$\Sigma = \{0, 1\}$

- $L(01 \cup 10) = L(01) \cup L(10) = \{01, 10\}$

- $L(0^*10^*) = L(0^*) \circ L(1) \circ L(0^*)$
  $= \{\varepsilon, 0, 00, \cdots\} \circ \{1\} \circ \{\varepsilon, 0, 00, \cdots\}$
  $= \{w \mid w$ contains a single $1\}$

- $L(\Sigma^* 1 \Sigma^*) = L(\Sigma^*) \circ L(1) \circ L(\Sigma^*)$
  $= (L(\Sigma))^* \circ L(1) \circ (L(\Sigma))^*$
  $= (\{0,1\})^* \circ \{1\} \circ (\{0,1\})^*$
  $= \{w \mid w$ has at least one $1\}$

- $L((\Sigma\Sigma)^*) = (L(\Sigma\Sigma))^* = (L(\Sigma) \circ L(\Sigma))^*$
  $= (\{0,1\} \circ \{0,1\})^*$
  $= \{w \mid w$ is a string of even length$\}$

- $L(0\Sigma^*0 \cup 1\Sigma^*1 \cup 0 \cup 1)$
  $= L(0\Sigma^*0) \cup L(1\Sigma^*1) \cup L(0) \cup L(1)$
  $= (L(0) \circ L(\Sigma^*) \circ L(0)) \cup (L(1) \circ L(\Sigma^*) \circ L(1))$
  $\cup L(0) \cup L(1)$
  $= \{w \mid w$ starts and ends with the same symbol$\}$

- $L((0 \cup \varepsilon)1^*) = L(0 \cup \varepsilon) \circ L(1^*)$
  $= L(0 \cup \varepsilon) \circ (L(1))^*$
  $= \{0, \varepsilon\} \circ \{\varepsilon, 1, 11, \cdots\}$
  $= \{0, 01, 011, \cdots, \varepsilon, 1, 11, \cdots\}$
  $= L(01^*) \cup L(1^*)$
  $= L(01^* \cup 1^*)$

- $L(1^*\emptyset)$
  $= L(1^*) \circ L(\emptyset)$
  $= \{\varepsilon, 1, 11, \cdots\} \circ \emptyset$
  $= \emptyset$

- $L(\emptyset^*) = (L(\emptyset))^* = (\emptyset)^* = \{\varepsilon\}$

- $L(R \cup \emptyset)$
  $= L(R) \cup L(\emptyset) = L(R) \cup \emptyset$
  $= L(R)$

- $L(R \circ \varepsilon)$
  $= L(R) \circ L(\varepsilon) = L(R) \circ \{\varepsilon\}$
  $= L(R)$

- $L(R \cup \varepsilon) = L(R) \cup L(\varepsilon) = L(R) \cup \{\varepsilon\}$
  (may not be equal to $L(R)$)

- $L(R \circ \phi) = L(R) \circ L(\phi) = L(R) \circ \phi$
  $= \phi$
  (may not be equal to $L(R)$)

< Lecture 9 >

Thm 1.54. A language is regular if and only if some regular expression describes it.

in ~~other~~ words,

1. Every language that is described by a regular expression is a regular language.
2. Every regular language is described by a regular expression.

in ~~other~~ words,

Language K is regular if and only if there is a regular expression R s.t. $L(R) = K$

$\leftarrow$ Given a language K, if there is a regular expression R s.t. $L(R) = K$, then K is regular

$\rightarrow$ If Language K is regular, then there is a regular expression R s.t. $L(R) = K$

$\leftarrow$ Every language that is described by a regular expression is a regular language.

Example :

$(ab \cup a)^*$

Q How to prove?

see the conversion in p 64.