# NLP: N-Grams

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## 1 Language Modeling Tasks

- Language idenfication / Authorship identification
- Machine Translation
- Speech recognition
- Optical character recognition (OCR)
- Context-sensitive spelling correction
- Predictive text (text messaging clients, search engines, etc)
- Generating spam
- Code-breaking (e.g. decipherment)

## 2 Kinds of Language Models

- Bag of words
- Sequences of words
- Sequences of tagged words
- Grammars
- Topic models

## 3 Language as a sequence of words

- What is the probability of seeing a particular sequence of words?
- Simplified model of langauge: words in order
- $p$(some words in sequence)

  - $p$("an interesting story") ?
  - $p$("a interesting story") ?
  - $p$("a story interesting") ?
- $p$(next | some sequence of previous words)
  - $p$("austin" | "university of texas at") ?
  - $p$("dallas" | "university of texas at") ?
  - $p$("giraffe" | "university of texas at") ?

Use as a language model

- Language ID: This sequence is most likely to be seen in what language?
- Authorship ID: What is the most likely person to have produced this sequence of words?
- Machine Translation: Which sentence (word choices, ordering) seems most like the target language?
- Text Prediction: What's the most likely next word in the sequence

Notation

- Probability of a sequence is a joint probability of words
- But the order of the words matters, so each gets its own feature
- $p(\text{"an interesting story"}) = p(w_{-3} = an,\ w_{-2} = interesting,\ w_{-1} = story)$
- $p(\text{"austin"} \mid \text{"university of texas at"})$
  $= p(w_0 = austin \mid w_{-4} = university,\ w_{-3} = of,\ w_{-2} = texas,\ w_{-1} = at)$
- So $w_0$ means "current word" and $w_{-1}$ is "the previous word".
- We will typically use the short-hand

# 4 Counting Words

- Terminology:
  - **Type:** Distinct word
    **Token:** Particular occurrence of a word
  - "the man saw the saw": 3 types, 5 tokens

What is a word? What do we count?

- Punctuation? Separate from neighboring words? Keep it at all?

- Stopwords?
- Lowercase everything?
- Distinct numbers vs. $\langle number \rangle$
- Hyphenated words?
- Lemmas only?
- Disfluencies? (um, uh)

## 5 Estimating Sequence Probabilities

- To build a statistical model, we need to set parameters.
- Our parameters: probabilities of various sequences of text
- Maximum Likelihood Estimate (MLE): of all the sequences of length N, what proportion are the relevant sequence?
- $p(\textit{university of texas at austin})$
  $= p(w_{-5} = university,\ w_{-4} = of,\ w_{-3} = texas,\ w_{-2} = at,\ w_{-1} = austin)$
  $= \frac{C(\text{“university of texas at austin”})}{C(\textit{all 5-word sequences})} = \frac{3}{25{,}000{,}000}$
- $p(\textit{a bank}) = \frac{C(\text{“a bank”})}{C(\textit{all 2-word sequences})} = \frac{609}{50{,}000{,}000}$
- $p(\textit{in the}) = \frac{C(\text{“in the”})}{C(\textit{all 2-word sequences})} = \frac{312{,}776}{50{,}000{,}000}$
- Long sequences are unlikely to have any counts:
  $p(\textit{the university of texas football team started the season off right by scoring a touchdown in the final seconds of play to secure a stunning victory over the out-of-town challengers})$
  $= \frac{C(\text{... that sentence ...})}{C(\textit{all 30-word sequences})} = \mathbf{0.0}$
- Even shorter sentences may not have counts, even if they make sense, are perfecty grammatical, and not improbable that someone might say them:
  - “*university of texas in amarillo*”
- We need a way of estimating the probability of a long sequence, even though counts will be low.

Make a “naïve” assumption?

- With naïve Bayes, we dealt with this problem by assuming that all features were independent.
- $p(w_{-5} = university,\ w_{-4} = of,\ w_{-3} = texas,\ w_{-2} = in,\ w_{-1} = amarillo) =$
  $p(w = university) \cdot p(w = of) \cdot p(w = texas) \cdot p(w = in) \cdot p(w = amarillo)$

- But this loses the difference between "university of texas in amarillo", which seems likely, and "university texas amarillo in of", which does not
- This amounts to a "bag of words" model

Use Chain Rule?

