We define FIB as

$$\mathrm{FIB}(n) = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ \mathrm{FIB}(n-1) + \mathrm{FIB}(n-2) & \text{otherwise} \end{cases}$$

and proceed to prove by induction that $\mathrm{FIB}(n)$ is the closest integer to $\phi^n/\sqrt{5}$.

Our base cases 0 and 1 are satisfied as follows:

$$\mathrm{FIB}(0) = \frac{\left(1 + \sqrt{5}\right)^0}{2} = \frac{1}{2} \approx 0$$

$$\mathrm{FIB}(1) = \frac{\left(1 + \sqrt{5}\right)^1}{2} = \frac{1 + \sqrt{5}}{2} \approx 1$$