Delay Scheduled Impulsive Control for Networked Control Systems

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***Abstract*—This paper presents a new design approach for networked control systems under the integral quadratic constraint (IQC) framework. Two types of network induced time-varying delays, that is, measurement delay and actuation delay, are considered. A novel delay scheduled impulsive (DSI) controller is proposed, which utilizes both plant state and the IQC dynamic state, as well as the real-time network-induced delay information for gain scheduling feedback control. Robust $\mathcal{L}_2$ stability analysis of the resulting impulsive closed-loop system is performed using dynamic IQCs combined with a clock-dependent storage function. Based on the analysis results, the synthesis conditions for the proposed DSI controller are established as a finite number of linear matrix inequalities by specifying a piecewise linear storage function, which can be solved effectively via convex optimization. Finally, an application to a dc motor system demonstrates the effectiveness and advantages of the proposed design approach.**

***Index Terms*—Gain-scheduling control, impulsive systems, integral quadratic constraints (IQCs), linear matrix inequalities (LMIs), networked control systems (NCSs), time-varying delay.**

I. Introduction

THE RECENT years have witnessed the accelerating development of networked control systems (NCSs) in the control community due to their widespread applications in modern real-world engineering problems over a broad range of areas, such as mobile sensor networks [1], remote surgery [2], automated traffic control [3], unmanned vehicles [4], and so on. Typically, an NCS can be described as a control system wherein the control loops are closed through a communication network. As depicted in Fig. 1, the defining feature of an NCS is that control and feedback signals are exchanged among the system components (such as sensors/samplers, actuators/zero-order-hold (ZOH) devices, and controllers) in the form of information packages through a network medium. Compared with conventional point-to-point interconnected feedback control systems, NCSs have advantages in many aspects: higher system operability, more efficient resource utilization and sharing, lower cost, reduced weight and power requirements, as well as simplicity for system installation and maintenance, etc.

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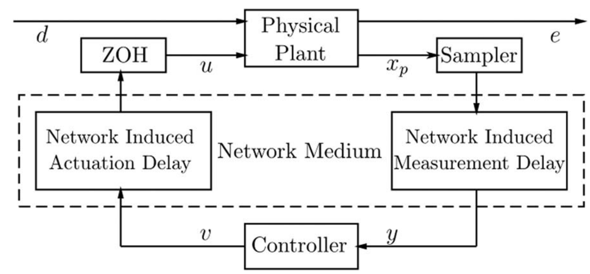

Fig. 1. Typical networked control system.

Despite many distinctive features provided by NCSs for distributed control system design, the insertion of the communication network in the feedback control loops makes the analysis and design of an NCS complex. Various challenging issues are encountered due to the network-induced time delays in control loops and/or possibility of data package losses, which give rise to many important research topics on NCSs, including modeling, stability analysis, control, and filtering design (see, for example, [6]–[8] and the references cited therein). Numerous effective methods have been reported in the literature aiming to tackle the aforementioned problems arising in NCSs (see, for example, [9]–[12]). In particular, three main approaches prevailing the current literature are worth mentioning: the discrete-time approach, the time-delay approach, and the impulsive/hybrid system approach. The first approach analyzes/designs the NCS by discretizing the continuous-time physical plants. This approach provides simple design conditions, while the main drawback lies in that discretization overlooks the knowledge about the intersampling behavior, and it can be hardly used for performance analysis, control of uncertain systems, and/or uncertain variable sampling network environments. The deficiency of the discrete-time method can be nicely overcome under the time-delay approach. However, as argued in [13]–[15], most existing results based on this approach rely on constructing complete forms of Lyapunov-Krasovskii functionals (LKFs) to improve the analysis condition of the NCSs. This could result in a rather complicated control synthesis problem, which could be very difficult to solve especially when more complicated LKFs are involved. The third approach based on the impulsive/hybrid system theory provides an effective tool for modeling NCSs, and it has been demonstrated to be successful due to its simplicity and accuracy in both analysis and synthesis of NCSs [10]–[12], [16]. Comprehensive comparisons among these three approaches can be found in [10].

In this paper, we will propose a new approach for NCS analysis and synthesis by combining two different control methods, that is, the linear impulsive control technique and dynamic integral quadratic constraints (IQCs)-based robust control technique. As a special type of hybrid system, linear impulsive systems, which are typically modeled by the combination of ordinary differential equations and instantaneous state jumps [17]–[20], have been intensively studied in the control community due to their capability in modeling highly complex systems involving discrete events and/or systems with large uncertainties, such as sampled-data systems [10], [16] and NCSs [11], [12]. Specifically, we will consider a class of NCSs with time-varying delays distributed to both channels from the continuous-time plant to the discrete-time controller (measurement delay) and from the discrete-time controller back to the continuous-time plant (actuation delay). For measurement delay, the associated delay behavior will be modeled as an impulsive effect acting on the nominal NCS dynamics. Therefore, it will be treated using hybrid control tools, such as the ranged dwell time techniques and clock-dependent storage functions [16]. On the other hand, for actuation delay, the considered NCS is first transformed to an equivalent linear fractional transformation (LFT) model, so that dynamic IQCs can be used to fully capture the nonlinear behavior of the time-varying delay. It is worth mentioning that the concept of dynamic IQCs was originally proposed by Megretski and Rantzer in [21] to model different types of nonlinearities and uncertainties, such as saturation, dead zone, delay, parametric or unmodeled dynamics, and so on. Later on, it was applied successfully for stability and stabilization of various uncertain dynamical systems (see, for instance, [22]–[25]). In particular, a library of IQC multipliers has been developed by Kao and Rantzer in [22] for continuous time-delay systems, and by Kao in [23] for the discrete-time case. However, incorporating IQCs with time-delay control techniques for analysis and design of NCSs has not been fully explored, and few results can be found in the literature along this topic. This motivates our current research work to solve the challenging NCS analysis and synthesis problems using the IQC mechanism.

The main contribution of this paper is that a novel delay scheduled impulsive (DSI) controller is presented for the networked systems under the proposed system setup. To this end, the DSI controller is constructed in a full-information feedback manner, that is, both plant states and the IQC dynamic states are used for feedback control, while real-time measurements of the network induced time-varying delays (measurement delay and actuation delay) are also used for controller gain scheduling. With this DSI control scheme, robust $\mathcal{L}_2$ stability analysis and controller synthesis problems are investigated utilizing the IQC mechanism and clock-dependent storage functions. In particular, the control synthesis conditions are formulated as a finite number of linear matrix inequalities (LMIs) by specifying the clock-dependent storage function in a piecewise linear quadratic form, which can be solved effectively using existing convex optimization algorithms [26]. It is worth mentioning that virtually all of the works from the literature address the stabilization problem for NCSs subject to time-varying delays by using robust linear time-invariant (LTI) controllers. The proposed DSI controller is more appealing in the sense that it utilizes the network induced delay information and takes it into account in the control design process.

The goal of this paper is not to develop sharper IQCs for better analysis results on the stability of NCSs, but to propose a simple yet systematic framework for NCS analysis and synthesis via a new DSI control strategy. The proposed design scheme is very general and can accommodate many different types of (less conservative) IQCs. Furthermore, it can be readily extended to systems with more complex settings (such as time-varying polytopic uncertain systems, switching systems, and so on) under more challenging control scenarios (such as output-feedback digital control, feedback with measurement noises, etc.).

The rest of this paper is organized as follows. Some preliminary results on IQCs, together with the problem statement, are given in Section II. A new network-based controller structure, that is, the DSI controller, is presented in Section III. Both problems of robust $\mathcal{L}_2$ stability analysis and $\mathcal{H}_\infty$ controller synthesis are addressed using dynamic IQCs in Sections IV and V, respectively. An application to a dc motor system will be provided in Section VI for illustration. The conclusion is finally drawn in Section VII.

