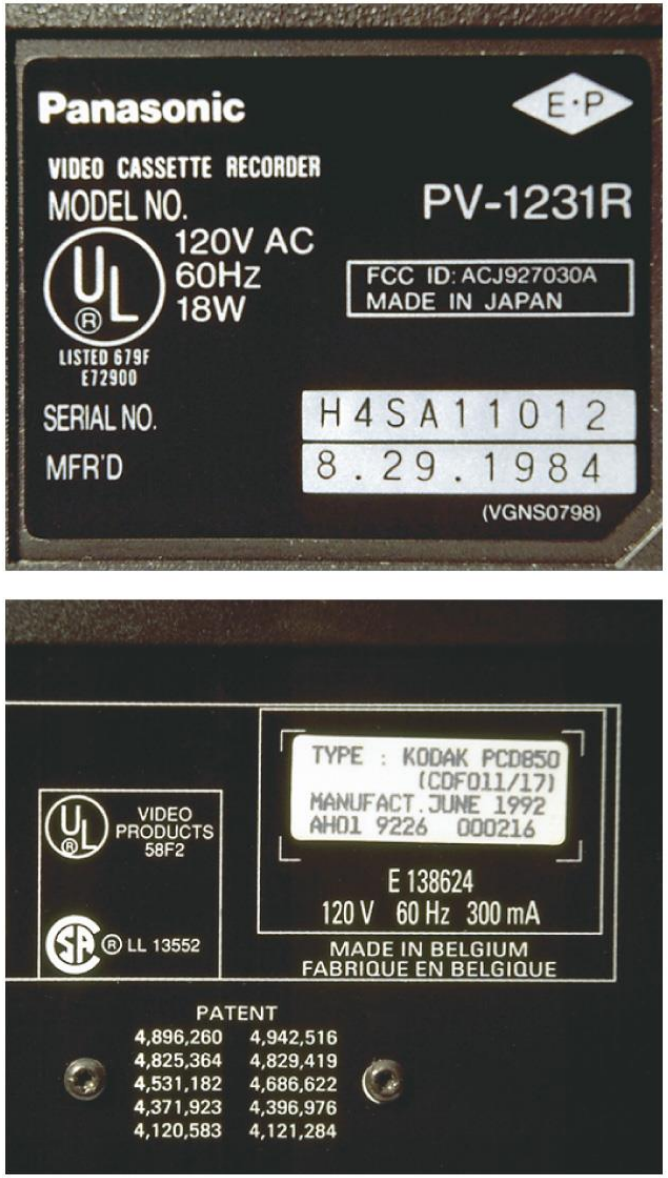
$$P = IV = \frac{V^2}{R} = I^2R$$

Electric Power

Problem-46: So, where does this power go? It is changed to heat in resistive materials.



Electric Power



Electric appliances are rated in watts, assuming standard household voltage.

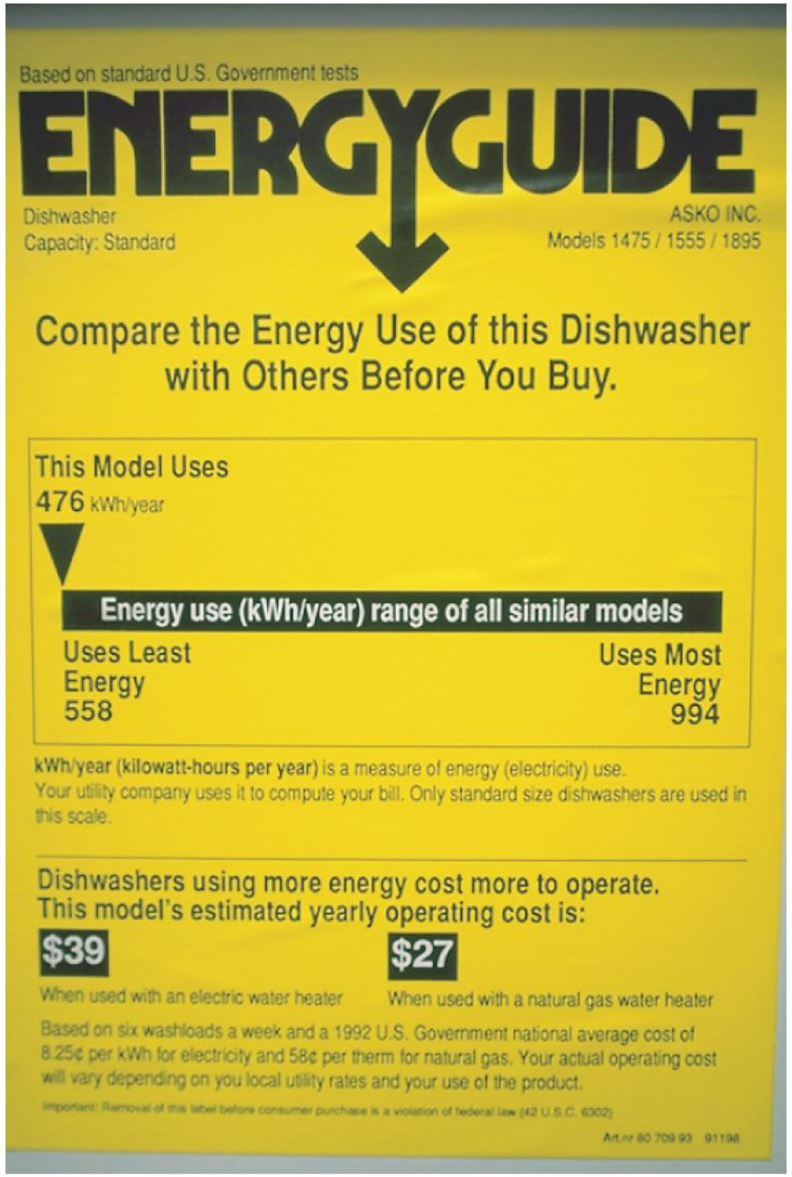
Electric Power

TABLE 17.2 Typical Power and Current Requirements for Various Household Appliances (120 V)

Appliance	*Power*	*Current*	*Appliance*	*Power*	*Current*
Air conditioner, room	1500 W	12.5 A	Heater, portable	1500 W	12.5 A
Air-conditioning, central	5000 W	41.7 A*	Microwave oven	900 W	5.2 A
Blender	800 W	6.7 A	Radio–cassette player	14 W	0.12 A
Clothes dryer	6000 W	50 A*	Refrigerator, frost-free	500 W	4.2 A
Clothes washer	840 W	7.0 A	Stove, top burners	6000 W	50.0 A*
Coffeemaker	1625 W	13.5 A	Stove, oven	4500 W	37.5 A*
Dishwasher	1200 W	10.0 A	Television, color	100 W	0.83 A
Electric blanket	180 W	1.5 A	Toaster	950 W	7.9 A
Hair dryer	1200 W	10.0 A	Water heater	4500 W	37.5 A*

*A high-power appliance such as this is typically wired to a 240-V house supply to reduce the current to half these values (Section 18.5).

Electric Power



The electric company typically bills us for kilowatt-hours (kWh), a unit of energy.

$$
\begin{aligned}
1\ \mathrm{kWh} &= (1000\ \mathrm{W})(3600\ \mathrm{s}) \\
&= (1000\ \mathrm{J/s})(3600\ \mathrm{s}) \\
&= 3.6 \times 10^{6}\ \mathrm{J}
\end{aligned}
$$

We can reduce our energy usage by buying efficient appliances.

Electric Power: Practice Problem

Problem-47: an electric heater draws 15.0 A on a 120 V line. How much power does it use and how much does it cost per 30 day month if it operates 3.0 h per day and the electric company charges 10.5 cents per kWh. For simplicity assume the current flows steadily in one direction.

(a) An electric heater draws 15.0 A on a 120 V line. How much power does it use.

$$P = IV$$

$$P = (15\ \text{A})(120\ \text{V}) = 1800\ \text{W} = 1.8\ \text{kW}$$

(b) How much does it cost per 30 day month if it operates 3.0 h per day and the electric company charges 10.5 cents per kWh.

$$\text{cost} = (1.8\text{kW})(30\ \text{days})\left(\frac{3\ \text{h}}{\text{day}}\right)\left(\frac{\$0.105}{\text{kWh}}\right)$$

$$\text{cost} = \$17.00$$

Electric Power: Practice Problem

(c) How much energy is a kilowatt hour (kWh)?

$$\left(1\text{ kW}\right)\left(1\text{ h}\right)=\left(1000\text{ W}\right)\left(3600\text{ s}\right)$$

$$=\left(1000\ \frac{\text{J}}{\text{s}}\right)\left(3600\text{ s}\right)$$

$$=3.6\times10^{6}\ \text{J}$$

So a kWh is a “funny” unit of energy. K (kilo) and h (hours) are lowercase, and W (James Watt) is uppercase.

Electric Power: Practice Problem

(d)How much energy did the electric heater use?

$$P_{\text{average}} = \frac{W_{\text{done by force}}}{\text{time}} = \frac{\text{Energy Transformed}}{\text{time}}$$

$$\text{Energy Transformed} = \left(P_{\text{average}}\right)\left(\text{time}\right)$$

$$\text{Energy Transformed} = \left(1800\,\frac{\text{J}}{\text{s}}\right)\left(30\ \text{days}\right)\left(\frac{3\ \text{h used}}{\text{day}}\right)\left(\frac{3600\ \text{s}}{\text{h}}\right)$$

$$\text{Energy Transformed} = 583{,}200{,}000\ \text{Joules used}$$

Electric Power: Practice Problem

$$\text{Energy Transformed} = 583,200,000 \text{ Joules used}$$

That's a ton of joules! Good bargain for \$17. That's about 34,000,000 joules per dollar (or 0.0000029¢/joule).

OK, "used" is not an SI unit, but I stuck it in there to help me understand. And joules don't come by the ton.

One last quibble. You know from energy conservation that you don't "use up" energy. You just transform it from one form to another.

Electric Power: Practice Problem

Problem-48: a typical lightning bolt can transfer 10^9 J of energy across a potential difference of perhaps $5x10^7$ V during a time interval of 0.2 s. Estimate the total amount of charge transferred, the current and the average power over the 0.2 s.

$$I_{avg} = \frac{\Delta Q}{\Delta t} \qquad P_{avg} = \frac{\Delta W}{\Delta t}$$

Electric Power: Practice Problem

Continuing with our energy transformation idea:

$$E_{transferred} = Q_{transferred}\ \Delta V_{i\to f}$$

$$Q_{transferred} = E_{transferred} / \Delta V_{i\to f}$$

$$Q_{transferred} = 10^9\ \mathrm{J} / 5\mathrm{x}10^7\ \mathrm{V}$$

$$\boxed{Q_{transferred} = 20\ \mathrm{C}}$$

That's a lot of charge (remember, typical charges are 10^{-6} C).

