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Control of Mobile Robots

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Proximity-graph

- We will focus on the case when the graph is a Δ-disk proximity graph, that is, where

$$\{i, j\} \in E \leftrightarrow x_i - x_j \leq \Delta$$

- If the robots are equipped with omnidirectional range sensors, they can only detect neighboring robots that are close enough

- Naturally, graphs are *dynamic*

Dynamic graphs

- What makes the multirobot problem challenging is that the agents' movements can no longer be characterized by purely combinatorial interaction conditions
- Suppose single integrator $\dot{x}_i(t) = u_i(t)$
- Control law in the form of $u_i(t) = \sum_{j \in N_\sigma(i)} f(x_i(t) - x_j(t))$
- Static interaction graph?

Subset of neighbors

Values of information

- **Not all information** has to be considered
- Symmetric indicator function $\sigma(i,j) = \sigma(j,i) \in \{0,1\}$

- Suppose *antisymmetric* control law
$$f\left(x_i(t) - x_j(t)\right) = -f\left(x_j(t) - x_i(t)\right), \forall\{v_i, v_j\}$$
- Range-based sensors: only based on **relative-states**

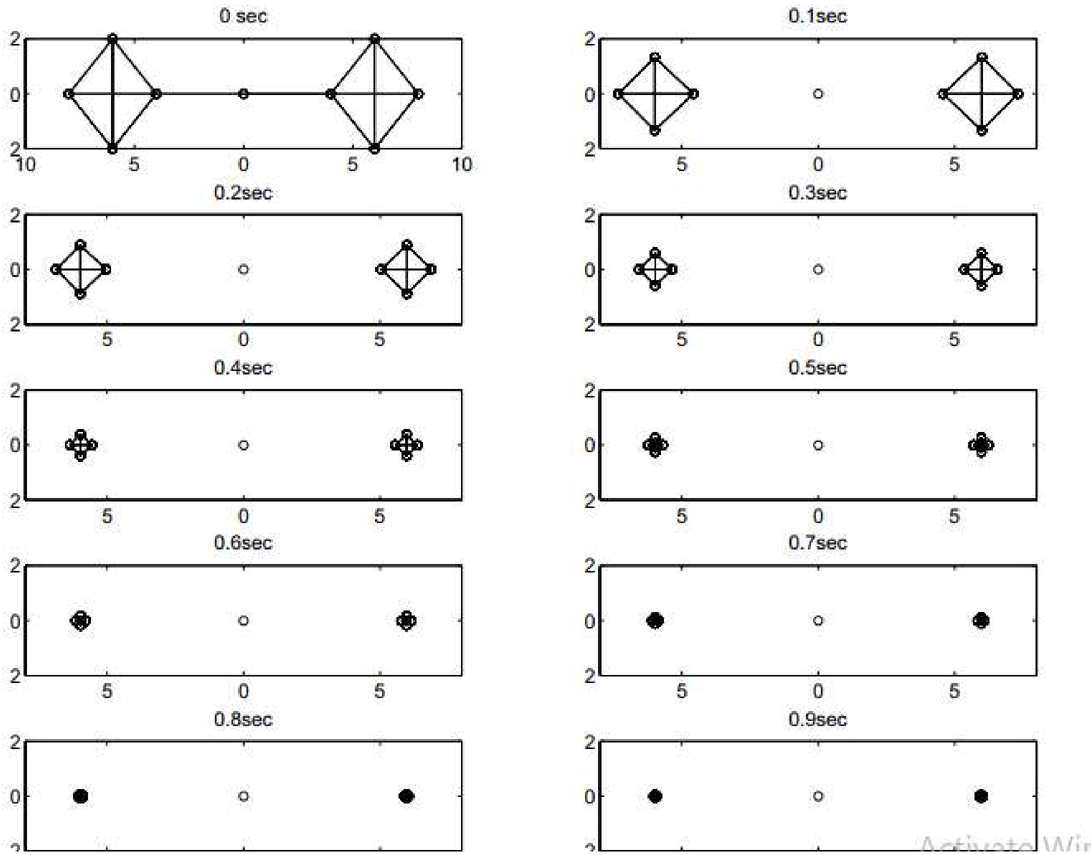
Cooperative robotics

- p-dimensional position of agent i be given by

$$x_i(t) = \left[x_{i,1}(t), \dots, x_{i,p}(t)\right]^T,\ i = 1, \dots, n$$

- If (interaction) graphs are always connected **in each time**, then consensus problem is solved
- **However, this is not always the case**

Dynamic interaction graphs



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Weighted graph-based feedback

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Nonlinear edge weight

- How nonlinear edge weights affect some properties
- Suppose

$$f\left(x_i(t) - x_j(t)\right) = \underbrace{-w\left(x_i(t) - x_j(t)\right)}_{\text{weighting function}}\left(x_i(t) - x_j(t)\right)$$

$$\dot{x}_i(t) = -\sum_{j \in N_i} w\left(x_i(t) - x_j(t)\right)\ \left(x_i(t) - x_j(t)\right)$$