Notation: $\mathbb{R}$ and $\mathbb{C}$ stand for the set of real and complex numbers, respectively. $\mathbb{R}_+$ stands for the set of positive real numbers. The set of positive (non-negative) integers is denoted by $\mathbb{N}_+$ ($\mathbb{N}$). $\mathbb{R}^{m\times n}$ ($\mathbb{C}^{m\times n}$) is the set of real (complex) $m\times n$ matrices, and $\mathbb{R}^n$ ($\mathbb{C}^n$) represents the set of real (complex) $n\times 1$ vectors. The identity matrix of dimension $n\times n$ is denoted by I_n. $\mathbb{S}^n$ and $\mathbb{S}^n_+$ are used to denote the sets of real symmetric $n\times n$ matrices and positive definite matrices, respectively. A block-diagonal matrix with matrices $X_1, X_2, \ldots, X_p$ on its main diagonal is denoted by $\mathrm{diag}\{X_1, X_2, \ldots, X_p\}$. Furthermore, we use the symbol $\star$ in LMIs to denote entries that follow from symmetry. For two integers $k_1 < k_2$, we denote $\mathbf{I}[k_1, k_2] = \{k_1, k_1+1, \ldots, k_2\}$. For $s \in \mathbb{C}$, $\bar{s}$ denotes the complex conjugate of s. For a matrix $M \in \mathbb{C}^{m\times n}$, M^T denotes its transpose and M^* denotes the complex conjugate transpose. $\mathbb{RL}_\infty$ denotes the set of rational functions with real coefficients that are proper and have no poles on the imaginary axis. $\mathbb{RH}_\infty$ is the subset of functions in $\mathbb{RL}_\infty$ that are analytic in the closed right half of the complex plane. $\mathbb{RL}_\infty^{m\times n}$ and $\mathbb{RH}_\infty^{m\times n}$ denote the sets of $m\times n$ matrices whose elements are in $\mathbb{RL}_\infty$ and $\mathbb{RH}_\infty$, respectively. The para-Hermitian conjugate of $G \in \mathbb{RL}_\infty^{m\times n}$, denoted as $G^\sim$, is defined by $G^\sim(s) := G(-\bar{s})^*$. For $x \in \mathbb{C}^n$, its norm is defined as $\|x\| := (x^* x)^{1/2}$. L^n_{2+} is the space of functions $u : [0,\infty) \to \mathbb{R}^n$ satisfying $\|u\|_2 := \left(\int_0^\infty u^T(t)u(t)d\right)^{1/2} < \infty$. Given $u \in L^n_{2+}$, u_T denotes the truncated function $u_T(t) = u(t)$ for $t \le T$ and $u_T(t) = 0$ otherwise. The extended space, denoted as L_{2e+}, is the set of functions u such that $u_T \in L_{2+}$ for all $T \ge 0$.

II. Problem Formulation and Preliminaries

A. *Networked Control System and Control Objective*

The structure of the considered NCS is shown in Fig. 1. It is seen that the physical plant is controlled by a remote controller through network communications. Assuming the controlled plant is connected with a sampler of a sampling period τ_s and a zero-order-hold (ZOH) device, and the network possesses time-varying delays. The mathematical model of the physical plant can be described as a continuous-time linear time-invariant (LTI) system

$$\begin{aligned}\dot{x}_p(t) &= A_p x_p(t) + B_{p1} d(t) + B_{p2} u(t)\\ e(t) &= C_p x_p(t) + D_{p1} d(t) + D_{p2} u(t)\end{aligned} \tag{1}$$

where $x_p \in \mathbb{R}^{n_x}$ is the plant state, $d \in \mathbb{R}^{n_d}$ is the generalized exogenous disturbance, $e \in \mathbb{R}^{n_e}$ is the error/performance output, and $u \in \mathbb{R}^{n_u}$ represents the control input signal. To facilitate the control design, the networked controlled system at the controller side can be represented by the following linear model with input and output delays:

$$\begin{aligned}\dot{x}_p(t) &= A_p x_p(t) + B_{p1} d(t) + B_{p2} v\left(t - \tau_1(t)\right)\\ e(t) &= C_p x_p(t) + D_{p1} d(t) + D_{p2} v(t - \tau_1(t)\\ y(t) &= x_p\left(t - \tau_2(t)\right)\end{aligned} \tag{2}$$

where $v \in \mathbb{R}^{n_u}$ and $y \in \mathbb{R}^{n_y}$ represent two signals from and to the controller. $\tau_1(t), \tau_2(t)$ represents the time-varying actuation delay and measurement delay, respectively. It is assumed that these two time-delay functions, together with the sampling period τ_s, satisfy

$$\tau_s \in [\underline{\tau}_s, \overline{\tau}_s] \tag{3}$$

$$\tau_1 \in \mathcal{T}_1 := \{\tau_1 : \mathbb{R}_+ \to [\underline{\tau}_1, \overline{\tau}_1], |\dot{\tau}_1(t)| \le r_1\} \tag{4}$$

$$\tau_2 \in \mathcal{T}_2 := \{\tau_2 : \mathbb{R}_+ \to [\underline{\tau}_2, \overline{\tau}_2], |\dot{\tau}_2(t)| \le r_2\} \tag{5}$$

where $0 < \underline{\tau}_s \le \overline{\tau}_s$, $0 \le \underline{\tau}_1 \le \overline{\tau}_1$, and $0 \le \underline{\tau}_2 \le \overline{\tau}_2$, as well as $r_1 \ge 0$ and $r_2 \ge 0$ are known constants. Note that no constraint is imposed on the upper bounds of r_1 and r_2; we thus also take into account cases of network-induced delays with unbounded variation rates, such as random delays, discrete-time delays, and so on. All of these cases with unbounded delay derivatives correspond to the fast-varying delay control problem, as has been extensively discussed in [5]. Moreover, it should be pointed out that when order-preserving communication/transmission channels are considered in the NCS, we typically have $r_1 = r_2 = 1$, see more detailed discussions in [27]. On the other hand, for delay scheduled control, the time-varying delays $\tau_1(t)$ and $\tau_2(t)$ are assumed to be unknown in advance but measurable in real time. All of the system matrices are known constant matrices with appropriate dimensions. In addition, for simplicity, it is also assumed that the network possesses the logical choice capability to ensure that only the latest data will be utilized.

In this paper, our objective is to design a control law using the IQC mechanism such that the overall networked control system is robustly stabilized against the network-induced delays $\tau_1(t)$ and $\tau_2(t)$, and achieves an $\mathcal{L}_2$-gain performance from the disturbance $d(t)$ to the error output $e(t)$, that is, $\|e(t)\|_2 < \gamma \|d(t)\|_2$ for some positive number γ under zero initial conditions.

B. Some Basic Definitions

Before proceeding further, some basic definitions of IQCs will be first recalled.

Definition 1 ([28]): Let $\Pi \in \mathbb{RL}_\infty^{(m_1+m_2)\times(m_1+m_2)}$ be a proper, rational function, called a "multiplier," such that $\Pi = \Psi^\sim W \Psi$ with $W \in \mathbb{R}^{n_z \times n_z}$ and $\Psi \in \mathbb{RH}_\infty^{n_z \times (m_1+m_2)}$. Then, two signals $q \in L_{2e+}^{m_1}$ and $p \in L_{2e+}^{m_2}$ satisfy the IQC defined by the multiplier Π, and (Ψ, W) is a hard IQC factorization of Π if the following inequality holds for all $T \ge 0$:

$$\int_0^T z^T(t) W z(t) dt \ge 0 \tag{6}$$

where $z \in \mathbb{R}^{n_z}$ denotes the filtered output of Ψ driven by inputs (q, p) with zero initial conditions, that is, $z = \Psi \begin{bmatrix} q \\ p \end{bmatrix}$. Moreover, a bounded, causal operator $\mathcal{S} : L_{2e+}^{m_1} \to L_{2e+}^{m_2}$ satisfies the IQC defined by Π if condition (6) holds for all $q \in L_{2e+}^{m_1}$, $p = \mathcal{S}(q)$ and all $T \ge 0$.

Note that the factorization of IQC multiplier $\Pi = \Psi^\sim W \Psi$ is not unique but can be computed with state-space methods [29]. Furthermore, it has been demonstrated that a broad class of IQC multipliers possesses a hard factorization [21]. More discussions about the hard IQCs as defined above can be found in [28] and [30]. The concept of hard IQC, together with the following factorization definition and lemma, is instrumental for using IQCs within the dissipation inequality framework.

Definition 2 ([28]): (Ψ, W) is called a J_{m_1,m_2}-spectral factorization of $\Pi = \Pi^\sim \in \mathbb{RL}_\infty^{(m_1+m_2)\times(m_1+m_2)}$ if $\Pi = \Psi^\sim W \Psi$, $W = \begin{bmatrix} X_1 & 0 \\ 0 & -X_2 \end{bmatrix}$, and $\Psi, \Psi^{-1} \in \mathbb{RH}_\infty^{(m_1+m_2)\times(m_1+m_2)}$, $X_1 \in \mathbb{S}_+^{m_1}$, $X_2 \in \mathbb{S}_+^{m_2}$.

Note that with a J_{m_1,m_2}-spectral factorization (Ψ, W), Ψ is always square, stable, and minimum phase [28].

Lemma 1 ([28]): Let $\Pi = \Pi^\sim \in \mathbb{RL}_\infty^{(m_1+m_2)\times(m_1+m_2)}$ be partitioned as $\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{12}^\sim & \Pi_{22} \end{bmatrix}$, where $\Pi_{11} \in \mathbb{RL}_\infty^{m_1 \times m_1}$ and $\Pi_{22} \in \mathbb{RL}_\infty^{m_2 \times m_2}$. Assuming that $\Pi_{11}(j\omega) > 0$ and $\Pi_{22}(j\omega) < 0$ for all $\omega \in \mathbb{R} \cup \{\infty\}$. Then, Π has a J_{m_1,m_2}-spectral factorization (Ψ, W), which is also a hard factorization of Π.

