Plan for remaining classes

13. I/o networks (23/5)
14. I/o networks (25/5)
15. Wrap-up (30/5)

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Input and Output Setup

Yurid E. Nugraha
TSP DTE FTEIC ITS

“Fundamental progress has to do with the reinterpretation of basic ideas.” — Alfred North Whitehead

Background

- Just as a stabilizing controller is typically a first step in the control design phase, **consensus** will provide the underlying cohesion of the network
- Consider situation when input and output nodes **are identical**
- Non input/output nodes: **floating nodes**
- Throughout the section, **undirected graphs** are considered

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Input and Output Setup

Basic input-output setup

Yurid E. Nugraha

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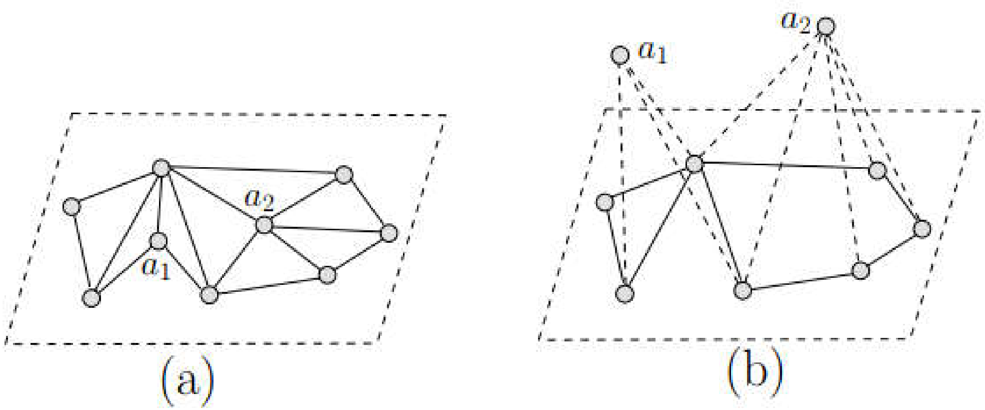
Basic i-o setup

- Floating graph G_f:the subgraph induced by the floating node set

$$V(G_f) \subseteq V(G)$$

after removing the input/output nodes as well as the edges between i/o nodes and between input/output nodes and floating nodes.



Basic i-o setup

- Node partition: i/o node ($V_i \subseteq V$) and floating node ($V_f \subseteq V$)
- Suppose D being incidence matrix and

$$A_f = D_f D_f^\top,\ A_i = D_i D_i^\top, B_f = D_f D_i^\top$$

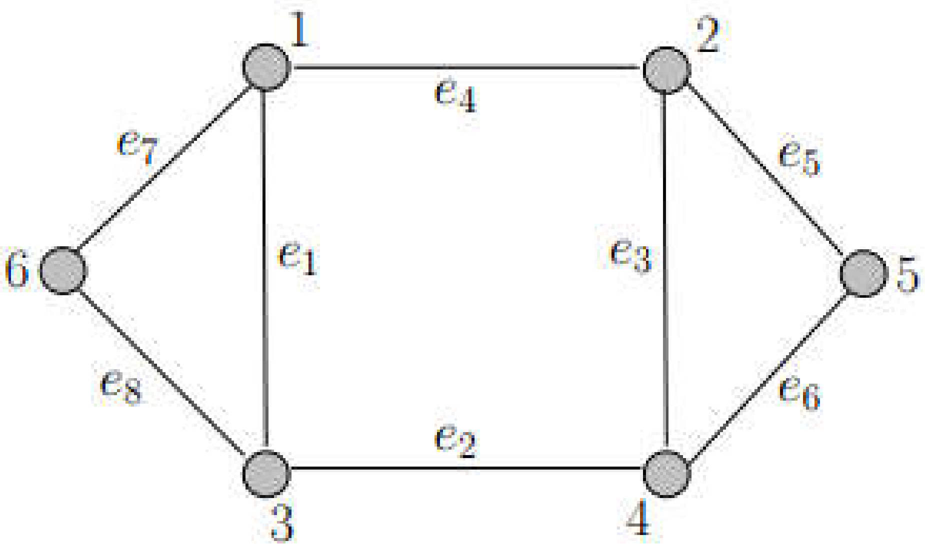
- Floating graph G_f:the subgraph induced by the floating node set

