<Lecture 10>

Proof :

For every regular expression R, we can construct a corresponding NFA.

But how?

There are infinitely many such R.

$\leftarrow$ Every language that is described by a regular expression is a regular language.

Proof by induction.

Base cases:

1. $R = a$ for some $a \in \Sigma$
   Then $L(R) = \{a\}$.

2. $R = \varepsilon$. Then $L(R) = \{\varepsilon\}$

3. $R = \emptyset$ Then $L(R) = \emptyset$

4. $R = R_1 \cup R_2$

Inductive step.

Q What to assume?

Assume that $L(R)$ is regular.

By the definition of regular expression,

R_1 and R_2 are regular expressions.

By induction hypothesis, each of $L(R_1)$ and $L(R_2)$ is regular. By Thm 1.25, $L(R_1) \cup L(R_2)$ is regular. Thus

$L(R_1) \cup L(R_2) = L(R_1 \cup R_2)$ is regular

5. $R = R_1 \circ R_2$

6. $R = R_1^*$

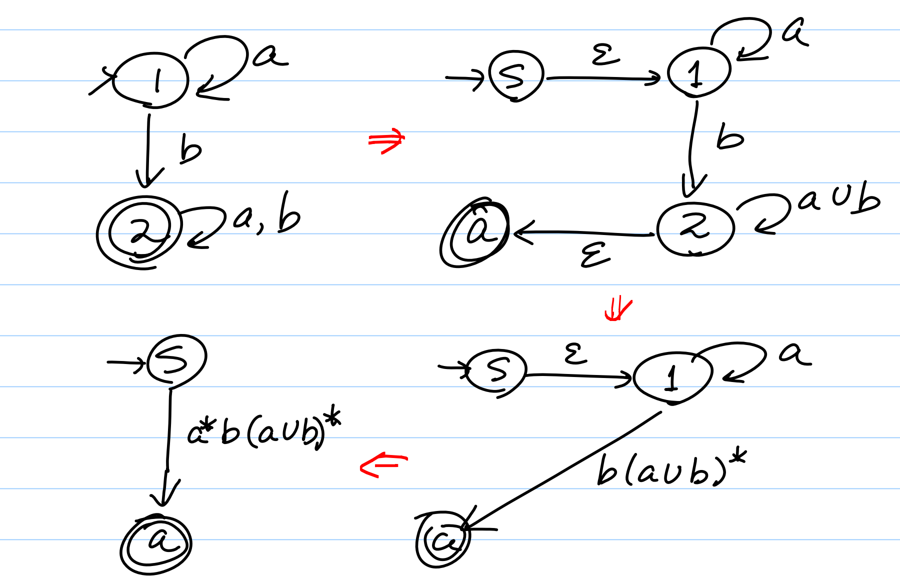
The other direction

→ Every regular language is described by a regular expression.

Take any regular language. and show that it can be described by a regular expression.

How?

Example : Guess !



Idea?

1. Since we're given a regular language, there is a DFA that recognizes it.

2. We will convert the DFA into a form where the edges are labelled with regular expressions.

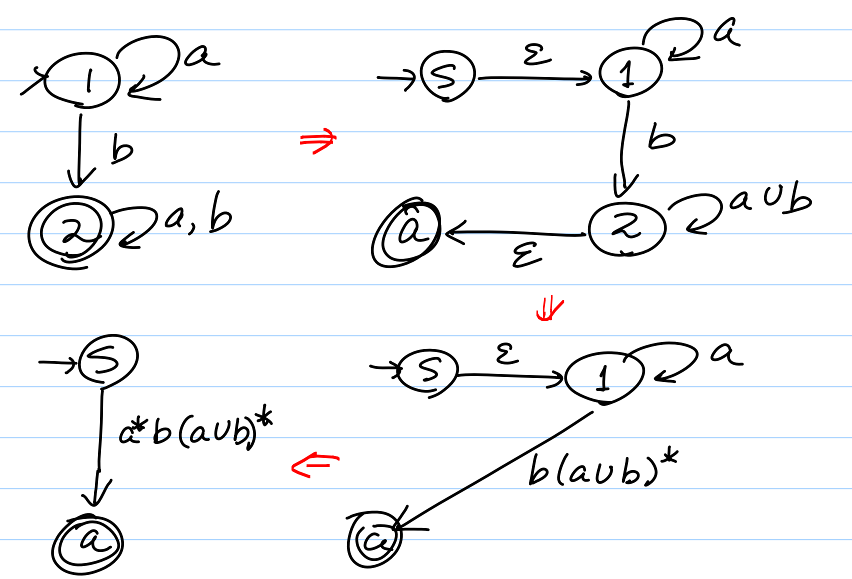
3. From this converted form, we'll remove states one by one until there remain only two states. The edge between them is the regular expression

GNFA (Generalized NFA)

- The edges are labeled with regular expressions rather than $\Sigma \cup \{\varepsilon\}$

- Furthermore, for simplicity, we assume that
  - the start state has edges going to every other state, but no incoming edge.
  - There is only one accept state, which has edges coming from every other state, but no outgoing edge
  - The other states has outgoing edges to all other states except for start state, and also to itself

Example: GNFA



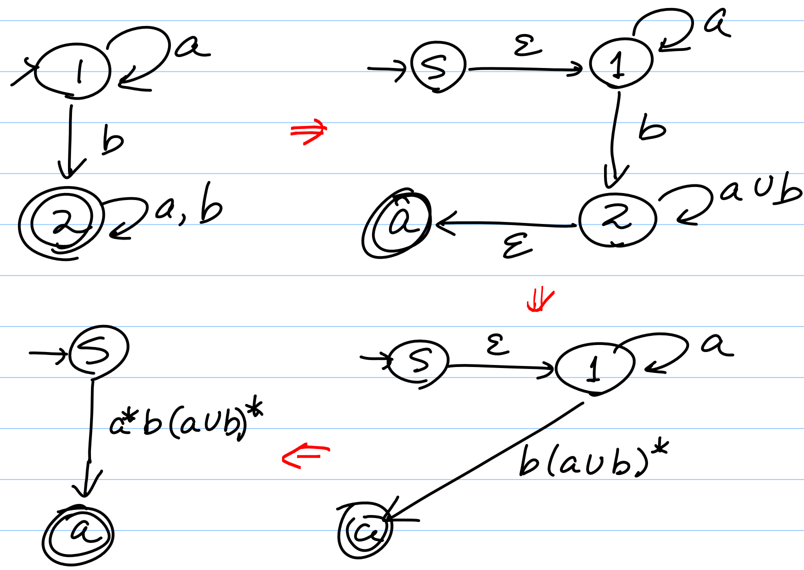
Formal.

Def. GNFA = $(Q, \Sigma, \delta, q_{start}, q_{accept})$

1. Q is a finite set of states
2. Σ is the input symbol
3. $\delta: (Q \setminus \{q_{accept}\}) \times (Q \setminus \{q_{start}\}) \to \mathcal{R}$
4. q_{start} is the start state
5. q_{accept} is the accept state.

How to get rid of a state from a GNFA?

Example :



How to get rid of a state from a GNFA?