IQCs, as a powerful tool for modeling a large variety of nonlinearities, have been demonstrated to be successful for robust stability analysis of various dynamical systems (see, for example, [21]–[23], [28], [30], [31]). A thorough review on this topic is out of the scope of this paper and we refer interested readers to the aforementioned references for more detailed discussions.

III. Delay Scheduled Impulsive Controller Structure

Prior to presenting the controller structure, a model transformation will be first performed on the original system (2) in order to single out the actuation delay nonlinearity from the nominal LTI dynamics. This will result in a new linear fractional transformation (LFT) model with a delay uncertainty block. Specifically, define $p(t) = \mathcal{S}_{\tau_1}(v(t)) := v(t - \tau_1(t)) - v(t)$, then we are able to rewrite the input-delayed system (2) as the following LFT system:

$$\begin{aligned}\dot{x}_p(t) &= A_p x_p(t) + B_{p1} d(t) + B_{p2} p(t) + B_{p2} v(t)\\ e(t) &= C_p x_p(t) + D_{p1} d(t) + D_{p2} p(t) + D_{p2} v(t)\\ y(t) &= x_p\left(t - \tau_2(t)\right)\\ p(t) &= \mathcal{S}_{\tau_1}\left(v(t)\right).\end{aligned} \tag{7}$$

Under this formulation, the nonlinear behavior of the time-varying delay $\tau_1(t)$ can thus be captured using dynamic IQCs as defined in Section II-B. To simplify the presentation and apply the IQC and dissipation theory [28], [32], we have the following assumption regarding the delay operator $\mathcal{S}_{\tau_1}$.

Assumption 1: $\mathcal{S}_{\tau_1}$ satisfies a collection of IQCs defined by $\{\Pi_i\}_{i=1}^{N_\pi} \in \mathbb{RL}_\infty^{2n_u \times 2n_u}$, where the multipliers $\{\Pi_i\}_{i=1}^{N_\pi}$ can be partitioned as $\begin{bmatrix}\Pi_{11,i} & \Pi_{12,i}\\ \Pi_{12,i}^{\sim} & \Pi_{22,i}\end{bmatrix}$ with $\Pi_{11,i}$ of dimension $n_u \times n_u$. Each multiplier satisfies $\Pi_{11,i}(j\omega) > 0$ and $\Pi_{22,i}(j\omega) < 0$ for all $\omega \in \mathbb{R} \cup \{\infty\}$. Furthermore, for each $i \in \mathbf{I}[1, N_\pi]$, Π_i has a J_{n_u,n_u}-spectral factorization (Ψ_i, W_i) in the form of $\Psi_i = \begin{bmatrix}\Psi_{11,i} & \Psi_{12,i}\\ 0 & I_{n_u}\end{bmatrix} \in \mathbb{RH}_\infty^{(n_u+n_u)\times(n_u+n_u)}$ and $W_i = \begin{bmatrix}X_i & 0\\ 0 & -X_i\end{bmatrix} \in \mathbb{R}^{(n_u+n_u)\times(n_u+n_u)}$, where $X_i \in \mathbb{S}_+^{n_u}$.

We stress that Assumption 1 does not cause any loss of generality. The IQCs are used to bound the input–output behavior of the delay operator $\mathcal{S}_{\tau_1}$. According to Lemma 1 and [28], the strict definiteness assumptions on $\{\Pi_{11,i}\}_{i=1}^{N_\pi}$ and $\{\Pi_{22,i}\}_{i=1}^{N_\pi}$ are typically adopted in the literature (e.g., [28], [30]). They guarantee the existence of a J_{n_u,n_u}-spectral factorization (Ψ_i, W_i) for each multiplier Π_i $(\forall i \in \mathbf{I}[1, N_\pi])$, such that Ψ_i is square, stable, and minimum phase. As such, the last part of Assumption 1 is not restrictive and is made in order to simplify the derivations. In fact, this assumption can be relaxed with more extensive and complicated formulae [13]–[15].

Under Assumption 1, the IQC-induced LTI system $\{\Psi_i\}_{k=1}^{N_\pi}$ for the delay nonlinearity $\mathcal{S}_{\tau_1}(v)$ in system (7) can be described in the following state-space form:

$$\begin{bmatrix}\dot{x}_\psi\\ z_i\end{bmatrix} = \begin{bmatrix}A_\psi & B_{\psi1} & B_{\psi2}\\ C_{\psi,i} & D_{\psi1,i} & D_{\psi2,i}\end{bmatrix}\begin{bmatrix}x_\psi\\ v\\ p\end{bmatrix}, \quad i \in \mathbf{I}[1, N_\pi] \tag{8}$$

where $x_\psi \in \mathbb{R}^{n_\psi}$ denotes the state vector of the operator $\{\Psi_i\}_{i=1}^{N_\pi}$ with $x_\psi(0) = 0$. $z_i \in \mathbb{R}^{n_z}$ with $n_z = 2n_u$ for all $i \in \mathbf{I}[1, N_\pi]$ are the operator outputs. In particular, Assumption 1 renders the associated output matrices with the following structure for all $i \in \mathbf{I}[1, N_\pi]$:

$$C_{\psi,i} = \begin{bmatrix}\bar{C}_{\psi,i}\\ 0\end{bmatrix}, \quad D_{\psi1,i} = \begin{bmatrix}\bar{D}_{\psi1,i}\\ 0\end{bmatrix}, \quad D_{\psi2,i} = \begin{bmatrix}\bar{D}_{\psi2,i}\\ I_{n_u}\end{bmatrix}.$$

Based on the above system setup, and in order to fulfill the control objective, we will propose a new IQC-based control synthesis scheme for the networked control system in Fig. 1 via full-information feedback. For full-information feedback control, we mean that in addition to the plant state x_p and the IQC dynamic state x_ψ, the two network-induced time-varying delays $\tau_1(t)$ and $\tau_2(t)$ are also assumed to be measurable online for scheduling controller gains. Specifically, the proposed control law can be written with the following delay-scheduled impulsive (DSI) controller structure:

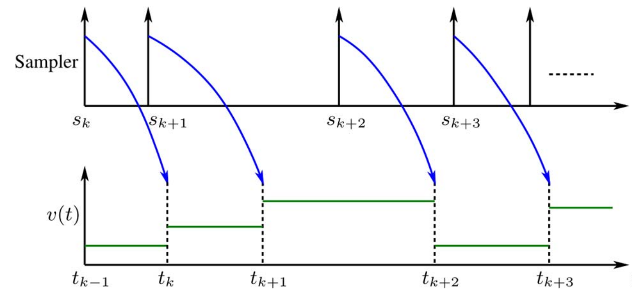

Fig. 2. Timing diagram of the NCS measurement channel.

$$v(t) = F_{c1}(\rho_k) x_p(t_k) + F_{c2}(\rho_k) x_\psi(t_k) + F_{c3}(\rho_k) v(t_{k-1}) \tag{9}$$

for all $t \in [t_k, t_{k+1})$ and $\rho_k := t_k - t_{k-1}$, where $\{t_k\}_{k\in\mathbb{N}_+}$ denotes the sequence of impulse instants, that is, the time instants when the controller updates its output $v(t)$ as soon as it receives a new sample/packet from the sensor. See the illustration in Fig. 2, where $\{s_k\}_{k\in\mathbb{N}_+}$ represents the sampling instants. The impulse time sequence $\{t_k\}_{k\in\mathbb{N}_+}$ is assumed to be strictly increasing and unbounded, that is, $t_k \to \infty$, excluding therefore any Zeno behavior. Furthermore, it should be noted that $\{t_k\}_{k\in\mathbb{N}_+}$ is defined at the controller side (i.e., from the y signal channel as shown in Fig. 1) but not from the output of the sampler. The benefits of doing so is that the sampling behaviors of the sampler (periodic/aperiodic), together with the networked communication medium over the measurement channel, can be modeled as an impulsive dynamics augmented to the physical plant. Thus, all associated delay information can be merged into the impulse sequence $\{t_k\}_{k\in\mathbb{N}_+}$, which will significantly simplify the controller design task. We assume that the impulse sequence $\{t_k\}_{k\in\mathbb{N}_+}$ obeys a ranged dwell-time constraint, that is, $\rho_k = t_k - t_{k-1} \in [T_{\min}, T_{\max}]$ with $0 \le T_{\min} \le T_{\max} < \infty$. As such, by taking into account the sampling time τ_s, it is straightforward to have $T_{\min} = \underline{\tau}_s + \underline{\tau}_2$ and $T_{\max} = \overline{\tau}_s + \overline{\tau}_2$. Moreover, $F_{c1} : \mathbb{R}_+ \to \mathbb{R}^{n_u \times n_x}$, $F_{c2} : \mathbb{R}_+ \to \mathbb{R}^{n_u \times n_\psi}$, and $F_{c3} : \mathbb{R}_+ \to \mathbb{R}^{n_u \times n_u}$ are the controller gains scheduled by the real-time measurement of ρ_k at each impulse time instant t_k.

