LAB 7 : Regression Part 2

In this Lab we will look into the shortcomings of Linear Regression and see how those problems can be solved using Logistic Regression. We will also explore Polynomian Regression

1. Polynomial Regression
2. Linear Regression on a specific pattern of data to observe shortcomings
3. Logistic Regression to solve those problems

In [ ]:
```
import numpy as np
import matplotlib.pyplot as plt
```

In [ ]:
```
start_number=-6
end_number=6
number_of_points=1000
weightage_of_noise=1
```

Polynomial Regression

1. Generate data using relation $y = 0.25x^3 + 1.25x^2 - 3x - 3$
2. Corrupt y by adding random noise (uniformly sampled)
3. Fit the generated curve using different polynomial order. (Using matrix inversion and gradient descent)

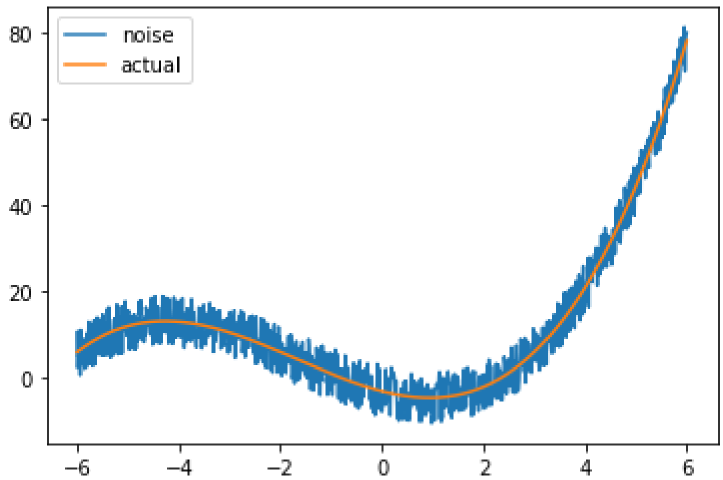
In [ ]:
```
## Use the Regression class defined in the previous lab
from turtle import color


def generate_polynomial_model_matrix_inversion(y,x):
    model=regression()
    weights=model.mat_inv(y=y,x_aug=x)
    weights=np.reshape(weights,newshape=(weights.shape[0],1))
    [y_pred,error]=model.error(w=weights,y=y,x=x)
    return [weights,y_pred,error]


class regression:
  # Constructor
    def __init__(self, name='reg'):
        self.name = name  # Create an instance variable

    def grad_update(self,w_old,lr,y,x):
        w=w_old+(2*lr/y.shape[0])*np.matmul(x,(y-np.matmul(x.T,w_old)))
        return w

    def error(self,w,y,x):
        y=np.reshape(y,newshape=(y.shape[0],1))
        number_of_elements=y.shape[0]
        y_pred=np.matmul(x.T,w)
        if(y_pred.shape[1]!=1):
            print("Incorrect dimesnions of y_pred")
            return -1
```

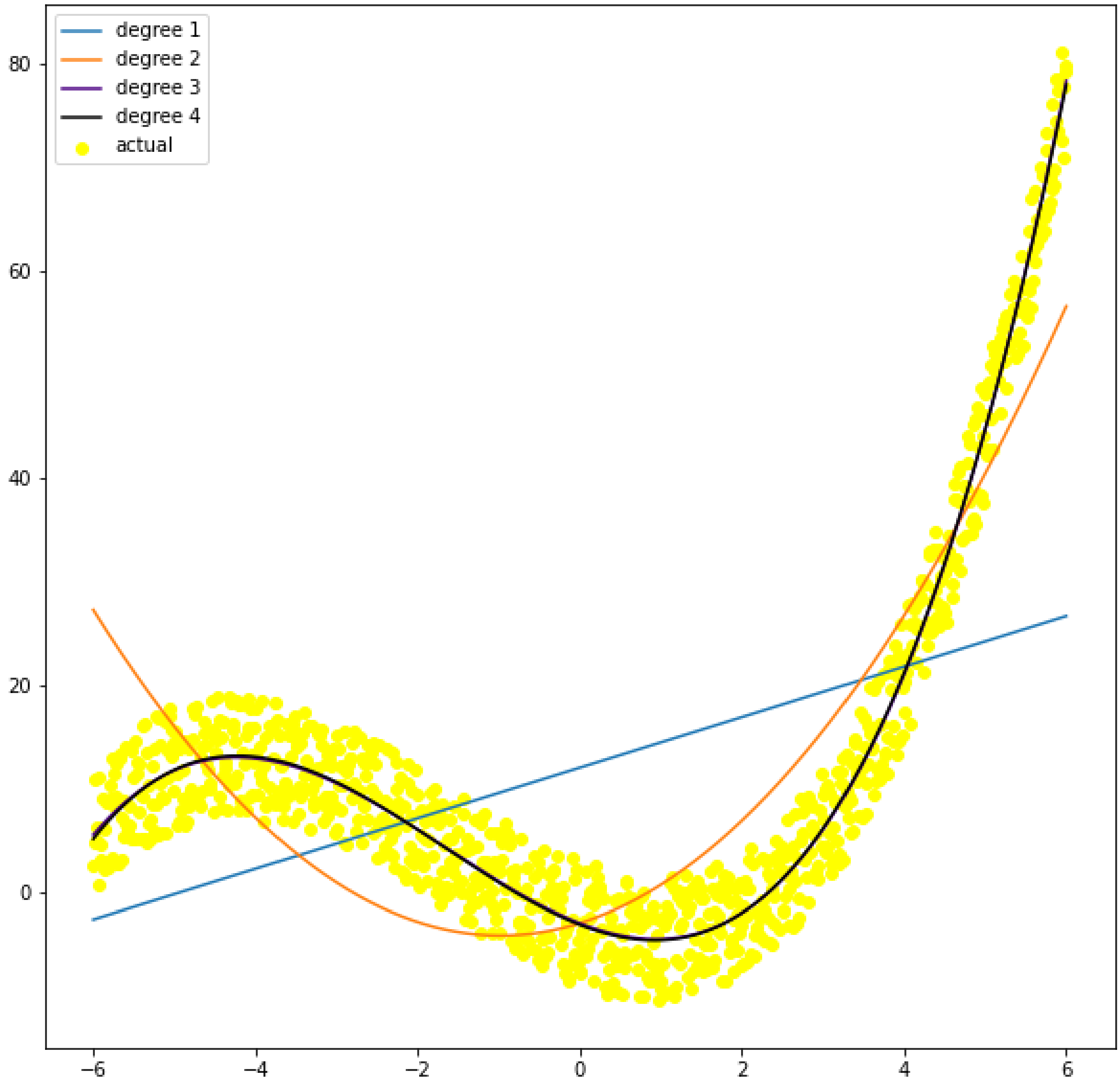
```
        squared_sum_error_matrix=(y-y_pred)**2
        sum_of_squares_of_error=np.sum(squared_sum_error_matrix)
        err=sum_of_squares_of_error**0.5/number_of_elements
        return [y_pred,err]

    def mat_inv(self,y,x_aug):
        weights=np.matmul(np.matmul(np.linalg.pinv(np.matmul(x_aug,x_aug.T)),x_aug
        return weights

    # By Gradien descent
    def Regression_grad_des(self,x,y,lr):
        w_old=np.zeros((x.shape[0],1))
        num_itr=1000
        err=[]
        y=np.reshape(y,newshape=(y.shape[0],1))
        if(y.shape[0]!=x.shape[1]):
            print("Incorrect dimensions of y")
            return -1
        for i in range(num_itr):
            w=self.grad_update(w_old=w_old,lr=lr,y=y,x=x)
            [ypred,error_new]=self.error(w,y,x)
            err.append(error_new)
            [y_pred,error_old]=self.error(w_old,y,x)
            dev=np.abs(error_new-error_old)
            if dev<=10**-7:
                w_old=w
                break
            w_old=w
        return w_old,err
    def Regression_grad_des_with_weights(self,x,y,lr,w_old):

        num_itr=10000
        # err=[]
        y=np.reshape(y,newshape=(y.shape[0],1))
        # if(x.shape[0]==5):
        #     num_itr=1
        if(y.shape[0]!=x.shape[1]):
            print("Incorrect dimensions of y")
            return -1

        for i in range(num_itr):
            w=self.grad_update(w_old=w_old,lr=lr,y=y,x=x)
            [ypred,error_new]=self.error(w,y,x)
            # err.append(error)
            [y_pred,error_old]=self.error(w_old,y,x)
            dev=np.abs(error_new-error_old)
            # print(error_new)
            if dev<=10**-5:
                w_old=w
                break
            w_old=w
        return w_old,error_new
```