$$V(G_f) \subseteq V(G)$$

after removing the i/o nodes as well as the edges between i/o nodes and between i/o nodes and floating nodes.

Basic i-o setup

- Example:



Basic i-o setup

43rd IEEE Conference on Decision and Control
December 14-17, 2004
Atlantis, Paradise Island, Bahamas

WeC05.5

On the Controllability of Nearest Neighbor Interconnections

Herbert G. Tanner
Mechanical Engineering Department
University of New Mexico
Albuquerque, NM 87131

***Abstract*—In this paper we derive necessary and sufficient conditions for a group of systems interconnected via nearest neighbor rules, to be controllable by one of them acting as a leader. It is indicated that connectivity seems to have an adverse effect on controllability, and it is formally shown why a path is controllable while a complete graph is not. The dependence of the graph controllability property on the size of the graph and its connectivity is investigated in simulation. Results suggest analytical means of selecting the right leader and/or the appropriate topology to be able to control an interconnected system with nearest neighbor interaction rules.**

I. Introduction

and LMIs are being used. Location optimization problems were examined within a cooperative control framework in [15]. Leader-follower local control laws with vision-based feedback were employed in [16] to stabilize a formation to a particular shape. More reactive, behavioral schemes were employed in [17], [18], [19], [20] to shape formations of vehicles. The investigation of "structural controllability" in [21] is close to the problem discussed in this paper.

In this paper, however, we consider the classical notion of controllability, for a group of autonomous agents in-

I-o agreement

- Agreement protocol:

$$\dot{x}_f(t) = -A_f x_f(t) - B_f u(t),$$
$$y(t) = -B_f^\top x_f(t),$$

- i-o indicator vectors $\delta_i: V(G_f) \to \{0,1\}^{n_f}$
- Input-to-state degree matrix

$$\Delta_f = \mathbf{diag}([d_i(1), \dots, d_i(n_f)]^\top)$$

- For example, if there is only one input in the network, i.e., $V(G_i) = \{n\}$:

$$B_f = -\delta_n \text{ and } \Delta_f = \mathbf{diag}([d_n(1), \dots, d_n(n-1)]^\top$$

Controllability and observability

- A system is said to be controllable at time t_0 if it is possible by means of an unconstrained control vector to transfer the system from any initial state $x(t_0)$ to any other state in a finite interval of time.
- A system is said to be observable at time t_0 if, with the system in state $x(t_0)$, it is possible to determine this state from the observation of the output over a finite time interval

Controllability and observability

- Consider

$$\dot{x}_f(t) = -A_f x_f(t) - B_f u(t),$$
$$y(t) = -B_f^\top x_f(t),$$

- Controllability allows the input nodes to be used as a *steering mechanism* for the states of the floating nodes by locally injecting continuous signals into the network.
- Similarly, observability at the output nodes of the network would allow a node to observe the state of the entire network by locally observing the states of its neighbors.

