CSE 591 Knowledge Representation and Reasoning, Fall 2009

4. Nonmonotonic Causal Theories

Multi-Valued Propositional Formulas

For definitions and notation, see *Nonmonotonic Causal Theories* by Giunchiglia *et al.* We will refer to that paper as *NMCT*.

In Problems 4.1, 4.2, by c, d, e we denote constants of a multi-valued propositional signature.

4.1 For any interpretation I,

$$I \models c = d \text{ iff } I(c) = I(d).$$

4.2 Each of the formulas

$$\begin{gathered} c = c \\ c = d \rightarrow d = c \\ (c = d \wedge d = e) \rightarrow c = e \end{gathered}$$

is satisfied by all interpretations.

4.3 Express each of the conditions below by a formula of the multi-valued propositional signature (3), (4) from *NMCT*, and determine whether your formula is entailed by the two formulas used in Section 2.1 of *NMCT* to characterize the possible states of the Monkey and Bananas domain:

$$\mathit{HasBananas} \rightarrow \mathit{Loc}(\mathit{Monkey}) = \mathit{Loc}(\mathit{Bananas})$$

and

$$\mathit{OnBox} \rightarrow \mathit{Loc}(\mathit{Monkey}) = \mathit{Loc}(\mathit{Box}).$$

(a) If the monkey is at location L_3 then he does not have the bananas.

(b) If the monkey has the bananas and is on the box then the bananas and the box are at the same location.

4.4 In the Monkey and Bananas example from Section 2.1 of *NMCT*, assume that there are two monkeys, rather than one. “Locations” are big, so that both monkeys can be at the same location at the same time. But at most one monkey can stand on the box at any time, and at most one monkey can have the bananas. Show how to modify the description of the domain to reflect this enhancement.

Causal Theories

4.5

(a) Find all models of the causal theory

$$\begin{array}{c} p \Leftarrow q \\ q \Leftarrow p \end{array}$$

assuming that its signature consists of the Boolean constants p, q.

(b) Answer the same question for the theory

$$\begin{array}{c} q \rightarrow p \Leftarrow \top \\ p \rightarrow q \Leftarrow \top. \end{array}$$

(c) Determine how the answers to these questions would be affected by adding the constraint

$$\bot \Leftarrow p \vee q.$$

4.6 Find all models of the causal theory

$$c = 1 \equiv c = 2 \Leftarrow \top$$

assuming that the underlying signature consists of one constant c, whose domain is

(a) $\{1, 2\}$,

(b) $\{1, 2, 3\}$,

(c) $\{1, 2, 3, 4\}$.

4.7

(a) If the signature of a causal theory T includes a Boolean constant p that does not occur in the heads of the rules of T, then T is inconsistent.

(b) Show that this assertion would not be true without the assumption that p is Boolean.

4.8 Causal theory T contains the rules

$$c = v \Leftarrow c = v$$

for every constant c in the signature of T and every $v \in Dom(c)$. Show that an interpretation I is a model of T iff, for every causal rule $F \Leftarrow G$ in T,

$$I \models G \rightarrow F.$$

4.9 For any causal theories T_1, T_2, T_3 of the same signature, if T_1 has the same models as T_2 then $T_1 \cup T_3$ has the same models as $T_2 \cup T_3$. True or false?

4.10

(a) Find all models of the causal theory

$$\begin{array}{l} p \Leftarrow q \\ p \Leftarrow r \\ q \Leftarrow \neg r \\ r \Leftarrow \neg q \\ \neg p \Leftarrow \neg p \\ \neg q \Leftarrow \neg q \end{array}$$

assuming that its signature consists of the Boolean constants p, q, r.

(b) How will the answer change if we add

$$\neg r \Leftarrow \neg r$$

to the list of rules?

Describing Actions in Causal Theories

4.11 Consider the enhancement of the example from Section 3.1 of *NMCT* in which a second truth-valued parameter q is introduced; the assumption that executing a in any state changes the value of p to $\mathbf{t}$ is limited to the states in which the value of q is $\mathbf{t}$. (Example: a is "publish a paper in a scientific journal"; p is "everybody is amazed"; q is "the author is a high school student.")

