<Lecture 4>

Q. How is concatenation different from cartesian product?

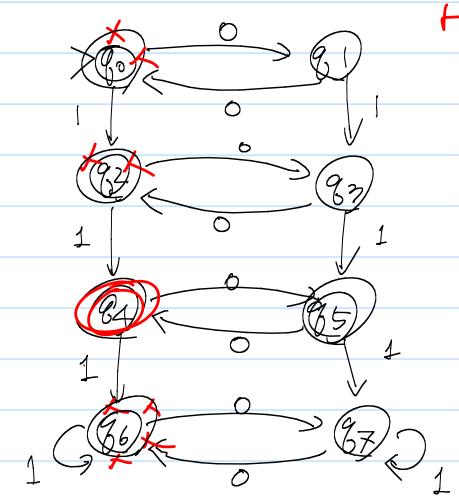
$A \times B = \{(x, y) \mid x \in A,\ y \in B\}$

$A \circ B = \{xy \mid x \in A \text{ and } y \in B\}$

Is the set of regular languages closed under intersection?

e.g. $\{w \mid w$ contains an even # of 0s ~~or~~ and contains exactly two 1s$\}$

How to modify?



Recall

Thm: The set of regular languages is closed under regular operations.

We just proved the case of $\cup$.

· What about $\circ$ or $*$?

Can we use the same "simulation" technique for $\cup$?

With $\circ$, say we break the input string w into $w_1 w_2$ and run M_1 on w_1 and then run M_2 on w_2?

Thm 1.26 For any regular language A and B, $A \circ B$ is regular.

Can we use the same "simulation" technique as with $\cup$?

Recall that

Given M_1 that recognizes A and M_2 that recognizes B, we need to construct M that recognizes $A \circ B$

say we break the input string w into $w_1 w_2$ and run M_1 on w_1 and then run M_2 on w_2?

However, this is possible:

Given M_1 that recognizes A and M_2 that recognizes B, we need to construct DFA M that recognizes $A \circ B$

But the construction is non-trivial!

Instead, we refer to Nondeterministic Finite Automata (NFA).

Thus our strategy is

- construct NFA M that recognizes $A \circ B$
- then, turn NFA M into equivalent DFA M'.

How NFA differs from DFA?

Def 1.37

A ~~DFA~~ NFA is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

- Q is a finite set called the "states"
- Σ is a finite set called the "alphabet"
- δ: ~~$Q \times \Sigma \to Q$~~ $Q \times (\Sigma \cup \{\varepsilon\}) \to \mathcal{P}(Q)$ is the "transition function"
- $q_0 \in Q$ is the start state
- $F \subseteq Q$ is the set of accept states

Q. What does new δ mean?

Power set

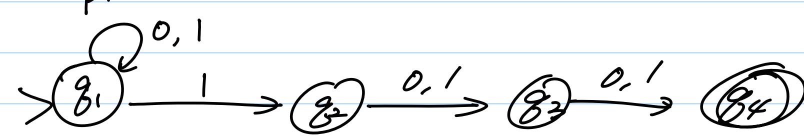
Def: Given a set A, the "power set" of A is the set of all subsets of A. (Denoted by $\mathcal{P}(A)$)

ex) $A = \{a, b, c\}$

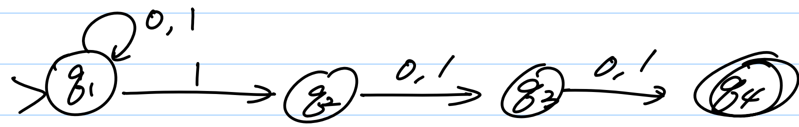
$\mathcal{P}(A) = ?$

$\{\phi, \{a\}, \{b\}, \{c\},$
$\{a, b\}, \{b, c\}, \{a, c\},$
$\{a, b, c\}\}$

Example:



$\delta: Q \times (\Sigma \cup \{\varepsilon\}) \to \mathcal{P}(Q)$



1. $Q = \{q_1, q_2, q_3, q_4\}$
2. $\Sigma = \{0, 1\}$
3. $\delta(q_1, 0) = \{q_1\}$ $\qquad$ $\delta(q_2, 0) = \{q_3\}$

   $\delta(q_1, 1) = \{q_1, q_2\}$ $\qquad$ $\delta(q_2, 1) = \{q_3\}$

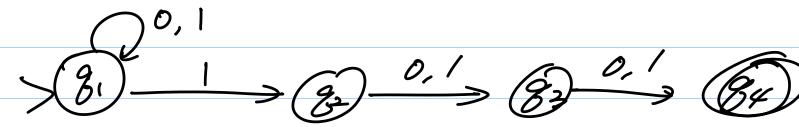
   $\delta(q_1, \varepsilon) = \emptyset$ $\qquad$ $\delta(q_2, \varepsilon) = \emptyset$

   $\delta(q_3, 0) = \{q_4\}$ $\qquad$ $\delta(q_4, 0) = \emptyset$

   $\delta(q_3, 1) = \{q_4\}$ $\qquad$ $\delta(q_4, 1) = \emptyset$

   $\delta(q_3, \varepsilon) = \emptyset$ $\qquad$ $\delta(q_4, \varepsilon) = \emptyset$
4. q_1 is the start state
5. $F = \{q_4\}$

Informally, NFA N accepts w if we can find a path from the start state to one of the accept states.



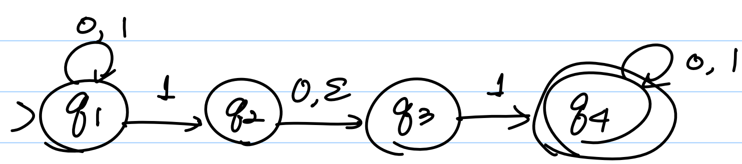
1 0 1

1 1

0 1 0 1 1 0

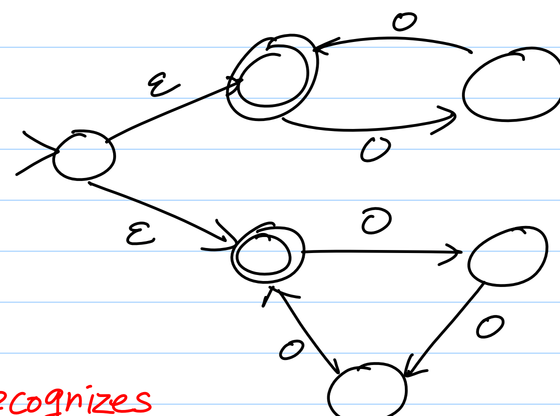
Edges may contain ε.

Q. What does it mean?



1 1

0 1 0 1 1 0



recognizes $\{ w \mid w = 0^k$ where k is a multiple of 2 or 3 $\}$