Edge tension

- Assumption: static interaction graph
- Edge vector between agents i and j: $l_{ij}(x) = x_i - x_j$
- Suppose **ϵ-interior** of a δ-constrained realization:

$$D_{G,\delta}^{\epsilon} = \{x \in \mathbb{R}^{pn} \mid \|l_{ij}\| \leq \delta - \epsilon, \forall \{i,j\} \in E\}$$

- An **edge tension** $\mathcal{V}_{ij}(\delta, x)$ is defined as (one example of 'consensus')

$$\mathcal{V}_{ij}(\delta, x) := \begin{cases} \dfrac{\|l_{ij}(x)\|^2}{\delta - \|l_{ij}(x)\|}, & \text{if } \{i,j\} \in E \\ 0, & \text{otherwise} \end{cases}$$

Edge tension $\mathcal{V}_{ij}(\delta, x)$

$$\mathcal{V}_{ij}(\delta, x) = \begin{cases} \dfrac{\left\|l_{ij}(x)\right\|^2}{\delta - \left\|l_{ij}(x)\right\|}, \text{if } \{i, j\} \in E \\ 0, \qquad\qquad\qquad \text{otherwise} \end{cases}$$

$$\frac{\partial \mathcal{V}_{ij}(\delta, x)}{\partial x_i} = \begin{cases} \dfrac{\left(2\delta - \left\|l_{ij}(x)\right\|\right)(x_i - x_j)}{\left(\delta - \left\|l_{ij}(x)\right\|\right)^2}, \text{if } \{i, j\} \in E \\ 0, \qquad\qquad\qquad \text{otherwise} \end{cases}$$

Edge tension $\mathcal{V}_{ij}(\delta, x)$

$$\mathcal{V}_{ij}(\delta, x) = \begin{cases} \dfrac{\|l_{ij}(x)\|^2}{\delta - \|l_{ij}(x)\|}, & \text{if } \{i, j\} \in E \\ 0, & \text{otherwise} \end{cases}$$

- $\mathcal{V}_{ij}(\delta, x)$ will be infinite if (when) $\|l_{ij}(x)\| = \delta$
- However, we can prevent the function from reaching infinity
- "All other edges have length zero"
- Total edge tension: $\mathcal{V}(\delta, x) = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\mathcal{V}_{ij}(\delta, x)$
- As $\mathcal{V}$ decreases, no edge distances will tend to δ

Main SIG theorem

Theorem 7.2:

Consider a connected SIG with initial condition $x_0 \in \mathcal{D}^{\epsilon}_{G,\delta}$. Then the multiagent system under control law

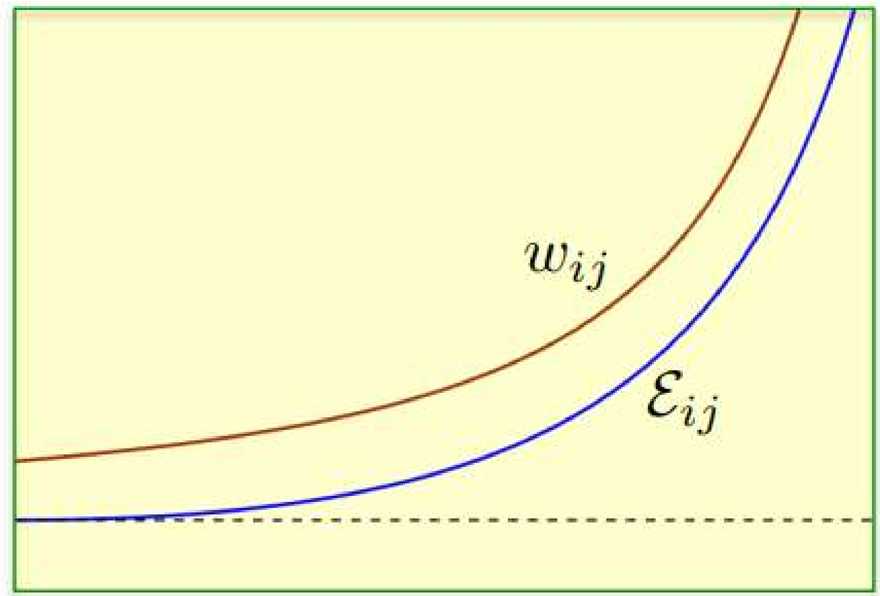
$$\dot{x}_i = -\sum_{j \in \mathcal{N}_i} \frac{\left(2\delta - \left\|l_{ij}(x(t))\right\|\right)}{\left(\delta - \left\|l_{ij}(x(t))\right\|\right)^2} (x_i(t) - x_j(t))$$

converges to static value $\bar{x}$

Main SIG theorem

- 'Connectivity maintenance':

$$\dot{x}_i = -\sum_{j \in \mathcal{N}_i} \frac{\left(2\delta - \left\| l_{ij}(x(t)) \right\|\right)}{\left(\delta - \left\| l_{ij}(x(t)) \right\|\right)^2} (x_i - x_j)$$



