

# Secure ZK Circuits with Formal Methods

Guest Lecturer: Yu Feng (UCSB & Veridise)



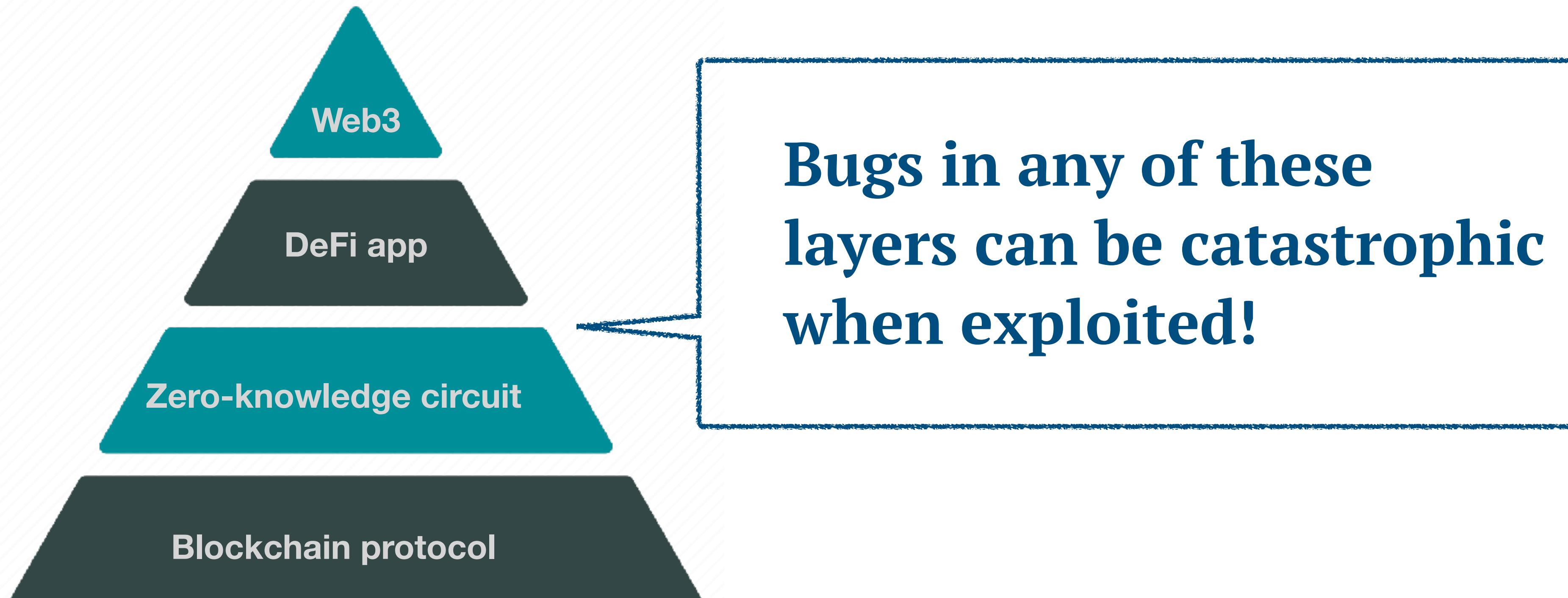
# Zero Knowledge Proofs

Instructors: Dan Boneh, Shafi Goldwasser, Dawn Song, Justin Thaler, Yupeng Zhang



# Motivation

Bugs in blockchain software are **extremely costly**



# Smart Contract Bugs

## Ethereum DeFi Protocol Beanstalk Hacked for \$182 Million—What You Need to Know

Beanstalk got jacked by a giant flash attack.



By [Jeff Benson](#)

Apr 18, 2022

2 min read



Beanstalk. Image: Shutterstock

Flash loan  
vulnerability  
in smart contract

# Blockchain Protocol Bugs

CRYPTO WORLD

## Solana suffered its second outage in a month, sending price plunging

PUBLISHED WED, JUN 1 2022 9:27 PM EDT

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**KEY POINTS**

- Solana fell more than 12% on Wednesday as the blockchain suffered its second outage in the last month.
- Investors who had been focused largely on ethereum began diversifying into Solana and other alternative blockchains during last year's crypto run-up.
- But the last year and a half has laid bare the trade-off as the blockchain network has suffered multiple outages.



