

5. $\left(\begin{array}{c} \text{שני ימים} \\ \text{בבית} \end{array} \right)$

הערות:
יש צורך קריאה קצרה יותר
אם נאמר

$$\frac{dz}{dy} = \frac{4t}{y} = \frac{4 \cos v}{6 \sin v} = 4 \cot v$$

$$t = e^x = U \cos V$$

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$$-\sin v$$

$$\Rightarrow \frac{\partial z}{\partial v} = 4U(\cos v \cot v - \sin v \ln(U \sin v))$$

$(2, \frac{\pi}{4})$: 120°

$$\sin \frac{\pi}{4} = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\tan \frac{\pi}{4} = \cot \frac{\pi}{4} = 1$$

$$\Rightarrow \frac{\partial z}{\partial v} = 4 \cdot \frac{\sqrt{2}}{2} \left(1 + \ln \left(2 \cdot \frac{\sqrt{2}}{2} \right) \right) = 2\sqrt{2} (1 + \ln(\sqrt{2})) = 3.808 \dots$$

$$\Rightarrow \frac{\partial z}{\partial V} = 4 \cdot 2 \left(\frac{\sqrt{2}}{2} \cdot 1 - \frac{\sqrt{2}}{2} \ln \left(2 \cdot \frac{\sqrt{2}}{2} \right) \right) = 4\sqrt{2} (1 - \ln(\sqrt{2})) = 3.696 \dots$$

המשפט השלישי $\int_a^b f(x) dx = F(b) - F(a)$

$75 > 16.5r$
 $0.396 + 588 \text{ s.x}$

$(9) \text{ grad } f(x,y) = \begin{pmatrix} 2y \\ 2x-6y \end{pmatrix} \Rightarrow \nabla f(5,5) = \begin{pmatrix} 10 \\ -20 \end{pmatrix}$
 $\Rightarrow \left(\frac{df}{ds} \right)_{\begin{pmatrix} 0.8 \\ 0.6 \end{pmatrix}, (5,5)} = \begin{pmatrix} 10 \\ -20 \end{pmatrix} \cdot \begin{pmatrix} 0.8 \\ 0.6 \end{pmatrix} = 8 - 12 = -4$
 $\vec{U} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \Rightarrow \frac{1}{5} \vec{U} = \begin{pmatrix} 0.8 \\ 0.6 \end{pmatrix}$
 $0.8^2 + 0.6^2 = 1$

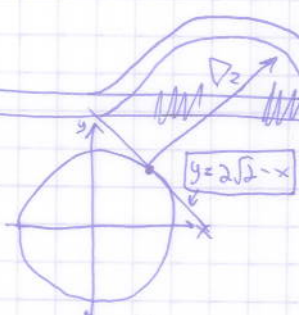
$(12) f(x,y) = \ln(x^2 y^2) \quad f_x = \frac{2x}{x^2 y^2}, \quad f_y = \frac{2y}{x^2 y^2}$
 $\text{grad } f(x,y) = \frac{2}{x^2 y^2} \begin{pmatrix} x \\ y \end{pmatrix}$

$\nabla f(x,y) = \frac{2}{1+0} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

$(15) f(x,y) = x^2 + y^2$
 $f_x = 2x, \quad f_y = 2y \Rightarrow \nabla f(x,y) = \begin{pmatrix} 2x \\ 2y \end{pmatrix} \Rightarrow \nabla f(1,1) = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

$\begin{pmatrix} 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 2 \end{pmatrix} = 4$
 $\begin{pmatrix} 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 2$
 $\begin{pmatrix} 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix} = 0$

II (1) (1)



$z = x^2 + y^2$

$\nabla z = \begin{pmatrix} \frac{\partial z}{\partial x} \\ \frac{\partial z}{\partial y} \end{pmatrix} = \begin{pmatrix} 2x \\ 2y \end{pmatrix} \Rightarrow \nabla z(\sqrt{2}, \sqrt{2}) = \begin{pmatrix} 2\sqrt{2} \\ 2\sqrt{2} \end{pmatrix}$

