Homework 3

Aaron Rosen

Tuesday, March 6

Problem 1 (Kozen HW3 #1)

(a)

$L = \{a^n b^m | n = 2m\}$ Let k be given. Let $x = a^{2k}$, $y = b^k$ and $z = E$. Note that $xyz = a^{2k}b^k \in L$. Let $y = uvw$, where $|u| = i$, $|v| = j$, $|w| = l$, and $v \neq E$. Then $k = i + j + l$ and $j > 0$.

$$xuv^r wz = a^{2(i+j+l)} uv^r w$$

When $r = 1$ then $|uvw| = |y| = k$ but choose any other $r > 1$ and $|y| > k$ and more importantly $|y| \neq \frac{1}{2}|x|$ which means the string is no no longer in the language.

(b)

$L = \{x \in \{a, b, c\}^* | x$ is a palendrome; i.e., $x =$rev$(x)\}$
Suppose for the sake of contradiction, L is regular. Then

$$L \cap a^* b^* a^* = \{a^n b^m a^n\}$$

Which we know is not regular, so L is not regular.

(c)

$L = \{x \in \{a, b, c\}^* |$ the length of x is a square $\}$
Let k be given and k is a square. Let $x \in L$ so $|x| = k$. Let's say $x = rt$, where $r = \{a, b, c\}$ and $|t| = k - 1$,

$$x = r^2 t \notin L$$

because if k is a square then $|x| = k - 1 + 2 = k + 1$ is not a square.

(d)

$P = \{x \in \{")","("\} | \forall$ "(" there is exactly one ")" following $\}$
Let $a =$"(" and $b =$ ")" then $L(a^*b^*) \cap P = L(a^n b^n)$ which we know is not regular so P must not be regular.

Problem 2 (Kozen HW3 #2)

(a)

$(01)^*||(10)^* = ((01) + (10) + (11)(00) + (00)(11))^*$

(b)

Problem 3 (Kozen HW3 #3)

(a)

In the first automaton only 7 and 8 are inaccessibly. In the second all of the states are accessible.

(b)

1	2	3
4	5	6
7	8	9

(c)