FIRST ORDER LOGIC AND PROBABILISTIC INFERENCING

Inference in Belief Networks

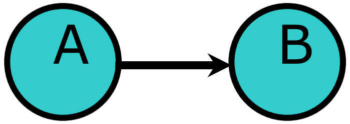
- Find $P(Q=q|\boldsymbol{E}= e)$
  - Q the query variable
  - $\boldsymbol{E}$ set of evidence variables

$$P(q \mid \mathbf{e}) = \frac{P(q, \mathbf{e})}{P(\mathbf{e})}$$

$X_1, \ldots, X_n$ are network variables except Q, **E**

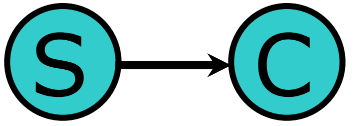
$$P(q, \mathbf{e}) = \sum_{x_1, \ldots, x_n} P(q, e, x_1, \ldots, x_n)$$

Basic Inference



$P(b) = ?$

Product Rule



- $P(C,S) = P(C|S)\,P(S)$

$S\Downarrow$ $C\Rightarrow$	*none*	*benign*	*malignant*
no	0.768	0.024	0.008
light	0.132	0.012	0.006
heavy	0.035	0.010	0.005

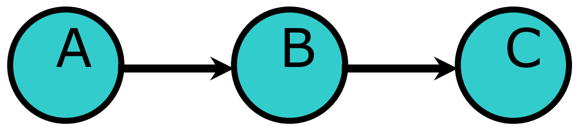
Marginalization

$S\Downarrow \quad C\Rightarrow$	*none*	*benign*	*malig*	total
no	0.768	0.024	0.008	.80
light	0.132	0.012	0.006	.15
heavy	0.035	0.010	0.005	.05
total	0.935	0.046	0.019	

P(Smoke)

P(Cancer)

Basic Inference



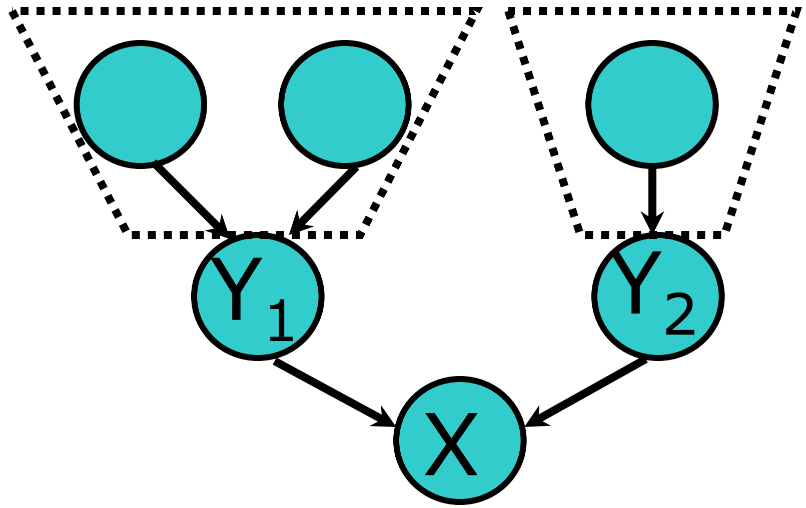
$$P(b) = \sum_a P(a, b) = \sum_a P(b \mid a)\, P(a)$$

$$P(c) = \sum_b P(c \mid b)\, P(b)$$

$$P(c) = \sum_{b,a} P(a, b, c) \qquad = \sum_{b,a} P(c \mid b)\, P(b \mid a)\, P(a)$$

$$= \sum_b P(c \mid b) \underbrace{\sum_a P(b \mid a)\, P(a)}_{P(b)}$$

Inference in trees



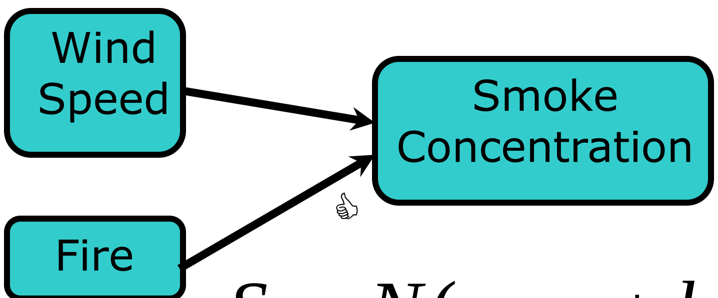
$$P(x) = \sum_{y_1, y_2} P(x \mid y_1, y_2)\, P(y_1, y_2)$$

because of independence of Y_1, Y_2:

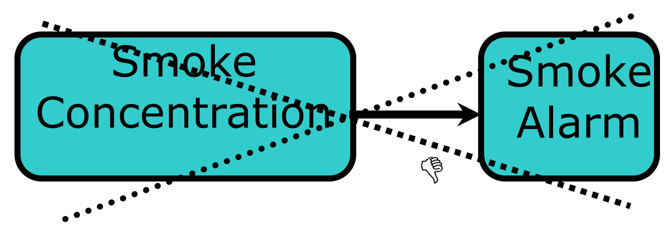
$$= \sum_{y_1, y_2} P(x \mid y_1, y_2)\, P(y_1)\, P(y_2)$$

Inference with continuous variables

- Gaussian networks: polynomial time inference regardless of network structure
- Conditional Gaussians:
  - discrete variables cannot depend on continuous



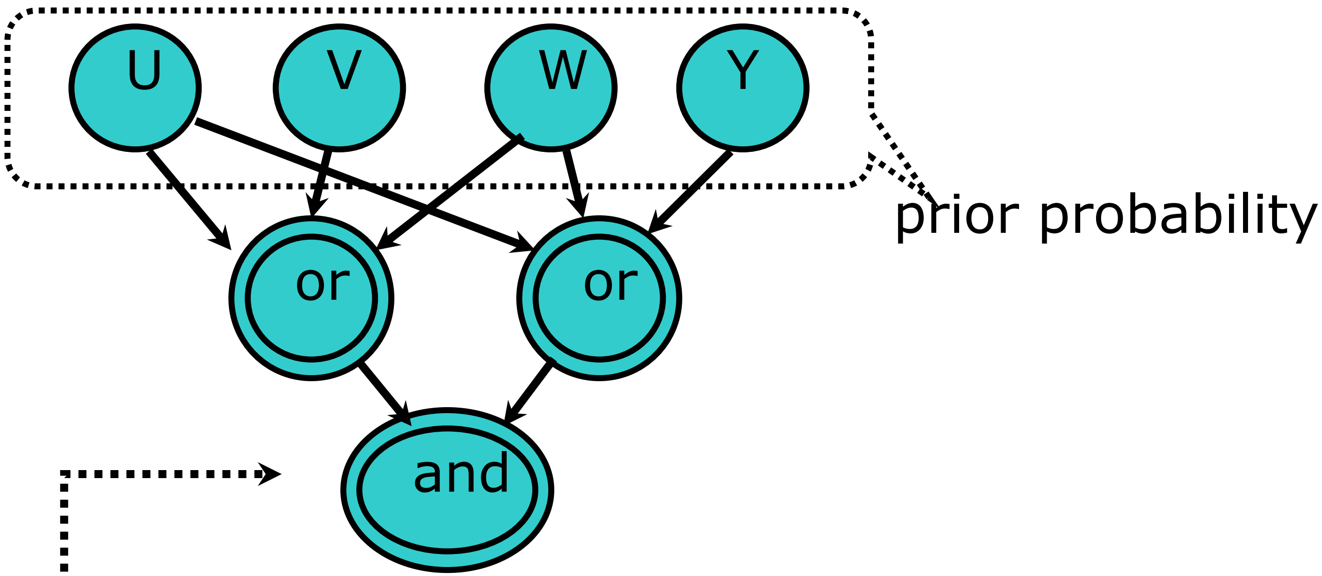
$$S \sim N(a_F w + b_F, \sigma_F^2)$$



- These techniques do not work for general hybrid networks.