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Control of Mobile Robots

Dynamic graphs

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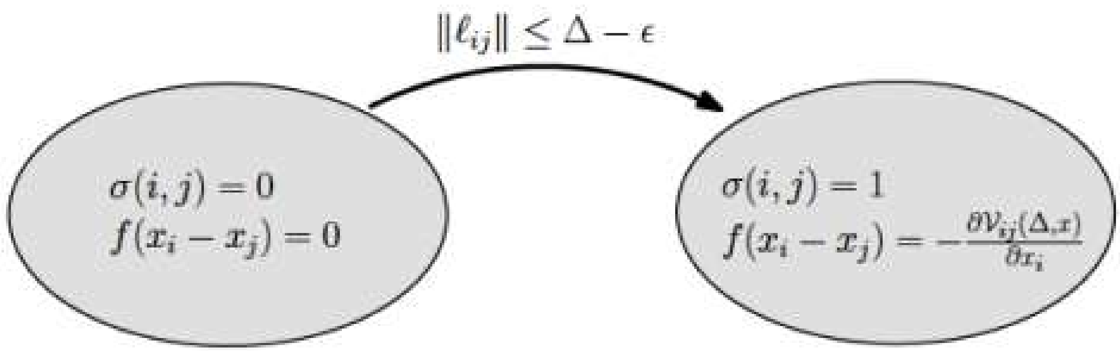
Dynamic graphs

- Let's continue using tension energy
- However, we cannot allow infinite tension energies in the definition of control laws
- Have to introduce 'hysteresis': note the one-way arrow
- Done through indicator function $\sigma(i,j)$



Dynamic graphs

- Let's continue using tension energy
- However, we cannot allow infinite tension energies in the definition of control laws

$$\sigma(i,j)[t^+] = \begin{cases} 0, \text{if } \sigma(i,j)[t^-] = 0 \text{ and } \|l_{ij}\| > \Delta - \epsilon, \\ 1, \text{otherwise.} \end{cases}$$

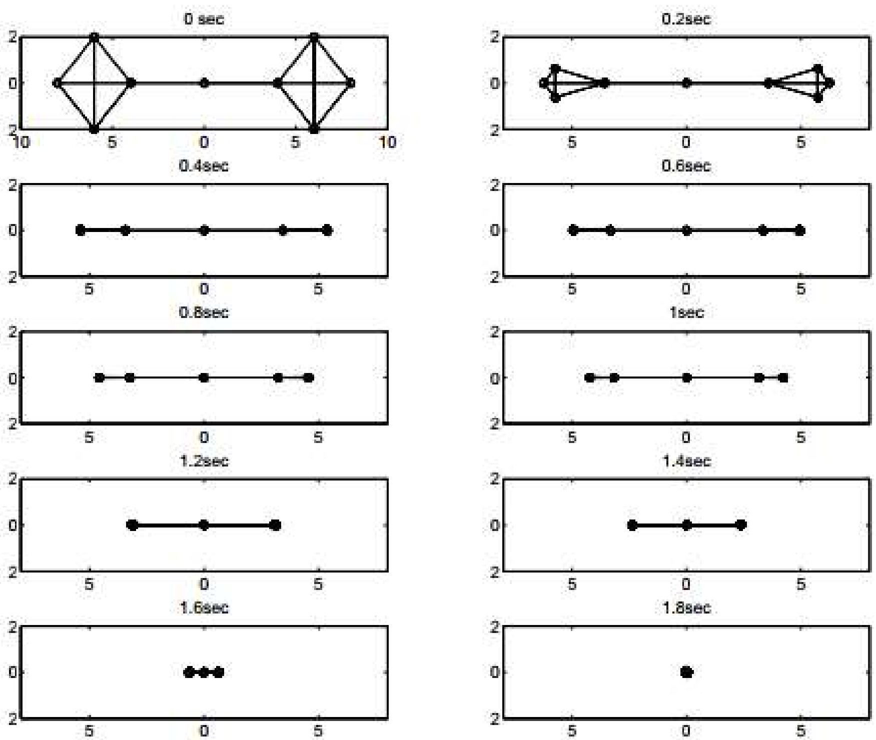
$$f(x_i - x_j) = \begin{cases} 0, \text{if } \sigma(i,j) = 0, \\ -\dfrac{\partial \mathcal{V}_{ij}(\Delta, x)}{\partial x_i}, \text{otherwise.} \end{cases}$$

