

# Homework 3

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## Problem 1 (Kozen HW3 #1)

(a)

$L = \{a^n b^m \mid n = 2m\}$  Let  $k$  be given. Let  $x = a^{2k}$ ,  $y = b^k$  and  $z = E$ . Note that  $xyz = a^{2k} b^k \in L$ . Let  $y = uvw$ , where  $|u| = i$ ,  $|v| = j$ ,  $|w| = l$ , and  $v \neq E$ . Then  $k = i + j + l$  and  $j > 0$ .

$$xuv^r wz = a^{2(i+j+l)} uv^r w$$

When  $r = 1$  then  $|uvw| = |y| = k$  but choose any other  $r > 1$  and  $|y| > k$  and more importantly  $|y| \neq \frac{1}{2}|x|$  which means the string is no longer in the language.

(b)

$L = \{x \in \{a, b, c\}^* \mid x \text{ is a palindrome; i.e., } x = \text{rev}(x)\}$   
Suppose for the sake of contradiction,  $L$  is regular. Then

$$L \cap a^* b^* a^* = \{a^n b^m a^n\}$$

Which we know is not regular, so  $L$  is not regular.

(c)

$L = \{x \in \{a, b, c\}^* \mid \text{the length of } x \text{ is a square}\}$   
Let  $k$  be given and  $k$  is a square. Let  $x \in L$  so  $|x| = k$ . Let's say  $x = rt$ , where  $r = \{a, b, c\}$  and  $|t| = k - 1$ ,

$$x = r^2 t \notin L$$

because if  $k$  is a square then  $|x| = k - 1 + 2 = k + 1$  is not a square.

(d)

$P = \{x \in \{", '\}^* \mid \forall " \text{ there is exactly one } " \text{ following } x\}$

Let  $a = "( "$  and  $b = ") "$  then  $L(a^*b^*) \cap P = L(a^n b^n)$  which we know is not regular so  $P$  must not be regular.

## Problem 2 (Kozen HW3 #2)

(a)

$$(01)^* \mid (10)^* = ((01) + (10) + (11)(00) + (00)(11))^*$$

(b)

## Problem 3 (Kozen HW3 #3)

(a)

(b)

(c)