Once we have the charge transferred, the current is easy.

$$I = \frac{\Delta Q}{\Delta t}$$

$$I = \frac{20\ \mathrm{C}}{0.2\ \mathrm{s}} = \boxed{100\ \mathrm{A}}$$

Electric Power: Practice Problem

Average power is just the total energy transferred divided by the total time.

$$\overline{P_F} = \frac{W_F}{t}$$

$$\overline{P} = \frac{E_{transferred}}{t}$$

$$\overline{P} = \frac{10^9\ \mathrm{J}}{0.2\ \mathrm{s}}$$

$$\overline{P} = 5\times10^9\ \mathrm{W}$$

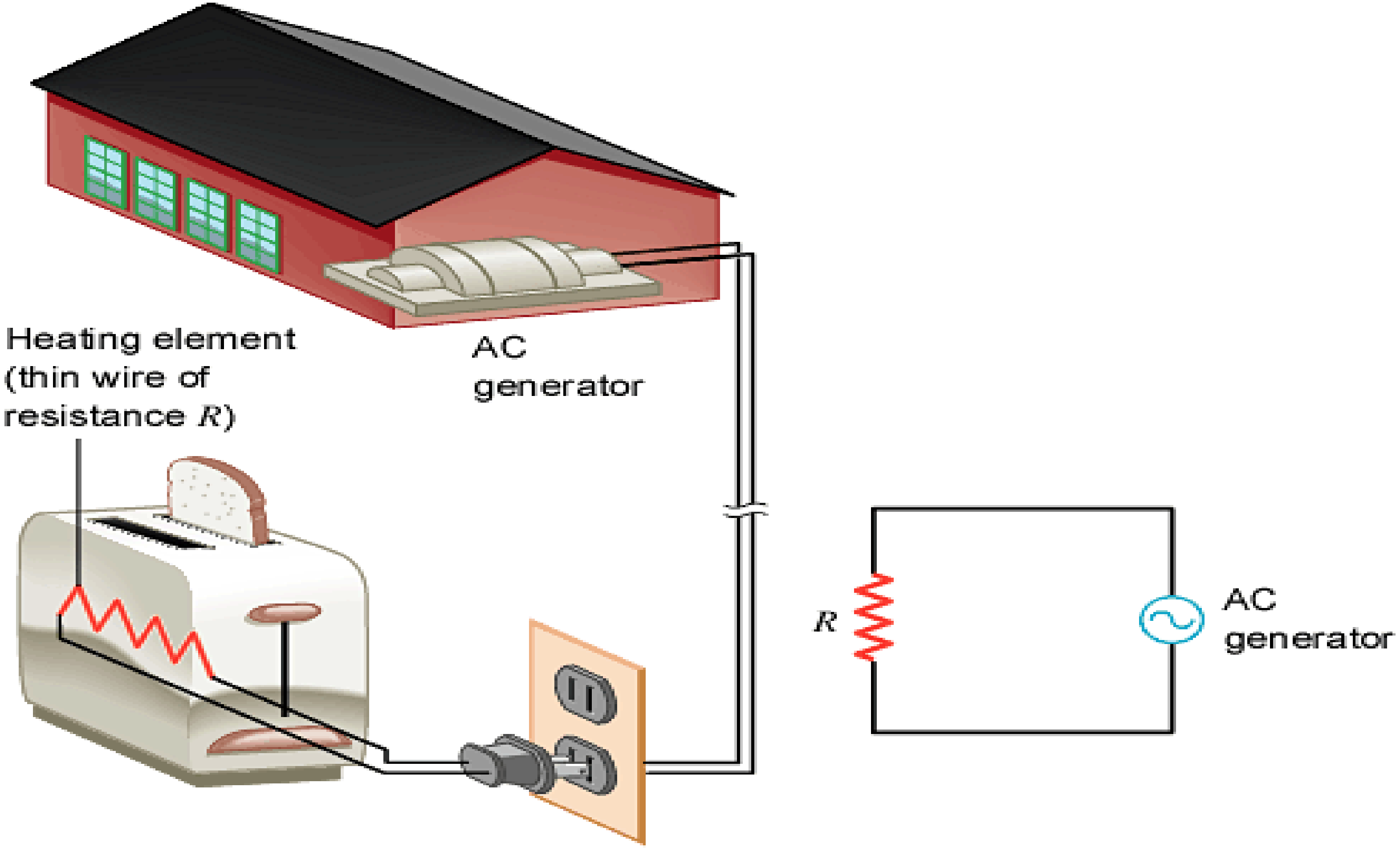
$$\boxed{\overline{P} = 5\ \mathrm{GW}}$$

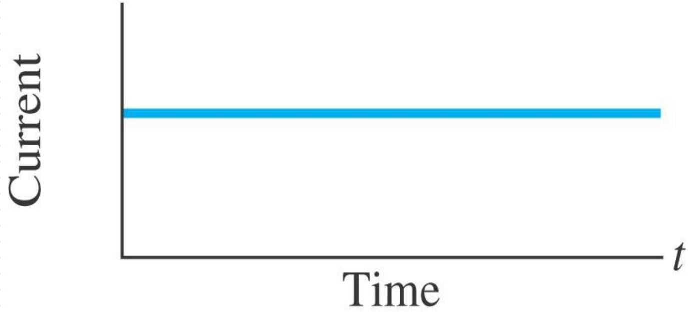
The numbers in this calculation differ substantially than the numbers in a homework problem(not necessarily assigned this semester). "This" lightning bolt carries relatively low current for a long time through a high potential difference, and transports a lot of energy.

In reality, there is no such thing as a universally-typical lightning bolt, so expect different results for different bolts.

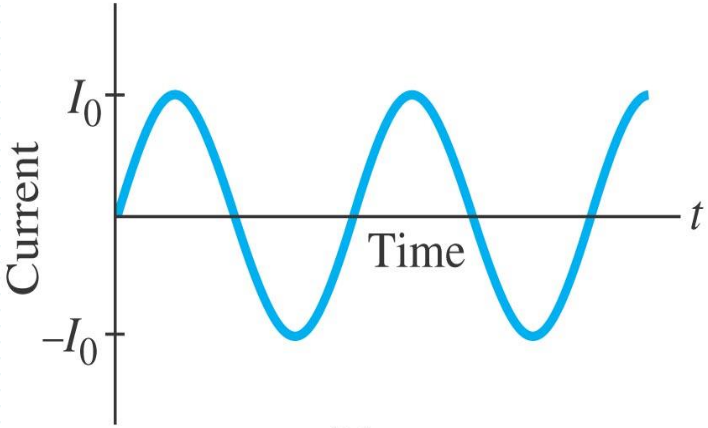
Holy ****, Batman. That's the power output of five enormous power plants!

Alternating Current





(a) DC



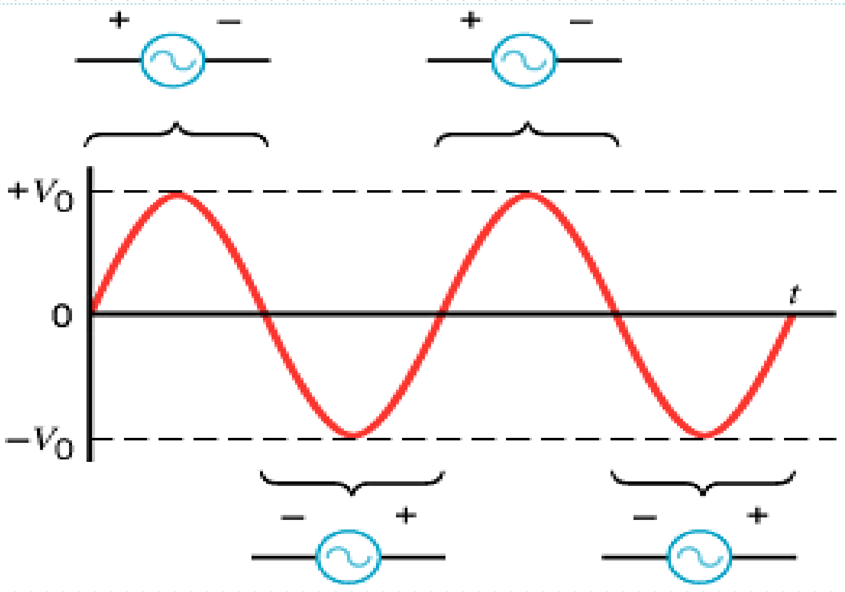
(b) AC

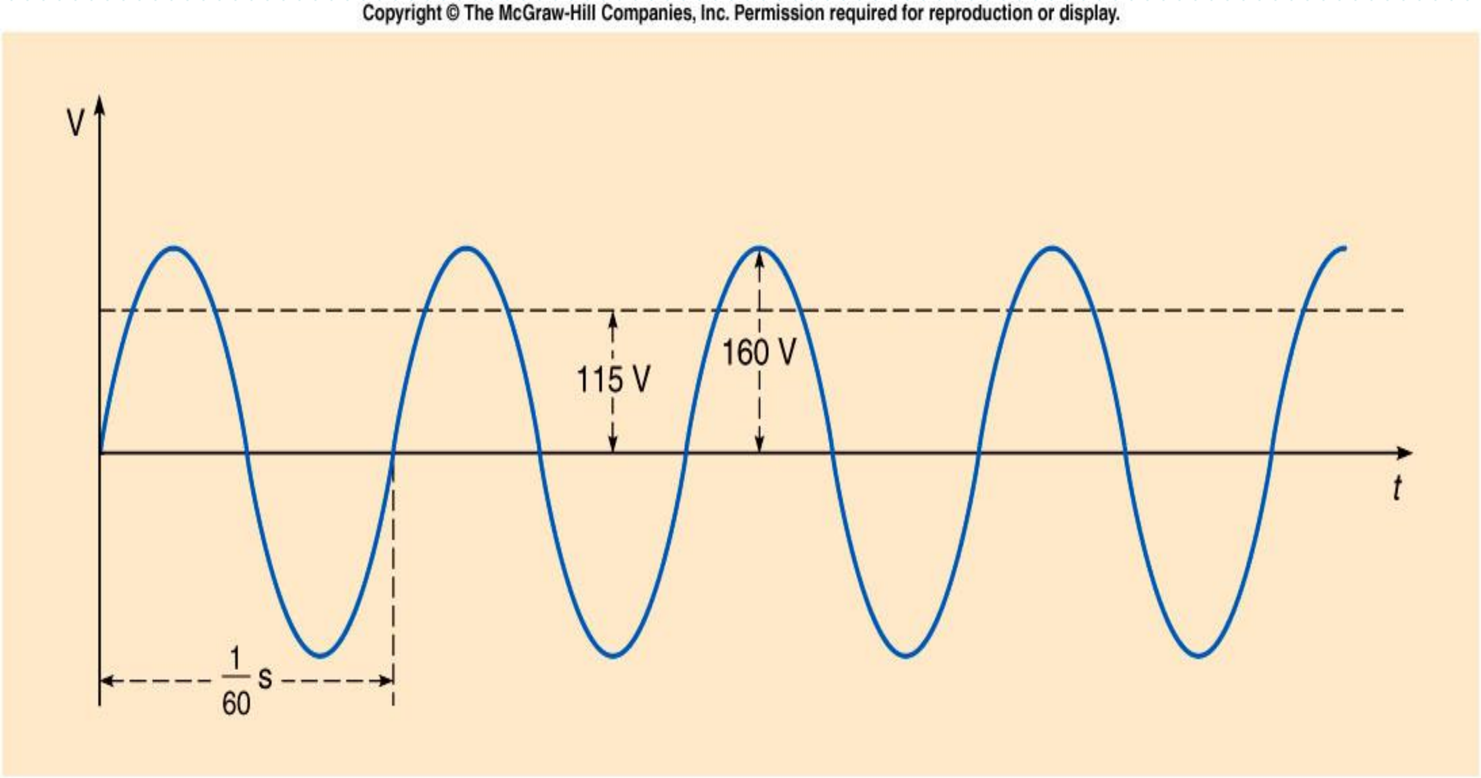
Alternating Current

Current from a power plant varies sinusoidally (alternating current, AC).

$$V = V_0 \sin 2\pi f t$$

Alternating Voltage from the outlet





Effective voltage ≈ 115 V, called the RMS value.

