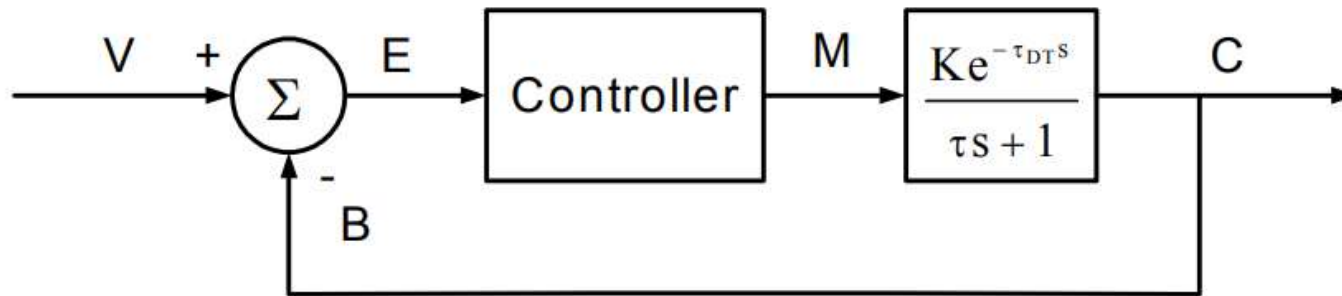


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Dead-time compensation

Yurid Eka Nugraha
TSP DTE FTEIC ITS

Control of a 1st-order process with dead time



The most commonly used model to describe the dynamics of chemical processes is the First-Order Plus Time Delay Model. By proper choice of τ_{DT} and τ , this model can be made to represent the dynamics of many industrial processes.

Dead-time

- Time delays or dead-times (DT's) between inputs and outputs are very common in industrial processes, engineering systems, economical, and biological systems.
- Transportation and measurement lags, analysis times, computation and communication lags all introduce DT's into control loops.
- DT's are also used to compensate for model reduction where high-order systems are represented by low-order models with delays.
- Two major consequences:
 - Complicates the analysis and design of feedback control systems
 - Makes satisfactory control more difficult to achieve

Effects of dead-time (time delay)

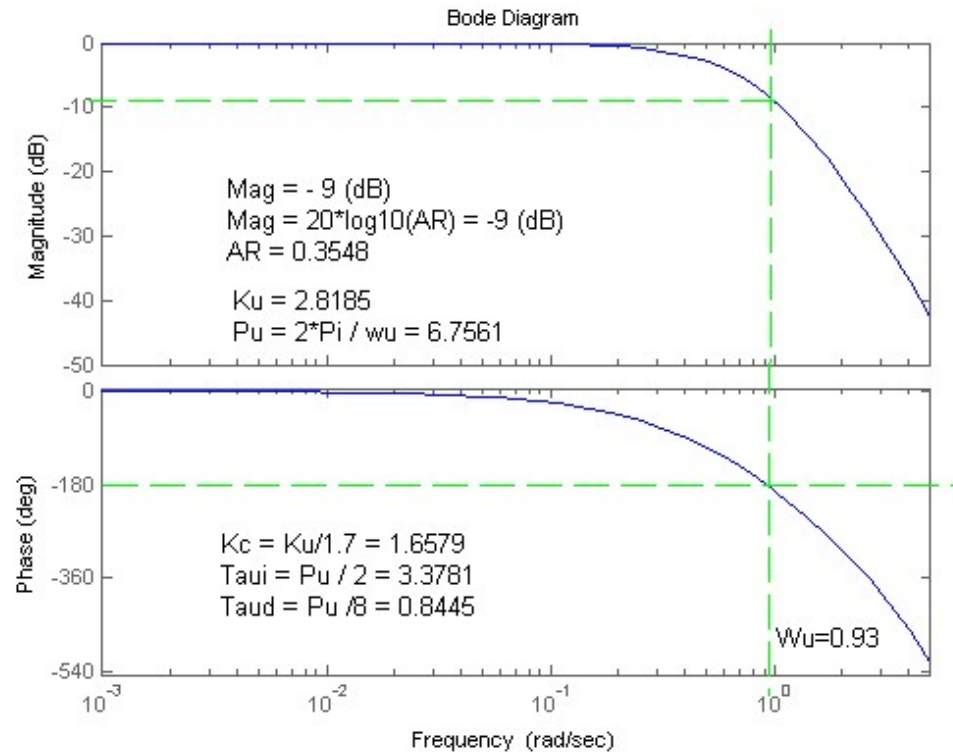
- Process with large dead time (relative to the time constant of the process) are difficult to control by pure feedback alone:
- Effect of disturbances is not seen by controller for a while
- Effect of control action is not seen at the output for a while. This causes controller to take additional compensation unnecessary
- This results in a loop that has inherently built in limitations to control



