

Exercise 3

$$\begin{array}{ll}
 \text{a)} & \begin{array}{l} | (P_1) = \text{john} \\ | (T) = \text{bike} \\ | (P_2) = \text{chris} \\ | (L) = \text{münster} \end{array} & \begin{array}{l} | (P_1) = \text{mary} \\ | (T) = \text{ski} \\ | (P_2) = \text{eve} \\ | (L) = \text{berlin} \end{array}
 \end{array}$$

$$\begin{array}{l}
 | (P_1) = \text{john} \\
 | (T) = \text{bob} \\
 | (P_2) = \text{bike} \\
 | (L) = \text{münster}
 \end{array}$$

$$\text{b)} \quad \bar{F} := \text{sells}(P_1, T) \wedge \text{buys}(P_2, T) \wedge \text{location}(P_1, L) \wedge \text{location}(P_2, L)$$

$$\begin{aligned}
 \uparrow(\bar{F}) &= \uparrow(\text{sells}(P_1, T) \wedge \text{buys}(P_2, T) \wedge \text{location}(P_1, L) \wedge \text{location}(P_2, L)) \\
 &= (\uparrow(P_1), \uparrow(T) \in I(\text{sells})) \wedge (\uparrow(P_2), \uparrow(T) \in I(\text{buys})) \wedge \\
 &\quad (\uparrow(P_1), \uparrow(L) \in I(\text{location})) \wedge (\uparrow(P_2), \uparrow(L) \in I(\text{location}))
 \end{aligned}$$

$$= (| (P_1), | (T) \in I(\text{sells})) \wedge (| (P_2), | (T) \in I(\text{buys})) \wedge \\
 (| (P_1), | (L) \in I(\text{location})) \wedge (| (P_2), | (L) \in I(\text{location}))$$

$$= (\text{john}, \text{bike}) \in I(\text{sells}) \wedge (\text{chris}, \text{bike}) \in I(\text{buys}) \wedge \\
 (\text{john}, \text{münster}) \in I(\text{location}) \wedge (\text{chris}, \text{münster}) \in I(\text{location})$$

$$= \text{true} \wedge \text{true} \wedge \text{true} \wedge \text{true} = \text{true}$$

Exercise 4

a)

$$F_1 := \neg \exists X \forall Y \left(\forall Z (p_1(X, Y, Z) \vee \exists X p_2(X)) \wedge p_3(X, Y) \right)$$

$$= \neg \exists X \forall Y \forall Z (p_1(X, Y, Z) \vee \exists X p_2(X) \wedge p_3(X, Y))$$

$$= \neg \exists X \forall Y \forall Z \neg \exists W \exists w (p_1(X, Y, Z) \vee p_2(w) \wedge p_3(X, Y))$$

b)

$$F_2 := \forall X \exists Y \exists Z \forall Q \exists R p_1(X, Y) \vee (\neg p_2(Q, R) \wedge p_3(Z))$$

$$= \forall X \forall Q p_1(X, Y) [Y/c(X)] \vee \\ (\neg p_2(Q, R) [R/c(X, Y, Q)] \wedge \\ p_3(Z) [Z/c(X, Y, Q, R)])$$

c)

$$F_3 := \forall X \forall Y \forall Z \left(\neg (p_1(Z) \wedge \neg p_2(Y, Z)) \wedge ((p_3(X, Y) \wedge p_4(Y, Z)) \vee p_5(X)) \right)$$

$$= \forall X \forall Y \forall Z \left(\neg (p_1(Z) \wedge \neg p_2(Y, Z)) \wedge \right. \\ \left. ((p_3(X, Y) \vee p_5(X)) \wedge (p_4(Y, Z) \vee p_5(X))) \right)$$

$$= \forall X \forall Y \forall Z \left(\neg (p_1(Z) \wedge \neg p_2(Y, Z)) \wedge \right. \\ \left. (\neg \neg (p_3(X, Y) \vee p_5(X)) \wedge (\neg \neg (p_4(Y, Z) \vee p_5(X)))) \right)$$

$$= \forall X \forall Y \forall Z \left(\neg (p_1(Z) \wedge \neg p_2(Y, Z)) \wedge \right. \\ \left. (\neg (\neg p_3(X, Y) \wedge \neg p_5(X)) \wedge (\neg (\neg p_4(Y, Z) \wedge \neg p_5(X)))) \right)$$