Flood-It

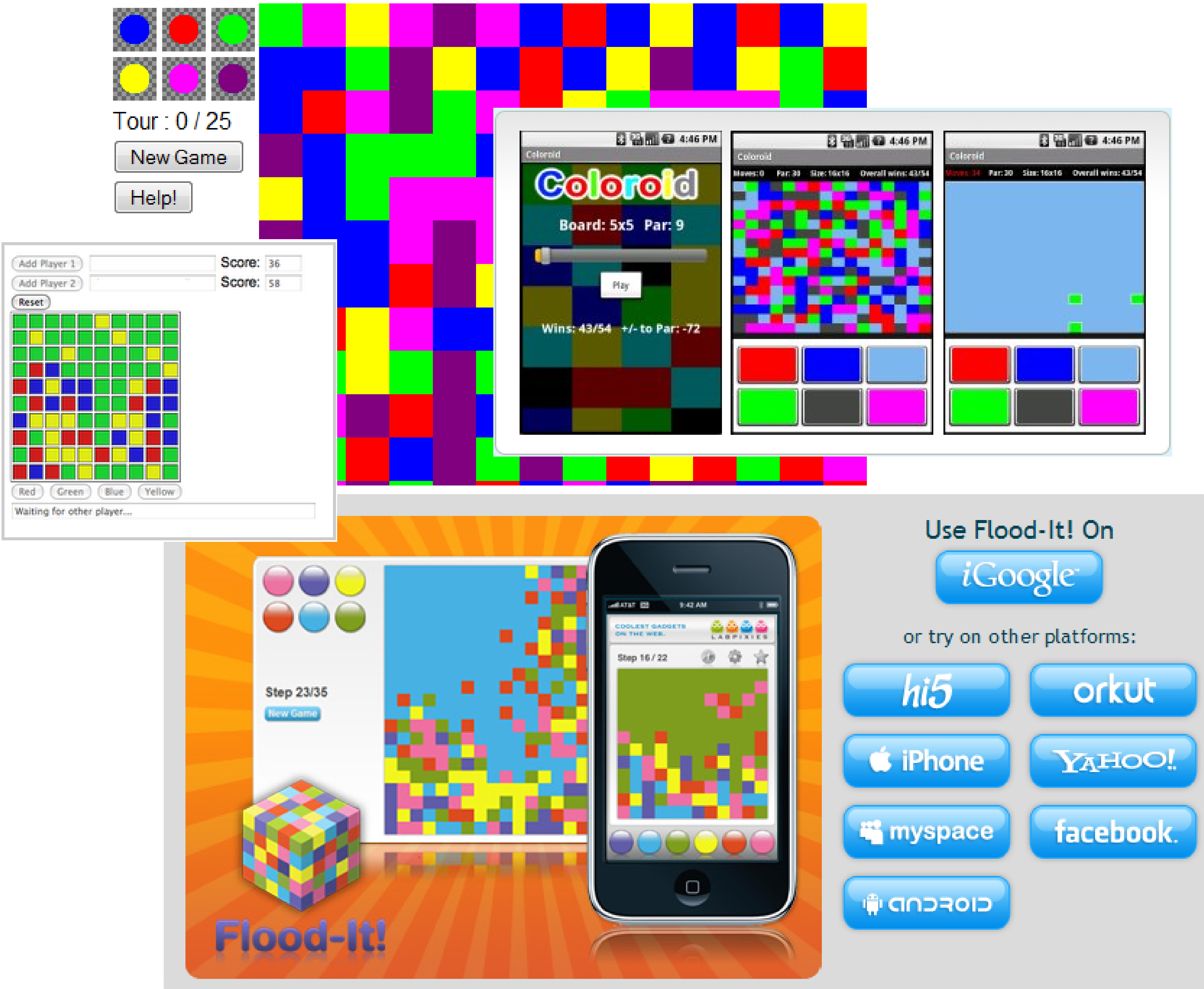
The Colourful Game of Board Domination

Bristol Algorithms Day, 15–16 Feb 2009

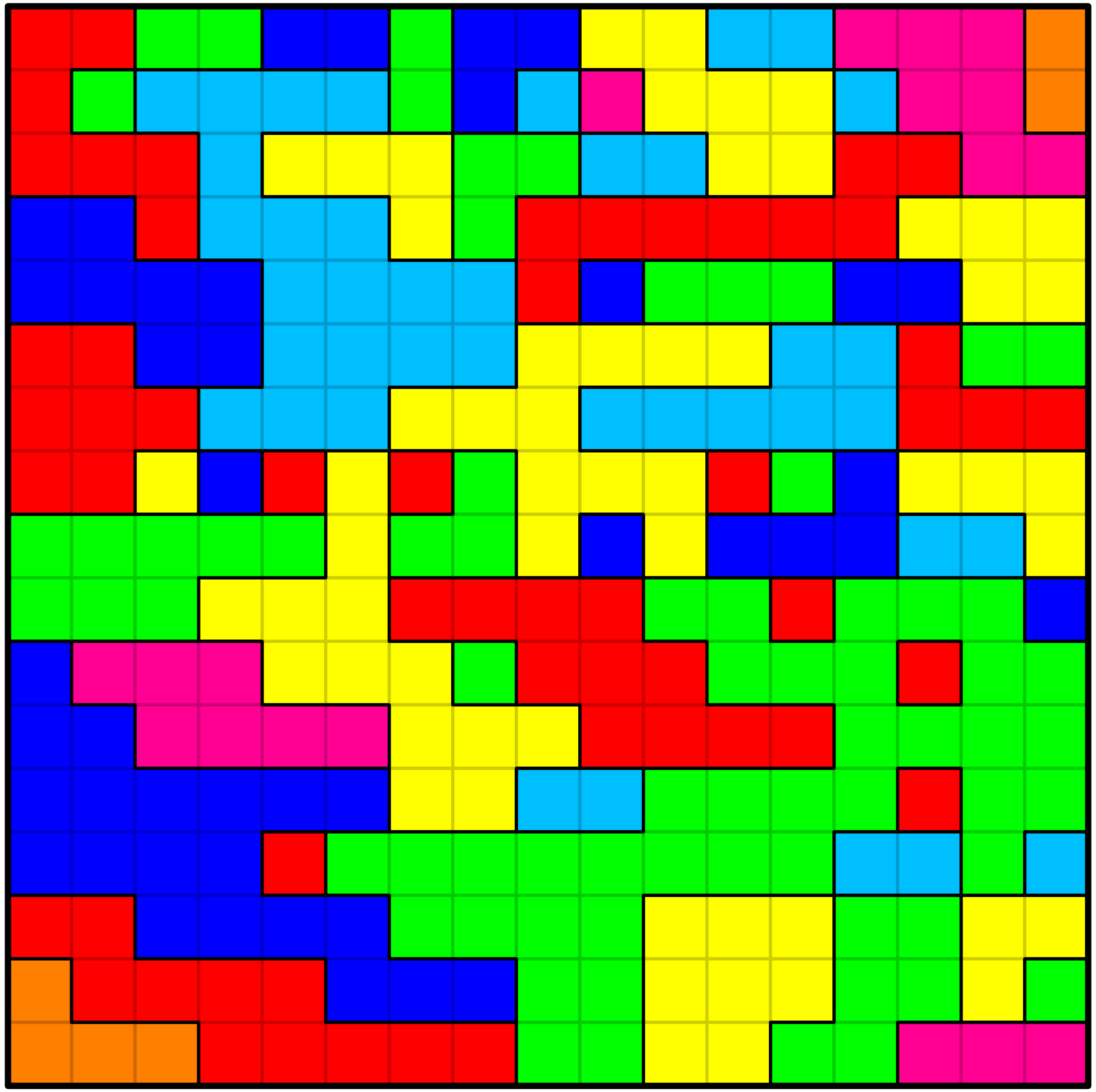
David Arthur, Raphaël Clifford, Markus Jalsenius, Ashley Montanaro and Benjamin Sach

Department of Computer Science

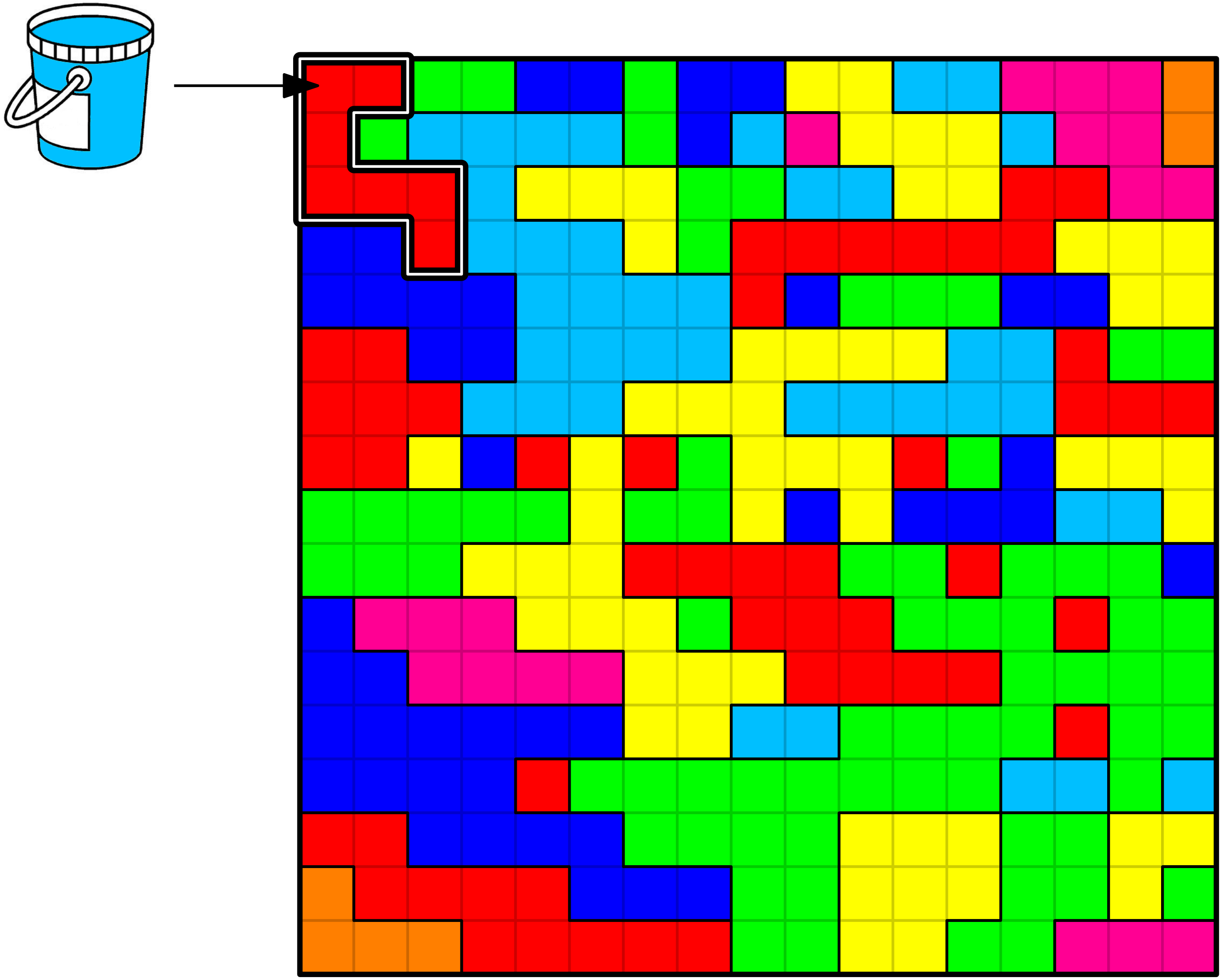
University of Bristol


Tour : 0 / 25
New Game
Help!
Add Player 1
Score: 36
Add Player 2
Score: 58
Reset
Red
Green
Blue
Yellow
Waiting for other player...
4:46 PM
Coloroid
Coloroid
Board: 5x5 Par: 9
Play
Wins: 43/54 +/- to Par: -72
Moves: 0 Par: 30 Size: 16x16 Overall wins: 43/54
Par: 30 Size: 16x16 Overall wins: 43/54
Step 23/35
New Game
Flood-It!
Step 16 / 22
Use Flood-It! On
iGoogle
or try on other platforms:
hi5
orkut
iPhone
YAHOO!
myspace
facebook
android

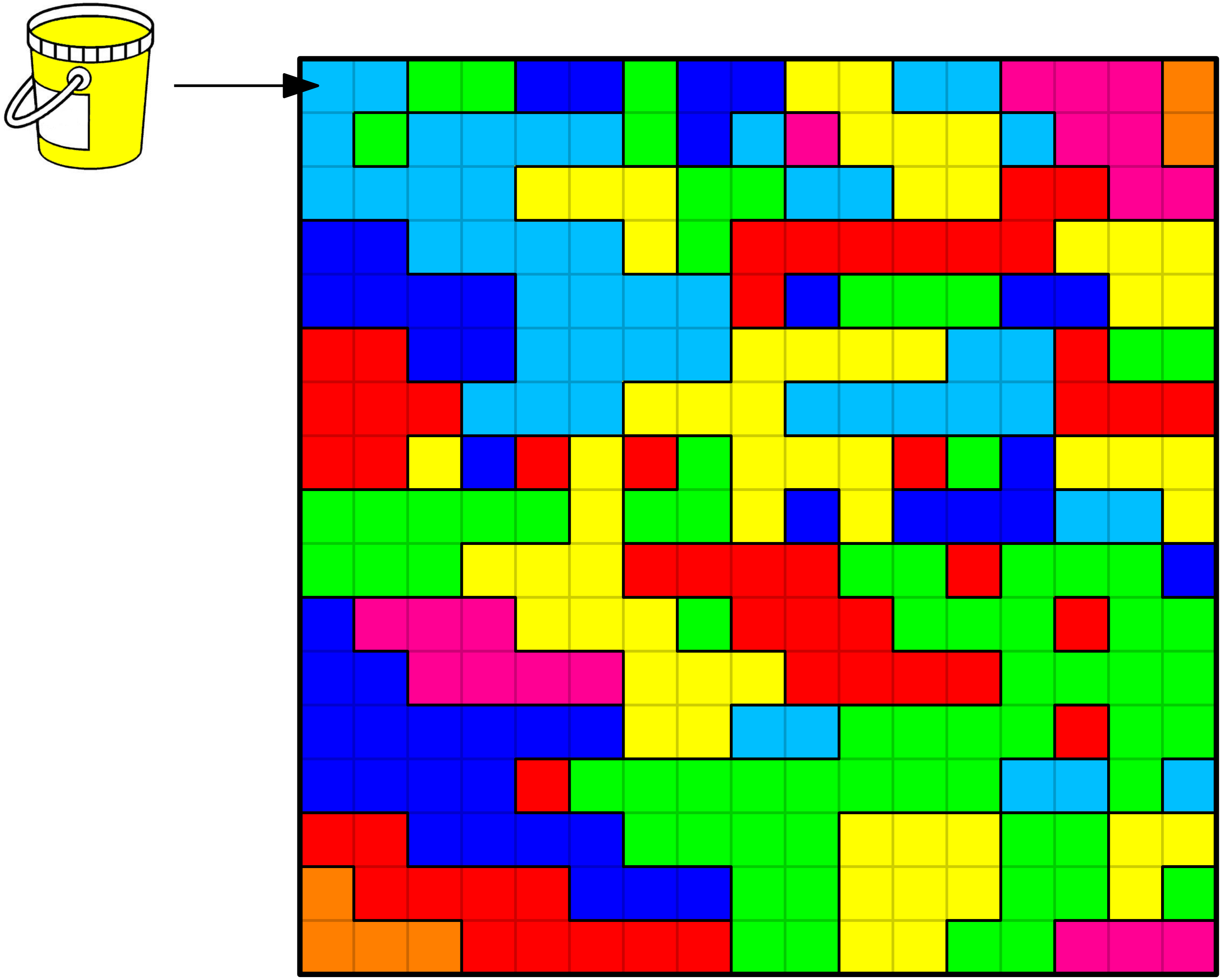
The Game



The Game



The Game



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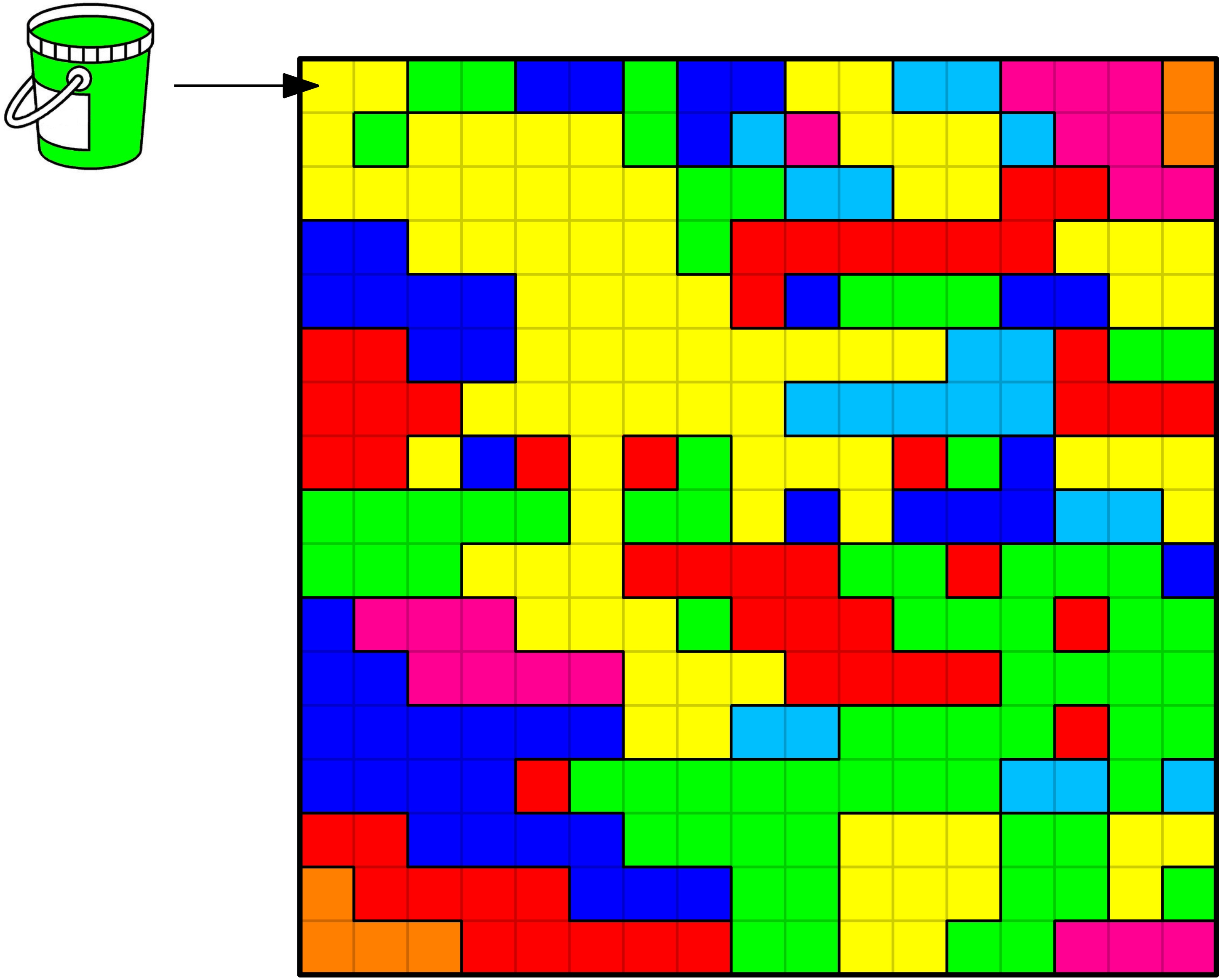

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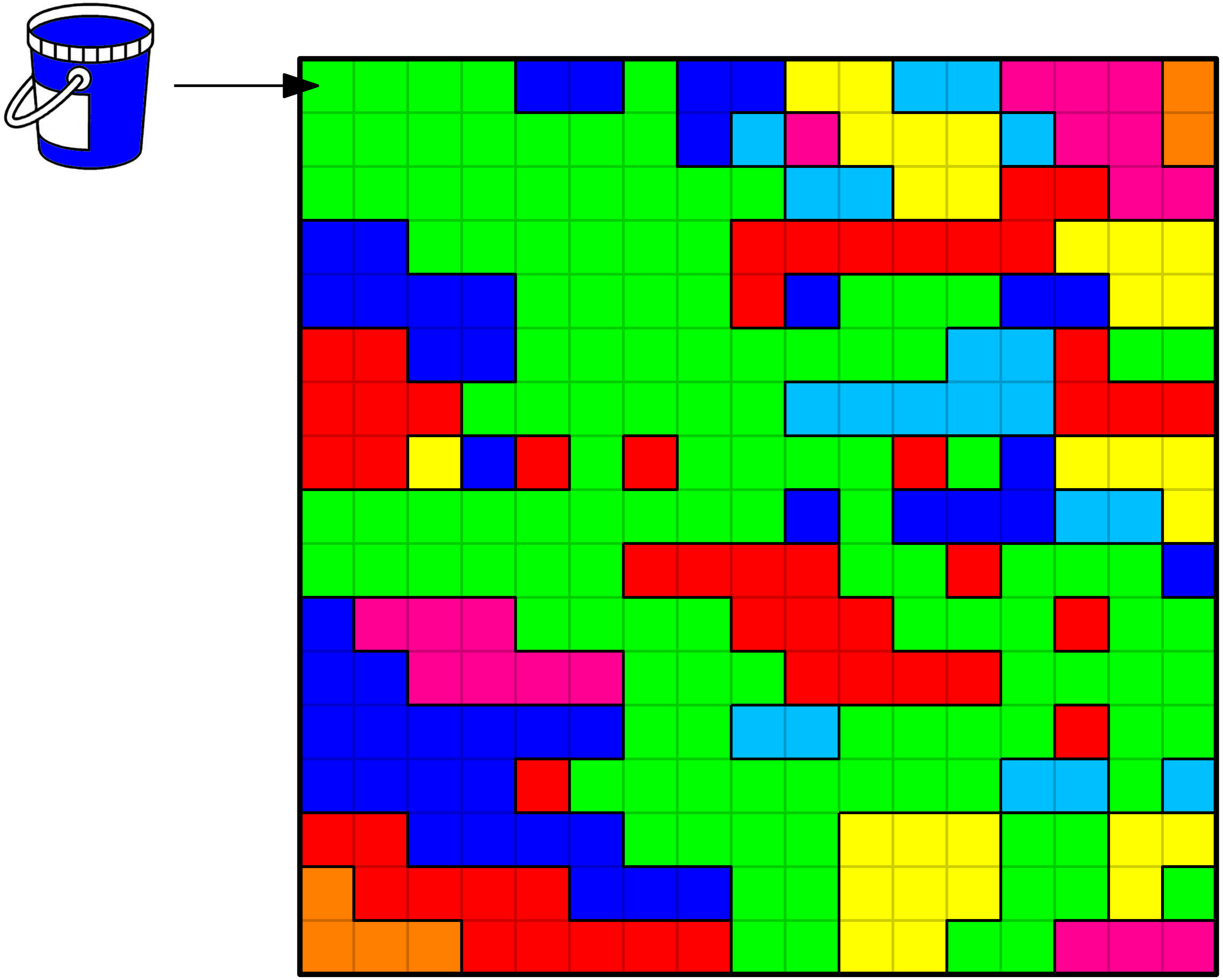

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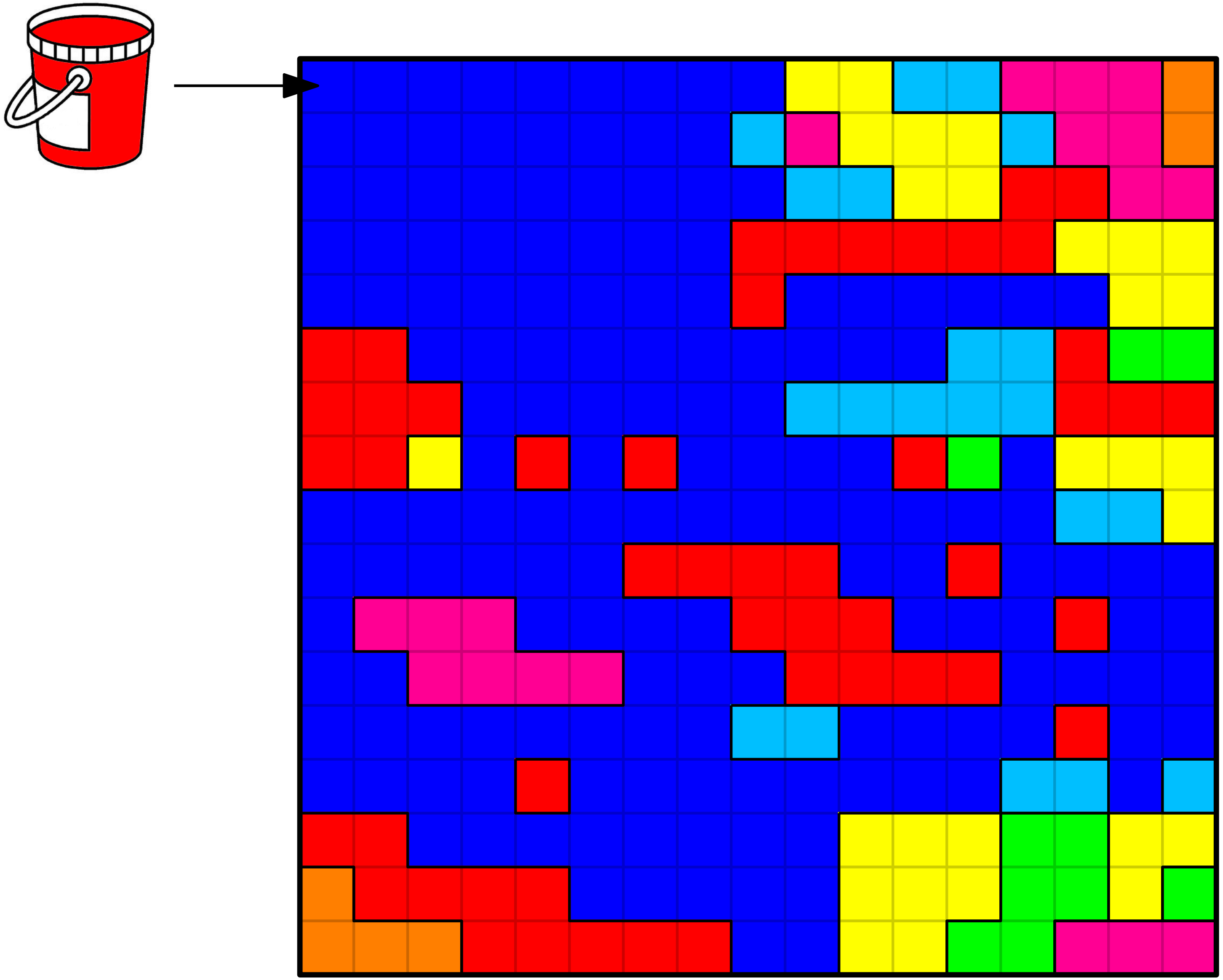

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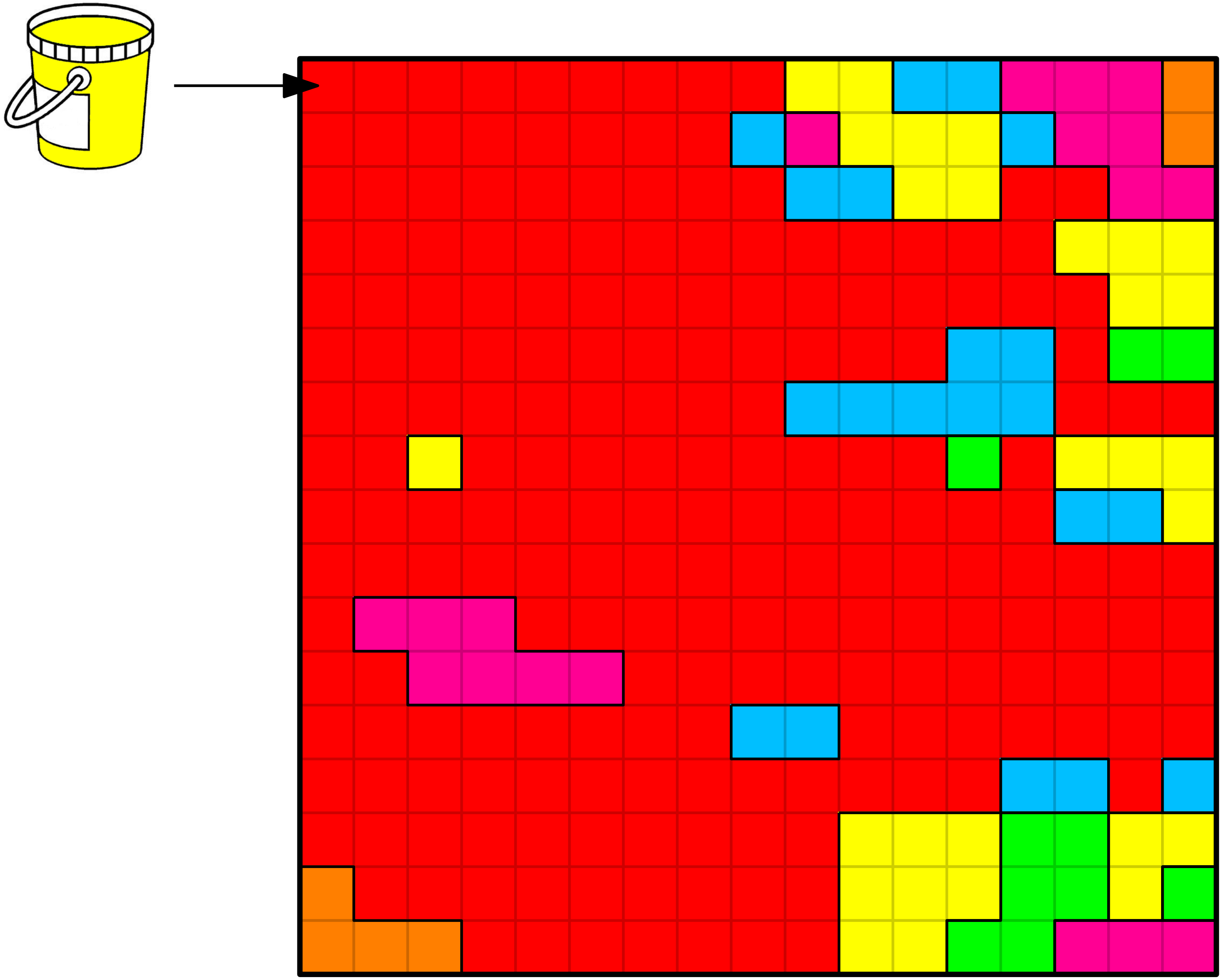

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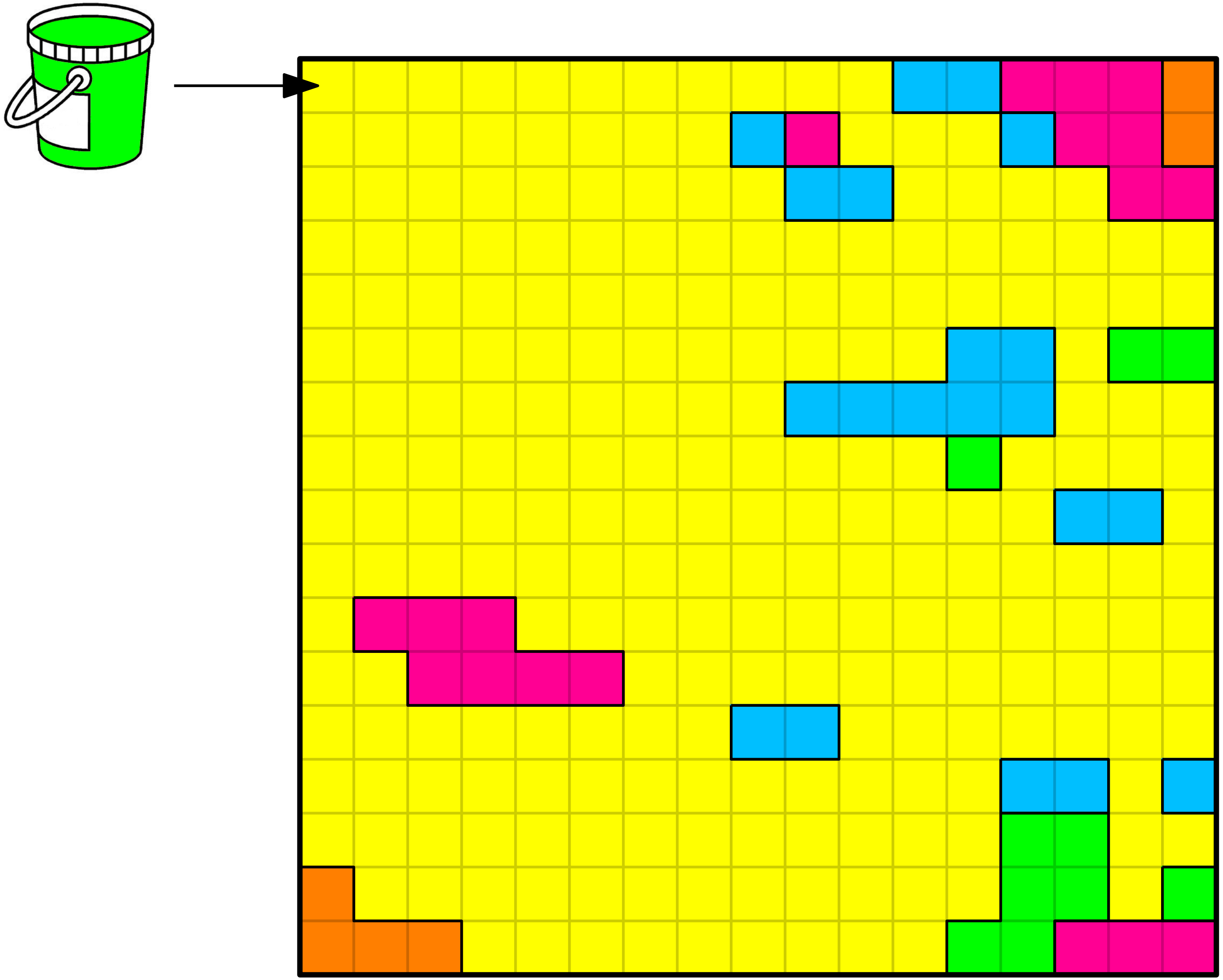

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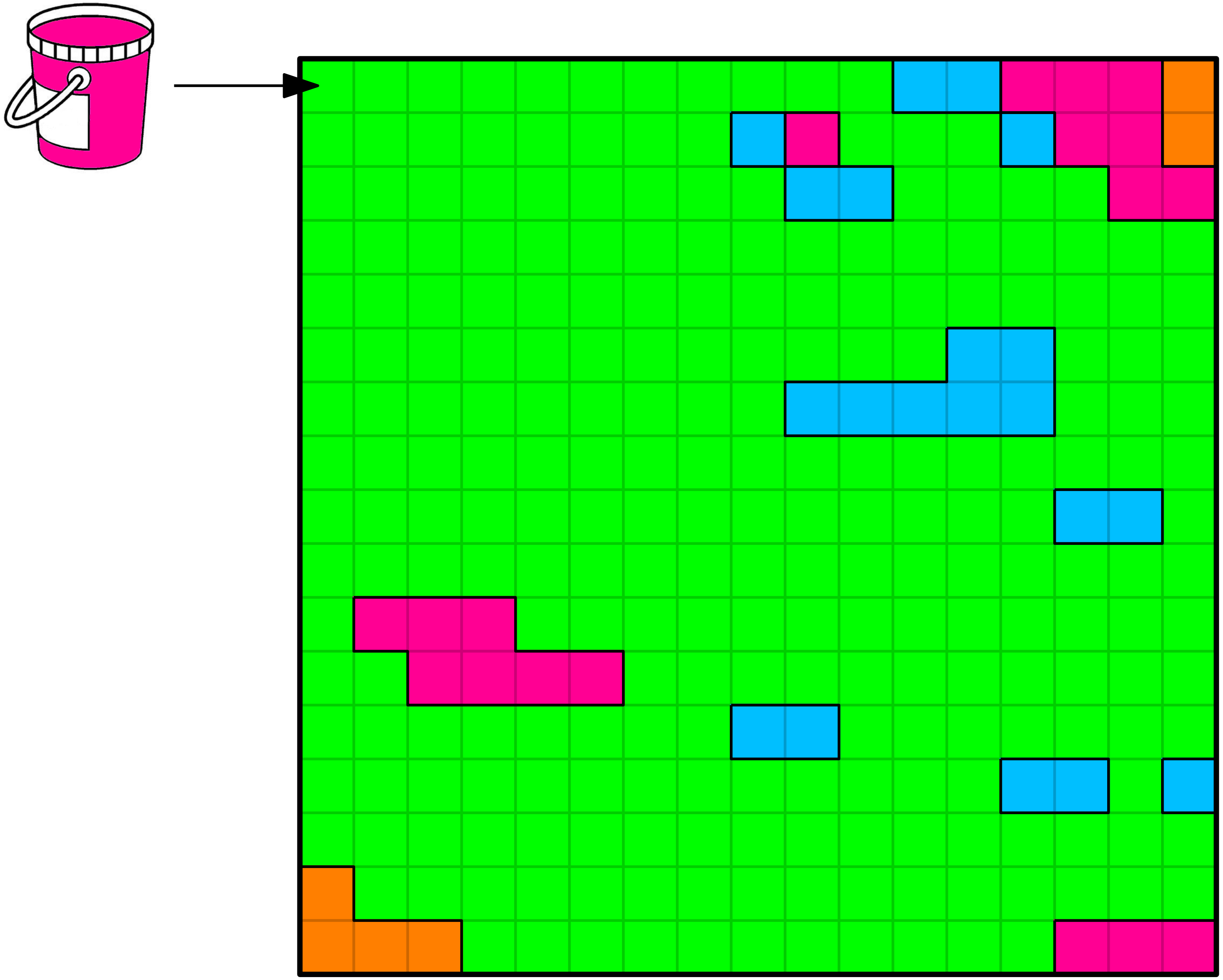