$$q_o(t) = q_i(t - \tau_{DT})u(t - \tau_{DT})$$

Example – A Bode Plot Example

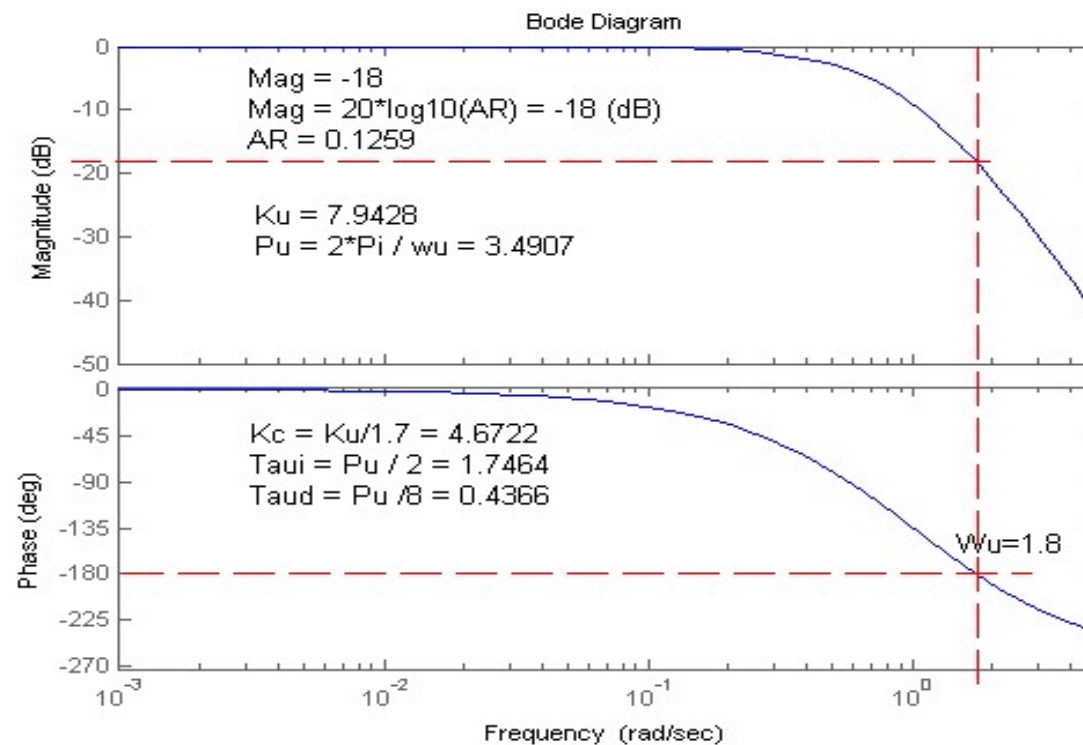
$$G(s) = \frac{e^{-s}}{(s+1)^3}$$



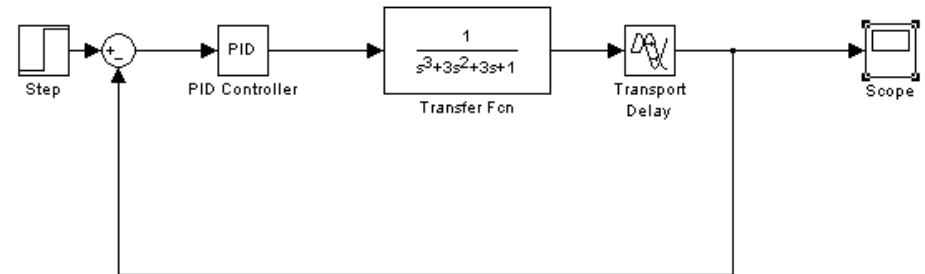
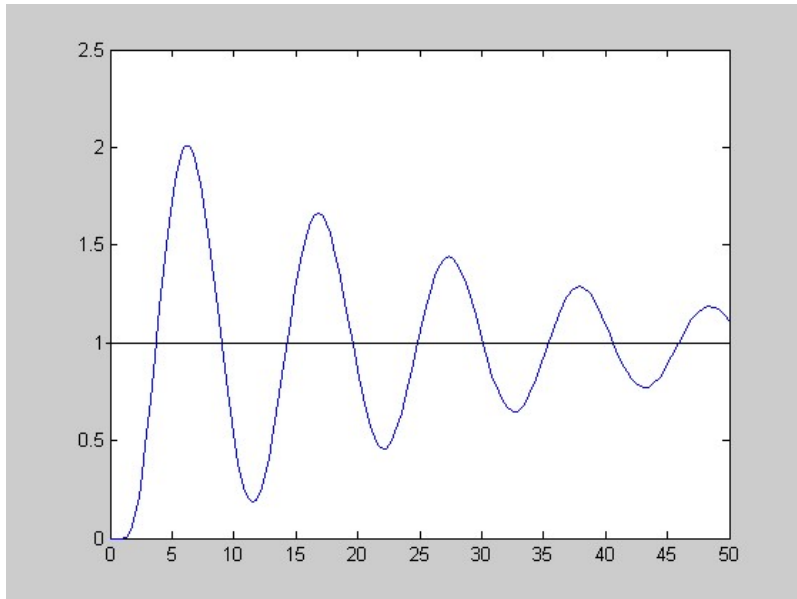
```
h=tf(1,[1 3 3 1],'inputdelay',1);
```

Example - continued, case without time delay

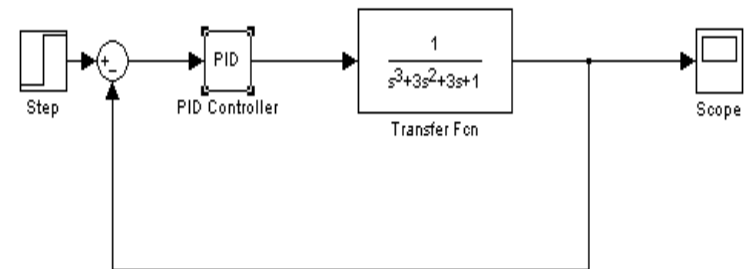
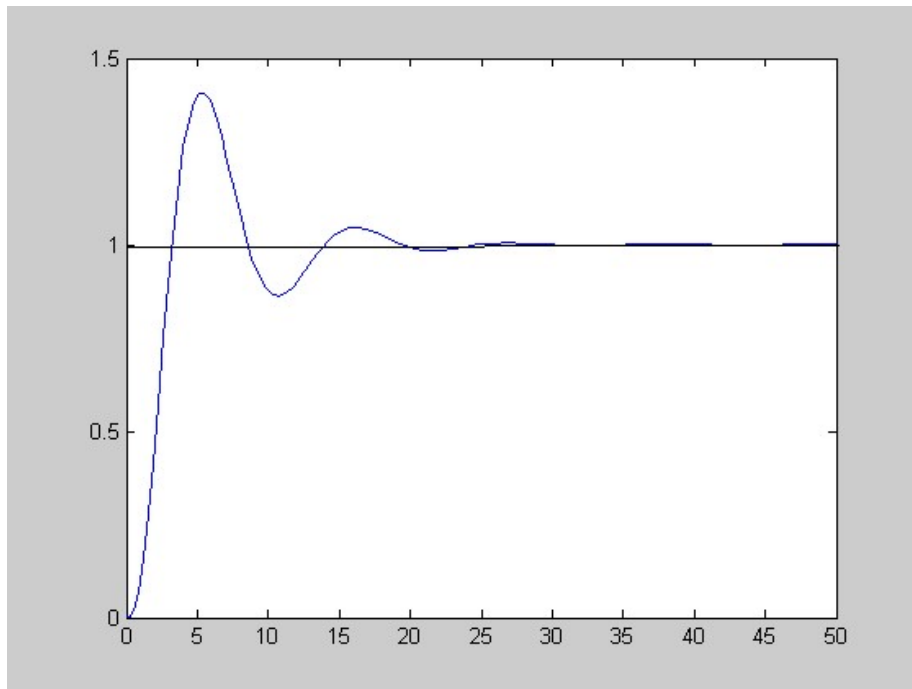
$$G(s) = \frac{1}{(s+1)^3}$$



PID Control Results - Case with delay ($K_p = 1$, $K_i = 1$, $K_d = 1$)



PID Control Results- Case without delay ($K_p = 1$, $K_i = 1$, $K_d = 1$)



Causes of Dead-Time

- Transportation lag (long pipelines)
- Sampling downstream of the process
- Slow measuring device
- Large number of first-order time constants in series (e.g. distillation column)
- Sampling delays introduced by computer control

Dead-Time approximations

- The simplest dead-time approximation can be obtained graphically or by taking the first two terms of the Taylor series expansion of the Laplace transfer function of a dead-time element, τ_{DT} .

$$f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots,$$

- The accuracy of this approximation depends on the dead time being sufficiently small relative to the rate of change of the slope of $q_i(t)$.

