ב"ה. אינפי א' תרגיל 8 | מרצה: דר' נגרי, מתרגל: דר' חדאד | מגיש: פרגל, ת.ז. 588+7+6 09 | (8-3) I,II

$$f(x)=\ln\left(x+\sqrt{x^2+a^2}\right)\quad |\ a\neq 0$$

I

1) תחום הגדרה:

הביטוי שתחת השורש תמיד (גדול מאפס), כך שהשורש לא מגדיר לנו! (נשאר רק הדרישה שהביטוי שבתוך הלוגריתם יהיה גדול מאפס.

$$\Rightarrow x+\sqrt{x^2+a^2}>0$$

נבדוק האם הביטוי יכול להיות אפס!

$$x+\sqrt{x^2+a^2}=0\Rightarrow -x=\sqrt{x^2+a^2}\Rightarrow x^2=x^2+a^2\Rightarrow 0=a^2$$

⇐ אין פ.

⇐ הביטוי תמיד חיובי ⇐ $\boxed{\text{הפונקציה מוגדרת רציפה וגזירה לכל } x \text{ ממשי}}$

$$\boxed{D_{f'}=D_f=\mathbb{R}=(-\infty,\infty)}$$

2)

$$f'(x)=\ln(U)=\frac{U'}{U}=\frac{1+\left(\sqrt{x^2+a^2}\right)'}{x+\sqrt{x^2+a^2}}=\frac{1+\frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+a^2}}}{x+\sqrt{x^2+a^2}}=\frac{\sqrt{x^2+a^2}+x}{\sqrt{x^2+a^2}\left(x+\sqrt{x^2+a^2}\right)}=\frac{1}{\sqrt{x^2+a^2}}$$

אחרי כמה סוגי ה' עבודות לא הגעתי "מהלך" מוקדם כמה סוגי פעולות, כי כדאי
היה לי המשפט טכנולוגי - מיותר!

3) הפונקציה תמיד עולה, כי הנגזרת שלה תמיד חיובית, ולכן היא חח"ע ולכן היא הפיכה

$$y=\ln\left(x+\sqrt{x^2+a^2}\right)\Rightarrow e^y=x+\sqrt{x^2+a^2}\Rightarrow e^y-x=\sqrt{x^2+a^2}\Rightarrow e^{2y}-2xe^y+\cancel{x^2}=\cancel{x^2}+a^2$$

$$\Rightarrow e^{2y}-a^2=2xe^y\Rightarrow x=\frac{e^{2y}-a^2}{2e^y}=\frac{e^y-a^2e^{-y}}{2}\Rightarrow f^{-1}(x)=\frac{e^x-a^2e^{-x}}{2}$$

4) עבור $a=1$ נקבל $\boxed{f^{-1}(x)=\sinh(x)}$

II

3)
$$\int\frac{(x+3)^2}{x}dx=\int\frac{x^2+x+3}{x}dx=\int x+1+\frac{3}{x}\,dx=\frac{x^2}{2}+x+3\ln|x|+C$$

6)
$$\int\sqrt[3]{x}\left(\sqrt{x}-2\right)dx=\int x^{\frac{1}{3}}\left(x^{\frac{1}{2}}-2\right)dx=\int x^{\frac{5}{6}}-2x^{\frac{1}{3}}\,dx=\frac{6x^{\frac{11}{6}}}{11}-\frac{2\cdot 3x^{\frac{4}{3}}}{4}+C=\frac{6}{5}x^{\frac{11}{6}}-\frac{3}{2}x^{\frac{4}{3}}+C$$

8)
$$\int\cos^2\frac{x}{2}\,dx$$

$$\cos 2\alpha=\cos^2\alpha-\sin^2\alpha$$
$$1=\cos^2\alpha+\sin^2\alpha$$
$$1+\cos 2\alpha=2\cos^2\alpha\Longrightarrow\cos^2\alpha=\frac{1}{2}+\frac{\cos 2\alpha}{2}\Rightarrow\cos^2\frac{x}{2}=\frac{1}{2}+\frac{\cos x}{2}$$

$$\int\cos^2\frac{x}{2}dx=\frac{1}{2}\int 1+\cos x\,dx=\frac{1}{2}(x+\sin x)+C=\boxed{\frac{x}{2}+\frac{\sin x}{2}+C}$$