Alternating Current

$$\langle \sin^2 \omega t \rangle = \frac{1}{2}$$

$$\begin{aligned}
\langle \sin^2 \omega t \rangle &= \frac{1}{T}\int_{-T/2}^{+T/2} \sin^2 \omega t \, dt \\
&\overset{1-2\sin^2\omega t = \cos 2\omega t}{=} \frac{1}{T}\int_{-T/2}^{+T/2} \left(\frac{1}{2} - \frac{1}{2}\cos 2\omega t\right) dt \\
&= \frac{1}{T}\left(\frac{1}{2}t\Big|_{-T/2}^{T/2} - \frac{1}{4\omega}\sin 2\omega t\Big|_{-T/2}^{+T/2}\right) \\
&\overset{T\to+\infty,\ \omega=\frac{2\pi}{T}}{=} \frac{1}{2} - \frac{1}{T}\cdot\frac{T}{8\pi}\sin\left(\frac{4\pi}{T}\cdot\frac{T}{2}\right)\cdot 2 \\
&= \frac{1}{2}
\end{aligned}$$

The voltage varies sinusoidally with time:

$$V = V_0 \sin 2\pi f t = V_0 \sin \omega t,$$

as does the current:

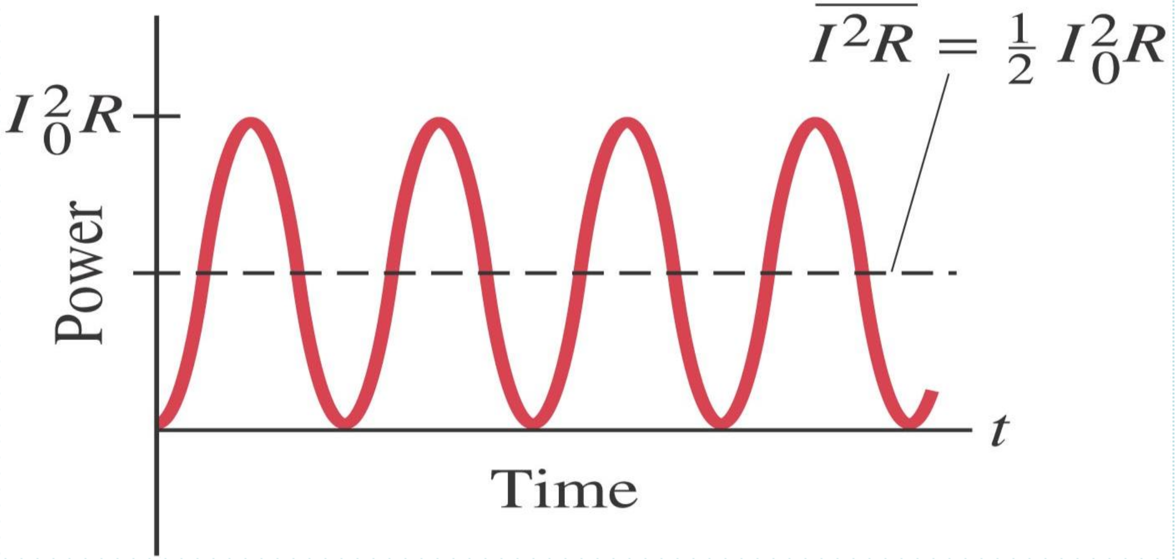
$$I = \frac{V}{R} = \frac{V_0}{R}\sin\omega t = I_0 \sin \omega t.$$

Multiplying the current and the voltage gives the power:

$$P = I^2 R = I_0^2 R \sin^2 \omega t.$$

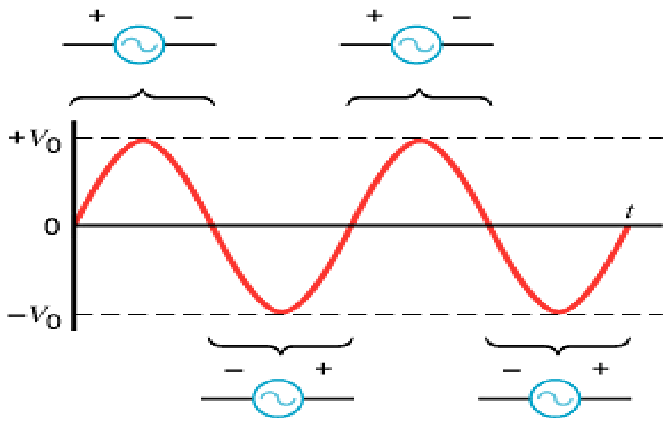
Usually we are interested in the average power:

$$\overline{P} = \tfrac{1}{2} I_0^2 R \qquad \overline{P} = \frac{1}{2}\frac{V_0^2}{R}.$$



Alternating Current: *RMS values*

The current and voltage both have average values of zero, so we square them, take the average, then take the square root, yielding the root-mean-square (rms) value:



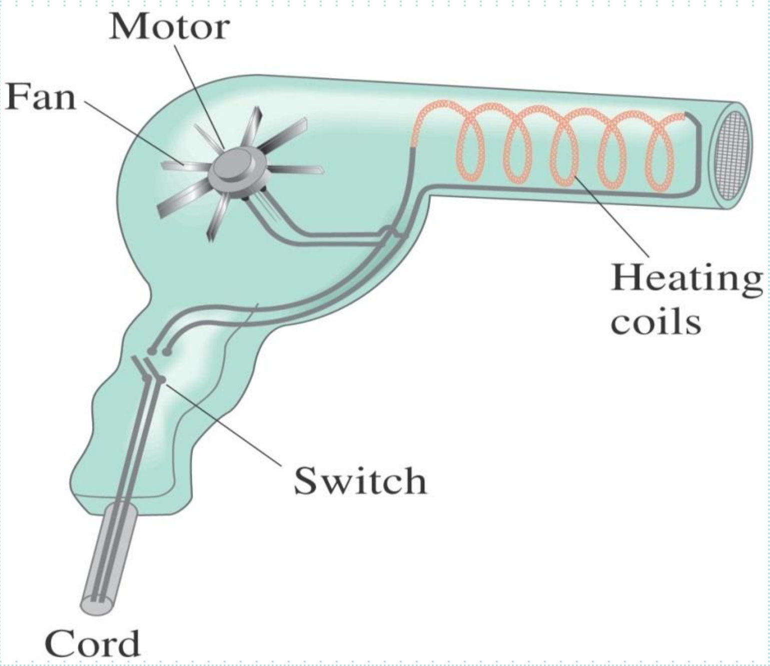
$$I_{rms} = \sqrt{\overline{I^2}} = \frac{I_0}{\sqrt{2}} = 0.707\, I_0,$$

$$V_{rms} = \sqrt{\overline{V^2}} = \frac{V_0}{\sqrt{2}} = 0.707\, V_0.$$

Practice Problem

Problem-49: Hair dryer.

(a) Calculate the resistance and the peak current in a 1000-W hair dryer connected to a 120-V line. (b) What happens if it is connected to a 240-V line in Britain?



Ans:A hair dryer. Most of the current goes through the heating coils, a pure resistance; a small part goes to the motor to turn the fan. Example 25–13.

Solution: (**a)** The rms current is the power divided by the rms voltage, or 8.33 A, so the peak current is 11.8 A. Then R = V/I = 14.4 Ω.

(b) Assuming the resistance would stay the same (probably wrong – the heat would increase it), doubling the voltage would mean doubling the current, and the power (P = IV) would increase by a factor of 4. This would not be good for the hair dryer.

Kirchoof's Current Law (KCL)

JUNCTION RULE: The sum of the currents entering any junction must be equal to the sum of the currents leaving that junction.

At junction *d* in the circuit

$$\boxed{i_1 + i_3 = i_2.}$$

This rule is often called *Kirchhoff's junction rule* (or *Kirchhoff's current law).*

For the left-hand loop,

$$\mathscr{E}_1 - i_1 R_1 + i_3 R_3 = 0.$$

For the right-hand loop,

$$-i_3 R_3 - i_2 R_2 - \mathscr{E}_2 = 0.$$

And for the entire loop,

$$\mathscr{E}_1 - i_1 R_1 - i_2 R_2 - \mathscr{E}_2 = 0.$$

The current into the junction must equal the current out (charge is conserved).

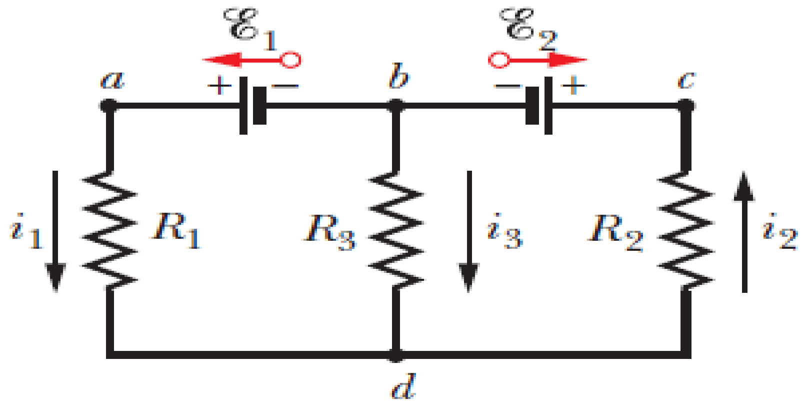
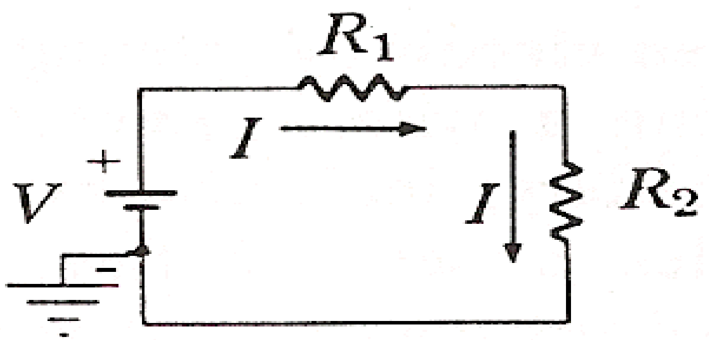


Fig. 27-9 A multiloop circuit consisting of three branches: left-hand branch *bad*, right-hand branch *bcd*, and central branch *bd*. The circuit also consists of three loops: left-hand loop *badb*, right-hand loop *bcdb*, and big loop *badcb*.