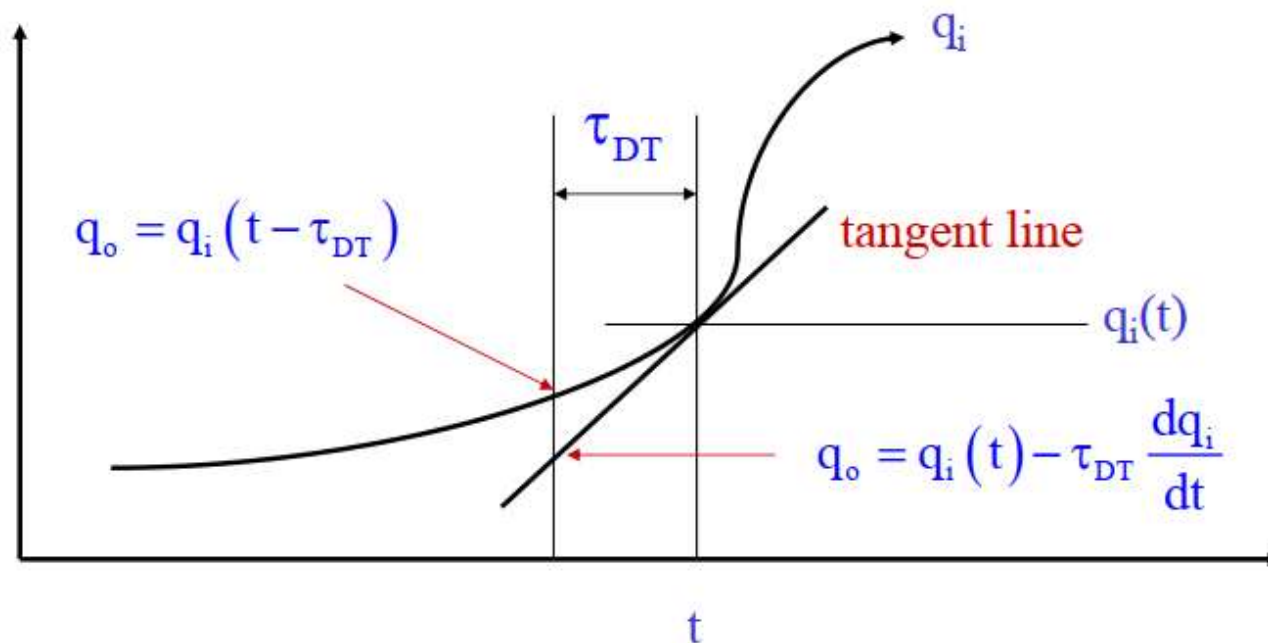
$$\frac{Q_o}{Q_i}(s) = e^{-\tau_{DT}s} \approx 1 - \tau_{DT}s \quad q_o(t) \approx q_i(t) - \tau_{DT} \frac{dq_i}{dt}$$

Dead-Time approximations

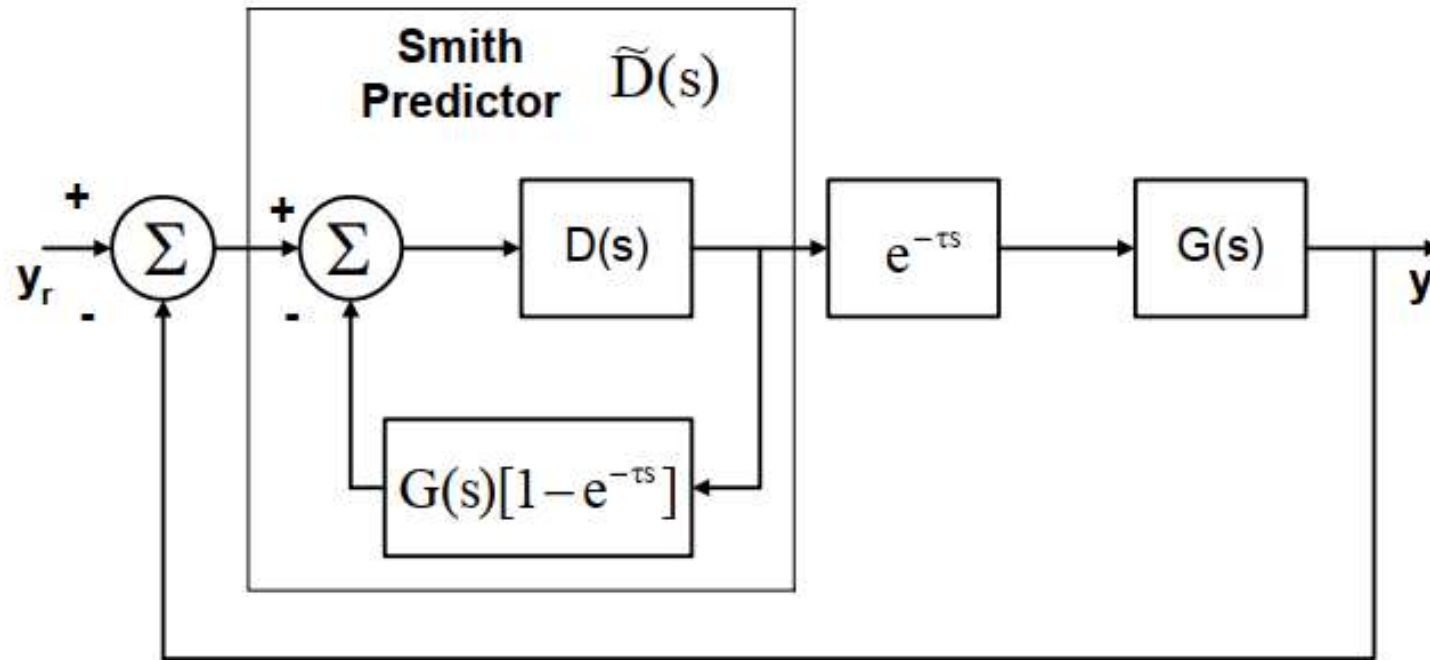
- If $q_i(t)$ were a ramp (constant slope), the approximation would be perfect for any value of τ_{DT} .
- When the slope of $q_i(t)$ varies rapidly, only small τ_{DT}' will give a good approximation.
- A frequency-response viewpoint gives a more general accuracy criterion; if the amplitude ratio and the phase of the approximation are sufficiently close to the exact frequency response curves of for the range of frequencies present in $q_i(t)$, then the approximation is valid.