Dynamic graphs

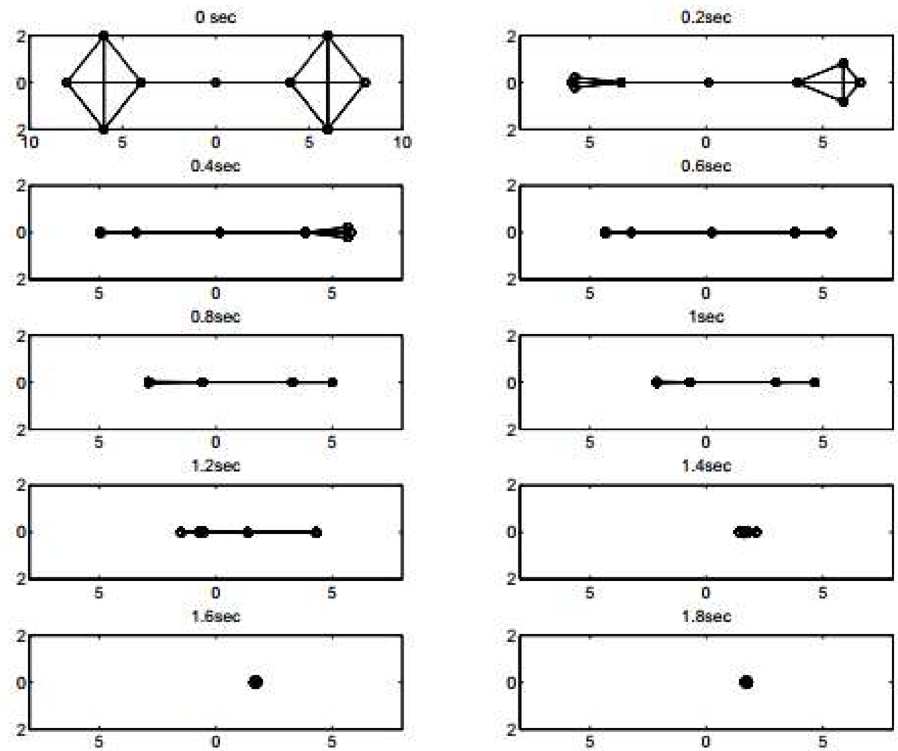
Theorem 7.3:

- *Consider an initial position $x_0 \in \mathcal{D}^{\epsilon}$, where $\epsilon > 0$ is the switching threshold*
- *Assume that the initial graph G_0 is connected*
- ***Then, by control law $u_i = -\sum_{j \in N(i)} \frac{\partial \mathcal{V}_{ij}(\Delta, x)}{\partial x_i}$, it is guaranteed that agents converge***

Dynamic graphs w/ weight and $f(x_i - x_j) = \frac{\partial \mathcal{V}_{ij}(\delta, x)}{\partial x_i}$



Agents still converges



Agents still converges w/ **vertex-weight matrix**

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Control of Mobile Robots

Formation control: revisited

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Formation control?

- Previously: showed a procedure for synthesizing control laws that preserve connectedness while **solving the rendezvous problem**

Goal:

Find a feedback law such that:

- F1: DIG converges to a graph that is a subgraph of the desired graph G_d in finite time. In other words, what we want is that $E_d \subseteq E(t)$ for all $t \geq T$, for some finite $T \geq 0$.
- F2: Pairwise distance $||l_{ij}(t)|| = ||x_i(t) - x_j(t)||$ converges to d_{ij} for $(i,j) \in E_d$
- F3: Feedback law utilizes only local information

Main result

Lemma 7.5:

Given an initial condition x_0 *such that* $y_0 = (x_0 - \tau_0) \in \mathcal{D}^{\epsilon}_{G_d,\Delta-\|d\|}$ *, the mobile agents adopting the decentralized control law*

$$\dot{x}_i = -\sum_{j\in\mathcal{N}_{(G_d)}(i)} \frac{2\left(\Delta - \|d_{ij}\|\right) - \|l_{ij}(t) - d_{ij}\|}{\left(\Delta - \|d_{ij}\| - \|l_{ij}(t) - d_{ij}\|\right)^2} (x_i(t) - x_j(t) - d_{ij})$$

are guaranteed to satisfy $x_i(t) - x_j(t) = l_{ij}(t) < \Delta$ *for all* $\{i, j\} \in E_d$.

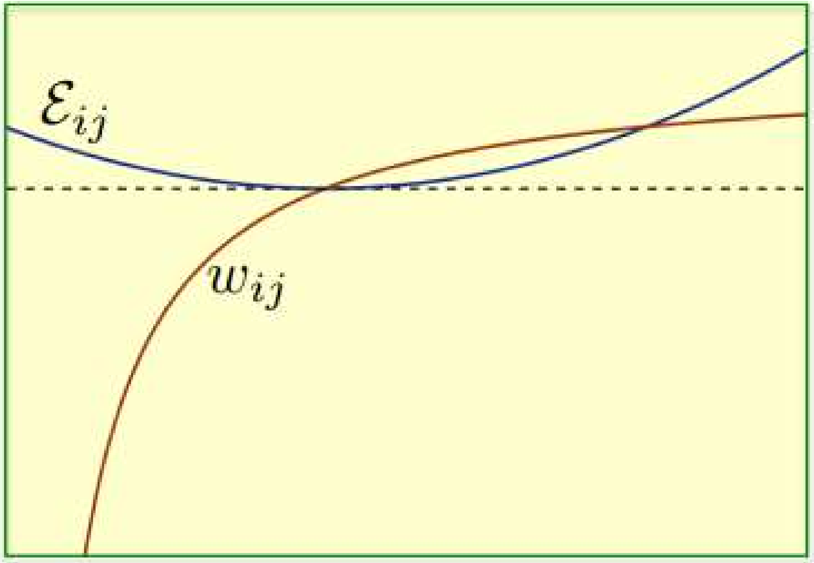
Theorem 7.6:

For all i, j*, the pairwise relative distances* $||l_{ij}(t)|| = ||x_i(t) - x_j(t)||$ *asymptotically converge to* $\|d_{ij}\|$ *for* $\{i, j\} \in E_d$.