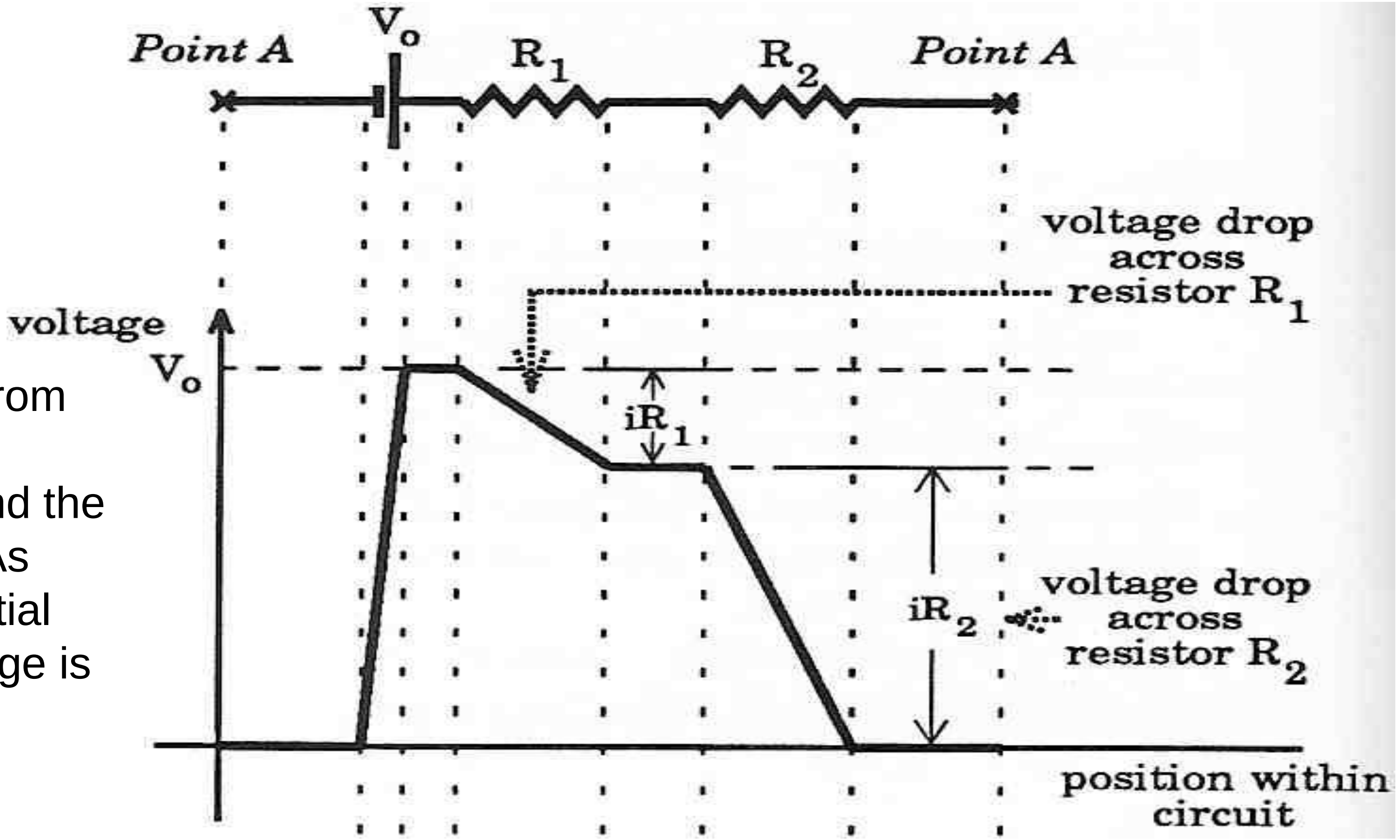
The former 3 equations solve this problem. The last one is the sum of that from the two loops.

Kirchhoff's Voltage Law (KVL)

"The sum of the potential changes around any closed loop is ZERO."



The rule is that as you move AWAY from the positive potential the potential is decreasing. So at ANY point if you find the change you get a negative number. As you move towards the positive potential the potential increases thus the change is positive.



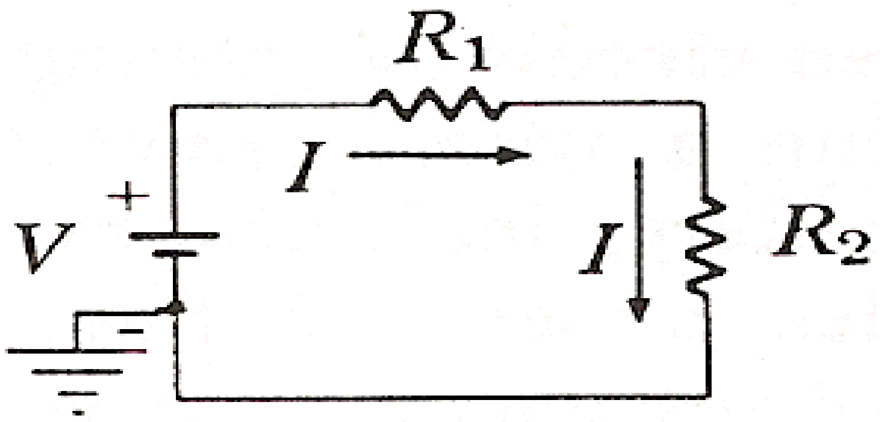
Kirchhoff's Voltage Law – Series Circuit

$$V_{Total} + (-IR_1) + (-IR_2) = 0$$

$$V_{Total} = IR_1 + IR_2, \quad V_{Total} = IR_{Total}$$

$$IR_{Total} = IR_1 + IR_2$$ ("I" Cancels)

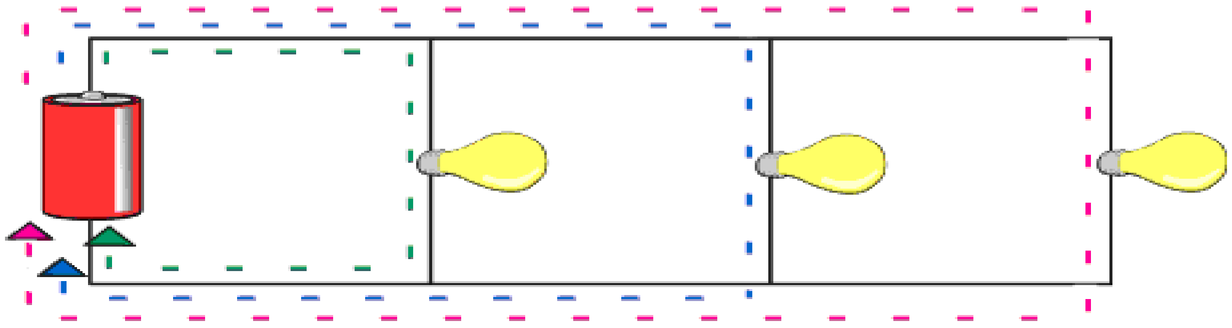
$$R_{Total} = R_1 + R_2 \rightarrow R_{Total(series)} = \sum_i^n R_i$$



Here we see that applying Kirchhoff's Voltage Law to this loop produces the formula for the ***effective*** resistance in a series circuit. The word ***effective*** or ***equivalent*** means the same thing as the TOTAL.

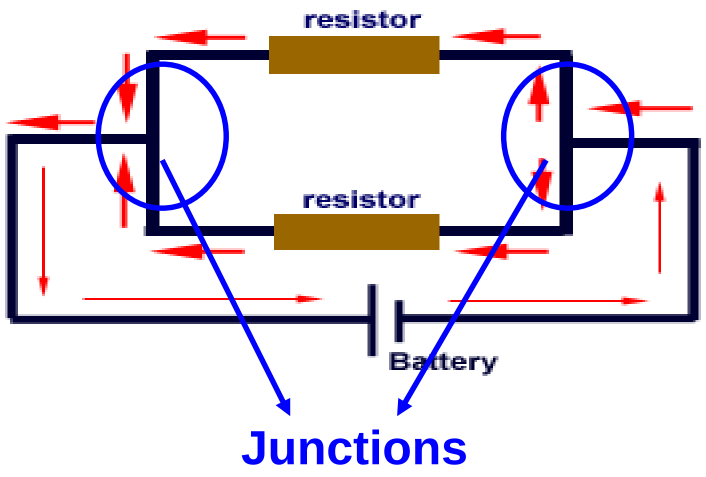
Parallel Circuit

In a parallel circuit, we have multiple loops. So the current splits up among the loops with the individual loop currents **adding** to the total current



It is important to understand that parallel circuits will all have some position where the current splits and comes back together. We call these **JUNCTIONS**.

The current going IN to a junction will always equal the current going OUT of a junction.



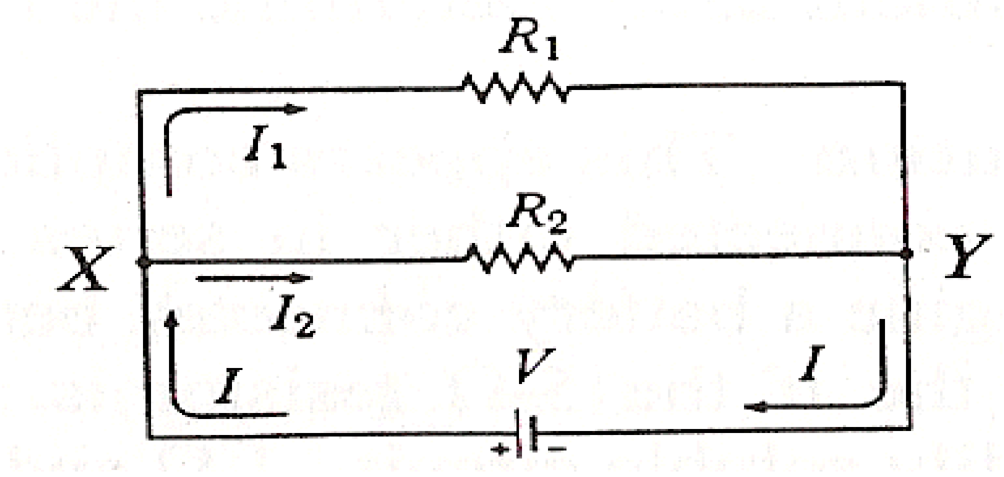
$$I_{(parallel)Total} = I_1 + I_2 + I_3$$

$$\text{Regarding Junctions:}$$

$$I_{IN} = I_{OUT}$$

Kirchhoff’s Current Law

“The sum of the currents flowing into a junction is equal to the sum of the currents flowing out.”



