Plan for remaining classes...

12. Quantized feedback control **(25/5, 8.30-10.10)**
13-14. Paper presentation **(2/6, 8.00-11.00)**
15. Wrap-up **(?/6, 8.30-10.10)**
16. Final exam **(~8/6)**

EE185524

Quantized Feedback Control Design

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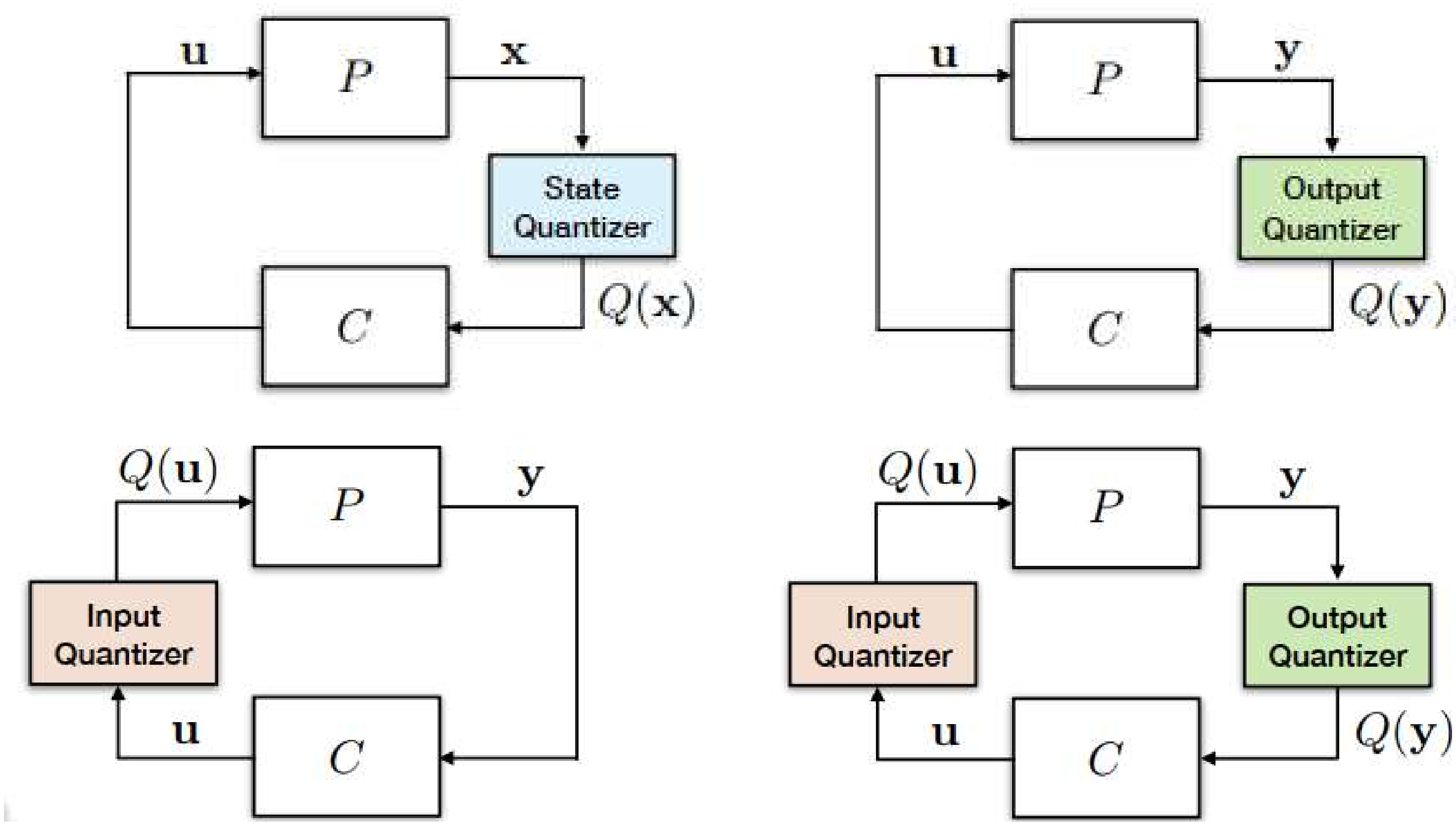
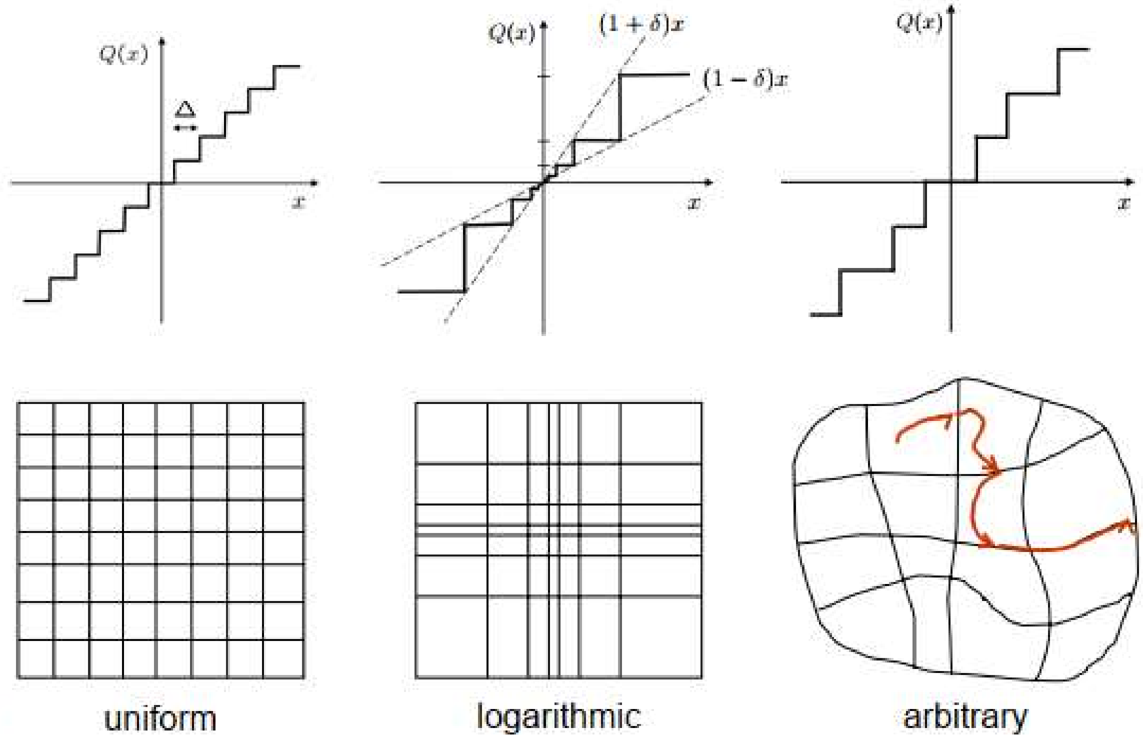
Possible setting