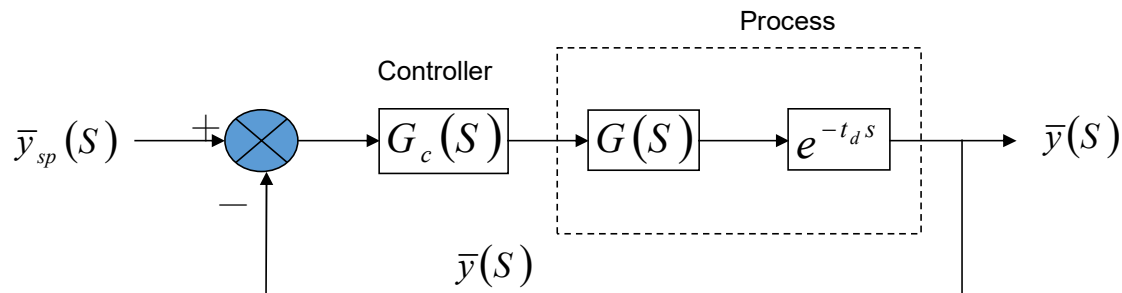
Dead-Time approximations



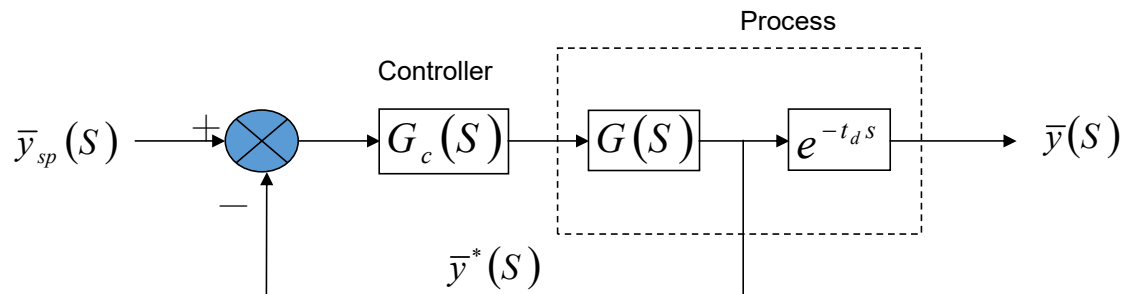
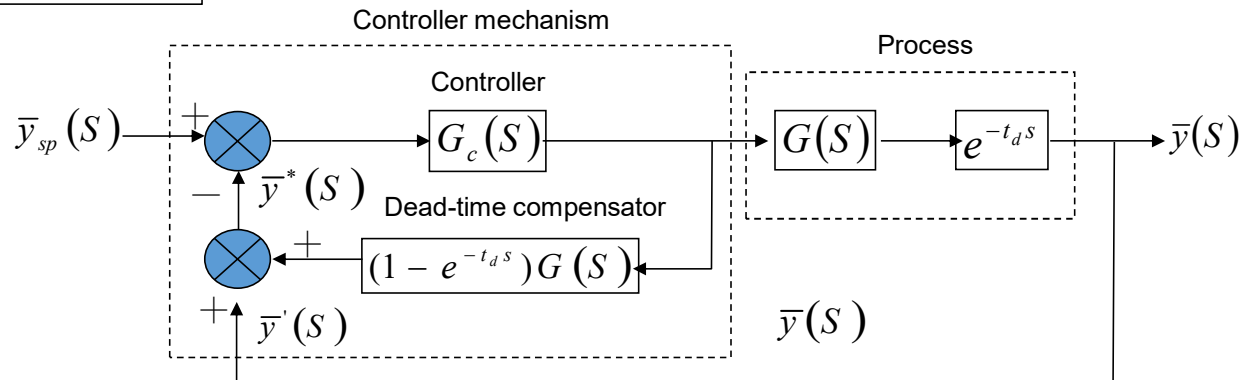
Smith predictor



Smith predictor



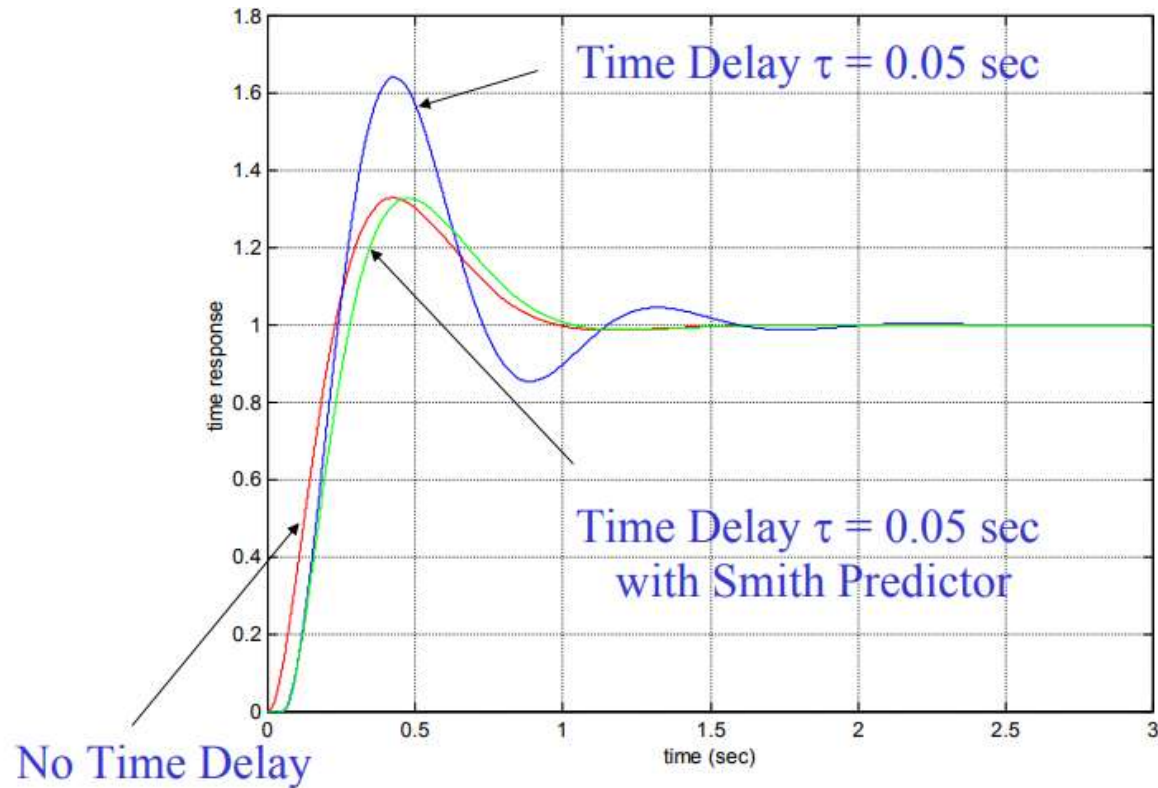
In this model, $G_c(s) = D(s)$



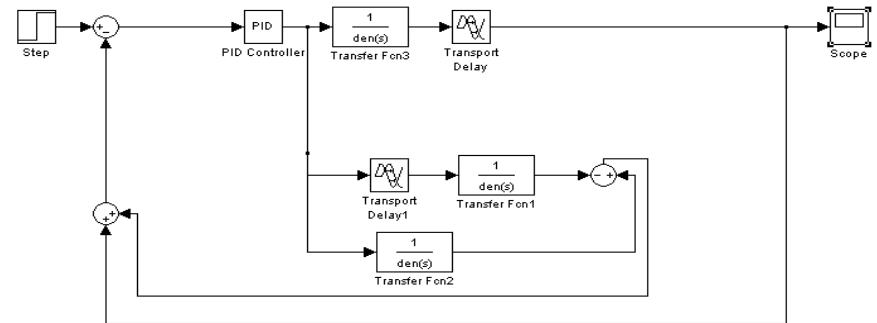
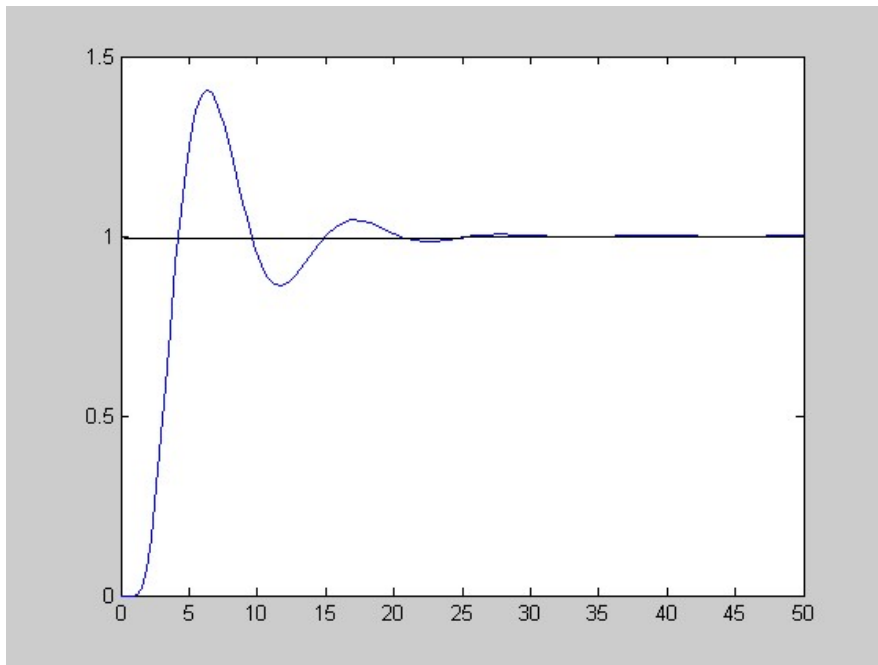
Smith predictor