$x = \sqrt{2}, y = \sqrt{2} \Rightarrow x^2 + y^2 = 2 \Rightarrow 2x + 2y = 0 \Rightarrow \frac{\partial z}{\partial x} = -\frac{\partial z}{\partial y}$
 $\Rightarrow y_0 = -\frac{x_0}{2} = -\frac{\sqrt{2}}{2} = -\frac{1}{\sqrt{2}}$
 $\Rightarrow y = -x - \frac{1}{\sqrt{2}} = -x - \frac{\sqrt{2}}{2}$
 $\Rightarrow 2\sqrt{2}(x-\sqrt{2}) + 2\sqrt{2}(y-\sqrt{2}) = 0 \Rightarrow y - \sqrt{2} = \sqrt{2} - x \Rightarrow y = 2\sqrt{2} - x$
 $\Rightarrow x + y = 2\sqrt{2}$

לפי (1) ו-2
0.967788.5.7

6, 5, 2 II

הצורה הכללית של המישור

2 נקודות
2 נקודות

2

$$f(x, y) = xy + y^2$$

$$\nabla f(x, y) = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{pmatrix} = \begin{pmatrix} y \\ x + 2y \end{pmatrix}$$

$$\Rightarrow \nabla f(3, 2) = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$$

הצורה הכללית של המישור

$$\left(\frac{-7\sqrt{53}}{53} \mid \frac{2\sqrt{53}}{53} \right) = \frac{\begin{pmatrix} -7 \\ 2 \end{pmatrix}}{\sqrt{7^2 + 2^2}}$$

$$\Rightarrow \vec{U} = \pm \frac{\sqrt{53}}{53} \begin{pmatrix} 7 \\ -2 \end{pmatrix}$$

הצורה הכללית של המישור

5

$$f(x, y) = x^2 - xy + y^2 \Rightarrow f(-1, 2) = 7$$

$$\nabla f = \begin{pmatrix} f_x \\ f_y \end{pmatrix} = \begin{pmatrix} 2x - y \\ 2y - x \end{pmatrix} \Rightarrow \nabla f(-1, 2) = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$$

$$\Rightarrow \ell: -4(x - (-1)) + 5(y - 2) = 0 \Rightarrow 5y - 4x - 10 - 4 = 0 \Rightarrow y = \frac{4}{5}x + \frac{14}{5}$$

$$x^2 - xy - y^2 - z^2 = 0 \Rightarrow f(x, y, z) = x^2 - xy - y^2 - z^2$$

6

$$\Rightarrow \nabla f(x, y, z) = (f_x, f_y, f_z) = (2x - y, -x - 2y, 2z)$$

$$\Rightarrow \nabla f(1, 1, 1) = (1, -3, 2)$$

$$\pi: f_x(x - x_0) + f_y(y - y_0) + f_z(z - z_0) = 0 \Rightarrow 1(x - 1) - 3(y - 1) + 2(z - 1) = 0$$

$$\Rightarrow x - 3y + 2z = 0$$

$$\ell \perp \pi \Rightarrow \ell: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} + t \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} \mid t \in \mathbb{R}$$

4, 7 | $\frac{1}{2} \ln \frac{1}{2}$ | $\frac{1}{2} \ln \frac{1}{2}$ | $\frac{1}{2} \ln \frac{1}{2}$

$\begin{pmatrix} -7 \\ -9 \end{pmatrix}$ ← זהו ג'לגור כ"א ו-כ"ב, כלומר בוקר 7/6

$$\frac{-1}{\sqrt{17}} \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ 0 & 1 \end{pmatrix}$

4

$$f_2 = 3(4x - y + z + 2)^2$$

$$\text{grad } f(x, y, z) = \begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix} \Rightarrow \nabla f \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 + 12 \cdot 2^2 \\ 6 - 3 \cdot 2^2 \\ 3 \cdot 2^2 \end{pmatrix} = \begin{pmatrix} 42 \\ -6 \\ 12 \end{pmatrix} = 18\sqrt{6} \begin{pmatrix} \frac{7\sqrt{6}}{18} \\ \frac{-\sqrt{6}}{18} \\ \frac{\sqrt{6}}{9} \end{pmatrix}$$

[illegible]