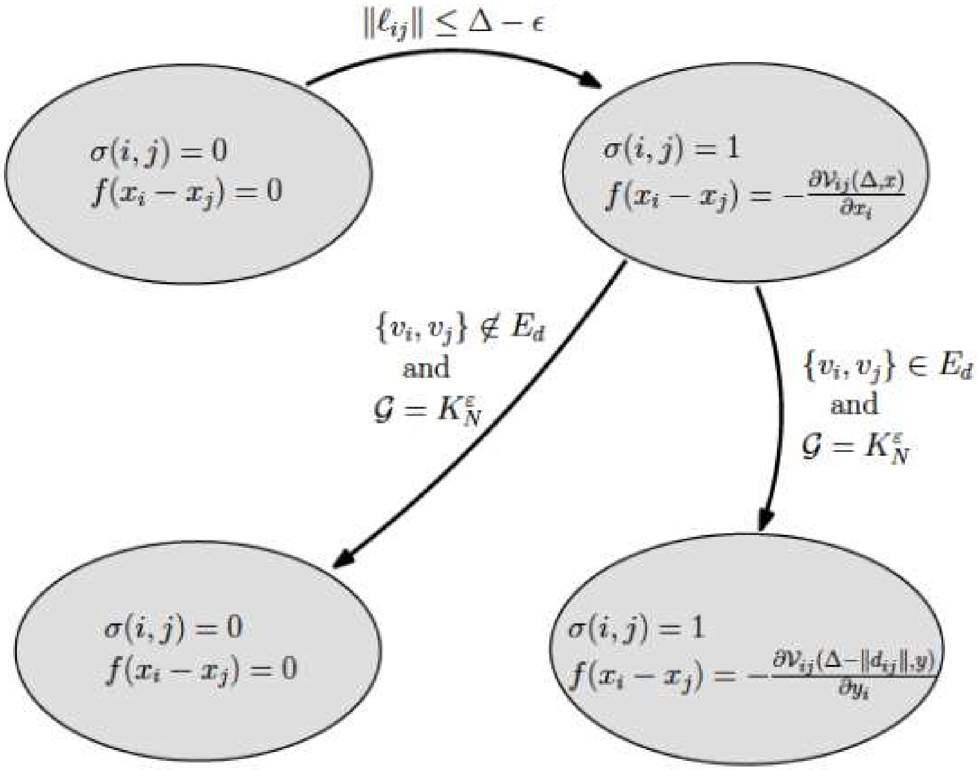
Formation control v2

Another version:

$$\dot{x}_i = -\sum_{j \in \mathcal{N}_{(G_d)}(i)} \frac{\left\| l_{ij}(t) - d_{ij} \right\|}{\left\| l_{ij}(t) \right\|} (x_i(t) - x_j(t))$$

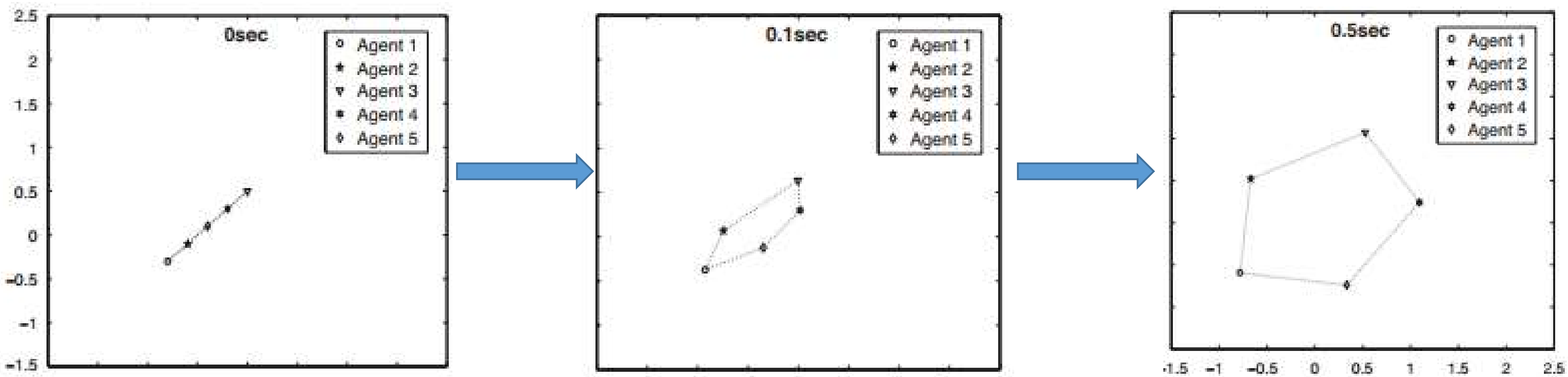


Hybrid rendezvous-to-formation strategies



- Switching based on agents' neighbors
- Notation:
  - $G_d = (V, E_d)$: graph of desired formation
  - K_n^{ϵ}: complete graph w/ ϵ-disk proximity graph

Formation control revisited



Desired formation: Pentagonal w/ $G_d = C_5$
Desired agent distances $\delta = 3.2\ \forall (i,j) \in E_d$

