

Tugas IIi sistem pengaturan Berjaringan

Muhammad Azriel Rizqifadiilah - 6022221047



Departemen teknik elektro

fakultas teknologi elektro dan informatika cerdas

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1. a). compute using the fact that .

Since , for a given , we can write the conditional distribution as:

will take the same values as y with the same probabilities. So, has the same distribution as for each fixed value of . Now we can rewrite the conditional entropy as:

Since has the same distribution as for each fixed , we can replace with :

The definition of the conditional entropy :

Therefore, we have shown that .

b). First, let's prove:

Since, we know that for all pairs such that . Now, we can rewrite the entropy of :

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Using the fact that for all pairs such that :

Now, we use the fact that and are independent:

Applying the log-sum inequality, we get:

Since :

Since entropy is non-negative, we can conclude:

Now, let's prove :

Following a similar process, we can prove this by starting with the entropy of :

Using the fact that for all pairs such that :

Now, we use the fact that and are independent:

Applying the log-sum inequality, we get:

Since :

Since entropy is non-negative, we can conclude:

In conclusion, when and are independent, and

c). holds when the random variables and are independent, and there is no "loss" of information when calculating the entropy of .

In the previous part of this question, showed that when and are independent, . However, equality holds when the log-sum inequality becomes equality. The log-sum inequality becomes an equality when the following condition is met:

for all pairs such that , where k is a constant. If and are independent and the condition above is met, then .

In summary, holds when and are independent, and the relationship between their probabilities satisfies the condition mentioned above.

1. In the case of a fair 6-sided dice, the probability distribution is uniform, meaning that each face has an equal probability of occurring. Thus, . To compute the entropy , we can substitute the probabilities into the entropy formula:

Since each term is the same, we can simplify the expression:

The -6 and 1/6 factors cancel out, leaving:

The base-2 logarithm of 6 is approximately 2.585. Therefore, the entropy H(X) of a fair 6-sided dice is approximately 2.585 bits. This means that, on average, about 2.585 bits of information are needed to describe the outcome of a roll of the fair 6-sided dice.

1. To compute the entropy for the given unfair 6-sided dice with the specified probability distribution, we can use the entropy formula:

Since and are all 0, their respective terms in the summation will also be 0. We only need to compute the terms for and .

So, the entropy for the given unfair 6-sided dice is 1.5 bits. Comparing this to the fair 6-sided dice entropy of approximately 2.585 bits, we can see that the entropy of the unfair dice is lower. This means that the unfair dice have less uncertainty or randomness associated with their outcomes compared to the fair dice. Since the unfair dice have fewer possible outcomes with non-zero probabilities (only 3 faces compared to 6 for the fair dice), less information is needed, on average, to describe the outcome of a roll of the unfair 6-sided dice.

1. Correlation: Correlation is a measure of the linear relationship between two continuous variables. It quantifies the strength and direction of the relationship, with values ranging from -1 to 1. A correlation of 1 indicates a perfect positive linear relationship, while a correlation of -1 indicates a perfect negative linear relationship. A correlation of 0 suggests no linear relationship between the variables. It's important to note that correlation only captures linear dependencies and might not be able to detect nonlinear relationships between variables.

Mutual information: Mutual information is a measure of the dependency between two variables, which can be either continuous or discrete. It quantifies the amount of information obtained about one variable through observing the other variable, and it is based on the concept of entropy from information theory. Mutual information can detect both linear and nonlinear relationships between variables, making it a more general measure of dependency.

1. Find the joint probability distribution and the marginal probability distributions and . Have a symmetric channel with 36 keys, where each key has equal probability of being selected. First find and for each and :

* : Since all keys have equal probability, for all .
* : The conditional probability distribution for given is:

If

If is the next character of

For all other characters,

Compute the joint probability distribution :

Since for all and is either or , the joint probability distribution will be:

Next find the marginal distribution . For each , sum up the joint probabilities over all :

Compute the mutual information :

For this problem, the distribution of input symbols doesn't affect the channel, so the maximum mutual information will occur when the input distribution is uniform (i.e., all input symbols are equally likely). Therefore, the channel capacity is equal to for this uniform distribution:

1. The main result of the paper is the establishment of connections between topological entropy, observability, robustness, and optimal control in networked control systems. The author presents novel criteria for observability, robustness, and optimal control of NCSs using topological entropy as the central measure.

Topological entropy, in this context, measures the complexity of a dynamical system, which is a crucial characteristic when designing and analysing networked control systems. It quantifies the rate at which information is produced or needed to describe the system's behaviour, essentially assessing the unpredictability and complexity of the system's evolution.

In summary, the main result of the paper is the establishment of connections between topological entropy and various aspects of networked control systems, including observability, robustness, and optimal control. The author shows that topological entropy plays a central role in determining the complexity and difficulty of these problems in NCSs. This result has important implications for designing and analysing networked control systems with different levels of complexity and robustness.