Remark 1: Note that the aforementioned discussions on network-induced measurement delays can be trivially extended to consider more complicated cases of data packet dropout (or communication outage) [10], [27]. Specifically, when data packet loss occurs, it can be incorporated in ρ_k because losing N packets would simply mean that the controller needs to wait $N\tau_s + \tau_2$ instead of $\tau_s + \tau_2$ seconds.

It should be noted that we have adopted two different approaches to deal with two different network-induced delays, that is, the actuation delay $\tau_1(t)$ and the measurement delay $\tau_2(t)$. The actuation delay $\tau_1(t)$ is treated as a nonlinear perturbation interconnecting to a nominal linear system [see (7)] and dynamic IQCs are used to characterize its input–output behavior, while the measurement delay information $\tau_2(t)$ is hidden in the impulsive dynamics as mentioned before. Both types of delay information will be used in the control law (9). Here, several comments are needed to clarify the use of the delay information for feedback control.

- First, regarding the actuation delay $\tau_1(t)$, we emphasize that the controller (9) is capable of memorizing all delayed (past) information of the control input signal over the time window $[t-\bar{\tau}_1, t]$. This stored information will be revoked and used to calculate the corresponding input $p = v(t-\tau_1(t)) - v(t)$ for the IQC-induced dynamics (8) online. Furthermore, it should be pointed out that since the IQC-induced LTI system (8) is artificially introduced for robustness analysis and control synthesis; thus, once the realization in (8) is fixed, the corresponding system state x_ψ can be readily computed online as the associated system input signals v and p are available in real time under the proposed control scheme.
- Second, as opposed to the actuation delay case, the use of measurement delay information $\tau_2(t)$ in controller implementation is more straightforward. As mentioned before, real-time measurement of $\tau_2(t)$ will be applied to schedule the controller parameters in (9) through the scheduling parameter ρ_k at each impulse time instant t_k.
- Finally, it should be noted that this type of delay controller with online delay scheduling is typically adopted in the literature, and also has wide applications in the control of physical systems (see, for example, [33], [34]). On the other hand, this delay-scheduled-type controller is different from the delay-memory-type controller considered in [14] and [15]. Specifically, the latter case is concerned with the design of controller with memory where the delay is involved as an operator in the controller structure, whereas the delay information is involved as a scheduling parameter for the former case.

By connecting the controller (9) to the controlled plant (7) and absorbing the IQC-induced dynamics (8), the resulting closed-loop system can be equivalently formulated as the following impulsive system:

$$\left.\begin{aligned}\dot{x}_{cl}(t) &= A_{cl}x_{cl}(t) + B_{cl0}p(t) + B_{cl1}d(t)\\ z_i(t) &= C_{cl0,i}x_{cl}(t) + D_{cl00,i}p(t) + D_{cl01,i}d(t)\\ e(t) &= C_{cl1}x_{cl}(t) + D_{cl10}p(t) + D_{cl11}d(t)\end{aligned}\right\},\ t \neq t_k$$

$$x_{cl}(t^+) = A_J(\rho_k)x_{cl}(t^-),\ t = t_k \tag{10}$$

where $x_{cl}(t) := [x_p^T(t)\ x_\psi^T(t)\ x_v^T(t)]^T$ $x_v(t) := v(t_k)$ for $t \in [t_k, t_{k+1})$, and

$$A_{cl} = \begin{bmatrix} A_p & 0 & B_{p2}\\ 0 & A_\psi & B_{\psi 1}\\ 0 & 0 & 0\end{bmatrix},\quad B_{cl0} = \begin{bmatrix} B_{p2}\\ B_{\psi 2}\\ 0\end{bmatrix}$$

$$B_{cl1} = \begin{bmatrix} B_{p1}\\ 0\\ 0\end{bmatrix}\ C_{cl0,i} = \begin{bmatrix}\bar{C}_{cl0,i}\\ 0\end{bmatrix} := \begin{bmatrix}[0 & C_{\psi,i} & D_{\psi 1,i}]\\ & 0 & \end{bmatrix}$$

$$C_{cl1} = \begin{bmatrix} C_p & 0 & D_{p2}\end{bmatrix},\ D_{cl00,i} = \begin{bmatrix}\bar{D}_{cl00,i}\\ I_{n_u}\end{bmatrix} := \begin{bmatrix} D_{\psi 2,i}\\ I_{n_u}\end{bmatrix}$$

$$D_{cl01,i} = \begin{bmatrix}\bar{D}_{cl01,i}\\ 0\end{bmatrix} := 0,\quad D_{cl10} = D_{p2},\quad D_{cl11} = D_{p1}$$

$$\begin{aligned}A_J(\rho_k) &= A_{J0} + B_J F_c(\rho_k)\\ &:= \begin{bmatrix} I_{n_x} & 0 & 0\\ 0 & I_{n_\psi} & 0\\ 0 & 0 & 0\end{bmatrix}\\ &\quad + \begin{bmatrix}0\\ 0\\ I_{n_u}\end{bmatrix}\begin{bmatrix}F_{c1}(\rho_k) & F_{c2}(\rho_k) & F_{c3}(\rho_k)\end{bmatrix}.\end{aligned} \tag{11}$$

IV. $\mathcal{L}_2$ Stability Analysis via IQCs

In this section, we will establish an $\mathcal{L}_2$ stability condition for the impulsive closed-loop system (10) by using the IQCs and dissipation theory [28] incorporated with a clock-dependent storage function [16].

Theorem 1: Consider the impulsive closed-loop system (10). If there exist a differentiable matrix function $P : [0, T_{\max}] \to \mathbb{S}_+^{n_x+n_\psi+n_u}$, positive definite matrices $X_i \in \mathbb{S}_+^{n_u}$ for all $i \in \mathbf{I}[1, N_\pi]$, and a positive scalar $\gamma \in \mathbb{R}_+$ such that

$$\begin{bmatrix} He\{P(\tau)A_{cl}\} + \dot{P}(\tau) & \star & \star & \star & \star\\ B_{cl0}^T P(\tau) & -\sum_{i=1}^{N_\pi} X_i & \star & \star & \star\\ B_{cl1}^T P(\tau) & 0 & -\gamma I_{n_d} & \star & \star\\ \Upsilon_{41} & \Upsilon_{42} & \Upsilon_{43} & -\Lambda^{-1} & \star\\ C_{cl1} & D_{cl10} & D_{cl11} & 0 & -\gamma I_{n_e}\end{bmatrix} < 0 \tag{12}$$

$$\begin{bmatrix} -P(\theta) & \star\\ P(0)A_J(\theta) & -P(0)\end{bmatrix} \leq 0 \tag{13}$$

for all $\tau \in [0, T_{\max}]$ and $\theta \in [T_{\min}, T_{\max}]$, where $\Upsilon_{41} := \begin{bmatrix}\bar{C}_{cl0,1}\\ \vdots\\ \bar{C}_{cl0,N_\pi}\end{bmatrix}$, $\Upsilon_{42} := \begin{bmatrix}\bar{D}_{cl00,1}\\ \vdots\\ \bar{D}_{cl00,N_\pi}\end{bmatrix}$, $\Upsilon_{43} := \begin{bmatrix}\bar{D}_{cl01,1}\\ \vdots\\ \bar{D}_{cl01,N_\pi}\end{bmatrix}$, and $\Lambda := \mathrm{diag}\{X_1, \ldots, X_{N_\pi}\}$. Then, the impulsive closed-loop system (10) is stable and achieves an $\mathcal{L}_2$ gain less than γ.

Proof: Motivated by [16], we define a clock-dependent storage function $V(x_{cl}, \tau) = x_{cl}^T P(\tau) x_{cl}$ for the impulsive system (10) with $P(\tau) > 0$ for all $\tau \in [0, T_{\max}]$, then condition (12) can be rewritten as follows via the Schur complement:

$$\begin{aligned}&\begin{bmatrix} He\{P(\tau)A_{cl}\} + \dot{P}(\tau) & \star & \star\\ B_{cl0}^T P(\tau) & 0 & \star\\ B_{cl1}^T P(\tau) & 0 & -\gamma I_{n_d}\end{bmatrix} + \frac{1}{\gamma}\begin{bmatrix} C_{cl1}^T\\ D_{cl10}^T\\ D_{cl11}^T\end{bmatrix}\\ &\times \begin{bmatrix} C_{cl1} & D_{cl10} & D_{cl11}\end{bmatrix} + \sum_{i=1}^{N_\pi}\begin{bmatrix}\bar{C}_{cl0,i}^T & 0\\ \bar{C}_{cl00,i}^T & I_{n_u}\\ \bar{D}_{cl01,i}^T & 0\end{bmatrix}\\ &\times \begin{bmatrix} X_i & 0\\ 0 & -X_i\end{bmatrix}\begin{bmatrix}\bar{C}_{cl0,i} & \bar{D}_{cl00,i} & \bar{D}_{cl01,i}\\ 0 & I_{n_u} & 0\end{bmatrix} < 0\end{aligned} \tag{14}$$

for all $\tau \in [0, T_{\max}]$. In light of the fact that $T_k = t_k - t_{k-1} \in [T_{\min}, T_{\max}]$ for all $k \in \mathbb{N}_+$, left and right multiply (14) by

