Homework 2

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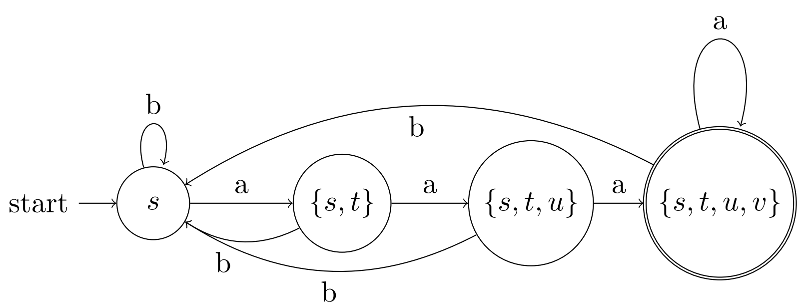
Problem 1 (Kozen HW2 #1)

$\delta(\{s\}, a) = \{s, t\}$
$\delta(\{s\}, b) = \{s\}$
$\delta(\{s, t\}, a) = \{s, t, u\}$
$\delta(\{s, t\}, b) = \{s, \}$
$\delta(\{s, t, u\}, a) = \{s, t, u, v\}$
$\delta(\{s, t, u\}, b) = \{s\}$
$\delta(\{s, t, u, v\}, a) = \{s, t, u, v\}$



Problem 2 (Kozen HW2 #2)

First we define a function ρ: RegEx $\to$ RegEx where
$\rho(E) = E$
$\rho(a) = a$
Assuming $\rho(\alpha) = \text{rev } \alpha$
$\rho(\alpha + \beta) = \text{rev } \alpha + \text{rev } \beta$
$\rho(\alpha\beta) = \text{rev } \beta \text{ rev } \alpha$
$\rho(\alpha^*) = (\text{rev}\alpha)^*$

Using this definition we can show that if $A \subseteq \Sigma^*$ is regular then so is rev A

We can use L: RegEx $\to P(\Sigma^*)$

$L(\rho(E)) = \{E\}$
$L(\rho(a)) = \{a\}$
$L(\rho(\alpha)) = L(\text{rev}\alpha)$
$L(\rho(\alpha + \beta)) = L(\text{rev}\alpha) \cup L(\text{rev}\beta)$
$L(\rho(\alpha\beta)) = L(\text{rev}\beta)L(\text{rev}\alpha)$
$L(\rho(\alpha^*)) = (L(\text{rev}\alpha))^*$
Since we've shown that every regular expression, when reversed is still regular, it follows that the set A of regular expressions will still be regular when reversed.

Problem 3 (Kozen HW3 #1)

(a)

$(b^*(aa)^*)^*$

(b)

$(a^*(bb)^*)^*$

(c)

$(b^*(aa)^*)^* + (a^*(bb)^*)^*b$

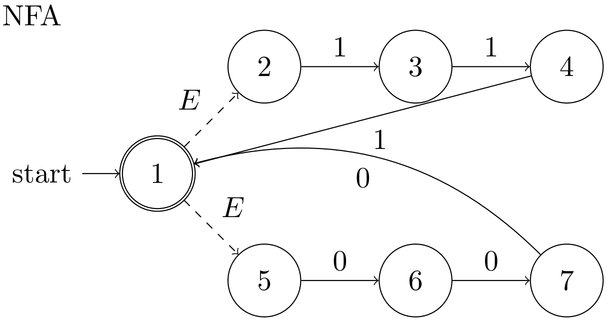
(d)

$((aa)^*(bb)^*)^*b$

Problem 4 (Kozen HW3 #2)

(a)