The logo of cryptocurrency platform Solana.  
Jaleeb Pursey / NurPhoto via Getty Images

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**Crypto.com Exchange:**  
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Crypto hedge fund Three Arrows Capital plunges into liquidation as market crash takes toll



One of the most prominent crypto hedge funds just defaulted on a \$670 million loan



Snoop Dogg on the current crypto winter and future of NFTs



El Salvador's \$425 million bitcoin experiment isn't saving the country's finances

DoS vulnerability  
in consensus  
protocol

# ZK Bugs are Coming

Zcash team fixes serious vulnerability that allowed counterfeiting

Malware and Vulnerabilities

• February 07, 2019 • Cyware Hacker News



Bug in  
arithmetic circuit  
implementing  
zkSNARK!

- The vulnerability was discovered by a cryptographer from Zcash Company in March 2018.
- Attackers could create fake Zcash coins in large numbers by exploiting this vulnerability.

# Formal Methods to Rescue

**Formal methods  
can eradicate these  
bugs**



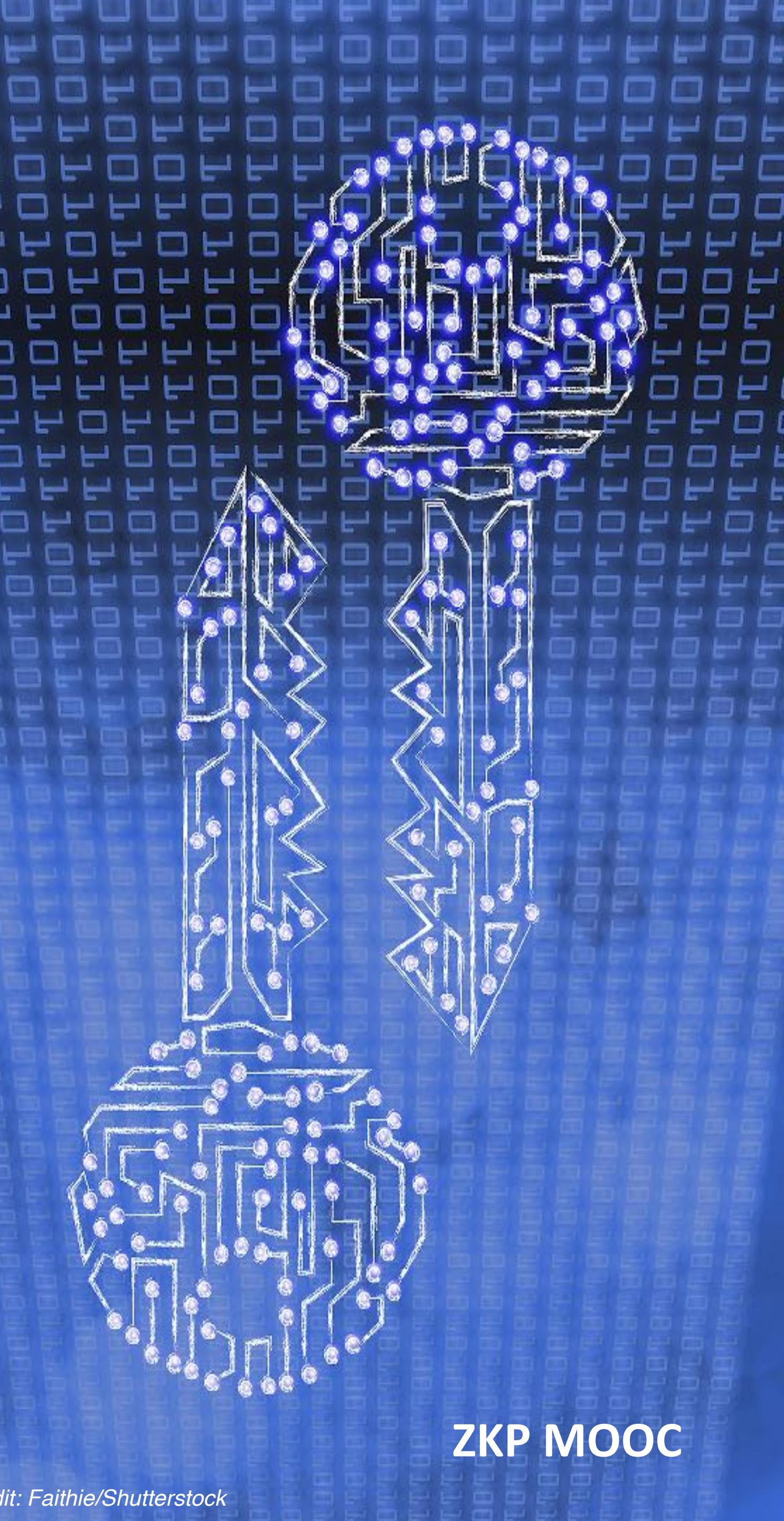
# Outline

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- Formal methods in a nutshell
- Formal methods for ZK I (static analysis)
- Formal methods for ZK II (SMT solver)
- Future work & Conclusion

