< Lecture 14 >

Chapter 2

Context-Free Grammars

- Context free, but not regular
  - (1) $\{0^n 1^n \mid n \geq 0\}$
  - (3) $\{0^m 1^n \mid m \neq n\}$
  - (4) $\{w \mid w$ has an equal # of 0s and 1s$\}$
  - (5) $\{w w^R \mid w \in \Sigma^*\}$: palindromes of even length
  - (7) $\{w \mid w \neq w^R, w \in \Sigma^*\}$: nonpalindromes (1.46 c)
  - (6) $\{w \mid w = w^R, w \in \Sigma^*\}$: palindromes (2.4 e)
  - (8) $\{w \mid w$ is a balanced string of parentheses$\}$
- Not even context-free :
  - $\{ww : w \in \Sigma^*\}$
  - $\{0^n 1^n 2^n \mid n \geq 0\}$

Context-Free Languages

A language is called "context-free" if it can be generated by a context-free grammar.

Context-free grammar:

e.g. $S \to 0S1$

$S \to \varepsilon$

* substitution rules / productions
* variables
* terminals

e.g. $S \to 0S1$

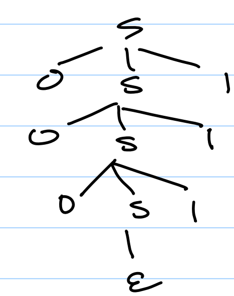
$S \to \varepsilon$

* derivation

$S \Rightarrow 0S1 \Rightarrow 00S11 \Rightarrow 000S111$

$\Rightarrow 000111$

* parse tree



$\{0^n 1^n \mid n \geq 0\}$

* The language of the grammar
(denoted by $L(G)$)

A language is called "Context-free" if it can be generated by a context-free grammar.

$S \to 0S1$ can be abbr as $S \to 0S1 \mid \varepsilon$
$S \to \varepsilon$

Formal definition of CFG.

A CFG (Context-Free Grammar) is a 4-tuple (V, Σ, R, S) where

1. V is a finite set of "variables".
2. Σ is a finite set of "terminals", disjoint from V.
3. R is a finite set of rules
4. $S \in V$ is the start variable.

u, v, w : strings

- uAv yields uwv if the gramma has a rule $A \to w$ (written $uAv \Rightarrow uwv$)
- u derives v ($u \overset{*}{\Rightarrow} v$) if
  - $u = v$ or
  - There is a sequence $u_1, \cdots u_k$ ($k \geq 0$) such that
    $u \Rightarrow u_1 \Rightarrow u_2 \Rightarrow \cdots \Rightarrow u_k \Rightarrow v$.
- The language of the grammar is
  $\{ w \in \Sigma^* \mid S \overset{*}{\Rightarrow} w \}$