Homework 3

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Problem 1 (Kozen HW3 #1)

(a)

 $L=\{a^nb^m|n=2m\}$ Let k be given. Let $x=a^{2k},\ y=b^k$ and z=E. Note that $xyz=a^{2k}b^k\in L$. Let y=uvw, where $|u|=i,\ |v|=j,\ |w|=l$, and $v\neq E$. Then k=i+j+l and j>0.

$$xuv^rwz = a^{2(i+j+l)}uv^rw$$

When r=1 then |uvw|=|y|=k but choose any other r>1 and |y|>k and more importantly $|y|\neq \frac{1}{2}|x|$ which means the string is no no longer in the language.

(b)

 $L = \{x \in \{a, b, c\}^* | x \text{ is a palendrome; i.e., } x = \text{rev}(x)\}$ Suppose for the sake of contradiction, L is regular. Then

$$L \cap a^*b^*a^* = \{a^nb^ma^n\}$$

Which we know is not regular, so L is not regular.

(c)

 $L = \{x \in \{a, b, c\}^* | \text{ the length of } x \text{ is a square } \}$ Let k be given and k is a square. Let $x \in L$ so |x| = k. Let's say x = rt, where $r = \{a, b, c\}$ and |t| = k - 1,

$$x = r^2 t \notin L$$

because if k is a square then |x| = k - 1 + 2 = k + 1 is not a square.

(d)

 $P = \{x \in \{"\}","("\}| \forall "(" \text{ there is exactly one "})" \text{ following } \}$ Let $a = "(" \text{ and } b = ")" \text{ then } L(a^*b^*) \cap P = L(a^nb^n) \text{ which we know is not regular so } P \text{ must not be regular.}$

Problem 2 (Kozen HW3 #2)

(a)

$$(01)^*||(10)^* = ((01) + (10) + (11)(00) + (00)(11))^*$$

(b)

Let M_A and M_B be machines:

$$M_A = (Q_A, \Sigma, \delta_A, S_A, F_A)$$

$$M_B = (Q_B, \Sigma, \delta_B, S_B, F_B)$$

where M_A is the macing accepting the language A and M_B is the machine accepting the language B.

We can define the macine that accepts the language A||B as:

$$M_{AB} = (Q_A \times Q_B, \Sigma, \delta_{AB}, S_A \times S_B, F_A \times F_B)$$
 where $p \in Q_A$ and $q \in Q_B$.

$$\delta_{AB}([p,q],a) = \{ [\delta_A(p,a), q], [p, \delta_B(q,a)] \}$$

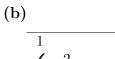
Lemma 1 Given an arbitrary state [p,q], applying the transition function with some $a \in \Sigma$ either moves state p via M_A 's transition function to some $p' \in Q_A$, leaving q in place, or moves state q via M_B 's transition function to some state $q' \in Q_B$, leaving p in place.

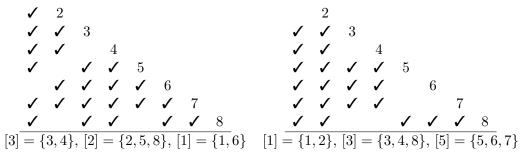
Starting at some state $[s_a, s_b] \in S_A \times S_B$ with some $x \in \Sigma^*$. According to lemma 1, for each letter of x, δ_{AB} will return some state in $Q_A \times Q_B$. If, after the last letter in x, $q' \in F_B$ and $p' \in F_A$ then $x \in L(A||B)$. Therefore A||B must be regular.

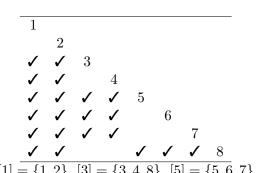
Problem 3 (Kozen HW3 #3)

(a)

In the first machine only 7 and 8 are inaccessible, in the second all states are accessible.







(c)

