

### Assignment 4: Quantization and Sampling

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1. (10%) Consider a discrete controller

$$y[k] = ay[k - 2] + by[k - 1] + x[k]$$

where  $a = 0.75$ ,  $b = -0.5$ ,  $x[0] = 0$ . For word length 5 bits, determine  $y_q[k]$ .

2. (15%) Describe clearly the advantages and disadvantages of non-uniform quantization. Give an example in what situations non-uniform quantization may help.
3. (15%) Explain and give an example of a signal that is sampled with frequency sampling  $\omega_s < 2\omega_0$ , i.e., does not satisfy Nyquist criterion. What are the consequences?
4. (15%) Convert the following real value into bits using uniform quantization and then reconstruct them into a real value again and calculate the error. Explain the steps.  
(a) 50 with 5-bit binary      (b) 0.05, 10-bit      (c) 35, 3-bit
5. (15%) Describe why second moment stability  $\sup_{k \in \mathbb{N}} \mathbb{E}(\|X_k\|^2) < \infty$  is important for networked systems as explained on the lecture. Why does it have to be second moment?
6. (30%) Consider a triple integrator system with transfer function  $G(s) = \frac{4}{s^3}$ . Design a controller (for a negative-feedback closed-loop system) in a discrete-time domain. Perform the sampling and reconstruction for the output signal  $y(t)$  and control signal  $u(t)$ . Try at least three different sampling period and compare the results. Provide explanation to your results.