- $D(s)$ is a suitable compensator for a plant whose TF, in the absence of time delay, is $G(s)$.
- With the compensator that uses the Smith Predictor, the closed-loop transfer function, except for the factor $e^{-\tau s}$, is the same as TF of the closed-loop system for the plant without the time delay and with the compensator $D(s)$.
- The time response of the closed-loop system with a compensator that uses a Smith Predictor will thus have the same shape as the response of the closed-loop system without the time delay compensated by $D(s)$; the only difference is that the output will be delayed by τ seconds.

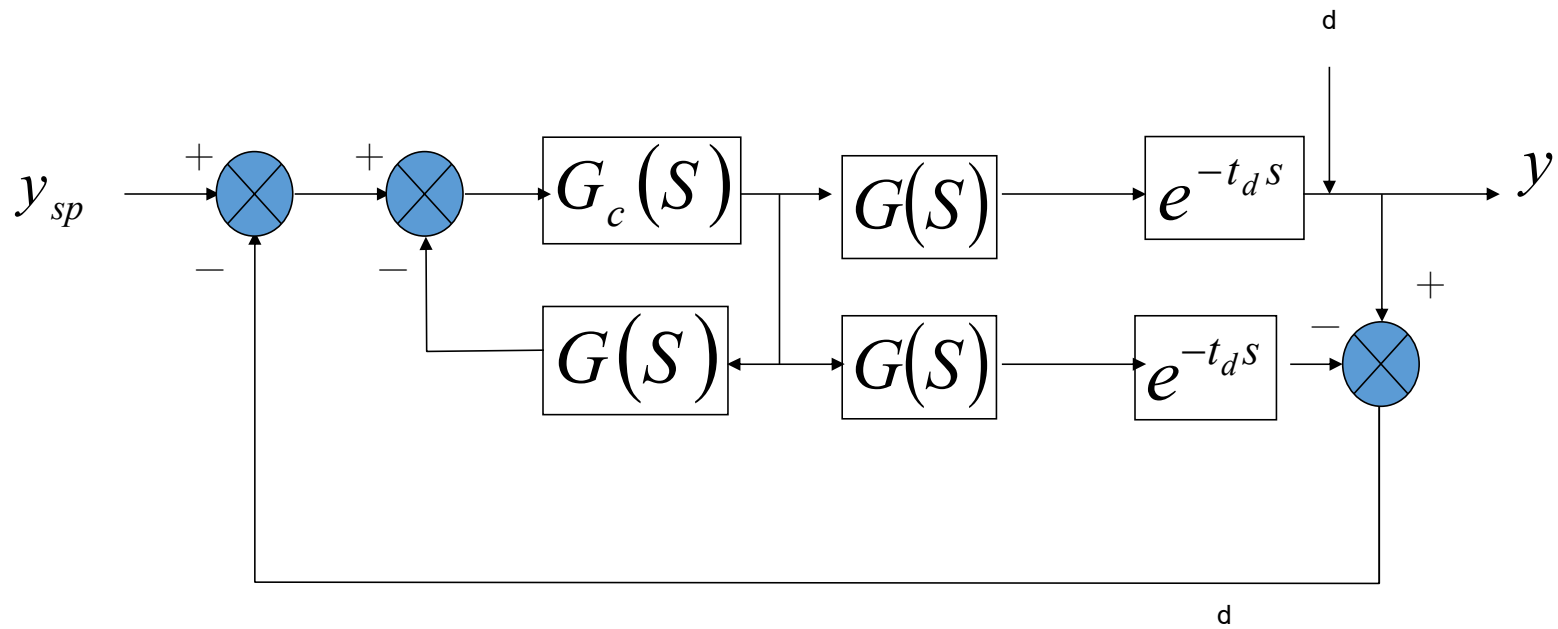
Step response example



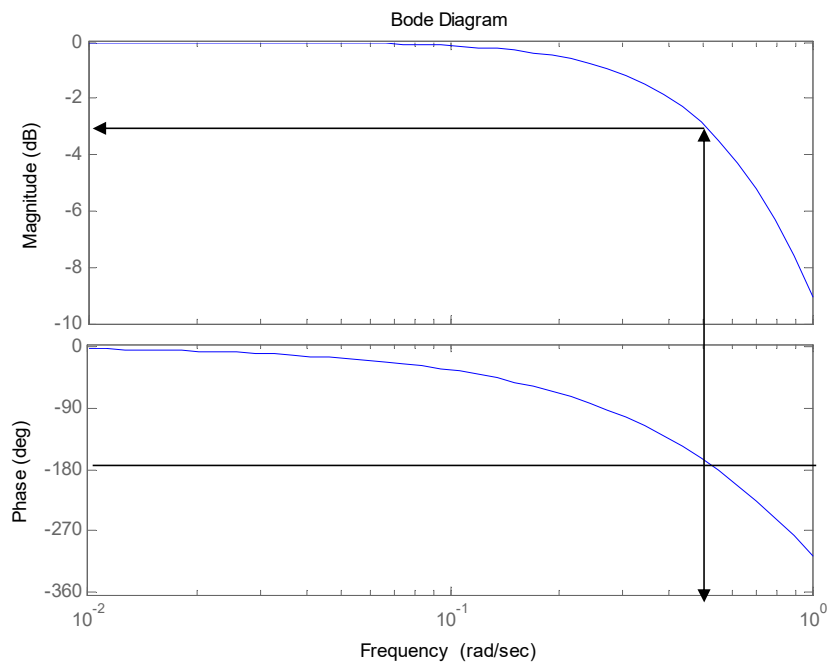
Control with Smith Predictor: ($K_p = 1$, $K_i = 1$, $K_d = 1$)