Controllability and observability

Proceedings of the 2006 American Control Conference
Minneapolis, Minnesota, USA, June 14-16, 2006

WeB20.1

Leader-Based Multi-Agent Coordination: Controllability and Optimal Control*

Meng Ji[†], Abubakr Muhammad[‡] and Magnus Egerstedt[†]

***Abstract*— In this paper, we consider the situation where a collection of leaders dictate the motion of the followers in heterogeneous multi-agent applications. In particular, the followers move according to a decentralized averaging rule, while the leaders' motion is unconstrained. Thus, the trajectories of the leaders can be viewed as exogenous control inputs, which allows us to state and study questions concerning controllability and optimal control.**

I. Introduction

In the rapidly expanding field of multi-agent robotics, two distinctly different approaches have emerged, depending on whether any distinct agents are allowed to take on leader roles. This paper focus on the leader based control problem and we investigate a number of control theoretic issues that arise when the followers' dynamics are governed by consensus-like local interaction rules. This means that only to be static throughout the paper. This result is moreover constructive in that it allows us to select leaders for the purpose of rendering the system controllable.

Once a set of leaders is selected, we apply optimal control techniques for driving the system between specified positions. It is shown that this problem is in fact equivalent to the problem of driving an invertible linear system in a quasi-static equilibrium process[1] .

The outline of this paper is as follows: In Section 2, we present some basic enabling results. These are followed by a controllability study in Section 3, where a sufficient condition for controllability relates the relative homology of a particular graph to the homology of the original network. The point-to-point transfer problem is considered in Section 4, where it is shown that an optimal transfer is always possible regardless of controllably properties.

Controllability and observability

Proposition:

- *This system is controllable and observable if and only if none of the eigenvectors of A_f are simultaneously orthogonal to all columns of B_f*
- *If A_f has an eigenvalue with* ***geometric multiplicity*** *greater than 1 then the system is uncontrollable (and unobservable)*
- *Given a connected graph, the system is controllable iff $L(G)$ and A_f do not share an eigenvalue.*

(the geometric multiplicity of an eigenvalue is the number of linearly independent eigenvectors associated with it)

Plan for remaining classes

14. I/o networks (25/5)
15. Topik khusus (30/5)
16. UAS (8/6 or 15/6 or sesuai jadwal dept)

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Input and Output Setup

Controllability: SISO case

Yurid E. Nugraha
TSP DTE FTEIC ITS

I-o agreement (SISO case)

- Connections between the controllability and observability of the **SISO agreement protocol** and the structural properties of the underlying network
- As the name suggests, there is only one input node
- Agreement protocol:

$$\dot{x}_f(t) = -A_f x_f(t) - B_f\, u(t),$$
$$y(t) = -B_f^\top x_f(t),$$

I-o agreement (SISO case)

Corollary:

The system is controllable iff none of the eigenvectors of A_f are orthogonal to 1.

Proposition:

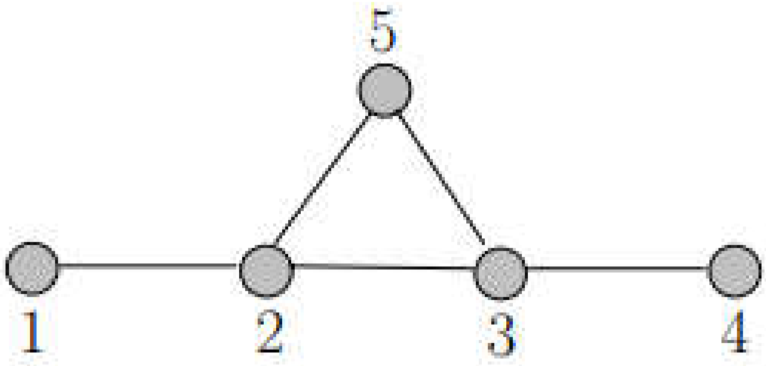
Suppose that system is uncontrollable. Then, there exists an eigenvector of $L(G)$ that has a zero component on the index that corresponds to the input.

Corollary:

Suppose that none of the eigenvectors of $L(G)$ have a zero component. Then the system is controllable for any choice of input node.

