

LSP-Exercise Sheet ①

Exercise 1

a) $\neg(x \wedge y) \Leftrightarrow \neg x \vee \neg y$

$$\begin{aligned}
 &= ((\neg(x \wedge y)) \Rightarrow (\neg x \vee \neg y)) \wedge ((\neg x \vee \neg y) \Rightarrow (\neg(x \wedge y))) \\
 &= (\neg(\neg(x \wedge y)) \vee (\neg x \vee \neg y)) \wedge (\neg(\neg x \vee \neg y) \vee (\neg(x \wedge y))) \\
 &= (\neg(\neg(x \wedge y)) \vee (\neg x \vee \neg y)) \wedge (\neg(\neg x) \vee (\neg(\neg y))) \\
 &= (x \wedge y) \vee \neg(x \wedge y) \wedge (x \wedge y) \vee \neg(x \wedge y) \\
 &= (x \wedge y) \vee \neg(x \wedge y)
 \end{aligned}$$

x	y	$x \wedge y$	$\neg x \vee \neg y$	$(x \wedge y) \vee \neg(x \wedge y)$
0	0	0	1	1
1	0	0	1	1
0	1	0	1	1
1	1	1	0	1

is satisfiable
and valid

b) $(y \wedge \neg z) \Rightarrow ((x \wedge y) \vee (x \wedge z))$

$$\begin{aligned}
 &= \neg(y \wedge \neg z) \vee ((x \wedge y) \vee (x \wedge z)) \\
 &= \neg y \vee z \vee ((x \wedge y) \vee (x \wedge z)) \\
 &\rightarrow \text{Absorption-Rule } z \vee (x \wedge z) = z
 \end{aligned}$$

$$\begin{aligned}
 &= \neg y \vee z \vee (x \wedge y) \\
 &= z \vee ((\neg y \vee x) \wedge (\neg y \vee y)) \\
 &= z \vee (\neg y \vee x)
 \end{aligned}$$

$$\neg y \vee y = 1$$

x	y	z	$\neg y$	$x \vee \neg y$	$x \vee z \vee \neg y \vee z$
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	0	0	0
1	0	0	1	1	1
0	1	1	0	0	1
1	0	1	1	1	1
1	1	0	0	1	1
1	1	1	0	1	1

is satisfiable, but
not valid

$$c) \neg((\neg x \wedge \neg z) \Rightarrow (\neg y \Rightarrow z))$$

$$= \neg(\neg(\neg x \wedge \neg z) \vee (\neg y \vee z))$$

$$= \neg(x \vee \neg z \vee y \vee z)$$

$$= \neg(x \vee y \vee 1)$$

$$= \neg(1) = \underline{\underline{0}}$$

$$\neg z \vee z = 1$$

$$x \vee 1 = 1$$

Not satisfiable or valid.