Alternate form



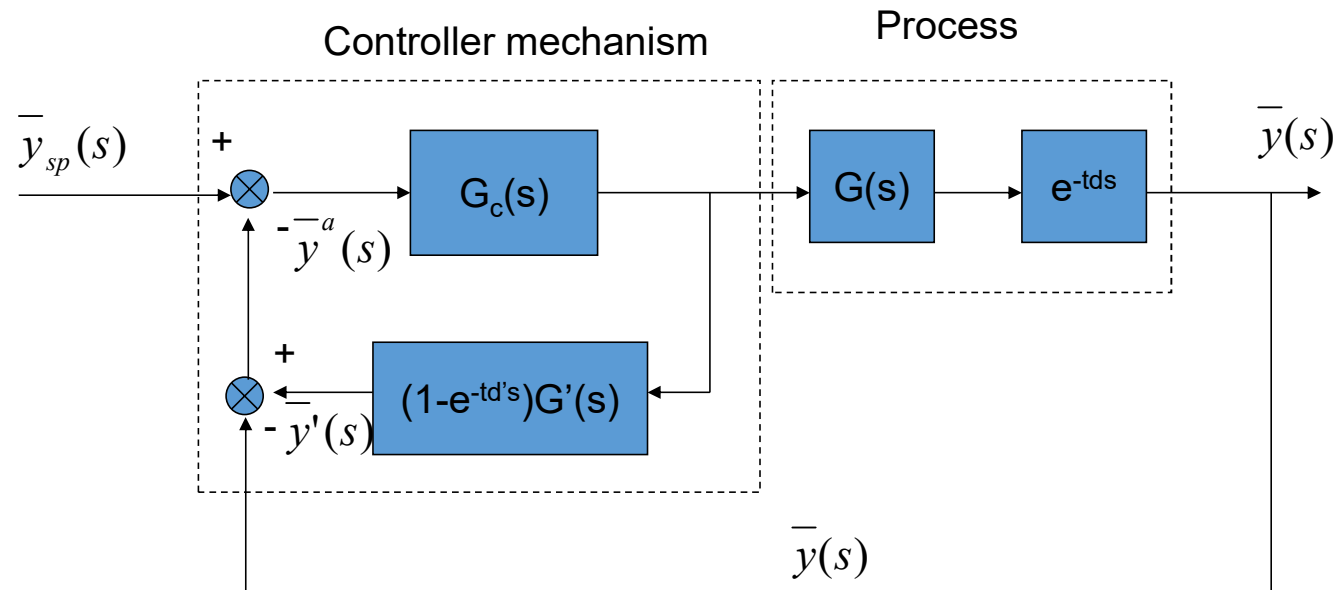
Example: Long time delay



$$G(s) = \frac{e^{-3s}}{(s^3 + 3s^2 + 3s + 1)}$$

$\text{Mag} = -3.3\text{dB} = 20\log_{10}(\text{AR});$
 $\text{AR} = 0.6839;$
 $W_u = 0.5; P_u = 2\pi/0.5 = 12.5664$
 $K_u = 1/0.6839 = 1.4622$
 $K_c = 0.8601$
 $\text{Tau}_i = P_u/2 = 6.2832$
 $\text{Tau}_d = P_u/8 = 1.5708$

The Effect of Modeling Error



$$y^*(s) = [G_c G e^{-t_d s} + (1 - e^{-t'_d s}) G_c G'] \bar{y}_{sp}$$

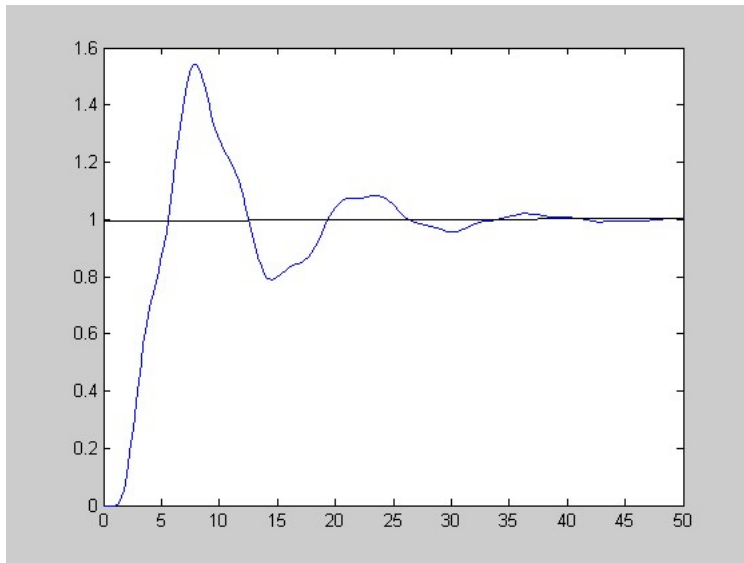
or

$$y^*(s) = G_c [G' + G e^{-t_d s} - G' e^{-t'_d s}] \bar{y}_{sp}$$

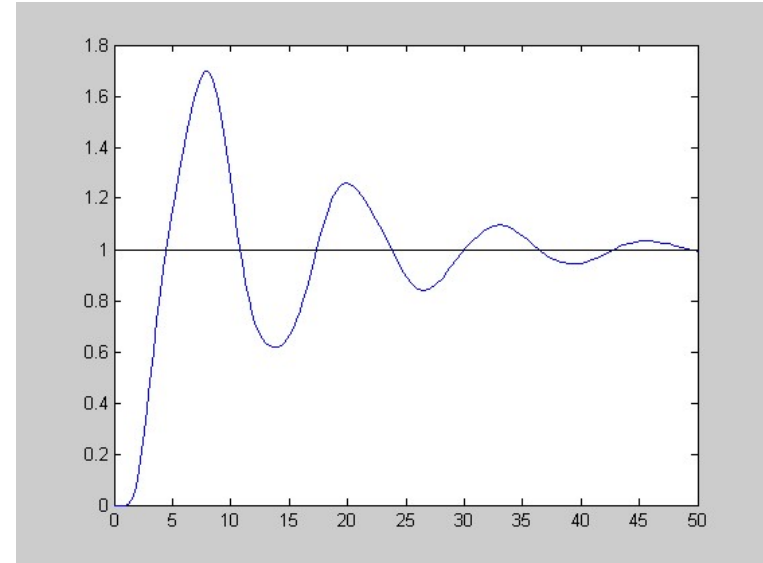
Errors in td' , G' both cause the control system to degrade