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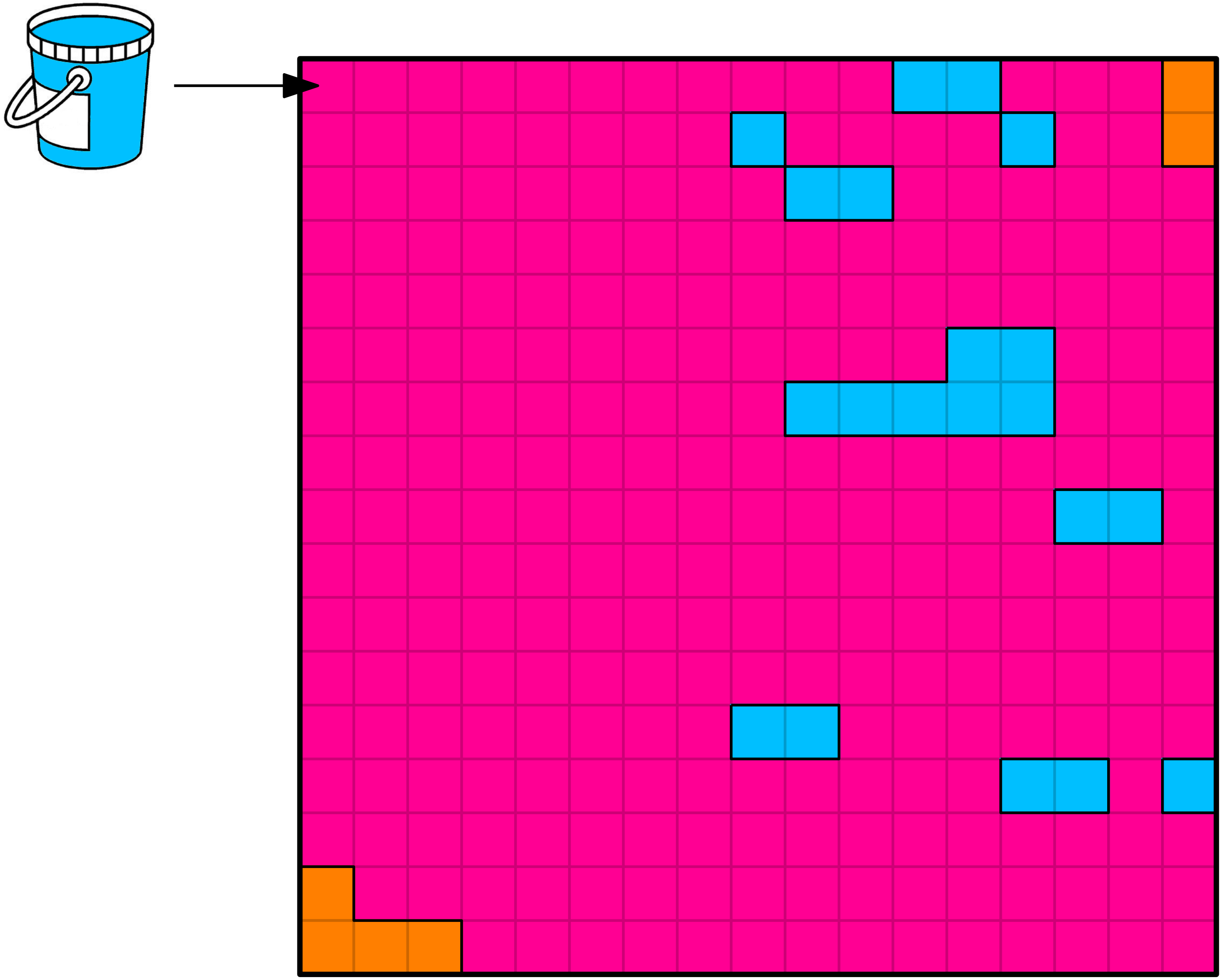

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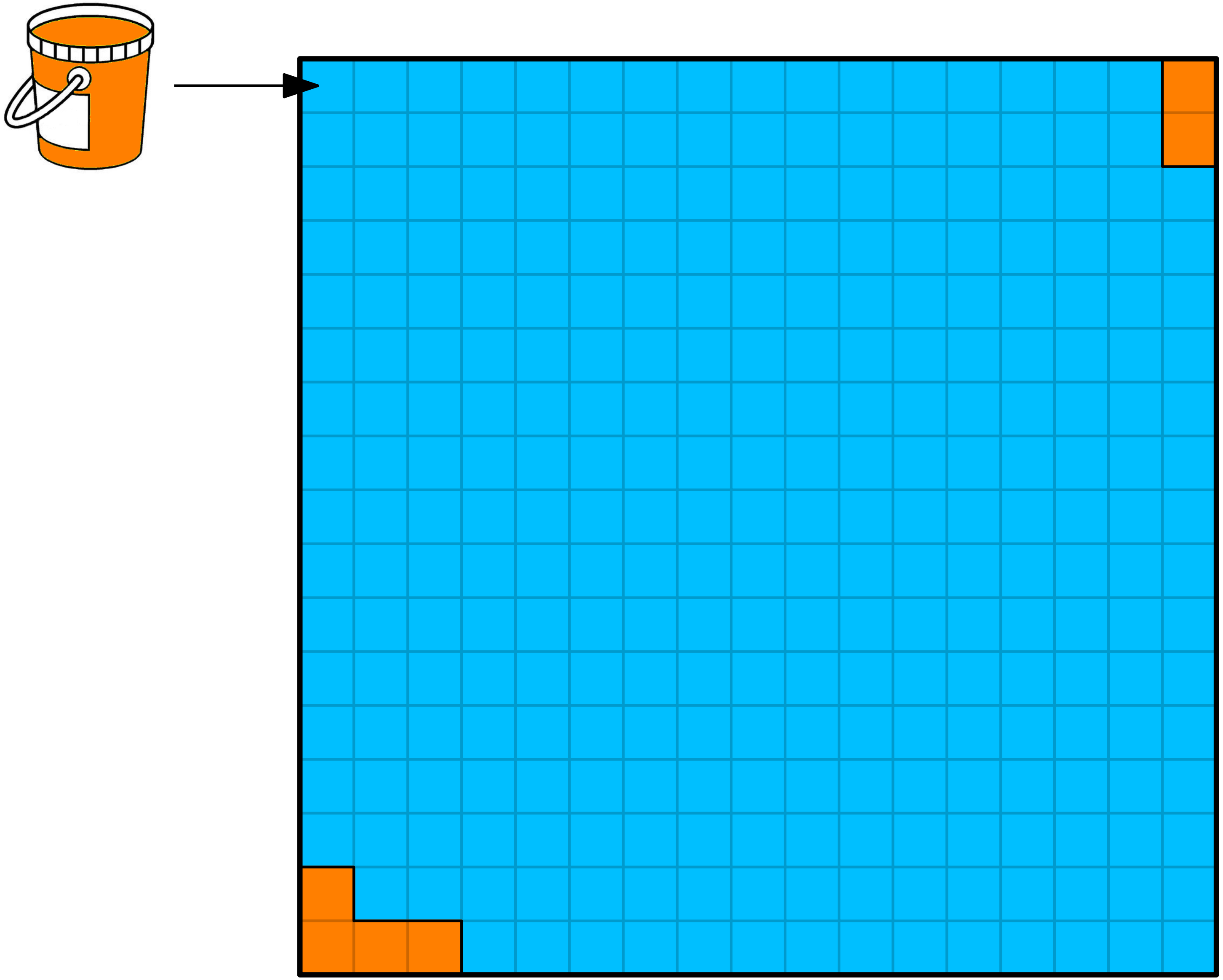
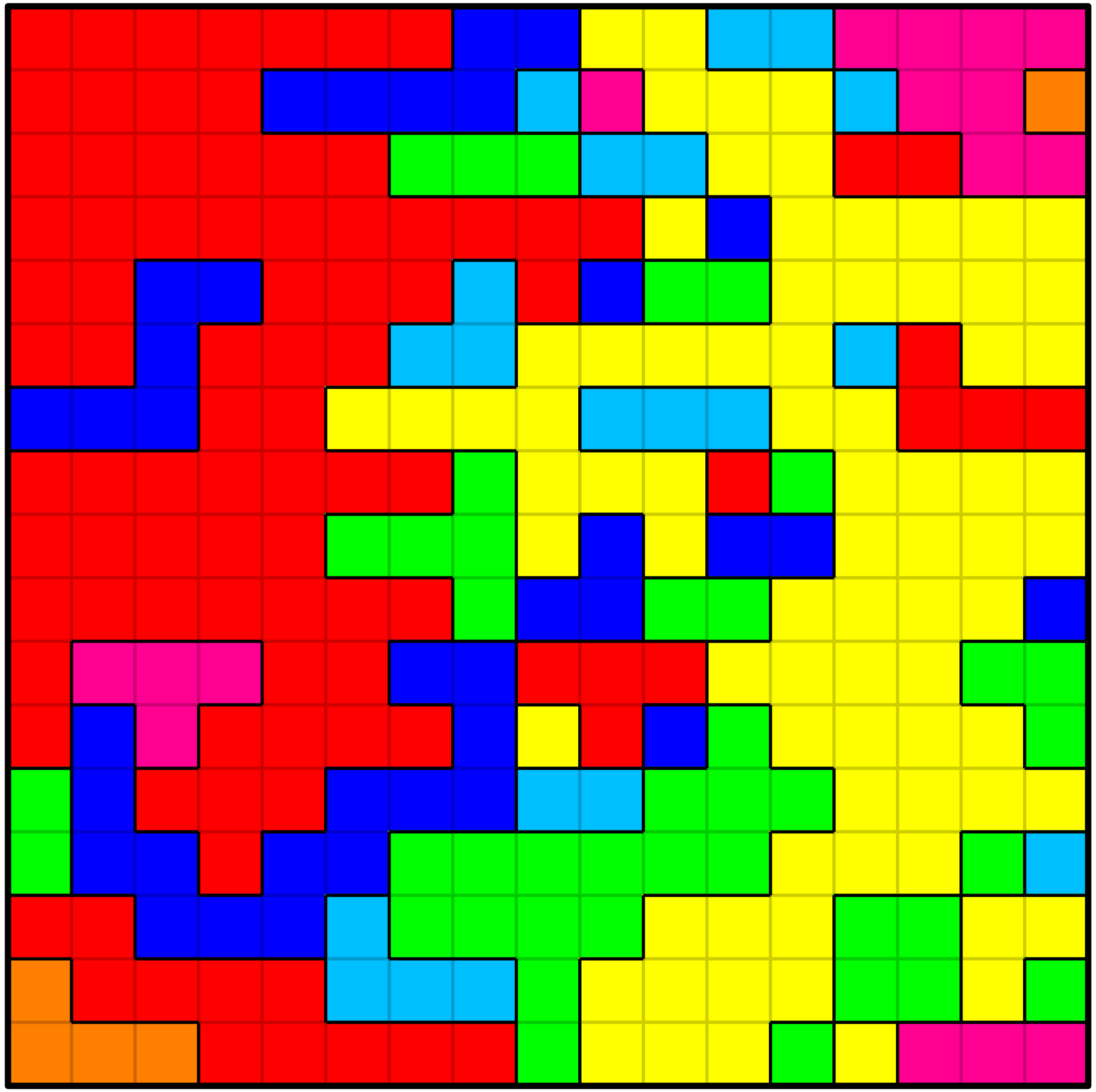


The Game

Greedy



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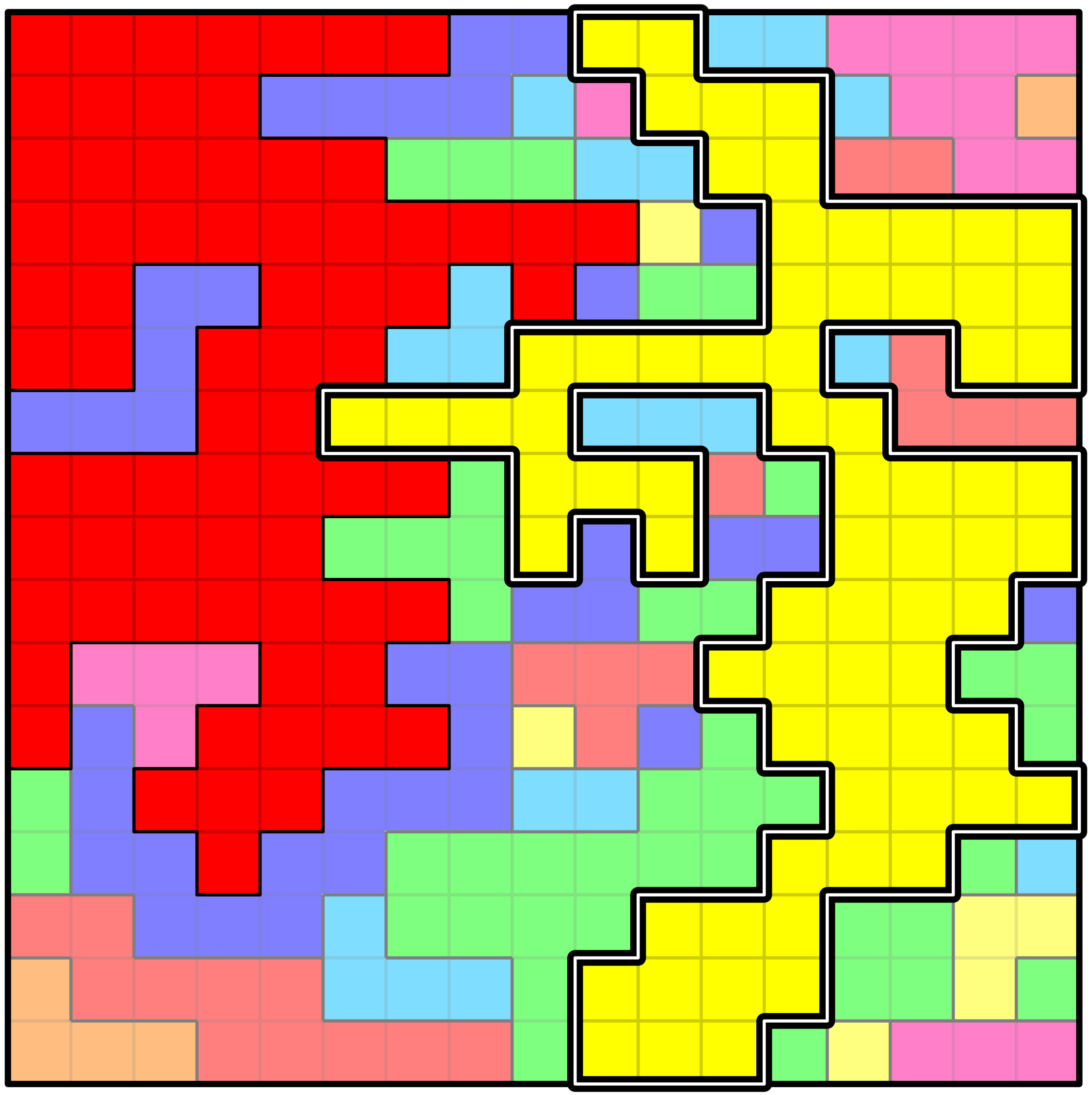

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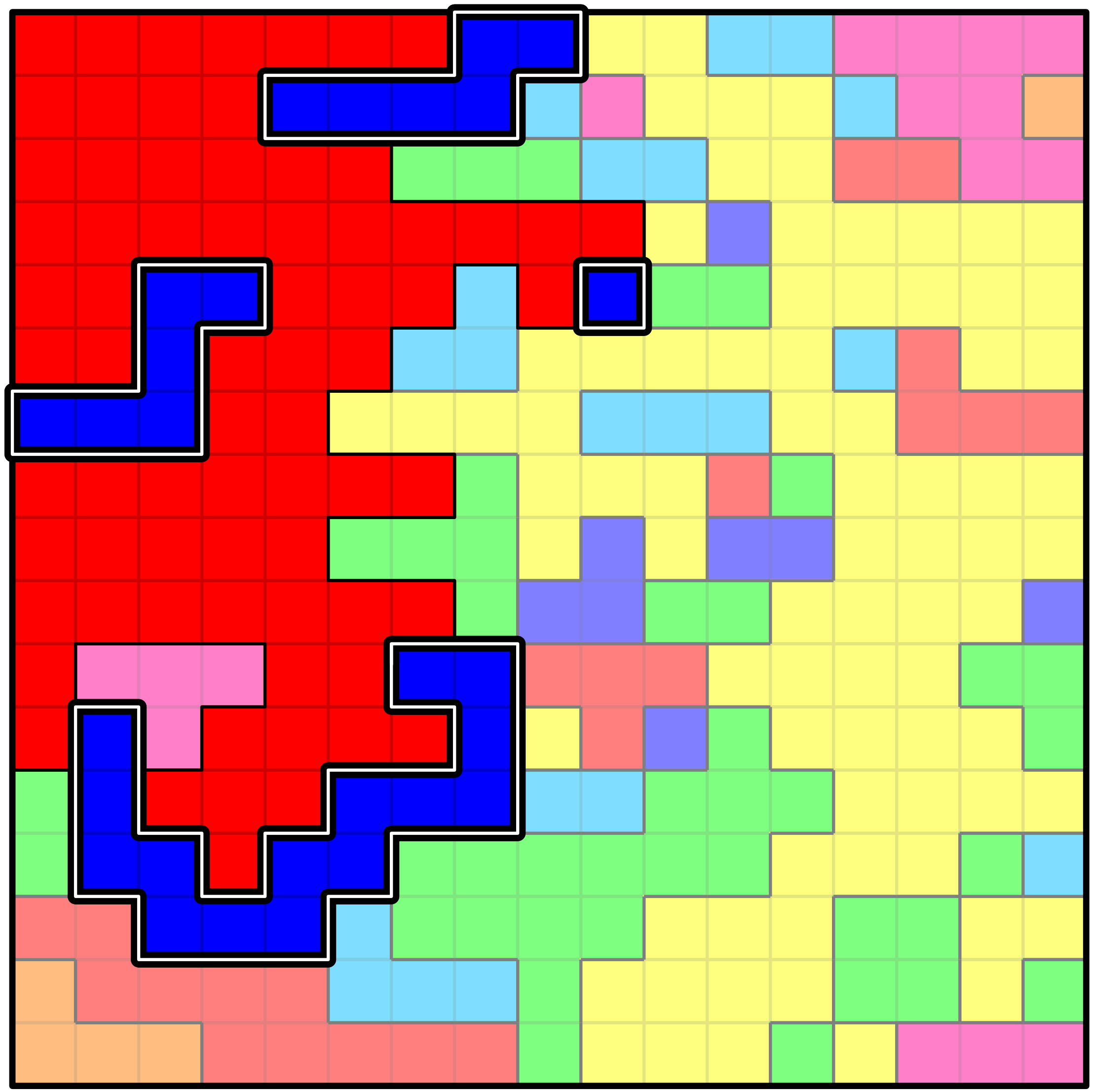

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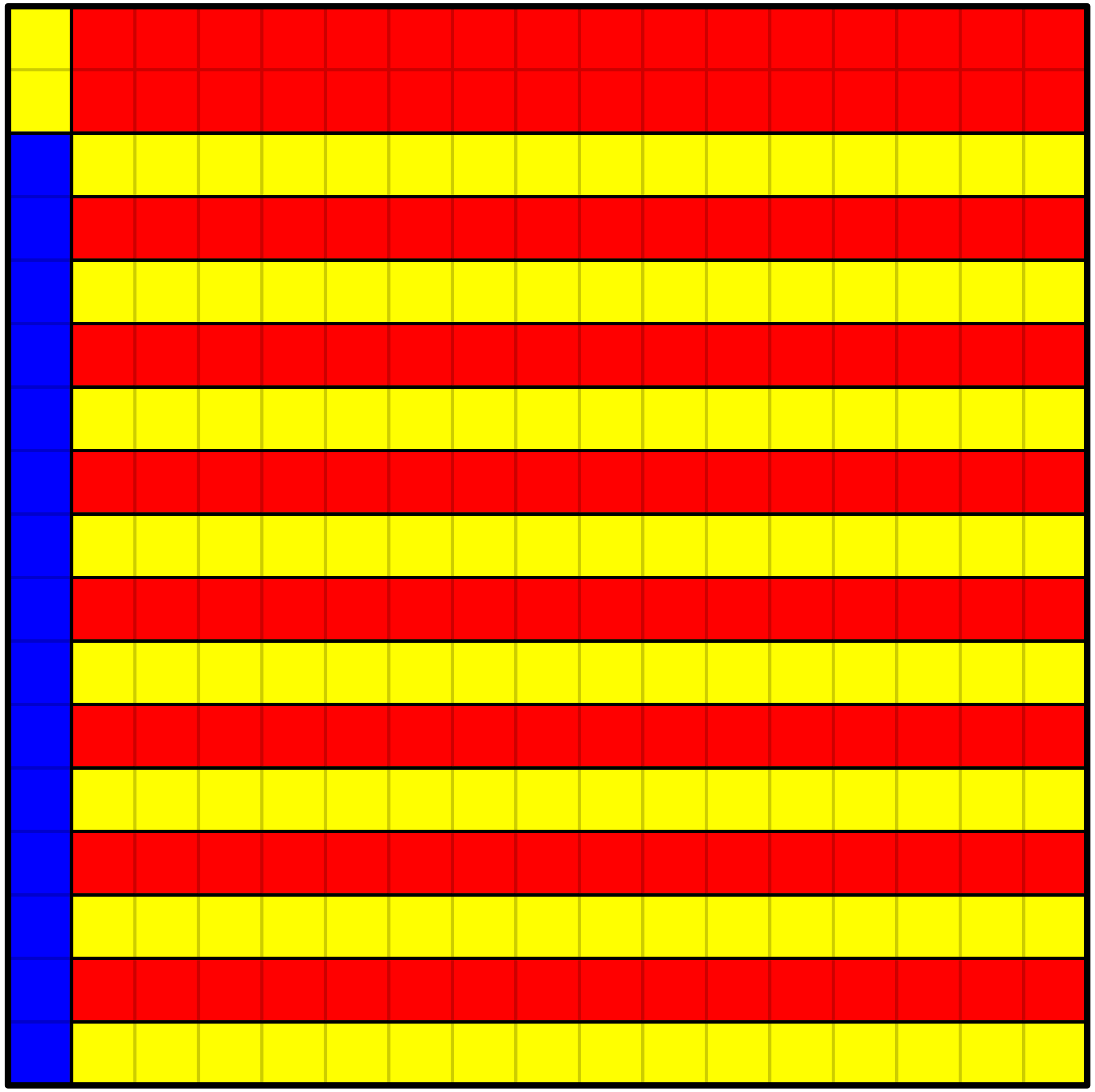

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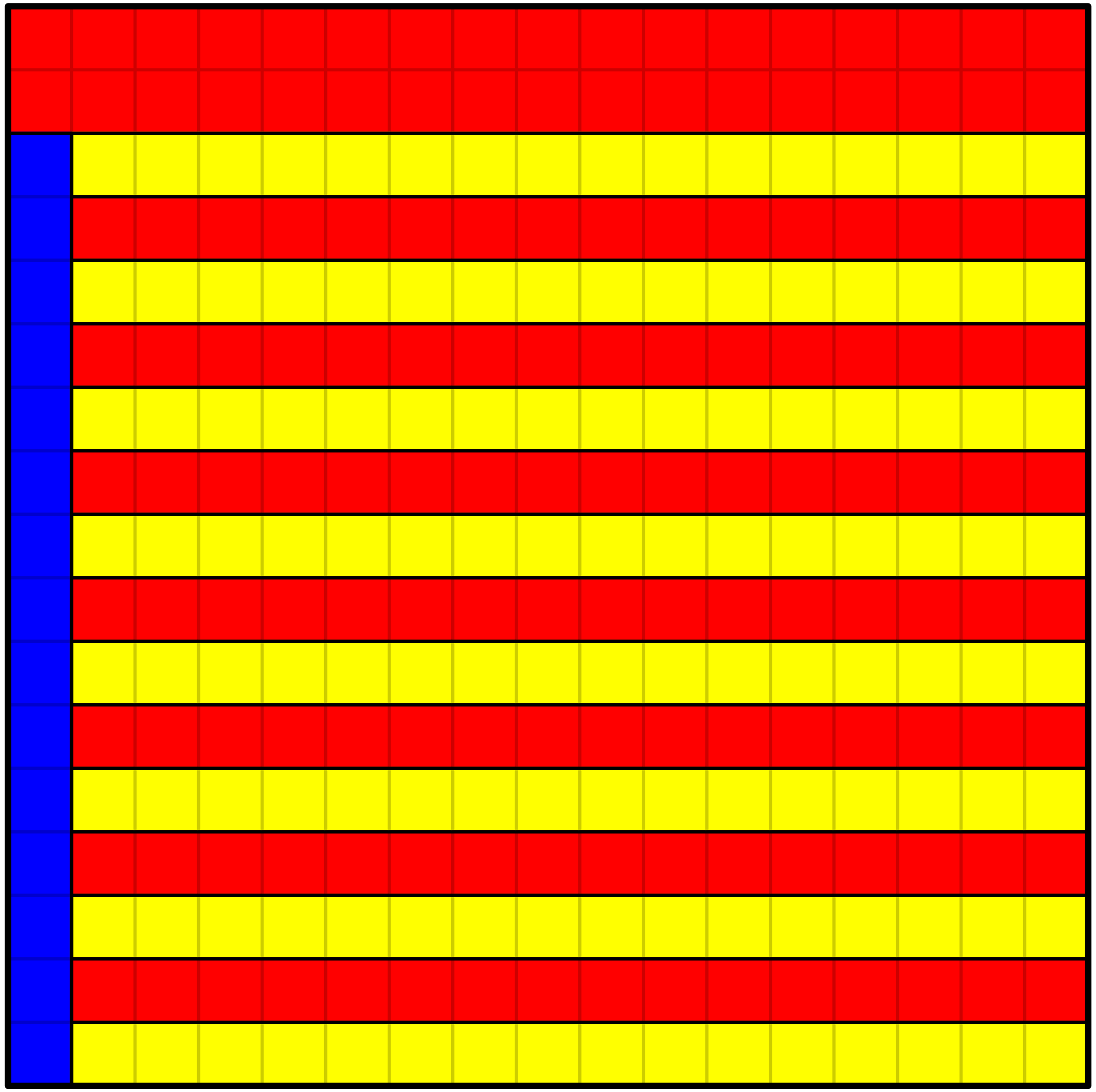

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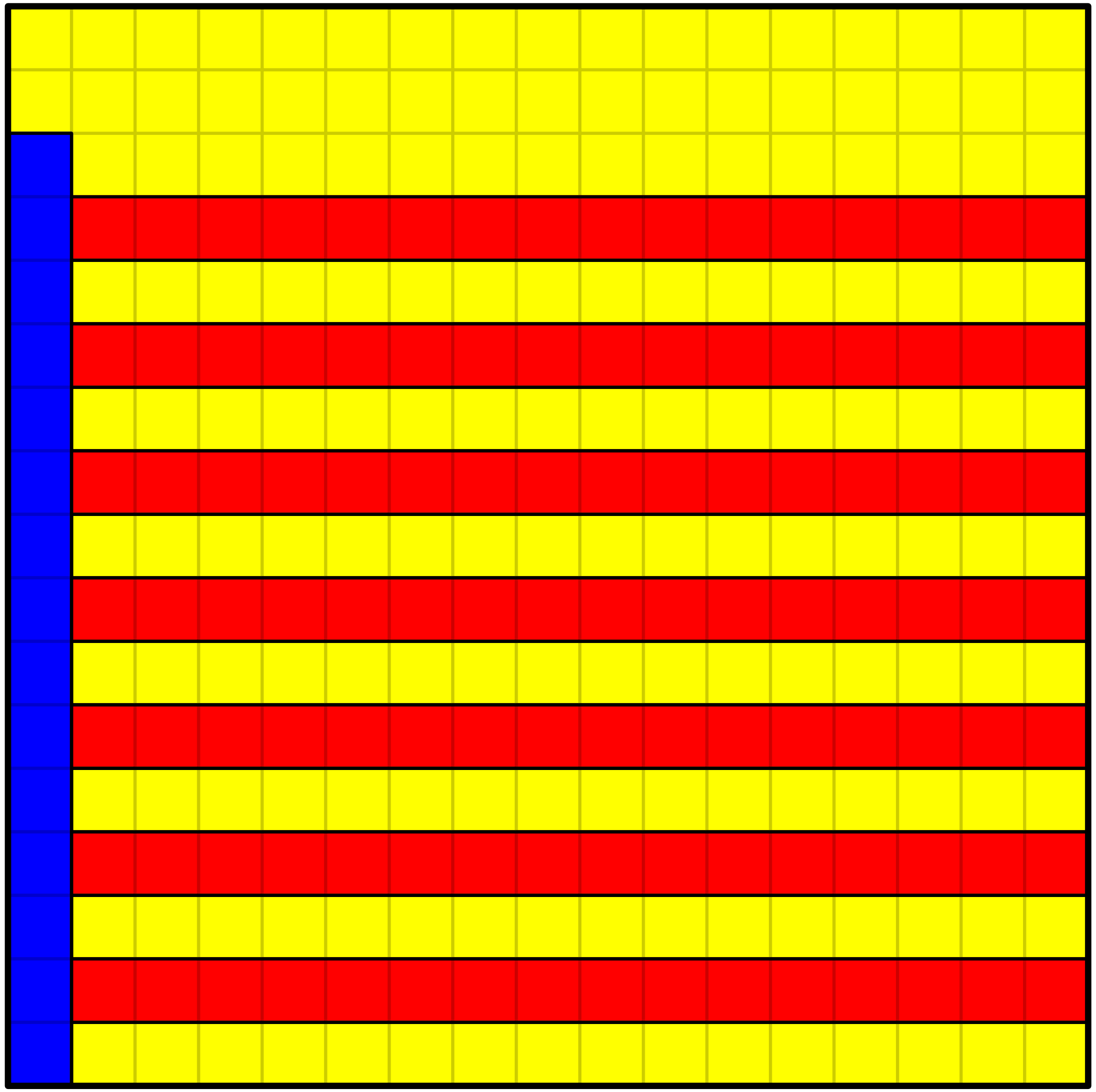

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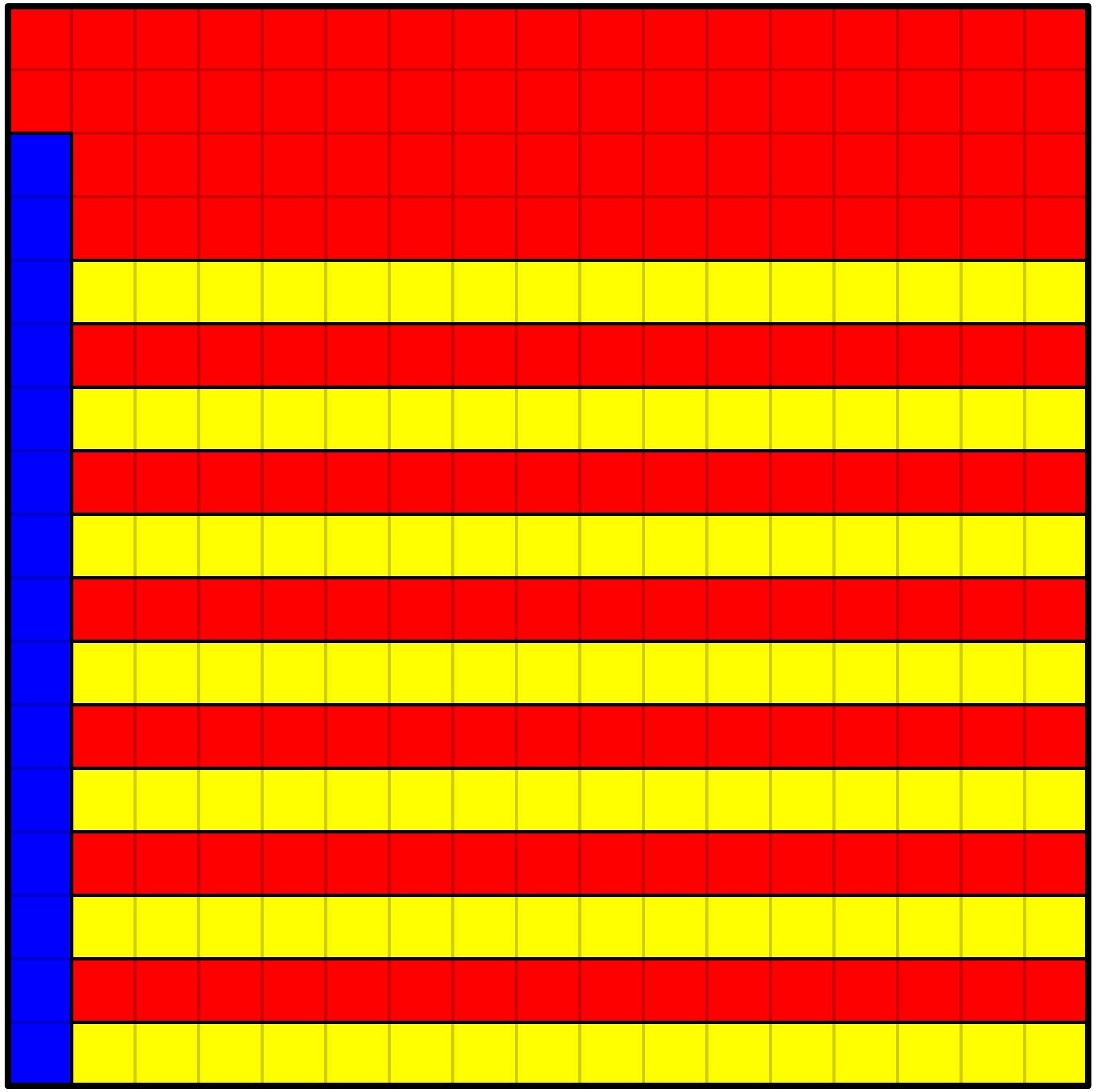

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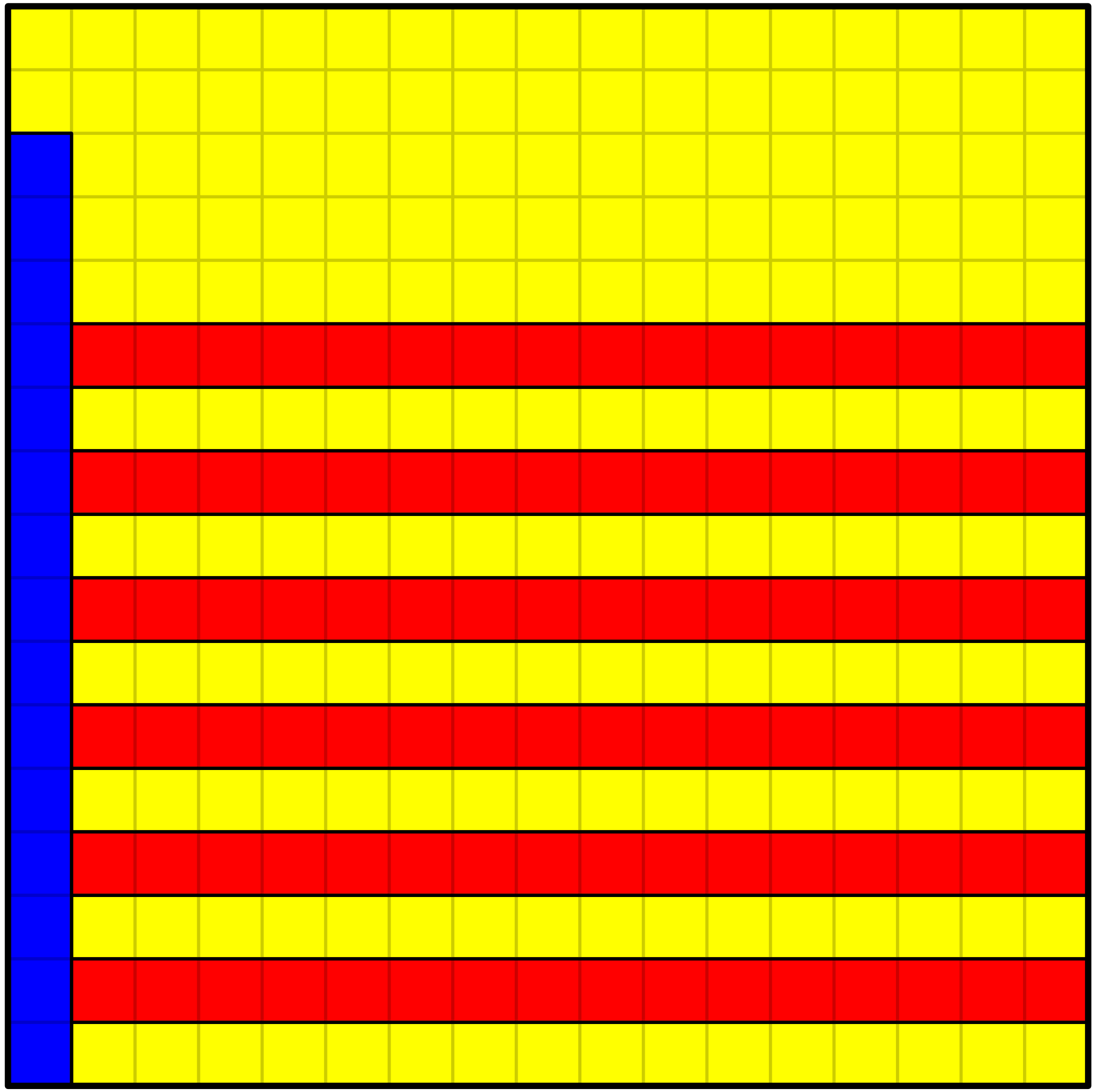