(וכמובן אין הגבלה)

II (10) $\int \tan^2 x\,dx = \int \frac{\sin^2 x}{\cos^2 x}\,dx = \int \frac{1-\cos^2 x}{\cos^2 x}\,dx = \cancel{\int \frac{1-\left(\frac{1}{2}+\frac{\cos 2\alpha}{2}\right)}{\frac{1}{2}+\frac{\cos 2\alpha}{2}}\,dx = \int \frac{\frac{1}{2}-\frac{\cos 2\alpha}{2}}{\frac{1}{2}+\frac{\cos 2\alpha}{2}}\,dx = \int \frac{1-\cos 2\alpha}{1+\cos 2\alpha}\,dx}$

$\cancel{\cos^2\alpha = \frac{1}{2}+\frac{\cos 2\alpha}{2}}$

$$= \int \frac{dx}{\cos^2 x} - \int dx = \boxed{\tan x - x + C}$$

(12) $\int \frac{dx}{\sin^2 2x} = \int \frac{dx}{4\sin^2 x\cos^2 x} = \cancel{\frac{1}{4}\int \frac{dx}{\sin^2 x(1-\sin^2 x)} = \frac{1}{4}\int \frac{dx}{\sin^2 x - \sin^4 x}} = \frac{1}{4}\int \frac{\sin^2 x + \cos^2 x}{\sin^2 x\cos^2 x}\,dx$

$$= \frac{1}{4}\int \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}\,dx = \frac{1}{4}\left(\tan x - \cot x + C\right) = \frac{1}{4}\left(\frac{\sin x}{\cos x} - \frac{\cos x}{\sin x}\right) + C = \frac{1}{2}\left(\frac{\sin^2 x - \cos^2 x}{2\sin x\cos x}\right) + C$$

$$= \frac{1\cdot(-\cos 2x)}{2\sin 2x} \cancel{+\cot 2x} + C = \boxed{-\frac{1}{2}\cot 2x + C}$$

III (3) $\int \frac{3}{x^2+16}\,dx = 3\int \frac{dx}{4^2+x^2} = \boxed{\frac{3}{4}\arctan\frac{x}{4} + C}$

(4) $\int \frac{x^4}{x^2+4}\,dx = \int \frac{x^4+4x^2-4x^2}{x^2+4}\,dx = \int x^2\cancel{\left(\frac{x^2+4}{x^2+4}\right)} - \frac{4x^2+16-16}{x^2+4}\,dx$

$$= \int x^2 - \frac{4\cancel{(x^2+4)}}{\cancel{x^2+4}} + \frac{16}{x^2+2^2}\,dx = \int x^2 - 4\,dx + 16\int \frac{dx}{2^2+x^2} = \boxed{\frac{x^3}{3} - 4x + 8\arctan\frac{x}{2} + C}$$

(6) $\int \frac{dx}{\sqrt{x^2-16}} = \int \frac{dx}{\sqrt{x^2-4^2}} = \boxed{\ln\left|x+\sqrt{x^2-4^2}\right| + C}$

(8) $\int \sin 2x\cos 3x\,dx = \int \frac{1}{2}\sin 5x - \frac{1}{2}\sin x\,dx = \frac{1}{2}\int \sin 5x - \sin x\,dx$

$t = 5x \Rightarrow dt = 5\,dx \Rightarrow dx = \frac{dt}{5}$

$$= \frac{1}{2}\left(-\frac{\cos 5x}{5} + \cos x + C\right) = \boxed{\frac{1}{2}\cos x - \frac{1}{10}\cos 5x + C}$$

(11) $\int \frac{x\,dx}{\sqrt[3]{x^2+7}} = \left[\begin{array}{l} t = x^2+7 \\ dt = 2x\,dx \Rightarrow x\,dx = \frac{dt}{2}\end{array}\right] = \int \frac{dt}{2\sqrt[3]{t}} = \frac{1}{2}\int t^{-\frac{1}{3}}\,dt = \frac{t^{\frac{2}{3}}}{2\cdot\frac{2}{3}} + C = \boxed{\frac{3}{4}(x^2+7)^{\frac{2}{3}} + C}$