When two resistors have BOTH ends connected together, with nothing intervening, they are connected in PARALLEL. The drop in potential when you go from X to Y is the SAME no matter which way you go through the circuit. Thus resistors in parallel have the same potential drop.

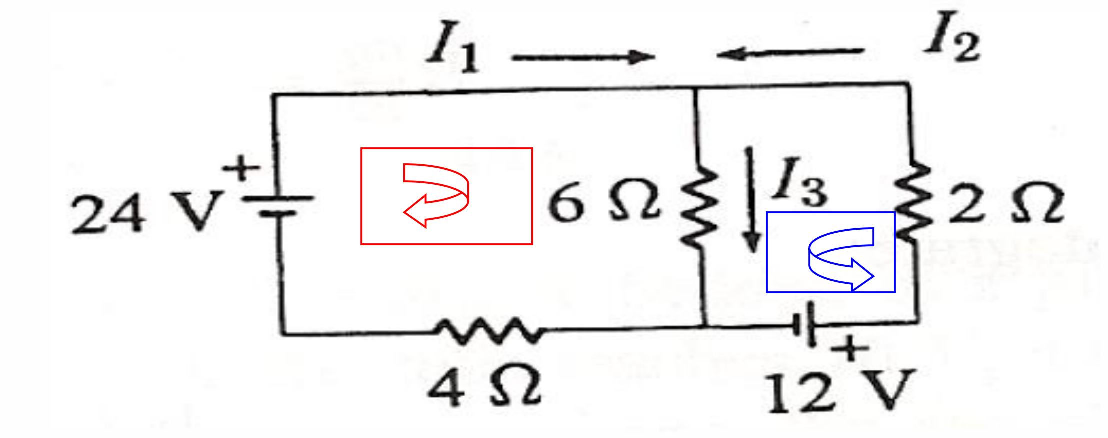
$$I_{Total} = I_1 + I_2$$

$$\frac{V_T}{R_T} = \frac{V_1}{R_1} + \frac{V_2}{R_2}$$

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_p} = \sum_{i}^{n} \frac{1}{R_i}$$

Problem-50: Applying Kirchhoff's Laws



Goal: Find the three unknown currents.

First decide which way you think the current is traveling around the loop. It is OK to be incorrect.

$$\text{Red Loop} \rightarrow V + (-I_3 6) + (-I_1 4) = 0$$

$$24 = 6I_3 + 4I_1$$

Using Kirchhoff's Voltage Law

$$\text{Blue Loop} \rightarrow V + (-I_2 2) + (-I_3 6) = 0$$

$$12 = 2I_2 + 6I_3$$

Using Kirchhoff's Current Law

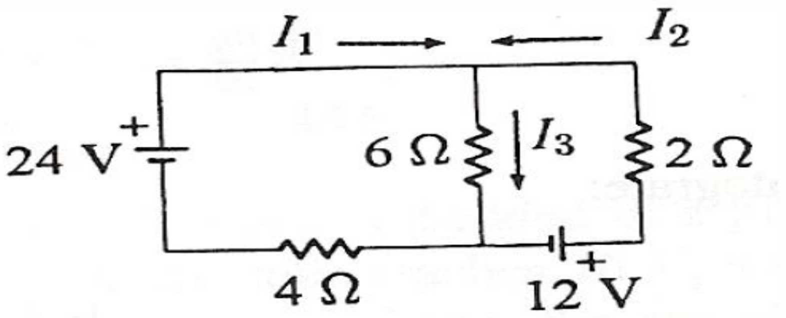
$$I_1 + I_2 = I_3$$

Applying Kirchhoff's Laws

$$24 = 6I_3 + 4I_1$$

$$12 = 2I_2 + 6I_3$$

$$I_3 = I_1 + I_2$$



$$\boxed{\begin{aligned} 24 &= 6(I_1 + I_2) + 4I_1 = 6I_1 + 6I_2 + 4I_1 = 10I_1 + 6I_2 \\ 12 &= 2I_2 + 6(I_1 + I_2) = 2I_2 + 6I_1 + 6I_2 = 6I_1 + 8I_2 \end{aligned}}$$

$$24 = 10I_1 + 6I_2 \rightarrow -6(24 = 10I_1 + 6I_2)$$

$$12 = 6I_1 + 8I_2 \rightarrow 10(12 = 6I_1 + 8I_2)$$

$$-144 = -60I_1 - 36I_2 \qquad 120 = 60I_1 + 80I_2$$

$$-24 = 44I_2$$

$I_2 =$ **-0.545 A**

A NEGATIVE current does NOT mean you are wrong. It means you chose your current to be in the wrong direction initially.

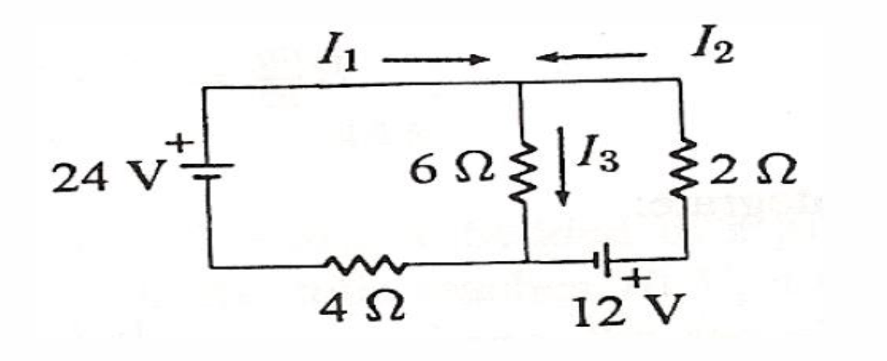
Applying Kirchhoff's Laws

$$12 = 2I_2 + 6I_3 \rightarrow 12 = 2(-0.545) + 6I_3$$

$I_3 =$ 2.18 A

$$24 = 6I_3 + 4I_1 \rightarrow 24 = 6(?) + 4I_1$$

$I_1 =$ 2.73 A



Instead of : $I_3 = I_1 + I_2$

It should have been : $I_1 = I_2 + I_3$

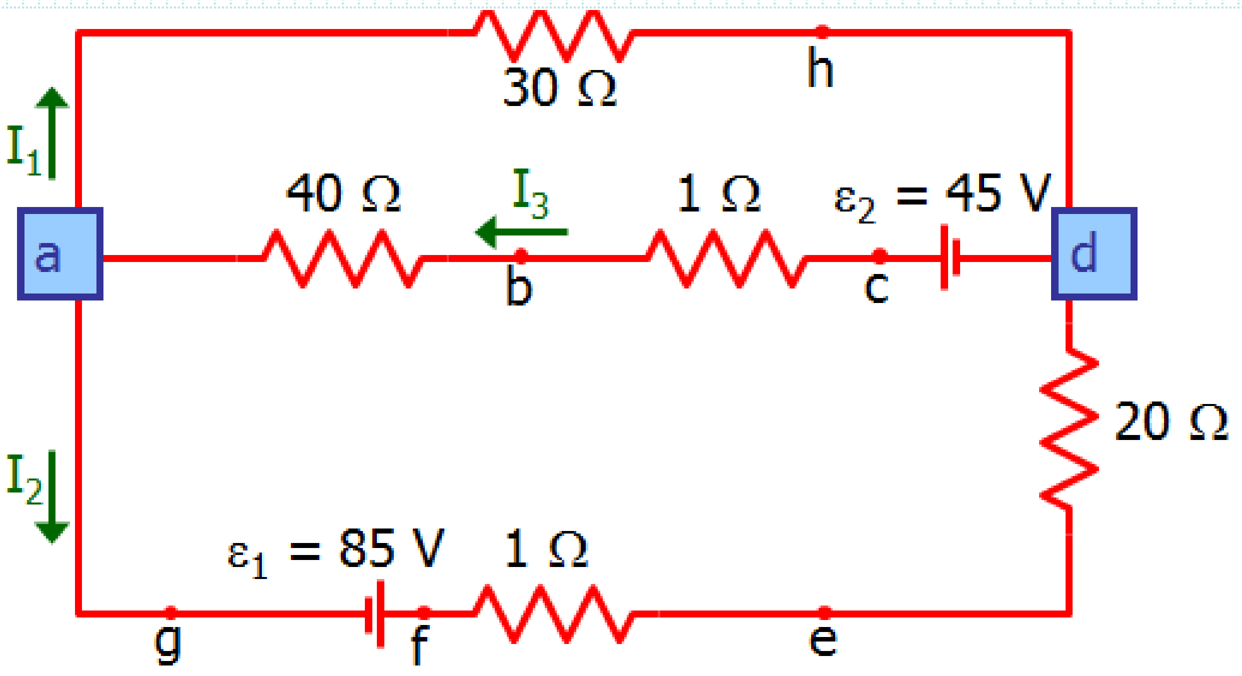
$$2.73 = 2.18 + 0.545$$

At a Glance the Kirchhoff's Rules

- Junction Rule
  - The sum of the currents entering any junction must equal the sum of the currents leaving that junction
    - A statement of Conservation of Charge
- Loop Rule
  - The sum of the potential differences across all the elements around any closed circuit loop must be zero
    - A statement of Conservation of Energy

Kirchoofs law: Practice Problem

Problem-51:



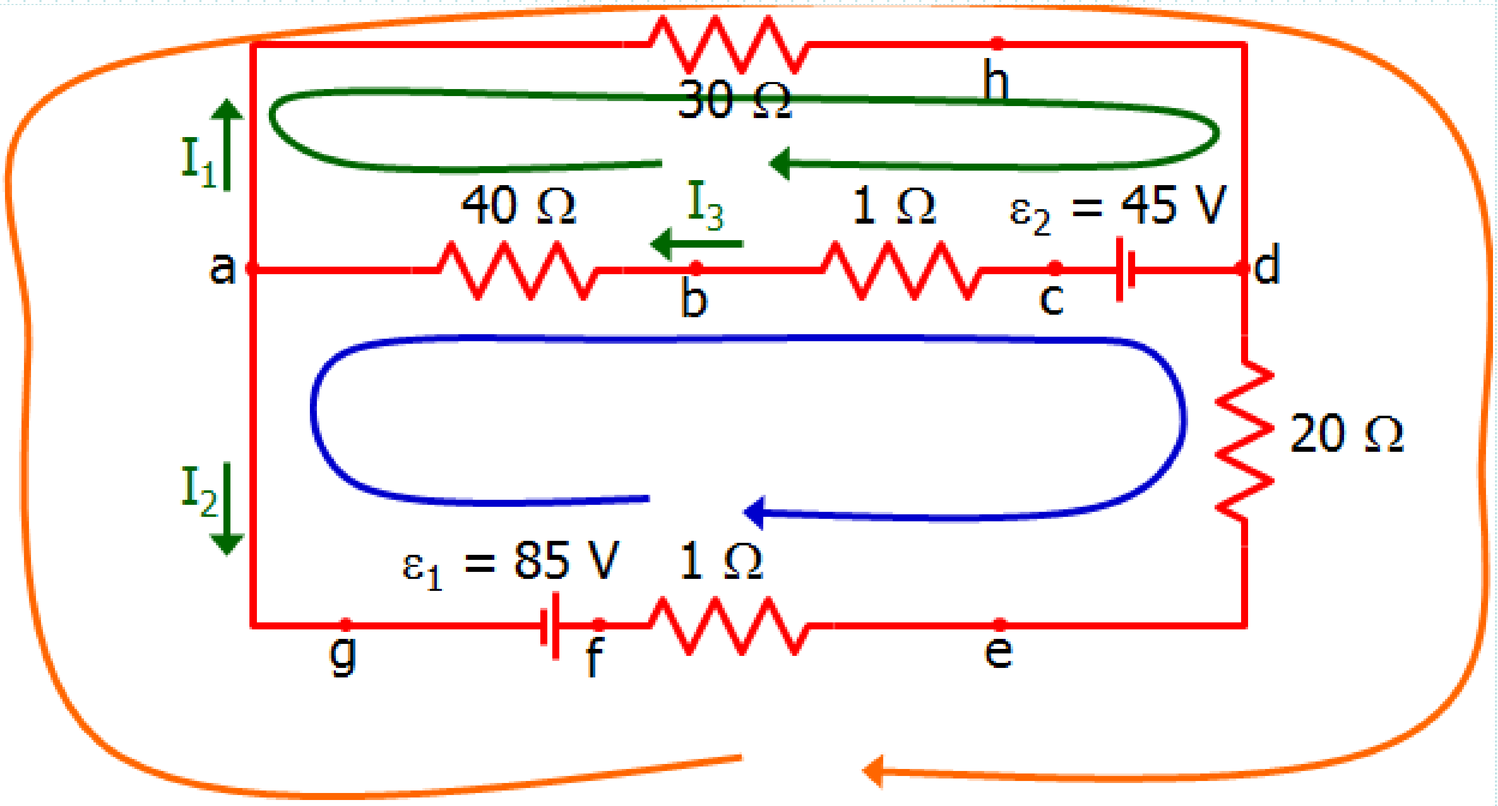
Back to our circuit: we have 3 unknowns (I_1, I_2, and I_3), so we will need 3 equations. We begin with the junctions.

Junction a: $I_3 - I_1 - I_2 = 0$ --eq. 1

Junction d: $-I_3 + I_1 + I_2 = 0$

Junction d gave no new information, so we still need two more equations.116

Kirchoofs law: Practice Problem

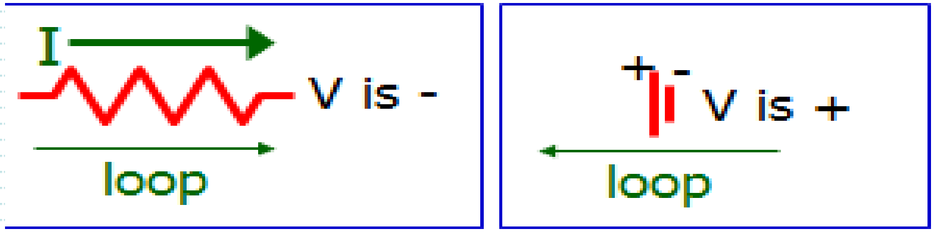


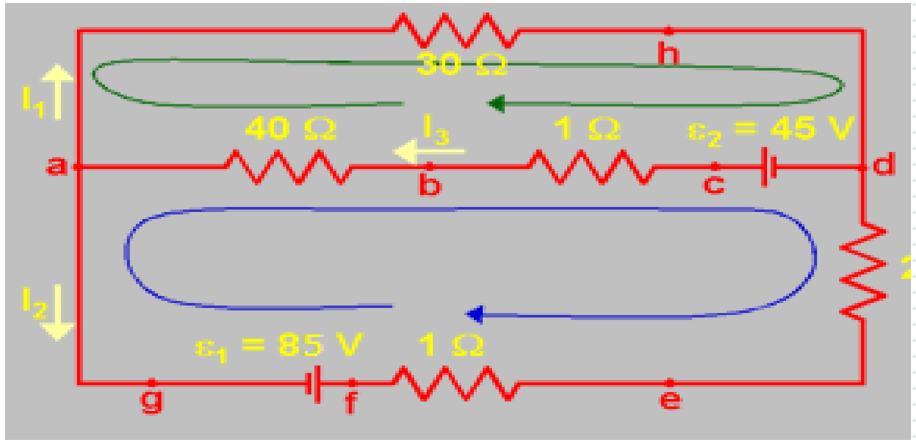
There are three loops. Loop 1. Loop 2. Loop 3.

Any two loops will produce independent equations. Using the third loop will provide no new information.

Kirchoofs law: Practice Problem

Reminders:





The "green" loop (a-h-d-c-b-a):

$$(-\,30\ I_1) + (+45) + (-1\ I_3) + (-\,40\ I_3) = 0$$

$$-\,30\ I_1 + 45 - 41\ I_3 = 0 \qquad \text{--eq. 2}$$

The "blue" loop (a-b-c-d-e-f-g):

$$(+\,40\ I_3) + (\,+1\ I_3) + (-45) + (+20\ I_2) + (+1\ I_2) + (-85) = ($$

$$41\ I_3\ \ -130 + 21\ I_2 = 0 \qquad \text{--eq. 3}$$

Three equations, three unknowns; the rest is "algebra."

Make sure to use voltages in V and resistances in Ω. Then currents will be in A.

Kirchoofs law: Practice Problem

Collect our three equations:

$$I_3 - I_1 - I_2 = 0$$

$$-30\ I_1 + 45 - 41\ I_3 = 0$$

$$41\ I_3\ -130 + 21\ I_2 = 0$$

Rearrange to get variables in "right" order:

$$-I_1 - I_2 + I_3 = 0$$

$$-30\ I_1 - 41\ I_3 + 45 = 0$$

$$21\ I_2 + 41\ I_3 - 130 = 0$$

Use the middle equation to eliminate I_1:

$$I_1 = (41\ I_3 - 45)/(-30)$$

There are many valid sets of steps to solving a system of equations. Any that works is accep

Two equations left to solve:

$$-(41\ I_3 - 45)/(-30) - I_2 + I_3 = 0$$

$$21\ I_2 + 41\ I_3 - 130 = 0$$

Might as well work out the numbers:

$$1.37\ I_3 - 1.5 - I_2 + I_3 = 0$$

$$21\ I_2 + 41\ I_3 - 130 = 0$$

$$-I_2 + 2.37\ I_3 - 1.5 = 0$$

$$21\ I_2 + 41\ I_3 - 130 = 0$$

Multiply the top equation by 21:

$$-21\ I_2 + 49.8\ I_3 - 31.5 = 0$$

$$21\ I_2 + 41\ I_3 - 130 = 0$$

Kirchoofs law: Practice Problem

Add the two equations to eliminate I_2:

$$\begin{array}{r} -21\ I_2 + 49.8\ I_3 - 31.5 = 0 \\ +\ (\ 21\ I_2 + 41\ I_3 - 130 = 0\ ) \\ \hline 90.8\ I_3 - 161.5 = 0 \end{array}$$

Solve for I_3:

$$I_3 = 161.5 / 90.8$$

$$I_3 = 1.78$$

Go back to the "middle equation" two slides ago for I_1:

$$I_1 = (41\ I_3 - 45)/(-30)$$

$$I_1 = -1.37\ I_3 + 1.5$$

$$I_1 = -(1.37)(1.78) + 1.5$$

$$I_1 = -0.94$$

Go back two slides to get an equation that gives I_2:

$$-I_2 + 2.37\ I_3 - 1.5 = 0$$

$$I_2 = 2.37\ I_3 - 1.5$$

$$I_2 = (2.37)(1.78) - 1.5$$

$$I_2 = 2.72$$

Summarize answers so your lazy professor doesn't have to go searching for them and get irritated (don't forget to show units in your answer):

$$\boxed{\begin{array}{c} I_1 = -0.94\ A \\ I_2 = 2.72\ A \\ I_3 = 1.78\ A \end{array}}$$

Are these currents correct? How could you tell? We'd better check our results.

Kirchoofs law: Practice Problem

$$I_3 - I_1 - I_2 = 0$$

$$-30\ I_1 + 45 - 41\ I_3 = 0$$

$$41\ I_3\ -130 + 21\ I_2 = 0$$

$$\boxed{\begin{aligned} I_1 &= -0.94\ A \\ I_2 &= 2.72\ A \\ I_3 &= 1.78\ A \end{aligned}}$$

$$1.78 - (-0.94) - 2.72 = 0 \quad ☑$$

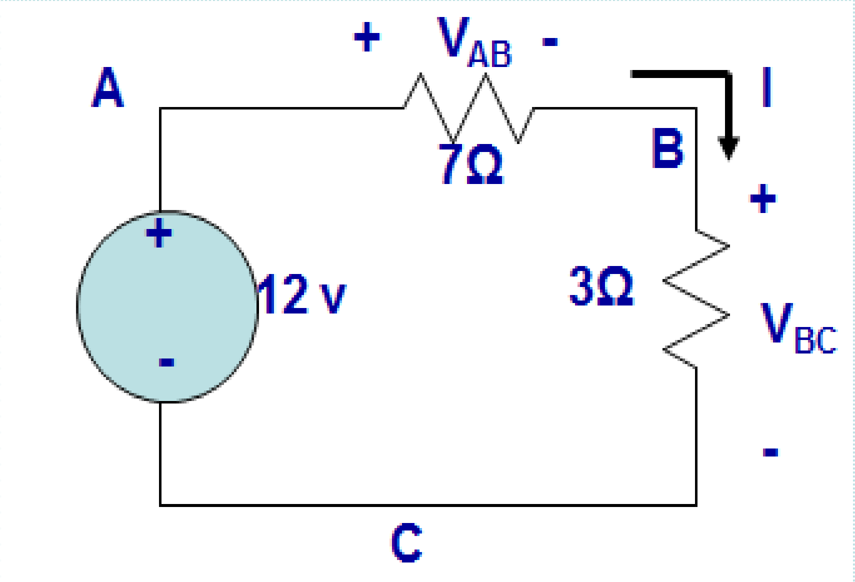
$$-30\ (-0.94) + 45 - 41\ (1.78) = 0.22 \quad ☑?$$

$$41\ (1.78)\ -130 + 21\ (2.72) = 0.10 \quad ☑?$$

Are the last two indication of a mistake or just round off error? Recalculating while retaining 2 more digits gives I_1=0.933, I_2=2.714, I_3=1.7806, and the last two results are 0.01 or less $\Rightarrow$ round off was the culprit.