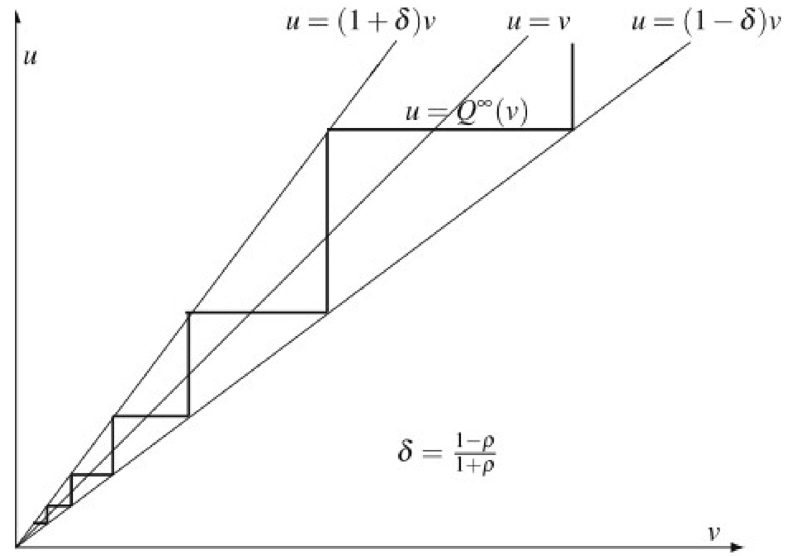


Quantizer geometry



Logarithmic quantizers

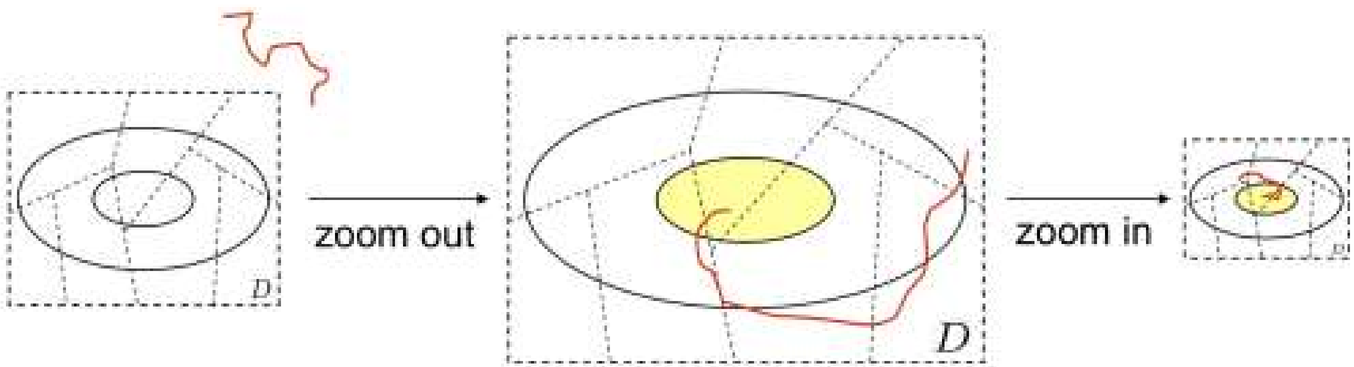
u_i: quantization level
ρ: quantization density
δ: sector bound