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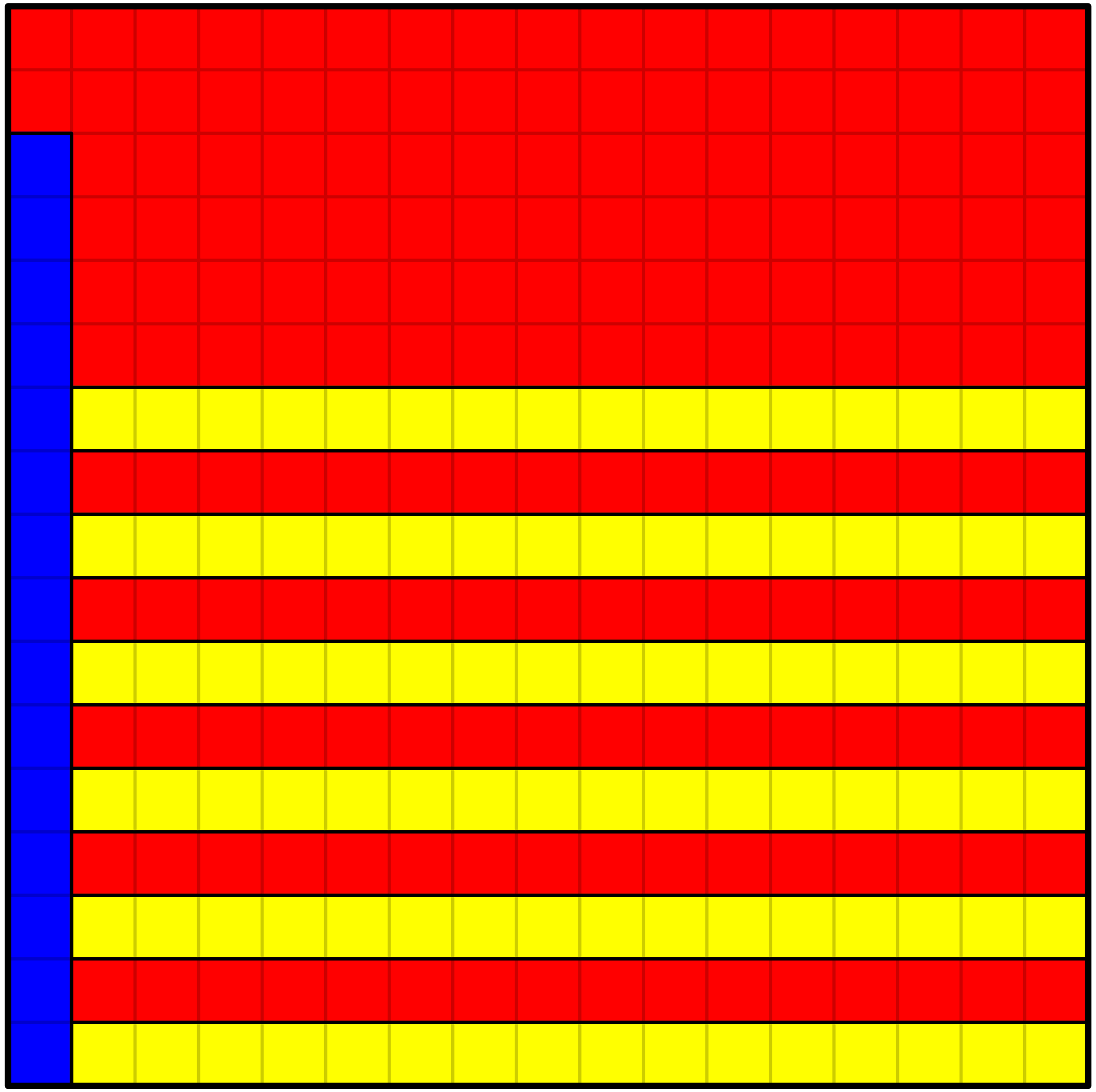

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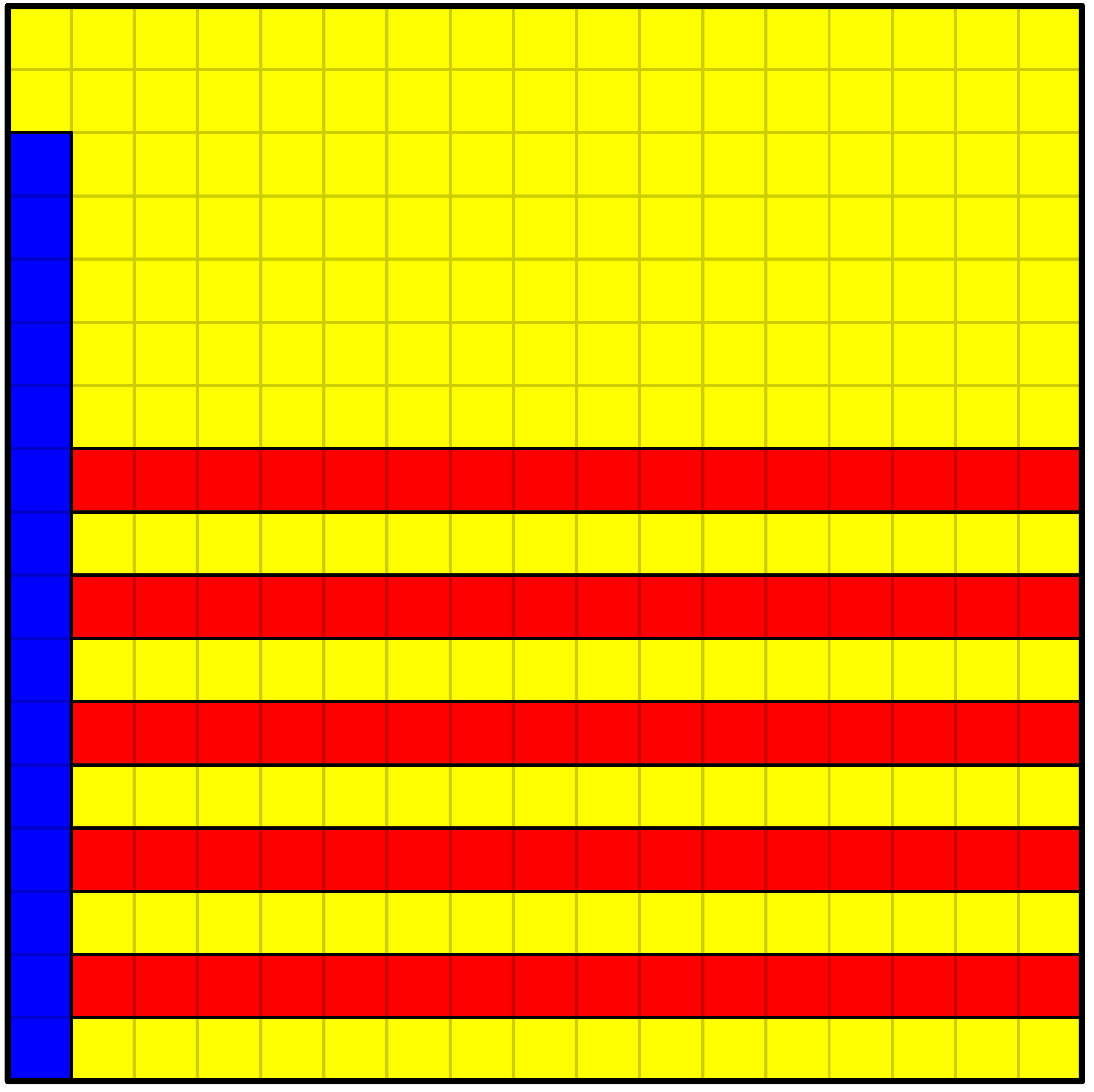

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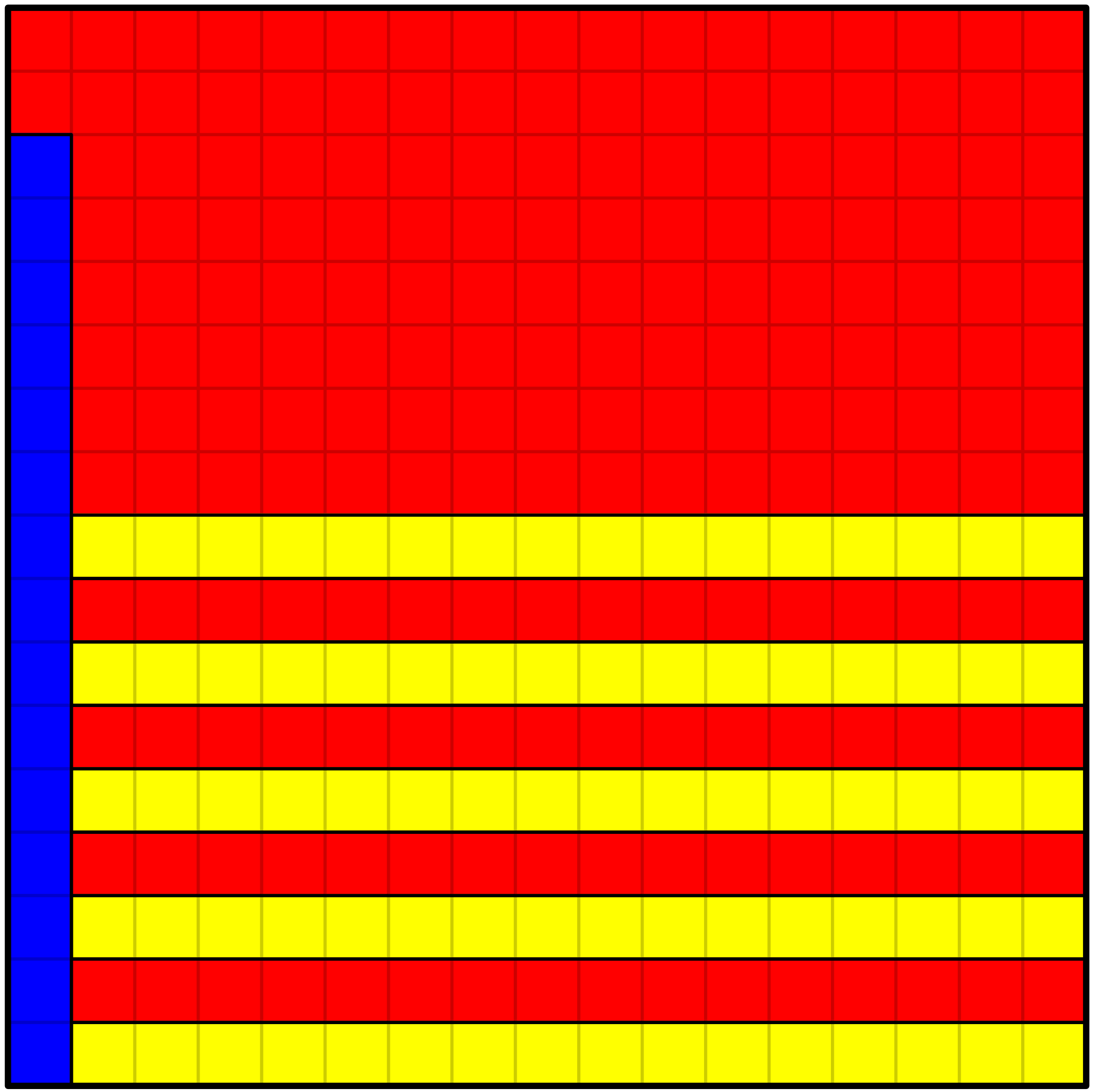

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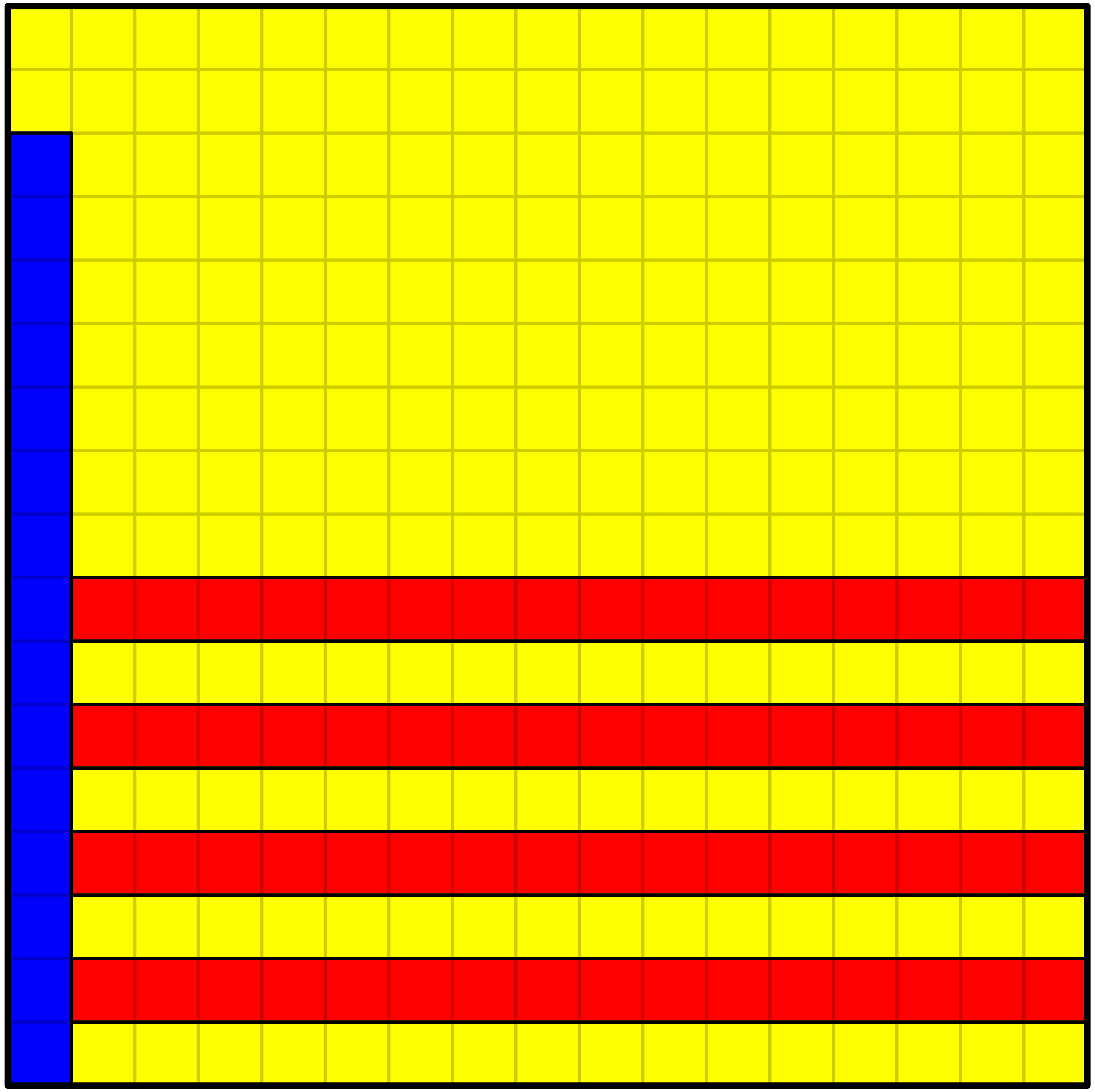

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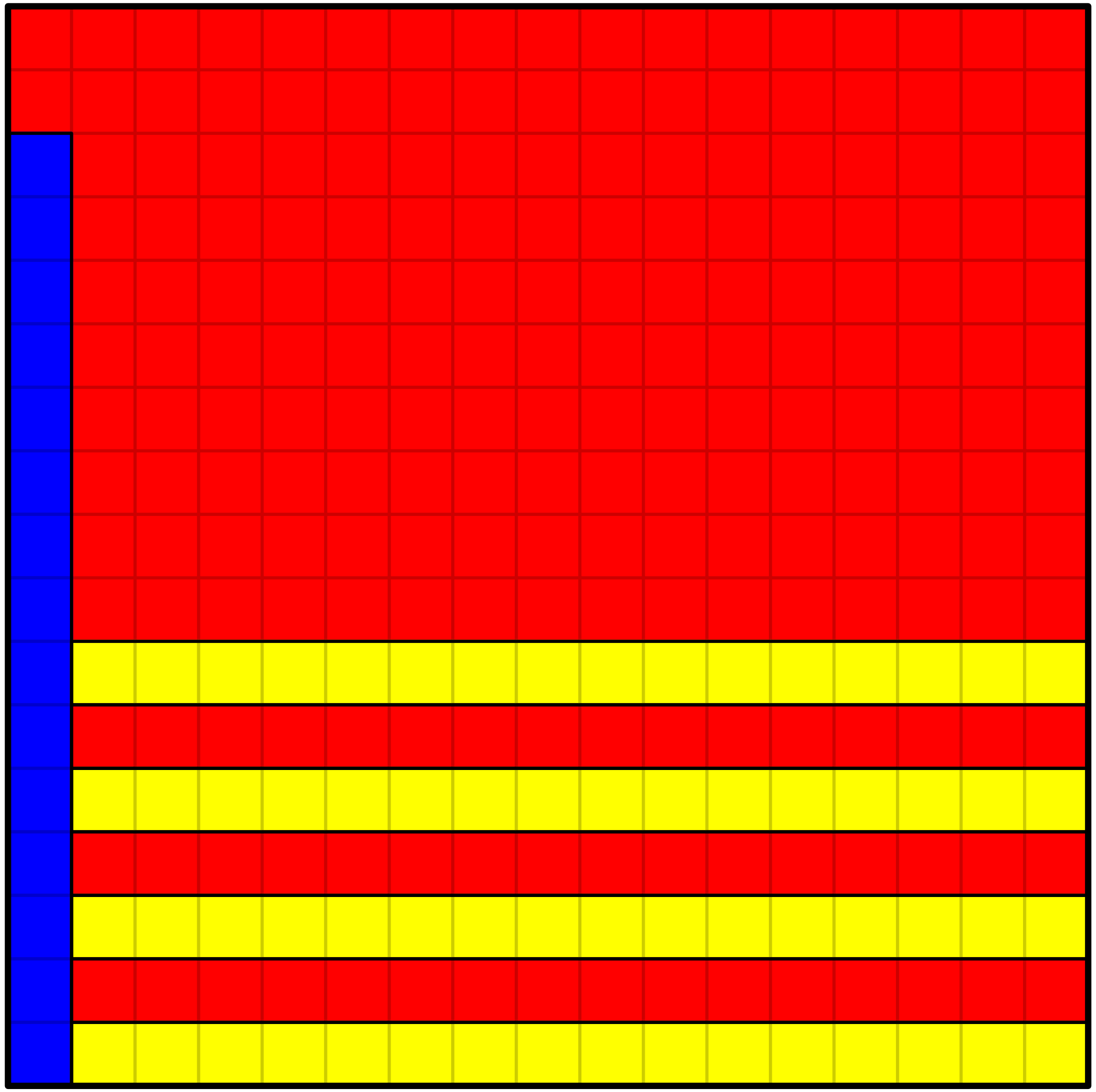

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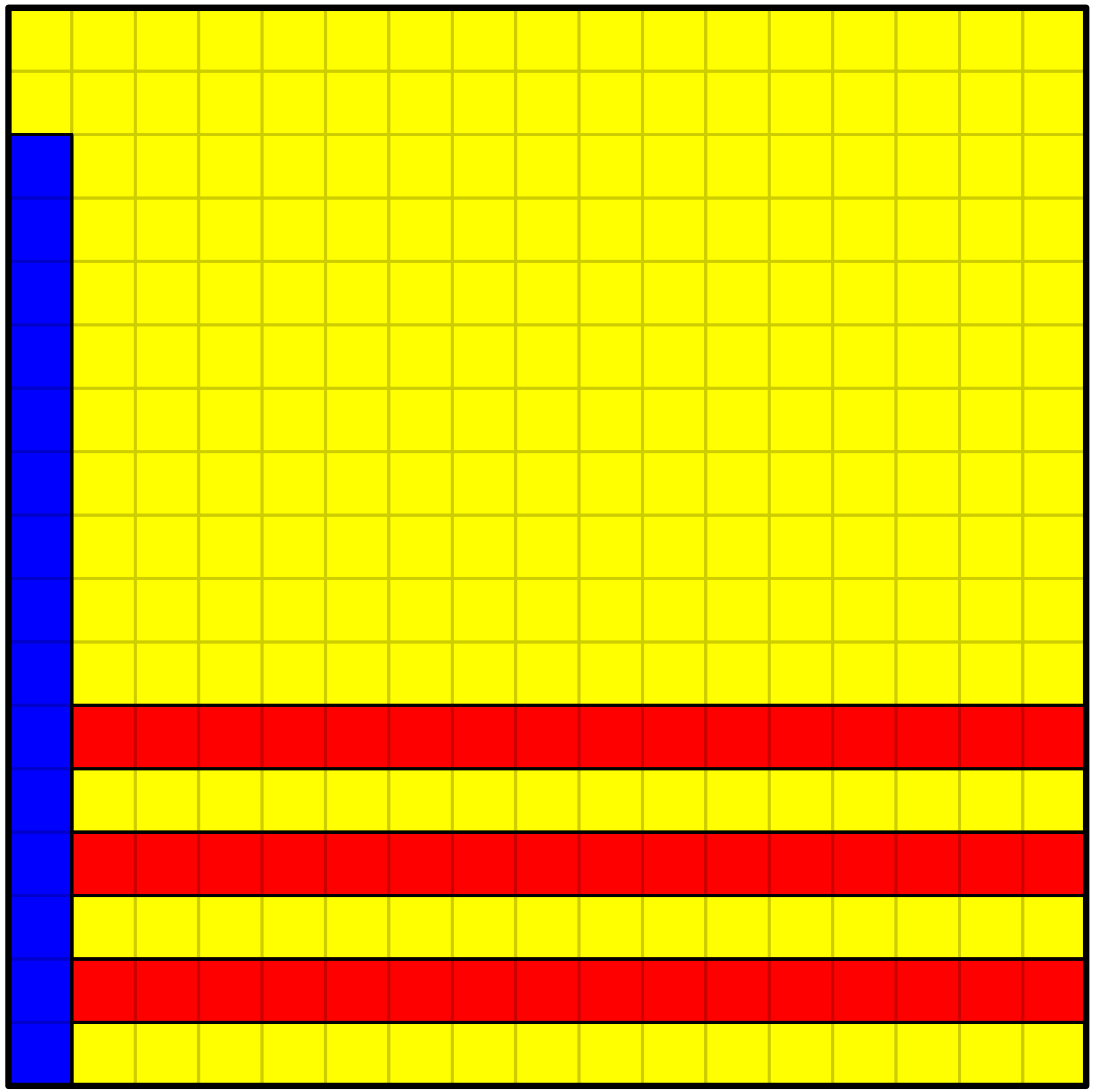

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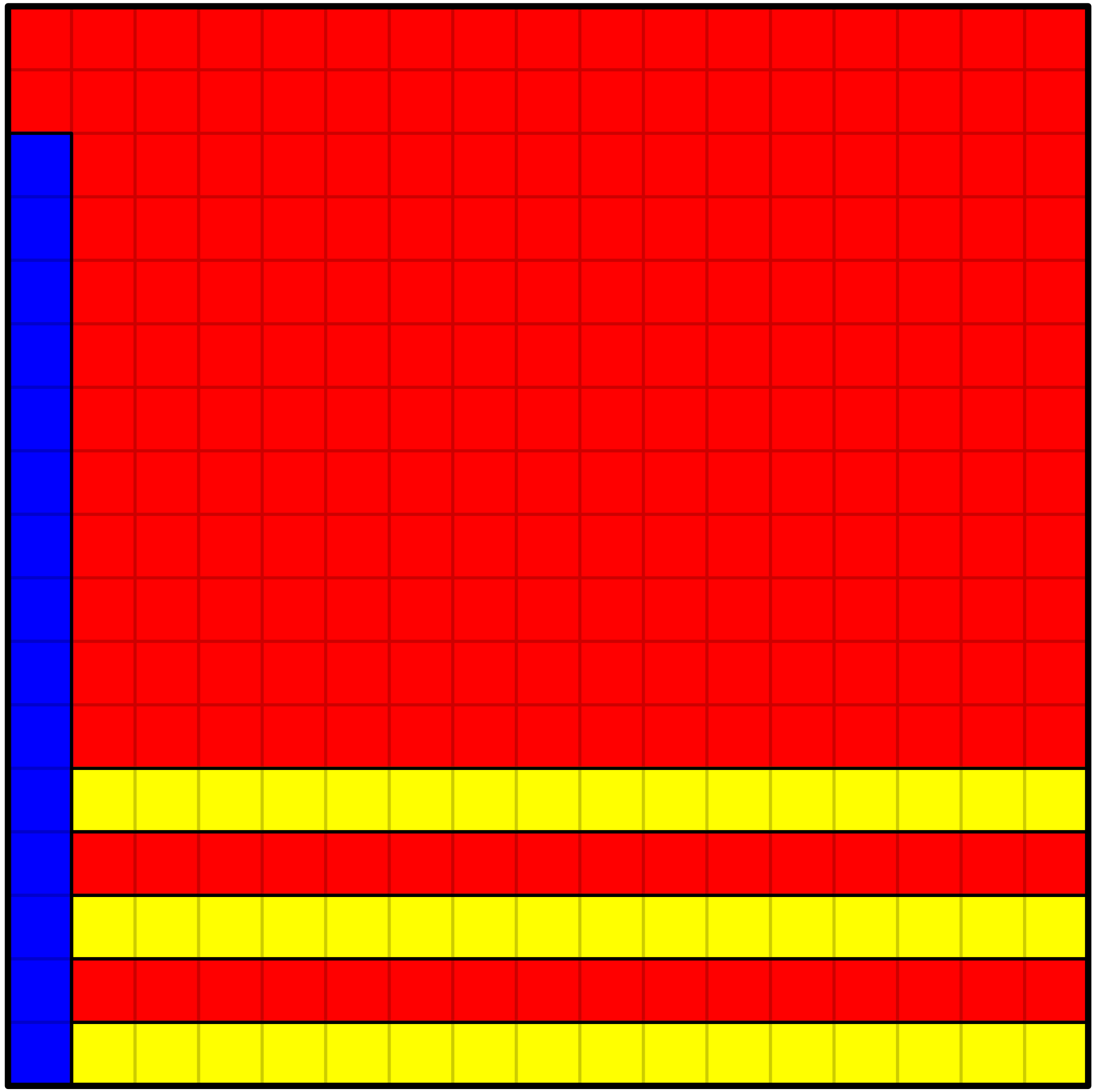

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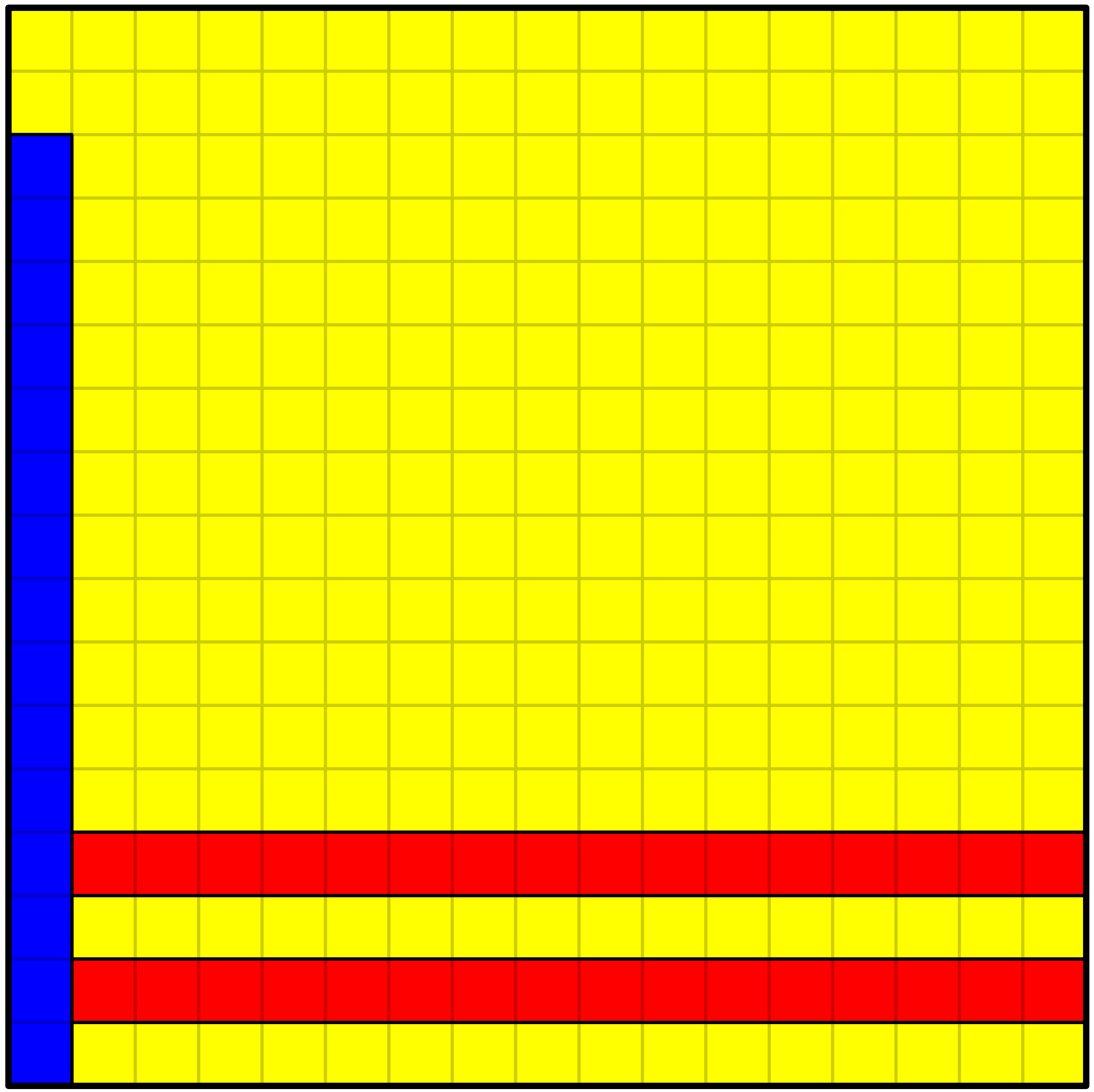

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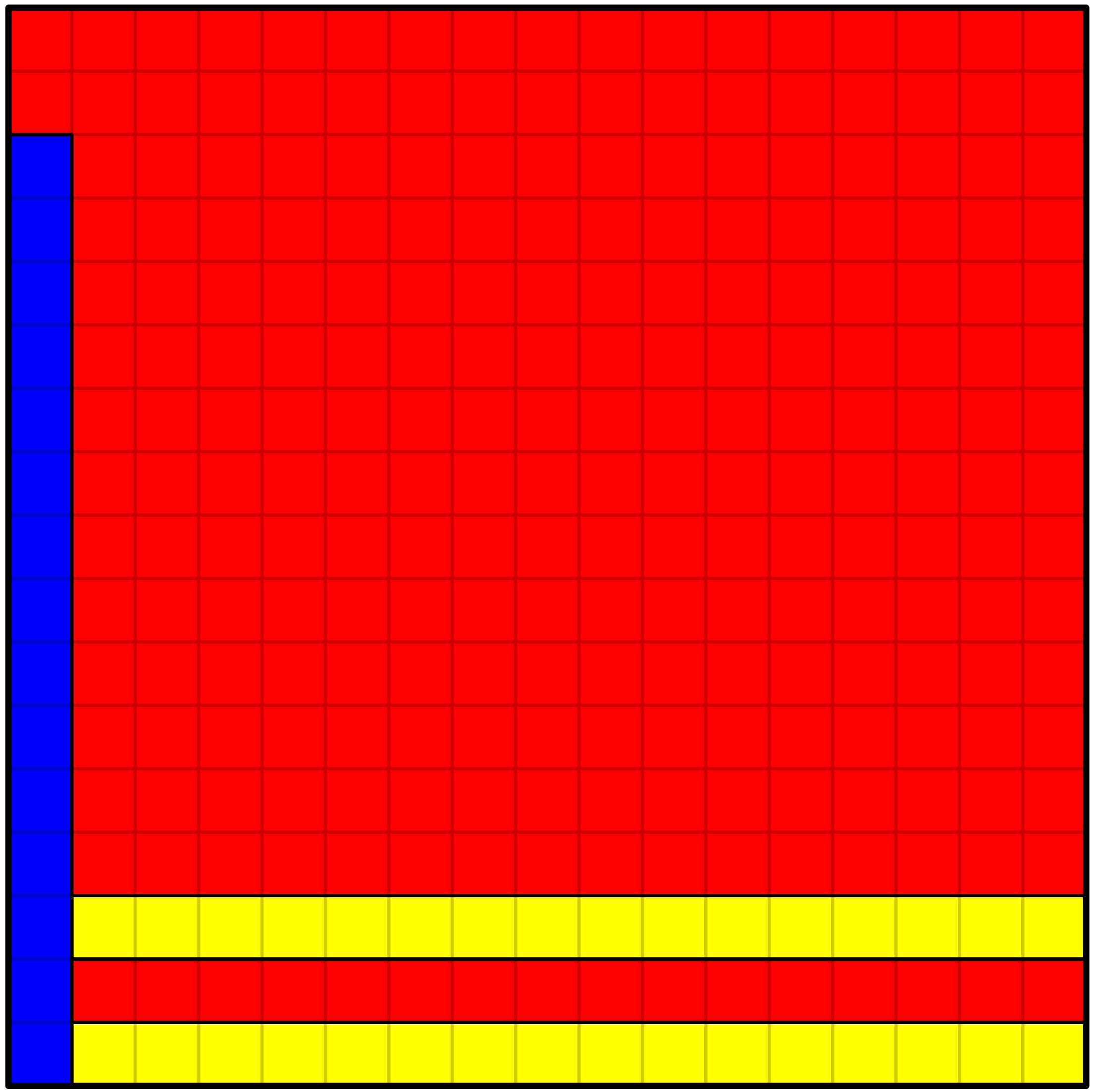