Kirchoofs law: Practice Problem

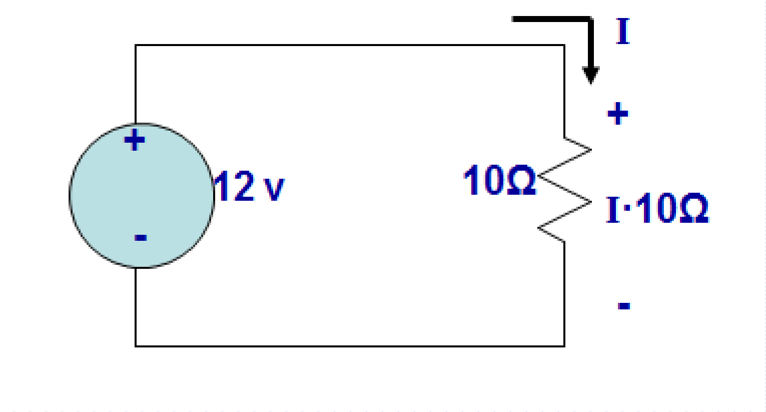
Problem-52: Q: Find the voltage and current using KVL and KCL.



Ans: V_{AB} = 8.4 v , V_{BC} = 3.6 v and current I=1.2 A

Kirchoofs law: Practice Problem

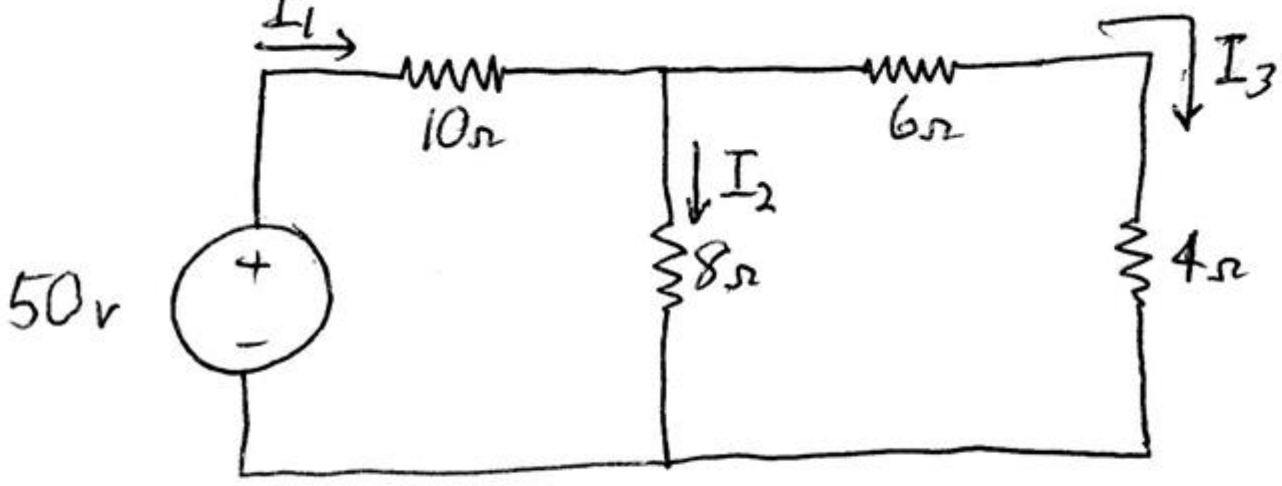
Problem-53: Q: Find the voltage and current using KVL and KCL.



Ans: V_{AB} =10x1.2 v and current I=1.2 A

Kirchoofs law: Practice Problem

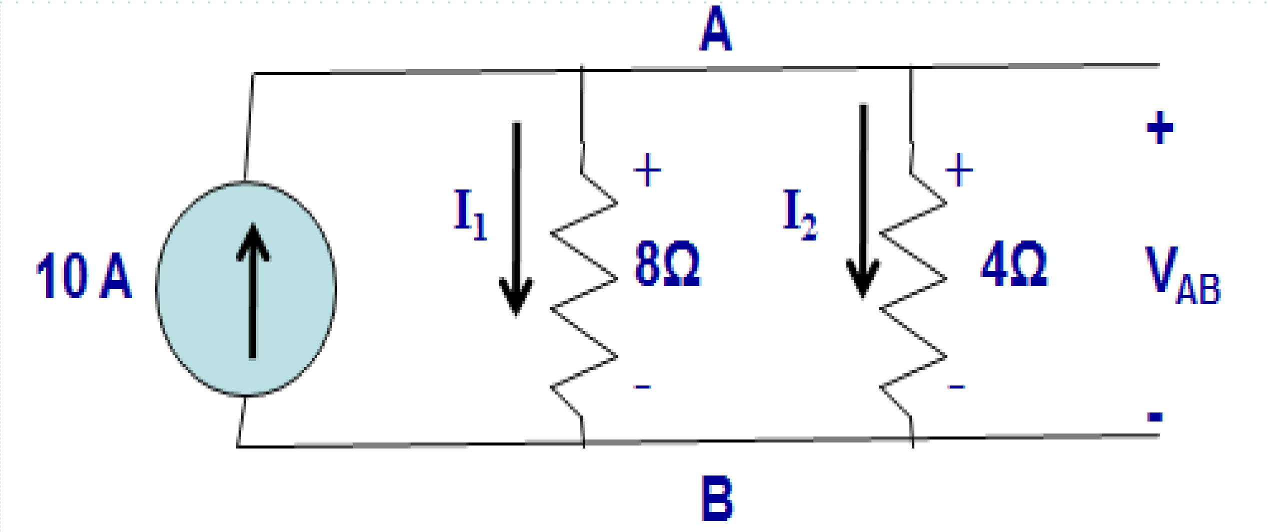
Problem-54: Q: Find the voltage and current using KVL and KCL.



Ans: Find V and current I_1=3.461 A, I_2=1.923 A and I_3=1.538 A

Kirchoofs law: Practice Problem

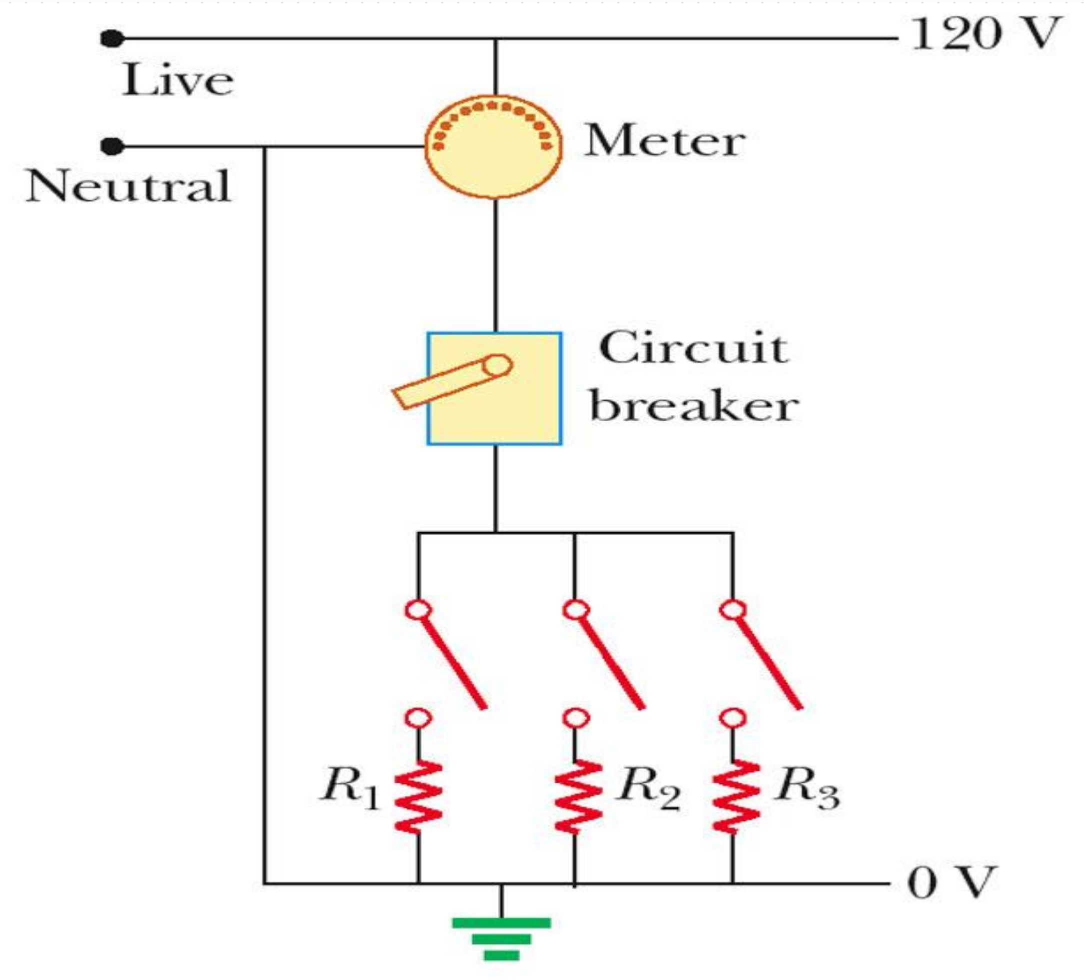
Problem-55: Q: Find the voltage and current using KVL and KCL.



Ans: Find Vab=26.67 V and current I_1=3.33 A, I_2=6.67A

Household Circuits

- The utility company distributes electric power to individual houses with a pair of wires
- Electrical devices in the house are connected in parallel with those wires
- The potential difference between the wires is about 120V



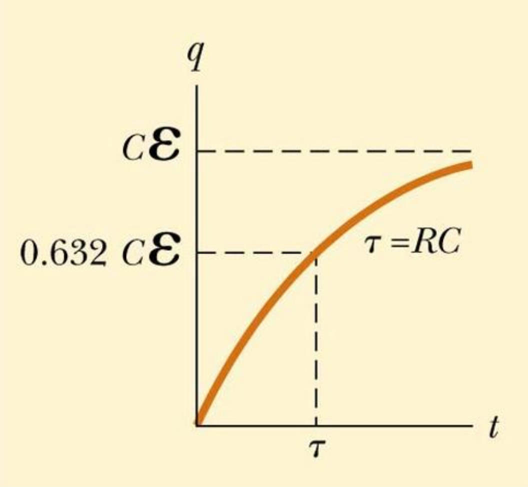
Electrical Safety: Effects of Various Currents

- 5 mA or less
  - Can cause a sensation of shock
  - Generally little or no damage
- 10 mA
  - Hand muscles contract
  - May be unable to let go a of live wire
- 100 mA
  - If passes through the body for just a few seconds, can be fatal

Electrical Safety: More Effects of Various Currents

Current	Effect
0.001 A (1 mA)	Can be felt
0.005 A (5 mA)	Painful
0.010 A (10 mA)	Involuntary muscle contractions (spasms)
0.015 A (15 mA)	Loss of muscle control
0.070 A (70 mA)	If through the heart, serious disruption. More than 1 second, probably fatal

RC Circuits,
Charging a Capacitor:



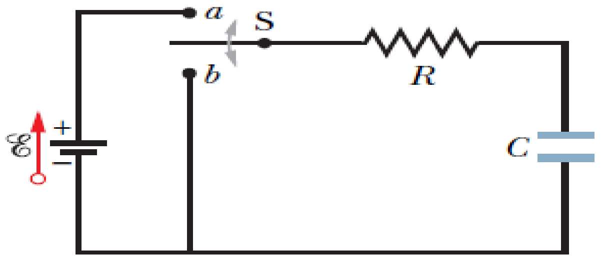
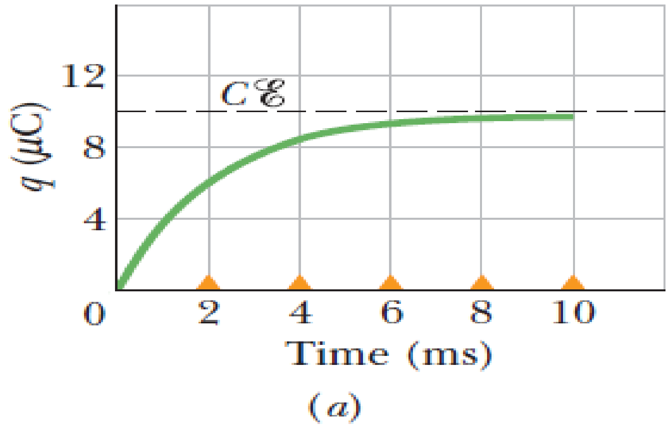


Fig. 27-15 When switch S is closed on *a*, the capacitor is *charged* through the resistor. When the switch is afterward closed on *b*, the capacitor *discharges* through the resistor.