- Long sequences are sparse, short sequences are less so
- Break down long sequences using the chain rule
- $p(university\ of\ texas\ in\ amarillo) =$
  $p(university) \cdot p(of \mid university) \cdot p(texas \mid university\ of) \cdot p(in \mid university\ of\ texas)$
  $\cdot\, p(amarillo \mid university\ of\ texas\ in)$
- "p(seeing 'university') times p(seeing 'of' given that the previous word was 'university') times p(seeing 'texas' given that the previous two words were 'university of') ..."
- $p(\text{university}) = \frac{C(\text{university})}{\sum_x C(x)} = \frac{C(\text{university})}{C(all\ words)}$; easy to estimate
  $p(\text{of} \mid \text{university}) = \frac{C(\text{university of})}{\sum_w C(\text{university } x)} = \frac{C(\text{university of})}{C(\text{university})}$; easy to estimate
  $p(\text{texas} \mid \text{university of}) = \frac{C(\text{university of texas})}{\sum_w C(u\text{university of } x)} = \frac{C(\text{university of texas})}{C(\text{university of})}$; easy to estimate
  $p(\text{in} \mid \text{university of texas}) = \frac{C(\text{university of texas in})}{\sum_w C(u\text{university of texas } x)} = \frac{C(\text{university of texas in})}{C(\text{university of texas})}$; easy to estimate
  $p(\text{amarillo} \mid \text{university of texas in}) = \frac{C(\text{university of texas in amarillo})}{\sum_w C(u\text{university of texas in } x)} = \frac{C(\text{university of texas in amarillo})}{C(\text{university of texas in})}$; **same problem**
- So this doesn't help us at all.

# 6 N-Grams

- We don't necessarily want a "fully naïve" solution
  - Partial independence: limit how far back we look
- "Markov assumption": future behavior depends only on recent history
  - $k^{th}$-order Markov model: depend only on $k$ most recent states
- **n-gram**: sequence of $n$ words
- **n-gram model**: statistical model of word sequences using n-grams.
- $p(university\ of\ texas\ in\ amarillo)$
  - 5+ -gram:
    $p(university) \cdot p(of \mid university) \cdot p(texas \mid university\ of) \cdot p(in \mid university\ of\ texas)$
    $\cdot\, p(amarillo \mid university\ of\ texas\ in)$
  - 3-gram (trigram):
    $p(university) \cdot p(of \mid university) \cdot p(texas \mid university\ of) \cdot p(in \mid of\ texas)$
    $\cdot\, p(amarillo \mid texas\ in)$

  - 2-gram (bigram):
    $p(university) \cdot p(of \mid university) \cdot p(texas \mid of) \cdot p(in \mid texas) \cdot p(amarillo \mid in)$
  - 1-gram (unigram) / bag-of-words model / full independence:
    $p(university) \cdot p(of) \cdot p(texas) \cdot p(in) \cdot p(amarillo)$

- Idea: reduce necessary probabilities to an estimatble size.

- Estimating bigrams:
  $p(university) \cdot p(of \mid university) \cdot p(texas \mid of) \cdot p(in \mid texas) \cdot p(amarillo \mid in) =$
  $\frac{C(\text{university})}{\sum_x C(x)} \cdot \frac{C(\text{university of})}{\sum_x C(\text{university } x)} \cdot \frac{C(\text{of texas})}{\sum_x C(\text{of } x)} \cdot \frac{C(\text{texas in})}{\sum_x C(\text{texas } x)} \cdot \frac{C(\text{in amarillio})}{\sum_x C(\text{in } x)} =$
  $\frac{C(\text{university})}{C(all\ words)} \cdot \frac{C(\text{university of})}{C(\text{university})} \cdot \frac{C(\text{of texas})}{C(\text{of})} \cdot \frac{C(\text{texas in})}{C(\text{texas})} \cdot \frac{C(\text{in amarillio})}{C(\text{in})}$

- All of these should be easy to estimate!