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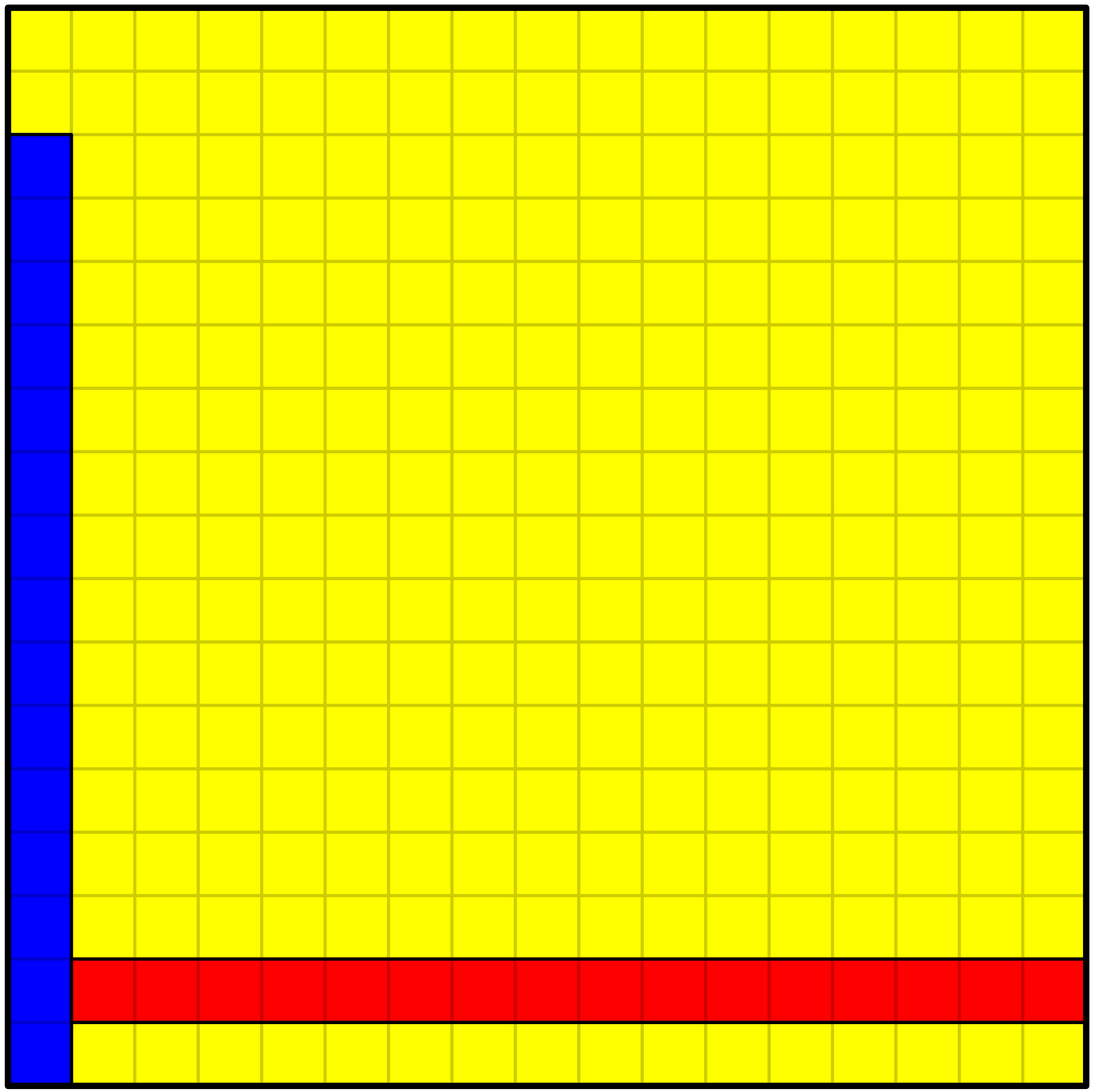

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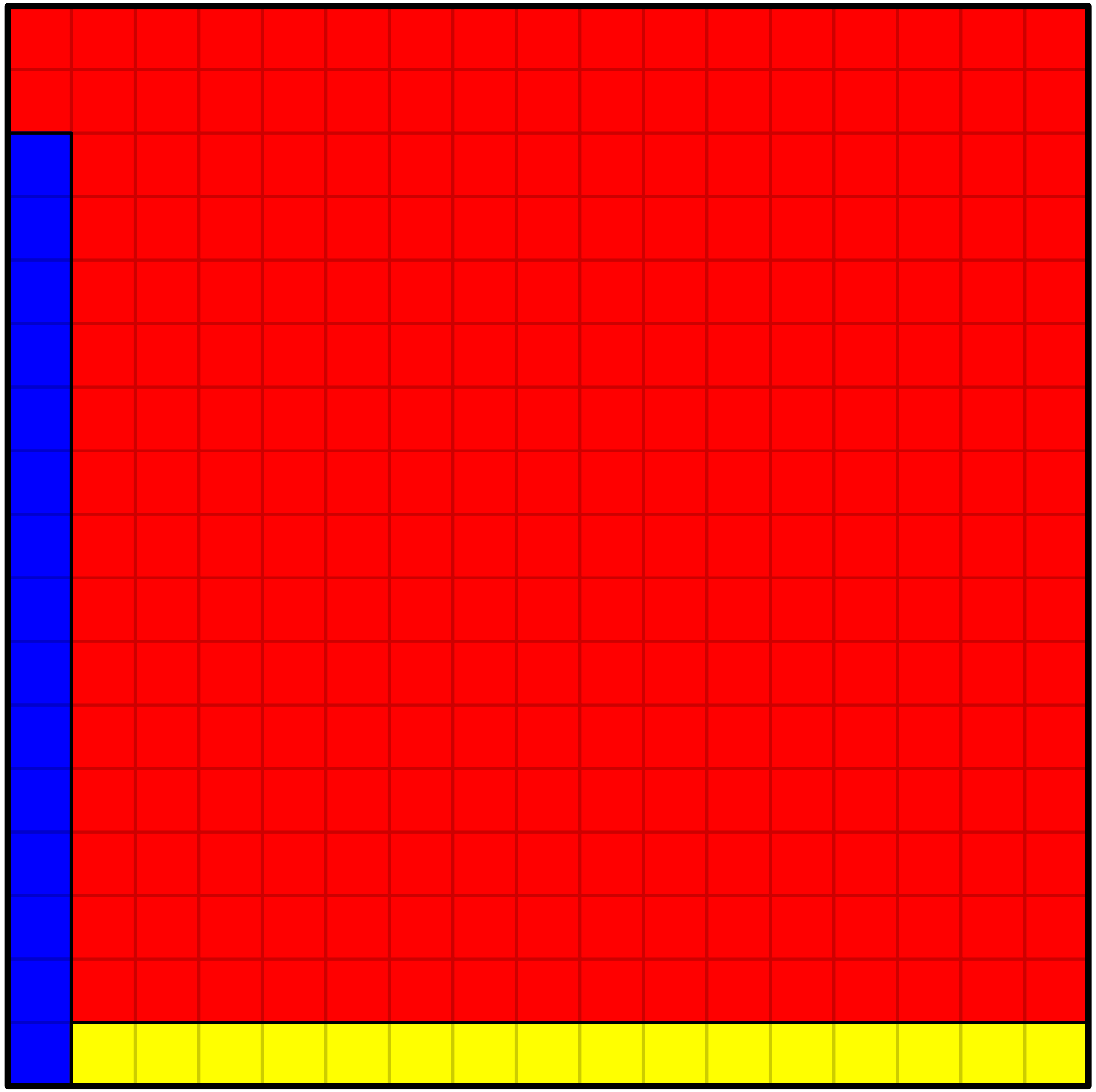

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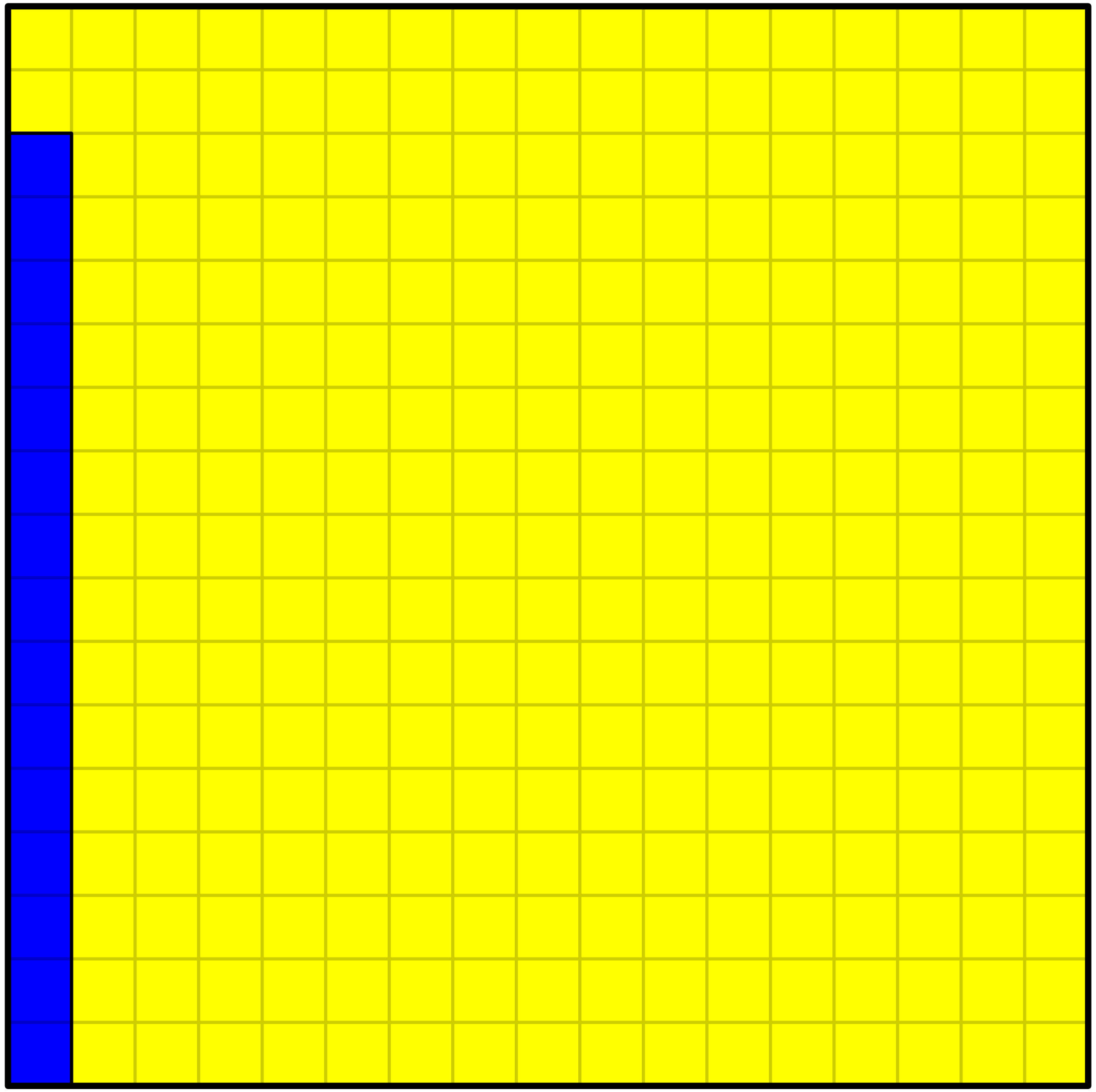

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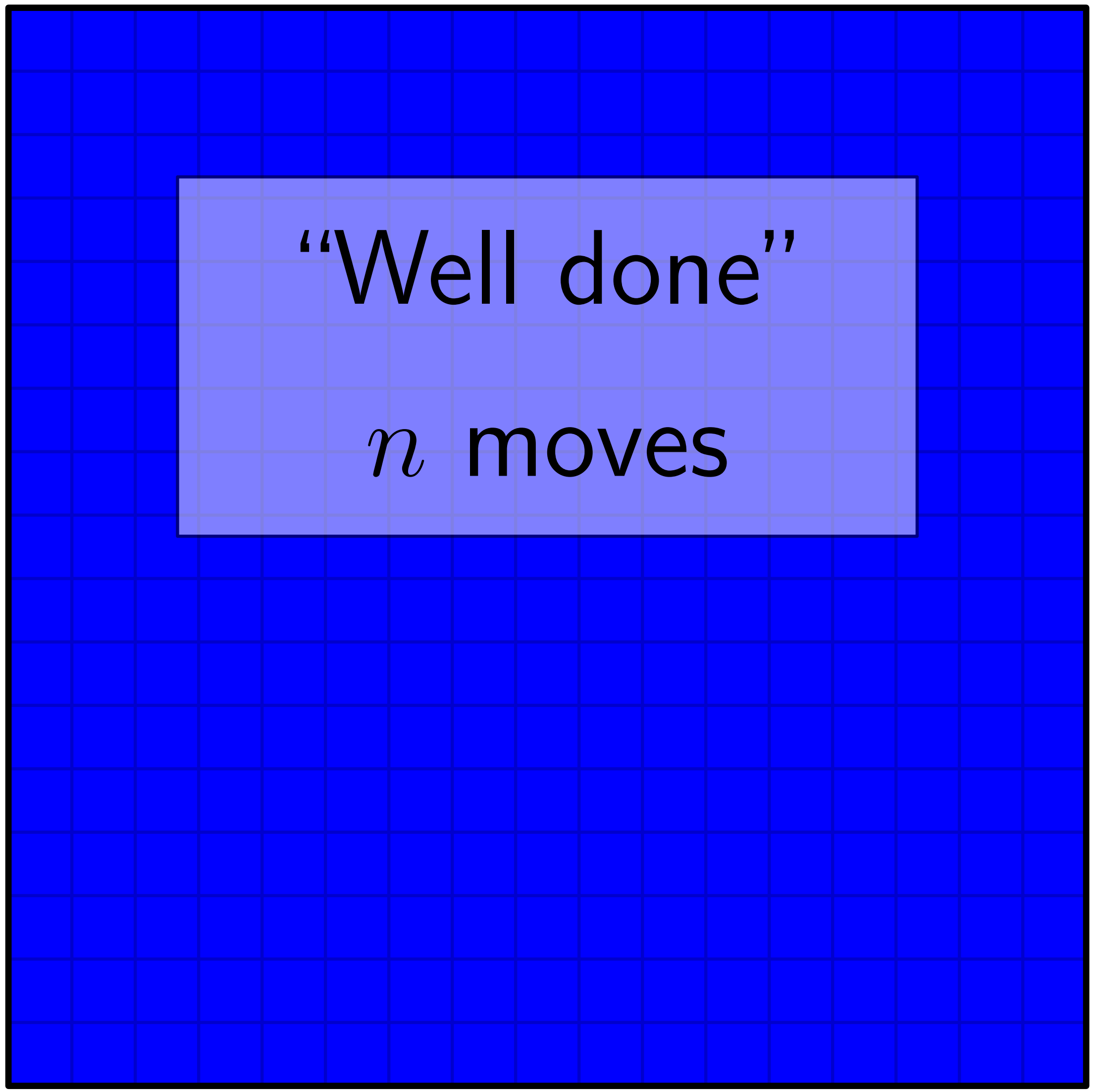

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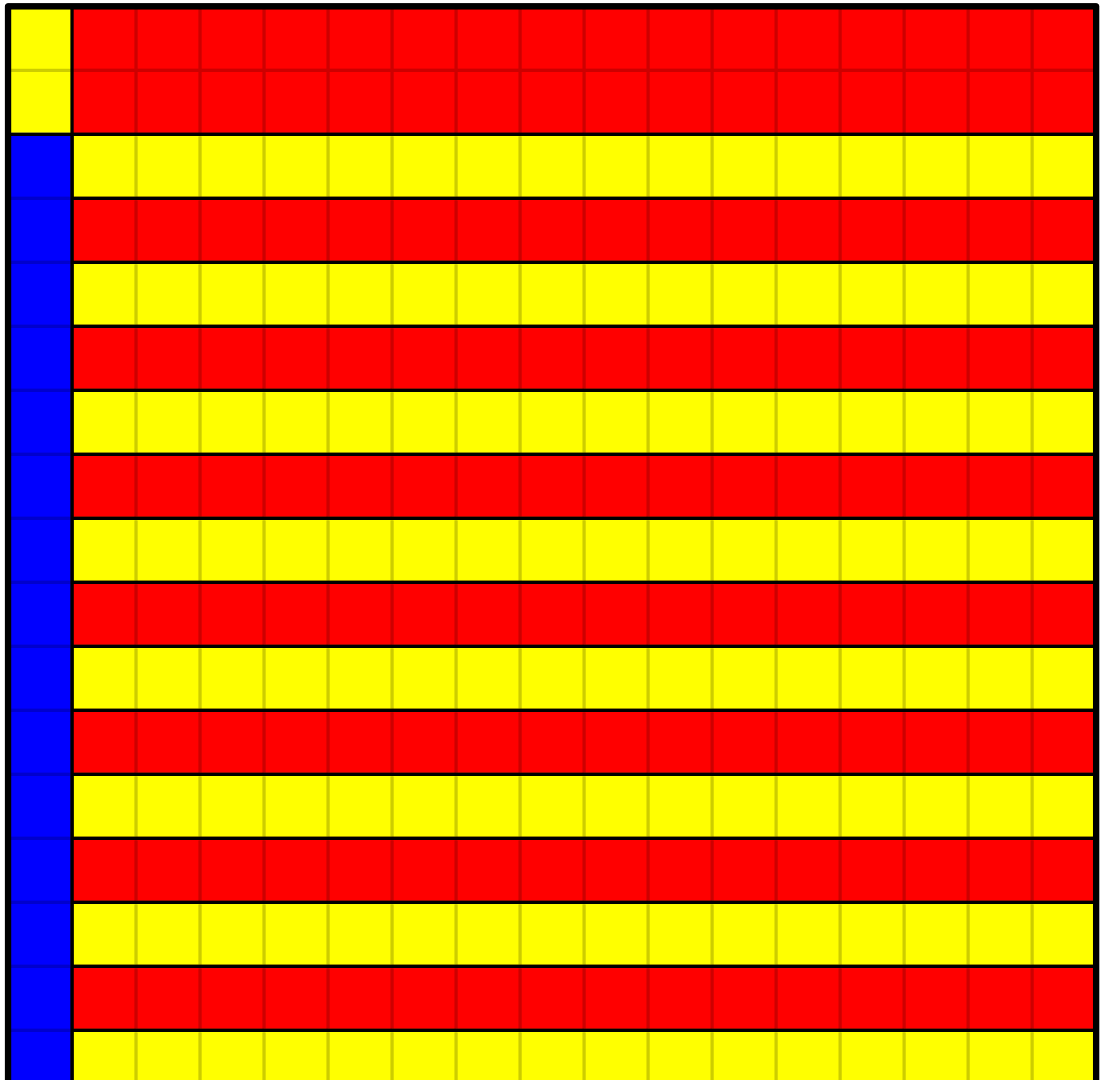

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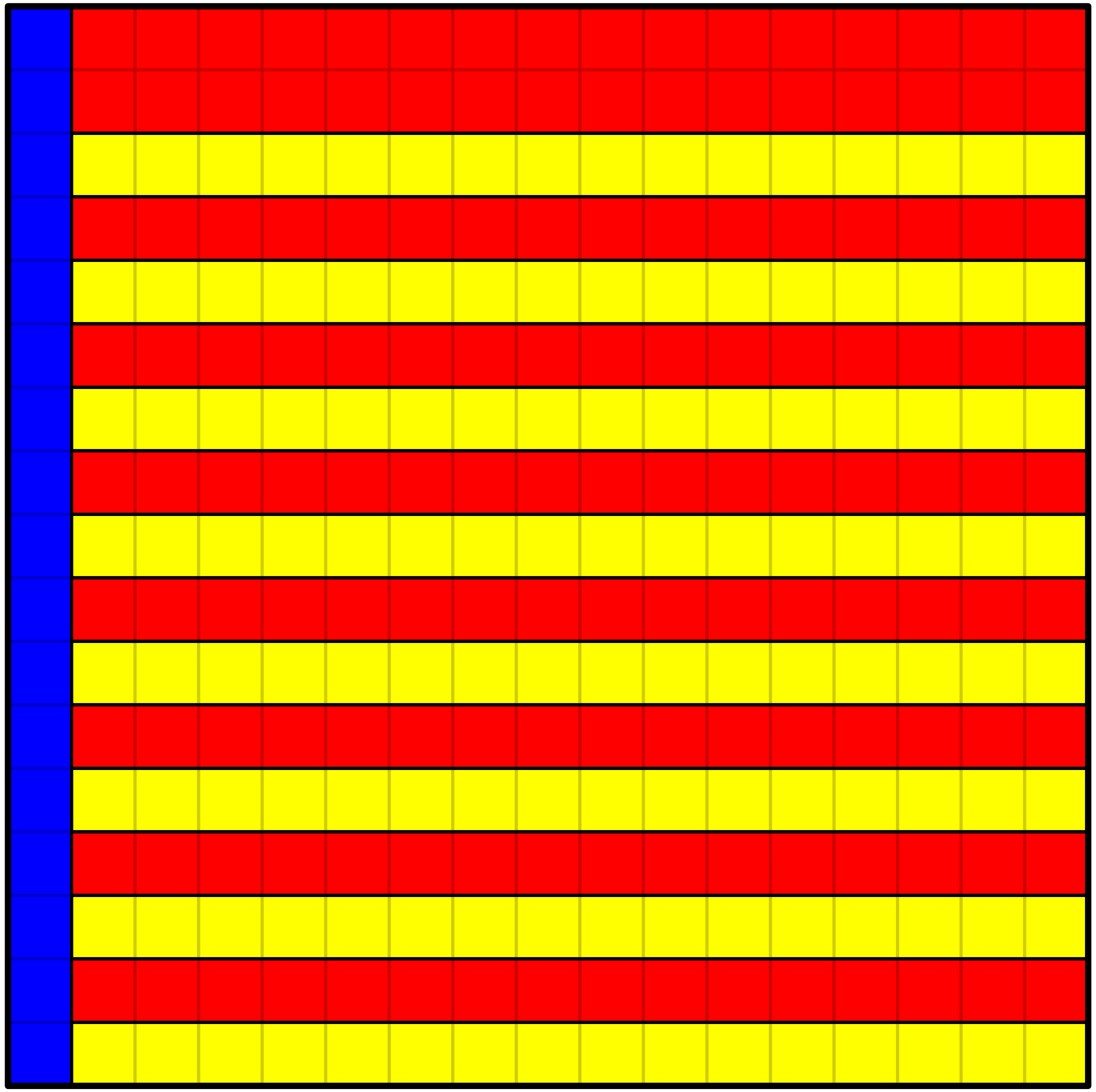

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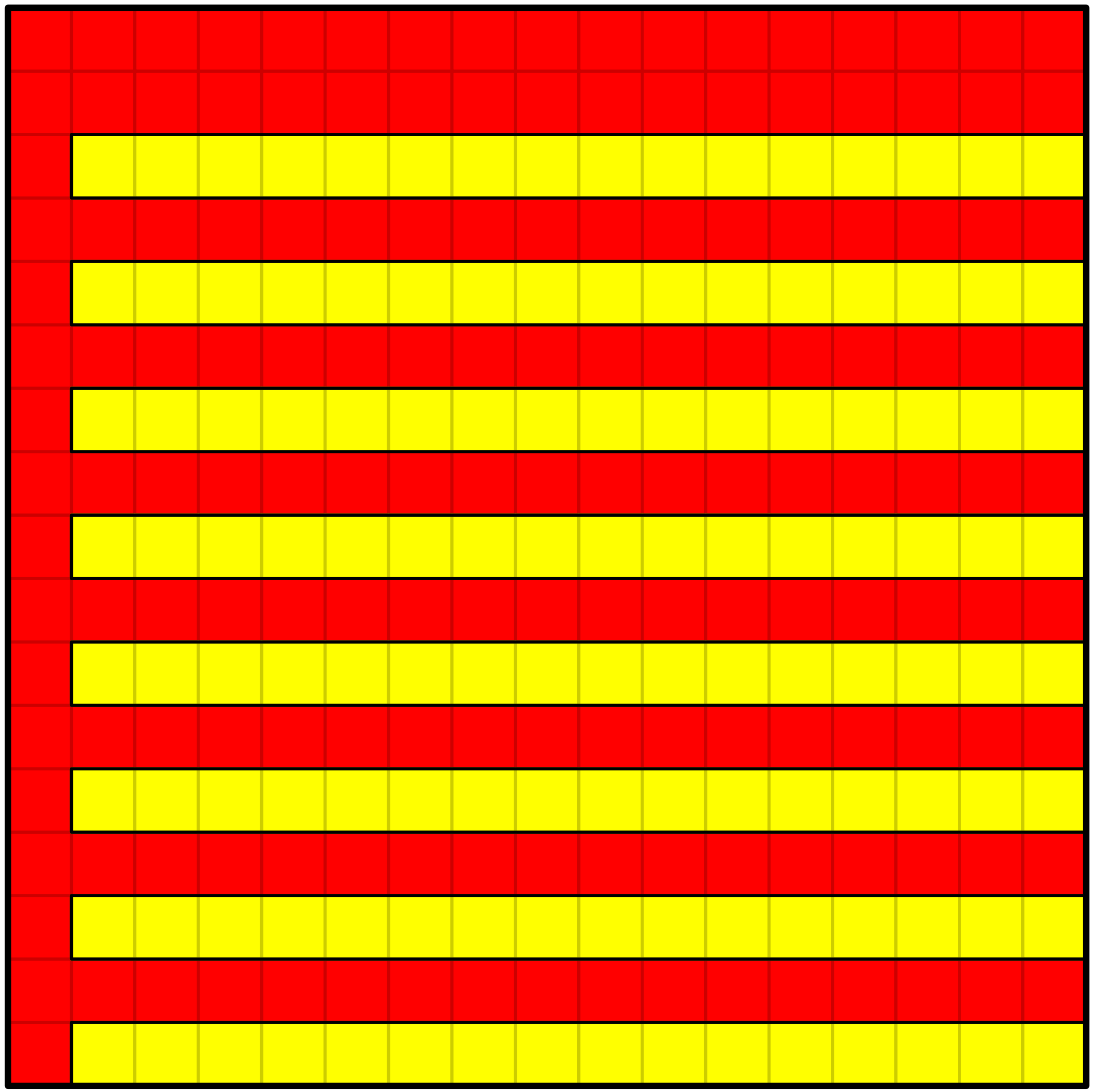

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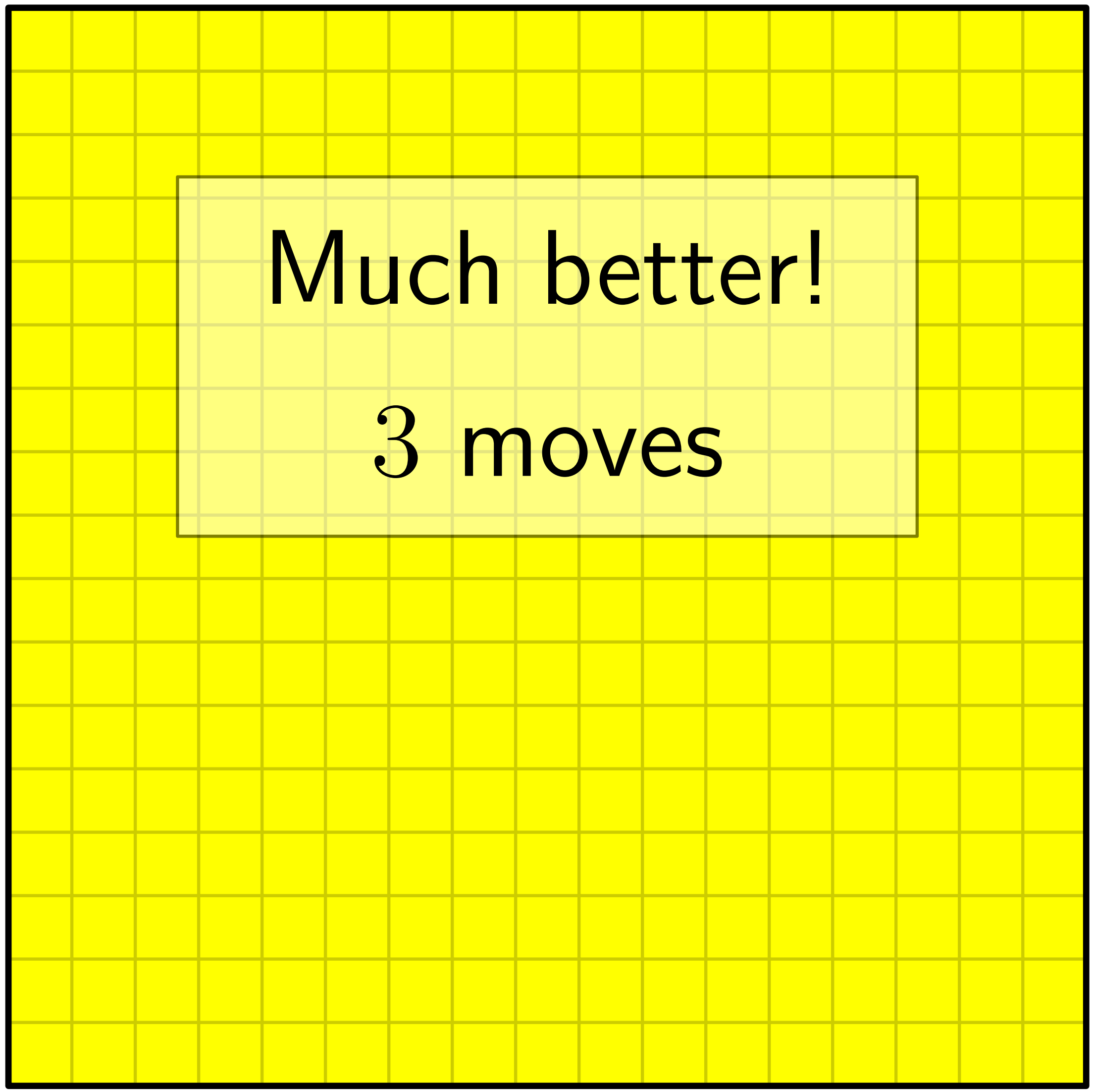
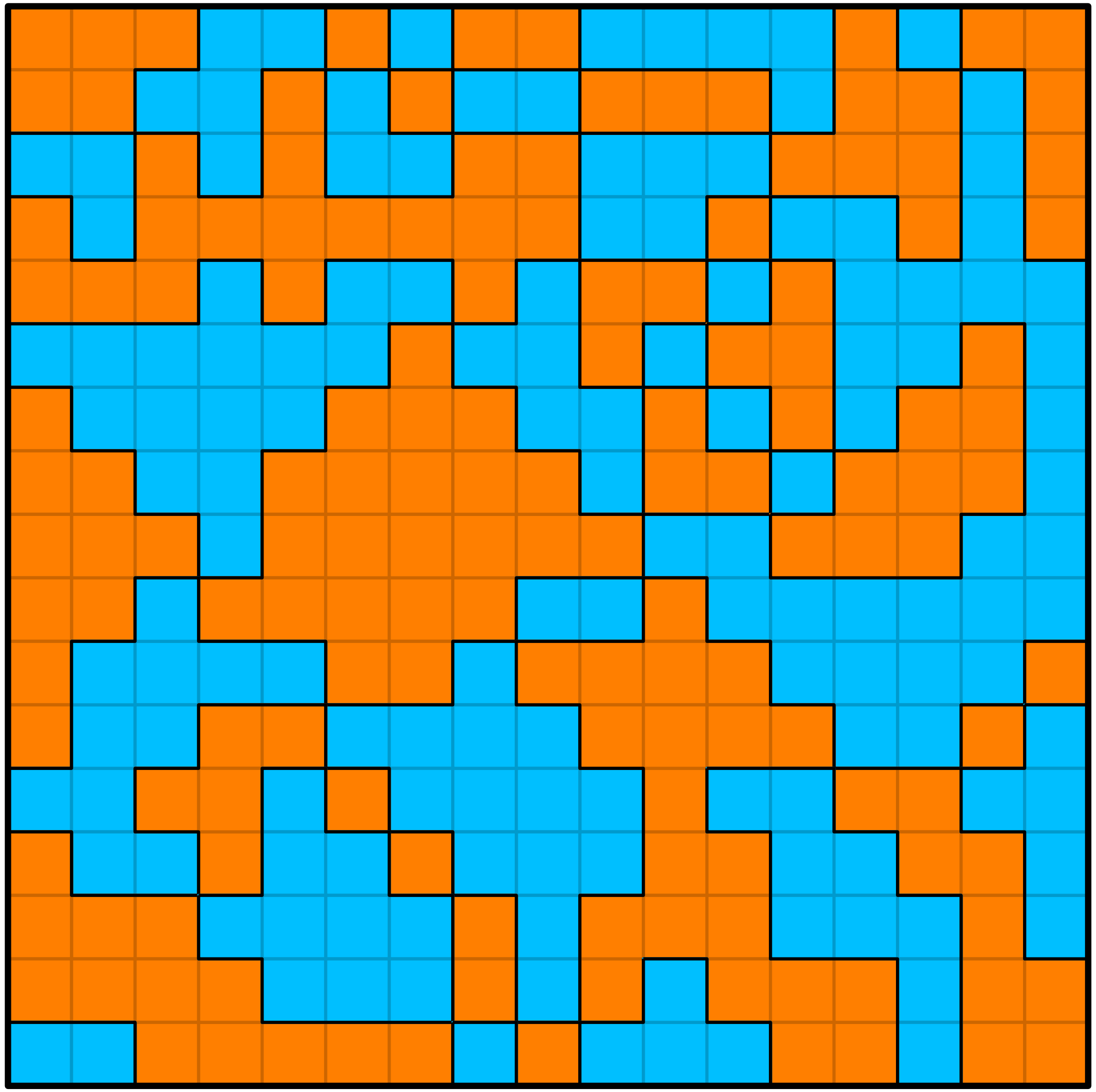


2 Colours



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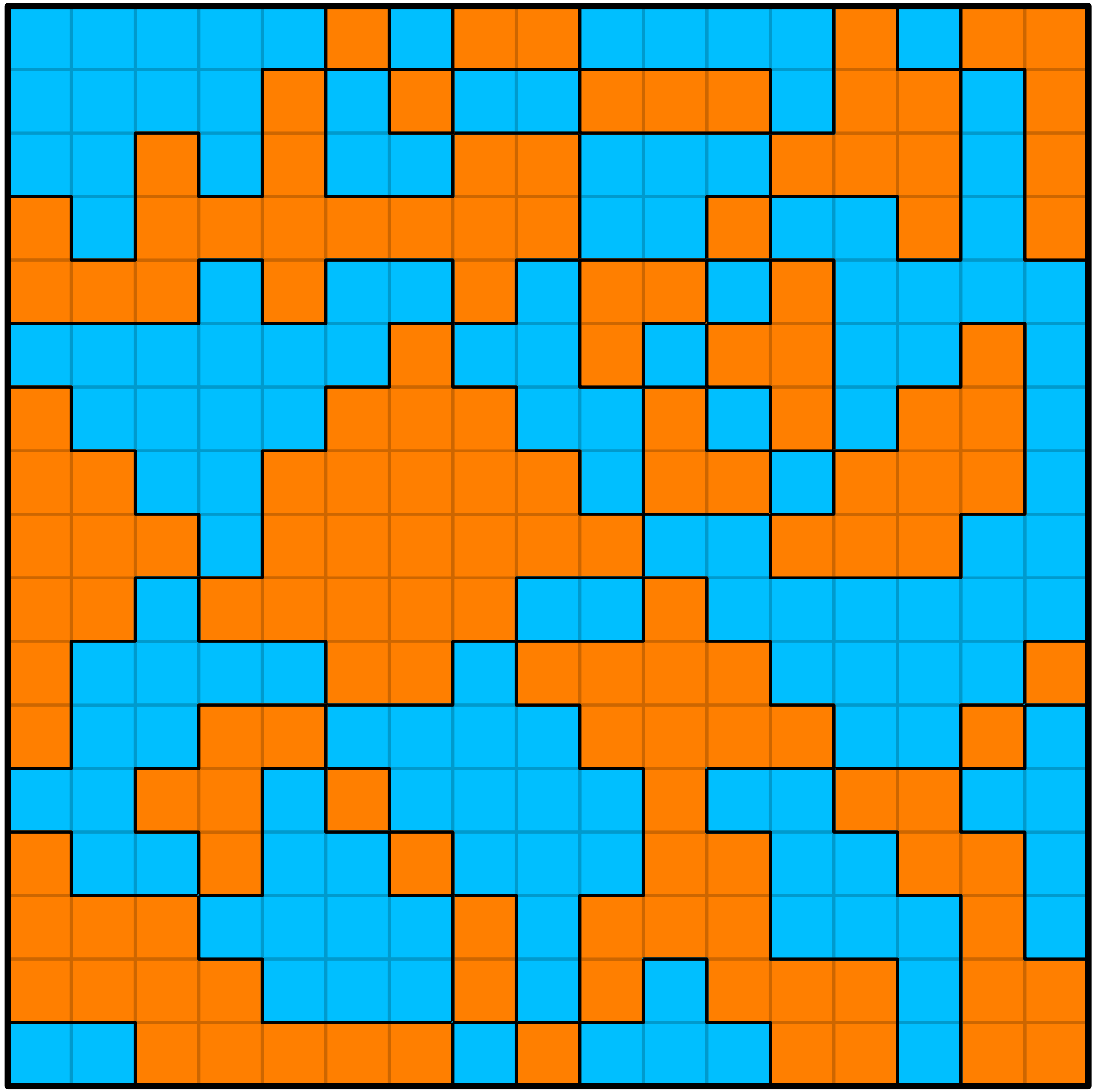