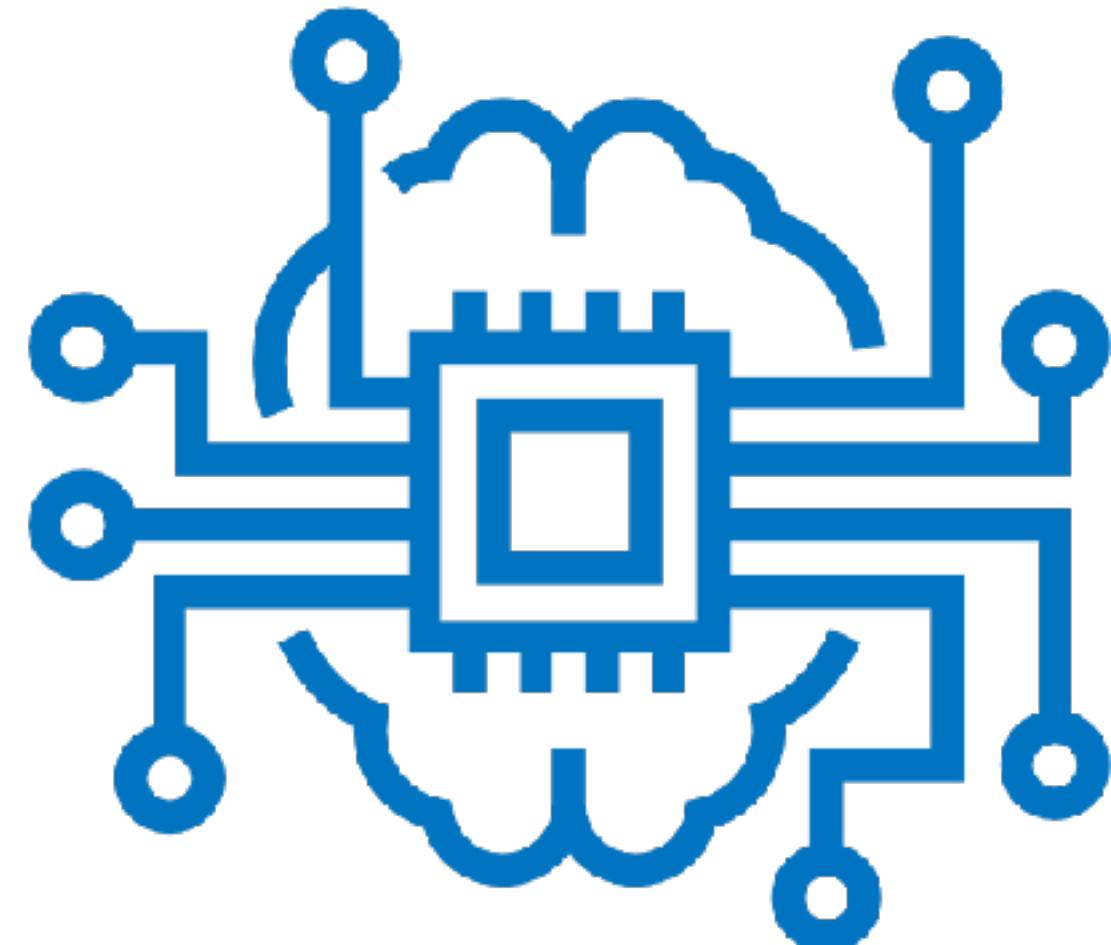
# Section 1

## Formal Methods in a Nutshell



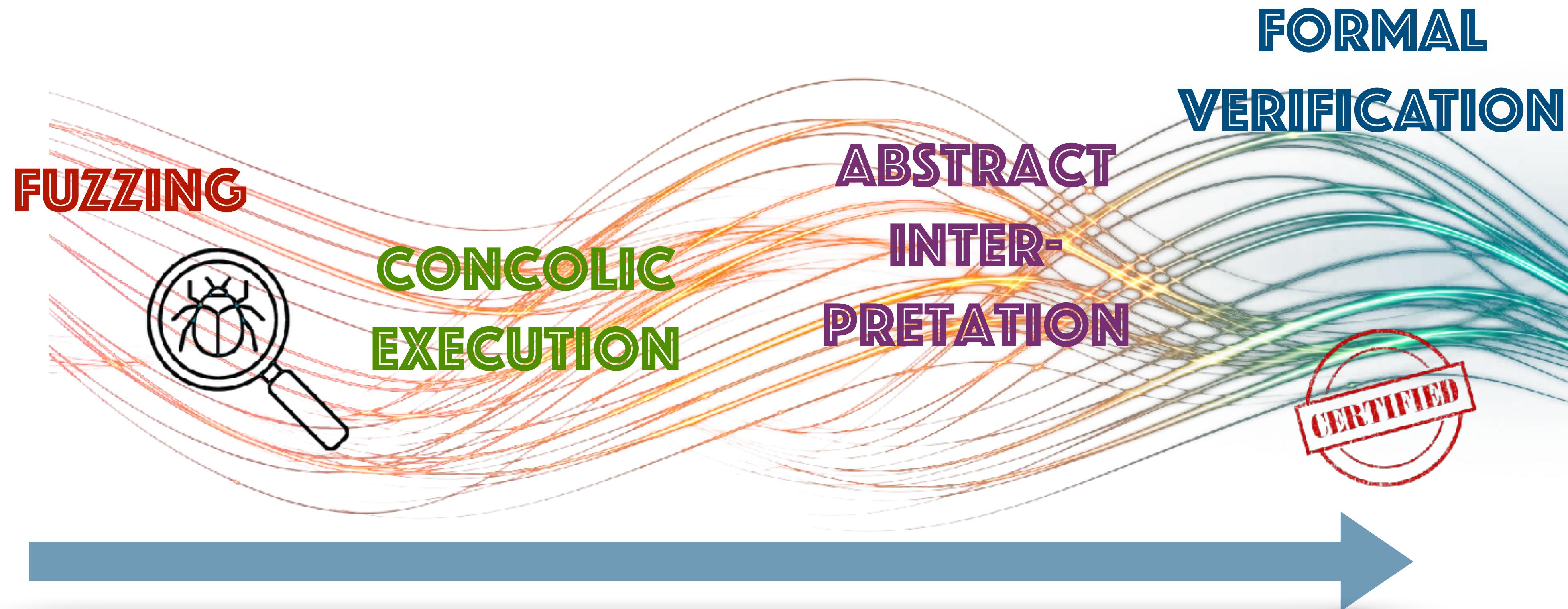
# What is Formal Methods