# 7 N-Gram Model of Sentences

- Sentences are sequences of words, but with starts and ends.

- We also want to model the likelihood of words being at the beginning/end of a sentence.

- Append special "words" to the sentence
  - $n$–1 $\langle b \rangle$ symbols to beginning
  - only one $\langle \backslash b \rangle$ to then end needed
    * $p(\langle \backslash b \rangle \mid . \langle \backslash b \rangle) = 1.0$ since $\langle \backslash b \rangle$ would always be followed by $\langle \backslash b \rangle$.

- "the man walks the dog ." (trigrams)
  - Becomes "$\langle b \rangle$ $\langle b \rangle$ the man walks the dog . $\langle \backslash b \rangle$"
  - $p(\langle b \rangle \langle b \rangle$ the man walks the dog . $\langle \backslash b \rangle) =$
    $p(\text{the} \mid \langle b \rangle \langle b \rangle)$
    $\cdot\, p(\text{man} \mid \langle b \rangle \text{ the})$
    $\cdot\, p(\text{walks} \mid \text{the man})$
    $\cdot\, p(\text{the} \mid \text{man walks})$
    $\cdot\, p(\text{dog} \mid \text{walks the})$
    $\cdot\, p(. \mid \text{the dog})$
    $\cdot\, p(\langle \backslash b \rangle \mid \text{dog .})$

- Can be generalized to model longer texts: paragraphs, documents, etc:
  - Good: can model ngrams that cross sentences (e.g. $p(w_0 \mid .)$ or $p(w_0 \mid ?)$)
  - Bad: more sparsity on $\langle b \rangle$ and $\langle \backslash b \rangle$

## 8 Sentence Likelihood Examples

Example dataset:

```
<b> the dog runs . </b>
<b> the dog walks . </b>
<b> the man walks . </b>
<b> a man walks the dog . </b>
<b> the cat walks . </b>
<b> the dog chases the cat . </b>
```

Sentence Likelihood with Bigrams:

$p(\langle b\rangle \text{ the dog walks . } \langle\backslash b\rangle)$

$$= p(\text{the} \mid \langle b\rangle) \cdot p(\text{dog} \mid \text{the}) \cdot p(\text{walks} \mid \text{dog}) \cdot p(. \mid \text{walks}) \cdot p(\langle\backslash b\rangle \mid .)$$
$$= \frac{C(\langle b\rangle \text{ the})}{\sum_x C(\langle b\rangle\ x)} \cdot \frac{C(\text{the dog})}{\sum_x C(\text{the } x)} \cdot \frac{C(\text{dog walks})}{\sum_x C(\text{dog } x)} \cdot \frac{C(\text{walks .})}{\sum_x C(\text{walks } x)} \cdot \frac{C(.\ \langle\backslash b\rangle)}{\sum_x C(.\ x)}$$
$$= \frac{C(\langle b\rangle \text{ the})}{C(\langle b\rangle)} \cdot \frac{C(\text{the dog})}{C(\text{the})} \cdot \frac{C(\text{dog walks})}{C(\text{dog})} \cdot \frac{C(\text{walks .})}{C(\text{walks})} \cdot \frac{C(.\ \langle\backslash b\rangle)}{C(.)}$$
$$= \tfrac{5}{6} \cdot \tfrac{4}{7} \cdot \tfrac{1}{4} \cdot \tfrac{3}{4} \cdot \tfrac{6}{6} = 0.83 \cdot 0.57 \cdot 0.25 \cdot 0.75 \cdot 1.0 = \mathbf{0.089}$$