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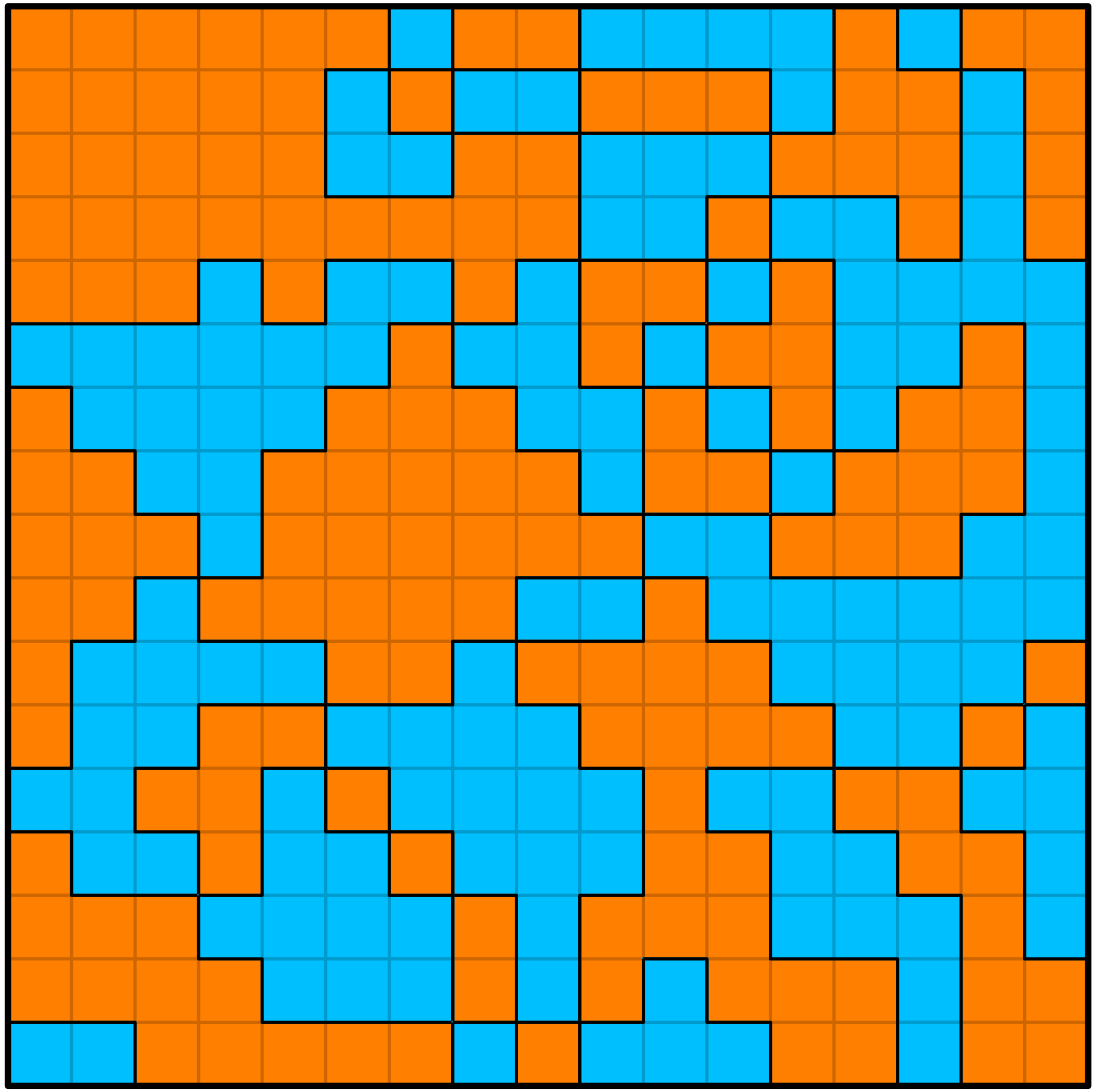

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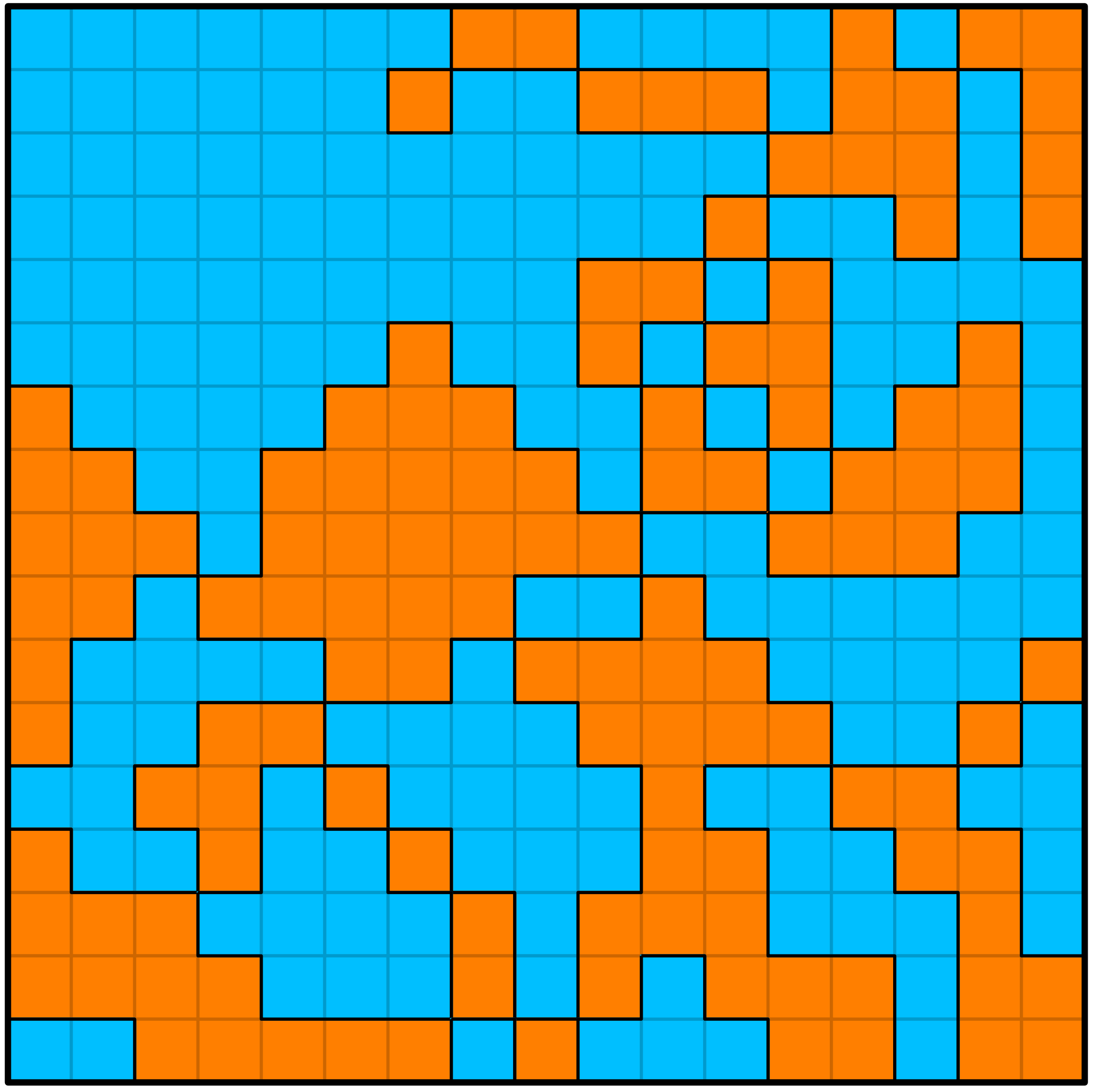

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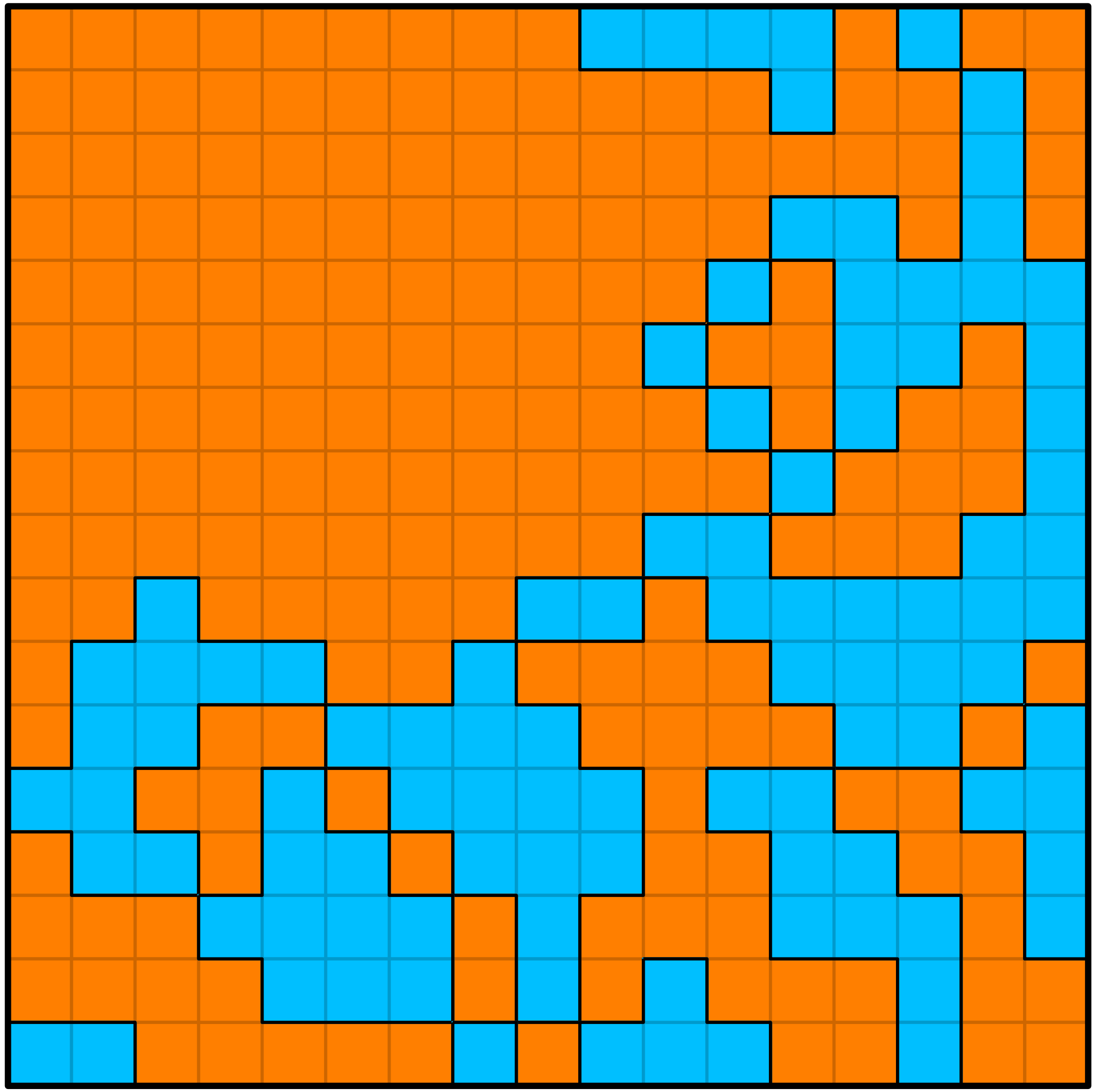

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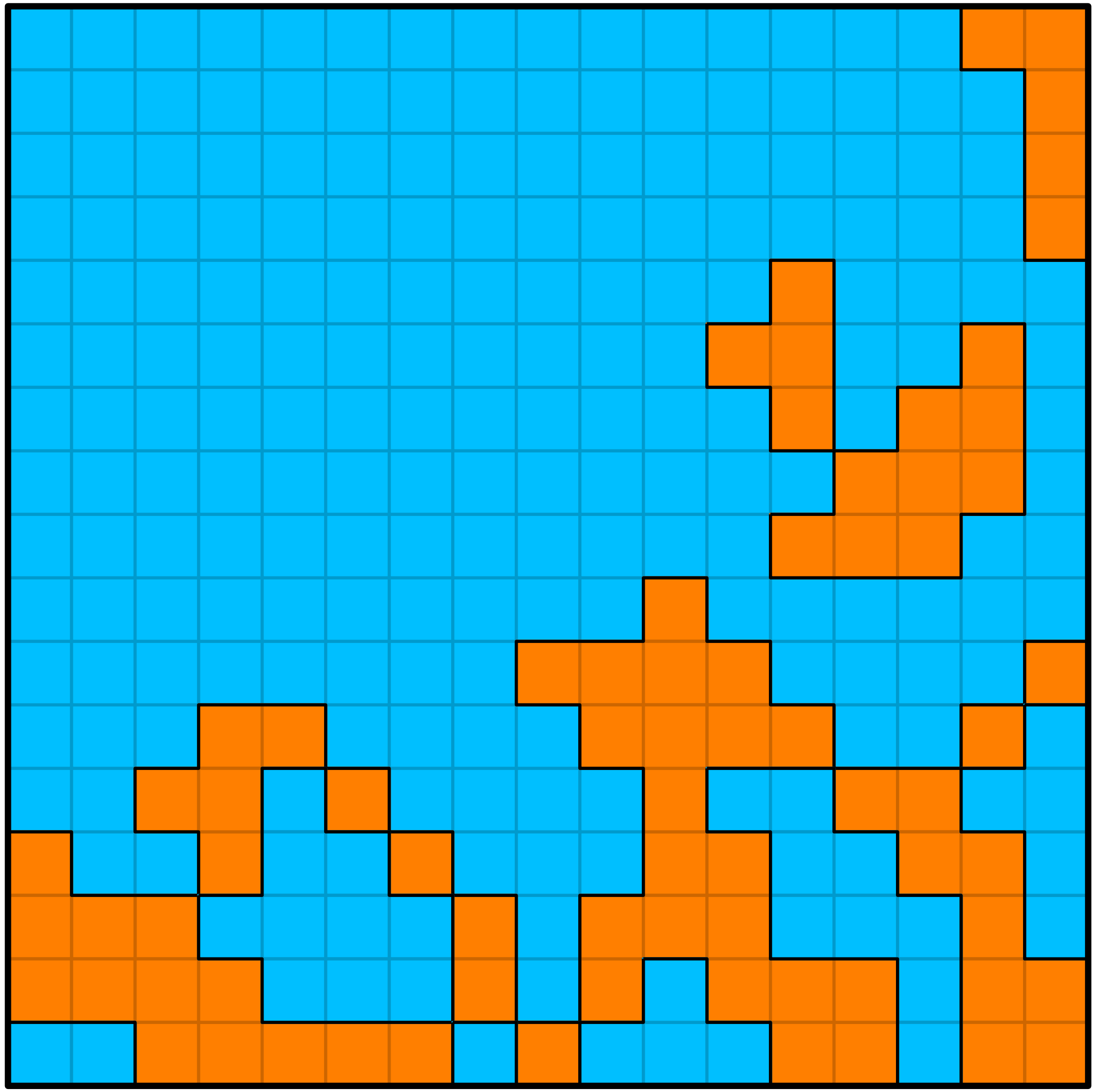

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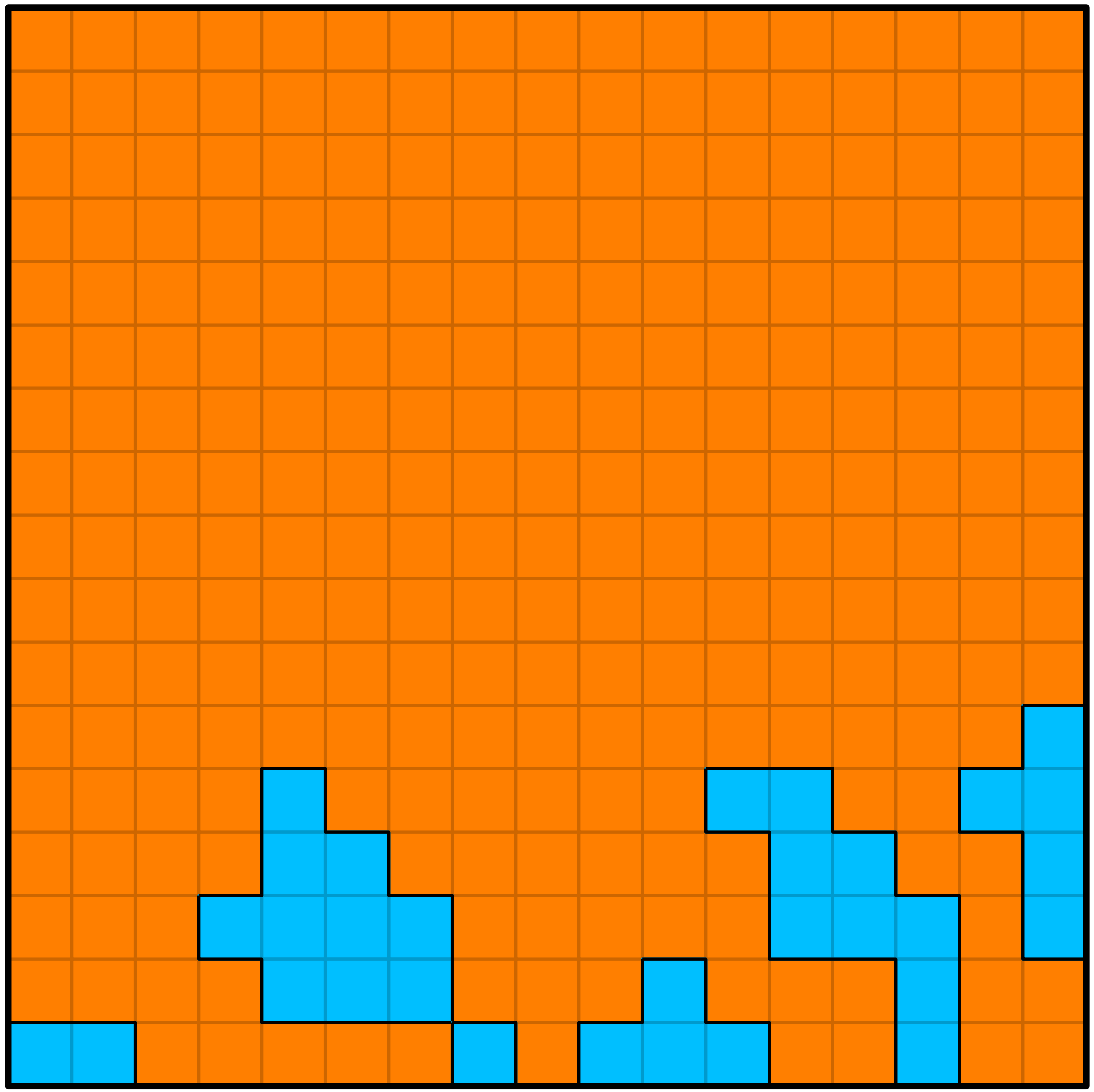

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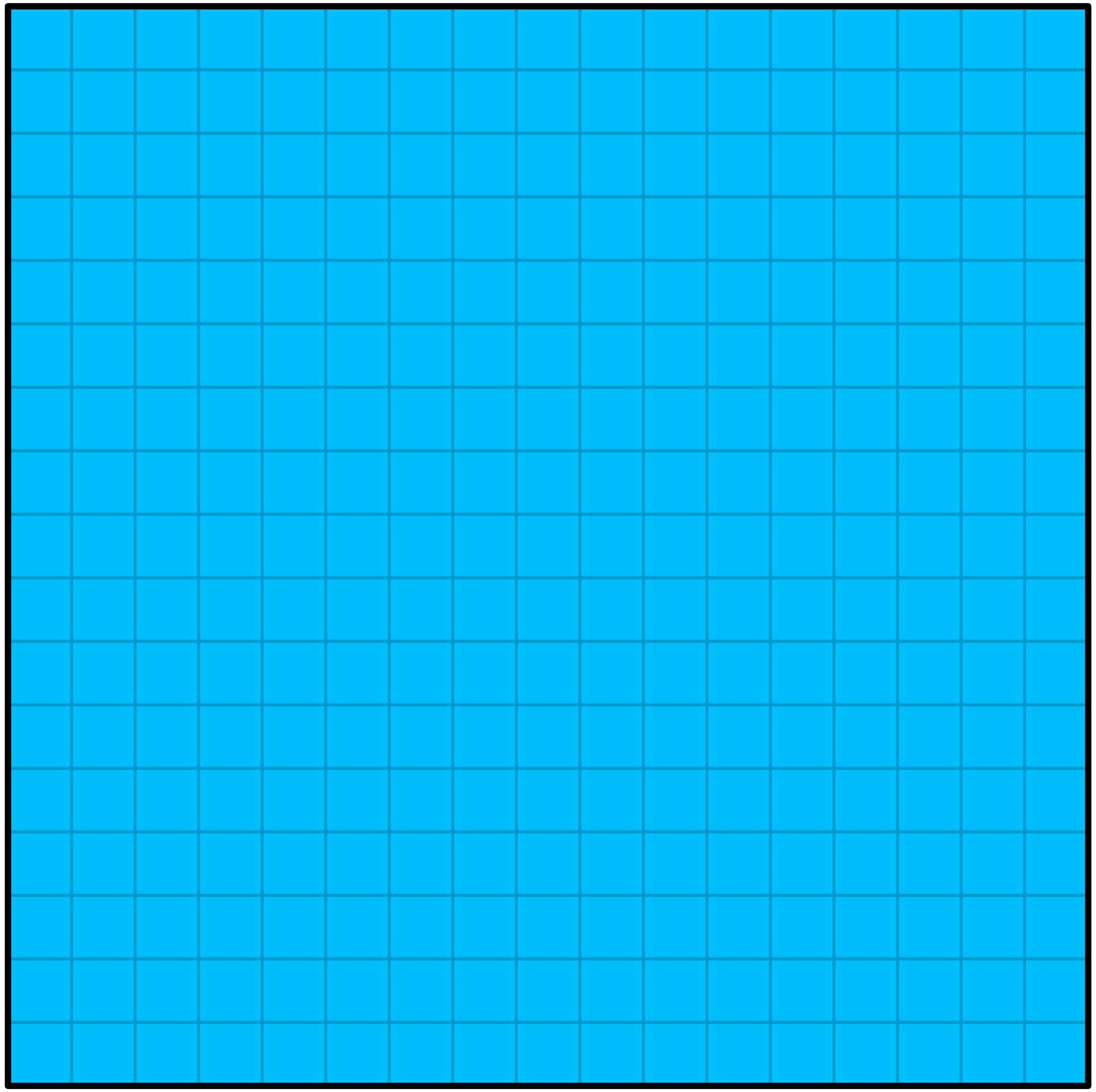


Shortest Common Supersequence

1. $a\ a\ b\ a\ c\ d\ c$
2. $c\ a\ b\ b\ a$
3. $b\ a\ d\ d\ d\ c\ a$
4. $a\ d\ d\ c\ a\ b$
5. $b\ a\ d\ c\ c\ d\ a\ a$
6. $d\ c\ a\ a\ b\ d\ c$
7. $c\ a\ b\ a\ d$
8. $b\ c\ d\ a\ a\ b\ c$

Shortest Common Supersequence

1. $a\ a\ b\ a\ c\ d\ c$
2. $c\ a\ b\ b\ a$
3. $b\ a\ d\ d\ d\ c\ a$
4. $a\ d\ d\ c\ a\ b$
5. $b\ a\ d\ c\ c\ d\ a\ a$
6. $d\ c\ a\ a\ b\ d\ c$
7. $c\ a\ b\ a\ d$
8. $b\ c\ d\ a\ a\ b\ c$

$$a\ a\ b\ c\ a\ d\ c\ a\ d\ a\ c\ b\ c\ a\ b\ d\ c\ a\ a$$

Shortest Common Supersequence

1. $a\ a\ b\ a\ c\ d\ c$
2. $c\ a\ b\ b\ a$
3. $b\ a\ d\ d\ d\ c\ a$
4. $a\ d\ d\ c\ a\ b$
5. $b\ a\ d\ c\ c\ d\ a\ a$
6. $d\ c\ a\ a\ b\ d\ c$
7. $c\ a\ b\ a\ d$
8. $\boxed{b}\boxed{c}\boxed{d}\boxed{a}\boxed{a}\boxed{b}\boxed{c}$

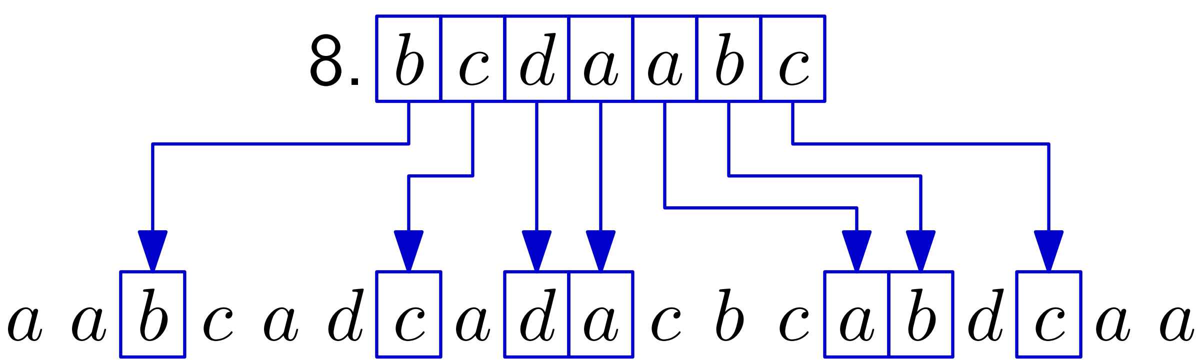
$$a\ a\ \boxed{b}\ c\ a\ d\ \boxed{c}\ a\ \boxed{d}\boxed{a}\ c\ b\ c\ \boxed{a}\boxed{b}\ d\ \boxed{c}\ a\ a$$

Shortest Common Supersequence

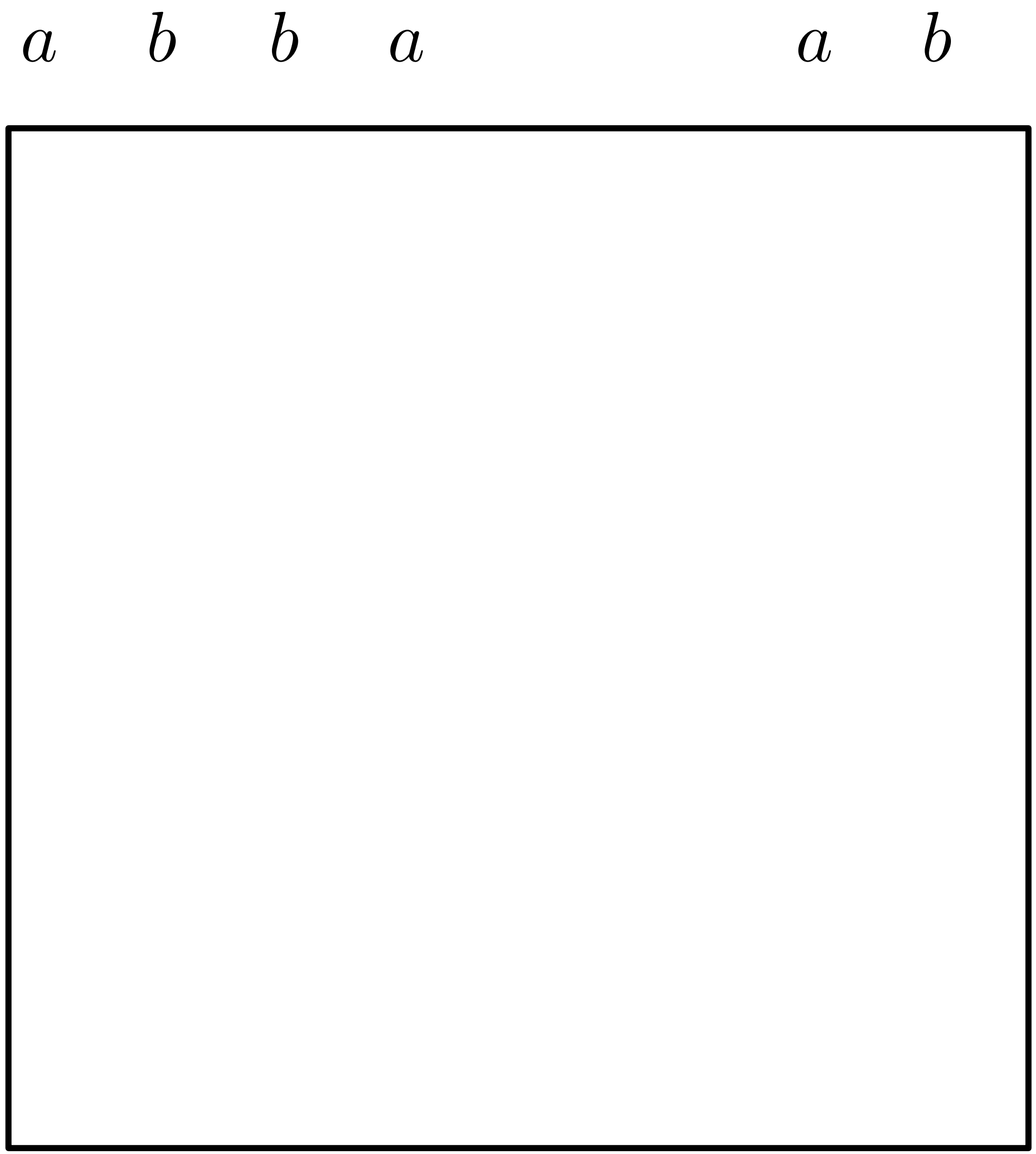
1. $a\ a\ b\ a\ c\ d\ c$
2. $c\ a\ b\ b\ a$

- **NP**-hard, even with a binary alphabet,
- no polynomial-time constant factor approximation algorithm, unless $\mathbf{P} = \mathbf{NP}$.

6. $d\ c\ a\ a\ b\ d\ c$
7. $c\ a\ b\ a\ d$
8. $b\ c\ d\ a\ a\ b\ c$



4 or More Colours



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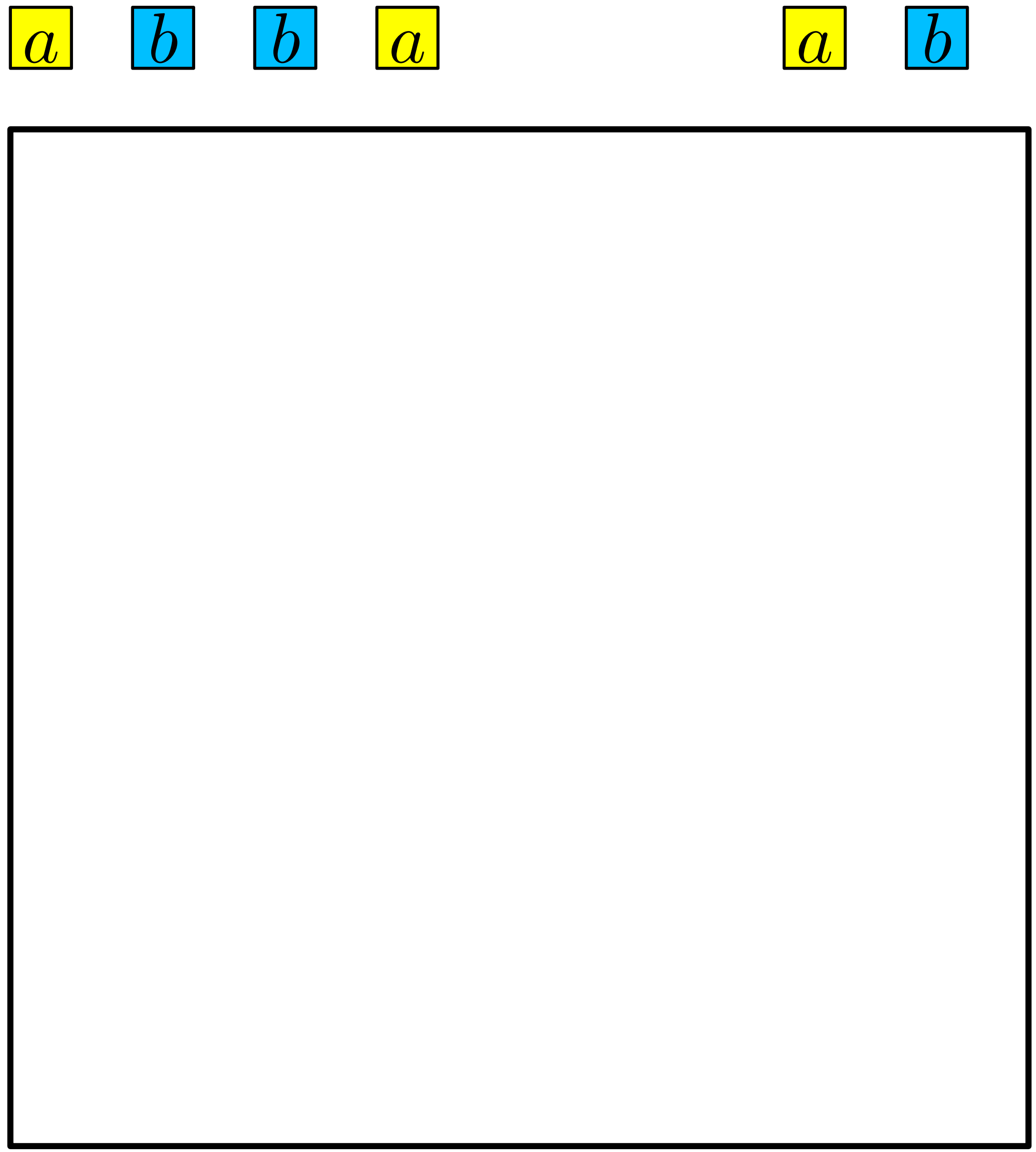

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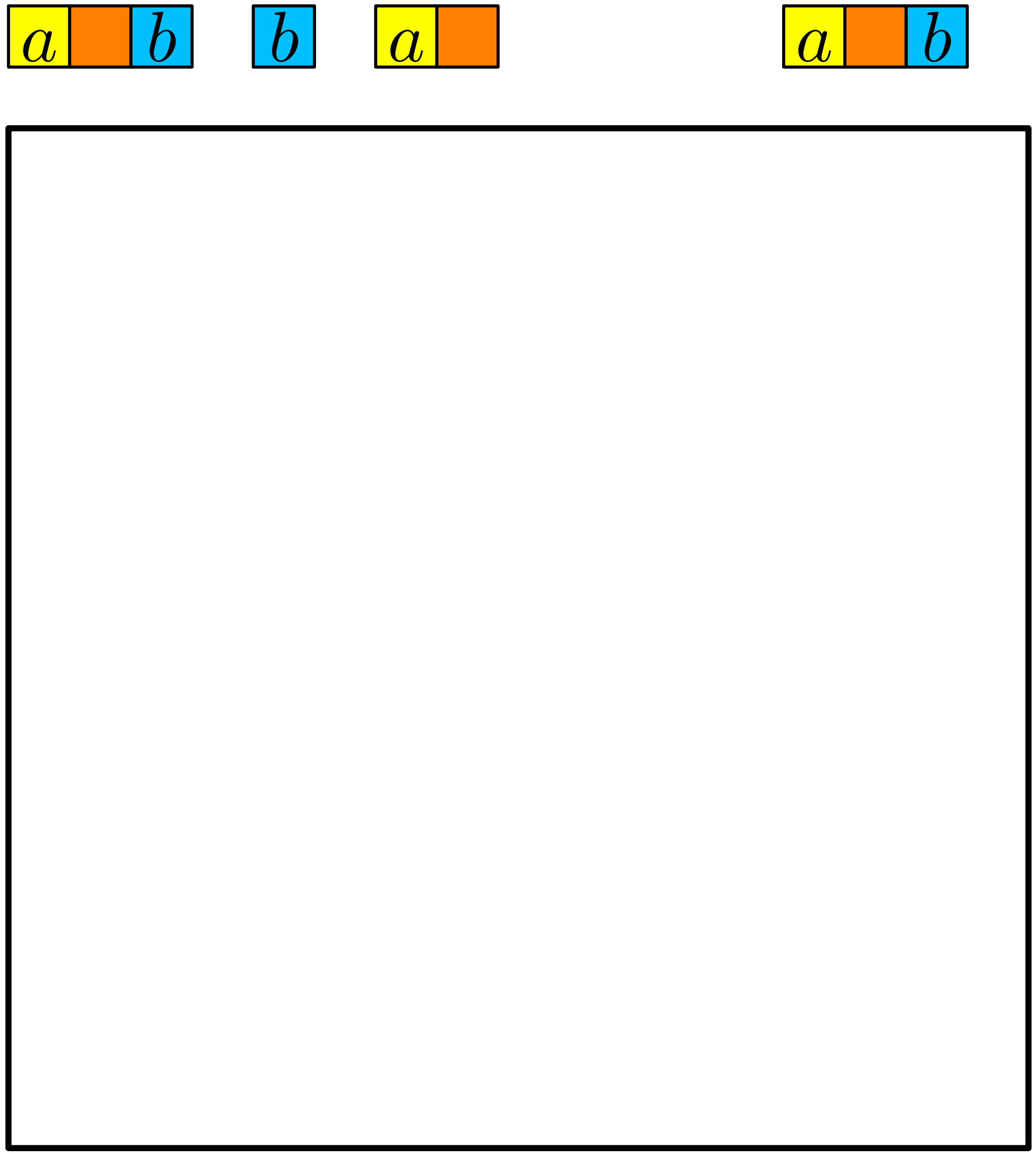