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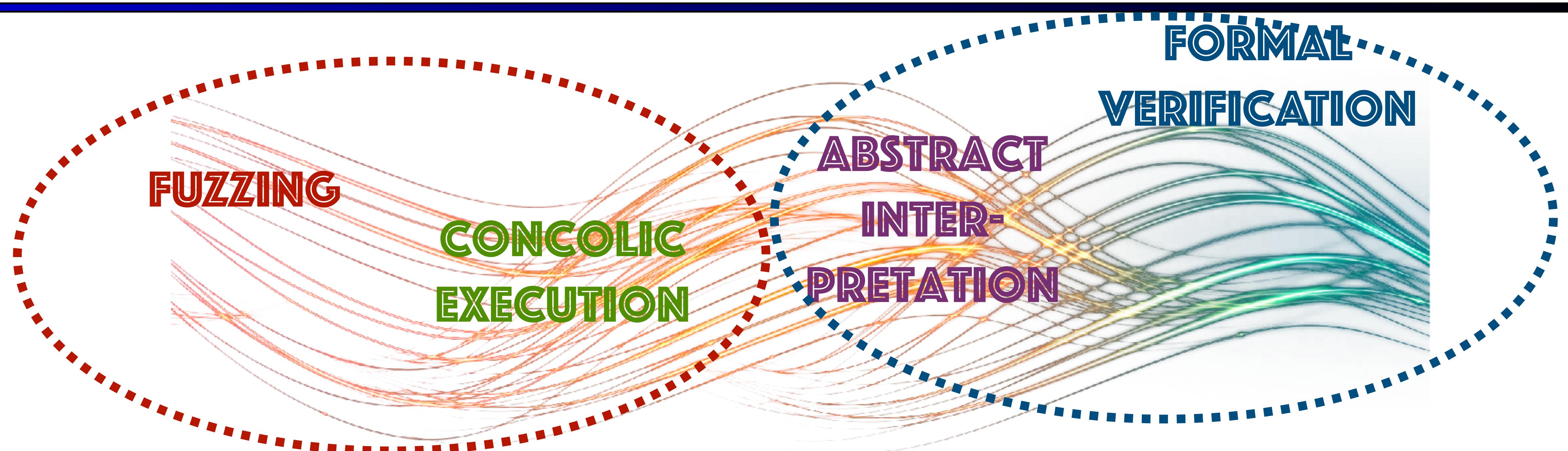
Set of mathematically rigorous techniques for **finding bugs** and **constructing proofs** about software