In [ ]:

```
x=np.linspace(start=start_number,stop=end_number,num=number_of_points)
y=0.25*x**3+1.25*x**2-3*x-3
noise=np.random.uniform(low=start_number,high=end_number,size=number_of_points)
y_cor=y+weightage_of_noise*noise
plt.plot(x,y_cor,label="noise")
plt.plot(x,y,label="actual")
plt.legend()
plt.show()
x=np.reshape(x,(1,1000))
```

```
#For degree 1
x_vector_for_degree_1_mv=np.r_[np.ones(shape=(1,number_of_points)),x]
[weights_for_degree_1,y_pred_for_degree_1,error_for_degree_1]=generate_polynomial_r


#For degree 2
x_vector_for_degree_2=np.r_[np.ones(shape=(1,number_of_points)),x,x**2]
[weights_for_degree_2,y_pred_for_degree_2,error_for_degree_2]=generate_polynomial_r

#For Degree 3
x_vector_for_degree_3=np.r_[np.ones(shape=(1,number_of_points)),x,x**2,x**3]
[weights_for_degree_3,y_pred_for_degree_3,error_for_degree_3]=generate_polynomial_r

#For degreee 4
x_vector_for_degree_4=np.r_[np.ones(shape=(1,number_of_points)),x,x**2,x**3,x**4]
[weights_for_degree_4,y_pred_for_degree_4,error_for_degree_4]=generate_polynomial_r

weights_of_all_degrees=[weights_for_degree_1,weights_for_degree_2,weights_for_degre
x=np.reshape(x,(1000,1))
fig=plt.figure(figsize=(10,10))
plt.scatter(x,y_cor,label="actual",color="yellow")
plt.plot(x,y_pred_for_degree_1,label="degree 1")
plt.plot(x,y_pred_for_degree_2,label="degree 2")
plt.plot(x,y_pred_for_degree_3,label="degree 3",color='indigo')
plt.plot(x,y_pred_for_degree_4,label="degree 4",color="black")
plt.legend()
plt.show()

print("For degree 1")
print("Optimal Weights are\n",weights_for_degree_1)
print("Error at optimal weight is:- ",error_for_degree_1)

print("\n\nFor degree 2")
print("Optimal Weights are\n",weights_for_degree_2)
print("Error at optimal weight is:- ",error_for_degree_2)

print("\n\nFor degree 3")
print("Optimal Weights are\n",weights_for_degree_3)
print("Error at optimal weight is:- ",error_for_degree_3)

print("\n\nFor degree 4")
print("Optimal Weights are\n",weights_for_degree_4)
print("Error at optimal weight is:- ",error_for_degree_4)
```





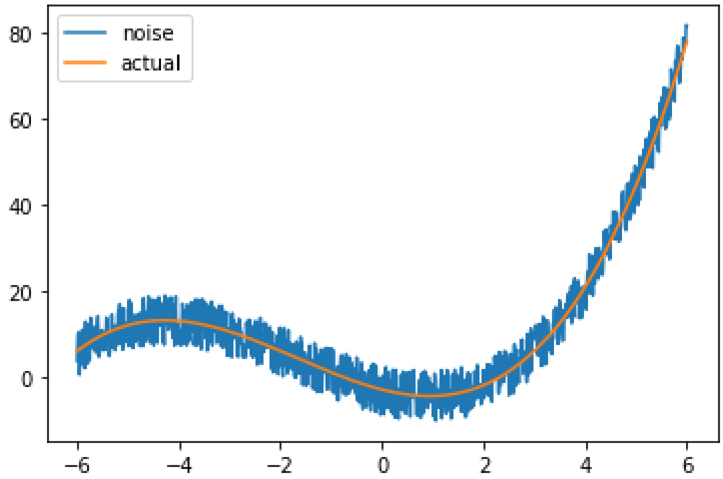
degree 1
degree 2
degree 3
degree 4
actual
80
60
40
20
0
−6
−4
−2
0
2
4
6

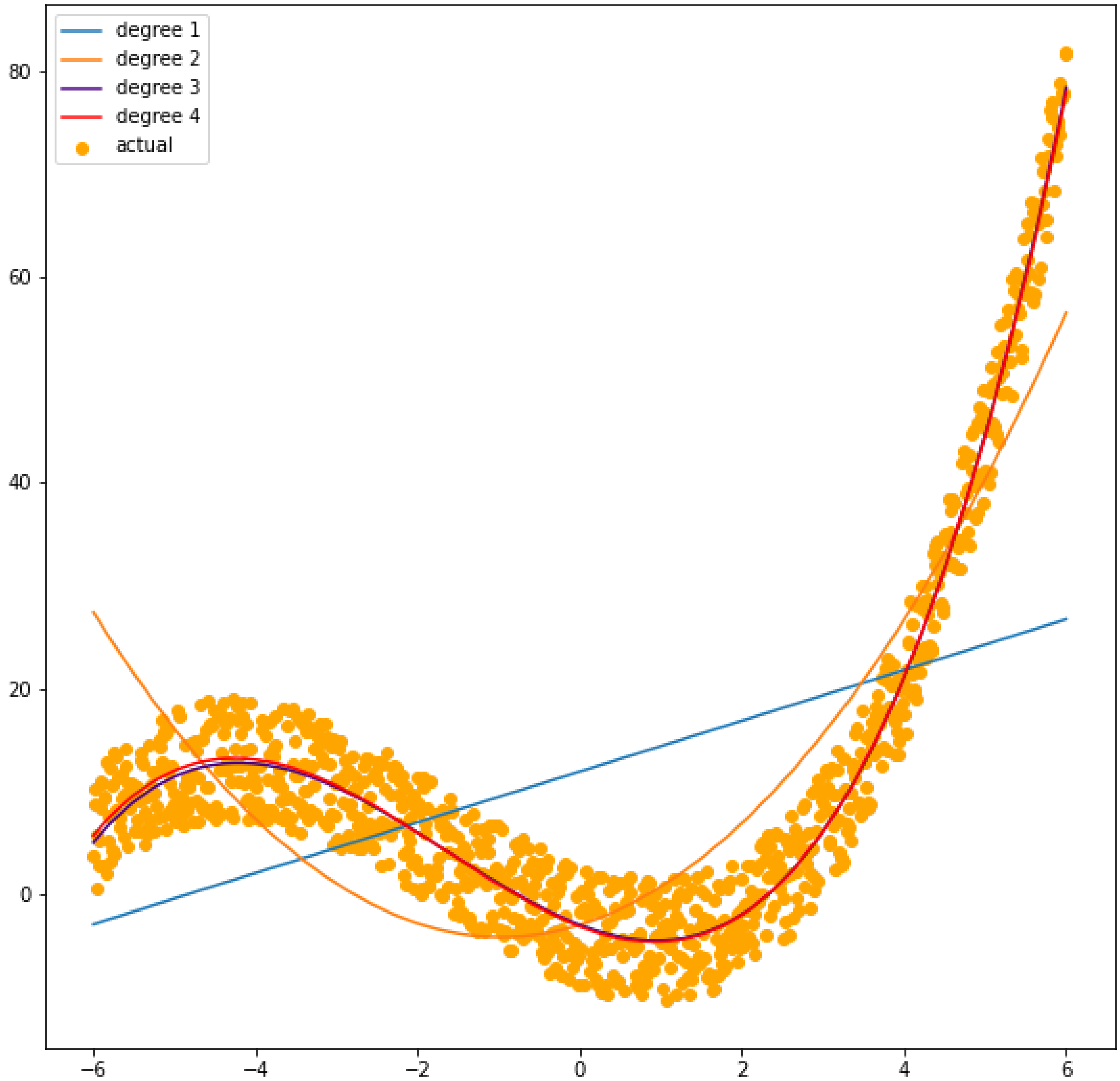
```
For degree 1
Optimal Weights are
 [[12.02637345]
 [ 2.44352656]]
Error at optimal weight is:-  0.5105606074778258


For degree 2
Optimal Weights are
 [[-2.98147932]
 [ 2.44352656]
 [ 1.24815559]]
Error at optimal weight is:-  0.2836973012238278


For degree 3
Optimal Weights are
 [[-2.98147932]
 [-3.02298638]
 [ 1.24815559]
 [ 0.25257399]]
Error at optimal weight is:-  0.10974755333498405


For degree 4
Optimal Weights are
 [[-3.13311122e+00]
 [-3.02298638e+00]
 [ 1.29019160e+00]
 [ 2.52573987e-01]
 [-1.35956093e-03]]
Error at optimal weight is:-  0.10966475705830644
```