(13) $\int \frac{e^x\,dx}{1+e^x} = \left[\begin{array}{l} t = e^x+1 \\ dt = e^x\,dx\end{array}\right] = \int \frac{dt}{t} = \ln|t| + C = \ln\left|e^x+1\right| + C = \boxed{\ln(e^x+1) + C}$

III

(15) $\int \frac{\sin 2x}{3+\cos 2x}\,dx = \begin{bmatrix} t=3+\cos 2x \\ dt=-2\sin 2x\,dx \end{bmatrix} = \int \frac{-dt}{2t} = -\int \frac{dt}{2t} = -\frac{1}{2}\ln|t|+C = \boxed{-\frac{1}{2}\ln|3+\cos 2x|+C}$

(17) $\int \frac{x+e^{2x}}{x^2+e^{2x}}\,dx = \begin{bmatrix} t=x^2+e^{2x} \\ dt=2x+2e^{2x}\,dx \end{bmatrix} = \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\ln|t|+C = \boxed{\frac{1}{2}\ln(x^2+e^{2x})+C}$

(19) $\int \frac{dx}{x+3\sqrt{x}} = \int \frac{dx}{\sqrt{x}(\sqrt{x}+3)} = \begin{bmatrix} t=\sqrt{x}+3 \\ dt=\frac{dx}{2\sqrt{x}} \Rightarrow \frac{dx}{\sqrt{x}}=2dt \end{bmatrix}$

$= 2\int \frac{dt}{t} = 2\ln|t|+C = \boxed{2\ln(\sqrt{x}+3)+C}$

(21) $\int \frac{(2x+3)\,dx}{(x^2+3x+1)^{10}} = \begin{bmatrix} t=x^2+3x+1 \\ dt=(2x+3)\,dx \end{bmatrix} = \int t^{-10}\,dt = \frac{t^{-9}}{-9}+C = \boxed{\frac{-1}{9(x^2+3x+1)^9}+C}$

(24) $\int \frac{\tan(\ln x)}{x}\,dx = \begin{bmatrix} t=\ln x \\ dt=\frac{dx}{x} \end{bmatrix} = \int \tan t\,dt = -\ln|\cos t|+C = \boxed{-\ln|\cos(\ln x)|+C}$

(26) $\int \sin^3 x\cos^2 x\,dx = \cancel{\int \sin^3 x(1-\sin^2 x)\,dx = \int \sin^3 x-\sin^5 x\,dx}$

$\cancel{\sin 2x = 2\sin x\cos x \Rightarrow \sin^2 2x = 4\sin^2 x\cos^2 x \Rightarrow \sin^3 x\cos^2 x = \sin x\frac{\sin^2 2x}{4}}$

$\cancel{\begin{bmatrix} t=\sin x \\ dt=\cos x\,dx \end{bmatrix} \Rightarrow dx=\frac{dt}{\cos x}=\frac{dt}{\sqrt{1-\sin^2 x}}}$ $\qquad (\sin^3 x)' = 3\sin^2 x\cos x$

$(\cos^3 x)' = 3\cos^2 x\sin x \qquad (1-\cos^2 x)(\sin x\cos^2 x)$

$\Rightarrow \int \sin^3 x\cos^2 x\,dx = \int (1-\cos^2 x)(\sin x\cos^2 x)\,dx = \int \sin x(\cos^2 x-\cos^4 x)\,dx = \begin{bmatrix} t=\cos x \\ dt=-\sin x\,dx \end{bmatrix}$

$= \int -(t^2-t^4)\,dt = \int t^4-t^2\,dt = \frac{t^5}{5}-\frac{t^3}{3}+C = \boxed{\frac{\cos^5 x}{5}-\frac{\cos^3 x}{3}+C}$

(27) $\int x^2\sqrt{x+3}\,dx = \cancel{\int \frac{x^2(x+3)}{\sqrt{x+3}}\,dx}\ \begin{bmatrix} t=x+3 \Rightarrow x=t-3 \\ dx=dt \end{bmatrix} = \int (t-3)^2\sqrt{t}\,dt = \int (t^2-6t+9)t^{\frac{1}{2}}\,dt$