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Coverage problem

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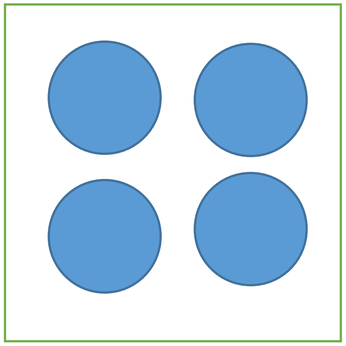
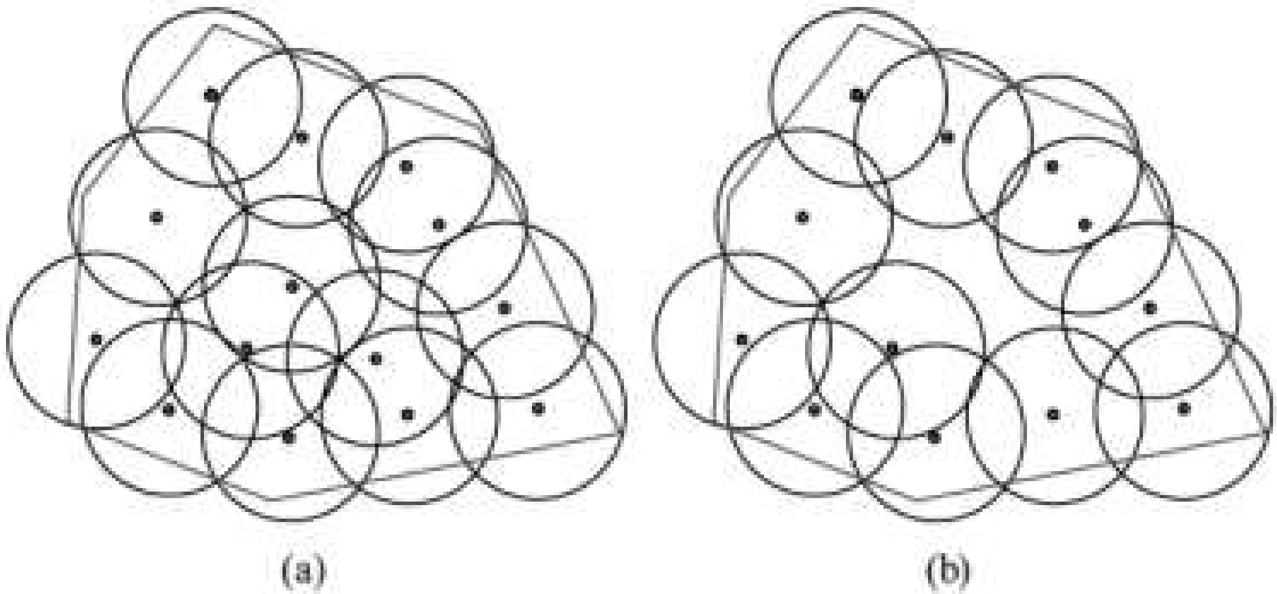
Coverage problem

- Focus on the concern that ensures a collection of mobile robot are placed in such a way that the area under consideration is completely covered
- **Geometry** will be important
  - of sensing areas,
  - of areas in which sensor nodes are deployed



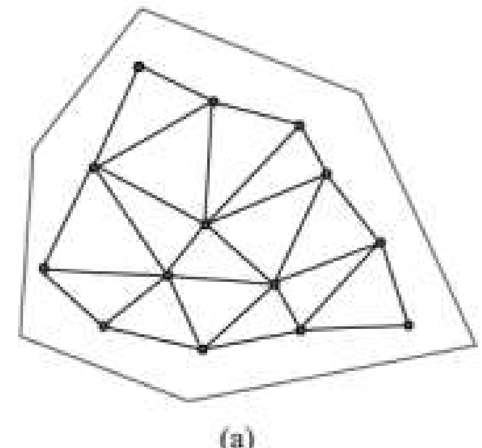
Coverage problem



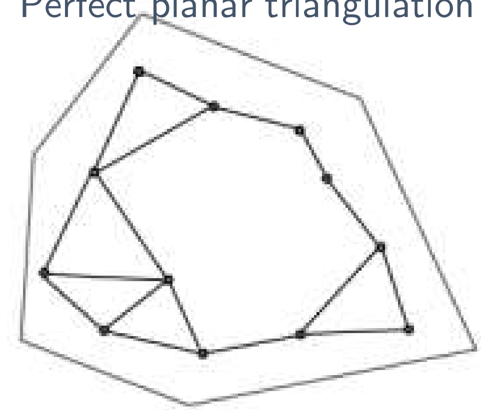
covered **not covered**

Triangulation

- Help understand how **areas** are contained by **edges**
- Triangular subgraph: any subgraph composed of three fully connected vertices
- Planar embedding: there exist $\zeta : V \to \mathbb{R}^2$ such that **no edges intersect**
- Perfect planar triangulation: **Outer face** is a cycle, **inner face** are all triangles