In [ ]:

```
# By Gradient Descent
lambdas=[0.01,0.001,0.0001,0.00000075]
def generate_polynomial_model_gradient_descent(y,x,degree):
    model=regression()
    weights,error=model.Regression_grad_des_with_weights(x=x,y=y,lr=lambdas[degree
    y_pred=x.T @ weights
    return [weights,y_pred,error]


## Write your code here
x=np.linspace(start=start_number,stop=end_number,num=number_of_points)
y=0.25*x**3+1.25*x**2-3*x-3
noise=np.random.uniform(low=start_number,high=end_number,size=number_of_points)
y_cor=y+weightage_of_noise*noise
plt.plot(x,y_cor,label="noise")
plt.plot(x,y,label="actual")
plt.legend()
plt.show()
x=np.reshape(x,(1,1000))


#For degree 1
x_vector_for_degree_1=np.r_[np.ones(shape=(1,number_of_points)),x]
[weights_for_degree_1,y_pred_for_degree_1,error_for_degree_1]=generate_polynomial_m


#For degree 2
x_vector_for_degree_2=np.r_[np.ones(shape=(1,number_of_points)),x,x**2]
[weights_for_degree_2,y_pred_for_degree_2,error_for_degree_2]=generate_polynomial_m

#For Degree 3
```

```
x_vector_for_degree_3=np.r_[np.ones(shape=(1,number_of_points)),x,x**2,x**3]
[weights_for_degree_3,y_pred_for_degree_3,error_for_degree_3]=generate_polynomial_r

# #For degreee 4
x_vector_for_degree_4=np.r_[np.ones(shape=(1,number_of_points)),x,x**2,x**3,x**4]
[weights_for_degree_4,y_pred_for_degree_4,error_for_degree_4]=generate_polynomial_r

x=np.reshape(x,(1000,1))
fig=plt.figure(figsize=(10,10))
plt.scatter(x,y_cor,label="actual",color="orange")
plt.plot(x,y_pred_for_degree_1,label="degree 1")
plt.plot(x,y_pred_for_degree_2,label="degree 2")
plt.plot(x,y_pred_for_degree_3,label="degree 3",color='indigo')
plt.plot(x,y_pred_for_degree_4,label="degree 4",color="red")
plt.legend()
plt.show()

print("For degree 1")
print("Optimal Weights are\n",weights_for_degree_1)
print("Error at optimal weight is:- ",error_for_degree_1)

print("\n\nFor degree 2")
print("Optimal Weights are\n",weights_for_degree_2)
print("Error at optimal weight is:- ",error_for_degree_2)

print("\n\nFor degree 3")
print("Optimal Weights are\n",weights_for_degree_3)
print("Error at optimal weight is:- ",error_for_degree_3)

print("\n\nFor degree 4")
print("Optimal Weights are\n",weights_for_degree_4)
print("Error at optimal weight is:- ",error_for_degree_4)
```




degree 1
degree 2
degree 3
degree 4
actual
80
60
40
20
0
−6
−4
−2
0
2
4
6

```
For degree 1
Optimal Weights are
 [[11.91806822]
 [ 2.46976818]]
Error at optimal weight is:-  0.5098152264155261


For degree 2
Optimal Weights are
 [[-2.95116219]
 [ 2.42444008]
 [ 1.24771728]]
Error at optimal weight is:-  0.2813887793752614


For degree 3
Optimal Weights are
 [[-2.95144163]
 [-2.99236917]
 [ 1.24084701]
 [ 0.2529559 ]]
Error at optimal weight is:-  0.1102700204762468


For degree 4
Optimal Weights are
 [[-3.10177941e+00]
 [-2.99274847e+00]
 [ 1.27730379e+00]
 [ 2.50220176e-01]
 [-8.31971261e-04]]
Error at optimal weight is:-  0.11050322863147566
```

Linear Regression

Generate the data as shown in the figure below

In [ ]:

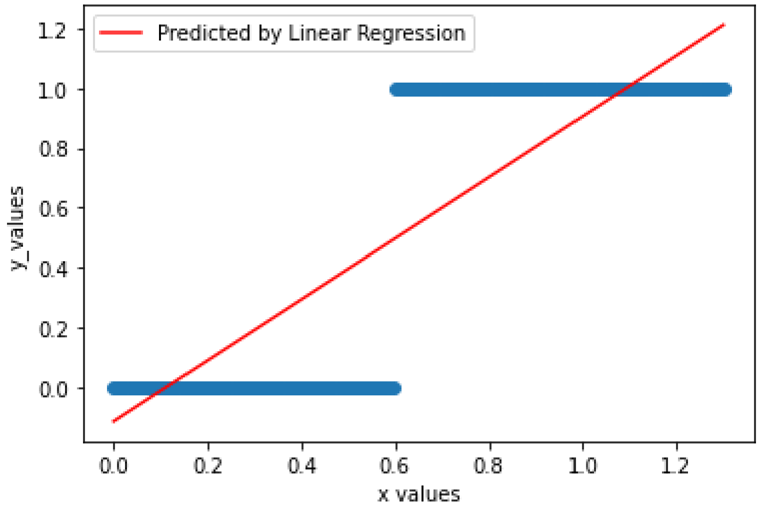
```
start_number=0
end_number=1.3
number_of_points=200
```

