Subspaces And Spaces Of Continuous Functions

1 Subspaces

If $(X, ||\cdot||)$ is a Normed Space and $Y \subset X$ then $(Y, ||\cdot||)$ is another Normed Space, with $||\cdot||_Y : Y \to [0,\infty)$ the restriction of $||\cdot||$ to Y defined by $||y||_Y = ||y||$, for all $y \in Y$.

2 Spaces Of Continuous Functions

The usual norm on $C([a,b])$ (defined as the space of continuous functions on the interval $[a,b]$) is the supremum (maximum) norm defined as

$$||f||_\infty := \sup_{x\in[a,b]} |f(x)|$$

A second example of a family of norms on this space is, for $p \in [1,\infty)$,

$$||f||_{L^p} := \left(\int_a^b |f(x)|^p dx\right)^{\frac{1}{p}}$$