- **Theorem:** Inference in a multi-connected Bayesian network is NP-hard.

Boolean 3CNF formula $\phi = (u \vee v \vee w) \wedge (u \vee w \vee y)$

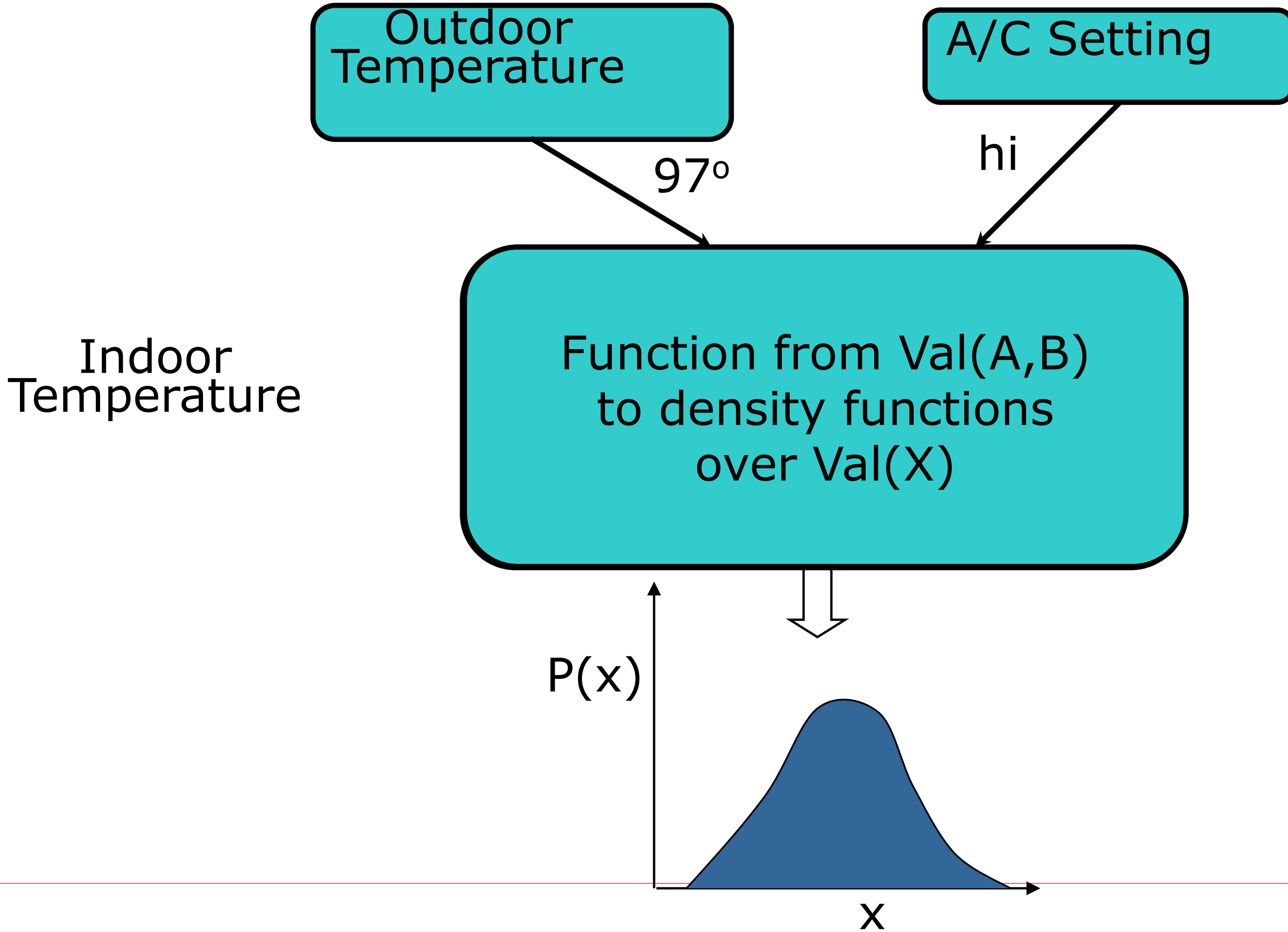


Probability ( ) = # satisfying assignments of ϕ

Summary

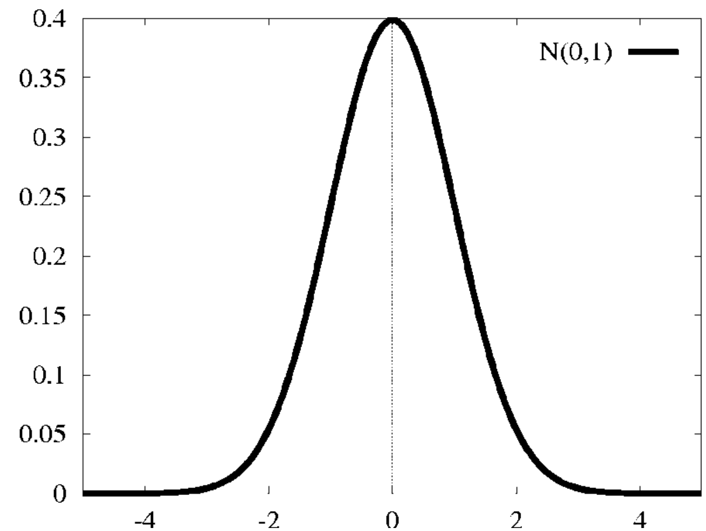
- Bayesian networks provide a natural representation for (causally induced) conditional independence
