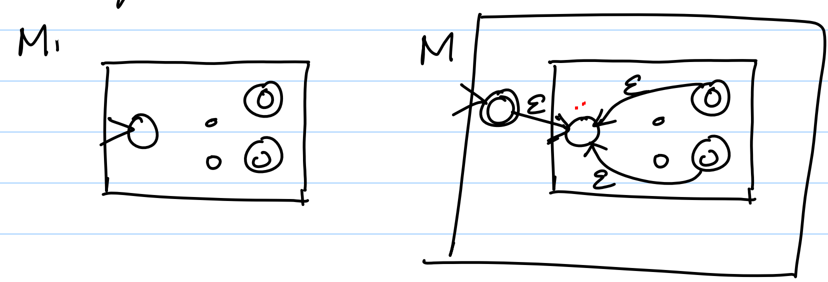
< Lecture 8 >

(3) Thm 1.47. For any regular language A, A^* is regular.

Given NFA M_1 that recognizes A and
Construct NFA M that recognizes A^*.

Informally:



Formally: Let $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ for A.
The following $M = (Q, \Sigma, \delta, q_0, F)$ recognizes A^*.

1. $Q = \{q_0\} \cup Q_1$
2. q_0 is the new start state.
3. $F = \{q_0\} \cup F_1$
4. For any $q \in Q$ and any $a \in \Sigma \cup \{\varepsilon\}$

$$\delta(q, a) = \begin{cases} \delta_1(q, a) & q \in Q_1 \setminus F_1 \\ \delta_1(q, a) & q \in F_1 \text{ and } a \neq \varepsilon \\ \delta_1(q, a) \cup \{q_1\} & q \in F_1 \text{ and } a = \varepsilon \\ \{q_1\} & q = q_0 \text{ and } a = \varepsilon \\ \emptyset & q = q_0 \text{ and } a \neq \varepsilon \end{cases}$$

Regular Expressions

- Each DFA and NFA recognizes a regular language.
- We'll learn another representation : regular expressions.
- We'll show that
  Regular expressions
  $\Leftrightarrow$ DFA
  $\Leftrightarrow$ NFA

A regular expression is defined recursively:

1. for every $a \in \Sigma$, a is a regular expression
2. ε, $\emptyset$ are regular expressions
3. if R_1 is a regular expression, so is R_1^*
4. if R_1, R_2 are regular expressions, so is $(R_1 \cup R_2)$ and $(R_1 \circ R_2)$.

Do not confuse
1. regular expressions vs regular language.

Given a regular expression R, the language represented by R, (denoted by $L(R)$) is defined as follows.

$L(a) = \{a\}$

$L(\varepsilon) = \{\varepsilon\}$

$L(\emptyset) = \emptyset$

$L(R_1 \cup R_2) = L(R_1) \cup L(R_2)$

$L(R_1 \circ R_2) = L(R_1) \circ L(R_2)$

$L(R_1^*) = (L(R_1))^*$

Recall

Def
- Union : $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
- Concatenation : $A \circ B = \{xy \mid x \in A \text{ and } y \in B\}$
- Star : $A^* = \{x_1 x_2 \cdots x_k \mid k \geq 0 \text{ and each } x_i \in A\}$

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special case

$A \cup \emptyset =$

$A \circ \emptyset =$

$\emptyset^* =$

Abbreviation

Σ (as a regular expression) stands for $a_1 \cup \cdots \cup a_n$ where $\Sigma = \{a_1, \cdots, a_n\}$.

$L(\Sigma) = ?$

(Do not confuse expression vs language)

R^+ stands for RR^*

$\Sigma = \{0, 1\}$

- $L(01 \cup 10) =$

- $L(0^* 1 0^*) =$

- $L(\Sigma^* 1 \Sigma^*) =$

- $L((\Sigma\Sigma)^*) =$

- $L(0\Sigma^*0 \cup 1\Sigma^*1 \cup 0 \cup 1)$

- $L((0 \cup \varepsilon)1^*) =$

- $L(1^*\emptyset)$

- $L(\emptyset^*)$

- $L(R \cup \emptyset)$

- $L(R \circ \varepsilon)$

- $L(R \cup \varepsilon) =$

- $L(R \circ \emptyset) =$