Class: Sistem Pengaturan Formasi dan Kolaborasi (EE185523)
Lecturer: Yurid E. Nugraha
Date and Time: 2023/06/16, 13.00-17.30
Rule: Take home

Final Exam (2022 Genap)

Rule: **Solve all six problems. In each problem, submit the m-file if needed.**

1. Consider a so-called leader-follower network with two leaders l_1, l_2 and three followers f_1, f_2, f_3. Assume that the leaders and followers all live on the real line and that the network topology is a line graph l_1-f_1-f_2-f_3-l_2. Moreover, let the dynamics be given by

$$\begin{aligned}\dot{x}_{f_1} &= \alpha_1((x_{l_1} - x_{f_1}) + (x_{f_2} - x_{f_1}))\\ \dot{x}_{f_2} &= \alpha_2((x_{f_1} - x_{f_2}) + (x_{f_3} - x_{f_2}))\\ \dot{x}_{f_3} &= \alpha_3((x_{f_2} - x_{f_3}) + (x_{l_2} - x_{f_3}))\\ \dot{x}_{l_1} &= 0\\ \dot{x}_{l_2} &= 0\end{aligned}$$

   where α_1, $\alpha_2, \alpha_3 > 0$. Where do x_1, x_2 end up as $t \to \infty$, if $x_3 = \beta$ and $x_4 = \gamma$, $\beta < \gamma$? Analyze according to the value of α_1, α_2, and α_3.

2. Consider an edge tension

$$\mathcal{V}_{ij}(||x_i - x_j||) = 1 - e^{(-||x_i - x_j||^2)}.$$

   Determine $\dot{x}_i(t)$ associated with negative gradient flow $-\partial \mathcal{V}_{ij}(\Delta, x)/\partial x$. Also, assuming the graph is undirected and connected at all times, what will the agents' states be at infinite time?

3. Consider n agents consisting of m leaders and $n - m$ followers all with scalar dynamics placed at $(0, 0)$ at $t = 0$. Assume that there is a goal in x_g that is known only to the leaders. Let the dynamics of *each* leader with state x_{il} be

$$\dot{x}_{il}(t) = \sum_{j \in N_{il}} (x_j(t) - x_{il}(t)) + c(x_g - x_{il}(t))$$

   for some positive weight $c > 0$. Moreover, assume that the followers are executing

$$\dot{x}_{if}(t) = \sum_{j \in N_{if}} (x_j(t) - x_{if}(t))$$

   Suppose each agent has a radius $\Delta = 5$ (radius will be used for designing proximity graph). What are the values of n, m, and x_g in order for the agents to stay connected from origin initial value to x_g?

4. Let $H_i, i = 1, 2, 3, 4$, be the rows of the 4 x 4 identity matrix in the observation scheme $z_i = H_i x + v_i$ for a four-node sensor network, observing state $x \in \mathbb{R}^4$. It is assumed that the nodes form a**cycle** graph and that v_i is a zero-mean, unit variance, Gaussian noise. Choose the weighting matrix W and the step size Δ which satisfies the condition for stability. Design the code. (Submit m-file)

5. Design a simple code in Matlab to simulate the single-input single-output networks which consist of 5 agents. You can use any graph topology and any position of the input node and floating node. Initial condition of the floating nodes as well as the the constant $x_i[k]$ are free. Use discrete-time protocol

$$x_f[k+1] - x_f[k] = 0.1(-A_f x_f[k] - B_f u[k])$$
$$y[k] = -B_f^\top x_f[k]$$

Determine the control signal $u[k]$ which can stabilize the floating nodes. Show the code and give some comments on the evolution of states of the agents. (Submit m-file)

6. Design a simple code in Matlab to convert a position of 7 agents in a two dimension into both Voronoi diagram and proximity graph. Input of the program should be just position $(x_1, y_1), \ldots, (x_7, y_7)$, and the output should be both Voronoi diagram and the proximity graph in two separate figures. Show the code. (Submit m-file)