<Lecture 12>

Pumping Lemma : Simplified (but not complete)

Let A be a regular language. Any "long" string s from A can be divided into three pieces $s = xyz$, so that

1. $xyyz \in A$
2. $|y| > 0$ (i.e., $y \neq \varepsilon$)

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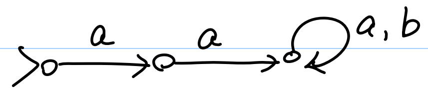
Example : $\Sigma = \{a, b\}$

$\{w \mid w$ has at least 2 a's$\}$



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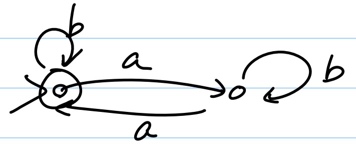
Example : $\{w \mid w$ has an even # of a's$\}$



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Example : $\{w \mid w$ has at most 2 a's $\}$

Pumping Lemma : Simplified (but not complete)

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$\{a^n b^n \mid n \geq 0\}$ does NOT have this property.

"Any long string s from A" formally means

"there exists a number p s.t. any string from A of length at least p."

Statement :
For any regular language A,
There exists a nonnegative integer p s.t.
every string s in A whose length is at least p can be divided into three pieces $s = xyz$, so that

1. $xyyz \in A$
2. $|y| > 0$ (i.e., $y \neq \varepsilon$)

Pumping Lemma : Full Version

For any regular language A,
There exists a nonnegative integer ("pumping length") s.t.
every string s in A whose length is at least p, can be divided into three pieces, $s = xyz$, that satisfies the following 3 conditions.

1. for each $i \geq 0$, $xy^iz \in A$
2. $|y| > 0$
3. $|xy| \leq p$

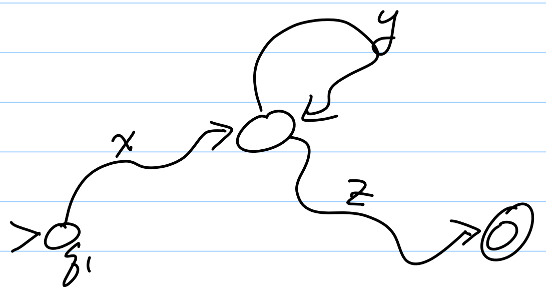
Proof of the pumping Lemma (Simpler one)

For any regular language A,
There exists a nonnegative integer p s.t.
every string s in A whose length is at least p can be divided into three pieces $s = xyz$, so that

1. $xyyz \in A$
2. $|y| > 0$ (i.e., $y \neq \varepsilon$)

- Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA that recognizes A
- Take any string in A whose length is $\geq |Q|$
- In the path that is associated with accepting s there must be a state that is visited more than once.

M



- Since xyz is accepted, $xyyz$ is also accepted
- moreover $|y| > 0$

Pumping Lemma : Full Version

For any regular language A,
There exists a nonnegative integer ("pumping length") s.t every string s in A whose length is at least p, can be divided into three pieces, $s = xyz$, that satisfies the following 3 conditions.

1. for each $i \geq 0$, $xy^iz \in A$
2. $|y| > 0$
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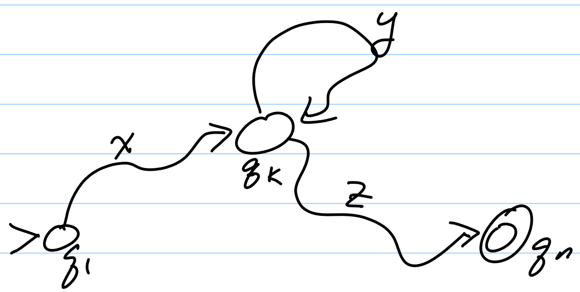

- Since xyz is accepted, $xyyz$ is also accepted. Also xz, $xyyyz$, $xyyyyz$ .... are accepted.
- moreover $|y| > 0$
- $|xy| \leq p$: assuming q_k is the first repetition.

Using the Pumping Lemma to Prove a Certain Language is NOT regular.

Wait!
The pumping lemma is about regular languages.

Prove that $A = \{0^n 1^n \mid n \geq 0\}$ is NOT regular.

Proof by contradiction.

Your opponent, say C, claims that A is regular.

I will prove that he is wrong

Me : C, let me trust you.

Since A is regular, by the pumping lemma, there is a pumping length, right?

Give me that number.

C : Okay, the pumping length is p.

Me : All right. Let's take $s = 0^p 1^p$.

Can you find a division xyz that satisfies the three conditions from the pumping lemma?

C : Sure, let me see...

....

....

....

....

....

Well, I gave up.

I tried every possibility, but none of them works.

Me : See, You're wrong!

A is NOT regular!!!

Formally,

Assume A is regular. According to the pumping lemma, there exists a nonnegative integer p such that every string s from A can be divided into 3 pieces, $s = xyz$, that satisfies the three conditions of the lemma.

Take $s = 0^p 1^p$. Consider <u>any</u> division of s into xyz such that $|xy| \leq p$ and $|y| > 0$.

Since $|xy| \leq p$, y does not contain 1,

Since $|y| > 0$, y contains at least one 0.

Clearly, $xyyz$ contains more 0's than 1's, so that it is not in A, which does not satisfy the first condition.

Consequently there is <u>NO</u> division xyz that satisfies the three conditions, which contradicts the claim that A is regular.

Q.E.D