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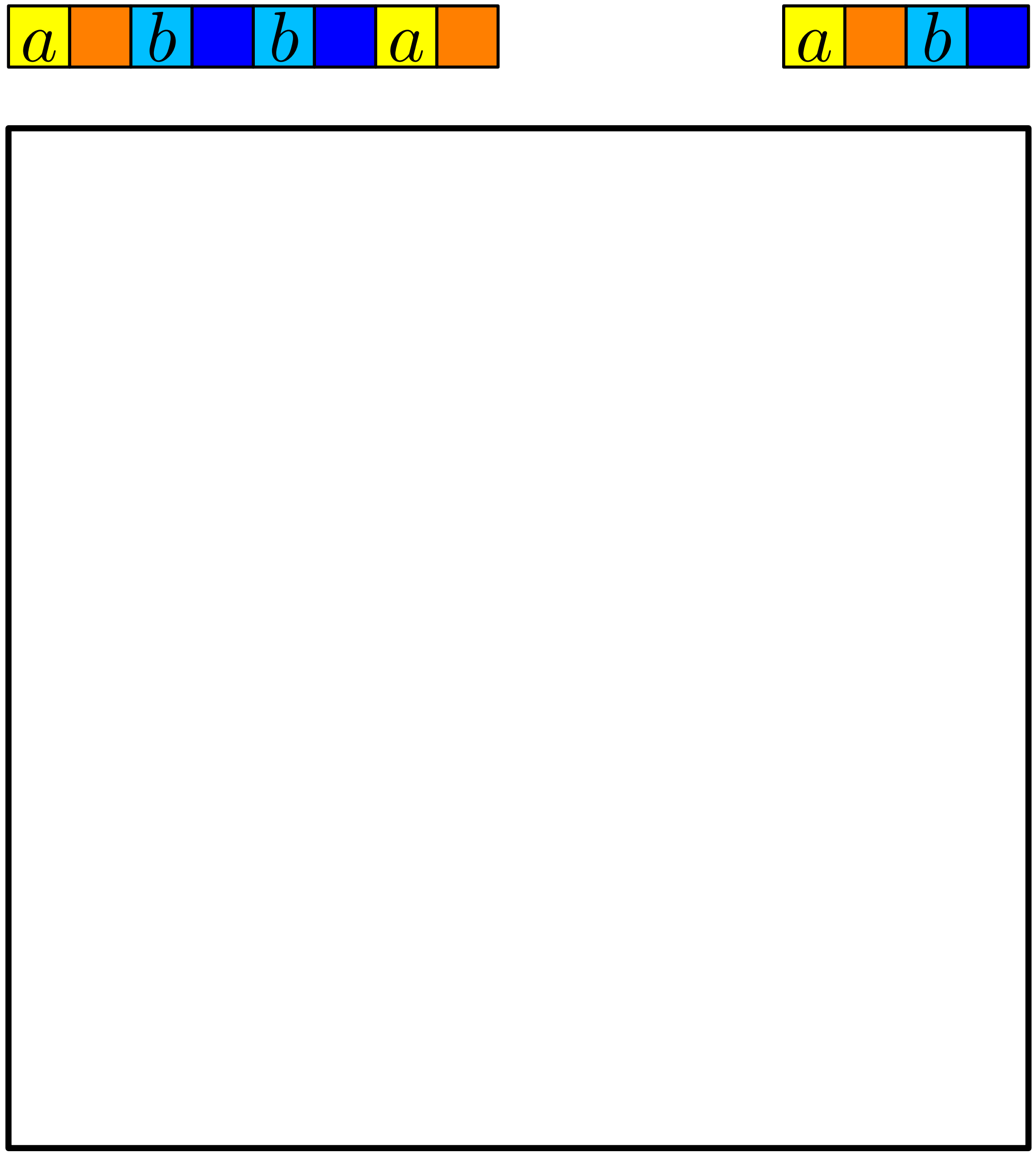

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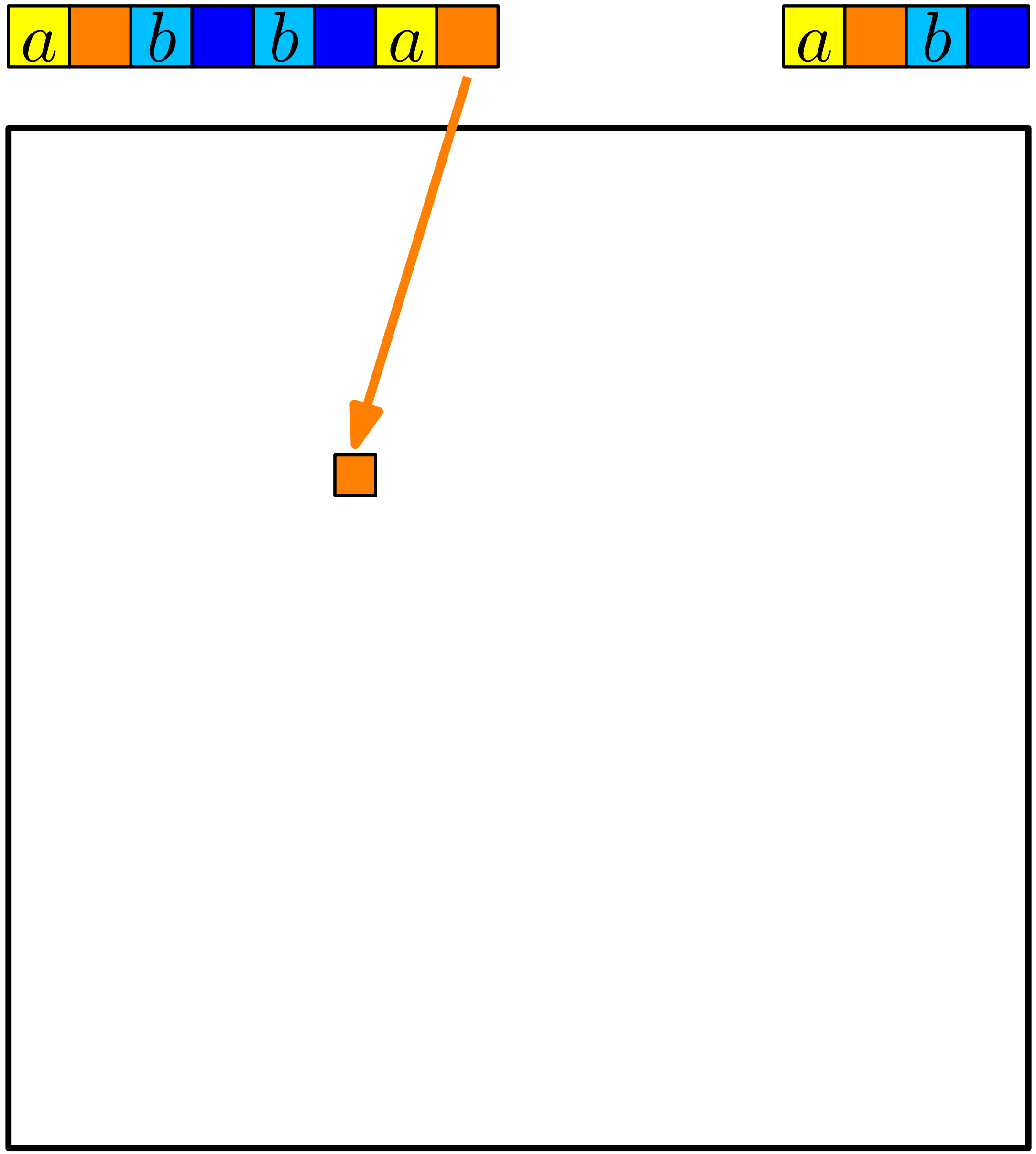

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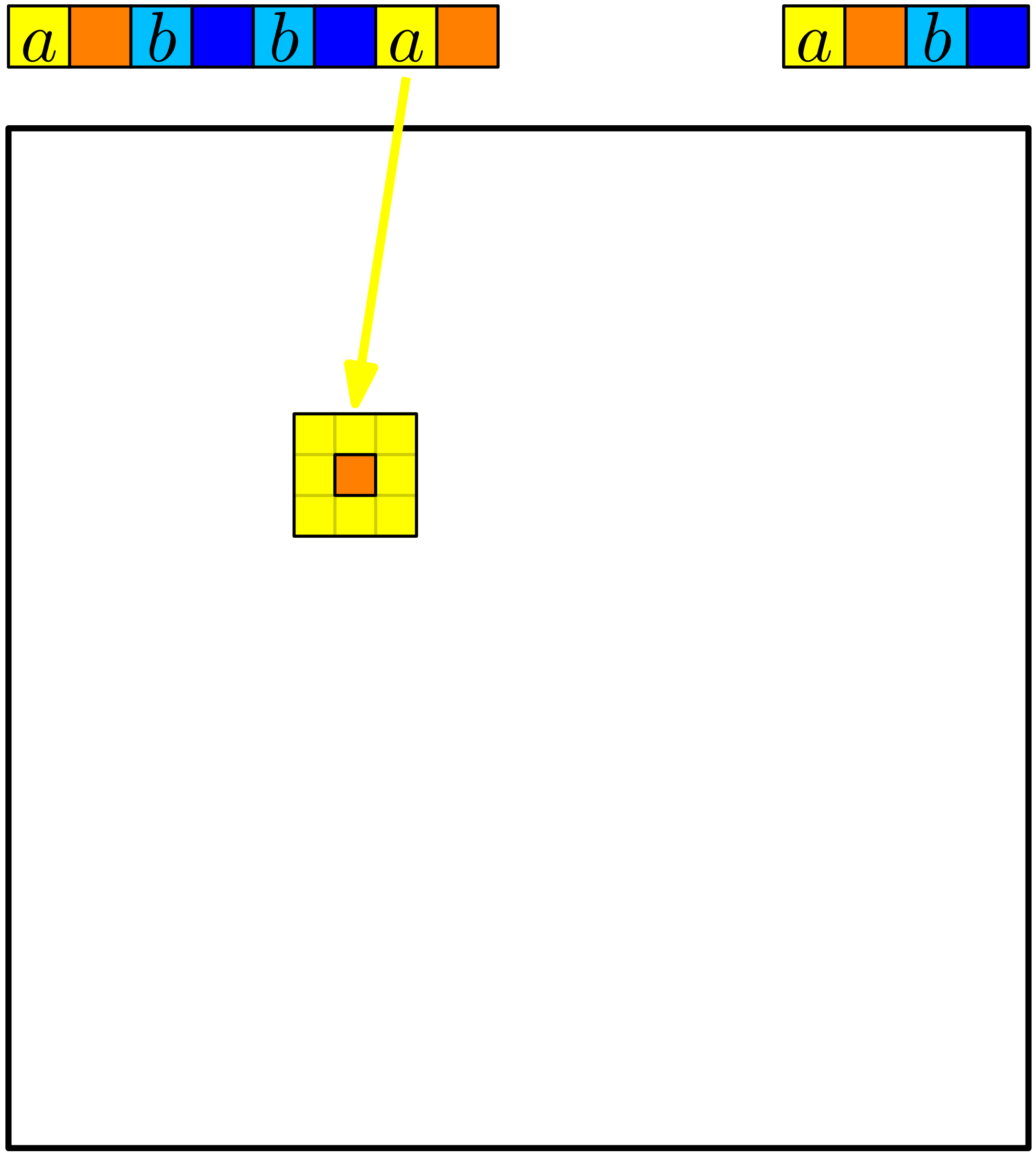

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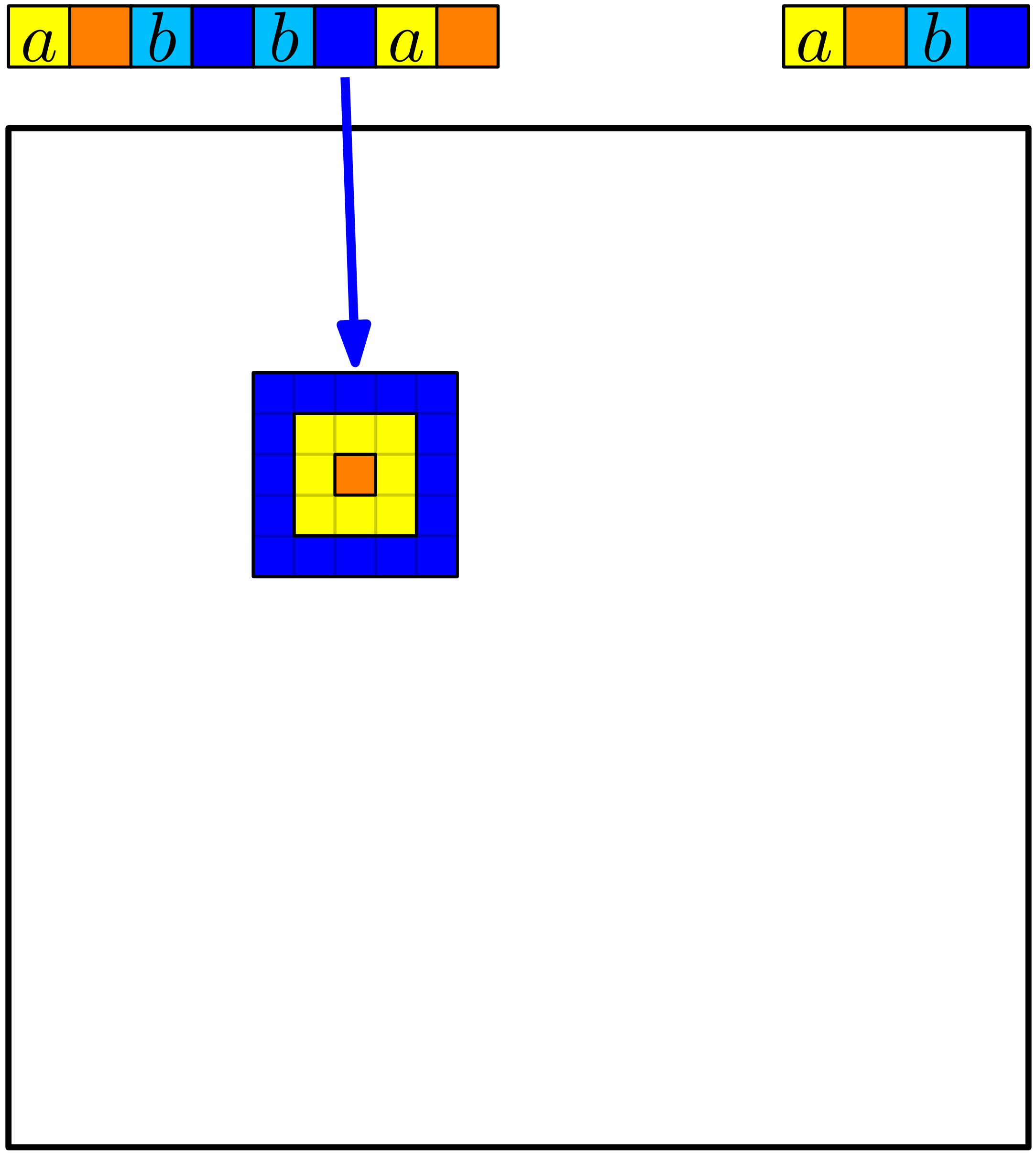

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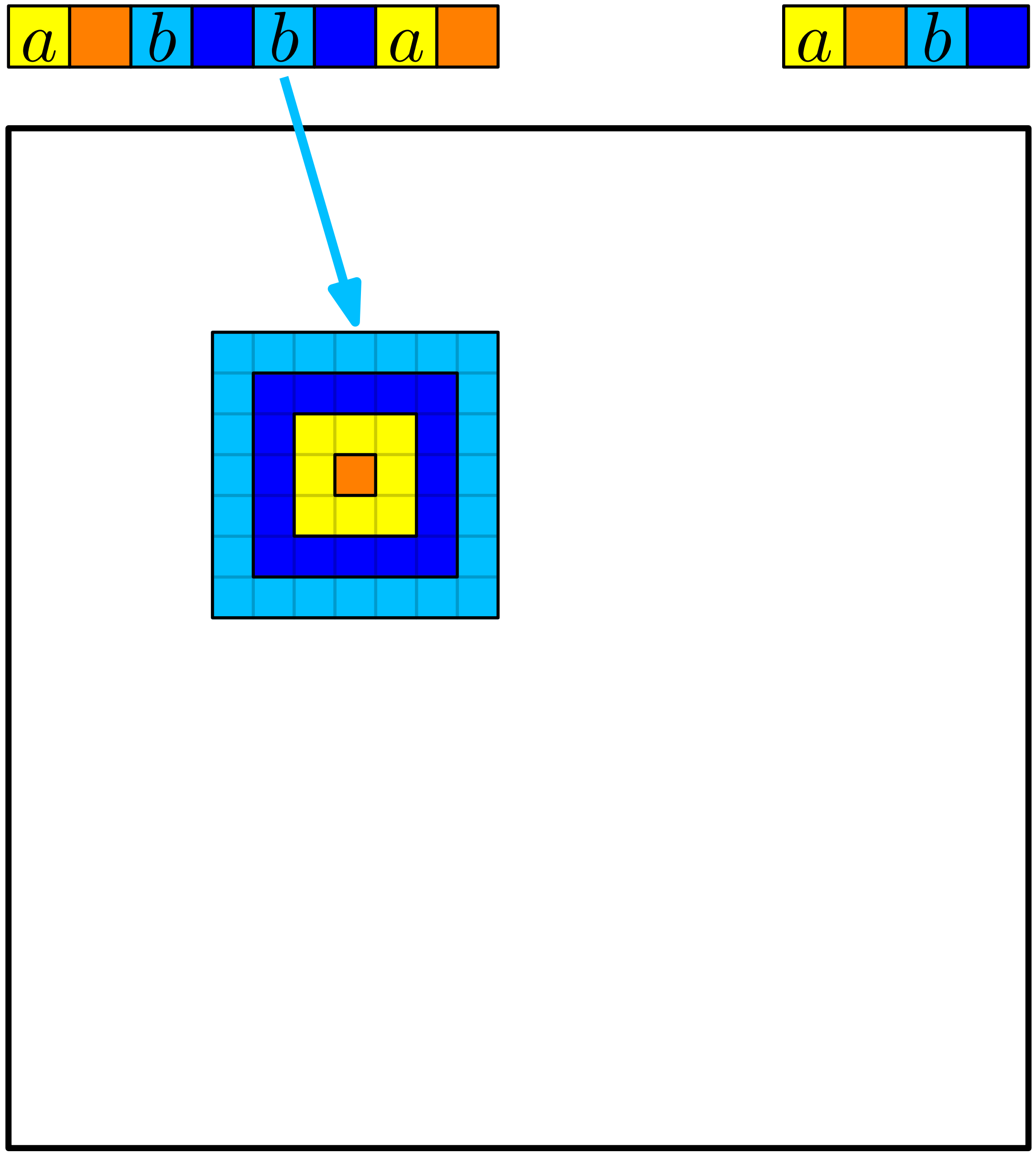

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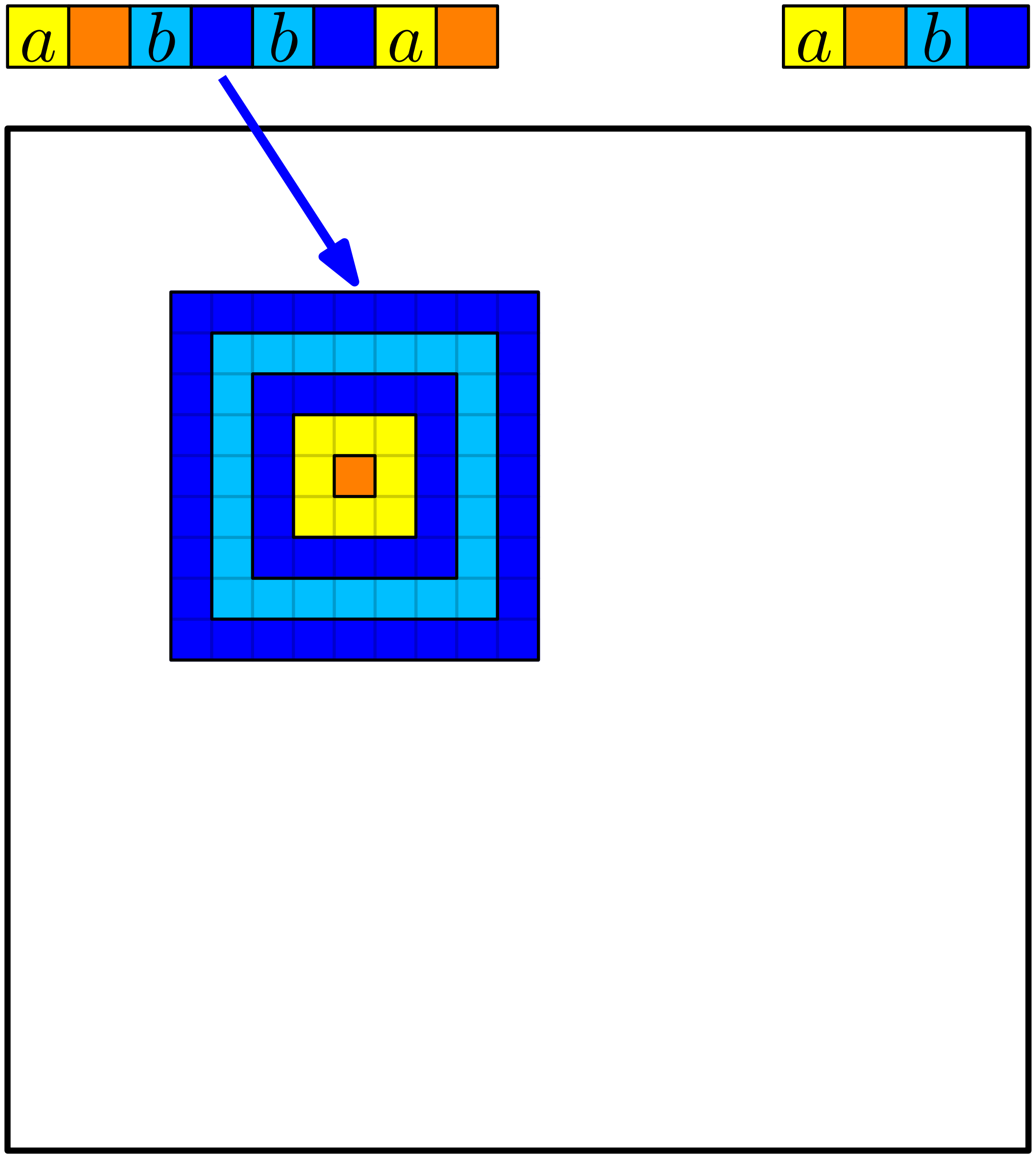

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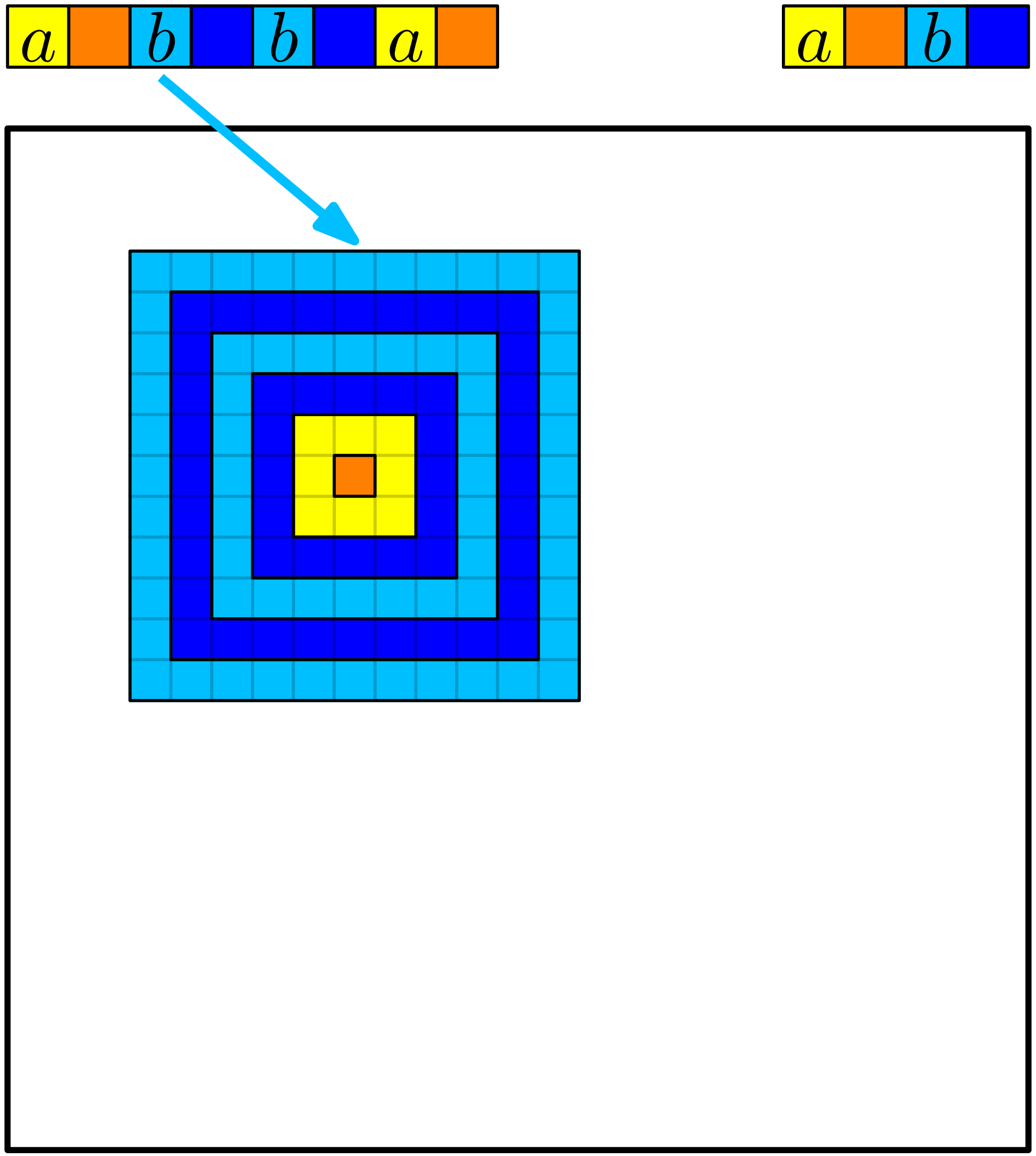
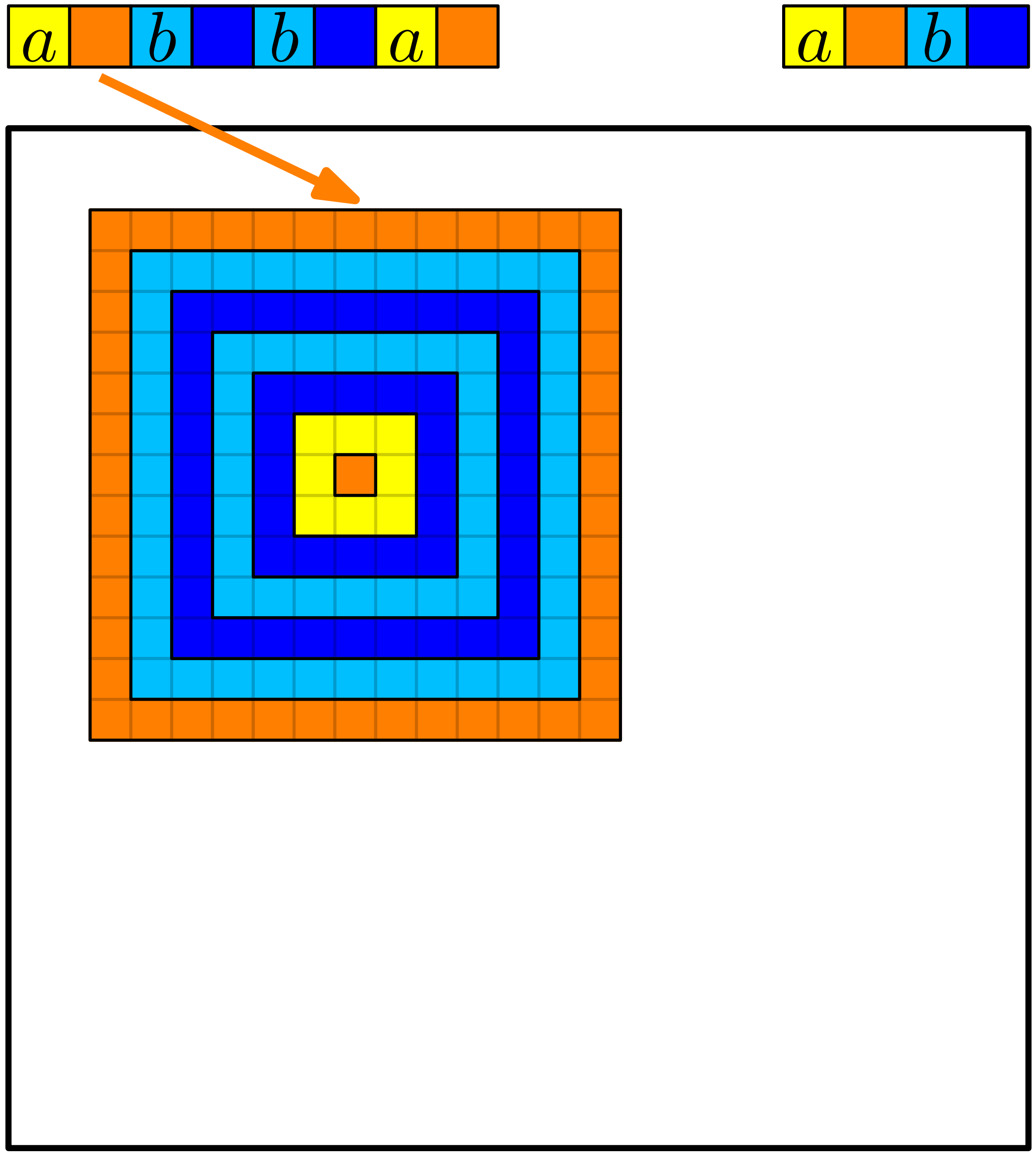


4 or More Colours



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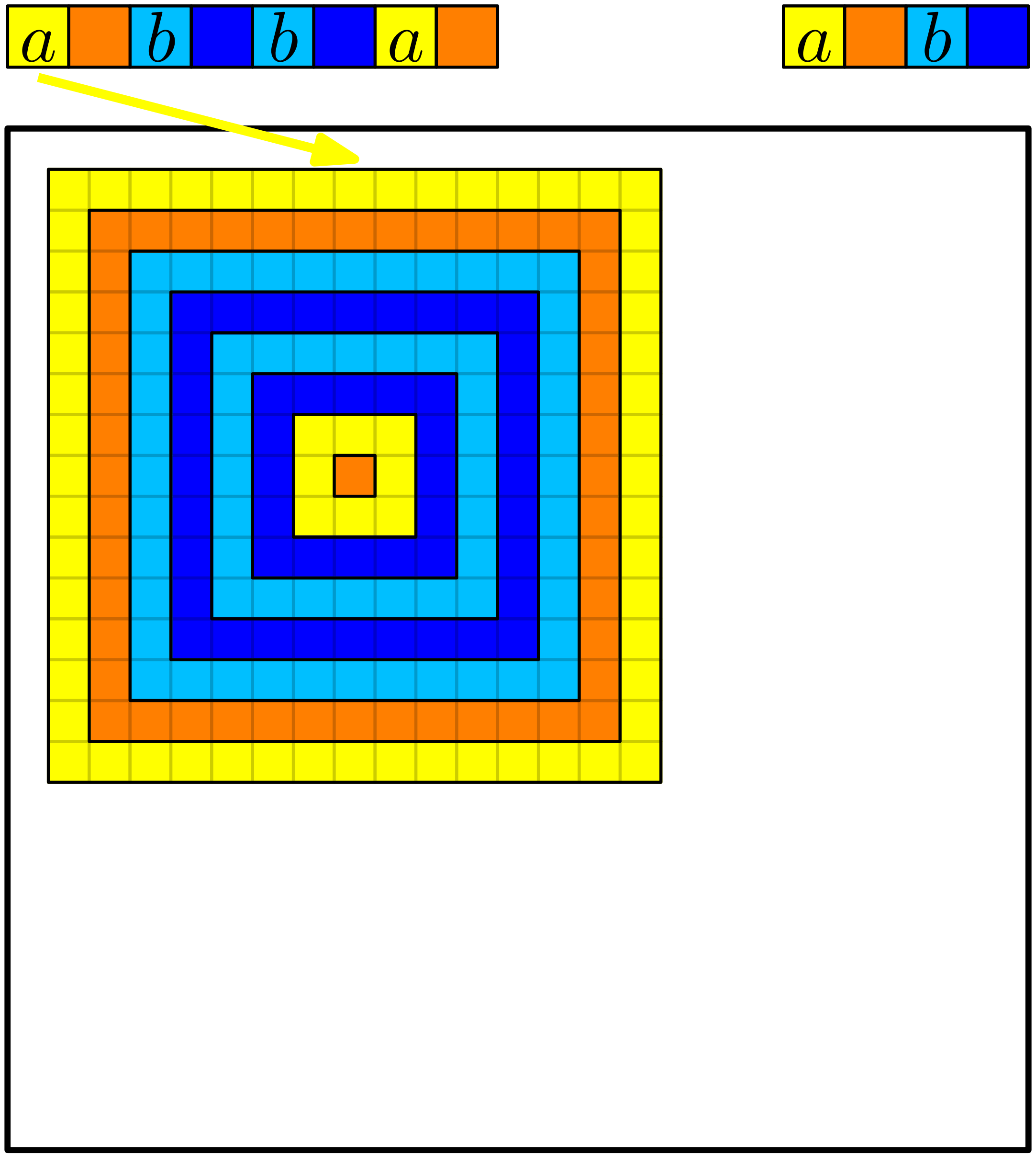
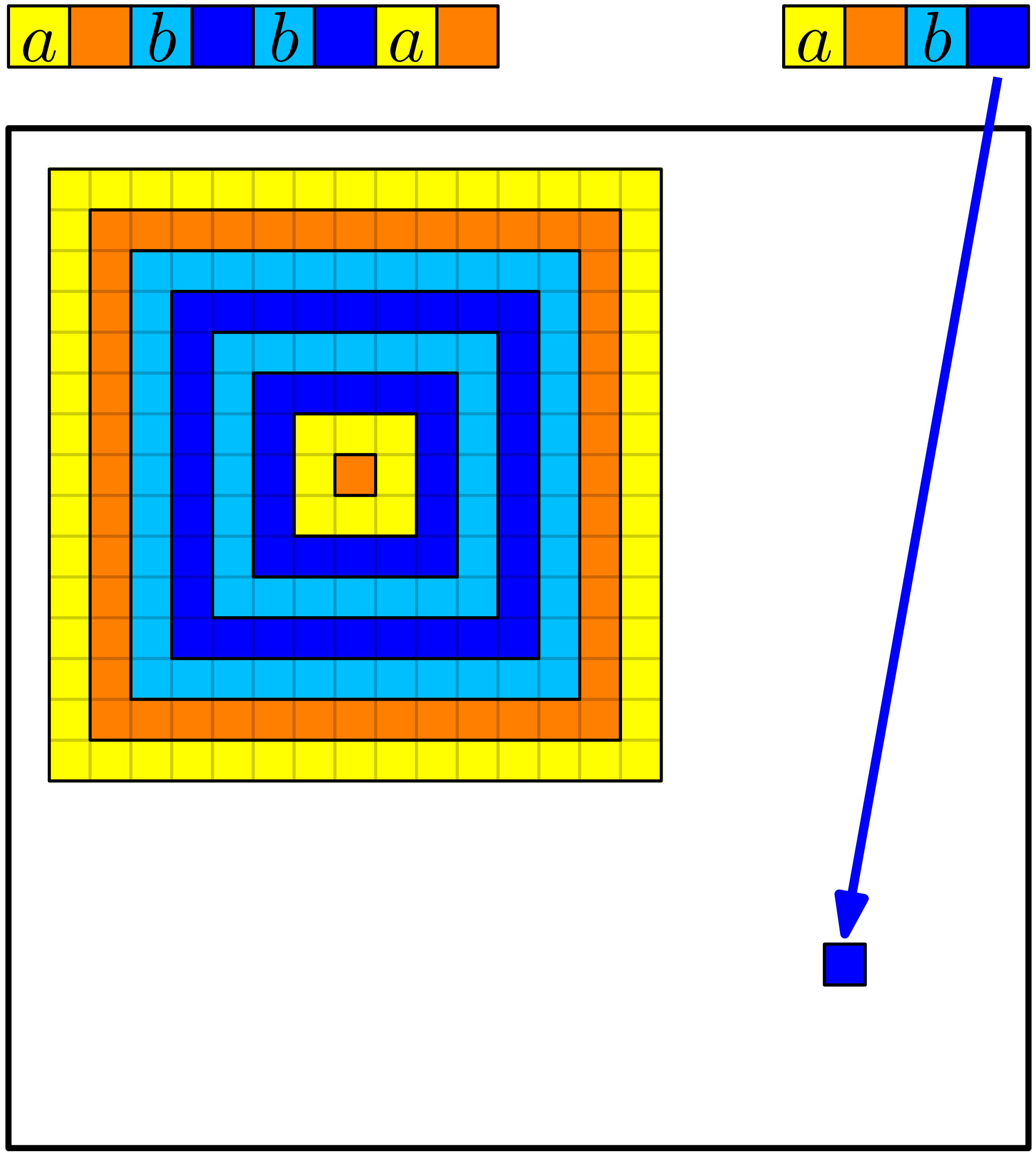
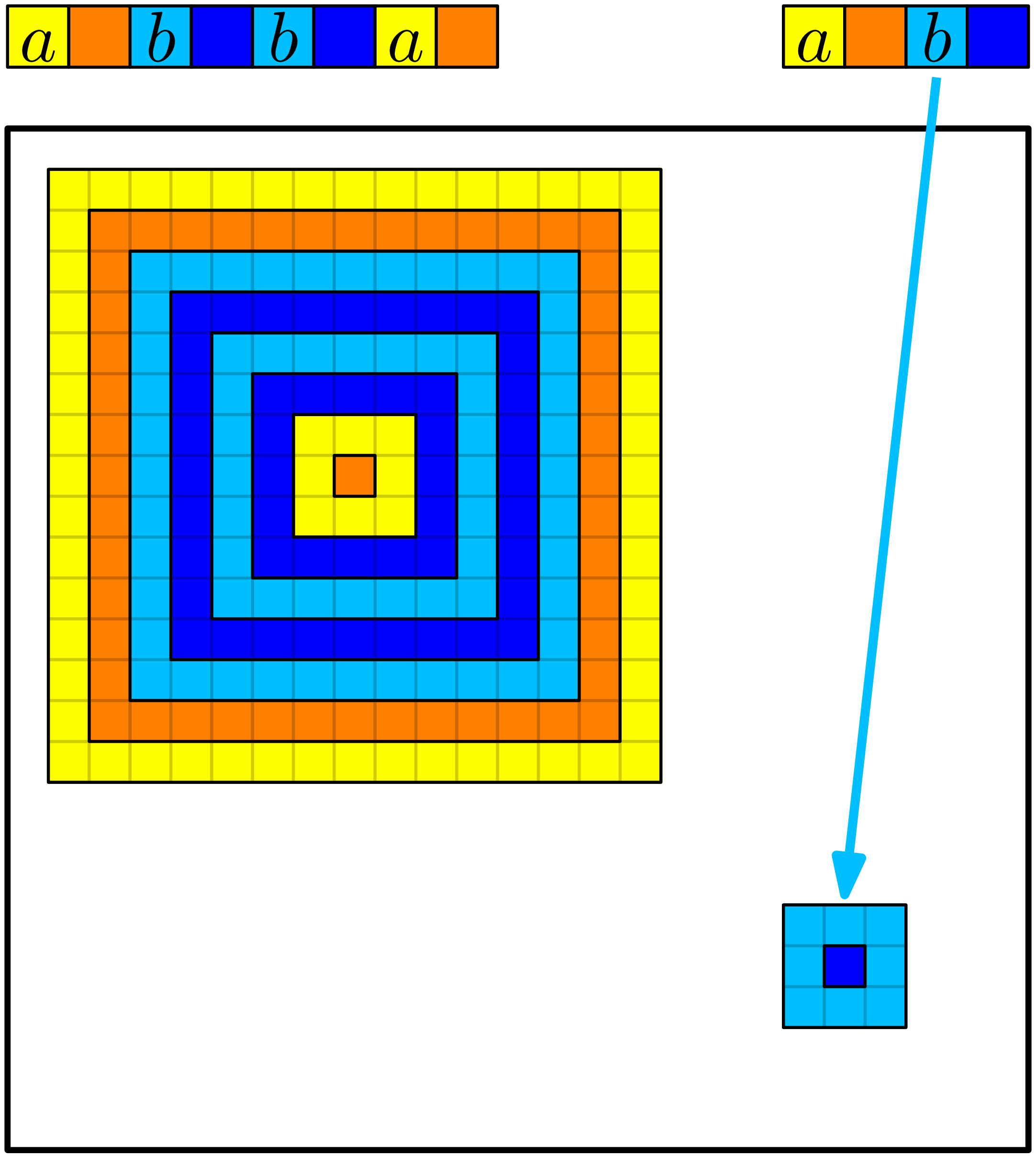