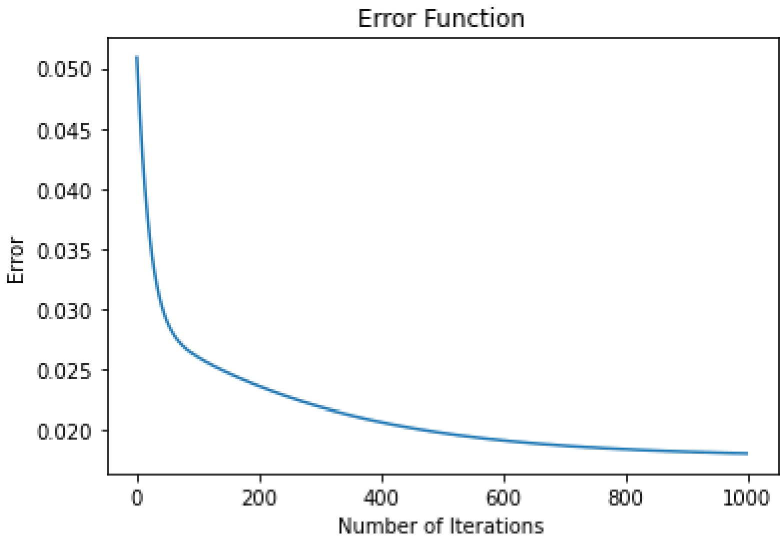
In [ ]:

```
## Write your code here
x=np.linspace(start=start_number,stop=end_number,num=number_of_points)
y=x
pos=np.where(y<0.6)
neg=np.where(y>=0.6)
y=np.array(y,dtype=int)
y[pos]=0
y[neg]=1
plt.scatter(x,y)
#For degree 1
x=np.reshape(x,newshape=(1,number_of_points))
x_vector_for_degree_1=np.r_[np.ones(shape=(1,number_of_points)),x]
reg=regression()
weights,err=reg.Regression_grad_des(x=x_vector_for_degree_1,y=y,lr=0.01)
x=np.reshape(x,newshape=(number_of_points,1))
print("optimal Weights:-\n",weights)
print("Error at Optimal Weights:-\n",err[-1])
y_pred=x_vector_for_degree_1.T @ weights
plt.xlabel("x values")
plt.ylabel("y_values")
```

```
plt.plot(x,y_pred,color="red",label="Predicted by Linear Regression")
plt.legend()
plt.show()
plt.xlabel("Number of Iterations")
plt.ylabel("Error")
plt.title("Error Function")
plt.plot(err)
plt.show()
```

```
optimal Weights:-
 [[-0.11389404]
 [ 1.01905114]]
Error at Optimal Weights:-
 0.01809237658323208
```





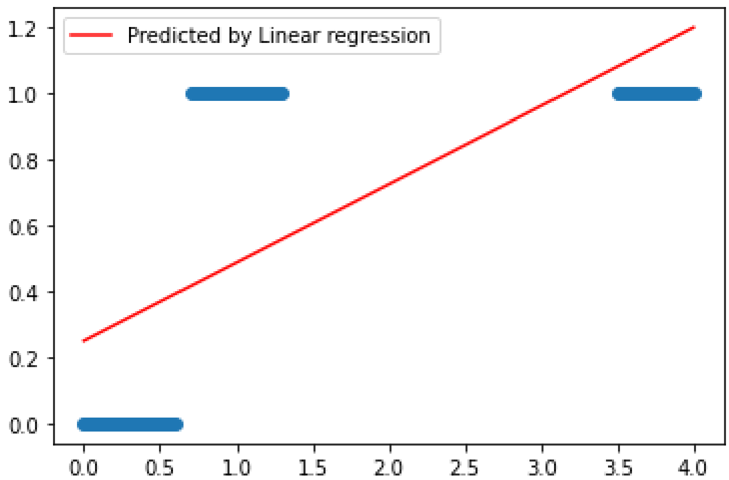
Drawback of Linear regression based Classificaton

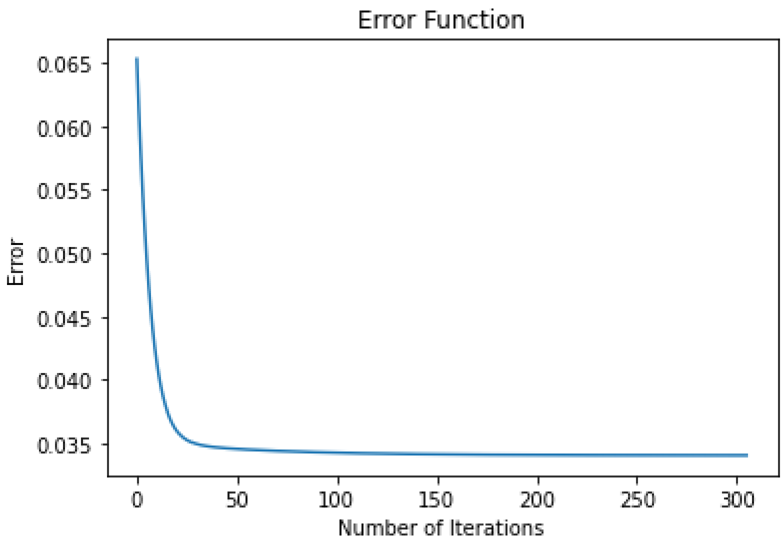
Generate the Data as shown in the figure and follow the same steps as above to fit a curve using regression class

In [ ]:
```
x1=np.linspace(start=0,stop=0.6,num=50)
y1=[0]*x1.shape[0]
x2=np.linspace(start=0.7,stop=1.3,num=40)
y2=[1]*x2.shape[0]
x3=np.linspace(start=3.5,stop=4,num=29)
y3=[1]*x3.shape[0]
```

```
x_total=np.concatenate((x1,x2,x3))
y_total=np.concatenate((y1,y2,y3))
plt.scatter(x_total,y_total)

x_total=np.reshape(x_total,newshape=(1,x_total.shape[0]))
x_vector_for_degree_1=np.r_[np.ones(shape=(1,x_total.shape[1])),x_total]
reg=regression()
weights,err=reg.Regression_grad_des(x=x_vector_for_degree_1,y=y_total,lr=0.01)
x_total=np.reshape(x_total,newshape=((x_total.shape[1]),1))
y_pred=x_vector_for_degree_1.T @ weights
print("Optimal Weights\n",weights)
print("Error at Optimal Weights",err[-1])

plt.plot(x_total,y_pred,color="red",label="Predicted by Linear regression")
plt.legend()
plt.show()
plt.xlabel("Number of Iterations")
plt.ylabel("Error")
plt.title("Error Function")
plt.plot(err)
plt.show()
```

```
Optimal Weights
 [[0.24924337]
 [0.23725208]]
Error at Optimal Weights 0.03406838578675097
```



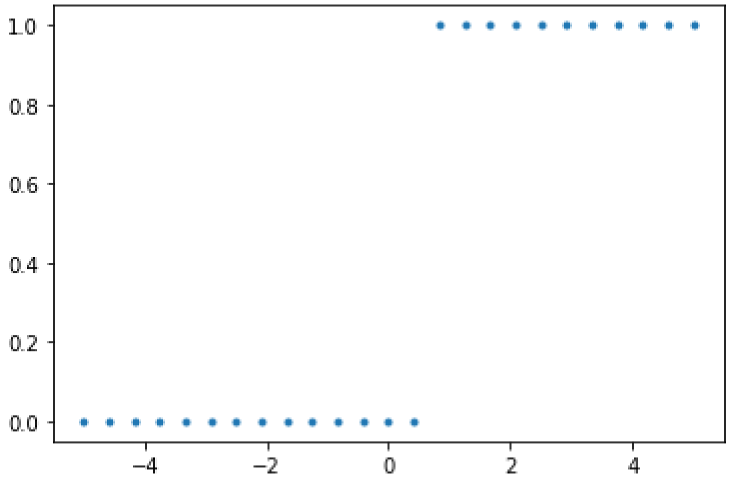


Logistic regression

Error Surface (Comparison between Logistic Loss and Mean Squared Error)

In [ ]:
```
import numpy as np
import matplotlib.pyplot as plt


x=np.linspace(-5,5,25)
y=np.zeros(x.shape)
y[np.where(x>0.7314)]=1

plt.plot(x,y,'.')
```