$= \int t^{2.5}-6t^{1.5}+9t^{0.5}\,dt = \frac{t^{3.5}}{3.5}-\frac{6t^{2.5}}{2.5}+\frac{9t^{1.5}}{1.5}+C = \boxed{\frac{2}{7}(x+3)^{3.5}-\frac{12}{5}(x+3)^{2.5}+6(x+3)^{1.5}+C}$

IV

(3) $\int x e^{-x} dx = \begin{bmatrix} U = x & U' = 1 \\ V' = e^{-x} & V = -e^{-x} \end{bmatrix} = -xe^{-x} + \int e^{-x} dx = \boxed{-xe^{-x} - e^{-x} + C}$

(6) $\int \frac{\ln x}{x^2} dx = \begin{bmatrix} U = \ln x & U' = \frac{1}{x} \\ V' = x^{-2} & V = -x^{-1} \end{bmatrix} = -\frac{\ln x}{x} + \int \frac{dx}{x^2} = -\frac{\ln x}{x} - \frac{1}{x} + C = \boxed{\frac{\ln x + 1}{-x} + C}$

(8) $\int \frac{x\,dx}{\cos^2 x} = \begin{bmatrix} U = x & U' = 1 \\ V' = \frac{1}{\cos^2 x} & V = \tan x \end{bmatrix} = x\tan x - \int \tan x\, dx = \boxed{x\tan x + \ln|\cos x| + C}$

(10) $\int x \sinh 2x\, dx = \begin{bmatrix} U = x & U' = 1 \\ V' = \sinh 2x & V = \frac{\cosh 2x}{2} \end{bmatrix} = \frac{x \cdot \cosh 2x}{2} - \int \frac{\cosh 2x}{2} dx = \boxed{\frac{x \cdot \cosh 2x}{2} - \frac{\sinh 2x}{4} + C}$

(14) $\int t^2 \sin\frac{t}{2} dt = \begin{bmatrix} U = t^2 & U' = 2t \\ V' = \sin\frac{t}{2} & V = -2\cos\frac{t}{2} \end{bmatrix} = -2t^2\cos\frac{t}{2} + \int 4t\cos\frac{t}{2} dt = \begin{bmatrix} U = 4t & U' = 4 \\ V' = \cos\frac{t}{2} & V = 2\sin\frac{t}{2} \end{bmatrix}$

$= -2t^2\cos\frac{t}{2} + 4t\sin\frac{t}{2} - 4\int \sin\frac{t}{2} dt = \boxed{4t\sin\frac{t}{2} - 2t^2\cos\frac{t}{2} + 8\cos\frac{t}{2} + C}$

*17 $\int e^{\sqrt{x}} dx = \begin{bmatrix} U = e^{\sqrt{x}} & U' = \frac{e^{\sqrt{x}}}{2\sqrt{x}} \\ V' = 1 & V = x \end{bmatrix} = x \cdot e^{\sqrt{x}} - \int \frac{x e^{\sqrt{x}}}{2\sqrt{x}} dx = xe^{\sqrt{x}} - \frac{1}{2}\int \sqrt{x} e^{\sqrt{x}} dx = \begin{bmatrix} U = e^{\sqrt{x}} & U' = \frac{e^{\sqrt{x}}}{2\sqrt{x}} \\ V' = \sqrt{x} & V = \frac{x^{1.5}}{1.5} \end{bmatrix}$

$= xe^{\sqrt{x}} - \frac{x^{1.5}}{3} e^{\sqrt{x}} + \int \frac{e^{\sqrt{x}}}{2\sqrt{x}} \cdot \frac{x^{1.5}}{3} dx = xe^{\sqrt{x}} - \frac{x^{1.5}}{3} e^{\sqrt{x}} + \frac{1}{6}\int$

*17 $\int e^{\sqrt{x}} dx = \begin{bmatrix} t = \sqrt{x} \Rightarrow x = t^2 \\ dx = 2t\,dt \end{bmatrix} = \int e^t \cdot 2t\,dt = \begin{bmatrix} U = 2t & U' = 2 \\ V' = e^t & V = e^t \end{bmatrix} = 2te^t - \int 2e^t dt$