The capacitor's charge grows as the resistor's current dies out.



$$\mathcal{E} - iR - \frac{q}{C} = 0.$$

$$i = \frac{dq}{dt}.$$ $\Rightarrow$ $$R\frac{dq}{dt} + \frac{q}{C} = \mathcal{E} \qquad \text{(charging equation).}$$

$$q = C\mathcal{E}(1 - e^{-t/RC}) \qquad \text{(charging a capacitor).}$$

$(\#\ q = 0,\ @\ t = 0)$

$$V_C = \frac{q}{C} = \mathcal{E}(1 - e^{-t/RC}) \qquad \text{(charging a capacitor).}$$

- A capacitor that is being charged initially acts like ordinary connecting wire relative to the charging current. A long time later, it acts like a broken wire.

RC Circuits, Time Constant:

The product *RC* is called the capacitive time constant of the circuit and is represented with the symbol τ:

$$\tau = RC \qquad \text{(time constant).}$$

At time $t = \tau = (RC)$, the charge on the initially uncharged capacitor increases from zero to:

$$q = C\mathscr{E}(1 - e^{-1}) = 0.63C\mathscr{E}.$$

RC Circuits,
Discharging a Capacitor:

Assume that the capacitor of the figure is fully charged to a potential V_0 equal to the emf of the battery.
At a new time $t = 0$, switch S is thrown from a to b so that the capacitor can discharge through resistance R.

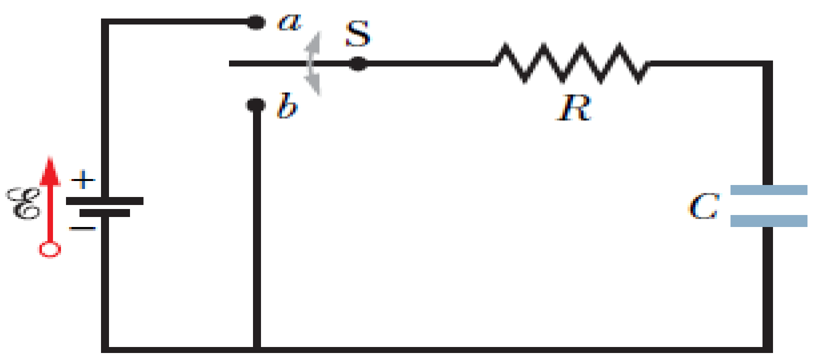


Fig. 27-15 When switch S is closed on a, the capacitor is *charged* through the resistor. When the switch is afterward closed on b, the capacitor *discharges* through the resistor.

$$R\frac{dq}{dt} + \frac{q}{C} = 0 \qquad \text{(discharging equation).}$$

$$q = q_0 e^{-t/RC} \qquad \text{(discharging a capacitor),}$$

$$i = \frac{dq}{dt} = -\frac{q_0}{RC}\, e^{-\frac{t}{RC}} = -\frac{\mathcal{E}}{R}\, e^{-t/\tau}$$

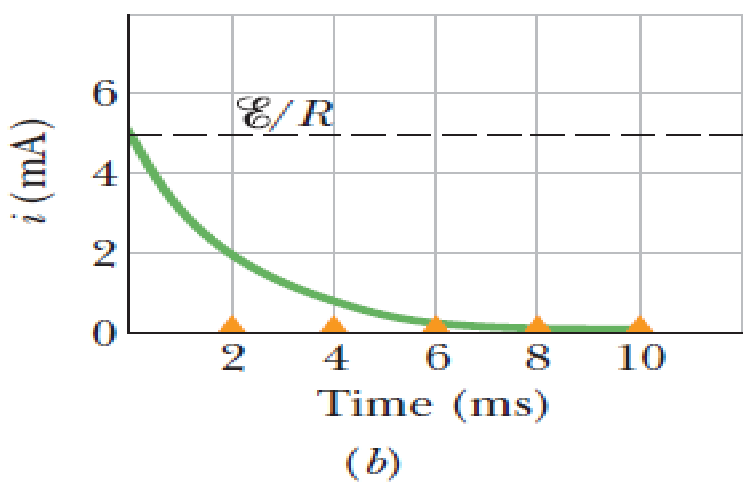
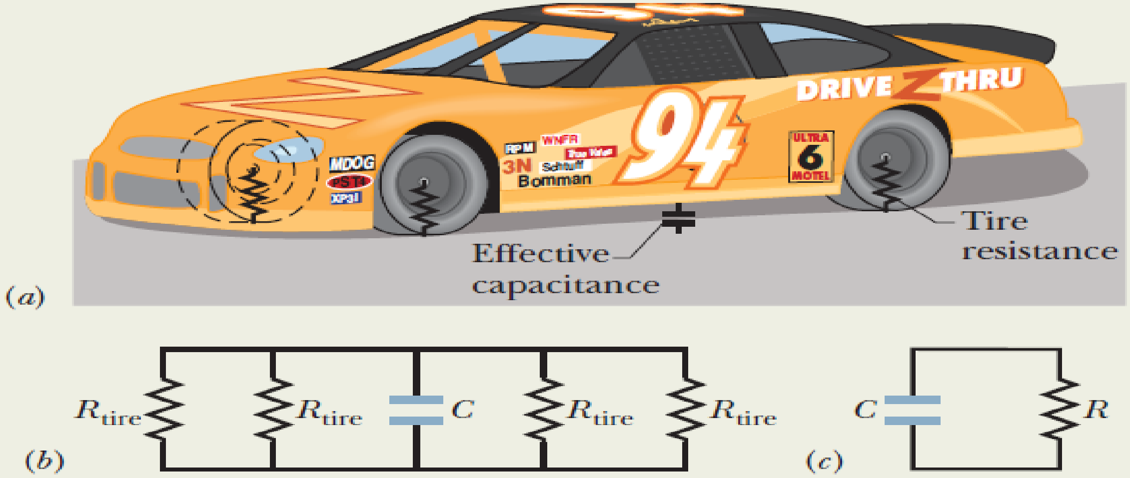


Fig. 27-16 (*b*) This shows the decline of the charging current in the circuit. The curves are plotted for $R = 2000\ \Omega$, $C = 1\ \mu F$, and $emf = 10\ V$; the small triangles represent successive intervals of one time constant τ.

Example, Discharging an RC circuit : Problem-56:

As a car rolls along pavement, electrons move from the pavement first onto the tires and then onto the car body. The car stores this excess charge and the associated electric potential energy as if the car body were one plate of a capacitor and the pavement were the other plate (Fig. 27-17*a*). When the car stops, it discharges its excess charge and energy through the tires, just as a capacitor can discharge through a resistor. If a conducting object comes within a few centimeters of the car before the car is discharged, the remaining energy can be suddenly transferred to a spark between the car and the object. Suppose the conducting object is a fuel dispenser. The spark will not ignite the fuel and cause a fire if the spark energy is less than the critical value $U_{fire} = 50$ mJ.

When the car of Fig. 27-17*a* stops at time $t = 0$, the car–ground potential difference is $V_0 = 30$ kV. The car–ground capacitance is $C = 500$ pF, and the resistance of *each* tire is $R_{tire} = 100$ GΩ. How much time does the car take to discharge through the tires to drop below the critical value U_{fire}?



Calculations: We can treat the tires as resistors that are connected to one another at their tops via the car body and at their bottoms via the pavement. Figure 27-17*b* shows how the four resistors are connected in parallel across the car's capacitance, and Fig. 27-17*c* shows their equivalent resistance R.

$$\frac{1}{R} = \frac{1}{R_{tire}} + \frac{1}{R_{tire}} + \frac{1}{R_{tire}} + \frac{1}{R_{tire}},$$

or

$$R = \frac{R_{tire}}{4} = \frac{100 \times 10^9\,\Omega}{4} = 25 \times 10^9\,\Omega. \qquad (27\text{-}44)$$

When the car stops, it discharges its excess charge and energy through R.

$$U = \frac{q^2}{2C} = \frac{(q_0 e^{-t/RC})^2}{2C}$$

$$= \frac{q_0^2}{2C} e^{-2t/RC}. \qquad (27\text{-}45)$$

$$U = \frac{(CV_0)^2}{2C} e^{-2t/RC} = \frac{CV_0^2}{2} e^{-2t/RC},$$

or

$$e^{-2t/RC} = \frac{2U}{CV_0^2}. \qquad (27\text{-}46)$$

Taking the natural logarithms of both sides, we obtain

$$-\frac{2t}{RC} = \ln\left(\frac{2U}{CV_0^2}\right),$$

$$t = -\frac{(25 \times 10^9\,\Omega)(500 \times 10^{-12}\,\text{F})}{2} \times \ln\left(\frac{2(50 \times 10^{-3}\,\text{J})}{(500 \times 10^{-12}\,\text{F})(30 \times 10^3\,\text{V})^2}\right)$$

$$= 9.4\ \text{s}. \qquad \text{(Answer)}$$

Review of Chapter

A battery produces emf; positive terminal is the anode, negative is the cathode.

emf is measured in volts; it is the number of joules the battery supplies per coulomb of charge.

An electric current can exist only in a complete circuit.

Resistance: $$R = \frac{V}{I}$$

Review of Chapter

Ohm’s law is obeyed if the resistance is constant:

$$V = IR$$

The resistance of an object depends on its length, cross-sectional area, and resistivity.

$$R = \rho\left(\frac{L}{A}\right)$$

Review of Chapter

Power is the rate at which work is done.

$$P = IV = \frac{V^2}{R} = I^2 R$$