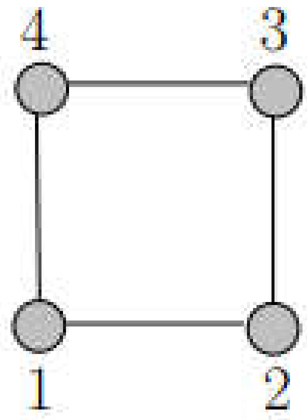
Controllability and graph symmetry

- Connections between the controllability and observability of the **SISO agreement protocol** and the structural properties of the underlying network



(a)

Symmetric only wrt 5



(b)

Symmetric wrt any node

Input symmetry through automorphism

Graph automorphism

文A 9 languages

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From Wikipedia, the free encyclopedia

In the mathematical field of graph theory, an **automorphism** of a graph is a form of symmetry in which the graph is mapped onto itself while preserving the edge–vertex connectivity.

Formally, an automorphism of a graph $G = (V, E)$ is a permutation σ of the vertex set V, such that the pair of vertices (u, v) form an edge if and only if the pair $(\sigma(u), \sigma(v))$ also form an edge. That is, it is a graph isomorphism from G to itself. Automorphisms may be defined in this way both for directed graphs and for undirected graphs. The composition of two automorphisms is another automorphism, and the set of automorphisms of a given graph, under the composition operation, forms a group, the automorphism group of the graph. In the opposite direction, by Frucht's theorem, all groups can be represented as the automorphism group of a connected graph – indeed, of a cubic graph.[1][2]

Proposition:

The system is input symmetric iff there is a nonidentity automorphism for G_f such that the input indicator vector remains invariant under its action.

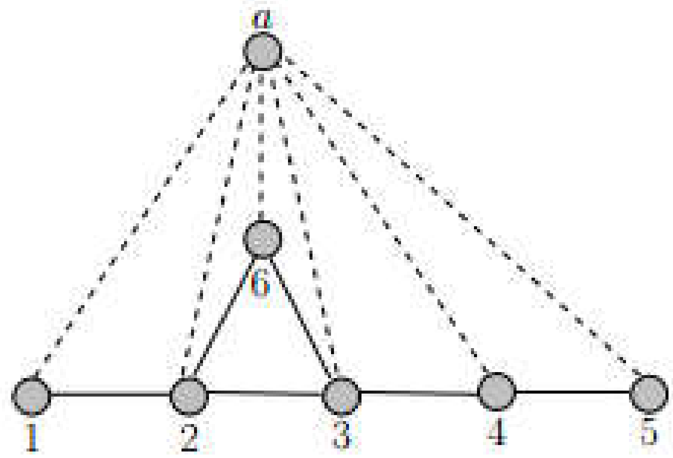
Controllability

Theorem:

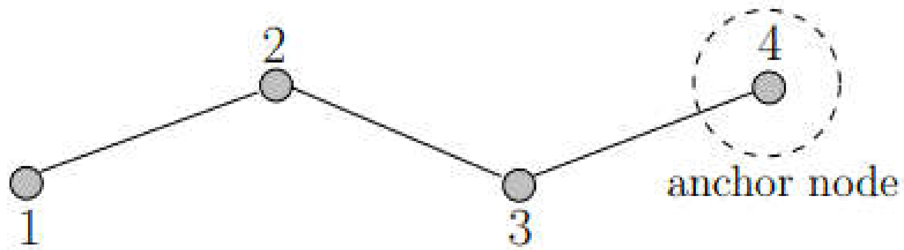
*The system is **uncontrollable** if it is **input symmetric**. Equivalently, the system is **uncontrollable** if the **floating graph admits a nonidentity automorphism** for which the input indicator vector remains invariant under its action.*