# Formal Methods Techniques on Spectrum



*Stronger guarantees*  
*More human effort*

# Classification of FM Techniques



**DYNAMIC**

Execute the program on interesting inputs  
& monitor what happens

**STATIC**

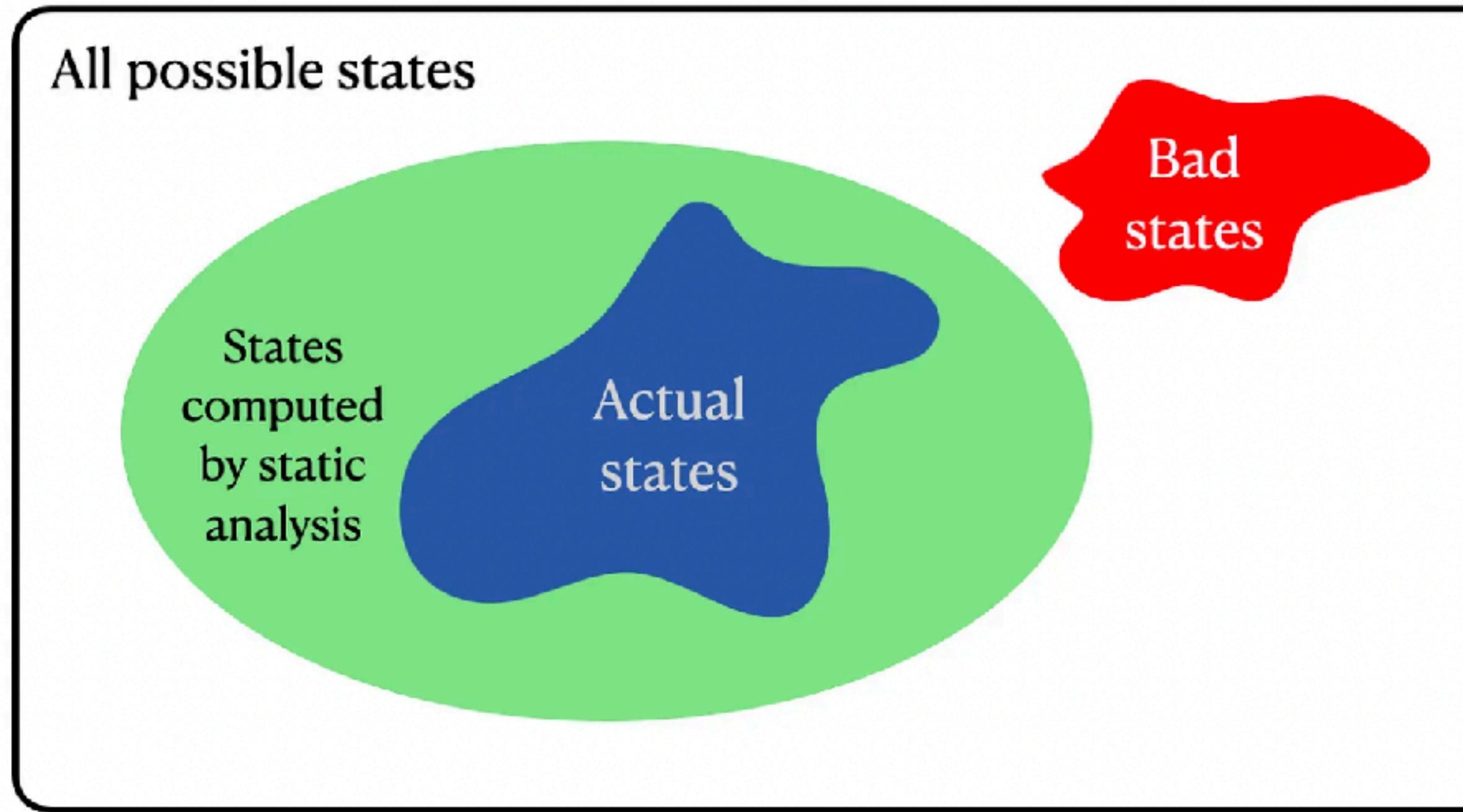
Analyze source code and  
reason about all executions

