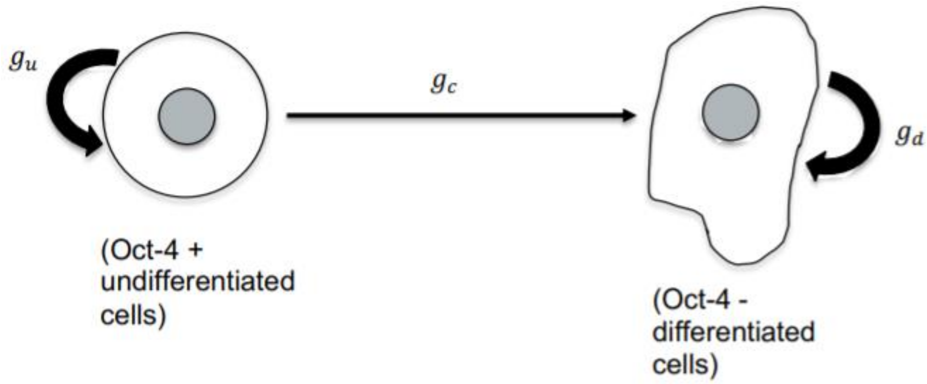
Edward Wu
585.609: Biomechanics of Cells and Stem Cells
Research Project

Develop and apply a cell population model of stem cell differentiation. Stem cells are determined by the presence of the factor, Oct-4. The dynamics of two, undifferentiated and differentiated, subpopulations are presented by the sketch:



where g_c, g_u, and g_d are, respectively, self-renewal rate of undifferentiated cells, differentiation rate, and self-renewal rate of differentiated cells in day-1 units. Mathematically, cell dynamics are described by the equations:

$$\frac{dU}{dt} = g_u U - g_c U$$

$$\frac{dD}{dt} = g_d D + g_c U$$

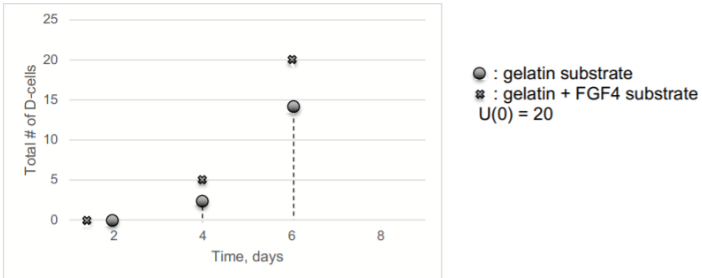
where $U(t)$ and $D(t)$ are the numbers of differentiated and differentiated cells, respectively.

1. Solve the system of ODEs and find analytical expressions for $U(t)$ and $D(t)$ if $U(0) = 20, D(0) = 0$

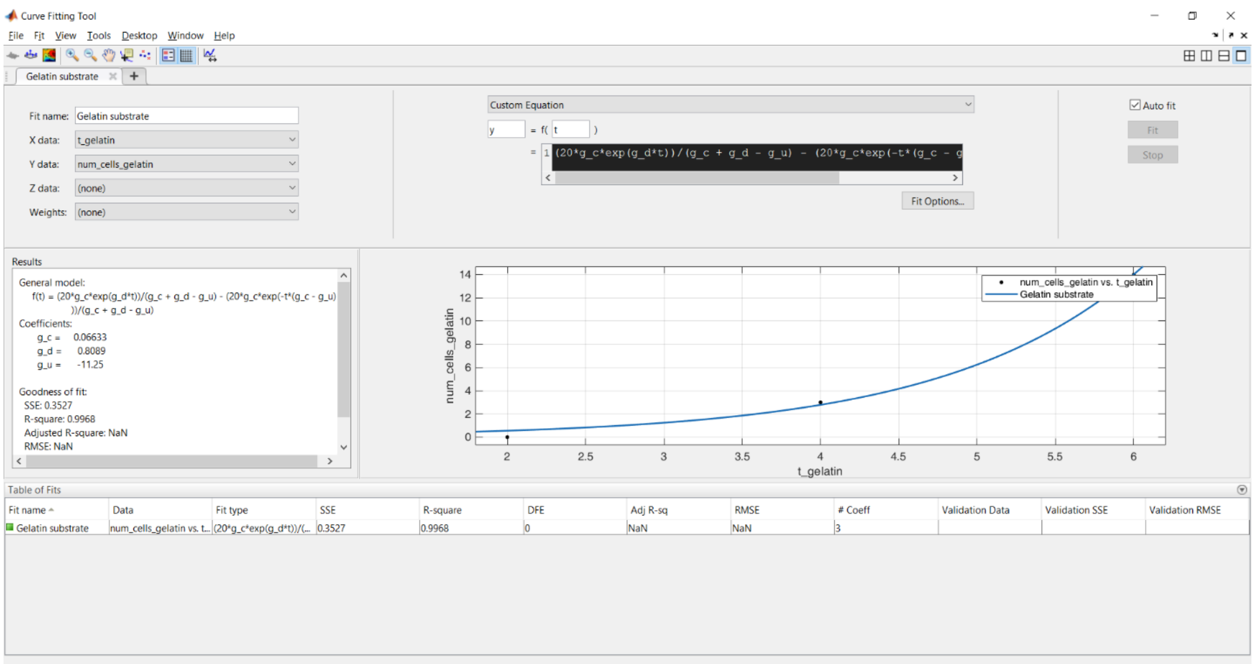
```
syms U(t) D(t) g_c g_u g_d

dU = diff(U) == (g_u*U) - (g_c*U);
dD = diff(D) == (g_d*D) + (g_c*U);
eqs = [dU; dD];

%Solve the system of ODEs
%Find analytical expressions for U(t) and D(t) if U(0) = 20, D(0) = 0
F = dsolve(eqs, U(0)==20, D(0)==0);

U(t) = F.U %U(t) = 20*exp(-t*(g_c - g_u))
D(t) = F.D %D(t) = (20*g_c*exp(g_d*t))/(g_c + g_d - g_u) - (20*g_c*exp(-
t*(g_c - g_u)))/(g_c + g_d - g_u)
```