Out[ ]:
```
[<matplotlib.lines.Line2D at 0x1efb68da670>]
```

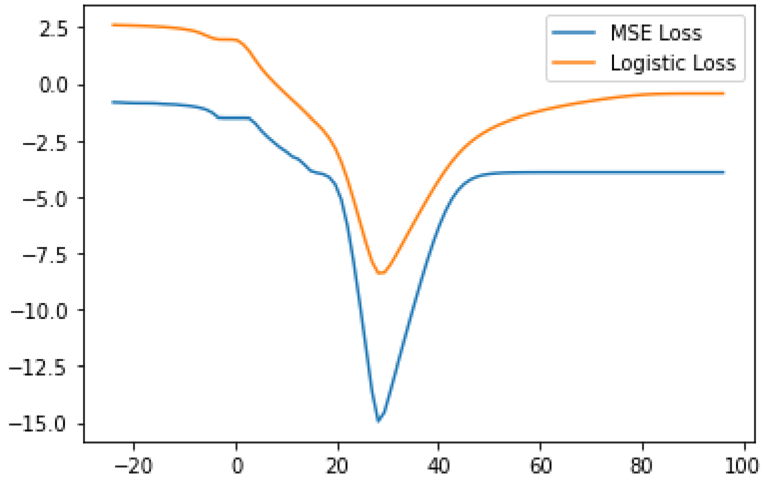


1. MSE= $\frac{1}{2N}\sum_{i=1}^{N}(y_i^p - y_i)^2$, where $y^p = \frac{1}{1+e^{-w^T x}}$
2. Logistic loss= $-\frac{1}{N}\sum_{i=1}^{N} y_i log(y_i^p) + (1-y_i)log(1-y_i^p)$

In [ ]:
```
def sigmoid(x):
  return 1/(1+np.exp(-x))

# search space (only w1 is searched, where as w0 is fixed)
w1_in=10/(x[1]-x[0])
w0=-w1_in*0.7314
w1=np.linspace(-w1_in,4*w1_in,100)


cost_fn_mse=[]
cost_fn_logis=[]
for i in range(w1.shape[0]):
  cost_fn_mse.append(np.sum((y-sigmoid(w0+w1[i]*x))**2)/(2*x.shape[0]))
  y_pred=sigmoid(w0+w1[i]*x)
  cost_fn_logis.append(np.sum((y*np.log(y_pred+0.0000001)+(1-y)*np.log(1-y_pred+0.(
```

In [ ]:
```
# Ploting of error surface
plt.figure()
plt.plot(w1,np.log(cost_fn_mse),label='MSE Loss')
plt.plot(w1,np.log(cost_fn_logis),label = 'Logistic Loss')
plt.legend()
```

Out[ ]:
```
<matplotlib.legend.Legend at 0x1efb6947370>
```

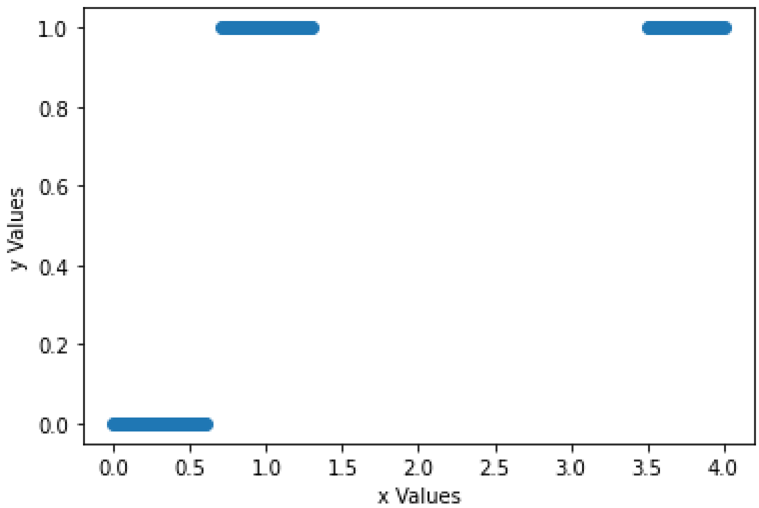


Solving the Outlier Issue

Generate the Data as shown in the figure

In [ ]:
```
x1=np.linspace(start=0,stop=0.6,num=50)
y1=[0]*x1.shape[0]
x2=np.linspace(start=0.7,stop=1.3,num=40)
y2=[1]*x2.shape[0]
x3=np.linspace(start=3.5,stop=4,num=29)
y3=[1]*x3.shape[0]
x_total=np.concatenate((x1,x2,x3))
y_total=np.concatenate((y1,y2,y3))
plt.xlabel("x Values")
plt.ylabel("y Values")
plt.scatter(x_total,y_total)
```

Out[ ]: <matplotlib.collections.PathCollection at 0x1efb69bdf40>



Define a Logistic Regression class

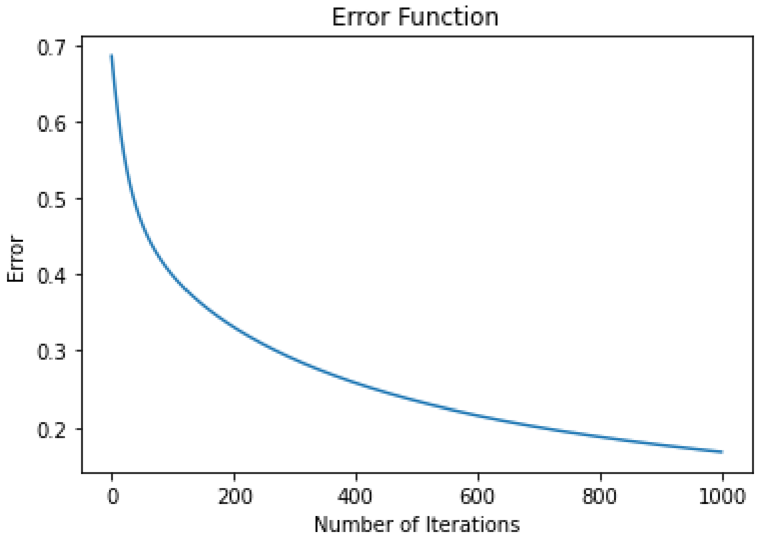
In [ ]:
```
class logis_regression:
  # Constructor
  def __init__(self, name='reg'):
    self.name = name  # Create an instance variable

  def logis(self,x,w_old):
    # write code here
    return 1/(1+np.exp(-x.T@w_old))
```

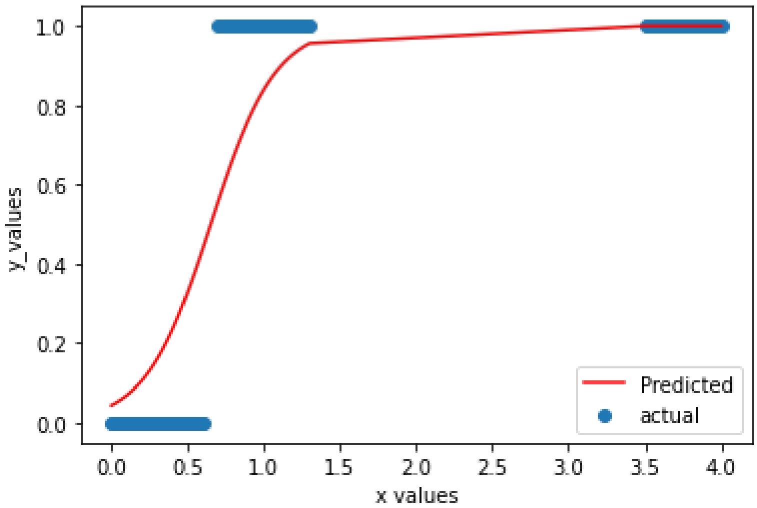
```
  def grad_update(self,w_old,lr,y,x):

    y=y.reshape((y.shape[0],1))
    w=w_old+(lr/x.shape[1])*(x@(y-self.logis(x,w_old)))
    return w

  def error(self,w,y,x):
    y=y.reshape((y.shape[0],1))
    y_pred=self.logis(x,w)
    return -np.sum((y*np.log(y_pred+0.0000001)+(1-y)*np.log(1-y_pred+0.0000001))/y
     # write code here

  def Regression_grad_des(self,x,y,lr):
    w_old=np.ones((x.shape[0],1))
    err=[]
    # print(w_old.shape)
    for i in range(1000):
      # write code here
      w_new=self.grad_update(w_old,lr,y,x)
      # print(str(i)+" iteration -",w_new,self.error(w_new,y,x))
      # print("/n")
      err.append(self.error(w_new,y,x))
      dev=np.abs(self.error(w_new,y,x)-self.error(w_old,y,x))
      if dev<=10**(-20):
        w_old=w_new
        break
      w_old=w_new

    return w_old,err
```