# Static Analysis via Abstract Interpretation

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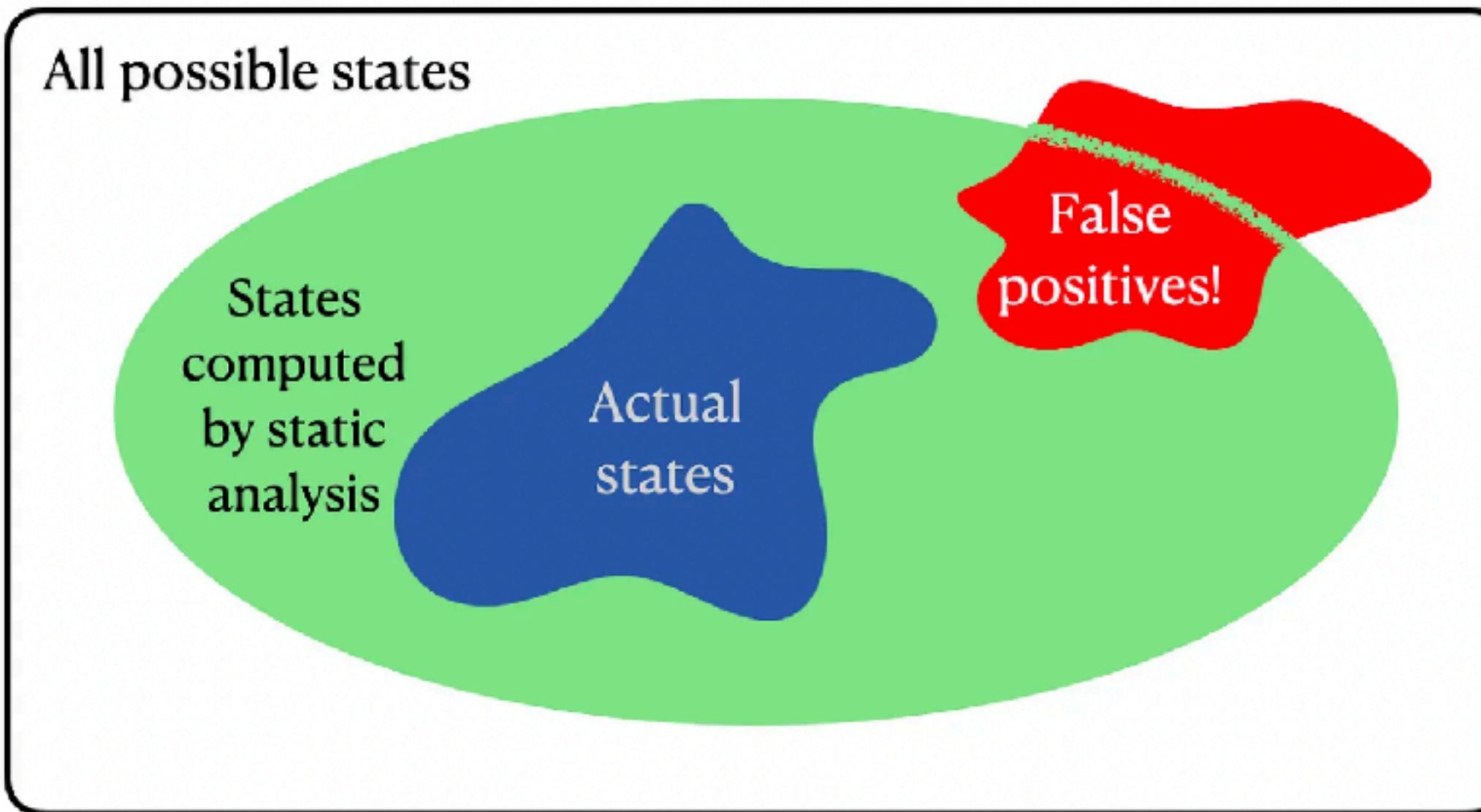
- Cannot reason about the exact program behavior due to undecidability
- Obtain a conservative over-approximation and this can be enough to prove program correctness
- Abstract interpretation is a framework for computing over-approximations of program states

