Propositional Logic

Reading: Handbook Ch1

Propositional Logic

- Signature: a nonempty set of symbols called atoms
- Formulas of σ: formed from atoms, 0-place connectives ($\bot$, $\top$), unary connective ($\neg$), binary connectives ($\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$)
  - every atom is a formula
  - both 0-place connectives are formulas
  - if F is a formula then -F is a formula
  - for any binary connective $\odot$, if F and G are formulas, then so is $(F \odot G)$

Structural Induction

- A way to prove ANY formula has some property P. Sufficient to prove that
  - every atom has property P
  - both 0-place connectives have property P
  - if a formula F has property P then so does $\neg F$
  - for any binary connective $\odot$, if formulas F and G have property P then so does $(F \odot G)$.
- Ex: The number of left and right parentheses are the same for any formula.

Example

- If the train arrives late and there are no taxis at the station then John is late for his meeting. John is not late for his meeting. The train did arrive late.
- True or False: there was a taxi at the station
- If the train arrives late and there are no taxis at the station then John is late for his meeting. John is not late for his meeting. The train did arrive late. No taxis were in the station.

DNF / CNF

- A literal is an atom or the negation of an atom
- A **simple conjunction** is a formula of the form $L_1 \wedge \ldots \wedge L_n$ where $L_1, \ldots, L_n$ are literals
- A formula is in **disjunctive normal form** if it has the form $C_1 \vee \ldots \vee C_m$ $(m>=1)$ where $C_{1,\ldots,}C_m$ are simple conjunctions
- A **simple disjunction** is a formula of the form $L_1 \vee \ldots \vee L_n$
- A formula is in **conjunctive normal form** if it has the form $D_1 \wedge \ldots \wedge D_m$ $(m>=1)$ where $D_{1,\ldots,}D_m$ are simple disjunctions

DNF / CNF

- Theorem: If the underlying signature is non-empty, then any formula is equivalent to a formula in DNF
- Theorem: If the underlying signature is non-empty, then any formula is equivalent to a formula in CNF