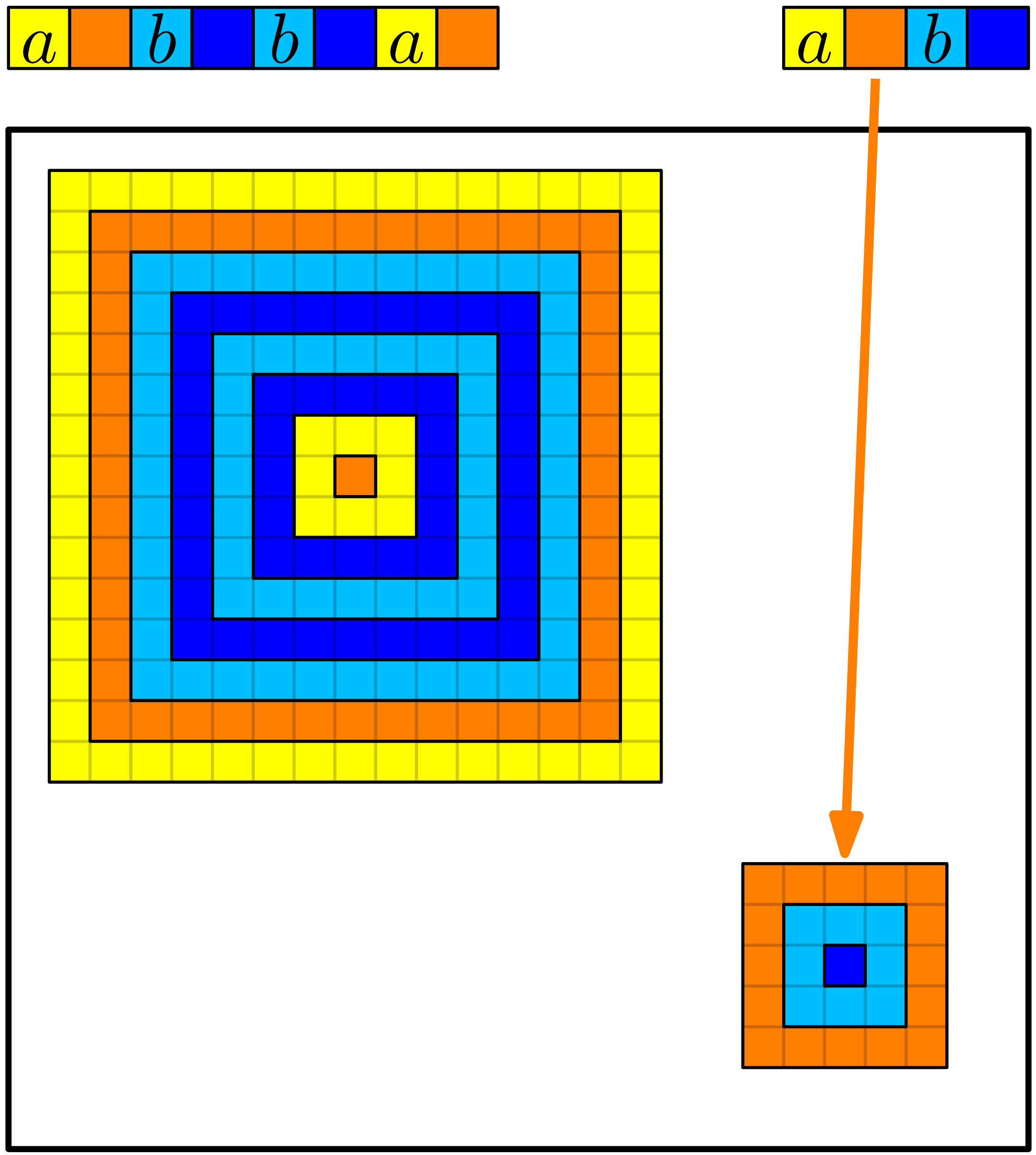
4 or More Colours



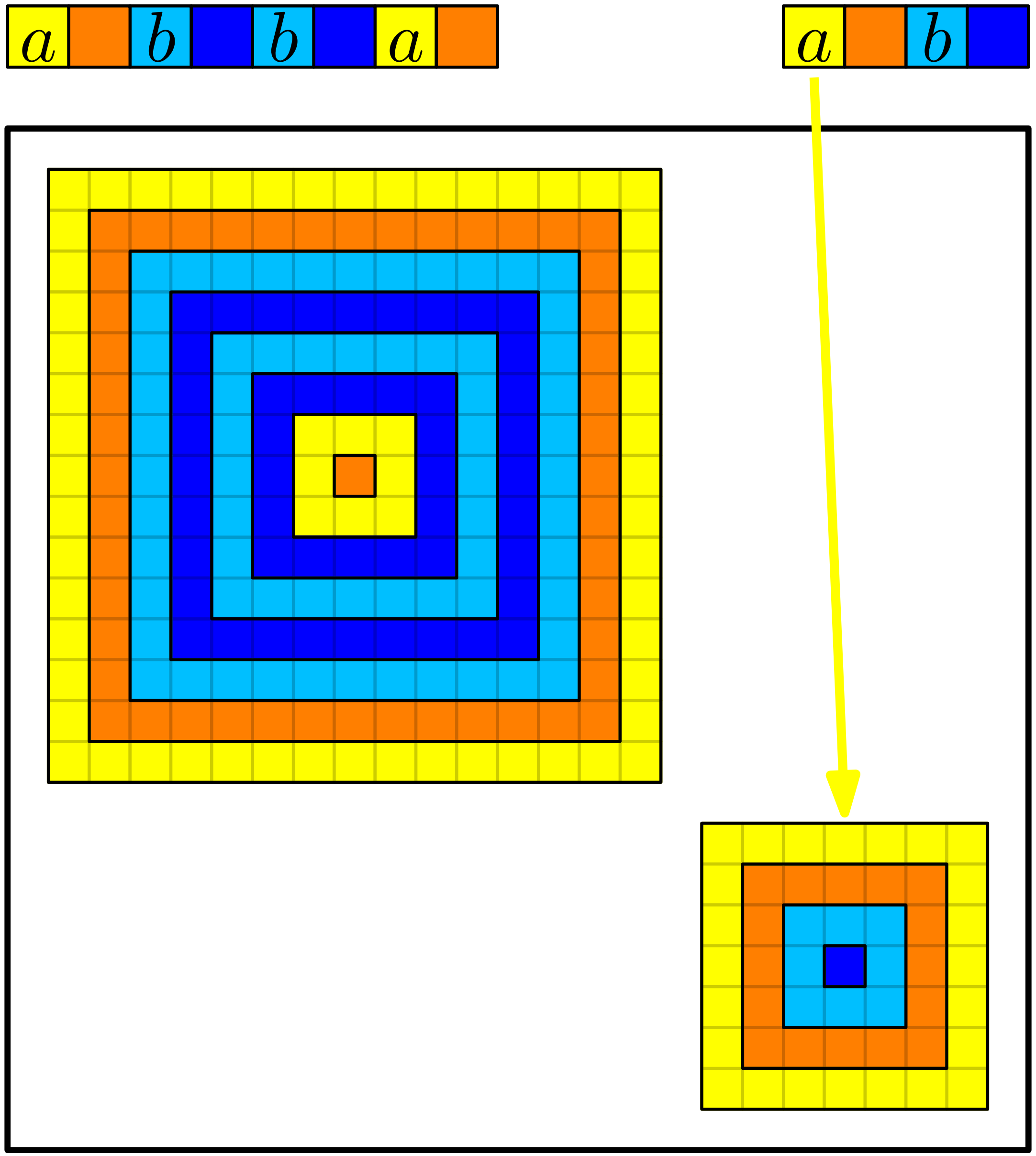
4 or More Colours



4 or More Colours



4 or More Colours



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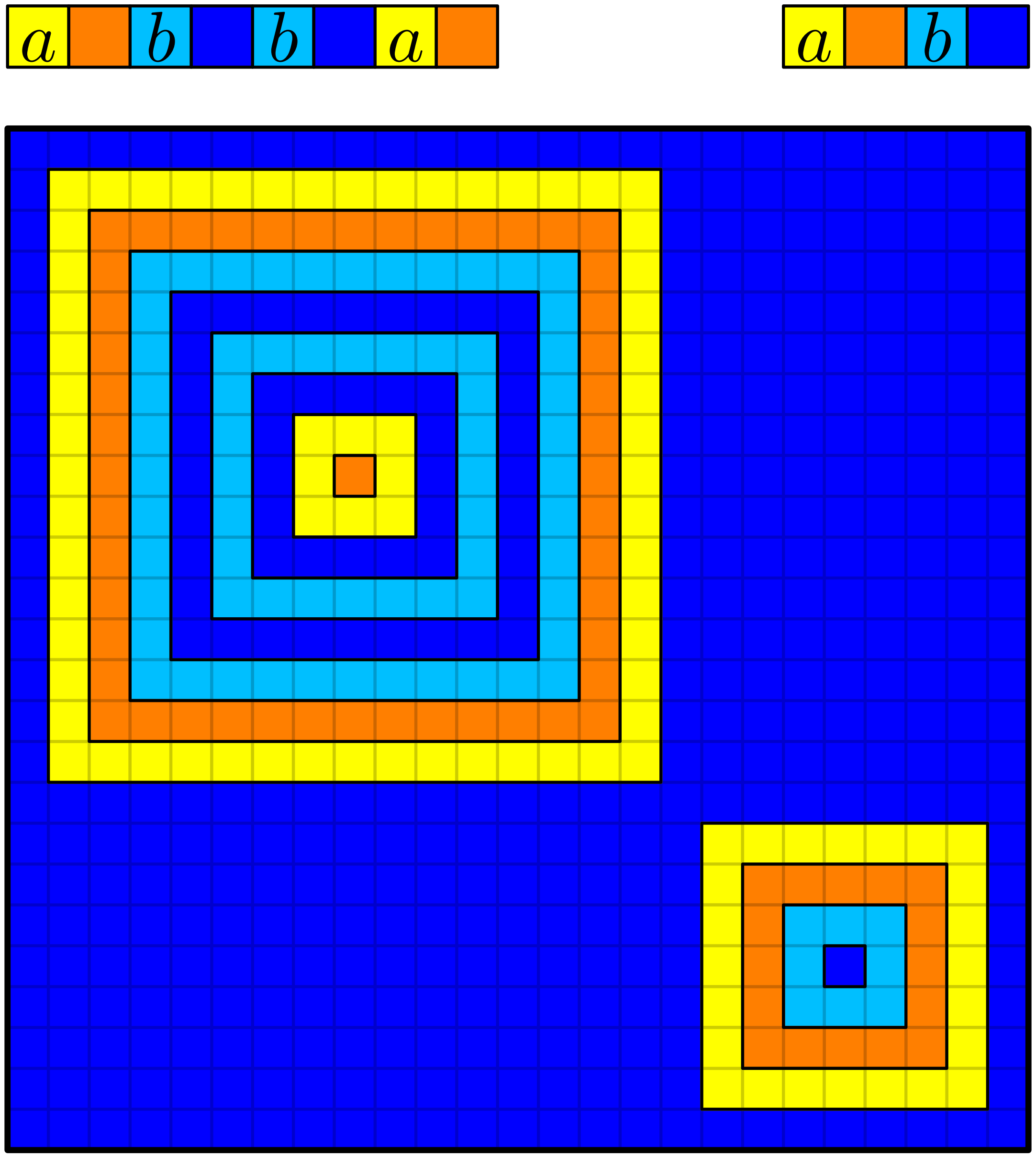
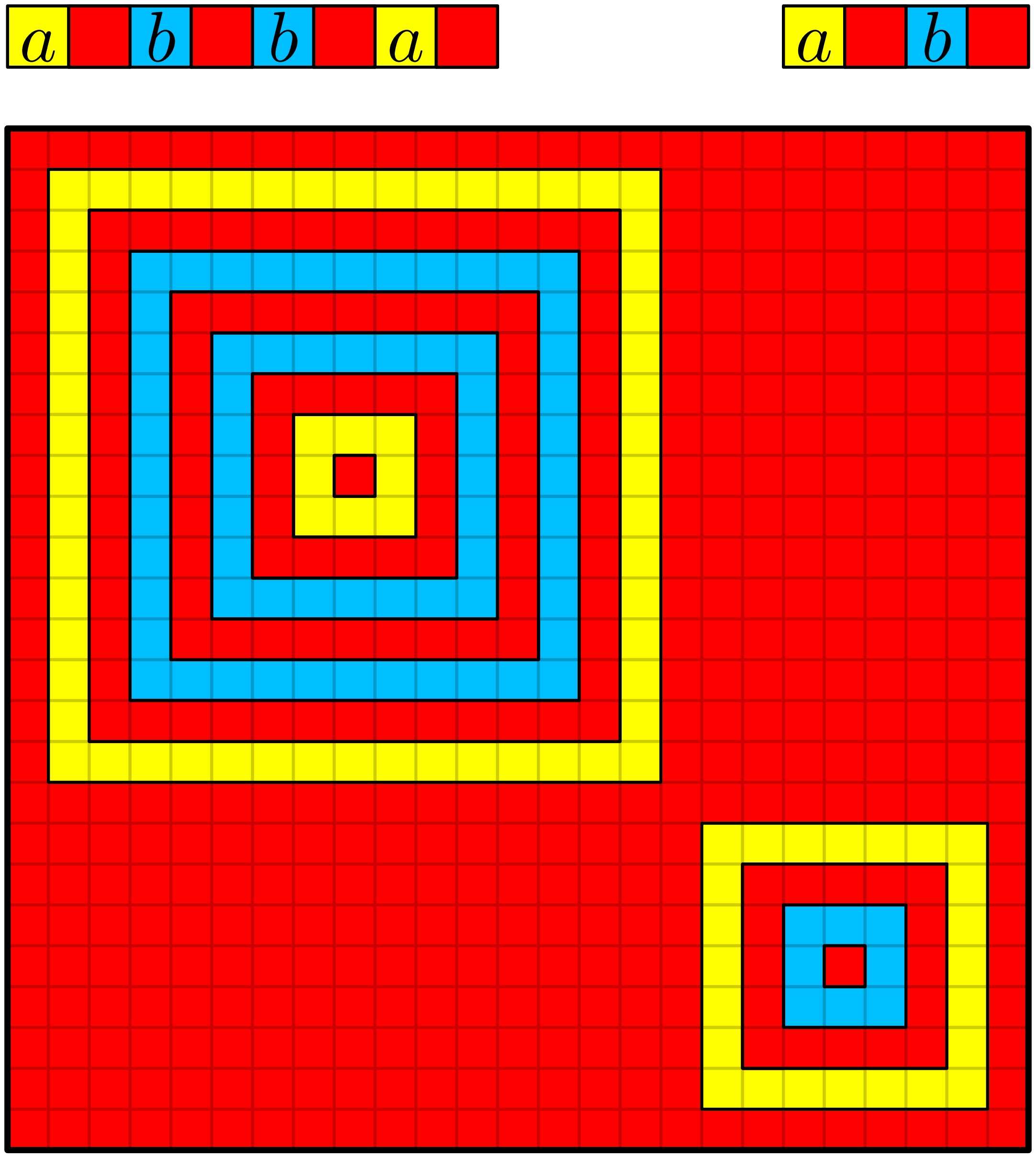
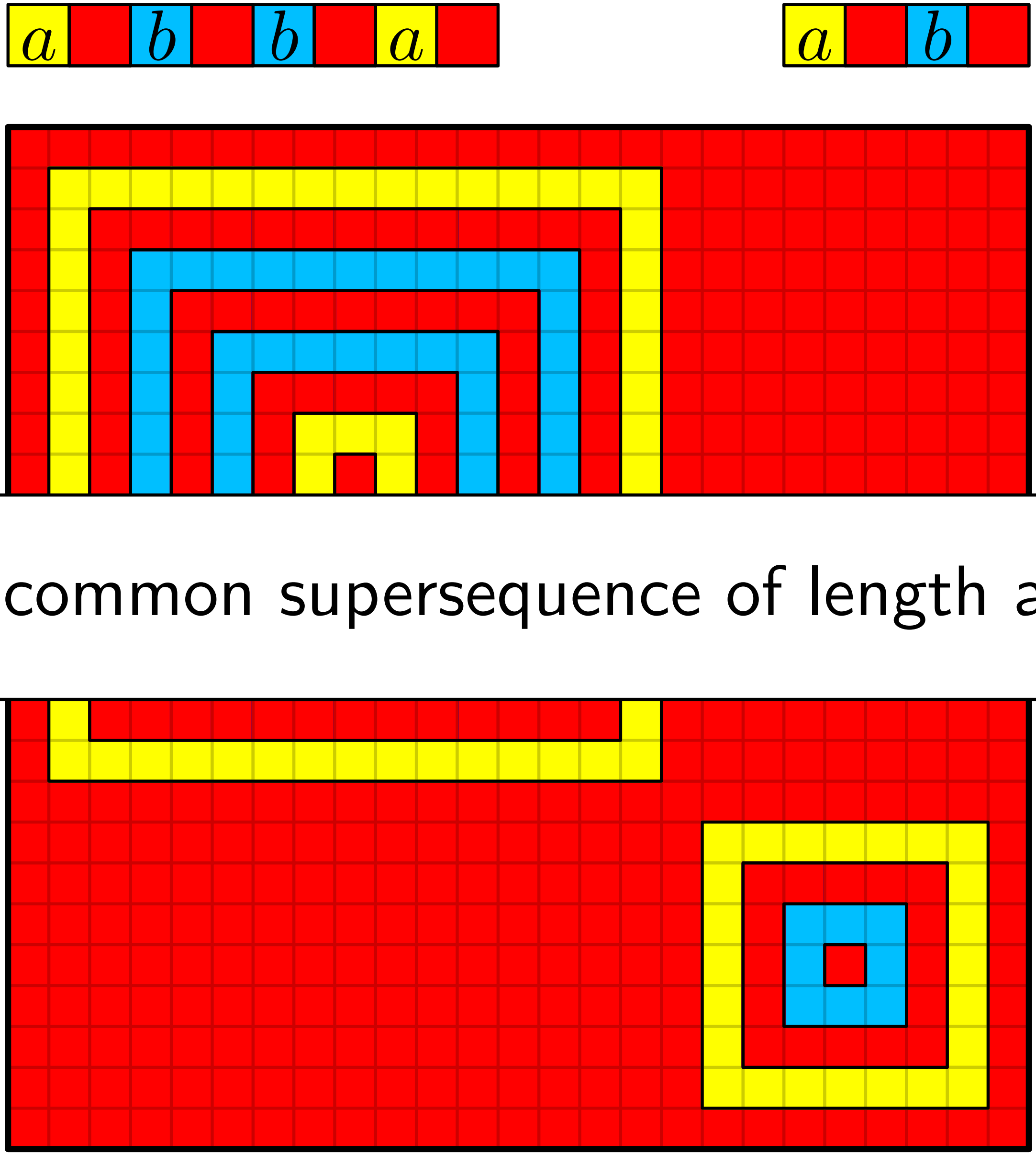


3 Colours

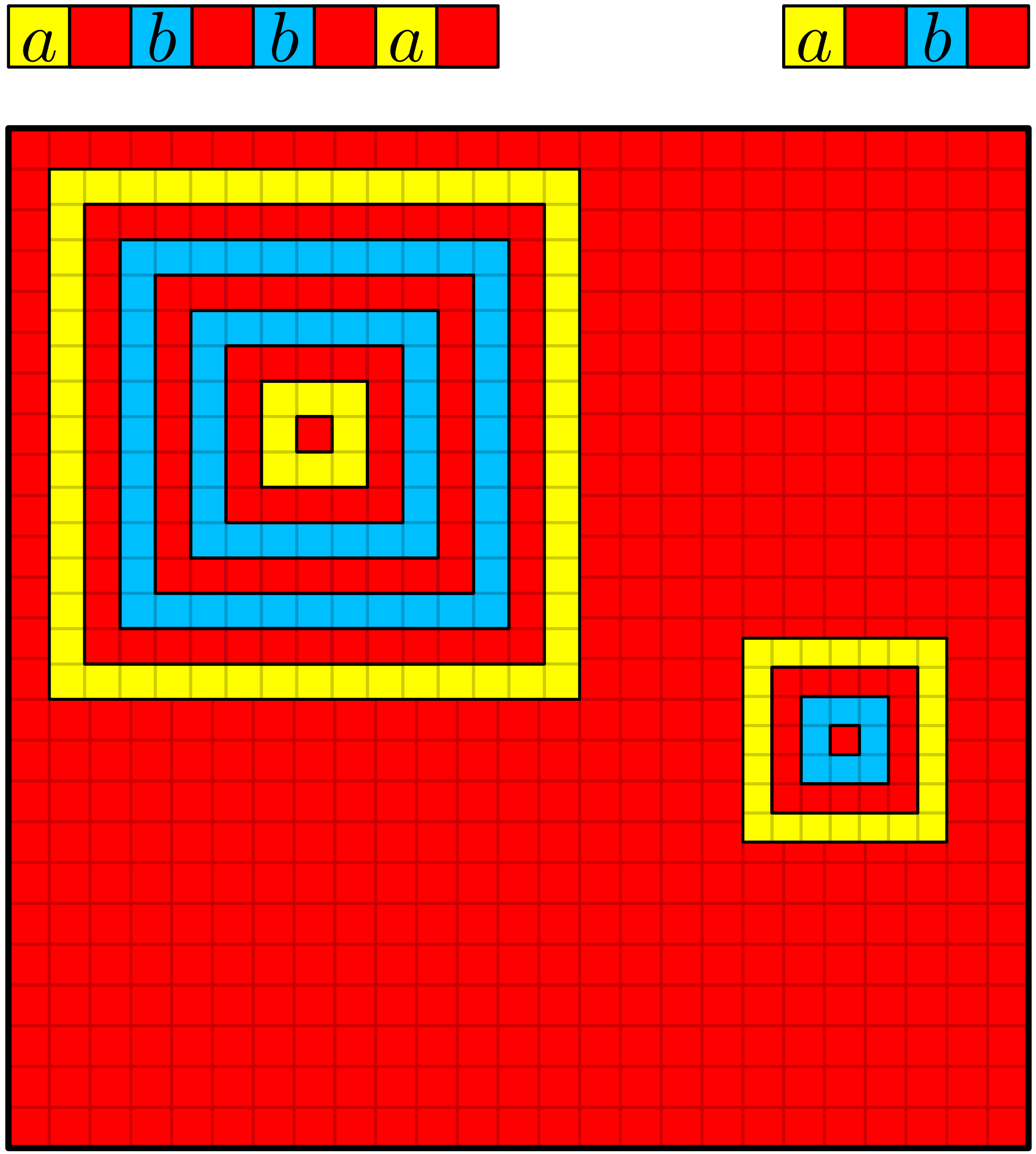


3 Colours

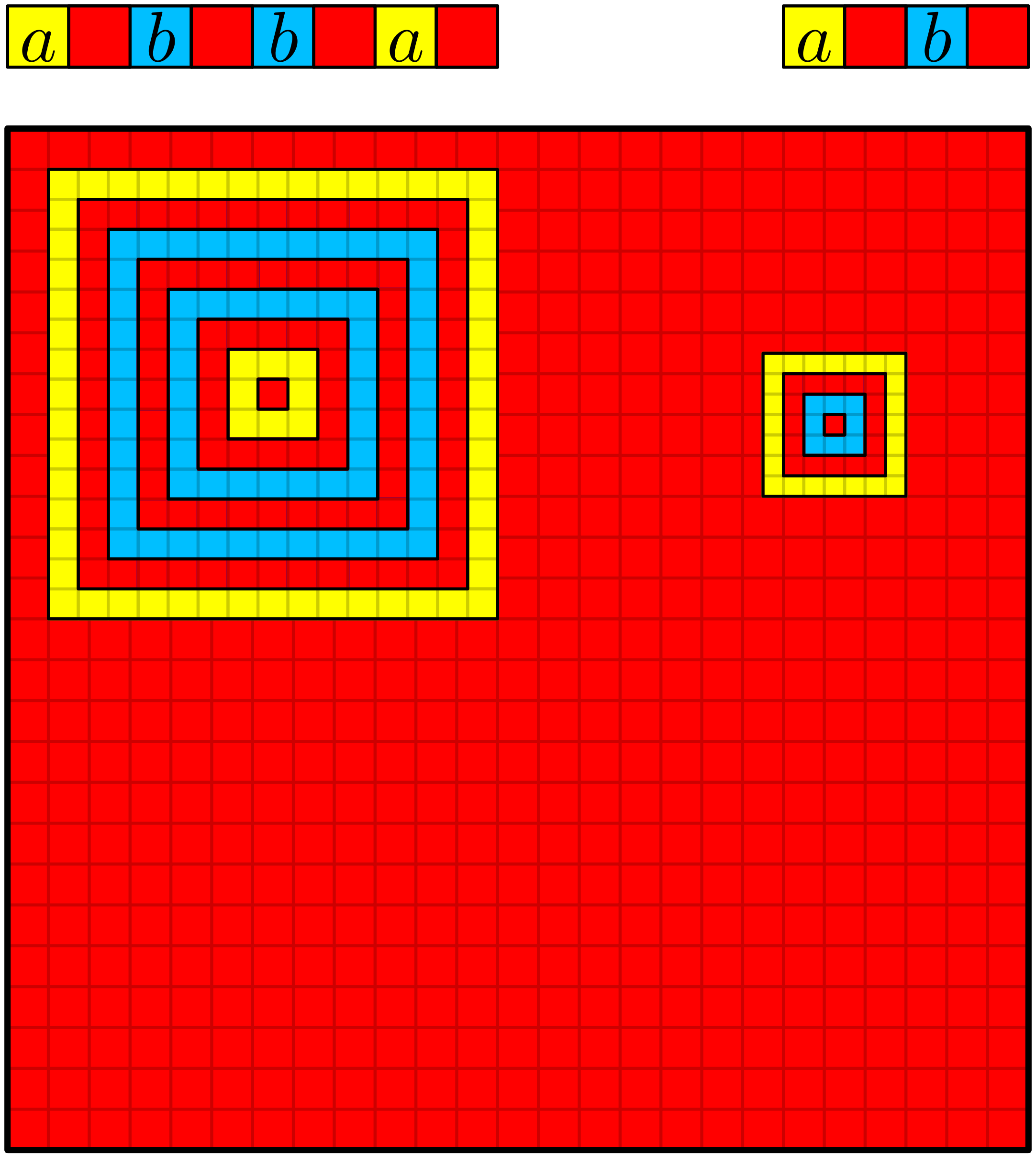


Is there a common supersequence of length at most 4?