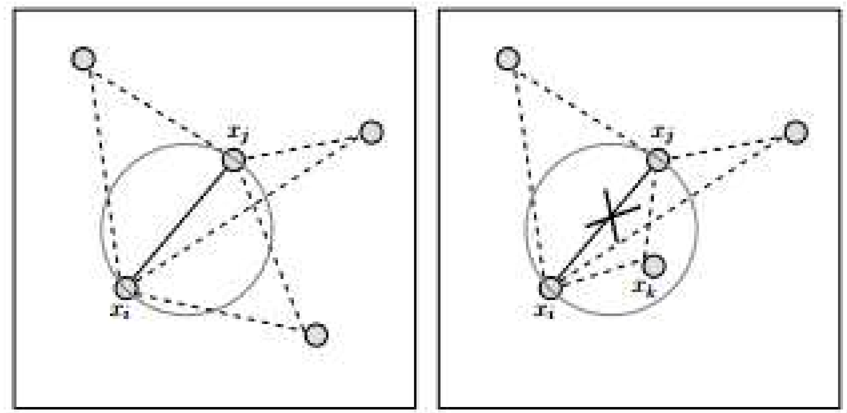
Perfect planar triangulation



Imperfect planar triangulation

Gabriel graphs

- The idea: to generate a proximity graph that is, to a large degree, a nearest neighbor interaction graph
- Gabriel graph: $G = (V, E)$, where $\{i, j\} \in E$ if and only if the interior angle $\angle(x_i, x_k, x_j)$ is **acute** for all other points x_k



(a) (b)

Some results on Gabriel graphs

Lemma:

Nearest neighbor edge is always present in the Gabriel graph, i.e., if $||x_i - x_j|| < ||x_i - x_k|| \; \forall k$, then $(i, j) \in E$

Lemma:

If $(i, j) \notin E$, then there exists v_k such that x_k is closer to both x_i and x_j than x_i and x_j are to each other, i.e., $||x_i - x_k|| < ||x_i - x_j||$ and $||x_j - x_k|| < ||x_i - x_j||$.

Some results on Gabriel graphs

Theorem:

- Any Gabriel graph is planar
- Any Gabriel graph is connected

As a consequence, we have obtained a combinatorial structure with **almost** the correct topology for achieving combinatorial coverage, that is, perfect, planar triangulations.

↓

"near-coverage"

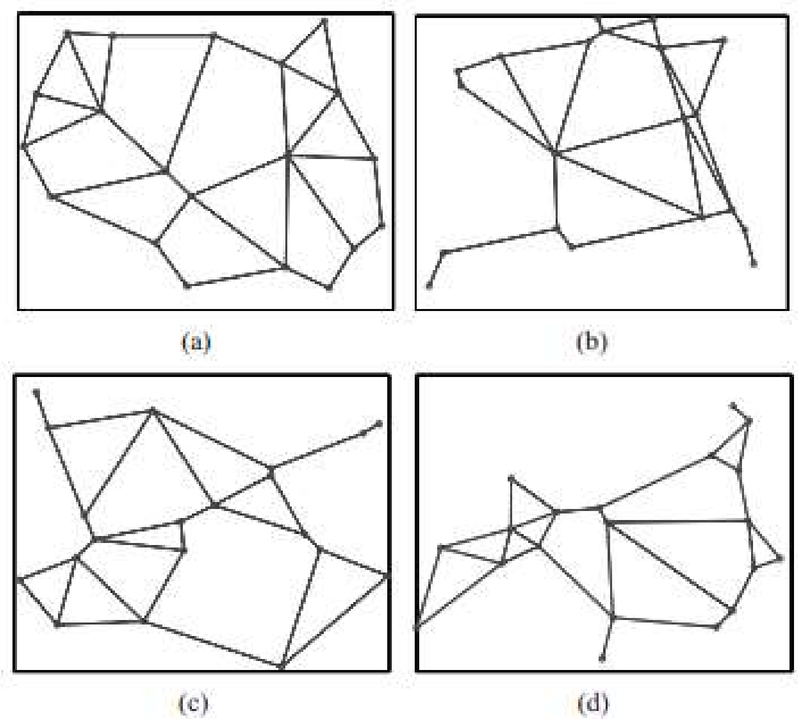
Gabriel graphs

- Corresponding control law:

$$\dot{x}_i(t) = -\sum_{j \in N_i} \nabla_{xi} U_{ij}$$
$$= \sum_{j \in N_i} \frac{(\|x_i(t) - x_j(t)\| - \Delta)\big(x_i(t) - x_j(t)\big)}{\|x_i(t) - x_j(t)\|}$$

- Edge potential:

