# Sum Rule For Series

## 1 Introduction

The Sum Rule is a basic rule for the convergence of series. It states that the series $\sum_{n=1}^{\infty}(a_n + b_n)$ converges if $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converges.

It is important to note that the reverse is not true. An example of this can be found at the bottom of the page.

## 2 Proof

Suppose that $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converge to the values $A$ and $B$ respectively.

By the definition of an infinite series as the limit of partial sums,

$$\sum_{n=1}^{\infty}(a_n + b_n) = \lim_{k\to\infty} \sum_{n=1}^{k}(a_n + b_n)$$

Since $\sum_{n=1}^{k}(a_n + b_n)$ is a finite sum, it can be separated into two separate sums,

$$\implies \sum_{n=1}^{\infty}(a_n + b_n) = \lim_{k\to\infty} \left( \sum_{n=1}^{k} a_n + \sum_{n=1}^{k} b_n \right)$$

$$\implies \sum_{n=1}^{\infty}(a_n + b_n) = \lim_{k\to\infty} \sum_{n=1}^{k} a_n + \lim_{k\to\infty} \sum_{n=1}^{k} b_n$$

$$\implies \sum_{n=1}^{\infty}(a_n + b_n) = A + B$$

This means that $\sum_{n=1}^{\infty}(a_n + b_n)$ converges if $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converges, as desired.

## 3 Counter-Example

Consider the two sequences, $a_n = n$ and $b_n = -n$. Clearly, the infinite sum of $a_n$ and $b_n$ both diverge. However, $a_n + b_n = 0$, and thus the series of the sum of the two sequences converges to 0