

Tugas I sistem pengaturan Formasi dan kolaborasi

Muhammad Azriel Rizqifadiilah - 6022221047



Departemen teknik elektro

fakultas teknologi elektro dan informatika cerdas

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1. Suppose is a simple graph with vertices. Choose vertices and of . The maximum number of edges in a simple graph are known with vertices is .

So, number of edges can be drawn for vertices. Thus, if we have more than edges than at least one edge should be drawn between the vertex, to some vertex . Hence, must be connected.

1. –
2. To prove (i) and (ii) as equivalent let assume that A diagraph with has no globally reachable node if and only if. It has two disjoint closed subsets of

Sufficient condition to prove than the above statement is follows directly from statement itself, now to prof the necessary condition by means constructive algorithm firstly select any node, say in and then as where every node in can reach and node in can reach , than +is closed also, since is no globally reachable.

Secondly, select any node, say in since is not empty, we can always find one check if the node is globally reachable in the induced subgraph .

and where every node in can reach than is closed and non-empty next select any node in and check if is globally reachable in the introduced subgraph . If the conclusion follows by the same arguments above and does not repeat this procedure again until this condition holds since the diagraph has finite number of nodes.

1. –
2. Theorem: If P is a doubly stochastic matrix associated with the transition probabilities of a Markov chain with N states, then the limiting-state probabilities are given by .

Proof: the limiting-state probabilities satisfy the condition to check the validity of the theorem, we observe that when we substitute in the above equation we obtain

This shows that satisfies the condition , which the limiting-state probabilities are required to satisfy. Conversely, from the above equation, we see that if the limiting-state probabilities are given by, then each column of sums to 1; that is, is doubly stochastic. This completes the proof.

1. (i) given A has positive diagonal elements the A is primitive. Since diagonal elements means degree of all elements in state space is unity that means aperiodic state, and now left with to show irreducibility + aperiodic (degree1) implies primitive. As definition **as per Pernon frobenius theory primitive matrix means if matrix is square having non-negative elements and some powers of it are positive. Since A is irreducible, there exits some such that.**

Hence for every sufficiently large , using Kolmogorov property

It means all integrals powers of A are also no negative.

(ii) A is primitive means A has no negative elements and some power of A have all positive elements

Clearly A is primitive because square matrix A has non-negative elements and some power say has all positive elements, but clearly see that in this example diagonal elements of are not all strictly positive. Hence with this counter example that disproved this statement is false.

(iii)

1. (i). Now diagraph of and are

|  |  |
| --- | --- |
| corresponding matrix | corresponding matrix |
| corresponding matrix | |

(ii).

All the entries of is nonzero and positive, is primitive.

All the entries of is non-zero positive then is primitive.

All the entries of are non-zero positive then is primitive.

(iii). are strongly connected and is not strongly connected. Since are strongly connected but is not strongly connected

If are irreducible then they are aperiodic, are aperiodic but is not aperiodic.

1. –