$[x_{cl}^T(t), p^T(t), d^T(t)]$ and $[x_{cl}^T(t), p^T(t), d^T(t)]^T$ for $t \in [t_k, t_{k+1})$ to show that V satisfies

$$\dot{V}(x_{cl}, \tau(t)) + \gamma^{-1} e^T(t) e(t) - \gamma d^T(t) d(t) + \sum_{i=1}^{N_\pi} z_i^T(t) W_i z_i(t) < 0, \quad \forall t \in [t_k, t_{k+1}) \tag{15}$$

where τ is a time-shift operator such that $\tau(t_k) = 0$. Furthermore, integrating both sides of the above inequality from $t = t_k$ to $t = t_{k+1}^-$, we obtain

$$V\left(x_{cl}\left(t_{k+1}^-\right), \tau\left(t_{k+1}^-\right)\right) - V\left(x_{cl}(t_k), 0\right) + \gamma^{-1} \int_{t_k}^{t_{k+1}^-} e^T(t) e(t) dt - \gamma \int_{t_k}^{t_{k+1}^-} d^T(t) d(t) dt + \sum_{i=1}^{N_\pi} \int_{t_k}^{t_{k+1}^-} z_i^T(t) W_i z_i(t) < 0. \tag{16}$$

Now, we examine the stability for the jump dynamics of the impulsive system (10). At the impulse instant t_k, condition (13) guarantees that

$$-P(\rho_k) + A_J^T(\rho_k) P(0) A_J(\rho_k) \le 0$$

for all $\rho_k := t_k - t_{k-1} \in [T_{\min}, T_{\max}]$. Again, since we have $t_k - t_{k-1} \in [T_{\min}, T_{\max}]$ for all $k \in \mathbb{N}_+$, the above inequality further implies that

$$V\left(x_{cl}\left(t_k^+\right), 0\right) - V\left(x_{cl}\left(t_k^-\right), \tau\left(t_k^-\right)\right) \le 0, \quad \forall k \in \mathbb{N}_+. \tag{17}$$

Together with condition (16), (17) yields by summing up the left-hand side of (16) from t_0 to t_{k+1}^- that

$$V\left(x_{cl}\left(t_{k+1}^-\right), \tau\left(t_{k+1}^-\right)\right) - V\left(x_{cl}(t_0), 0\right) + \gamma^{-1} \int_{t_0}^{t_{k+1}^-} e^T(t) e(t) dt - \gamma \int_{t_0}^{t_{k+1}^-} d^T(t) d(t) dt + \sum_{i=1}^{N_\pi} \int_{t_0}^{t_{k+1}^-} z_i^T(t) W_i z_i(t) < 0. \tag{18}$$

Since Assumption 1 ensures that $\int_0^T z_i^T(t) W_i z_i(t) \ge 0$ for all $T \ge 0$, the above condition indicates with $t_0 = 0$ that

$$V\left(x_{cl}\left(t_{k+1}^-\right), \tau\left(t_{k+1}^-\right)\right) - V\left(x_{cl}(0), 0\right) + \gamma^{-1} \int_{0}^{t_{k+1}^-} e^T(t) e(t) dt - \gamma \int_{0}^{t_{k+1}^-} d^T(t) d(t) dt < 0. \tag{19}$$

With zero initial conditions and non-negativity of $V(x_{cl}(\infty), \tau(\infty))$, as $t_{k+1} \to \infty$, the above condition gives

$$\int_0^\infty e^T(t) e(t) dt < \gamma^2 \int_0^\infty d^T(t) d(t) dt \tag{20}$$

which implies that an $\mathcal{L}_2$ disturbance attenuation level γ is achieved, that is, $\|e(t)\|_2 < \gamma \|d(t)\|_2$ and, thus, the closed-loop system (10) is stable. ■

V. Controller Synthesis

Based on the IQC analysis results established in Theorem 1, we are now in position to examine the synthesis problem for the proposed DSI controller.

Theorem 2: Consider the open-loop system (2). If there exist a differentiable matrix function $Q : [0, T_{\max}] \to \mathbb{S}_+^{n_x + n_\psi + n_u}$, a matrix function $\hat{F}_c : [T_{\min}, T_{\max}] \to \mathbb{R}^{n_u \times (n_x + n_\psi + n_u)}$, positive definite matrices $Y_i \in \mathbb{S}_+^{n_u}$ for all $i \in \mathbf{I}[1, N_\pi]$, a rectangular matrix $Y \in \mathbb{R}^{n_u \times n_u}$, and a positive scalar $\gamma \in \mathbb{R}_+$ such that the conditions (21) and (22) (shown at the bottom of the page) hold for all $\tau \in [0, T_{\max}]$ and $\theta \in [T_{\min}, T_{\max}]$, where $\hat{\Upsilon}_{41} := \begin{bmatrix} [0 & C_{\psi,1} & D_{\psi1,1}] \\ & \vdots & \\ [0 & C_{\psi,N_\pi} & D_{\psi1,N_\pi}] \end{bmatrix}$, $\hat{\Upsilon}_{42} := \begin{bmatrix} D_{\psi2,1} \\ \vdots \\ D_{\psi2,N_\pi} \end{bmatrix}$, and $\hat{\Lambda} := \operatorname{diag}\{Y_1, \ldots, Y_{N_\pi}\}$. Then, the network-controlled plant (2) is stabilized by the DSI controller (9) and achieves an $\mathcal{L}_2$ gain less than γ. Moreover, the controller coefficient matrices $F_{c1}(\rho_k)$, $F_{c2}(\rho_k)$ and $F_{c3}(\rho_k)$ can be calculated online at each

$$\begin{bmatrix} He\left\{\begin{bmatrix} A_p & 0 & B_{p2} \\ 0 & A_\psi & B_{\psi1} \\ 0 & 0 & 0 \end{bmatrix} Q(\tau)\right\} - \dot{Q}(\tau) & \star & \star & \star & \star \\ Y^T \begin{bmatrix} B_{p2}^T & B_{\psi2}^T & 0 \end{bmatrix} & \sum_{i=1}^{N_\pi}(Y_i - Y - Y^T) & \star & \star & \star \\ \begin{bmatrix} B_{p1}^T & 0 & 0 \end{bmatrix}^T & 0 & -\gamma I_{n_d} & \star & \star \\ \hat{\Upsilon}_{41} Q(\tau) & \hat{\Upsilon}_{42} Y & 0 & -\hat{\Lambda} & \star \\ \begin{bmatrix} C_p & 0 & D_{p2} \end{bmatrix} Q(\tau) & D_{p2} Y & D_{p1} & 0 & -\gamma I_{n_e} \end{bmatrix} < 0 \tag{21}$$

$$\begin{bmatrix} -Q(\theta) & \star \\ \begin{bmatrix} I_{n_x} & 0 & 0 \\ 0 & I_{n_\psi} & 0 \\ 0 & 0 & 0 \end{bmatrix} Q(\theta) + \begin{bmatrix} 0 \\ 0 \\ I_{n_u} \end{bmatrix} \hat{F}_c(\theta) & -Q(0) \end{bmatrix} \le 0 \tag{22}$$

impulse time instant t_k by $[F_{c1}(\rho_k) \quad F_{c2}(\rho_k) \quad F_{c3}(\rho_k)] = \hat{F}_c(\rho_k)Q^{-1}(\rho_k)$ with $\rho_k = t_k - t_{k-1}$ ($\forall\, k \in \mathbb{N}_+$).

Proof: Based on Theorem 1, using the fact that $-Y_i^{-1} \leq -Y^{-T}(Y + Y^T - Y_i)Y^{-1}$ with $Y_i := X_i^{-1}$ for all $i \in \mathbf{I}[1, N_\pi]$ and any nonsingular matrix $Y \in \mathbb{R}^{n_u \times n_u}$, we obtain the sufficient condition to (12) as (23) (shown at the bottom of the page), for all $\tau \in [0, T_{\max}]$. Then, substituting all of the system matrices of (11) into the above condition, and performing a congruence transformation with matrix $\mathrm{diag}\{Q(\tau) := P^{-1}(\tau), Y, I_{n_d}, I_{N_\pi n_u}, I_{n_e}\}$ on it, one can readily derive condition (13). Similarly, performing a congruence transformation with matrix $\mathrm{diag}\{Q(\theta), Q(0)\}$ on condition (13) will arrive at condition (22) by denoting

$$\hat{F}_c(\theta) = \begin{bmatrix} F_{c1}(\theta) & F_{c2}(\theta) & F_{c3}(\theta) \end{bmatrix} Q(\theta).$$

This ends the proof. ■

Remark 2: The proof of Theorem 2 involves a relaxation from condition (12) to condition (23) via $-Y_i^{-1} \leq -Y^{-T}(Y + Y^T - Y_i)Y^{-1}$. This may introduce some degree of conservatism to the resulting synthesis conditions only when multiple IQC multipliers are employed. That is to say, the controller synthesis conditions (21) and (22) are completely equivalent to the analysis ones (12) and (13) when using a single IQC multiplier.

