Class: Sistem Pengaturan Formasi dan Kolaborasi (EE185523)
Lecturer: Yurid E. Nugraha
Deadline: 2023/04/11

Assignment 3: Introduction to Formation Control

(You can choose which problems to solve **except Problem 1, which is mandatory**.)

1. (**Weight: 30%**) The problem of specifying, achieving, and maintaining formations has a rich history and a number of different control strategies have been proposed to this end. Pick **two** of the papers below and summarize each paper in about 400 words.

   - P. Ogren, M. Egerstedt, and X. Hu, A control Lyapunov function approach to multiagent coordination, *IEEE Transactions on Robotics and Automation*, vol. 18, pp. 847 – 851, 2002.
   - M. Ji and M. Egerstedt, Distributed coordination control of multiagent systems while reserving connectedness. *IEEE Transactions on Robotics*, vol. 23, pp. 693–703, 2007.
   - A. Muhammad and M. Egerstedt. On the structural complexity of multi-agent robot formations. *Proceedings of the American Control Conference*, June 2004.
   - R. Funada et al., Coordination of Robot Teams Over Long Distances: From Georgia Tech to Tokyo Tech and Back-An 11,000-km Multirobot Experiment. *IEEE Control Systems Magazine*, vol. 40, pp. 53-79, August 2020.

2. (**25%**) Recall that a condition for rigidity is that

$$(x_i - x_j)^\top (\dot{x}_i - \dot{x}_j) = 0 \quad \forall (i, j \in \mathcal{E}_f),$$

   where $\mathcal{E}_f$ being the edges of the formation graph. This relation can be rewritten as $R(q)\dot{q}$, where

$$q = \begin{bmatrix} x_1 \\ \dots \\ x_n \end{bmatrix}.$$

   For planar (two-dimensional) case, analyze the **rank** of the matrix R and the condition of rank required to guarantee the rigidity of the graph.

3. (**25%**) Given an undirected and connected graph, let

$$\dot{x}_i = -\Sigma_{j \in \mathcal{N}_i} (||x_i - x_j|| - k_{ij})(x_i - x_j),$$

   where $k_{ij} = k_{ji}$ is the desired separation between agents i and j. If the desired interagent separations are feasible, analyze whether the dynamics above is stable when $x_i - x_j \approx k_{ij}$.

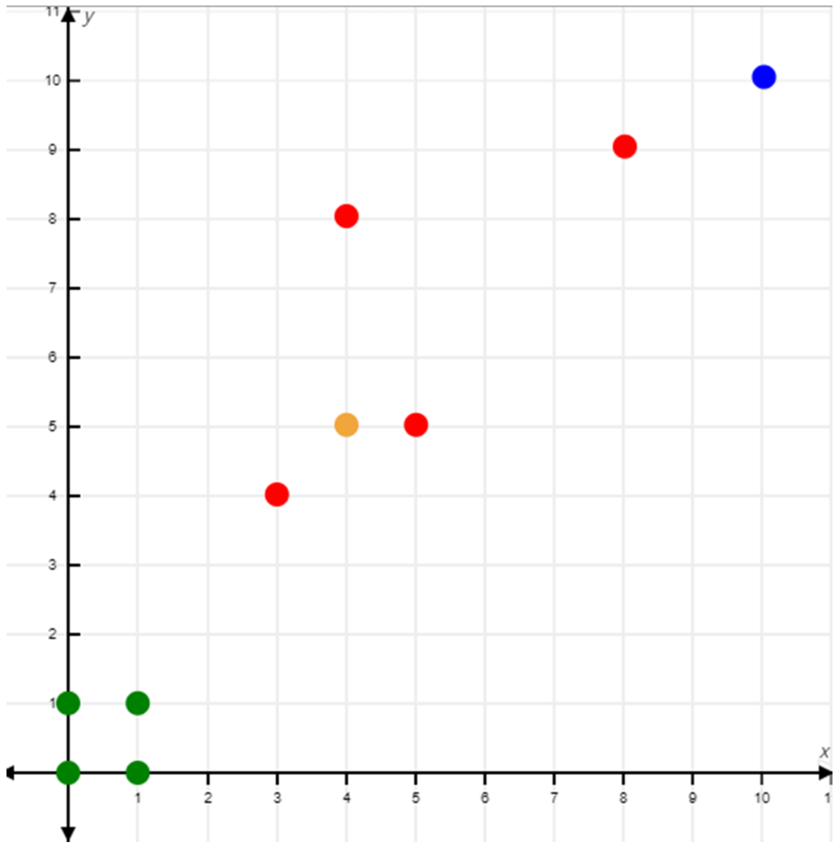

Figure 1: Agents, goals, and obstacles for Problem 4

4. (**25%**) Consider a group of four agents that moves together in two dimension and want to keep two formations described as below:

   - Square with distance/side 1 and
   - Line 1-2-3-4 with distance of each agent is 1.

   Suppose that the four agents attempt to move from initial points in green to $(10, 10)$ and have to go through point $(4, 5)$ with obstacles in red as specified in Figure 1. It is known that the agents prefer to be in a square formation whenever possible.

   Assume that an agent can only move vertically or horizontally at most one unit distance per time step. Approximate how agents may move, design a feasible function $\mathcal{E}$, and show how the changing of formation works. (other design parameters are free)

5. (**25%**) Discuss and provide examples of the application of Hungarian algorithm in the context of graph labeling and agent assignment. Compare it to other algorithm and discuss the speed.

6. (**25%**) In multi-robot setting, the interaction graph is often determined by **proximity**, i.e., two agents are said to be connected if they are physically close. Discuss the challenge of doing formation control with this proximity graph assumption. Is it possible, to, say, switch formation quickly for formations that require robots to be physically far from each other?