Augment the data and fit the curve by obtaining optimal weights (Using Gradient Descent)

In [ ]:
```
x_total=np.reshape(x_total,newshape=(1,x_total.shape[0]))
x_aug=np.r_[np.ones((1,x_total.shape[1])),x_total]
print(x_aug.shape)
reg=logis_regression()
weights,err = reg.Regression_grad_des(x_aug,y_total,0.1)
print("Optimal Weights\n",weights)
print("Error at Optimal Weights",err[-1])
plt.xlabel("Number of Iterations")
plt.ylabel("Error")
plt.title("Error Function")
plt.plot(err)
plt.show()

plt.plot(x_total.T,sigmoid(x_aug.T@weights),label="Predicted",color="red")
plt.xlabel("x values")
plt.ylabel("y_values")
plt.scatter(x_total,y_total,label="actual")
plt.legend()
plt.show()
```

```
(2, 119)
Optimal Weights
 [[-3.08150721]
 [ 4.74466168]]
Error at Optimal Weights 0.16802572159243115
```

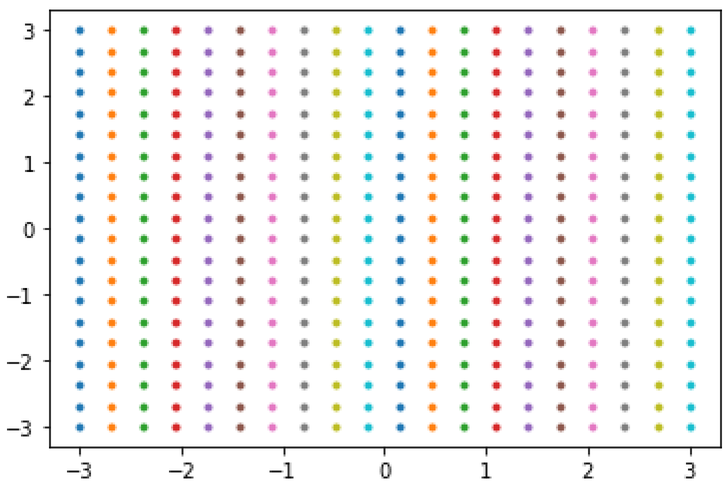




Classification of circularly separated data using logistic regression

In [ ]:
```
x1=np.linspace(-3,3,20)
x2=np.linspace(-3,3,20)

x11,x22=np.meshgrid(x1,x2)
plt.plot(x11,x22,'.')
print(x11.shape)
```

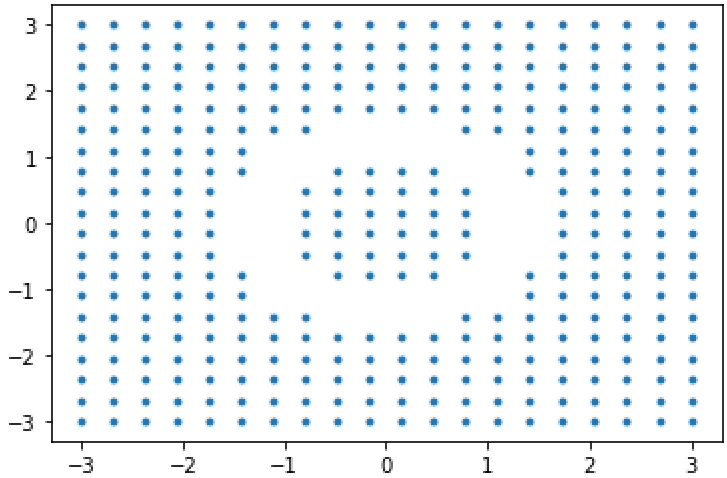
```
(20, 20)
```



Using the above data generate circular data

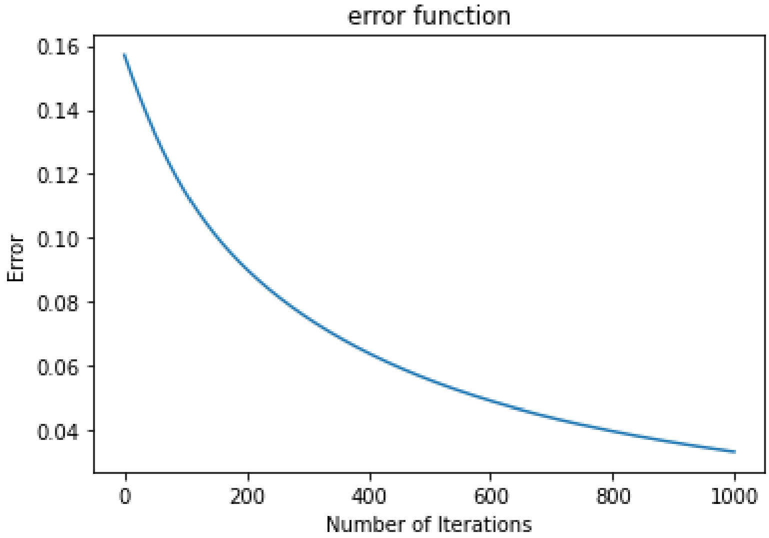
In [ ]:
```
# Write code here
x1 = x11.flatten()
x1 = x1.reshape((x1.shape[0],1))
x2 = x22.flatten()
x2 = x2.reshape((x2.shape[0],1))
x = np.c_[x1, x2] # create all ordered pairs
print(x.shape)
in_pt_ind = np.where((x[:,0]**2 + x[:,1]**2) <= 1)
out_pt_ind = np.where((x[:,0]**2 + x[:,1]**2) >= 2.5)
# print(in_pt_ind[0])
x_in = x[in_pt_ind[0], :]
x_out = x[out_pt_ind[0], :]

x = np.r_[x_in,x_out] # our dataset

plt.figure()
plt.plot(x[:,0], x[:,1], '.')
plt.show()
```

```
(400, 2)
```



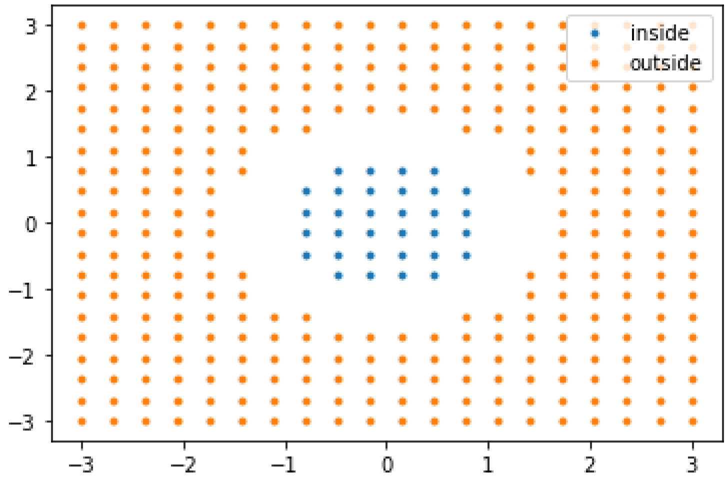
As in case of circularly separated data, the boundary is nonlinear, so squared feature is taken.

