Grounding

Grounding

- Assume that the underlying predicate signature consists of a finite nonempty set of object constants and at least one predicate constant. (no function constants of arity > 0)
- A ground formula is a formula which contains neither variables nor equality.
- For any sentence F, the result of its *grounding* is the ground formula F^g defined as follows:

Undecidability of FOL

- Theorem: The validity problem of first-order logic (given any sentence F, is F logically valid?) is undecidable.
  - Proof: By reducing PCP to validity problem
- Corollary: The satisfiability problem of first-order logic (given a sentence F, is F satisfiable?) is undecidable:
  - Proof: By reducing the validity problem to it.
- We need to restrict FOL in a meaningful way!

Grounding

- A way to reduce reasoning in FOL to reasoning in PL.
- Function constants are not usually allowed.
- Restrict attention to certain domains in which complete knowledge of the objects is given.
  - UNA, DCA

F^g

Assume no function constants of positive arity.

- if F is atomic formula (not containing equality), then $F^g = F$;
- $(t_1 = t_2)^g$ = TRUE if t_1 equals t_2, and FALSE otherwise;
- $(\neg F)^g = \neg F^g$;
- for every binary connective $\odot$, $(F \odot G)^g = F^g \odot G^g$;
- $\forall x\, F(x)^g = \wedge_c F^g(c)$; $\exists x\, F(x)^g = \vee_c F^g(c)$;
  - c ranges over the object constants of the underlying signature.

UNA, DCA

- UNA : the set of the formula $c \neq d$ for all pairs of distinct object constants c, d.
- DCA :

$$\forall x \bigvee_c x = c.$$

Theorems

Theorem: For any (first-order) sentence F,

Γ, UNA, DCA $\models$ F

iff Γ, UNA, DCA $\models F^g$

Corollary: For any set of Γ of sentences and any sentence F,

Γ, UNA, DCA $\models$ F

iff Γ^g, UNA, DCA $\models F^g$

Grounding and Propositional Logic

- Let σ^g be the propositional signature whose atoms are ground atomic formulas of σ.
- Accordingly, the concepts of an interpretation, satisfaction, model, satisfiability and entailment, when applied to ground formulas, can be understood in the sense of propositional logic

Theorem: For any set Gamma of ground formulas, the following conditions are equivalent.

(i) the union of Gamma with UNA and DCA is satisfiable;

(ii) Gamma is satisfiable in the sense of propositional logic.

Corollary: For any set Gamma of ground formulas and any ground formula F,

Gamma, UNA, DCA |= F

iff Gamma entails F in the sense of propositional logic.

(The use of grounding is justified when the axiom set contains both UNA and DCA).