$p(\langle b\rangle \text{ the cat walks the dog . } \langle\backslash b\rangle)$

$$= p(\text{the} \mid \langle b\rangle) \cdot p(\text{cat} \mid \text{the}) \cdot p(\text{walks} \mid \text{cat}) \cdot p(\text{the} \mid \text{walks}) \cdot p(\text{dog} \mid \text{the}) \cdot p(. \mid \text{dog}) \cdot p(\langle\backslash b\rangle \mid .)$$
$$= \frac{C(\langle b\rangle \text{ the})}{\sum_x C(\langle b\rangle\ x)} \cdot \frac{C(\text{the cat})}{\sum_x C(\text{the } x)} \cdot \frac{C(\text{cat walks})}{\sum_x C(\text{cat } x)} \cdot \frac{C(\text{walks the})}{\sum_x C(\text{walks } x)} \cdot \frac{C(\text{the dog})}{\sum_x C(\text{the } x)} \cdot \frac{C(\text{dog .})}{\sum_x C(\text{dog } x)} \cdot \frac{C(.\ \langle\backslash b\rangle)}{\sum_x C(.\ x)}$$
$$= \frac{C(\langle b\rangle \text{ the})}{C(\langle b\rangle)} \cdot \frac{C(\text{the cat})}{C(\text{the})} \cdot \frac{C(\text{cat walks})}{C(\text{cat})} \cdot \frac{C(\text{walks the})}{C(\text{walks})} \cdot \frac{C(\text{the dog})}{C(\text{the})} \cdot \frac{C(\text{dog .})}{C(\text{dog})} \cdot \frac{C(.\ \langle\backslash b\rangle)}{C(.)}$$
$$= \tfrac{5}{6} \cdot \tfrac{1}{7} \cdot \tfrac{1}{1} \cdot \tfrac{1}{4} \cdot \tfrac{4}{7} \cdot \tfrac{1}{4} \cdot \tfrac{6}{6} = 0.83 \cdot 0.14 \cdot 1.0 \cdot 0.25 \cdot 0.57 \cdot 0.25 \cdot 1.0 = \mathbf{0.004}$$

$p(\langle b\rangle \text{ the cat runs . } \langle\backslash b\rangle)$

$$= p(\text{the} \mid \langle b\rangle) \cdot p(\text{cat} \mid \text{the}) \cdot p(\text{runs} \mid \text{cat}) \cdot p(. \mid \text{runs}) \cdot p(\langle\backslash b\rangle \mid .)$$
$$= \frac{C(\langle b\rangle \text{ the})}{\sum_x C(\langle b\rangle\ x)} \cdot \frac{C(\text{the cat})}{\sum_x C(\text{the } x)} \cdot \frac{C(\text{cat runs})}{\sum_x C(\text{cat } x)} \cdot \frac{C(\text{runs .})}{\sum_x C(\text{runs } x)} \cdot \frac{C(.\ \langle\backslash b\rangle)}{\sum_x C(.\ x)}$$
$$= \frac{C(\langle b\rangle \text{ the})}{C(\langle b\rangle)} \cdot \frac{C(\text{the cat})}{C(\text{the})} \cdot \frac{C(\text{cat runs})}{C(\text{cat})} \cdot \frac{C(\text{runs .})}{C(\text{runs})} \cdot \frac{C(.\ \langle\backslash b\rangle)}{C(.)}$$
$$= \tfrac{5}{6} \cdot \tfrac{1}{7} \cdot \tfrac{\mathbf{0}}{1} \cdot \tfrac{1}{1} \cdot \tfrac{6}{6} = 0.83 \cdot 0.14 \cdot \mathbf{0.0} \cdot 1.0 \cdot 1.0 = \mathbf{0.000}$$

- Longer sentences have lower likelihoods.
  - This makes sense because longer sequences are harder to match exactly.
- Zeros happen when an n-gram isn't seen.

# 9 Handling Sparsity

How big of a problem is sparsity?

- Alice's Adventures in Wonderland
  - Vocabulary (all word types) size: $V = 3{,}569$
  - Distinct bigrams: 17,149; $\frac{17{,}149}{|V|^2}$, or 99.8% of possible bigrams unseen
  - Distinct trigrams: 28,540; $\frac{17{,}149}{|V|^3}$, or 99.9999994% of possible trigrams unseen
- If a sequence contains an unseen ngram, it will have likelihood zero: an impossible sequence.
- Many legitimate ngrams will simply be absent from the corpus.
- This does not mean they are impossible.
- Even ungrammatical/nonsense ngrams should not cause an entire sequence's likelihood to be zero.
- Many others will be too infrequent to estimate well.