Smith Predictor with Modeling Error

$$\text{Plant} = e^{-s} / (s^3 + 3s^2 + 3s + 1)$$

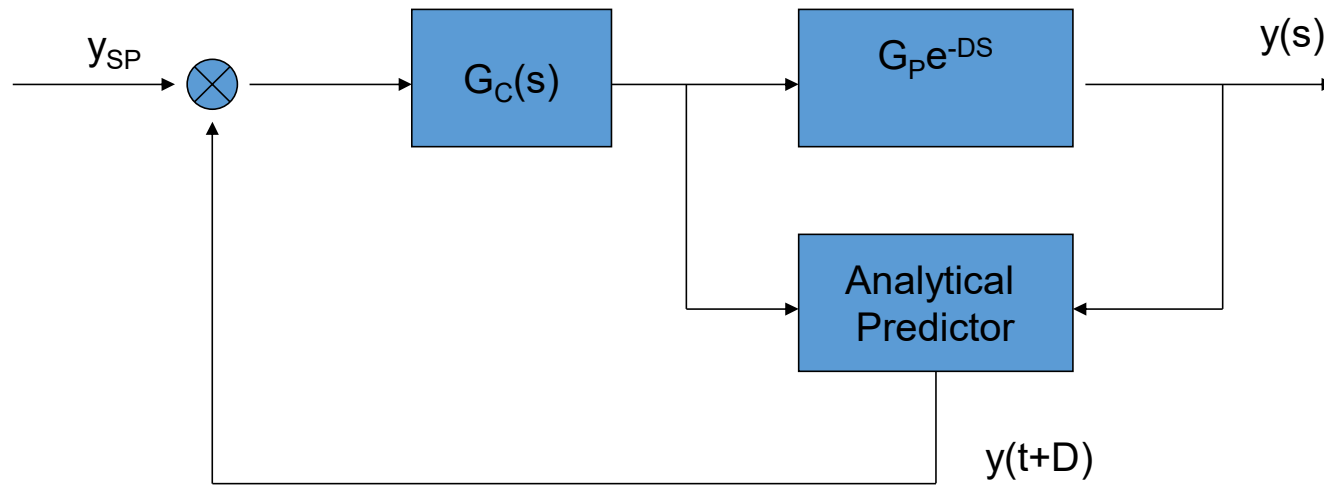


$$\text{Model} = e^{-s} / (s^3 + s^2 + s + 1)$$



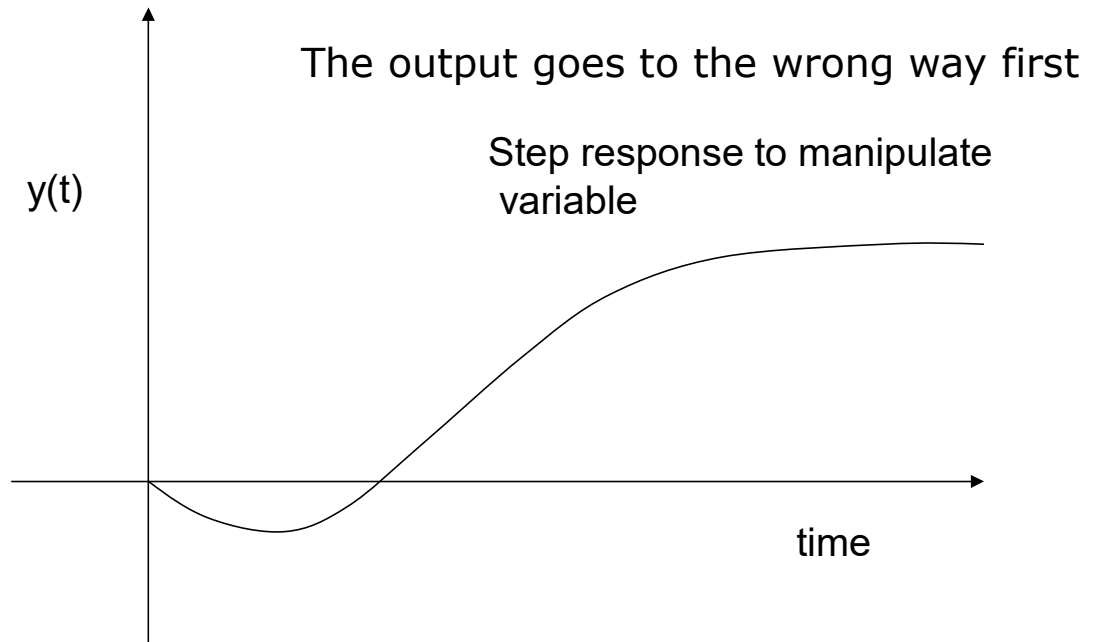
$$\text{Model} = e^{-0.5s} / (s^3 + s^2 + s + 1)$$

Analytical Predictor



Analytical Predictor is a model that projects what y will be D units into the future (on-line model identification)

Processes with inverse response



Example:
$$y(s) = \frac{K_1}{\tau_1 s + 1} - \frac{K_2}{\tau_2 s + 1}$$

This can occur in (1) reboiler level response to change in heat input to the reboiler. (2) Some tubular reactor exit temperature to inlet flow rate.

Inverse response is caused by a RHP zero

Example: Inverse response of concentration and temperature to a change in process flow of 0.15 ft³/min