$= 2te^t - 2e^t + C = 2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + C = \boxed{2(\sqrt{x} - 1)e^{\sqrt{x}} + C}$

*20 $\int e^x \sin 2x\, dx = \begin{bmatrix} U = e^x & U' = e^x \\ V' = \sin 2x & V = -\frac{1}{2}\cos 2x \end{bmatrix} = -\frac{1}{2}e^x\cos 2x + \frac{1}{2}\int e^x \cos 2x\, dx = \begin{bmatrix} U = \cos 2x & U' = -2\sin 2x \\ V' = e^x & V = e^x \end{bmatrix}$

$= -\frac{1}{2}e^x \cos 2x + \frac{1}{2} \begin{bmatrix} U = e^x & U' = e^x \\ V' = \cos 2x & V = \frac{1}{2}\sin 2x \end{bmatrix} = -\frac{1}{2}e^x\cos 2x + \frac{1}{4}e^x \sin 2x - \frac{1}{4}\int e^x \sin 2x$

$\Rightarrow \frac{5}{4}\int e^x \sin 2x\, dx = \frac{1}{4}e^x(\sin 2x - 2\cos 2x) \Rightarrow \int e^x \sin 2x\, dx = \boxed{\frac{1}{5}e^x(\sin 2x - 2\cos 2x) + C}$

בס"ד אינפי 1 תרגיל 8 | מרצה: דר' לנדר, מתרגל: דר' חבלין | [illegible] ת.ז. 039677588 | V ② - ⑩ (1

V ② $\int \frac{4x^2-3x-4}{x(x-1)(x+2)}dx$

$$\frac{4x^2-3x-4}{x(x-1)(x+2)} = \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x+2)} = \frac{A(x-1)(x+2)+B(x+2)x+C(x-1)x}{x(x-1)(x+2)}$$

$$4x^2-3x-4 = A(x-1)(x+2)+Bx(x+2)+Cx(x-1)$$

$$x=0 \Rightarrow -4 = A(-1)(2)+B\cdot 0+C\cdot 0 \Rightarrow A=2$$

$$x=1 \Rightarrow -3 = 2\cdot 0+B\cdot 1(3)+C\cdot 0 \Rightarrow B=-1$$

$$x=-2 \Rightarrow 4\cdot 4+3\cdot 2-4 = A\cdot 0+B\cdot 0+C(-3)(-2) \Rightarrow 6C=18 \Rightarrow C=3$$

$$\Rightarrow \int \frac{4x^2-3x-4}{x(x-1)(x+2)}dx = \int \frac{2}{x} - \frac{1}{x-1} + \frac{3}{x+2}\,dx = 2\ln|x|-\ln|x-1|+3\ln|x+2|+C$$

$$= \ln x^2 - \ln|x-1| + \ln|x+2|^3 + C = \boxed{\ln\left(x^2\left|\frac{(x+2)^3}{x-1}\right|\right)+C}$$

⑥ $\int \frac{3x-4}{x^2+2x+5}dx = \int \frac{3(x+1)-7}{(x+1)^2+4}dx = \begin{bmatrix} t=x+1 \\ dt=dx \end{bmatrix} = \int \frac{3t-7}{t^2+4}dt =$

$\boxed{(x+1)^2 = x^2+2x+1}$

$$= \frac{3}{2}\int \frac{2t\,dt}{t^2+4} - 7\int \frac{dt}{t^2+2^2} = \frac{3}{2}\ln(t^2+4) - \frac{7}{2}\arctan\frac{t}{2}+C = \boxed{\frac{3}{2}\ln(x^2+2x+5) - \frac{7}{2}\arctan\frac{x+1}{2}+C}$$

