Logic Specification Programming - Sheet #2

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Exercise 5

$$
\begin{aligned}
&\forall X \forall Y \forall Z((p_1(X,Y) \vee \neg p_2(X,Y)) \wedge (p_1(Y,Z) \vee \neg p_2(Z) \vee p_3(Y)) \wedge p_3(Z)) \\
&= \forall X \forall Y \forall Z((p_1(X,Y) \vee \neg p_2(X,Y)) \wedge (p_1(Y,Z) \vee \neg (p_2(Z) \wedge p_3(Y)) \wedge p_3(Z)) \\
&= \forall X \forall Y \forall Z(p_2(X,Y) \Rightarrow p_1(X,Y)) \wedge ((p_2(Z) \wedge p_3(Y) \Rightarrow p_1(Y,Z)) \wedge p_3(Z))
\end{aligned}
$$

Prolog Program

$$
\begin{aligned}
p_1(X,Y) &:- p_2(X,Y) \\
p_1(Y,Z) &:- p_2(Z), p_3(Y) \\
p_3(Z)
\end{aligned}
$$

Exercise 6

$$
\begin{aligned}
T^1_{P_0} = \{&bought(bob, eraser)\\
&bought(bob, pencil)\\
&bought(judy, pencil)\\
&category(novel, books)\\
&category(cookbook, books)\\
&category(pencil, stationary)\\
&category(notebook, stationary)\\
&interest(alice, books)\\
&interest(judy, books)\}
\end{aligned}
$$

$$
\begin{aligned}
T^2_{P_0} = \{&interest(bob, stationary)\\
&interest(judy, stationary)\\
&recommendation(judy, eraser)\\
&recommendation(alice, novel)\\
&recommendation(alice, cookbook)\\
&recommendation(judy, novel)\\
&recommendation(judy, cookbook)\} \cup T^1_{P_0}
\end{aligned}
$$

$$
\begin{aligned}
T^3_{P_0} = \{&recommendation(bob, pencil)\\
&recommendation(judy, pencil)\\
&recommendation(bob, notebook)\\
&recommendation(judy, notebook)\} \cup T^2_{P_0}
\end{aligned}
$$

$fix(T_p) = \sqcup\{T_p^i(\emptyset) | i \in 1, 2, 3\}$

Exercise 7

a) $L_1 = \{(p(c_1, X), p(Y, c_2))\}$

$$
\begin{aligned}
& unify(p(c_1, X), p(Y, c_2)) \\
\overset{\textbf{rule 6}}{=} & unify((c_1, Y) : (X, c_2)) \\
& \sigma = Y/c_1 \\
\overset{\textbf{rule 4}}{=} & unify((X, c_2)\hat{\sigma}) \circ \sigma \\
& \sigma' = X/c_2 \\
\overset{\textbf{rule 3}}{=} & unify(\epsilon) \circ \sigma' \circ \sigma \\
\overset{\textbf{rule 1}}{=} & id \circ \sigma' \circ \sigma \\
= & id \circ Y/c_1 \circ X/c_2
\end{aligned}
$$

The unification was successful.

b) $L_2 = \{(x, p(x)), (c_1(x), c_2(c_3))\}$

$$unify((x, p(x)), (c_1(x), c_2(c_3)))$$

No rule applicable, therefore the unification was unsuccessful.

c) $L_3 = \{(p(c_1(X), Z, c_2(c_2(c_0))), p(Y, c_3(X, Y), c_2(X)))\}$

$$
\begin{aligned}
& unify(p(c_1(X), Z, c_2(c_2(c_0))), p(Y, c_3(X, Y), c_2(X))) \\
& \overset{\textbf{rule 6}}{=} unify((c_1(X), Y) : (Z, c_3(X, Y)) : (c_2(c_2(c_0)), c_2(X))) \\
& \sigma = Y/c_1(X) \\
& \overset{\textbf{rule 3}}{=} unify((Z, c_3(X, c_1(X))) : (c_2(c_2(c_0)), c_2(X)) \circ \hat{\sigma}) \circ \sigma \\
& \sigma' = Z/c_3(X, c_1(X)) \\
& \overset{\textbf{rule 4}}{=} unify((c_2(c_2(c_0)), c_2(X)) \circ \hat{\sigma}) \circ \sigma' \circ \sigma \\
& \sigma'' = X/c_2(c_0) \\
& \overset{\textbf{rule 3}}{=} unify(\epsilon) \circ \sigma'' \circ \sigma' \circ \sigma \\
& \overset{\textbf{rule 1}}{=} id \circ \sigma'' \circ \sigma' \circ \sigma \\
& = id \circ X/c_2(c_0) \circ Z/c_3(X, c_1(X)) \circ Y/c_1(X)
\end{aligned}
$$

The unification was successful.

Exercise 8

a) $1.? - friends(alice, P) \rightarrowtail_{P_1 1\{P/bob\}}$
$2.? - \square$
First answer substitution is $\sigma = [P/bob]$**.**
Intermediate result: $friends(alice, bob)$**.**
$3.? - friends(alice, P) \rightarrowtail_{P_1 1\{P/charlie\}}$
$4.? - \square$
Second answer substitution is $\sigma = [P/charlie]$**.**
Intermediate result: $friends(alice, charlie)$**.**
No further rules applicable.
$M_{SLD}(P_1, friends(alice, P)) = \{friends(alice, bob), friends(alice, charlie)\}$

b) $1.? - social\_net(charlie, P) \rightarrowtail_{P_1 5\{X/charlie, Y/P\}}$
$2.? - friends(charlie, P) \rightarrowtail_{P_1 3\{P/dave\}}$
$3.? - \square$
First answer substitution is $\sigma = [P/dave]$**.**
Intermediate result: $social\_net(charlie, dave)$**.**
$4.? - social\_net(charlie, P) \rightarrowtail_{P_1 6\{X/charlie, Y/P\}}$
$5.? - friends(charlie, Z), social\_net(Z, P) \rightarrowtail_{P_1 3\{Z/dave\}}$
$7.? - social\_net(dave, P) \rightarrowtail_{P_1 4\{P/ted\}}$
$8.? - friends(dave, ted) \rightarrowtail_{P_1 4\{\}}$
$9.? - \square$
Second answer substitution is $\sigma = [P/ted]$**.**
Intermediate result: $social\_net(charlie, ted)$**.**
No further rules applicable.
$M_{SLD}(P_1, social\_net(charlie, P)) = \{social\_net(charlie, dave), social\_net(charlie, ted)\}$