Class: Sistem Pengaturan Berjaringan (EE185524)
Lecturer: Yurid E. Nugraha
Deadline: 2023/04/19

Tugas 3: Information theory

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1. (**Weight: 25%**) Let X, Y be random variables and let $Z = X + Y$.
   a) Show that $H(Z|X) = H(Y|X)$.
   b) When X and Y are independent, show that $H(X) \leq H(Z)$ and $H(Y) \leq H(Z)$.
   c) When does $H(Z) = H(X) + H(Y)$ holds?

2. (**10%**) Let X be an fair 6-sided dice with probability distribution. Compute the entropy $H(X)$.

3. (**10%**) Let X be an unfair 6-sided dice with probability distribution defined by $P(X = 1) = \frac{1}{2}$, $P(X = 2) = \frac{1}{4}$, $P(X = 3) = 0$, $P(X = 4) = 0$, $P(X = 5) = \frac{1}{4}$, and $P(X = 6) = 0$. Compute the entropy and compare with your result for Problem 2 above.

4. (**15%**) Explain the difference between mutual information and correlation.

5. (**15%**) Consider a *noisy typewriter* with 36 keys (A to Z and 1 to 0) where each of the characters of the alphabet and numbers are either transmitted exactly with probability 0.5, or replaced by the next character with probability 0.5. For example, A may be transmitted correctly as A with probability 0.5, or as B with probability 0.5. Suppose that the input is denoted by X and the output by Y. Obtain the mutual information $I(X;Y)$ and the channel capacity $C = \max_{p(x)} I(X;Y)$.

6. (**25%**) Read a paper "Analysis and synthesis of networked control systems: Topological entropy, observability, robustness and optimal control" by Andrey V. Savkin. Discuss what the main result of the paper and explain clearly how topological entropy affects the optimal control problem.