<Lecture 3>

<Regular Operations>

- The idea is to build a new regular language using existing regular languages

Def
- Union : $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
- Concatenation : $A \circ B = \{xy \mid x \in A \text{ and } y \in B\}$
- Star : $A^* = \{x_1 x_2 \cdots x_k \mid k \geq 0 \text{ and each } x_i \in A\}$

Example : $A = \{a, b, c\}$, $B = \{d, e\}$

$A \cup B = \{$

$A \circ B = \{$

$A^* = \{$

In general, we say that a set A is closed under operation $\odot$ if the result of applying $\odot$ for each element in A belongs to A as well.

Example : $N = \{1, 2, 3 \ldots\}$

Thm : The set of regular languages is closed under regular operations.

I.e, for any regular languages A and B, $A \cup B$, $A \circ B$, A^* are all regular!

That's why $\cup$, $\circ$, $*$ are called regular operations.

Thm 1.25. For any regular language A and B, $A \cup B$ is regular

Q. How to prove this?

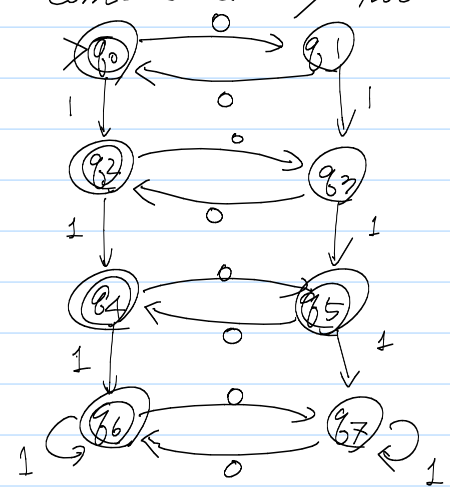
Idea:
We need to construct machine M that simulates both machines M_1 and M_2

But how?

Q. What if we run M_1 on w, and if accepts, run M_2 on w?

Recall P1.6 l (HW1)

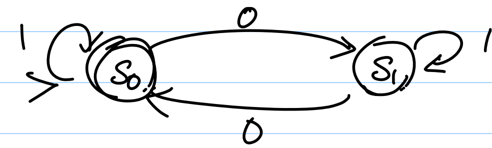
l. $\{w \mid w$ contains an even # of 0s or contains exactly two 1s$\}$



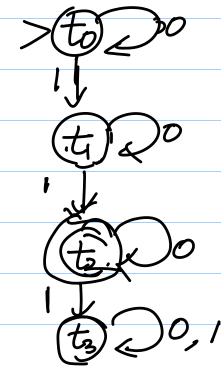
0 1 0 1 0

r

$A = \{w \mid w$ contains an even number of 0s$\}$



$B = \{w \mid w$ contains exactly two 1s$\}$



	0	1	0	1	0
q	t_1	t_1	t_0	t_0	t_1
r	t_0	t_1	t_1	t_2	t_2

How to combine them?

Given $M_1=(Q_1, \Sigma, \delta_1, q_1, F_1)$ that recognizes A, and $M_2=(Q_2, \Sigma, \delta_2, q_2, F_2)$ that recognizes B, we need to construct machine $M=(Q, \Sigma, \delta, q_0, F)$ that simulates M_1 and M_2 together?

How?

Q. How many states do we need?

Q. What should be the start state q_0?

Q. What should be the accept states?

Q What should be the transition function?

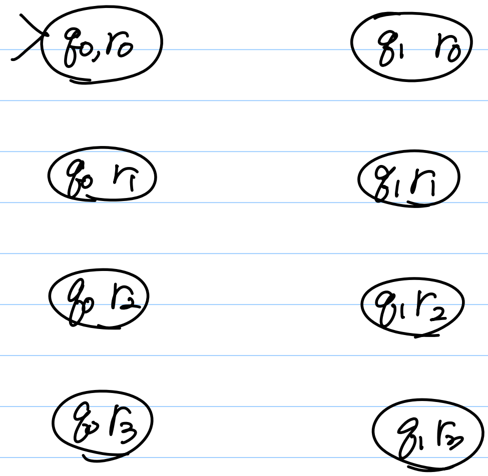
1. States:
   $Q = \{(r_1, r_2) \mid r_1 \in Q_1$ and $r_2 \in Q_2\}$

   Ex) $\{(s_0, t_0), (s_1, t_0),$
   $(s_0, t_1), (s_1, t_1),$
   $(s_0, t_2), (s_1, t_2),$
   $(s_0, t_3), (s_1, t_3)\}$

2. q_0 is the "common start state"
   : $q_0 = (q_1, q_2)$

   Ex) (s_0, t_0)

{Show the construction}



3. accept states?

$F = \{ (r_1, r_2) \mid r_1 \in F_1 \text{ or } r_2 \in F_2 \}$

$= (F_1 \times Q_2) \cup (Q_1 \times F_2)$

$\neq F_1 \times F_2$.

Ex)

$\{(s_0, t_0), (s_1, t_0),$
$(s_0, t_1), (s_1, t_1),$
$(s_0, t_2), \boxed{(s_1, t_2)},$
$(s_0, t_3), (s_1, t_3)\}$

4. transition function.

Q. How many transitions (edges)?

$\delta((r_1, r_2), a) = (\delta_1(r_1, a), \delta_2(r_2, a))$

{consider each transition by M_1 and M_2}