< Lecture 23>

Encoding

- Input to a TM is a string of symbols
- But algorithms work on many different objects such as graphs, languages, TM, etc

- Need to choose an encoding for objects

- Can be done in many reasonable ways

- Sometimes distinguish between
  $$X \quad \text{and} \quad \langle X \rangle$$
  (object)   (encoding)

Example 3.23

Let A be the language consisting of strings representing undirected graphs

A graph is <u>connected</u> if every node can be reached from any other node by traveling along edges
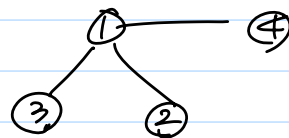
$$A = \{\langle G \rangle \mid G \text{ is a connected graph}\}$$
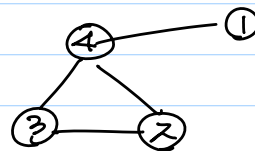

High level description of TM that decides

$$A = \{\langle G \rangle \mid G \text{ is a connected graph}\}$$

On input $\langle G \rangle$, encoding of graph G
- select first node of G and mark it.
- repeat until no new nodes marked:
- For each node in G, mark if attached by an edge to a node already marked.
- scan nodes of G to determine whether they are all marked. If so, accept, otherwise reject.

Some details

Q: How is G encoded?

$$\langle G \rangle = \underbrace{(1,2,3,4)}_{\text{nodes}} \underbrace{((1,4),(2,3),(3,4),(2,4))}_{\text{edges}}$$

More detailed algorithm

On input M checks that input is valid graph encoding

- two lists
- first is list of numbers
- second is list of pairs
- first list contains no duplicates (element distinctness subroutine)
- every node in second list appears in first

Now ready to start "step one"

On input $\langle G \rangle$, encoding of graph G

1 mark first node with a dot on leftmost digit.
2 loop:
- M scans list and "underlines" undotted node n_1.
- M rescans and "underlines" dotted node n_2.
- M scans edges.
- M tests each edge if it is (n_1, n_2).
- If so, dot n_1, remove underlines, and restart Step 2.
- If not, check next edge. When no more edges, move underline to next dotted n_2.
- when no more dotted vertexes, move underlines: new n_1 is next undotted node and new n_1 is first dotted node. Repeat Step 2. When no more undotted nodes, go to Step 3.
3 M scans the list of nodes. If all dotted, accept, otherwise reject.

Even more detailed description possible!