- Topology + CPTs = compact representation of joint distribution
- Generally easy for domain experts to construct

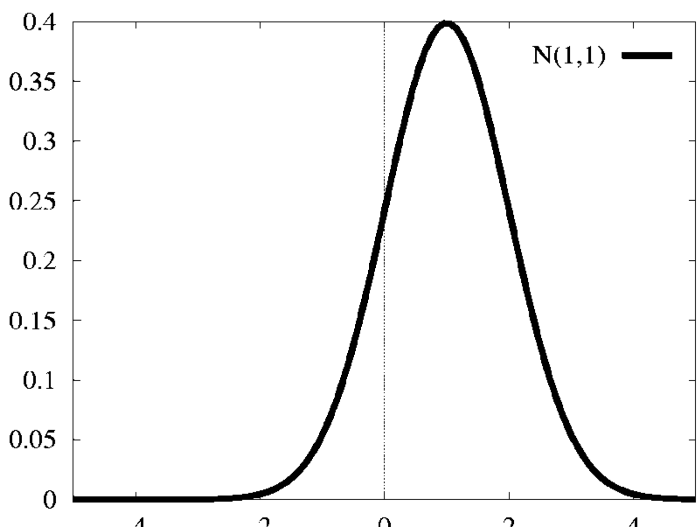
Continuous variables



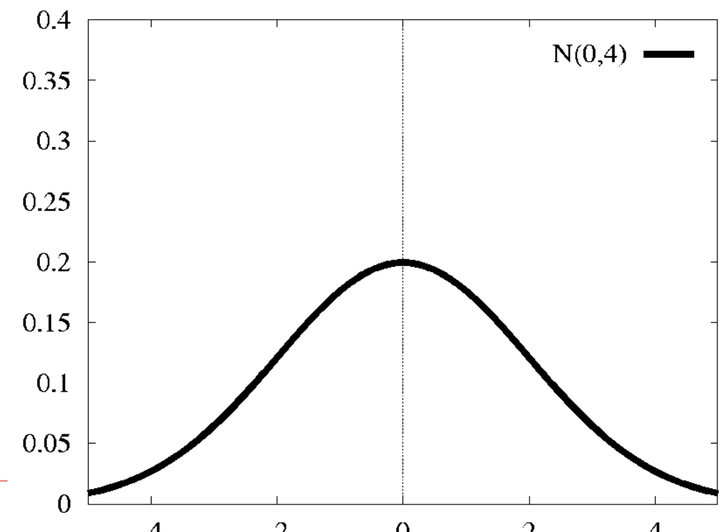
Gaussian (normal) distributions

$$\underbrace{P(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(\frac{-(x-\mu)^2}{2\sigma} \right)}_{N(\mu,\ \sigma)}$$