Using `dsolve`, we obtain $U(t) = 20e^{-(g_c - g_u)t}$ and $D(t) = \frac{20g_c}{g_c + g_d - g_u} e^{g_d t} - 20g_c e^{-t(\frac{g_c - g_u}{g_c + g_d - g_u})}$

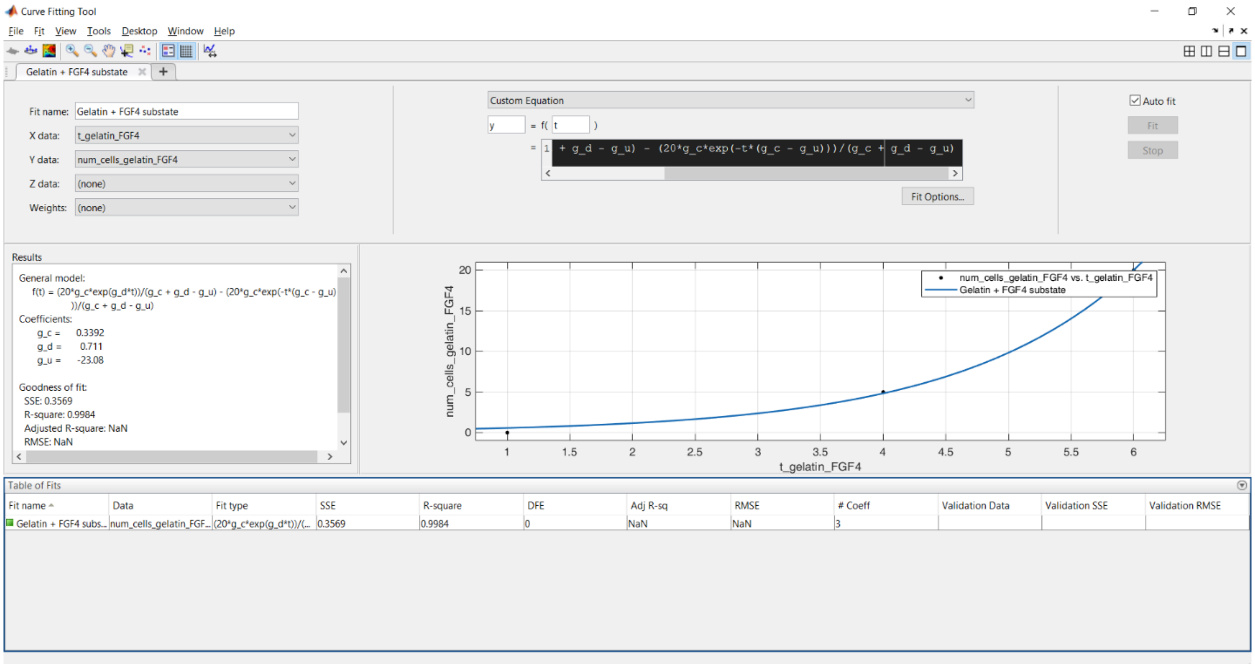
2. Fit the $D(t)$ function to the experimental data below and determine the g_c, g_u, and g_d rates for two substrates.




```
t_gelatin = [2 4 6];
num_cells_gelatin = [0 3 14];
t_gelatin_FGF4 = [1 4 6];
num_cells_gelatin_FGF4 = [0 5 20];
```



Using experimental data for gelatin substate, we obtain $g_c = 0.06633$, $g_u = -11.25$, and $g_d = 0.8089$



Using experimental data for gelatin and FGF4 substate, we obtain $g_c = 0.3392$, $g_u = -23.08$, and $g_d = 0.711$

3. Predict the numbers, $U(t = 8)$ and $D(t = 8)$

```
t = 8;
g_c = 0.06633;
g_d = 0.8089;
g_u = -11.25;

U_8_gelatin = 20*exp(-t*(g_c - g_u))
D_8_gelatin = (20*g_c*exp(g_d*t))/(g_c + g_d - g_u) - (20*g_c*exp(-t*(g_c -
g_u)))/(g_c + g_d - g_u)
```

Using $g_c = 0.06633$, $g_u = -11.25$, and $g_d = 0.8089$, $U(t = 8) = 9.6399 \times 10^{-39}$ and $D(t = 8) = 70.7060$

```
g_c = 0.3392;
g_d = 0.711;
g_u = -23.08;

U_8_gelatin_FGF4 = 20*exp(-t*(g_c - g_u))
D_8_gelatin_FGF4 = (20*g_c*exp(g_d*t))/(g_c + g_d - g_u) - (20*g_c*exp(-
t*(g_c - g_u)))/(g_c + g_d - g_u)
```

Using $g_c = 0.3392$, $g_u = -23.08$, and $g_d = 0.711$, $U(t = 8) = 8.5980 \times 10^{-81}$ and $D(t = 8) = 83.0218$