<Lecture 18>

Formal Definition of PDA

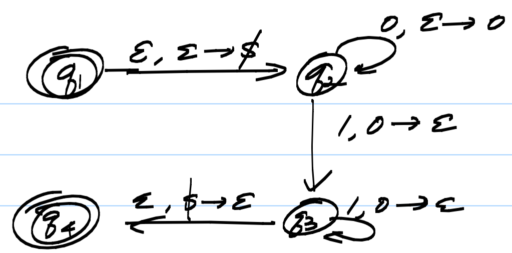
PDA is a 6-tuple $(Q, \Sigma, \Gamma, \delta, q_0, F)$

1. Q is a finite set of states
2. Σ is a finite set of input alphabet
3. Γ is a finite set of stack alphabet
4. $\delta : Q \times (\Sigma \cup \{\varepsilon\}) \times (\Gamma \cup \{\varepsilon\}) \rightarrow \mathcal{P}(Q \times (\Gamma \cup \{\varepsilon\}))$
5. $q_0 \in Q$ is the start state
6. $F \subseteq Q$ is the set of accept states.

PDA M accepts $w = w_1 w_2 \cdots w_m$ $(w_i \in (\Sigma \cup \{\varepsilon\}))$ if there are a sequence of states $r_0, \cdots r_m \in Q$ and a string $s_0 \cdots s_m \in \Gamma^*$ s.t

1. $r_0 = q_0$, and $s_0 = \varepsilon$
2. For $i = 0, \cdots, m-1$, $(r_{i+1}, b) \in \delta(r_i, w_{i+1}, a)$ where $s_i = at$ and $s_{i+1} = bt$ for some $a, b \in (\Gamma \cup \{\varepsilon\})$, and $t \in \Gamma^*$
3. $r_m \in F$

At each state, one of the possible actions can be executed.



q_1 : without reading input and stack, push \$, go to q_2 ; or accept if input is empty

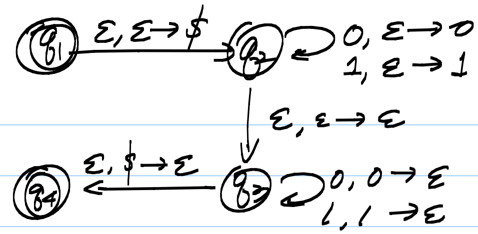
q_2 – upon reading 0 from input, push 0, and remain at q_2
- upon reading 1 from input, and 0 from the stack pop up, and go to q_3

q_3 – upon reading 1 from input and 0 from the stack, pop up and remain at q_3
- upon reading \$ from the stack, without reading input, go to q_4

q_4 – accept if input is empty

- Input alphabet and stack alphabet may be different
- Note Nondeterminism. If no actions are possible, that branch dies
- Can we skip pushing \$?

Ex 2.18



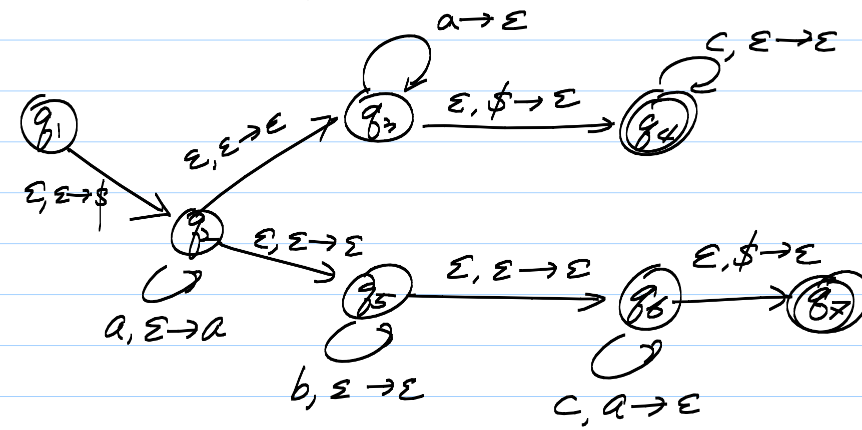
q_1 : without reading input and stack, push \$, go to q_2 ; or accept if input is empty

q_2 — upon reading an input symbol, push it and remain.
— without reading input and stack, go to q_3

q_3 — upon reading the same symbol from both input and stack, pop it and remain
— upon reading \$ from the stack, without reading input, go to q_4

q_4 — accept if input is empty.

$\{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i=j \text{ or } i=k\}$



aabb ?

aacc ?

abc ?