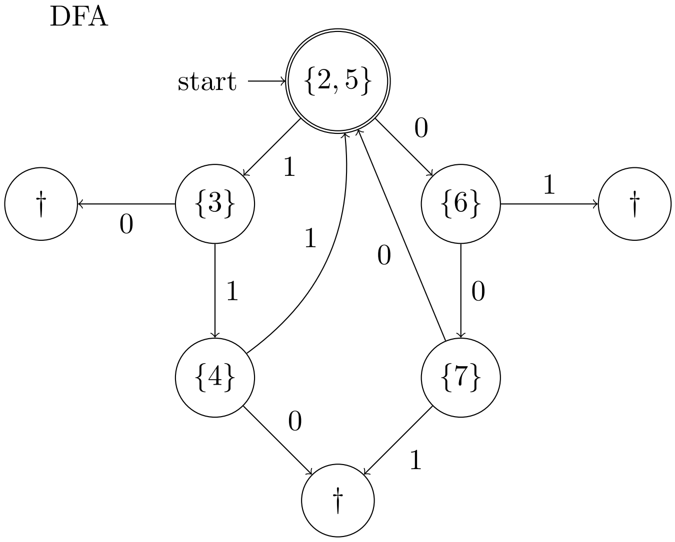
NFA



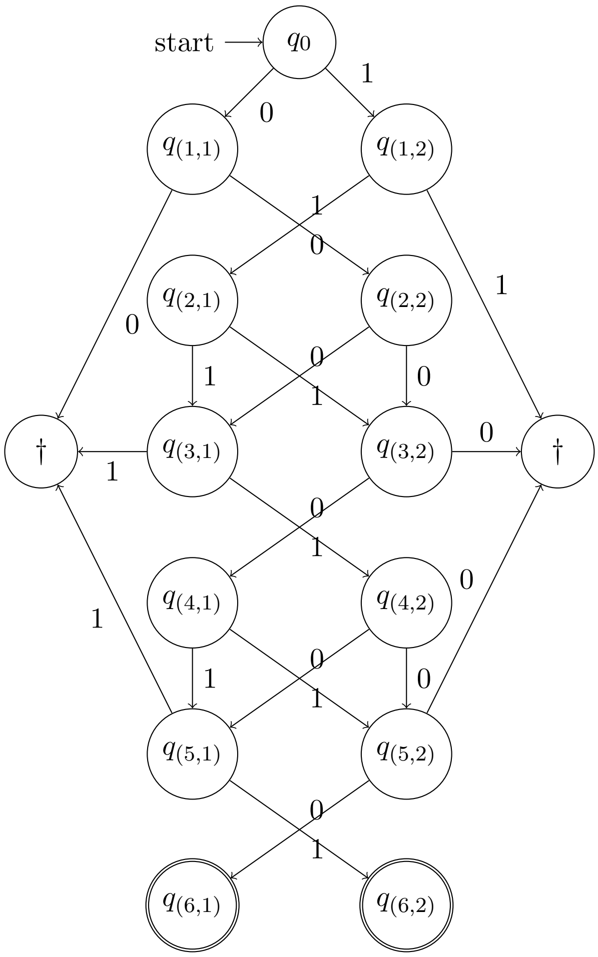
Delta Function
$\Delta(\{1\}, E) = \{2, 5\}$
$\Delta(\{2, 5\}, 1) = \{3\}$
$\Delta(\{2, 5\}, 0) = \{6\}$

$\Delta(\{3\}, 1) = \{4\}$
$\Delta(\{6\}, 0) = \{7\}$
$\Delta(\{4\}, 1) = \{1\}$
$\Delta(\{7\}, 0) = \{1\}$

DFA

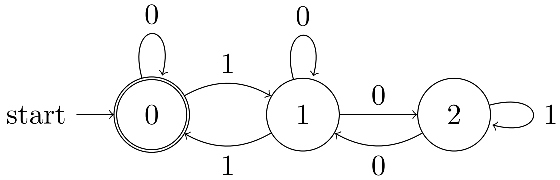


(b)



(c)

NFA

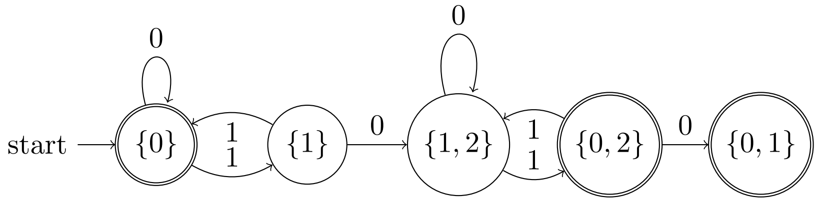


Delta Function

$\Delta(\{0\}, 0) = \{0\}$
$\Delta(\{0\}, 1) = \{1\}$
$\Delta(\{1\}, 0) = \{1, 2\}$
$\Delta(\{1\}, 1) = \{0\}$
$\Delta(\{1, 2\}, 0) = \{1, 2\}$
$\Delta(\{1, 2\}, 1) = \{0, 2\}$

$\Delta(\{0,2\},0) = \{0,1\}$
$\Delta(\{0,2\},1) = \{1,2\}$

DFA



Problem 5 (Kozen HW3 #3)

Let us define automata $M = (Q, \Sigma, \delta, s, F)$ as the DFA for A. Now we can define an NFA
$\hat{M} = (\hat{Q}, \Sigma, \Delta, \hat{S}, \hat{F})$ where
$\hat{Q} = Q^5$
$\hat{S} = \{[p, q, s, q, p] | p, q \in Q\}$ where p and q are guesses.
$\hat{F} = \{[v, w, f, w, v] | v, w \in Q, f \in F\}$
$\Delta((p, q, r, t, u), a) = \{[p, q, \delta(r, b), \delta(t, a), \delta(u, c)] | b, c \in \Sigma\}$

Applying the Δ function to the start state we get
$\Delta(\hat{S}, y) = \{[p, q, \delta(s, x), \delta(q, z), \delta(p, y)] | p, q \in Q, x, z \in \Sigma^{|y|}\}$

Using this we can prove that $L(\hat{M})$ =Middlethirds$L(M)$

$y \in L(\hat{M}) \Leftrightarrow$
$\Delta(\hat{S}, y) \cap \hat{F} \neq \emptyset \Leftrightarrow$
$\{[p, q, \delta(s, x), \delta(q, z), \delta(p, y)] | p, q \in Q, x, z \in \Sigma^{|y|}\} \cap \{[v, w, f, w, v] | v, w \in Q, f \in F\} \neq \emptyset \Leftrightarrow$
$\exists x, z \in \Sigma^{|y|}, p, q \in Q$ such that $p = \delta(s, x), q = \delta(p, y)$ and $\delta(q, z) \in F \Leftrightarrow$
$\exists x, z \in \Sigma^{|y|}$ such that $\delta(\delta(\delta(s, x), y, z) \in F \Leftrightarrow$
$\exists x, z \in \Sigma^{|y|}$ such that $\delta(s, xyz) \in F \Leftrightarrow$
$\exists x, z \in \Sigma^{|y|}$ such that $xyz \in L(M) \Leftrightarrow$
$y \in$ Middlethirds$L(M)$