# Static Analysis via Abstract Interpretation

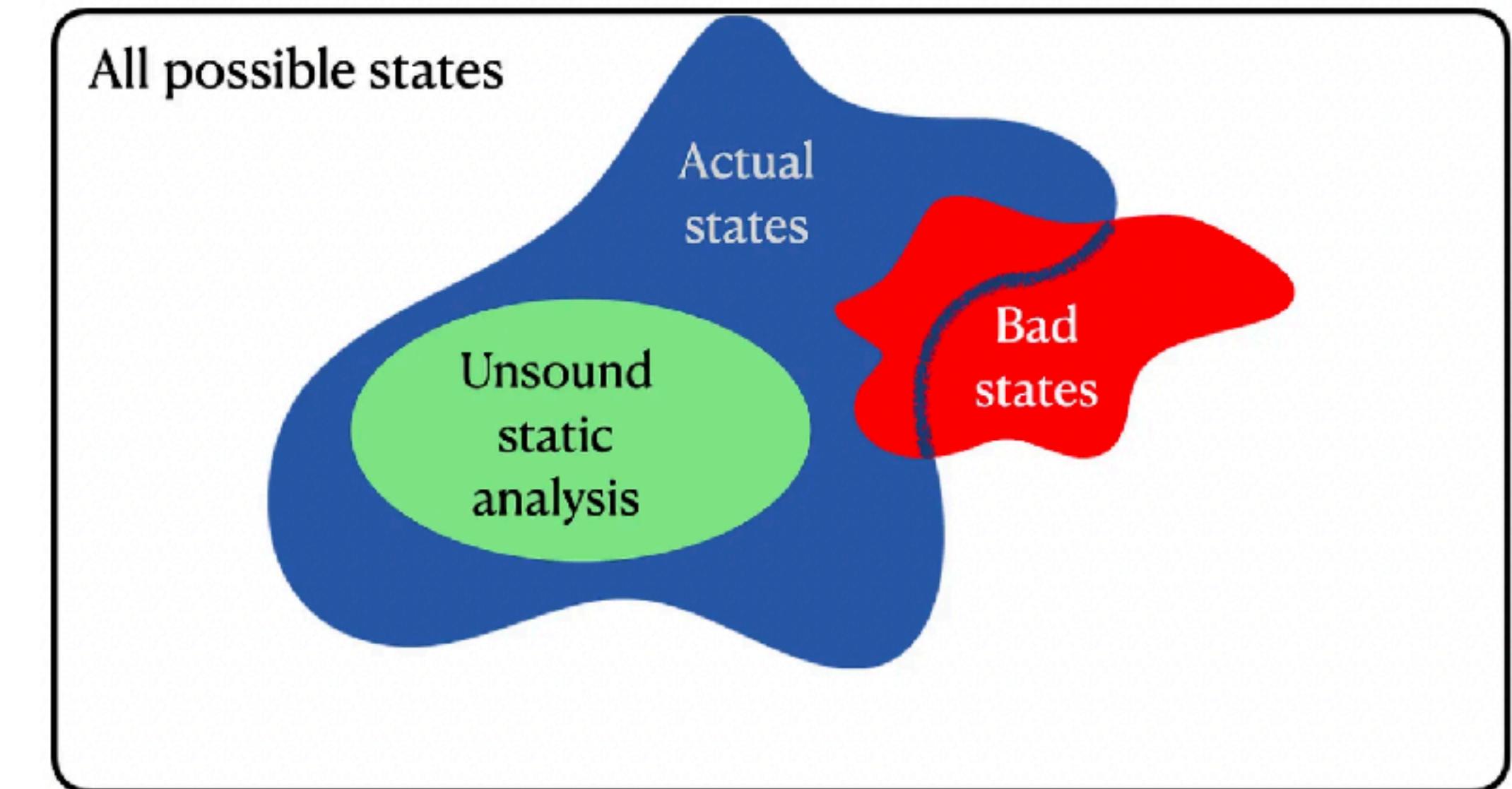


*Program is safe: intersection between the green and red region is empty*

# Static Analysis via Abstract Interpretation



False Positives



False Negatives

# Concrete Interpretation



- Concrete interpreter operates over **concrete values**
- $x=y+z$ .  $x$  evaluates to 3 if  $y=1$  and  $z=2$

# Static Analysis via Abstract Interpretation



- Abstract interpreter operates over **abstract values**
- Integer  $x$  can have an abstract value of interval  $[a,b]$
- $x = y + z \quad y \in [a, b] \wedge z \in [c, d] \implies x \in [a + c, b + d]$

# Static Analysis via Abstract Interpretation

**Idea:** Emulate all possible program paths

```
if(flag)
    x = 1;
else
    x = -1;
```

When in doubt, conservatively assume either path could be taken and merge information for different paths

$$x \in [-1, 1]$$

# Detect Reentrancy via Abstract Interpretation

```
1 contract Attacker {  
2   Victim v;  
3  
4   function exploit() {  
5     v = Victim(0x123);  
6     v.withdraw(10);  
7   }  
8  
9   function() payable {  
10    v.withdraw(10);  
11  }  
12 }
```

(2) attack program

```
1 contract Victim {  
2  
3   ...  
4   function withdraw(uint a) {  
5     1 msg.sender.call.value(a);  
6     2 balances[msg.sender] -= a;  
7   }  
8 }
```

(3) victim program

**External function call followed by a storage update**

# Other Vulnerabilities & Tools

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- Integer overflow
- Transaction order dependency
- Flashloan attack
- Related tools:
  - Slither (TrailOfBits)
  - Vanguard (Veridise)
  - Sailfish (Oakland'22), Securify (CCS'19)