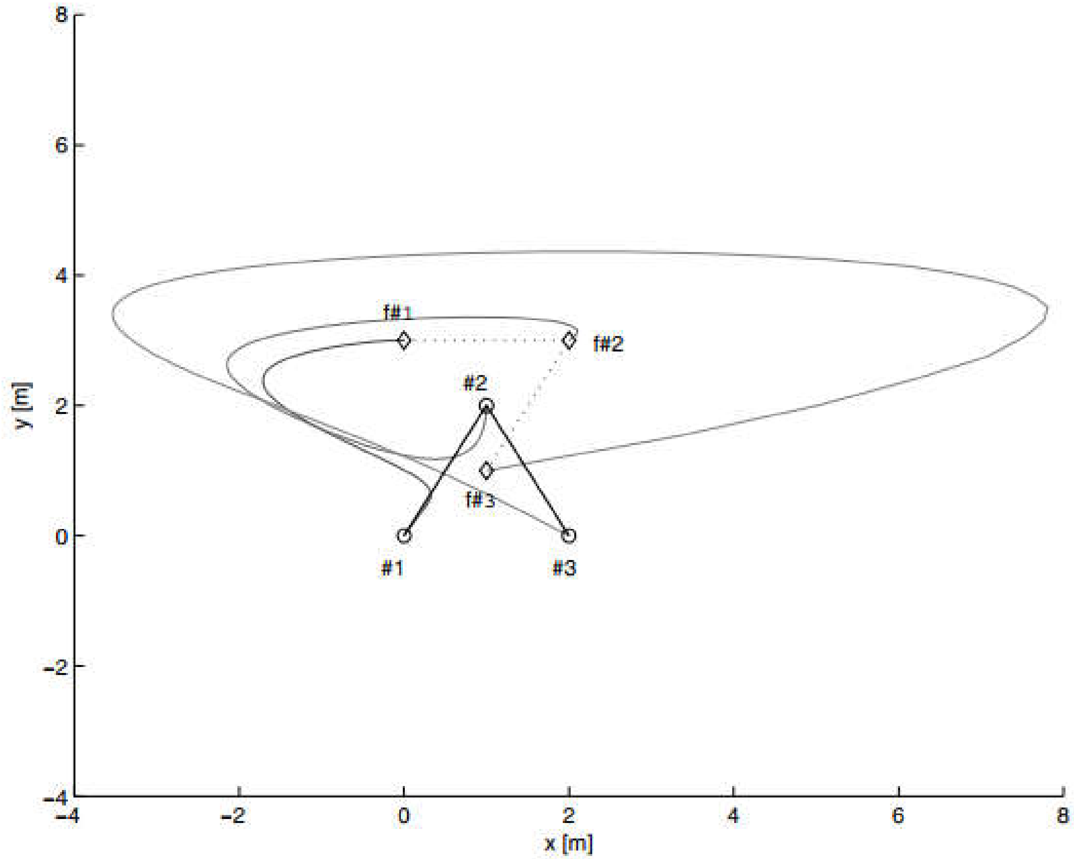
Proposition:

Input symmetry is not a necessary condition for system uncontrollability.



Controllability





Observability from a single observer post

In this setting, consider the system

$$\dot{x}_f(t) = -A_f x_f(t),$$
$$y(t) = -B_f^\top x_f(t),$$

Proposition:

The system is unobservable if it is output symmetric. Equivalently, it is unobservable if the floating graph admits a nonidentity automorphism for which the output indicator vector remains invariant under its action.