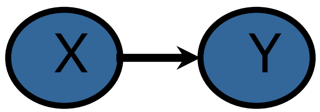
different mean



different variance

Gaussian networks

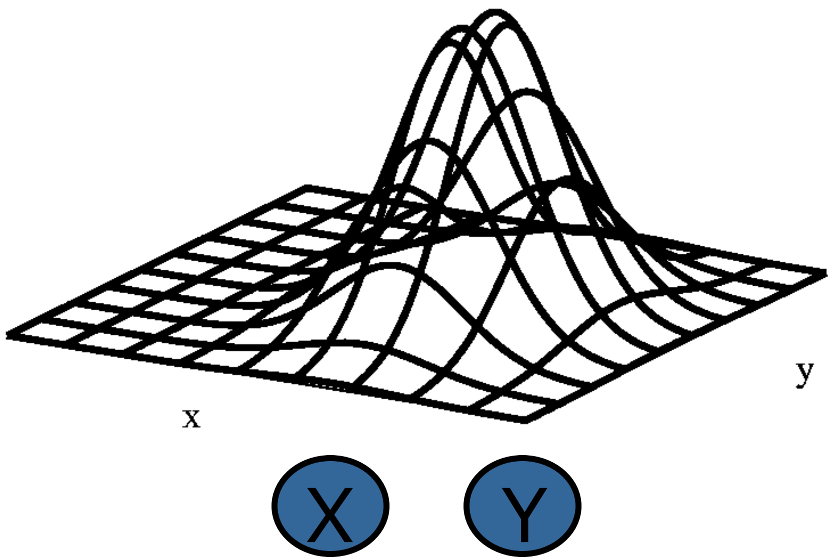
$$X \sim N(\mu, \sigma_X^2)$$

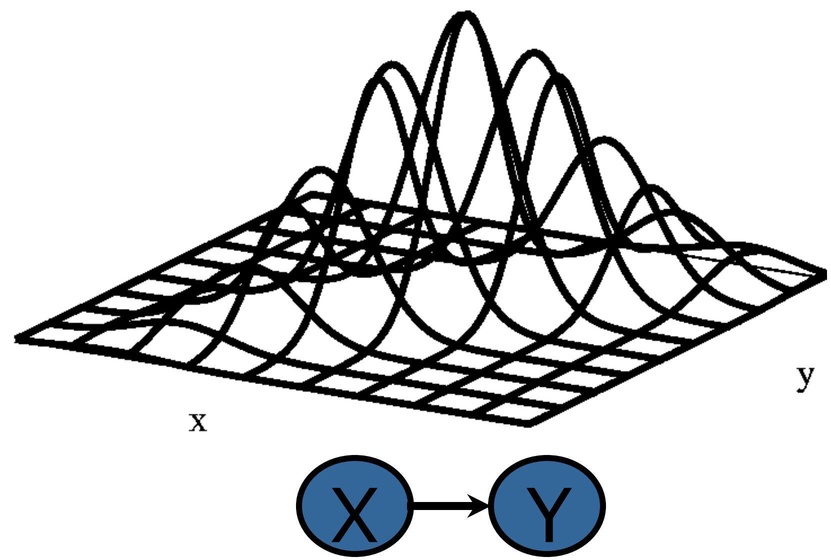


$$Y \sim N(ax + b, \sigma_Y^2)$$

Each variable is a linear function of its parents, with Gaussian noise

Joint probability density functions:





D-Separation

- X is d-separated from Y if, **for all paths** between X and Y **there exists** an intermediate node Z for which:
  - The connection is serial or diverging and there is evidence for Z.
  - The connection is converging and Z (nor any of its descendants) have received any evidence.

- X is independent of Y given Z for some conditional probabilities if and only if X is d-separated from Y given Z.

Example of Independence Questions

- If there was evidence for B, which probabilities would change?
- If there was evidence for N, which probabilities would change?
- If there was evidence for M and N, which variables probabilities would change?
- Etc…