In Theorem 2, the synthesis conditions (21) and (22) are formulated as parameter-dependent LMIs for all $\tau \in [0, T_{\max}]$ and $\theta \in [T_{\min}, T_{\max}]$. This may cause some computational difficulties in practice. The first one is to solve an infinite numbers of LMIs, one typical method to obtain an approximate solution is based on parameter space gridding [35], which, however, could be very involved and time-consuming. The second difficulty follows the first one in that the resulting gain-scheduling controller could be complicated (typically requires performing interpolation among a family of LTI controllers [35]) and is unreliable/risky to implement.

In order to overcome these two possible deficiencies and provide an easier-to-implement and reliable controller, we will propose the following method using a special type of clock-dependent storage function (i.e., piecewise linear functions). Specifically, we partition the parameter space $[0, T_{\min}]$ into $N_1 \in \mathbb{N}_+$ portions such that $[0, T_{\min}] = \cup_{\ell=0}^{N_1-1}[\tau_\ell, \tau_{\ell+1}]$ with $\tau_0 = 0$, $\tau_{N_1} = T_{\min}$, $\tau_{\ell+1} - \tau_\ell = \delta_{\ell+1} > 0$, and $\sum_{\ell=1}^{N_1} \delta_\ell = T_{\min}$. Similarly, partition the parameter space $[T_{\min}, T_{\max}]$ into $N_2 \in \mathbb{N}_+$ portions such that $[T_{\min}, T_{\max}] = \cup_{\ell=N_1}^{N_1+N_2-1}[\tau_\ell, \tau_{\ell+1}]$ with $\tau_{N_1+N_2} = T_{\max}$, $\tau_{\ell+1} - \tau_\ell = \delta_{\ell+1} > 0$ and $\sum_{\ell=N_1+1}^{N_1+N_2} \delta_\ell = T_{\max} - T_{\min}$. Then, assign Q_ℓ to each vertex at τ_ℓ for all $\ell \in \mathbf{I}[0, N_1 + N_2]$. Within each time interval, $Q(\tau)$ is specified in the following piecewise linear form:

$$Q(\tau) = Q_\ell + (Q_{\ell+1} - Q_\ell)\frac{\tau - \tau_\ell}{\delta_{\ell+1}}, \quad \tau \in [\tau_\ell, \tau_{\ell+1}). \tag{24}$$

A similar definition is also applied to $\hat{F}_c(\theta)$ over the parameter space $[T_{\min}, T_{\max}]$, i.e.,

$$\hat{F}_c(\theta) = \hat{F}_{c,\ell} + (\hat{F}_{c,\ell+1} - \hat{F}_{c,\ell})\frac{\theta - \theta_\ell}{\delta_{\ell+1}}, \quad \theta \in [\theta_\ell, \theta_{\ell+1}) \tag{25}$$

where $\ell \in \mathbf{I}[N_1, N_1 + N_2 - 1]$, and θ_ℓ is defined as the same as τ_ℓ for all $\ell \in \mathbf{I}[N_1, N_1 + N_2 - 1]$ such that $[T_{\min}, T_{\max}] = \cup_{\ell=N_1}^{N_1+N_2-1}[\theta_\ell, \theta_{\ell+1}]$ with $\theta_{N_1+N_2} = T_{\max}$, $\theta_{\ell+1} - \theta_\ell = \delta_{\ell+1} > 0$, and $\sum_{\ell=N_1+1}^{N_1+N_2} \delta_\ell = T_{\max} - T_{\min}$. It is clear that the defined matrix function in (24) is differentiable for any $\tau \in [\tau_\ell, \tau_{\ell+1})$ with $\ell \in \mathbf{I}[0, N_1 + N_2 - 1]$, that is, $\dot{Q}(\tau) = (Q_{\ell+1} - Q_\ell)/\delta_{\ell+1}$. As such, we have the following corollary to formulate the DSI control synthesis problem as finite numbers of LMIs by using the above-defined piecewise linear matrix functions.

Corollary 1: Consider the open-loop system (2). If there exist positive definite matrices $Q_\ell \in \mathbb{S}_+^{n_x+n_\psi+n_u}$ ($\forall\, \ell \in \mathbf{I}[0, N_1 + N_2]$), $Y_i \in \mathbb{S}_+^{n_u}$ ($\forall\, i \in \mathbf{I}[1, N_\pi]$); rectangular matrices $\hat{F}_{c,\ell_2} \in \mathbb{R}^{n_u \times (n_x+n_\psi+n_u)}$, $Y \in \mathbb{R}^{n_u \times n_u}$; and a positive scalar $\gamma \in \mathbb{R}_+$ such that conditions (26) and (27), shown at the bottom of the next page, and (28) hold for all $\ell_1 \in \mathbf{I}[0, N_1 + N_2 - 1]$ and $\ell_2 \in \mathbf{I}[N_1, N_1 + N_2]$

$$\begin{bmatrix} -Q_{\ell_2} & \star \\ \begin{bmatrix} I_{n_x} & 0 & 0 \\ 0 & I_{n_\psi} & 0 \\ 0 & 0 & 0 \end{bmatrix} Q_{\ell_2} + \begin{bmatrix} 0 \\ 0 \\ I_{n_u} \end{bmatrix} \hat{F}_{c,\ell_2} & -Q(0) \end{bmatrix} \leq 0 \tag{28}$$

where $\hat{\Upsilon}_{41}$, $\hat{\Upsilon}_{42}$, and $\hat{\Lambda}$ have the same definitions as in Theorem 2. Then, the network-controlled plant (2) is stabilized by the DSI controller (9) and achieves an $\mathcal{L}_2$ gain less than γ. Moreover, the controller coefficient matrices $F_{c1}(\rho_k)$, $F_{c2}(\rho_k)$, and $F_{c3}(\rho_k)$ can be calculated online at each impulse time instant t_k by

$$\begin{bmatrix} F_{c1}(\rho_k) & F_{c2}(\rho_k) & F_{c3}(\rho_k) \end{bmatrix} = \hat{F}_c(\rho_k)Q^{-1}(\rho_k) \tag{29}$$

with $\rho_k = t_k - t_{k-1}$ ($\forall\, k \in \mathbb{N}_+$), and $Q(\rho_k)$, $\hat{F}_c(\rho_k)$ are defined in (24) and (25), respectively.

Proof: The proof follows immediately the one of Theorem 2 combined with the piecewise linearity of $Q(\tau)$ and $\hat{F}_c(\theta)$. First, we observe from (24) and (25) that $Q(\tau)$ and $\hat{F}_c(\theta)$ can be rewritten as follows by denoting $\lambda_{11} = (\tau - \tau_{\ell_1})/\delta_{\ell_1+1}$, $\lambda_{12} = 1 - ((\tau - \tau_{\ell_1})/\delta_{\ell_1+1})$, and $\lambda_{21} = (\theta - \theta_{\ell_2})/\delta_{\ell_2+1}$,

$$\begin{bmatrix} He\{P(\tau)A_{cl}\} + \dot{P}(\tau) & \star & \star & \star & \star \\ B_{cl0}^T P(\tau) & \sum_{i=1}^{N_\pi} -Y^{-T}(Y + Y^T - Y_i)Y^{-1} & \star & \star & \star \\ B_{cl1}^T P(\tau) & 0 & -\gamma I_{n_d} & \star & \star \\ \hat{\Upsilon}_{41} & \hat{\Upsilon}_{42} & 0 & -\hat{\Lambda} & \star \\ C_{cl1} & D_{cl10} & D_{cl11} & 0 & -\gamma I_{n_e} \end{bmatrix} < 0 \tag{23}$$

$\lambda_{22} = 1 - ((\theta - \theta_{\ell_2})/\delta_{\ell_2+1})$ for all $\ell_1 \in \mathbf{I}[0, N_1 + N_2 - 1]$ and $\ell_2 \in \mathbf{I}[N_1, N_1 + N_2]$

$$Q(\tau) = \lambda_{11} Q_{\ell_1+1} + \lambda_{12} Q_{\ell_1}$$
$$\hat{F}_c(\theta) = \lambda_{21} \hat{F}_{c,\ell_2+1} + \lambda_{22} \hat{F}_{c,\ell_2}.$$

Apparently, we have $0 \le \lambda_{11} < 1$, $0 \le \lambda_{21} < 1$, $0 < \lambda_{12} \le 1$, $0 < \lambda_{22} \le 1$, and $\lambda_{11} + \lambda_{12} = \lambda_{21} + \lambda_{22} = 1$. Consequently, multiplying λ_{12} and λ_{11} to conditions (26) and (27) (shown at the bottom of the page), respectively, and then summing up the results, we get exactly the same condition (21) in Theorem 2 with definition (24). The sufficiency of conditions (22)–(28) can also be proved similarly based on definitions (24) and (25). ■

With Corollary 1 and the piecewise linearity in (24) and (25), all of the design variables appear linearly in the synthesis conditions (26)–(28), which means that the synthesis problem for the proposed DSI controller (9) is convex and formulated in terms of a finite number of LMIs. They can be posed as an optimization problem as below to obtain an optimal solution in the sense of the smallest $\mathcal{L}_2$ gain γ without involving any approximation (as opposed to the well-known gridding technique [35]), in turn, ensuring reliability of the corresponding controller realization (29)

$$\min_{Q_{\ell_1}, \hat{F}_{c,\ell_2}, Y, Y_i, \ \forall \ell_1 \in \mathbf{I}[0, N_1+N_2-1], \ell_2 \in \mathbf{I}[N_1, N_1+N_2], i \in \mathbf{I}[1, N_\pi]} \quad \gamma$$
$$\text{s.t.} \qquad (26)-(28). \tag{30}$$

Remark 3: It should be pointed out that the result of Corollary 1 could be more conservative than that of Theorem 2. Nevertheless, the conservatism can be reduced or even overcome by choosing larger N_1 and N_2 at the price of more computational efforts. Similar proposals can be found in [36] and [37].