$$U_{ij} := \frac{1}{2}\big(\|x_i - x_j\| - \Delta\big)^2$$

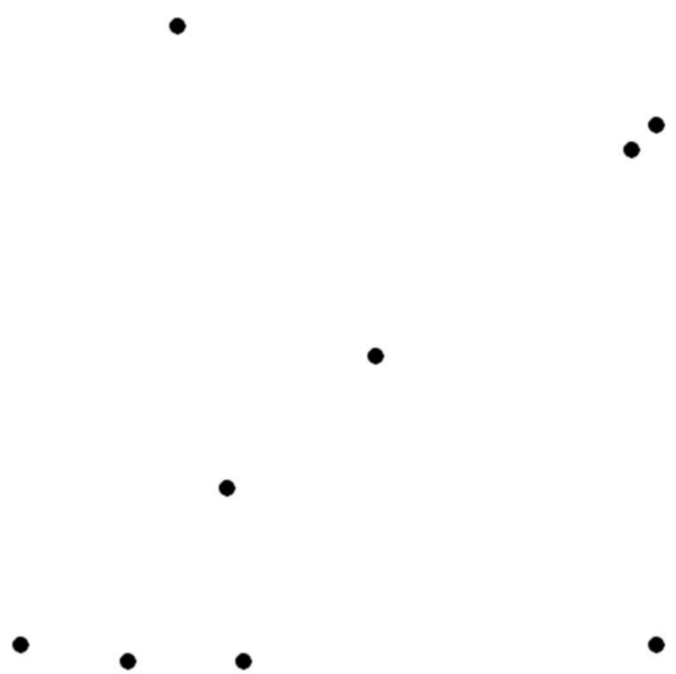


(Gabriel graphs associated with 20 randomly placed nodes)

Voronoi-based coverage algorithm

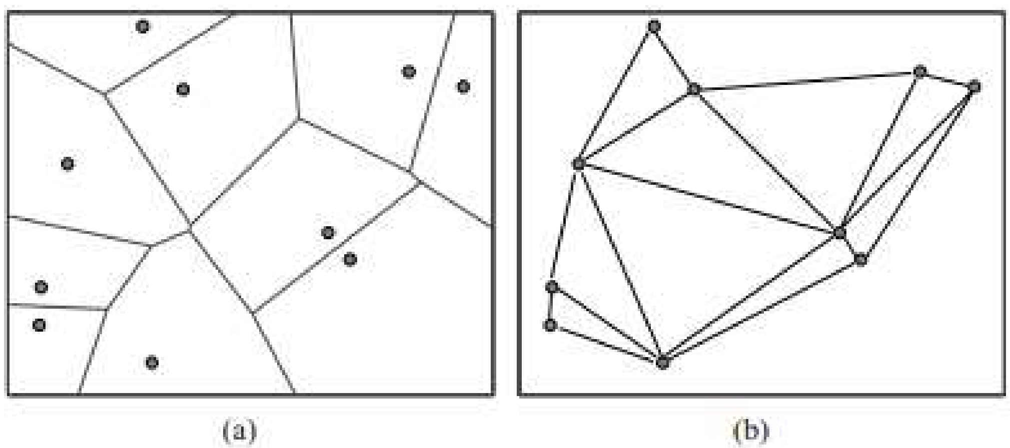
- What was needed to produce these structures was the ability to go beyond close-range interactions and incorporate longer-range interactions if needed
- These **longer-range** interactions were based on the relative placements of the nodes
- One can adjust this viewpoint by basing the longer-range interactions on the areas covered by the sensor nodes directly

Voronoi-based coverage algorithm



- Tessellation: a shape is repeated over and over again covering a plane without any gaps or overlaps
- Tesselation of Ω by Voronoi partition $\mathcal{V} = \{V_i(x)\}: V_i(x) = \{q \in \Omega \mid \|q - x_i\| \leq \|q - x_j\| \ \forall j \neq i\}$

Voronoi-based locational cost function

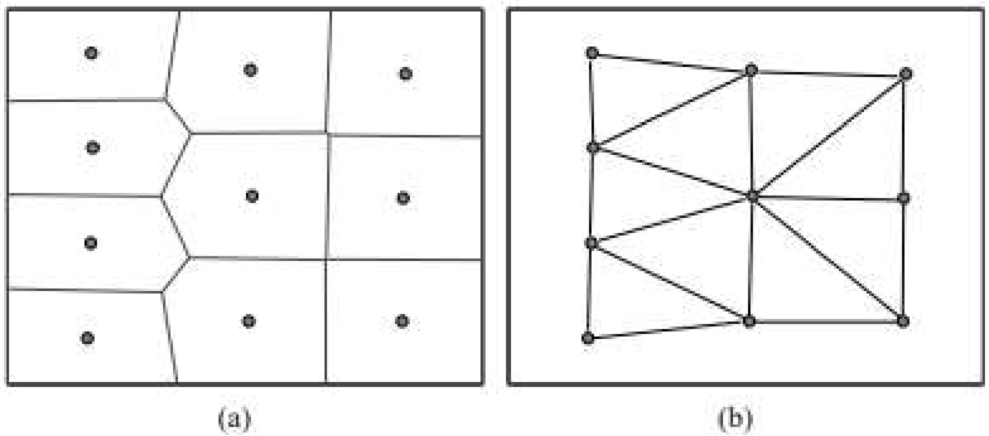


Optimization problem:

$$H_V = H\big(x, V(x)\big)$$
$$= \int_{\Omega} \min_{i\in\{1,\ldots,n\}} \|q - x_i\|^2 dq$$

Initial placement of agents (Voronoi partition: (a), associated graph: (b))

Voronoi-based locational cost function



After running gradient descent method

Some notes:

- For its computation, the Voronoi region $V_i(x)$ must be computed.
- Thus, agent i needs also to know the relative location of all agents whose Voronoi cells are adjacent to V_i.
- This is where the long range interactions may needed since there are no guarantees that, e.g., these agents are within a certain distance of each other.