< Lecture 13 >

Prove that $B = \{w \mid w$ has an equal # of 0s and 1s$\}$ is not regular

Assume that B is regular.

Since the set of regular languages is closed under intersection (i.e, if L_1 and L_2 are regular, then $L_1 \cap L_2$ is regular),

$B \cap L(0^* 1^*)$ is regular.

However $B \cap L(0^* 1^*) = \{0^n 1^n \mid n \geq 0\}$ which is not regular, contradiction.

Prove that $B = \{w \mid w$ has an equal # of 0s and 1s$\}$

a direct proof ?

Assume that B is regular. By the pumping lemma, there exists a pumping length p.

Let $s = (01)^p$.

If we show that s can't be divided into xyz that satisfies the three conditions, we are done.

Is it?

Prove that $B = \{w \mid w$ has an equal # of 0s and

Let $s = 0^p 1^p$

Consider any division into xyz such that $|xy| \leq p$, $|y| > 0$.

y contains 0s only, and not empty

$xyyz = 0^{p+|y|} 1^p$, which is not in B.

Contradiction.

Prove that $C = \{ww \mid w \in \{0,1\}^*\}$ is not regular

Assume C is regular. Let p be the pumping length.

Let $s = 0^p 1 0^p 1$

Consider any division of s into xyz s.t. $|xy| \leq p$, and $|y| > 0$.

$xyyz = 0^{p+|y|} 1 0^p 1 \notin C$,

Contradiction.

$(s = 0^p 0^p ?)$