EE185523

Coordination and Control of Unicycles

Yurid E. Nugraha
TSP DTE FTEIC ITS

Unicycles

- Convenient models in a wide range of applications, including in aerospace (unmanned aerial vehicles) and biology (fish locomotion)
- Position of unicycles i in $\mathbb{R}^2$: $[x_i, y_i]^T$, which can be represented by a complex number

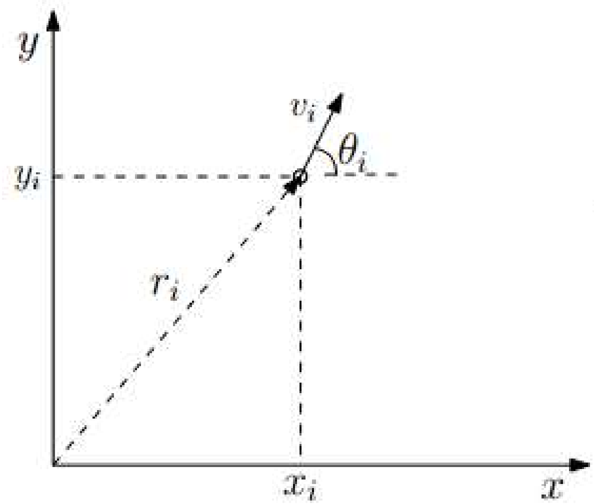
$$r_i(t) = x_i(t) + jy_i(t), t \geq 0, i \in \{1, \dots, n\}$$

- Since $\dot{x}_i(t) = v_i \cos\theta_i(t)\,, y_i(t) = v_i \sin\theta_i(t)$, and $\dot{\theta}_i(t) = \omega_i(t)$
- Assuming $\omega_i(t) = \dot{\theta}_i = u_i(t)$, kinematic:

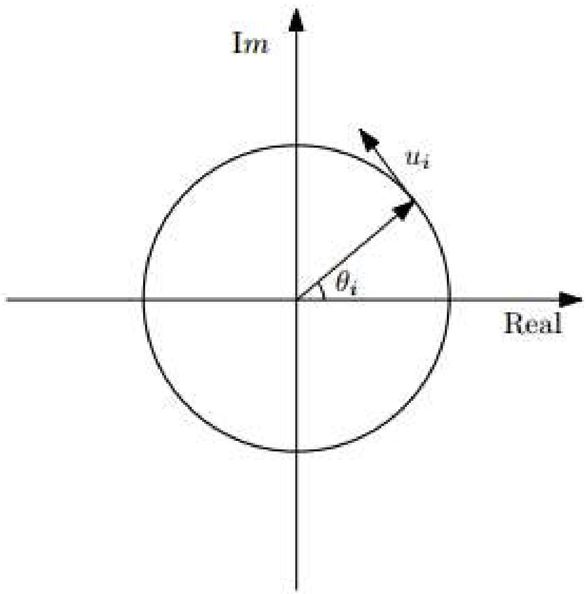
$$\dot{r}_i(t) = v_i e^{j\theta_i(t)}$$

Unicycles

By normalizing the speed of unicycle to 1, we can study the dynamics in the unit disk of a complex plane



cartesian



complex plane

Unicycle behaviors

- Agents' states: $\theta(t) = [\theta_1(t), \theta_2(t), \dots, \theta_n(t)]^T$, $u(t) = [u_1(t), u_2(t), \dots, u_n(t)]^T$, $e^{j\theta(t)} = [e^{j\theta_1(t)}, \dots, e^{j\theta_n(t)}]$
- Goal: to explore (undirected) **local interaction rules among the multiple unicycles** that lead to coordinated behavior among them.
  - Synchronization: the **heading angles** for the unicycles assume a common value,
  - Balanced behavior: the **center of mass** of the evolution of the unicycles remain constant,
  - Spacing: the unicycles rotate around a **pre-specified center(s)**, and
  - Symmetrical phase patterns: where the unicycles rotate about **a given center** with a certain regularity in their phase differences.

Navigation function

- Suppose $\dot{r}_i(t) = e^{j\theta i}$ as "state"
- Navigation function:
  - **Average state** (m^{th} order): $p_m(\theta) = \frac{\mathbf{1}^T e^{jm\theta}}{nm}$
  - **Potential** (m^{th} order): $U_m(\theta) = \frac{n}{2}|p_m(\theta)|^2$
  - **Average angle** ψ_m such that $p_m(\theta) = |p_m(\theta)|e^{j\psi_m}$

Balanced configuration

- **Balanced** configuration: when $p_m(\theta) = 0$
- **Synchronized** configuration of order m: when $\forall i, j$,

$$\theta_i = \theta_j \; mod \; (2\pi/m)$$

$$\Downarrow$$

Gradient control law $u_i(t) = -k\nabla_i U_1(\theta) = -\frac{k}{n}\sum_{j=1}^{n} \sin(\theta_j(t) - \theta_i(t))$