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Input and Output Setup

Controllability: MIMO case

Yurid E. Nugraha
TSP DTE FTEIC ITS

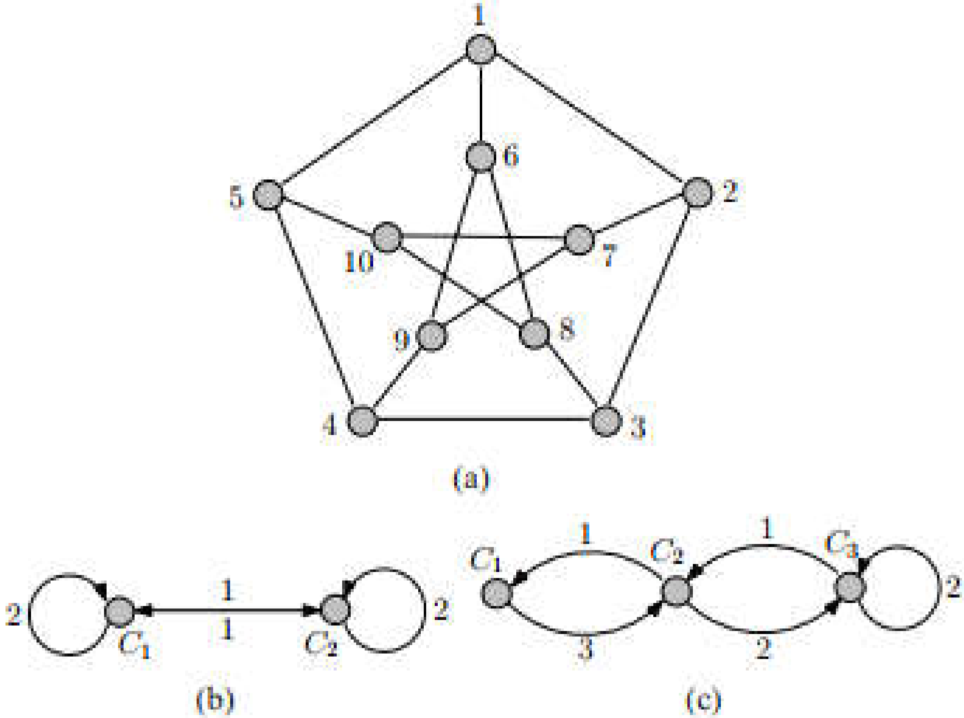
I-o agreement (MIMO case)

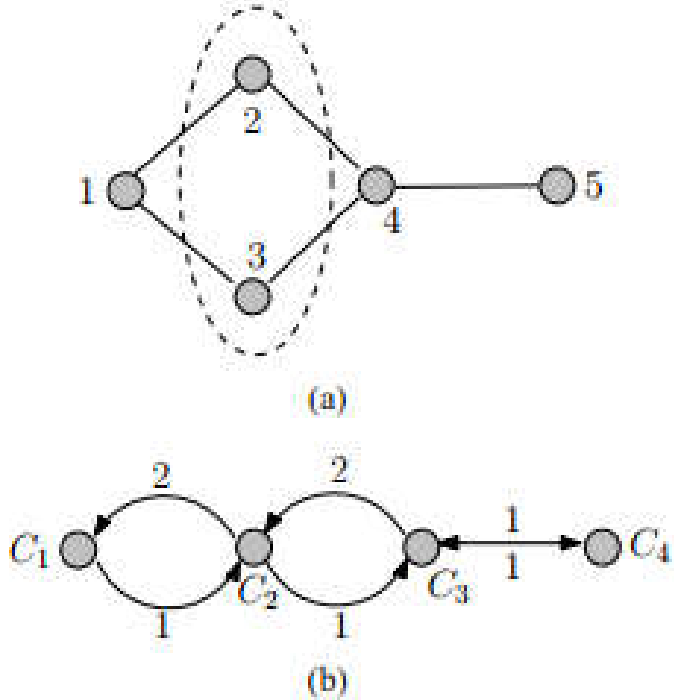
- Examine the graph theoretic connection between network topology and controllability for the agreement protocol equipped with **multiple inputs and outputs**.
- Agreement protocol:

$$\begin{aligned} \dot{x}_f(t) &= -A_f x_f(t) - B_f u(t), \\ y(t) &= -B_f^\top x_f(t), \end{aligned}$$

I-o agreement (MIMO case)