VI. Example

In this section, an application to a simple dc motor problem will be used to demonstrate the IQC-based design procedure and the effectiveness of the proposed DSI control strategy (9).

TABLE I
DC Motor Parameters [38]

Parameter	Description	Value
R_a	armature resistant	4.67 Ω
L_a	armature inductance	0.17 H
J	moment of inertia	42.6×10^{-6} Kg-m^2
f	viscous-friction coefficient	47.3×10^{-6} N-$m/rad/sec$
K	torque constant	0.0147 N-m/A
K_b	back-EMF constant	0.0147 V-sec/rad

The continuous-time linear model of the dc motor is borrowed from [38], which can be written in the form of (1) with the system matrices given by

$$A_p = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{K_b}{L_a} \\ \frac{K}{J} & -\frac{f}{J} \end{bmatrix}, \quad B_{p1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad B_{p2} = \begin{bmatrix} \frac{1}{L_a} \\ 0 \end{bmatrix}$$
$$C_p = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad D_{p1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad D_{p2} = \begin{bmatrix} 0 \\ 10 \end{bmatrix} \tag{31}$$

where the control input u is the armature voltage of the dc motor, and the state variables $x_p := [i_a \ w]^T$ with i_a and w being the armature current and shaft rotational speed, respectively. The associated motor parameters are adopted from [38] and given as in Table I.

Our goal is to apply the proposed network-based control scheme to achieve optimal $\mathcal{H}_\infty$ stabilization for the aforementioned dc motor system. To this end, we assume that the dc motor plant is connected to a digital controller through a communication network (as shown in Fig. 1). The sampler is assumed to be with a periodic sampling period of $\tau_s = 0.01$ s, and the actuation and measurement delays satisfy (4) and (5) with $(\underline{\tau}_1, \overline{\tau}_1, r_1) = (0, 0.5, 0.1)$ and $(\underline{\tau}_2, \overline{\tau}_2, r_2) = (0.1, 0.3, \infty)$, respectively. In particular, $\tau_2(t)$ takes the form of $\tau_2(t) = m(t)\tau_s$, where $m(t) \in \mathbb{N}_+$ for all $t \ge 0$ is a positive integer randomly introduced to the measurement channel.

With such a system setup and following the proposed control methodology, the two network-induced delays $\tau_1(t)$ and $\tau_2(t)$ will be treated in two different ways. For the actuation delay $\tau_1(t)$, we select two dynamic IQC multipliers $\{\Pi_i\}_{i=1}^2$ from

$$\begin{bmatrix} He\left\{\begin{bmatrix} A_p & 0 & B_{p2} \\ 0 & A_\psi & B_{\psi1} \\ 0 & 0 & 0 \end{bmatrix} Q_{\ell_1}\right\} - \frac{Q_{\ell_1+1} - Q_{\ell_1}}{\delta_{\ell_1+1}} & \star & \star & \star & \star \\ Y^T \begin{bmatrix} B_{p2}^T & B_{\psi2}^T & 0 \end{bmatrix} & \sum_{i=1}^{N_\pi}(Y_i - Y - Y^T) & \star & \star & \star \\ \begin{bmatrix} B_{p1}^T & 0 & 0 \end{bmatrix}^T & 0 & -\gamma I_{n_d} & \star & \star \\ \hat{\Upsilon}_{41} Q_{\ell_1} & \hat{\Upsilon}_{42} Y & 0 & -\hat{\Lambda} & \star \\ \begin{bmatrix} C_p & 0 & D_{p2} \end{bmatrix} Q_{\ell_1} & D_{p2} Y & D_{p1} & 0 & -\gamma I_{n_e} \end{bmatrix} < 0 \tag{26}$$

$$\begin{bmatrix} He\left\{\begin{bmatrix} A_p & 0 & B_{p2} \\ 0 & A_\psi & B_{\psi1} \\ 0 & 0 & 0 \end{bmatrix} Q_{\ell_1+1}\right\} - \frac{Q_{\ell_1+1} - Q_{\ell_1}}{\delta_{\ell_1+1}} & \star & \star & \star & \star \\ Y^T \begin{bmatrix} B_{p2}^T & B_{\psi2}^T & 0 \end{bmatrix} & \sum_{i=1}^{N_\pi}(Y_i - Y - Y^T) & \star & \star & \star \\ \begin{bmatrix} B_{p1}^T & 0 & 0 \end{bmatrix}^T & 0 & -\gamma I_{n_d} & \star & \star \\ \hat{\Upsilon}_{41} Q_{\ell_1+1} & \hat{\Upsilon}_{42} Y & 0 & -\hat{\Lambda} & \star \\ \begin{bmatrix} C_p & 0 & D_{p2} \end{bmatrix} Q_{\ell_1+1} & D_{p2} Y & D_{p1} & 0 & -\gamma I_{n_e} \end{bmatrix} < 0 \tag{27}$$

[22] to characterize the associated delay operator $\mathcal{S}_{\tau_1}(v(t))$ in (7), i.e.,

$$\Pi_1(s) = \begin{bmatrix} |\phi(s)|^2 & 0 \\ 0 & -1 \end{bmatrix}, \quad \Pi_2(s) = \begin{bmatrix} |\varphi(s)|^2 & 0 \\ 0 & -1 \end{bmatrix} \tag{32}$$

where $\phi(s) = k_1((\bar{\tau}^2 s^2 + c_1\bar{\tau}s)/(\bar{\tau}^2 s^2 + a_1\bar{\tau}s + k_1 c_1)) + \epsilon$, $\varphi(s) = k_2((\bar{\tau}^2 s^2 + c_2\bar{\tau}s)/(\bar{\tau}^2 s^2 + a_2\bar{\tau}s + b_2)) + \delta$, with $k_1 = 1 + (1/\sqrt{1-r})$, $a_1 = \sqrt{2k_1 c_1}$, c_1 being any positive real number such that $c_1 < 2k_1$, $k_2 = \sqrt{8/(2-r)}$, $a_2 = \sqrt{6.5 + 2b_2}$, $b_2 = \sqrt{50}$, $c_2 = \sqrt{12.5}$, and ϵ, δ are two arbitrarily small positive numbers. In this example, we select $c_1 = 1$, $\epsilon = 10^{-7}$, $\delta = 0.0001$. It is easy to see that the selected multipliers $\{\Pi_i\}_{i=1}^2$ satisfy Assumption 1. Thus, applying the J_{n_u,n_u}-spectral factorization [28], [30] to the above multipliers, we obtain

$$\Psi_1(s) = \begin{bmatrix} \frac{\frac{k_1(c_1-a_1)}{\bar{\tau}}s - k_1^2 c_1/\bar{\tau}^2}{s^2 + \frac{a_1}{\bar{\tau}}s + k_1 c_1/\bar{\tau}^2} + k_1 + \epsilon & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Psi_2(s) = \begin{bmatrix} \frac{\frac{k_2(c_2-a_2)}{\bar{\tau}}s - k_2 b_2/\bar{\tau}^2}{s^2 + \frac{a_2}{\bar{\tau}}s + b_2/\bar{\tau}^2} + k_2 + \delta & 0 \\ 0 & 1 \end{bmatrix}. \tag{33}$$