Gradient control law

Gradient control law $u_i(t) = -k\nabla_i U_1(\theta) = -\frac{k}{n}\sum_{j=1}^{n} \sin(\theta_j(t) - \theta_i(t))$

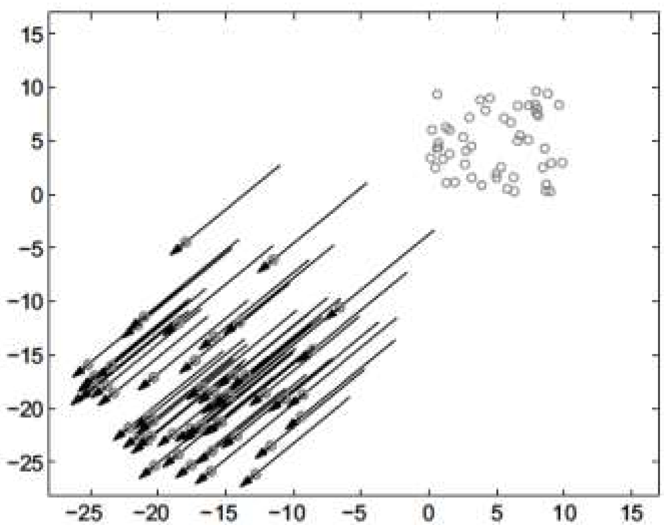
$\Downarrow$

Steers unicycle group towards minimum of $U_1(\theta)$ when $k > 0$

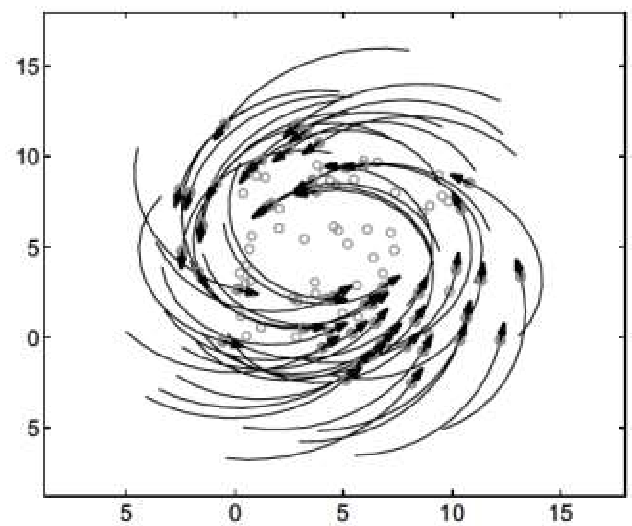
Steers unicycle group towards maximum of $U_1(\theta)$ when $k < 0$

Critical points that do not correspond to minimum or maximum of $U_1(\theta)$ are unstable

Gradient control law



synchronization



balanced configuration

Coordinated behaviors

By using critical points of the Laplacian-based potentials for synthesizing distributed control laws

$$W_m(\theta) = \frac{1}{2}\left(e^{jm\theta}\right) * L(G)e^{jm\theta}$$

$$\Downarrow$$

Gradient control law $u_i(t) = -\frac{k\left(\partial W_m(\theta)\right)}{\partial \theta_i} = mk \sum_{j \in N_i} \sin \mathrm{m}(\theta_j(t) - \theta_i(t))$

ensures that the unicycles are steered toward the synchronized configuration of order m

Technical results

Theorem: *Global* ***minimum*** *of $W_m(\theta)$ is the* ***synchronized*** *configuration of order m, whereas the global* ***maximum*** *of $W_m(\theta)$ is the* ***balanced*** *configuration of order m. In either case, a gradient law^ provides a distributed control strategy to attain these configurations with $k > 0$ for reaching synchronization, and $k < 0$ for reaching a balanced configuration.*

Spacing

- Consider $u_i(t) = \omega_0$, where ω_0 being a nonzero constant
- Unicycles travel centered at

$$c_i(t) = r_i(t) + \frac{j}{\omega_0} e^{j\theta_i(t)}$$

with radius $\rho_0 = \frac{1}{|\omega_0|}$

- Consider variable $q_i(t) = -j\omega_0 c_i(t) = e^{j\theta_i(t)} - j\omega_0 r_i(t)$
- Characterizes information on the heading and the centered rotation

Technical results

Theorem: *Construct potential* $S\big(q(t)\big) = \frac{1}{2}\langle q(t), L(G)q(t)\rangle$*. Then, the gradient-based control*

$$u_i(t) = \omega_0 + k\big\langle L(G)_i q(t), je^{j\theta_i(t)}\big\rangle$$

steers the group of unicycles toward agreement on their centers of rotation