Quantizers

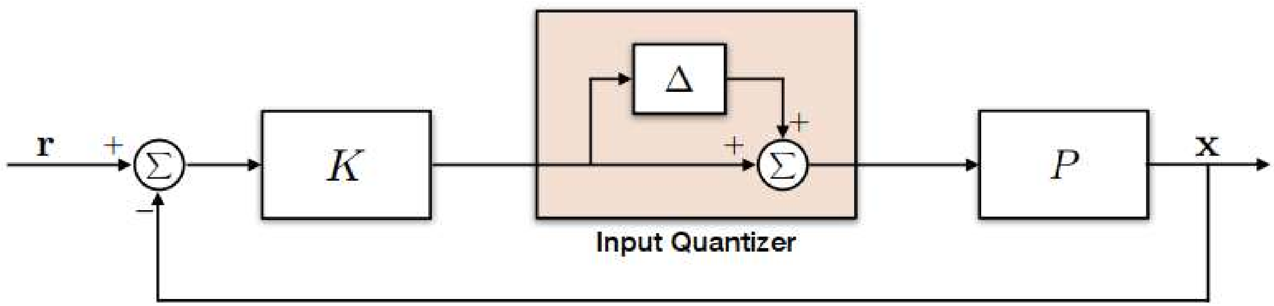
- In the information-theoretic approach, since a finite number of bits are transmitted, proper quantizers should be designed to quantize the continuous numbers
- By dynamic scaling, the performance of static quantizers can be improved.
- A smaller scaling factor provides a fine quantization near the origin while a larger one ensures large numbers fall within the domain for quantization
- Zoom out: a larger scaling factor, ultimate bound is achieved
- Zoom in: a smaller scaling factor, recompute partition for smaller region

Quantizers



State feedback quantization

- For a logarithmic quantizer where $\delta^- = \delta^+ = \delta$, the problem of coarsest quantization is equivalent to finding the maximum δ



State feedback quantization

Sector bound method used to establish 3 results:

1. Quantized state feedback stabilization $\equiv$ Feedback quadratic stabilization subject to a sector bound uncertainty
2. The optimal quantizer structure is logarithmic
3. The solution of the logarithmic quantizer has an explicit solution

Two types of result: quantized control vs measurement

State feedback quantization

Consider following system:

$$x_{k+1} = Ax_k + Bu_k$$
$$y_k = Cx_k$$

Two types of result: quantized control vs measurement

State feedback quantization

Theorem:

Suppose (A, C) is observable. The LTI system can be quadratically stable via quantized controller. The coarsest quantization sector bound δ_{sup} for quadratic stabilization of quantized measurement feedback can be achieved. The largest sector bound δ_{sup} is given by

$$\delta_{\text{sup}} = \frac{1}{\inf_{\bar{K}} \|G\|_\infty}$$

where $\bar{G}(z) = \left(1 - G(z)H(z)\right)^{-1} G(z)H(z)$, $H(z) = C_c(zI - A_c)^{-1}B_c + D_c$.

If $G(z)$ has relative degree equal to 1 and no unstable zeros, then the coarsest sector bound δ_{sup} for quantized state feedback can be reached via quantized output feedback

Output feedback quantization

- Generalize the technique discussed for quantized state feedback to quantized output feedback
- Quantized control vs measurement

Quantized control

Theorem:

Suppose (A, C) is observable. The coarsest quantization sector bound δ_{sup} for quadratic stabilization of state feedback can also be achieved by output feedback. In particular the corresponding output feedback controller can be chosen as an observer-based controller:

$$x_{c,k+1} = Ax_{c,k} + Bu_k + L(y_k - Cx_{c,k})$$
$$u_k = Q(v_k)$$
$$v_k = Kx_{c,k}$$

Quantized control

Sketch of the proof:

Choose L such that the observer is deadbeat (i.e., $x_k - x_{c,k} \neq 0$ only for a finite number of steps N .This can be always done because (A, C) is observable. Then, after N steps, the output feedback controller is the same as state feedback controller

A deadbeat controller has the property that for a given input type, such as a step, the error between input and output,

$$e(kT) = y_{in}(kT) - y(kT)$$

will always show $e(kT) = 0\ (k > n)$ for a particular n which depends upon the plant.

Summary

- Sector bound method transform quantized state/output feedback stabilization to Feedback quadratic stabilization subject to a sector bound uncertainty
- Finite word length of uniform quantizer controllers may introduce limit cycles (sustained oscillations) at the controller output.
- Sample step is related to the variance of quantization controller output