Add-$\lambda$ Smoothing

- Add some constant $\lambda$ to every count, including unseen ngrams
- $V$ is the Vocabulary — all word types — including $\langle \backslash b \rangle$ (if necessary $n > 1$)
- $p(w_0 \mid w_{1-n} \ldots w_{-1}) = \frac{C(w_{1-n} \ldots w_{-1}\ w_0)+1}{\sum_x (C(w_{1-n} \ldots w_{-1}\ x)+1)} = \frac{C(w_{1-n} \ldots w_{-1}\ w_0)+1}{(\sum_x C(w_{1-n} \ldots w_{-1}\ x))+|V|} = \frac{C(w_{1-n} \ldots w_{-1}\ w_0)+1}{C(w_{1-n} \ldots w_{-1})+|V|}$
- Add $|V|$ to the denominator to account for the fact that there is an extra count for every $x$
- In practice it over-smoothes, even when $\lambda < 1$

Good-Turing Smoothing

- Estimate counts of things you haven't seen from counts of things you have
- Estimate probability of things which occur $c$ times with the probability of things which occur $c + 1$ times
- $c^* = (c+1)\frac{N_{c+1}}{N_c}$

  $p^*_{GT}(\textit{things with freq 0}) = \frac{N_1}{N}$
- $p_{GT}(w_0 \mid w_{1-n} \ldots w_{-1}) = \frac{C^*(w_{1-n} \ldots w_{-1}\ w_0)}{C^*(w_{1-n} \ldots w_{-1})}$

Stupid Backoff

- If p(n-gram)=0, use p((n-1)-gram)

- Works shockingly well for huge datasets

Interpolation

- Mix n-gram probability with probabilities from lower-order models

$$\begin{aligned}\hat{p}(w_0 \mid w_{-2}\ w_{-1}) &= \lambda_3 \cdot p(w_0 \mid w_{-2}\ w_{-1}) \\ &= \lambda_2 \cdot p(w_0 \mid w_{-1}) \\ &= \lambda_1 \cdot p(w_0)\end{aligned}$$

- $\lambda_i$ terms used to decide how much to smooth
- $\sum_i \lambda_i = 1$, because they are proportions
- Use *dev* dataset to tune $\lambda$ hyperparameters
- Also useful for combining models trained on different data:
  - Can interpolate "customized" models with "general" models
  - Baseline English + regional English + user-specific English
  - Little in-domain data, lots of out-of-domain

Knesser-Ney Smoothing

- Intuition: interpolate based on "openness" of the context
- Words seen in more contexts are more likely to appear in others
- Even if we haven't seen $w_0$ following the context, if the context is "open" (supports a wide variety of "next words"), then it is more likely to support $w_0$
- Boost counts based on $|\{x : C(w_{1-n}\ ...\ w_{-1}\ x) > 0\}|$, the number of different "next words" seen after $w_{1-n}\ ...\ w_{-1}$

## 10 Out-of-Vocabulary Words (OOV)

Add-$\lambda$

- If ngram contains OOV item, assume count of $\lambda$, just like for all other ngrams.
- Probability distributions become invalid. We can't know the full vocabulary size, so we can't normalize counts correctly.

$\langle unk \rangle$

- Create special token $\langle unk \rangle$
- Create a fixed lexicon $L$

  - All types in some subset of training data?
  - All types appearing more than $k$ times?
- $V = L + \langle unk \rangle, \quad |V| = |L| + 1$
- Before training, change any word not in $L$ to $\langle unk \rangle$
- Then train as usual as if $\langle unk \rangle$was a normal word
- For new sentence, again replace words not in $L$ with $\langle unk \rangle$before using model
- Probabilities containing $\langle unk \rangle$ measure likelihood with *some rare word*
- Problem: the "rare" word is no longer rare since there are many $\langle unk \rangle$tokens
  - Ngrams with $\langle unk \rangle$will have higher probabilities than those with any particular rare word
  - Not so bad when comparing same sequence under multiple models. All will have inflated probabilities.
  - More problematic when comparing probabilities of different sequences under the same model
    * p(i **totes** know) < p(i **totally** know) < p(i ⟨**unk**⟩ know)