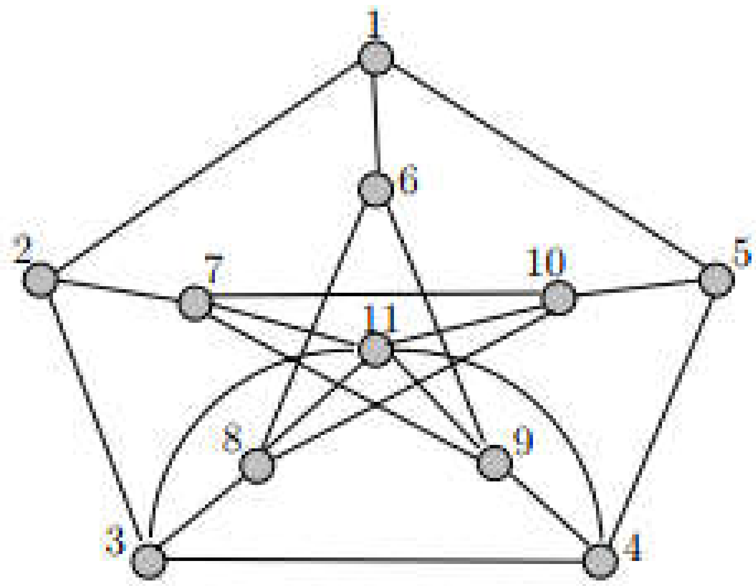
- NEP: Nontrivial equitable partition





I-o agreement (MIMO case)

- Peterson graph:



Corollary:

If G is disconnected, a necessary condition for the system to be controllable is that all of its connected components are controllable.

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Input and Output Setup

Agreement reachability

Yurid E. Nugraha
TSP DTE FTEIC ITS

Agreement reachability

- Examine the graph theoretic connection between network topology and controllability for the agreement protocol equipped with **multiple inputs and outputs**.
- Agreement protocol:

$$\begin{aligned} \dot{x}_f(t) &= -A_f x_f(t) - B_f u(t), \\ y(t) &= -B_f^\top x_f(t), \end{aligned}$$

Steering to the agreement subspace

- Controlled agreement: $\dot{x}(t) = -Q_r L(G) x(t) = -L_r(G) x(t)$,
- Disagreement protocol: $\dot{\zeta}(t) = -L_r(G)\,\zeta(t)$, where $\zeta(t)$ is projection of the followers' state $x_f(t)$ onto the subspace orthogonal to the agreement subspace span$\{1\}$
- Additional definition:

$$A_f = P_f^\top L(G) P_f, B_f = P_f^\top L(G) T_{fl}$$

Proposition:

For a single input node, the matrix $L_r(G)$ has a real spectrum and the same number of zero and positive eigenvalues as $L(G)$

Rate of convergence

- Define

$$\mu_2 = \min \frac{\zeta^\top \bar{L}_r(G)\zeta}{\zeta^\top \zeta}$$

$L_r(G)$: reduced identity matrix, $\bar{L}_r(G) = \frac{L_r(G)+L_r^\top(G)}{2}$

Proposition:

The rate of convergence of the disagreement dynamics is bounded by $\mu_2(L_r(G))$ and $\lambda_2(L(G))$, when the input node transmits a constant signal.

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Input and Output Setup

Network feedback

Yurid E. Nugraha
TSP DTE FTEIC ITS

Network feedback

- By viewing individual nodes as inputs, it becomes imperative to investigate how this point of view can be used to make the network perform useful things
- "how should the herding dogs move in order to maneuver the herd in the desired way"?
- Terminology: **leaders: input, followers: floating**

Network feedback

Lemma:

If the graph is connected, then the matrix A_f is positive definite.

Theorem:

Given fixed leader positions x_l, the quasi-static equilibrium point under the follower dynamics is

$$x_f = -A_f^{-1} B_f x_l,$$

which is globally asymptotically stable.

Summary

- What control theoretic properties one can infer from a network by looking solely at the network topology
- For controllability, the question becomes that of determining whether it is possible to "drive" the states of all the floating nodes by adjusting the value of the input nodes
- How the symmetry structure of a network directly relates to the controllability of the corresponding input system.