⑩ $\int \frac{4x+6}{x^3-8}dx = \cancel{\int \frac{4x+6}{(x-2)(x^2+2x+4)}dx}$

$$\frac{4x+6}{x^3-8} = \frac{4x+6}{(x-2)(x^2+2x+4)} = \frac{A}{x-2} + \frac{Mx+N}{x^2+2x+4} \Rightarrow 4x+6 = A(x^2+2x+4)+(Mx+N)(x-2)$$

$$x=0 \Rightarrow 6 = 4A-2N \Rightarrow N=2A-3$$

$$x=2 \Rightarrow 8+6 = A(4+4+4)+(Mx+N)(0) \Rightarrow 14 = 12A \Rightarrow A=\frac{7}{6} \Rightarrow N = -\frac{2}{3}$$

$$x=1 \Rightarrow 4+6 = A(1+2+4)+(M+N)(-1) \Rightarrow 10 = \frac{49}{6}+\frac{2}{3}-M \Rightarrow M = \frac{49}{6}+\frac{2}{3}-10 = -\frac{7}{6}$$

$$\Rightarrow \int \frac{4x+6}{x^3-8}dx = \int \frac{\frac{7}{6}}{x-2} + \frac{-\frac{2}{3}-\frac{7}{6}x}{x^2+2x+4}dx = \frac{7}{6}\int\frac{dx}{x-2}$$

(לא הספקתי)

ב"ה. | אינפי 2 תרגיל 8 | מרצה: [illegible] מתרגל: [illegible] | ת.ז. [illegible] | V (10)(2)-

פתרון- (10) (מצאתי הבעיה?)

~~$$\int \frac{4x+6}{x^3-8} = 3.5\int \frac{dx}{x-2} - 1.75\int \frac{2x-2}{x^2-2x+4}\,dx + 0.5\int \frac{dx}{(x-1)^2+\sqrt{3}^2}$$~~

~~$$= 3.5\ln|x-2| - 1.75\ln|x^2-2x+4| + \frac{0.5}{\sqrt{3}}\arctan\frac{x-1}{\sqrt{3}} + C$$~~

[illegible]

$$\int \frac{4x+6}{x^3-8}\,dx = \frac{7}{6}\int \frac{dx}{x-2} - \frac{1}{6}\int \frac{7x+4}{x^2+2x+4}\,dx$$

$$= \frac{7}{6}\ln|x-2| - \frac{7}{12}\int \frac{2x+2-\frac{6}{7}}{x^2+2x+4}\,dx = \frac{7}{6}\ln|x-2| - \frac{7}{12}\ln|x^2+2x+4| + \frac{1}{2}\int \frac{dx}{(x+1)^2+\sqrt{3}^2}$$

$$\boxed{= \frac{7}{12}\ln\left|\frac{(x-2)^2}{x^2+2x+4}\right| + \frac{\sqrt{3}}{6}\arctan\frac{x+1}{\sqrt{3}} + C}$$

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(13) $$\int \frac{x^2-3x-8}{(x^2+4x+5)(x+1)}\,dx$$

$$\frac{x^2-3x-8}{(x^2+4x+5)(x+1)} = \frac{A}{x+1} + \frac{Mx+N}{x^2+4x+5}$$

$$\Rightarrow A(x^2+4x+5) + (Mx+N)(x+1) = x^2-3x-8$$

$$\Rightarrow x^2(A+M) + x(4A+M+N) + (N+5A) = x^2-3x-8$$

$$\Rightarrow \begin{cases} x^2: & A+M = 1 \Rightarrow M = 1-A \\ x: & 4A+M+N = -3 \\ 1: & 5A+N = -8 \Rightarrow N = -8-5A \end{cases} \Rightarrow 4A+1-A-8-5A = -3 \Rightarrow A = -2 \Rightarrow \begin{cases} M = 3 \\ N = 2 \end{cases}$$

$$\Rightarrow \int \frac{x^2-3x-8}{(x^2+4x+5)(x+1)}\,dx = \int \frac{-2}{x+1} + \frac{3x+2}{x^2+4x+5}\,dx = -2\int \frac{dx}{x+1} + \frac{3}{2}\int \frac{2x+4-4+\frac{4}{3}}{x^2+4x+5}\,dx$$

$$= -2\ln|x+1| + \frac{3}{2}\ln|x^2+4x+5| - 4\int \frac{dx}{(x+2)^2+1} = \boxed{\frac{3}{2}\ln|x^2+4x+5| - 2\ln|x+1| - 4\arctan(x+2) + C}$$