These transfer function matrices can be readily transformed into their respective state-space representations. Consequently, the associated IQC-induced dynamics can be expressed in the form of (8) with the system matrices given by

$$A_\psi = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1 c_1}{\bar{\tau}^2} & -\frac{a_2}{\bar{\tau}} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{b_2}{\bar{\tau}^2} & -\frac{a_2}{\bar{\tau}} \end{bmatrix}, \quad B_{\psi 1} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$
$$\bar{C}_{\psi,1} = \begin{bmatrix} -\frac{k_1^2 c_1}{\bar{\tau}^2} & \frac{k_1(c_1-a_1)}{\bar{\tau}} & 0 & 0 \end{bmatrix}, \quad \bar{D}_{\psi 1,1} = k_1 + \epsilon$$
$$\bar{C}_{\psi,2} = \begin{bmatrix} 0 & 0 & -\frac{b_2 k_2}{\bar{\tau}^2} & \frac{k_2(c_2-a_2)}{\bar{\tau}} \end{bmatrix}, \quad \bar{D}_{\psi 1,2} = k_2 + \delta$$
$$B_{\psi 2} = 0, \quad \bar{D}_{\psi 2,1} = 0, \quad \bar{D}_{\psi 2,2} = 0. \tag{34}$$

Note that the above selection of the IQC multipliers is only for illustration purposes, more sophisticated IQC multipliers can be utilized to pursue better results. We stress that the proposed IQC-based design scheme provides a systematic yet simple design framework for NCS synthesis in the sense that different types of IQCs, once available and appropriate, can be readily incorporated as an individual module into the synthesis process with little effort. On the other hand, for the measurement delay $\tau_2(t)$, in order to fit into the design framework of Corollary 1 with piecewise linear matrix functions, we partition the associated delay parameter spaces $[0, T_{\min}] = [0, 0.1]$ and $[T_{\min}, T_{\max}] = [0.1, 0.3]$ evenly into $N_1 = 10$ and $N_2 = 20$ portions, respectively, such that $\delta_\ell = \tau_s = 0.01$ for all $\ell \in \mathbf{I}[1, N_1 + N_2]$.

Then, under the proposed DSI control design scheme, utilizing the above system data to solve the LMI optimization problem (30), it yields an optimal solution with $\gamma = 3.9269$, and 31 and 21 numbers of matrices with respect to the decision variables Q_ℓ and $\hat{F}_{c,\ell_2}$ for all $\ell \in \mathbf{I}[0, N_1 + N_2]$ and $\ell_2 \in \mathbf{I}[N_1, N_1 + N_2]$. Afterwards, the controller gain matrices $F_{c1}(\rho_k), F_{c2}(\rho_k), F_{c3}(\rho_k)$ can be calculated online using (29). In order to demonstrate the advantages of the proposed DSI network control scheme, the synthesized results are compared to those obtained under a robust controller

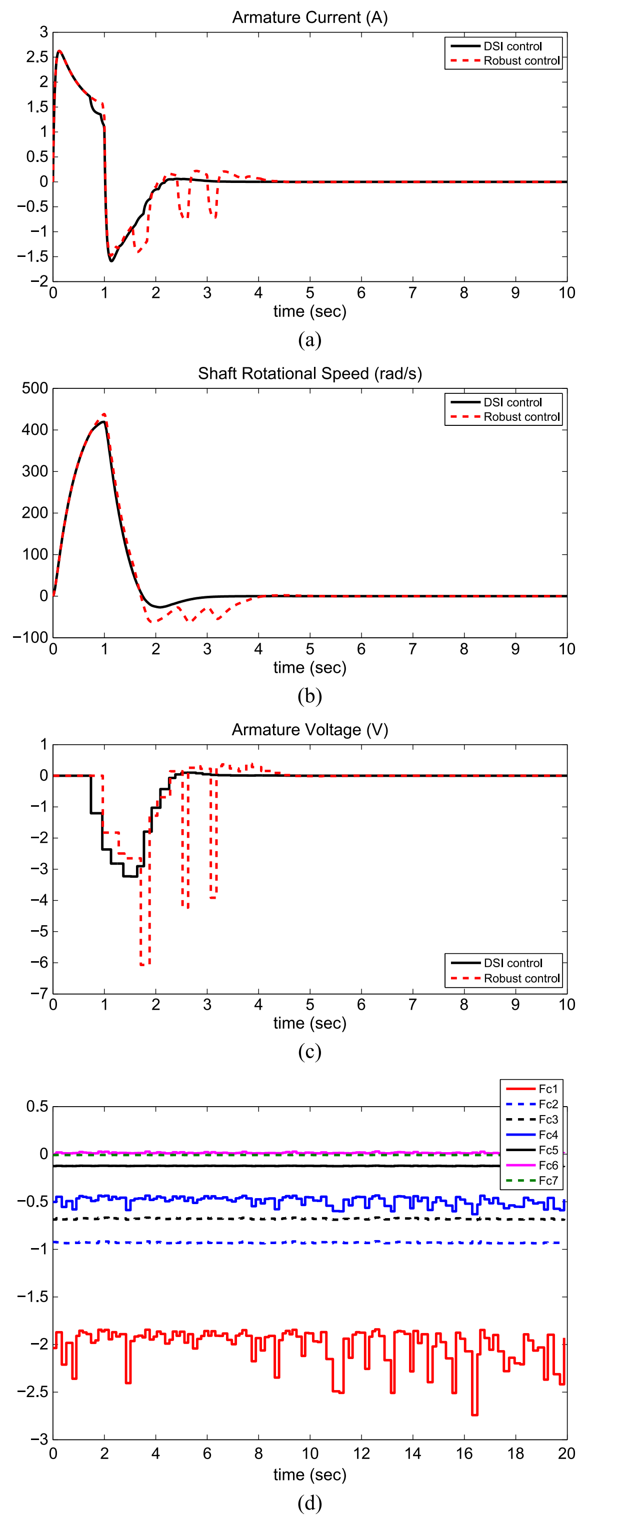

Fig. 3. Simulation results. (a) DC motor states $x_{p1} = i_a$. (b) DC motor states $x_{p2} = w$. (c) Control input u. (d) Controller gains F_c.

(with constant nondelay-scheduled controller gains) by adopting the design method in [16, Theor. 4.1]. The corresponding optimized $\mathcal{L}_2$ gain is obtained as $\gamma = 5.3271$, which is larger than the one from the proposed DSI control method, and the associated robust controller gains are given by $K = [-0.1415 \quad -0.0058 \quad 0.0235]$.

Remark 4: The robust impulsive control design method of [16], which is only concerned with the stabilization issue for sampled-data systems though, can be readily extended for $\mathcal{H}_\infty$ control of NCSs as considered in this paper. Furthermore, since the robust control law in [16] is static and time-invariant, the network effects on both measurement and actuation channels can be merged together as in a one-channel feedback NCS [11]; thus, the robust controller can still be synthesized under the same design framework of [16] by lumping the actuation delay into the measurement delay.

Further comparisons are also carried out through time-domain simulations. With zero initial conditions and an impulse disturbance of magnitude 80 starting from $t = 0$ and lasting for 1 s, we simulate the closed-loop behavior of the networked control system using the synthesized robust and DSI controllers, respectively. For simulation purposes, the actuation delay $\tau_1 \in \mathcal{T}_1$ is assumed to vary as a function of time and oscillates around its nominal value at 0.4 in the form of $\tau_1(t) = 0.4 + 0.1\cos(t)$, while the measurement delay is randomly generated and satisfies $\tau_2 \in \mathcal{T}_2$. The simulation results have been plotted in Fig. 3. It is observed from Fig. 3(a) and (b) that both controllers are indeed capable of robustly stabilizing the closed-loop system under the networked control environment, and both plant states converge rapidly to zero when the disturbance signal vanishes. Moreover, compared with the robust control method, the proposed DSI control scheme renders better disturbance rejection performance with not only smaller plant state responses but also fewer control efforts [Fig. 3(c)]. This performance gain can be attributed to the incorporation of the network-induced delay information to the DSI controller structure. The DSI controller gain trajectories are also depicted in Fig. 3(d) for illustration.

VII. Conclusion

In this paper, a new control design approach has been presented for networked control systems. Network-induced delays both from the continuous-time plant to the discrete-time controller (measurement delay) and from the discrete-time controller back to the continuous-time plant (actuation delay) have been considered. The proposed approach is developed under the IQC framework combined with linear impulsive control techniques. Specifically, the two network-induced delays have been treated in different ways. For actuation delays, the considered NCS is first transformed to an equivalent linear fractional transformation (LFT) model, such that dynamic IQCs can be used to fully characterize the behavior of the actuation delay nonlinearities, while the delay behavior exhibiting over the measurement channel is modeled as an impulsive effect acting on the nominal NCS dynamics. Based on this formulation, a novel network-based controller has been constructed with full-information feedback, that is, both plant states and the IQC-induced dynamics states are utilized for feedback control. Moreover, real-time measurement of both network-induced actuation and measurement delays are also used for controller gain scheduling. This leads to the so-called delay scheduled impulsive (DSI) controller. Both robust $\mathcal{L}_2$ stability analysis and controller synthesis problems have been investigated using dynamic IQCs combined with a clock-dependent storage function. In particular, the synthesis conditions are formulated as a finite number of LMIs by specifying a piecewise linear storage function, which can be solved using existing convex optimization algorithms. The proposed design scheme provides a simple yet systematic framework for analysis and synthesis of NCSs. Its effectiveness has been demonstrated through an application to a dc motor system.

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