Class: Sistem Pengaturan Formasi dan Kolaborasi (EE185523)
Lecturer: Yurid E. Nugraha
Deadline: 2023/03/08

Assignment 2: Consensus

(You can choose which problems to solve; however, the maximum grade given will still be 100%.)

1. (**Weight: 30%**) Consensus (also sometimes called *agreement protocol*) is one of most common feature of multi-agent systems that has been discussed by many research papers. There are several breakthrough journal papers published in mid 2000s that discusses consensus in the context of control systems for the first time. Pick one of those papers and write a summary (about 400-500 words) on the content.

2. (**40%**) Simulate the continuous consensus protocol $\dot{x}(t) = -Lx(t)$ for a graph on five vertices. Write your code and compare the rate of convergence of the states as the number of edges increases (which eigenvalue dictates rate of convergence?). Does the convergence of the protocol always improve when the graph contains more edges? Provide an analysis to support your observation. (You can use matlab with command, e.g., 'ss' and 'initial')

3. (**30%**) Consider a group of five moving small robots that interact to each other according to a graph with adjacency matrix A. Their position $x[k]$ is modeled as discrete consensus model $x[k+1] = Ax[k]$, where

$$A = \begin{bmatrix} 0.15 & 0.15 & 0.1 & 0.2 & 0.4 \\ 0 & 0.55 & 0 & 0 & 0.45 \\ 0.3 & 0.05 & 0.05 & 0 & 0.6 \\ 0 & 0.4 & 0.1 & 0.5 & 0 \\ 0 & 0.3 & 0 & 0 & 0.7 \end{bmatrix}.$$

   Answer the following:

   (a) Draw the condensation of the associated digraph.

   (b) Do they finally converge to the same position $\lim_{k\to\infty} x[k]$? Support your answer with an analysis.

   (c) For $x_0 = [4\ 6\ 1\ 9\ 3]^\top$, compute the final position.

4. (**30%**) Recall the definition $\exp(A) = \sum_{k=0}^{\infty} \frac{1}{k!} A^k$ for any square matrix A. Complete the following tasks:

   (a) Show that $A = \mathrm{diag}(a_1, \ldots, a_n)$ implies $\exp(A) = \mathrm{diag}(e^{a_1}, \ldots, e^{a_n})$,

   (b) Show that $AB = BA$ implies $\exp(A + B) = \exp(A)\exp(B)$, and

   (c) Show that $\frac{d}{dt}\exp(At) = A\exp(At) = \exp(At)A$.

5. (**40%**) Consider the following weighted digraph and their associated non-negative adjacency matrices A and Laplacian matrices L of appropriate dimensions. Consider the associated discrete-time consensus $x[k+1] = Ax[k]$ and continuous-time Laplacian flows $\dot{x}(t) = -Lx(t)$. Analyze whether the discrete and/or continuous-time systems converge as time goes to infinity. If they converge, what value do they converge to? Show the simulation results and the code to accompany your analysis.

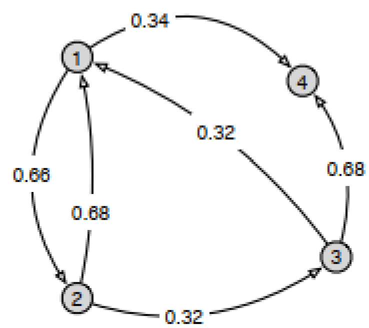

Figure 1: Digraph for Problem 4