In [ ]:
```
# perform logistic regression
y_in = np.zeros((x_in.shape[0]))
# print(y_in)
y_out = np.ones((x_out.shape[0]))
# print(y_out)
y = np.r_[y_in,y_out]
y_new=y.reshape((y.shape[0],1))

x_sq = (x.T)**2
x_aug = np.r_[np.ones((1, x.shape[0])),x_sq]

reg = logis_regression()
lr = 0.1
w_pred,err = reg.Regression_grad_des(x_aug, y_new, lr)
plt.figure()
plt.plot(err)
plt.xlabel("Number of Iterations")
plt.ylabel("Error")
plt.title("error function")
plt.show()
```



Plot classification using 0.5 as threshold

In [ ]:
```
#write code here
y_pred = reg.logis(x_aug,w_pred)

in_pred, _ = np.where(y_pred < 0.5)
out_pred, _ = np.where(y_pred >= 0.5)

x_in_pred = x[in_pred,:]
x_out_pred = x[out_pred,:]

plt.figure()
plt.plot(x_in_pred[:,0], x_in_pred[:,1], '.',label="inside")
plt.plot(x_out_pred[:,0], x_out_pred[:,1], '.',label="outside")
plt.legend()
plt.show()
```



Multiclass logistic regression

1. Generate 1D data with 3 classes

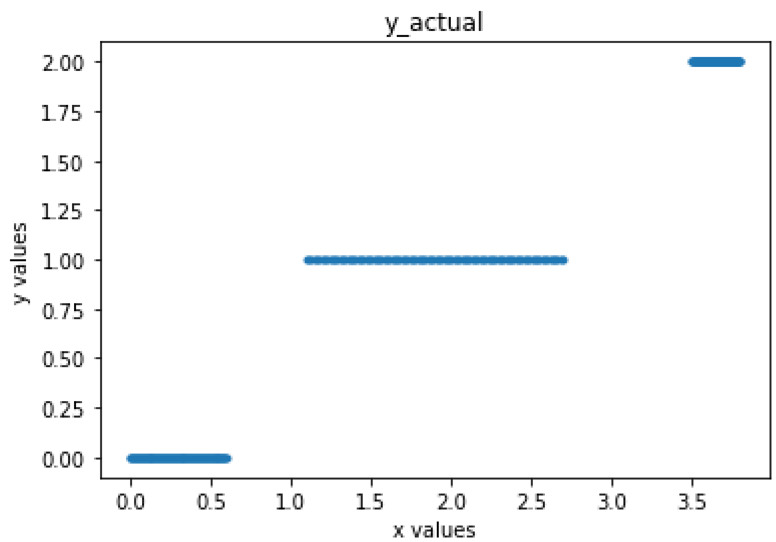
One vs rest classification

1. Lets take a polynomial of order 2 (by seeing the data distribution)

In [ ]:
```
## Write your code here

import numpy as np
import matplotlib.pyplot as plt

x1=np.linspace(0,0.6,100)
x2=np.linspace(1.1,2.7,100)
x3=np.linspace(3.5,3.8,100)

x=np.concatenate((x1,x2,x3))
print(x.shape)

y1=np.zeros(x1.shape)
y2=np.ones(x2.shape)
y3=np.tile([2],x3.shape)

y=np.concatenate((y1,y2,y3))

plt.figure()
plt.plot(x,y,'.')
plt.xlabel("x values")
plt.ylabel("y values")
plt.title("y_actual")
```

```
(300,)
```

Out[ ]:
```
Text(0.5, 1.0, 'y_actual')
```



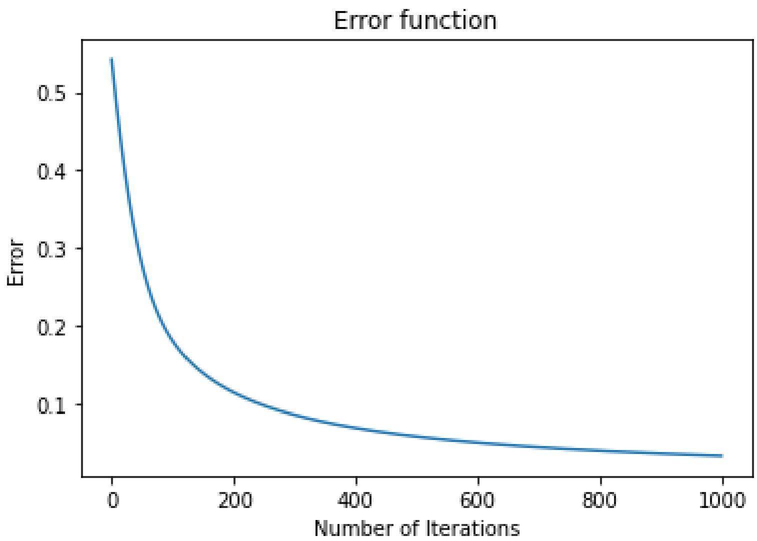
In [ ]:
```
# def data_transform(X,degree):
#   X_new=[]
#   for i in range(degree +1):
#     # write code here to generate a polynomial

def data_transform(X,degree):
  X_new=[]
  for i in range(degree +1):
    X_new.append(X**i)
  X_new = np.concatenate(X_new)
  return X_new

x_aug=data_transform(x[np.newaxis,:],2)
```

In [ ]:
```
# plot for classification
from turtle import title

def plot_op(x,y_pred):
```

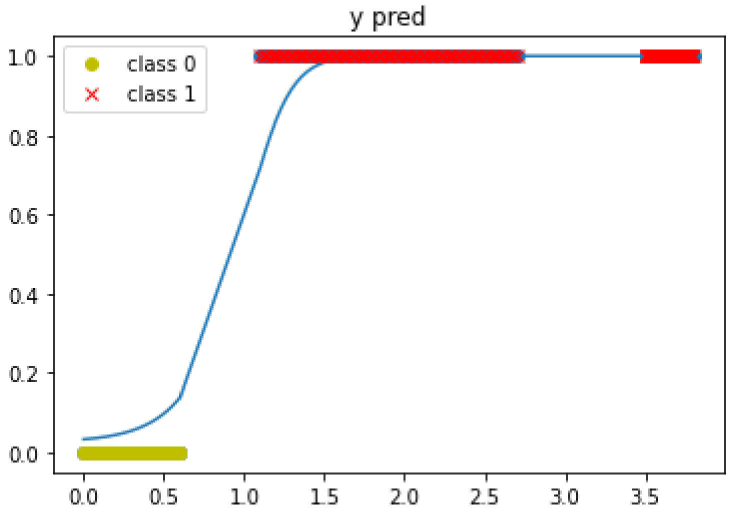
```
    ind0,_=np.where(y_pred<0.5)
    ind1,_=np.where(y_pred>=0.5)
    x0=x[ind0]
    x1=x[ind1]
    plt.plot(x0,np.zeros((x0).shape),'o',color='y',label="class 0")
    plt.plot(x1,np.ones((x1).shape),'x',color='r',label="class 1")
    plt.legend()
```

Using the above function for plotting, plot the curve using different configurations

In [ ]:
```
# take class 0 as '0' and other to '1'
## Write your code here
y_copy=np.array(y!=0,dtype=int)
reg=logis_regression()
weights,err = reg.Regression_grad_des(x_aug,y_copy,0.1)

plt.plot(err)
plt.xlabel("Number of Iterations")
plt.ylabel("Error")
plt.title("Error function")
plt.show()
y_pred0=sigmoid(x_aug.T@weights)
plt.scatter(x,y_copy)
plt.plot(x,y_pred0)
plot_op(x=x,y_pred=y_pred0.reshape(y_pred0.shape[0],1))
plt.title("y pred")
print("Optimal Weights\n",weights)
print("Error at Optimal Weights",err[-1])
```