# 11 Evaluation

Extrinsic

- Use the model in some larger task. See if it helps.
- More realistic
- Harder

Intrinsic

- Evaluate on a test corpus
- Easier

Perplexity

- Intrinsic measure of model quality
- How well does the model "fit" the test data?
- How "perplexed" is the model whe it sees the test data?
- Measure the probability of the test corpus, normalize for number of words.
- $PP(W) = \sqrt[|W|]{\frac{1}{p(w_1 \ w_2 \ \ldots \ w_{|W|}}}$

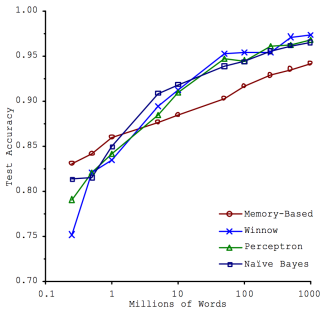
## 12 How much data?

Choosing $n$

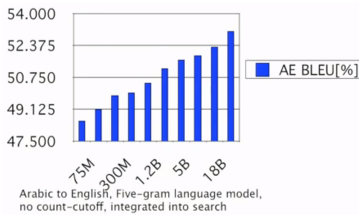
- Large $n$
  - More context for probabilities:
    $p$(phone) vs
    $p$(phone | cell) vs
    $p$(phone | your cell) vs
    $p$(phone | off your cell) vs
    $p$(phone | turn off your cell)
  - Long-range dependencies
- Small $n$
  - Better generalization
  - Better estimates
  - Long-range dependencies

How much training data?

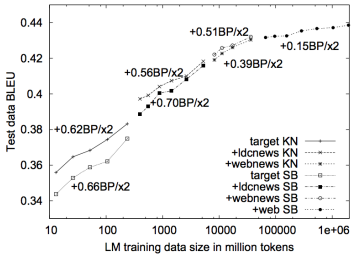
- *As much as possible.*
- More data means better estimates
- Google N-Gram corpus uses 10-grams




(a) Data size matters more than algorithm (Banko and Brill, 2001)




(b) Results keep improving with more data (Norvig: Unreasonable ...)




(c) With enough data, stupid backoff approaches Knesser-Ney accuracy (Brants et al., 2007)

## 13 Generative Models

- Generative models are designed to model how the data *could have been generated.*
- The best parameters are those that would most likely generate the data.

- MLE maximizes that likelihood that the training data was generated by the model.
- As such, we can *actually generate* data from a model.
- Trigram model:
  - General:

    For each sequence:
    1. Sample a word $w_0$ according to $w_0 \sim p(w_0)$
    2. Sample a second word $w_1$ according to $w_1 \sim p(w_1 \mid w_0)$
    3. Sample a next word $w_k$ according to $w_k \sim p(w_k \mid w_{k-2}\ w_{k-1})$
    4. Repeat step 3 until you feel like stopping.
  - Sentences:

    For each sentence:
    1. Sample a word $w_0$ according to $w_0 \sim p(w_0 \mid \langle b \rangle\ \langle b \rangle)$
    2. Sample a second word $w_1$ according to $w_1 \sim p(w_1 \mid w_0\ \langle b \rangle)$
    3. Sample a next word $w_k$ according to $w_k \sim p(w_k \mid w_{k-2}\ w_{k-1})$
    4. Repeat until $\langle \backslash b \rangle$ is drawn.
- Longer $n$ generates more coherent text
- Too-large $n$ just ends up generating sentences from the training data because most counts will be 1 (no choice of next word).
- Naïve Bayes was a generative model too!

  For each instance:
  1. Sample a label $l$ according to $l \sim p(Label = l)$
  2. For each feature $F$: sample a value $v$ according to $v \sim p(F = v \mid Label = l)$
- We will see many more generative models throughout this course

## 14 Citations

Some content adapted from:

- `http://courses.washington.edu/ling570/gina_fall11/slides/ling570_class8_smoothing.pdf`