Homework 3

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Problem 1 (Kozen HW3 #1)

(a)

$L = \{a^n b^m | n = 2m\}$ Let k be given. Let $x = a^{2k}$, $y = b^k$ and $z = E$. Note that $xyz = a^{2k}b^k \in L$. Let $y = uvw$, where $|u| = i$, $|v| = j$, $|w| = l$, and $v \neq E$. Then $k = i + j + l$ and $j > 0$.

$$xuv^r wz = a^{2(i+j+l)}uv^r w$$

When $r = 1$ then $|uvw| = |y| = k$ but choose any other $r > 1$ and $|y| > k$ and more importantly $|y| \neq \frac{1}{2}|x|$ which means the string is no no longer in the language.

(b)

$L = \{x \in \{a, b, c\}^* | x \text{ is a palendrome; i.e., } x = \text{rev}(x)\}$
Suppose for the sake of contradiction, L is regular. Then

$$L \cap a^*b^*a^* = \{a^n b^m a^n\}$$

Which we know is not regular, so L is not regular.

(c)

$L = \{x \in \{a, b, c\}^* | \text{ the length of } x \text{ is a square }\}$
Let k be given and k is a square. Let $x \in L$ so $|x| = k$. Let's say $x = rt$, where $r = \{a, b, c\}$ and $|t| = k - 1$,

$$x = r^2 t \notin L$$

because if k is a square then $|x| = k - 1 + 2 = k + 1$ is not a square.

(d)

$P = \{x \in \{")","("\} | \forall "(" \text{ there is exactly one } ")" \text{ following } \}$
Let $a = "("$ and $b = ")"$ then $L(a^*b^*) \cap P = L(a^n b^n)$ which we know is not regular so P must not be regular.

Problem 2 (Kozen HW3 #2)

(a)

$(01)^*||(10)^* = ((01) + (10) + (11)(00) + (00)(11))^*$

(b)

Let M_A and M_B be machines:

$M_A = (Q_A, \Sigma, \delta_A, S_A, F_A)$
$M_B = (Q_B, \Sigma, \delta_B, S_B, F_B)$

where M_A is the macing accepting the language A and M_B is the machine accepting the language B.
We can define the macine that accepts the language $A||B$ as:

$M_{AB} = (Q_A \times Q_B, \Sigma, \delta_{AB}, S_A \times S_B, F_A \times F_B)$ where $p \in Q_A$ and $q \in Q_B$.

$\delta_{AB}([p, q], a) = \{[\delta_A(p, a), q], [p, \delta_B(q, a)]\}$
Lemma 1 Given an arbitrary state $[p, q]$, applying the transition function with some $a \in \Sigma$ either moves state p via M_A's transition function to some $p' \in Q_A$, leaving q in place, or moves state q via M_B's transition function to some state $q' \in Q_B$, leaving p in place.

Starting at some state $[s_a, s_b] \in S_A \times S_B$ with some $x \in \Sigma^*$. According to lemma 1, for each letter of x, δ_{AB} will return some state in $Q_A \times Q_B$. If, after the last letter in x, $q' \in F_B$ and $p' \in F_A$ then $x \in L(A||B)$. Therefore $A||B$ must be regular.

Problem 3 (Kozen HW3 #3)

(a)

In the first machine only 7 and 8 are inaccessible, in the second all states are accessible.

(b)

1							
✓	2						
✓	✓	3					
✓	✓		4				
✓		✓	✓	5			
	✓	✓	✓	✓	6		
✓	✓	✓	✓	✓	✓	7	
✓		✓	✓		✓	✓	8

$[3] = \{3, 4\}$, $[2] = \{2, 5, 8\}$, $[1] = \{1, 6\}$

1							
	2						
✓	✓	3					
✓	✓		4				
✓	✓	✓	✓	5			
✓	✓	✓	✓		6		
✓	✓	✓	✓			7	
✓	✓			✓	✓	✓	8

$[1] = \{1, 2\}$, $[3] = \{3, 4, 8\}$, $[5] = \{5, 6, 7\}$

(c)