# From Abstract Interpretation to Formal Verification

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- Identify **specific** types of bugs & security vulnerabilities
- Can't guarantee program is free from **logical errors**
- Formal verification to rescue
- Need a **specification** to describe how the program is supposed to behave

# Formal Specifications

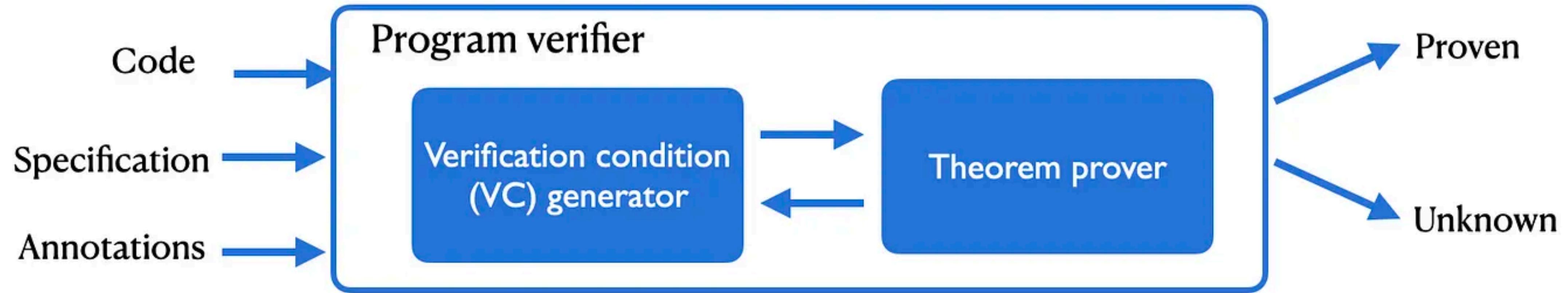


**Formal specification:** Precise mathematical description of intended program behavior, typically in some formal logic

- $((\text{finish}(\text{bid}, \text{msg}.\text{value} = X \wedge \text{msg}.\text{sender} = L))$   
 $\wedge \Diamond \text{finish}(\text{close}, L \neq \text{winner})$   
 $\rightarrow \Diamond \text{send}(\text{to} = L \wedge \text{amt} = X))$

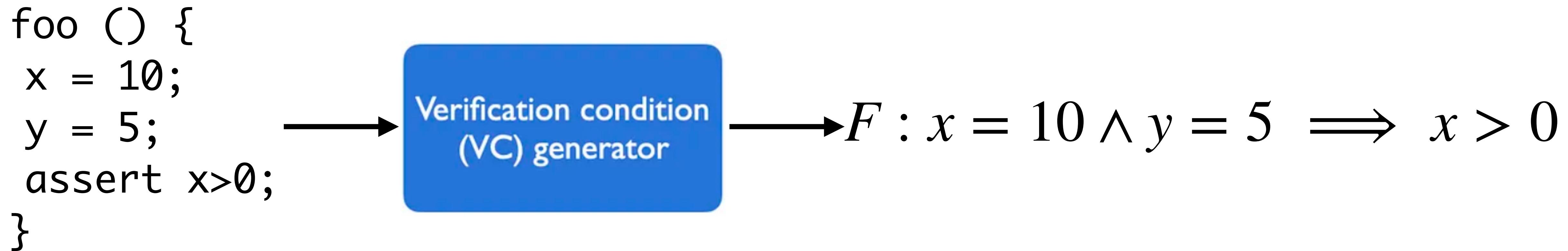
If auction closes with me  
not being the winner, I  
should eventually get  
back my bid

# Overview of Formal Verification



- Code: Source code or byte code
- Specification: A formal description of the property to be verified
- Human annotations (optional): Loop invariants, Contract invariants

# VC Generation



**Assertion always hold iff F is valid!**

# When Formal Verification Fails...

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- Logical error (true positive)
- Insufficient human input, loop iterations, missing lemmas
- Incompleteness of theorem prover
  - Linear integer arithmetic ✓
  - Finite fields over large prime, non-linear arithmetic ✗

# Bounded vs Unbounded Verification

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- Unbounded verification so far
- Bounded verification with weaker guarantees
  - Restrict input size/space
  - Bounded loop iterations
- Tools: Certora prover, K-framework, Mythril, Sailfish, etc.

# Different Flavors of Static Analysis

**Formal verification checks program  
against provided specification**

## Abstract Interpretation

- ✍ Looks for known types of bugs
- ✓ Doesn't require specifications



## Formal Verification

- ✓ Can find (prove absence of) any bug
- ✍ Requires specifications