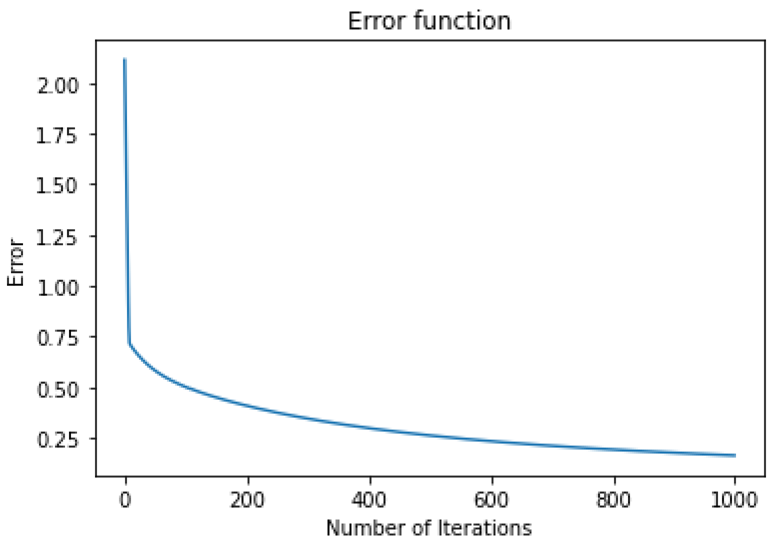



```
Optimal Weights
 [[-3.37710198]
 [ 0.95274342]
 [ 2.70116121]]
Error at Optimal Weights 0.03273105792625165
```

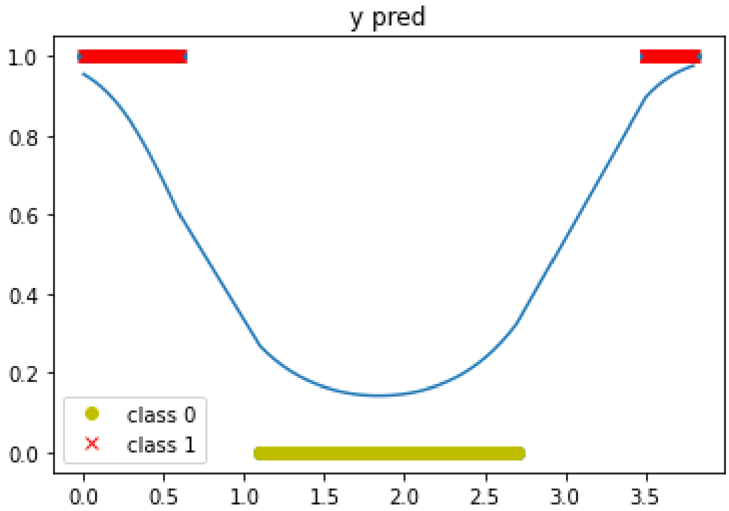


In [ ]:

```
# take class 1 as '0' and other to '1'
## Write your code here
y_copy=np.array(y!=1,dtype=int)
# print(y_copy)
weights,err = reg.Regression_grad_des(x_aug,y_copy,0.1)
plt.plot(err)
plt.xlabel("Number of Iterations")
plt.ylabel("Error")
plt.title("Error function")
plt.show()
y_pred1=sigmoid(x_aug.T@weights)
plt.scatter(x,y_copy)
plt.plot(x,y_pred1)
plot_op(x=x,y_pred=y_pred1.reshape(y_pred1.shape[0],1))
plt.title("y pred")
print("Optimal Weights\n",weights)
print("Error at Optimal Weights",err[-1])
```

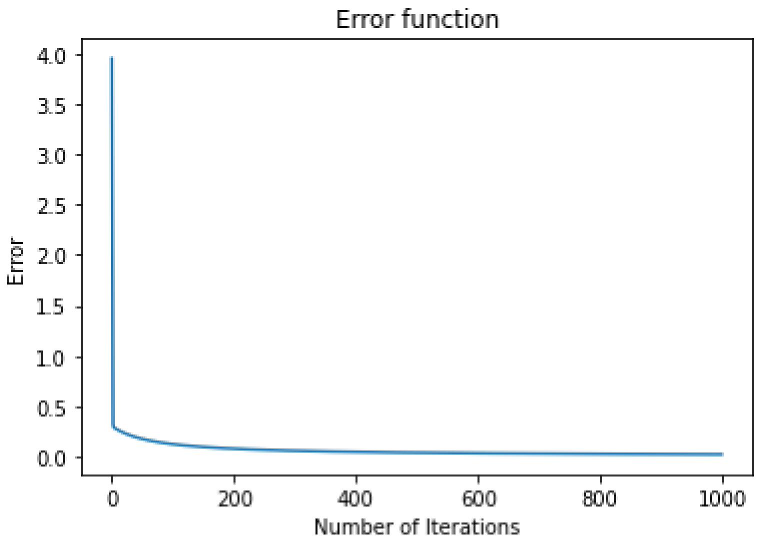



```
Optimal Weights
 [[ 3.03156849]
 [-5.24177546]
 [ 1.424629  ]]
Error at Optimal Weights 0.16359283007696068
```

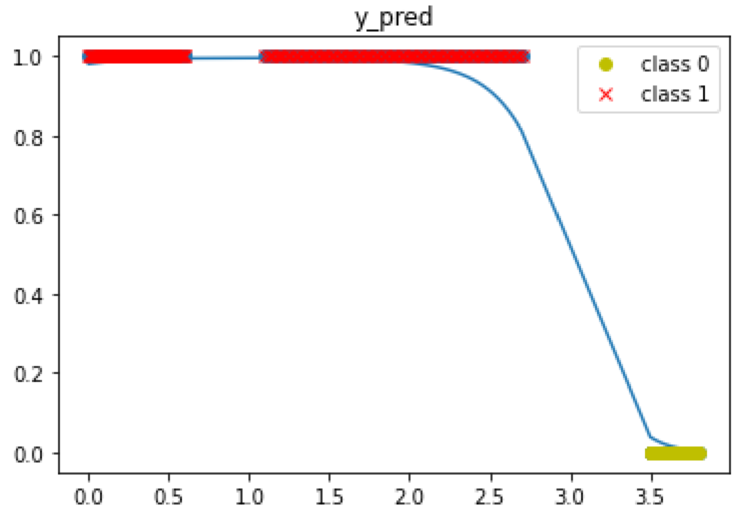


In [ ]:

```
# Take class 2 as '0' and other to '1'
## Write your code here
y_copy=np.array(y!=2,dtype=int)
weights,err = reg.Regression_grad_des(x_aug,y_copy,0.1)
plt.plot(err)
plt.xlabel("Number of Iterations")
plt.ylabel("Error")
plt.title("Error function")
plt.show()
y_pred2=sigmoid(x_aug.T@weights)
plt.scatter(x,y_copy)
plt.plot(x,y_pred2)
plot_op(x=x,y_pred=y_pred2.reshape(y_pred2.shape[0],1))
plt.title("y_pred")
print("Optimal Weights\n",weights)
print("Error at Optimal Weights",err[-1])
```




```
Optimal Weights
 [[ 3.88557735]
 [ 2.90894296]
 [-1.41013816]]
Error at Optimal Weights 0.021116425656068975
```



In [ ]:
```
# final classification
## Write your code here
index = np.where(y_pred0<0.5)[0]
x_0 = x[index]
index = np.where(y_pred1<0.5)[0]
x_1 = x[index]
index = np.where(y_pred2<0.5)[0]
x_2 = x[index]

plt.figure()
plt.plot(x_0, np.zeros(x_0.shape), '.',label="class 0")
plt.plot(x_1, np.ones(x_1.shape), '.',label="class 1")
plt.plot(x_2, np.tile([2], x_2.shape), '.',label="class 2")
plt.legend()
plt.show()
```