(a) How does this change affect causal theory (19)–(22), its completion, and the simplified form of the completion computed in Section 3.1?

(b) Write a formula expressing that the value of p in state s_1 can be $\mathbf{t}$ only if at least one of the parameters p, q had the value $\mathbf{t}$ in state s_0. Show that this formula is entailed by your causal theory.

4.12 Consider the enhancement of the example from Section 3.1 of *NMCT* in which a second action a' is introduced; the effect of this action is opposite to the effect of a.

(a) How does this change affect causal theory (19)–(22), its completion, and the simplified form of the completion computed in Section 3.1?

(b) Write a formula expressing that actions a and a' cannot be executed concurrently. Show that this formula is entailed by your causal theory.

(c) Answer the questions from part (a) for causal theory (23), describing histories of length m. How many models does your theory have?

(d) Write a formula expressing that if a value of p was $\mathbf{f}$ in state s_0 and $\mathbf{t}$ in state s_m then action a was performed at least once between the two states. Show that this formula is entailed by the causal theory from part (c).

4.13 Express the given assertions by formulas of causal theory MB_1 (defined in Section 3.2 of *NMCT*) and show that they are entailed by that theory.

(a) Walking always changes the location of the monkey, but never changes the location of the box.

(b) If the location of the monkey changed then he either walked or pushed the box.

4.14 Consider the enhancement of the example from Section 3.1 of *NMCT* in which causal law (19) is made defeasible. How does this change affect the completion of the causal theory and its simplified form computed in Section 3.1?

Language C+

4.15 Describe the causal theories from Problems 11, 12 and 14 in language $\mathcal{C}+$, and draw the corresponding transition systems.

4.16 Consider the action description in Figure 5 of *NMCT*.

(a) Enhance this description to reflect the assumption that, by default, turkeys are fat, but Turkey 1 is an exception.

(b) In state s_0 all turkeys are alive, and the gun is aimed at Turkey 1. Write a formula expressing that, under these assumptions, all fat turkeys will be alive in state s_1. Show that this formula is entailed by your enhanced action description.

4.17 Consider the action description in Figure 6 of *NMCT*. In state s_0 Jack is at home and has his car with him. Then he goes to work. Show that these assumptions don't imply any specific conclusion about the location of Jack's car in state s_1.

4.18 Consider the action description in Figure 9 of *NMCT*. In state s_0 David had no long publications. Then he published a paper, 3 page long. Write a formula expressing that, under this assumption, in state s_1 David still has no long publications. Show that this formula is entailed by the action description in Figure 9.

CCalc Programming

4.19 Solve all exercises in the CCalc page:

`http://www.cs.utexas.edu/~tag/cc/tutorial/toc.html`

4.20 Solve Problems 4.11—14 using CCalc.

4.21 Solve Problems 3.7, 3.10, 3.11 using CCalc.

Solve the following problems using CCalc.

4.22 (a) A robot with two grippers wants to move three balls from Room 1 to Room 2. Use CCalc to find a plan for doing that. (b) Assume, in addition, that the balls need to be painted in the process: originally they are all white, and at the end one should be red, one blue and one white. Red paint and blue paint are available in Room 3.

4.23 (a) You have a few quarters and a few dollar bills. A dollar changer is available. Use CCalc to find a plan for putting a given number of quarters into a parking meter. (b) Assume, in addition, that the meter has a handle that needs to be turned every time after putting a quarter in the slot.

4.24 A man wants to move a wolf, a sheep, and a box of cabbage to the other side of the lake. He can move one at each step by crossing in a boat. Notice that wolves eat sheep and sheep eat cabbages when no man is around. Use CCalc to help the man.

4.25 Modify the CCalc description of the blocks world available at

(`http://www.cs.utexas.edu/~tag/ccalc/examples/`)

to reflect the following assumptions. Moving blocks is performed by two agents—Big Robot and Little Robot. Each of them can move only one block at a time. Some blocks are heavy, and Little Robot is not capable of moving heavy blocks.