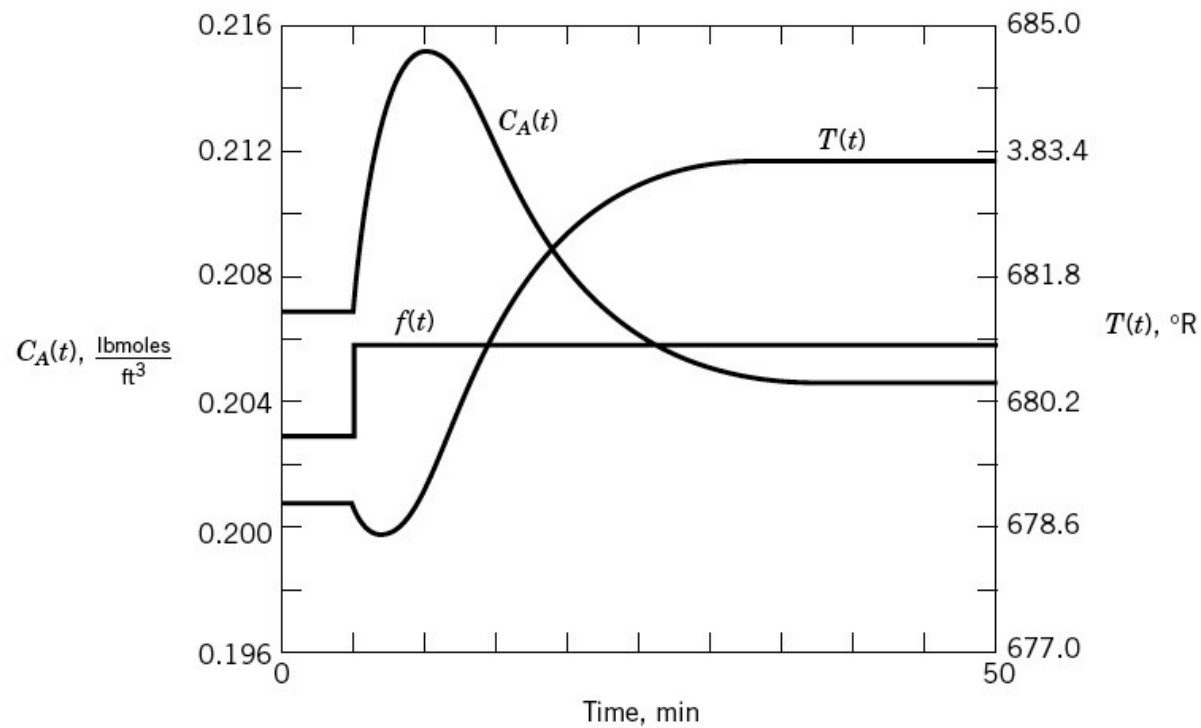
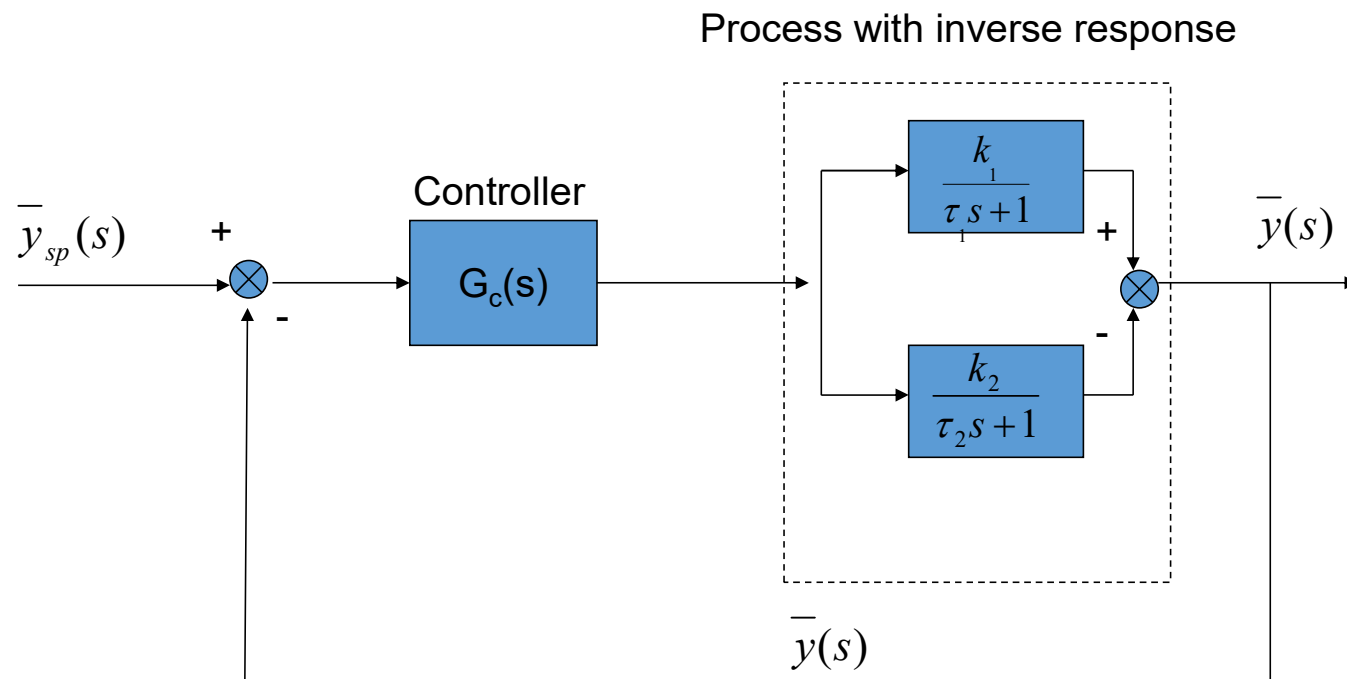
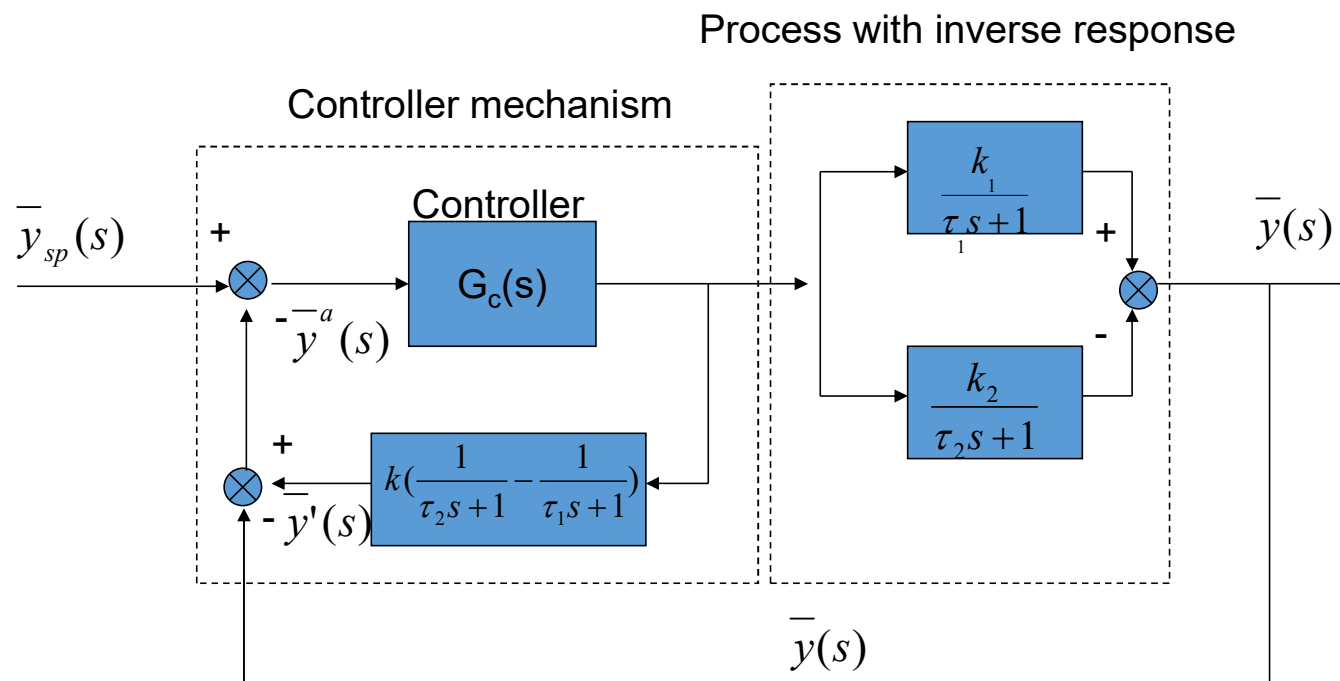


Figure 4-4.8 Inverse response of concentration and temperature to a change in process flow of 0.15 ft³/min.





Open-loop response analysis

$$y(s) = G_c(s) \frac{(K_1\tau_2 - K_2\tau_1)s + K_1 - K_2}{(\tau_1s + 1)(\tau_2s + 1)} y_{sp}(s)$$

$$\text{RHP zero at } s = -\frac{(K_1 - K_2)}{K_1\tau_2 - K_2\tau_1}$$

Note: if $\frac{\tau_1}{\tau_2} > \frac{K_1}{K_2} > 1$, then s is positive

$$\text{If we add to the open loop response } y'(s) = G_c K \left(\frac{1}{\tau_2s + 1} - \frac{1}{\tau_1s + 1} \right) y_{sp}(s)$$

$$\text{then } y^*(s) = y(s) + y'(s) = G_c(s) \frac{[(K_1\tau_2 - K_2\tau_1) + K(\tau_1 - \tau_2)]s + (K_1 - K_2)}{(\tau_1s + 1)(\tau_2s + 1)} y_s$$

$$\text{and if } K > \frac{K_2\tau_1 - K_1\tau_2}{\tau_1 - \tau_2} \quad \text{the zero is in the LHP}$$