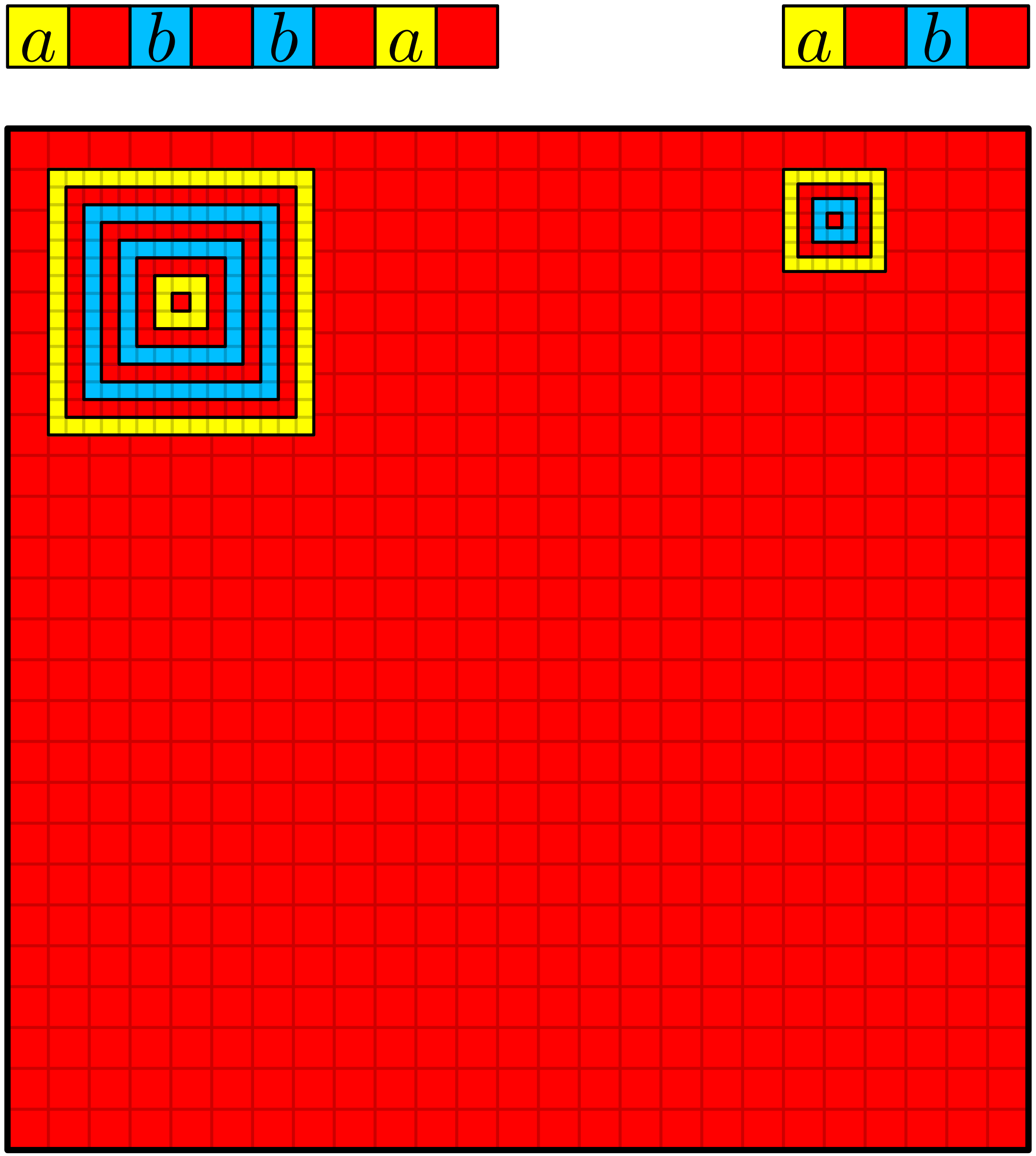
3 Colours



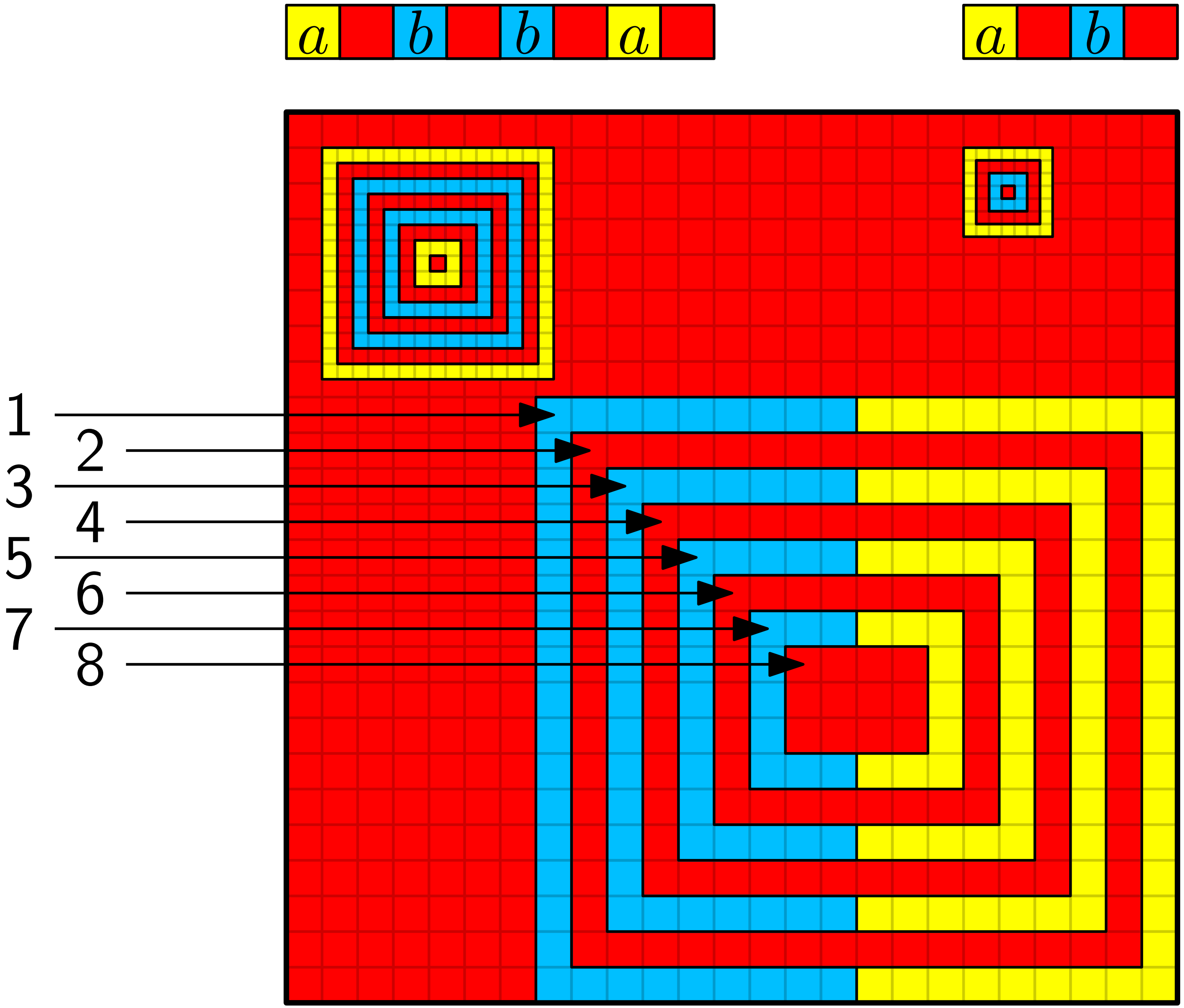
3 Colours



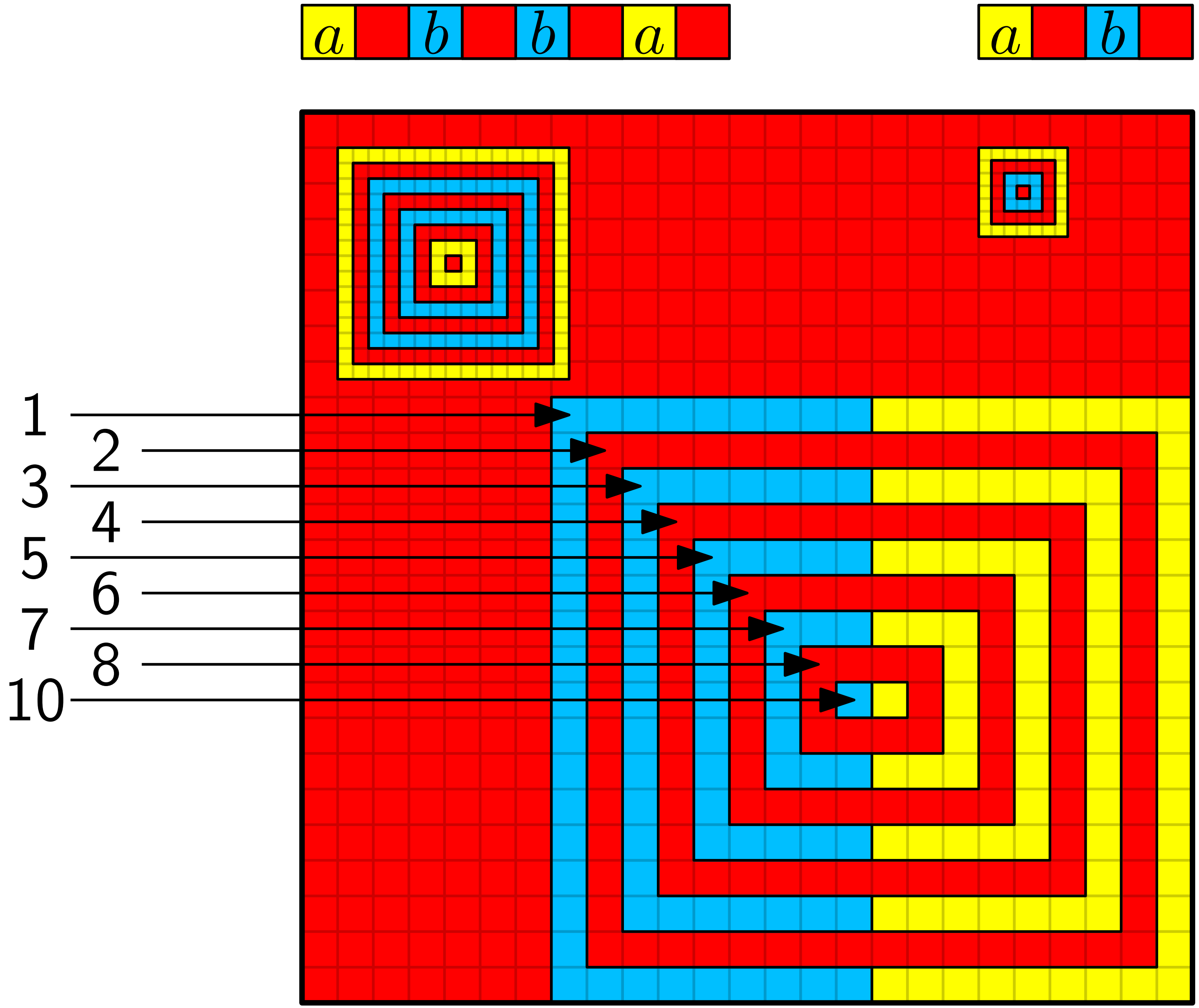
3 Colours



3 Colours

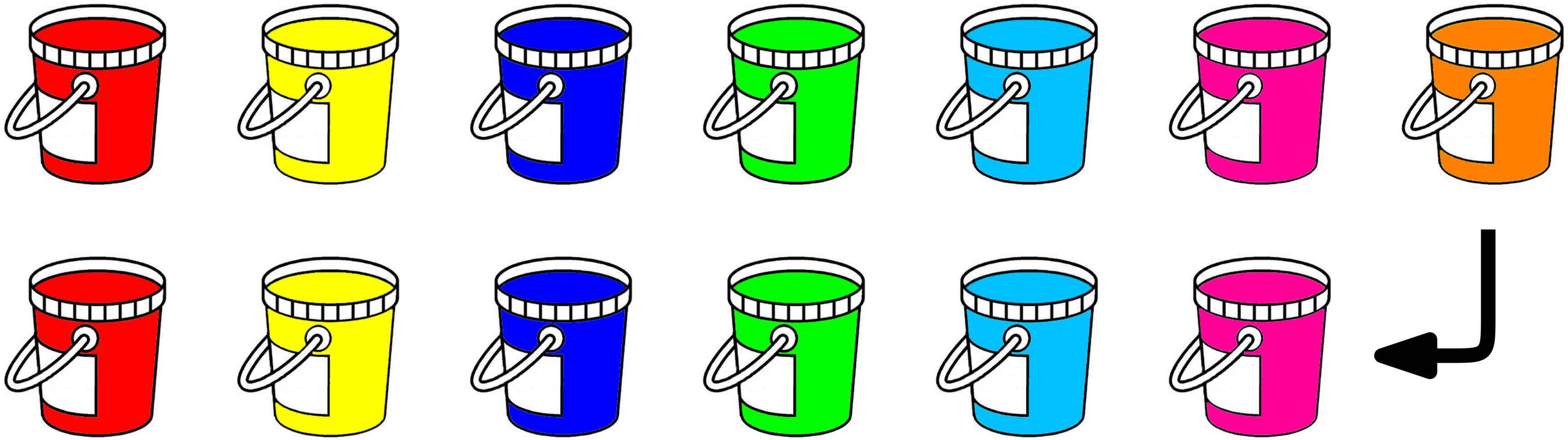


3 Colours

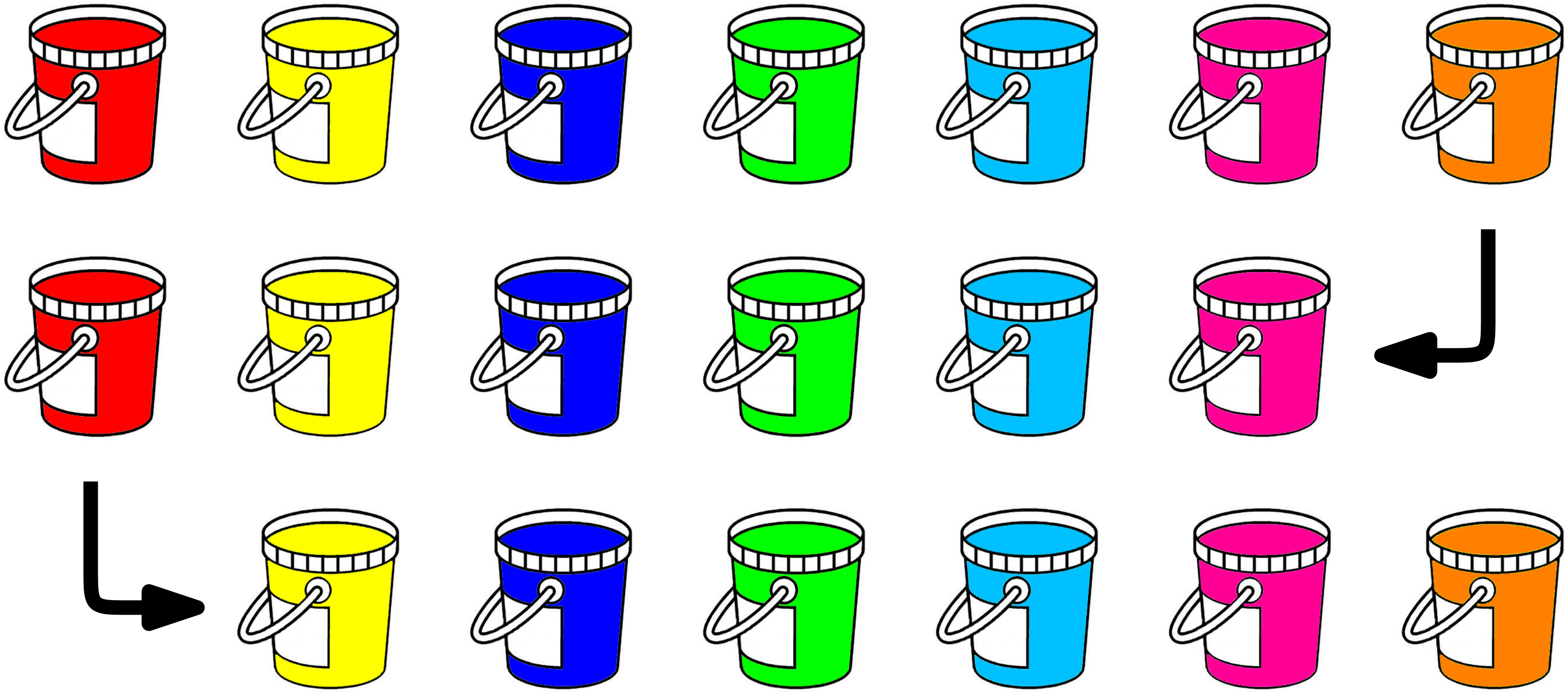


Approximation

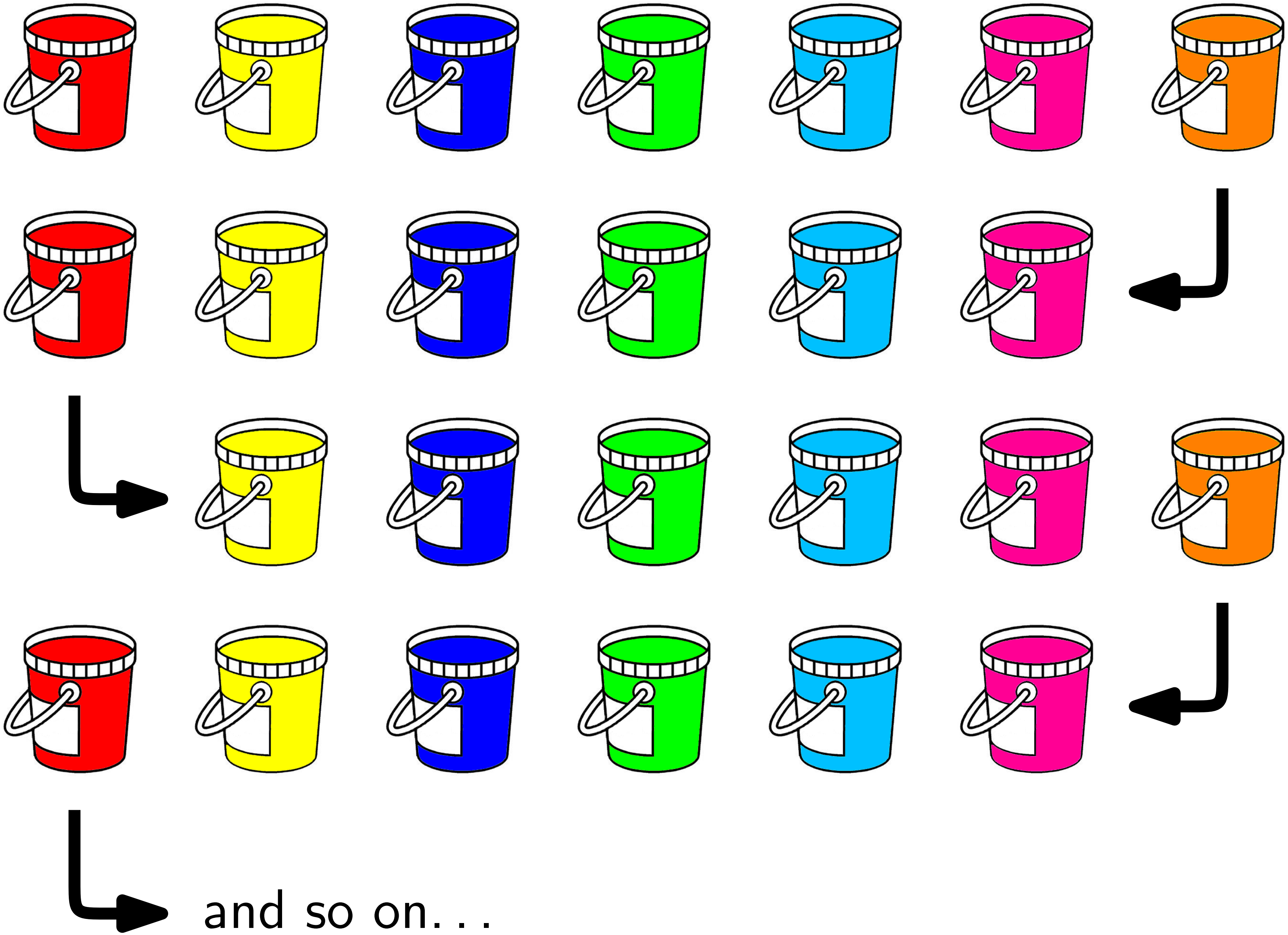
Approximation



Approximation

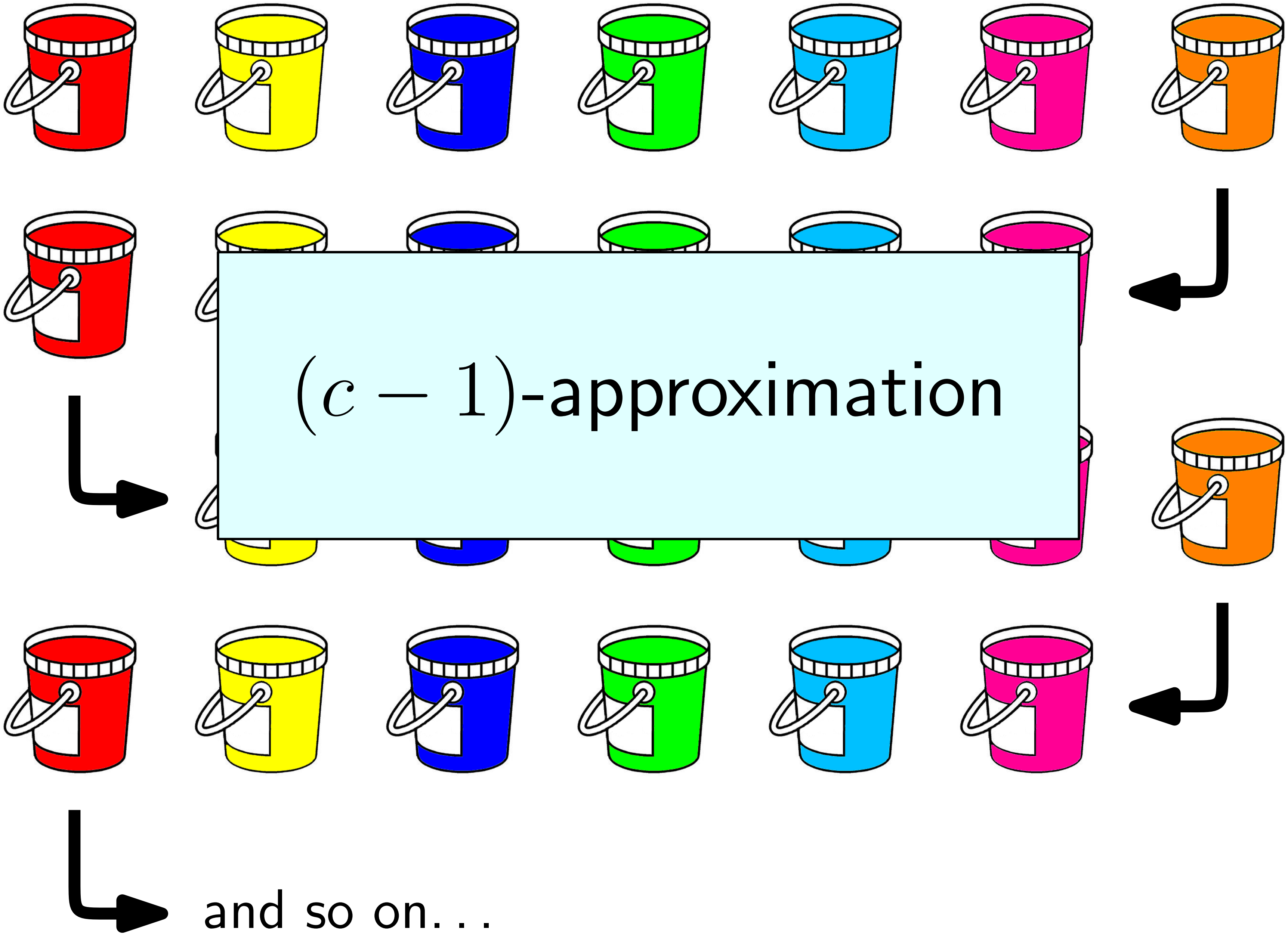


Approximation

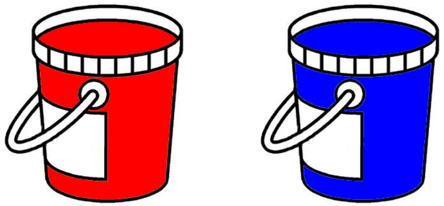


and so on...

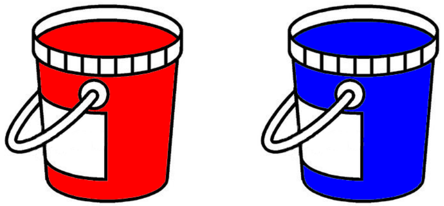
Approximation



Randomised Approximation

Flooding sequence:  • • •

Randomised Approximation

Flooding sequence:  • • •

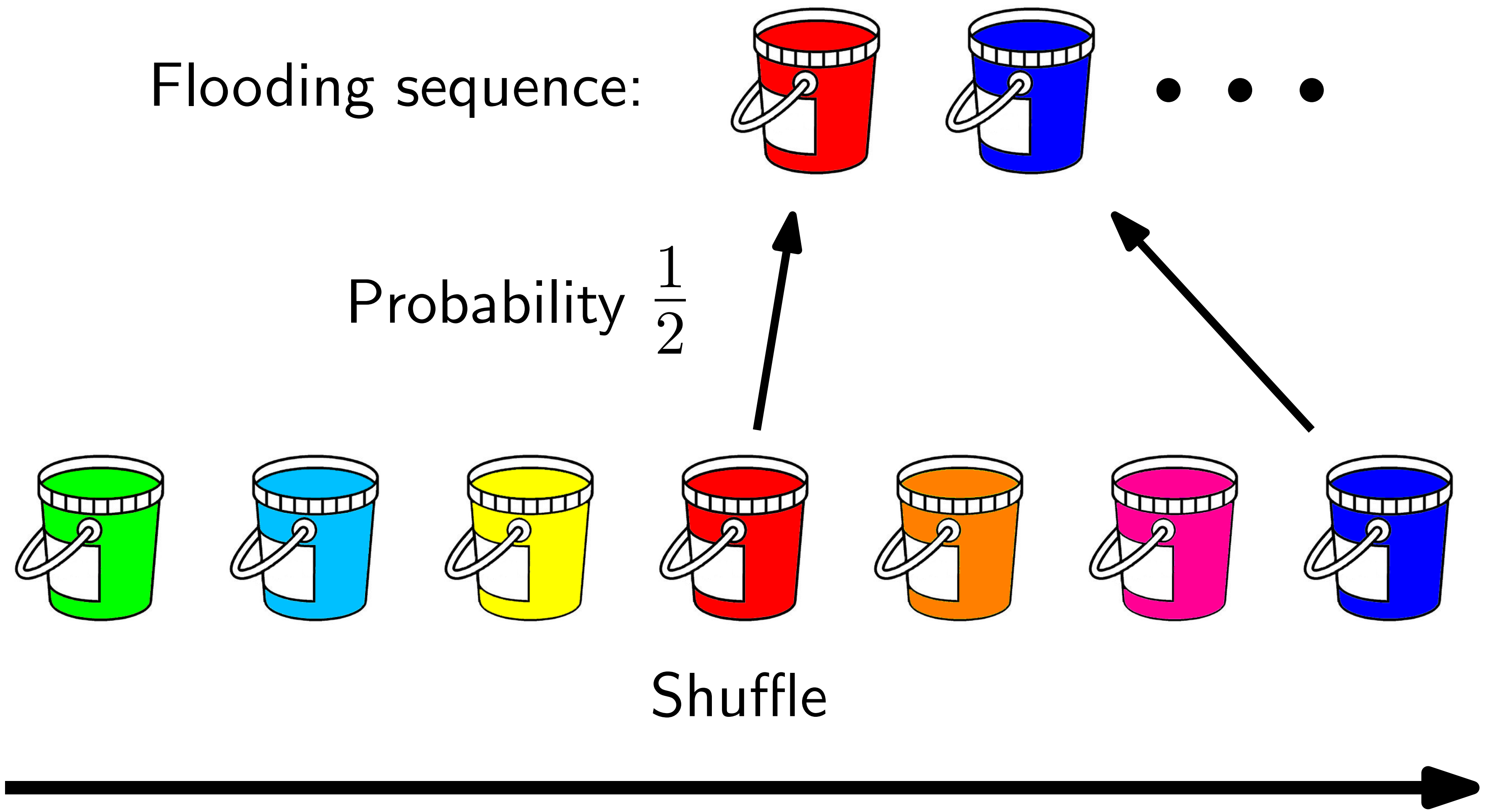
Shuffle

Randomised Approximation

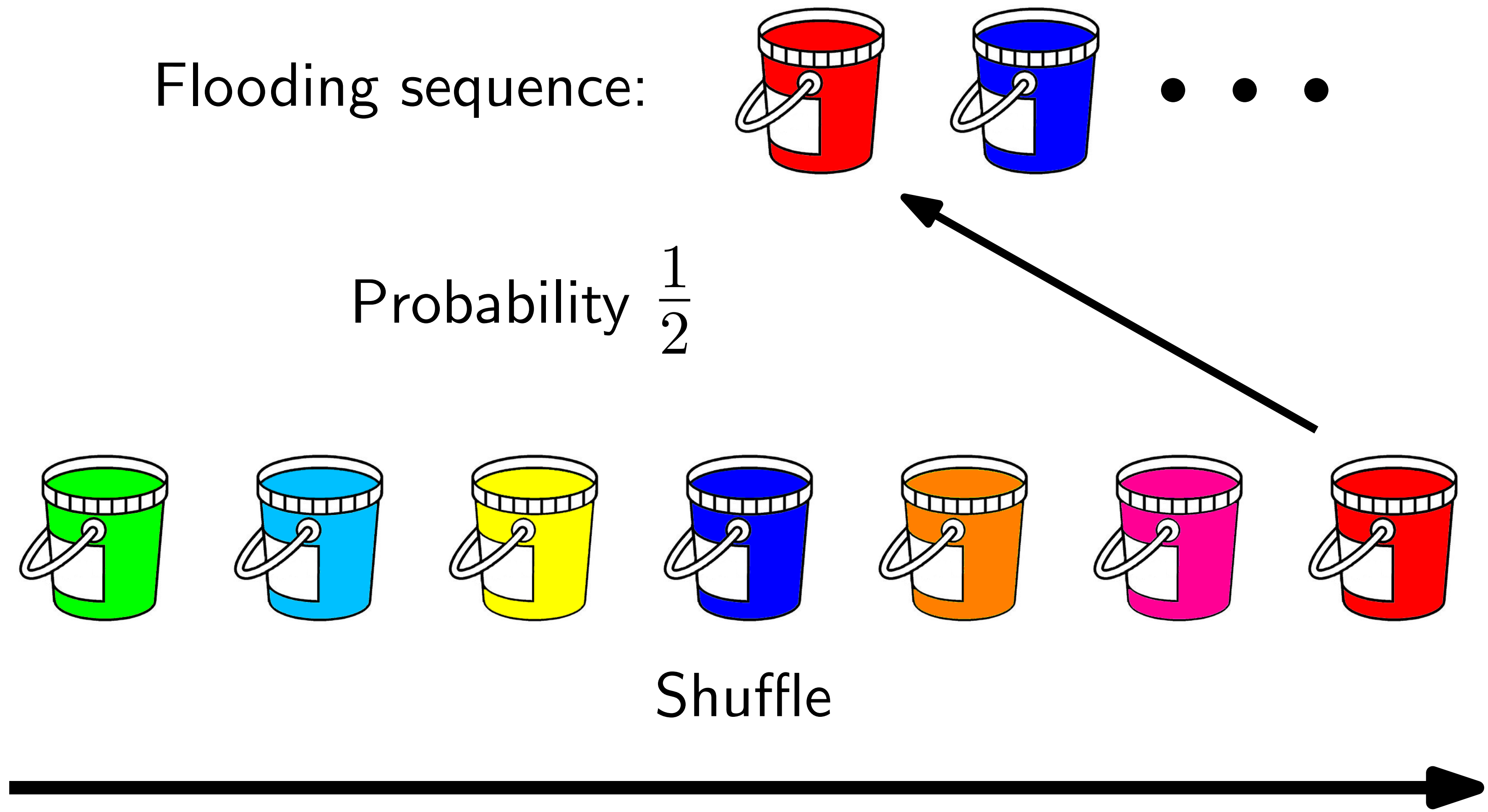
Flooding sequence: 

Shuffle

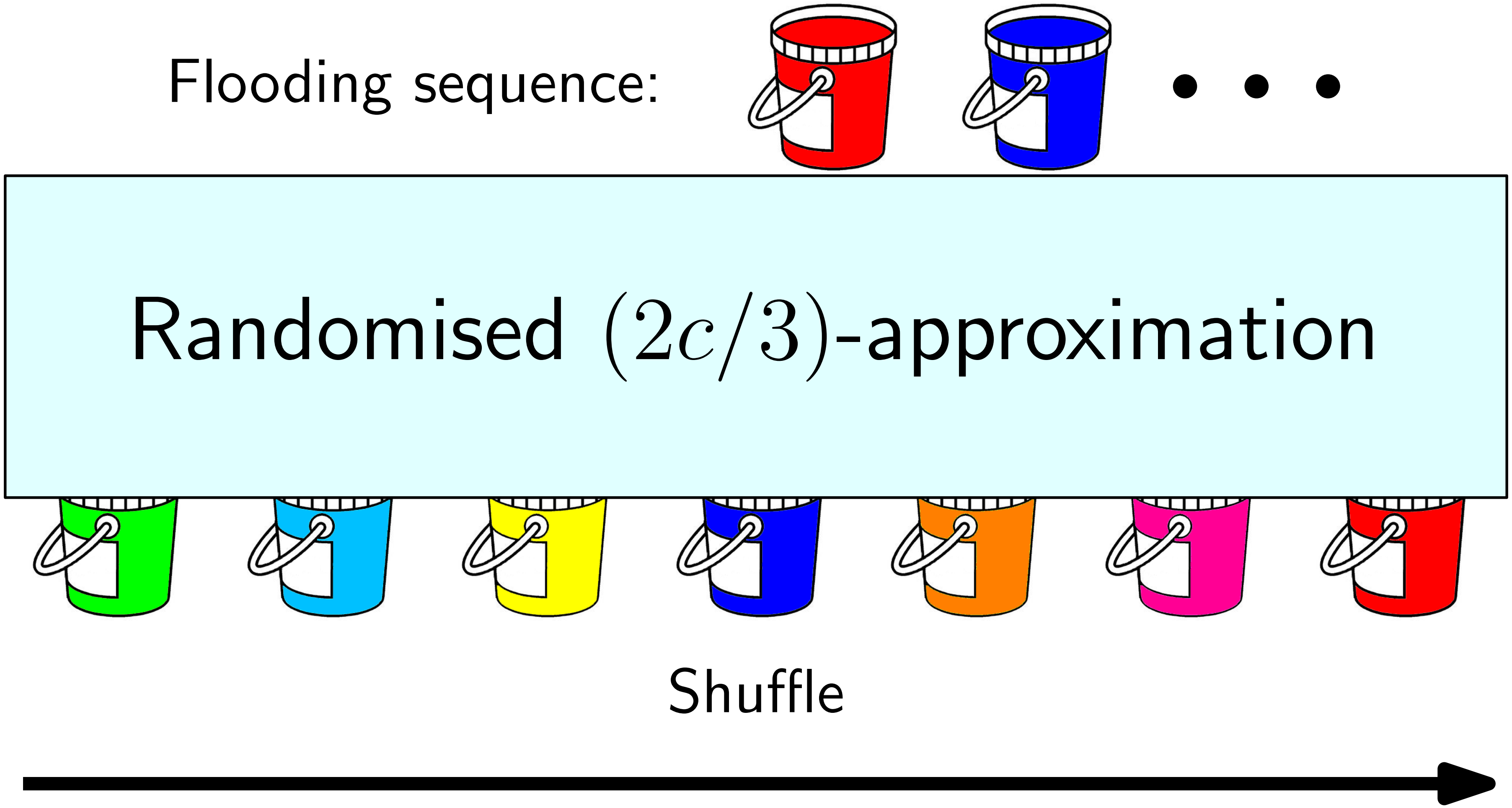
Randomised Approximation



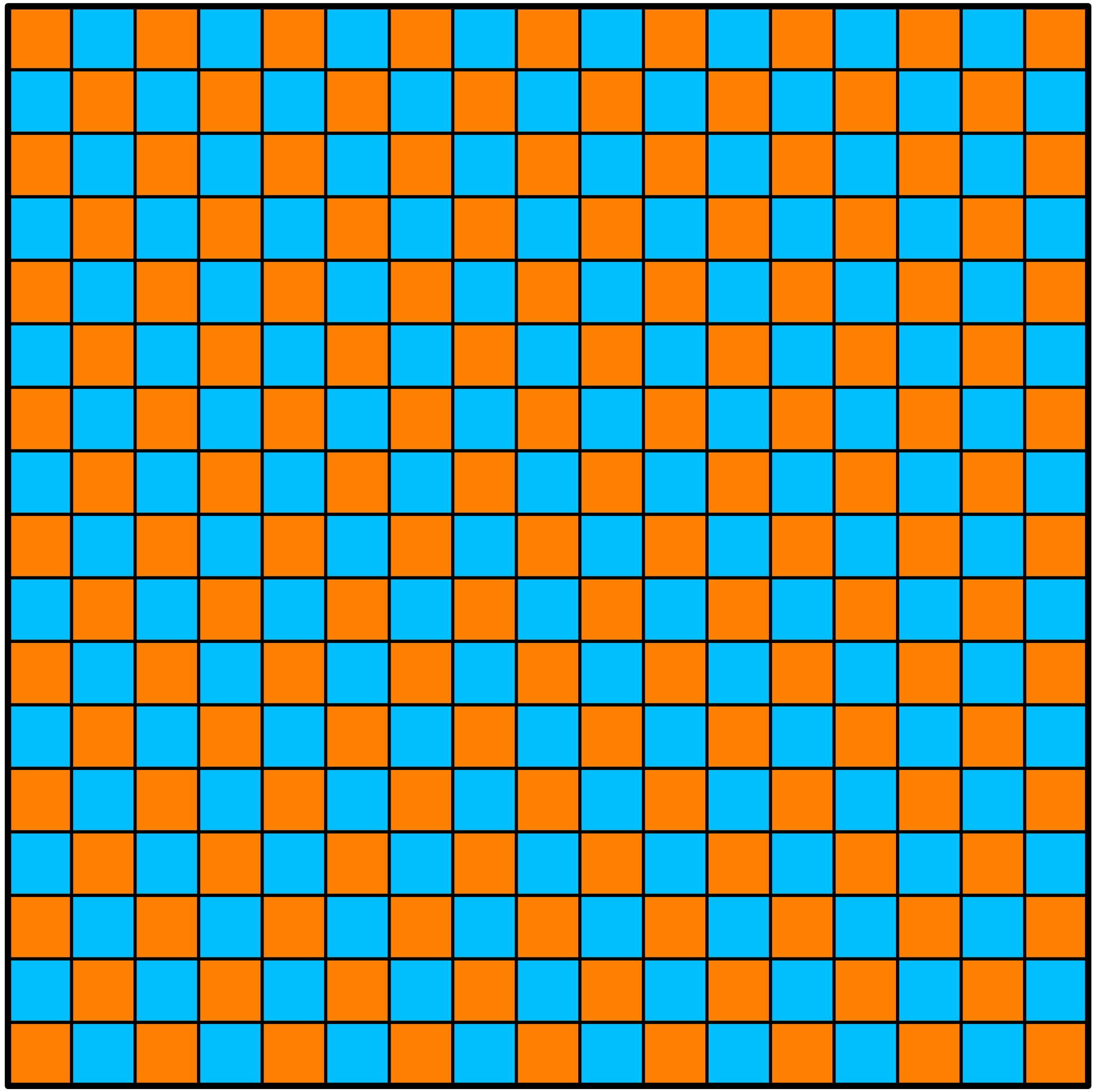
Randomised Approximation



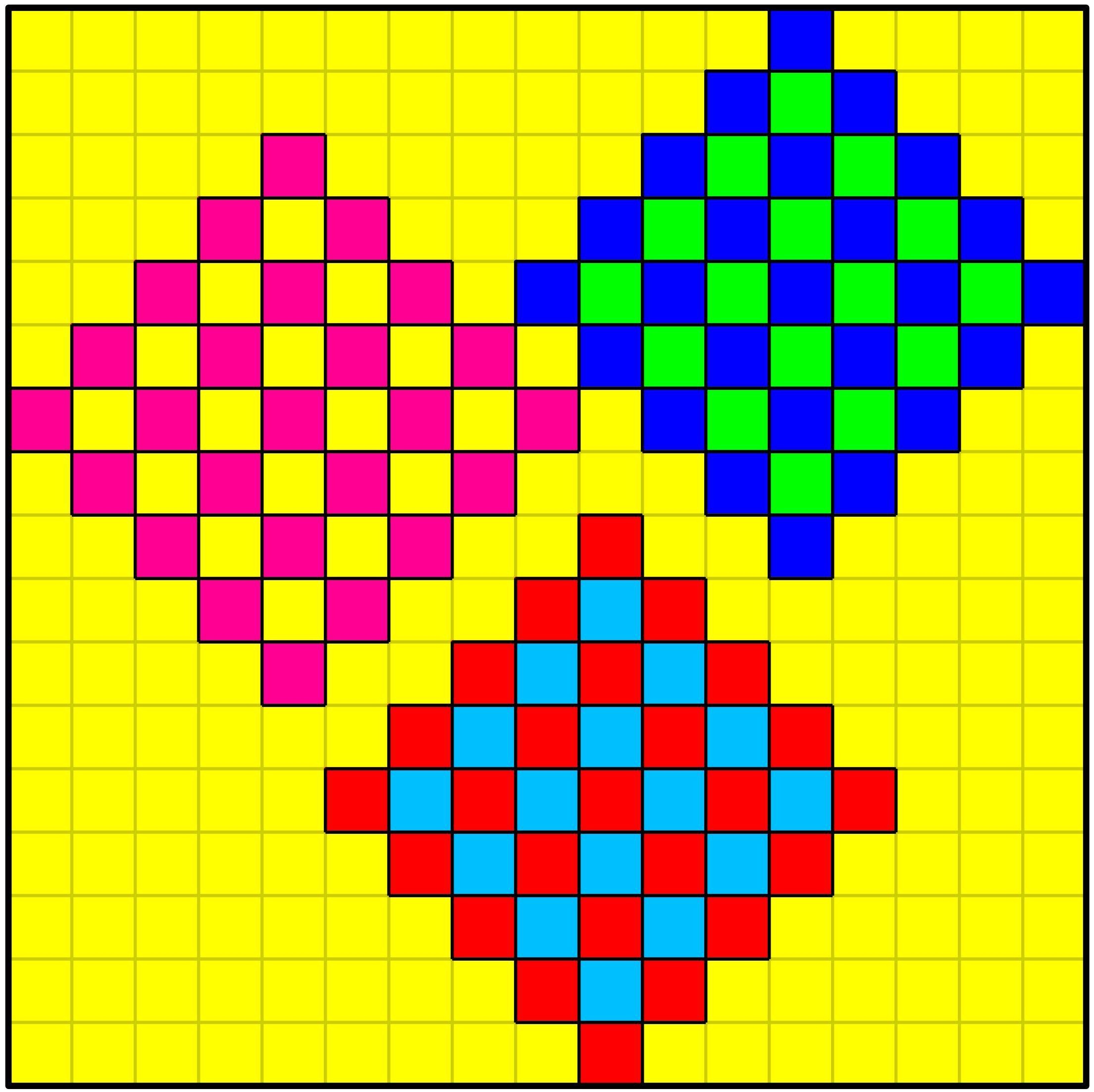
Randomised Approximation



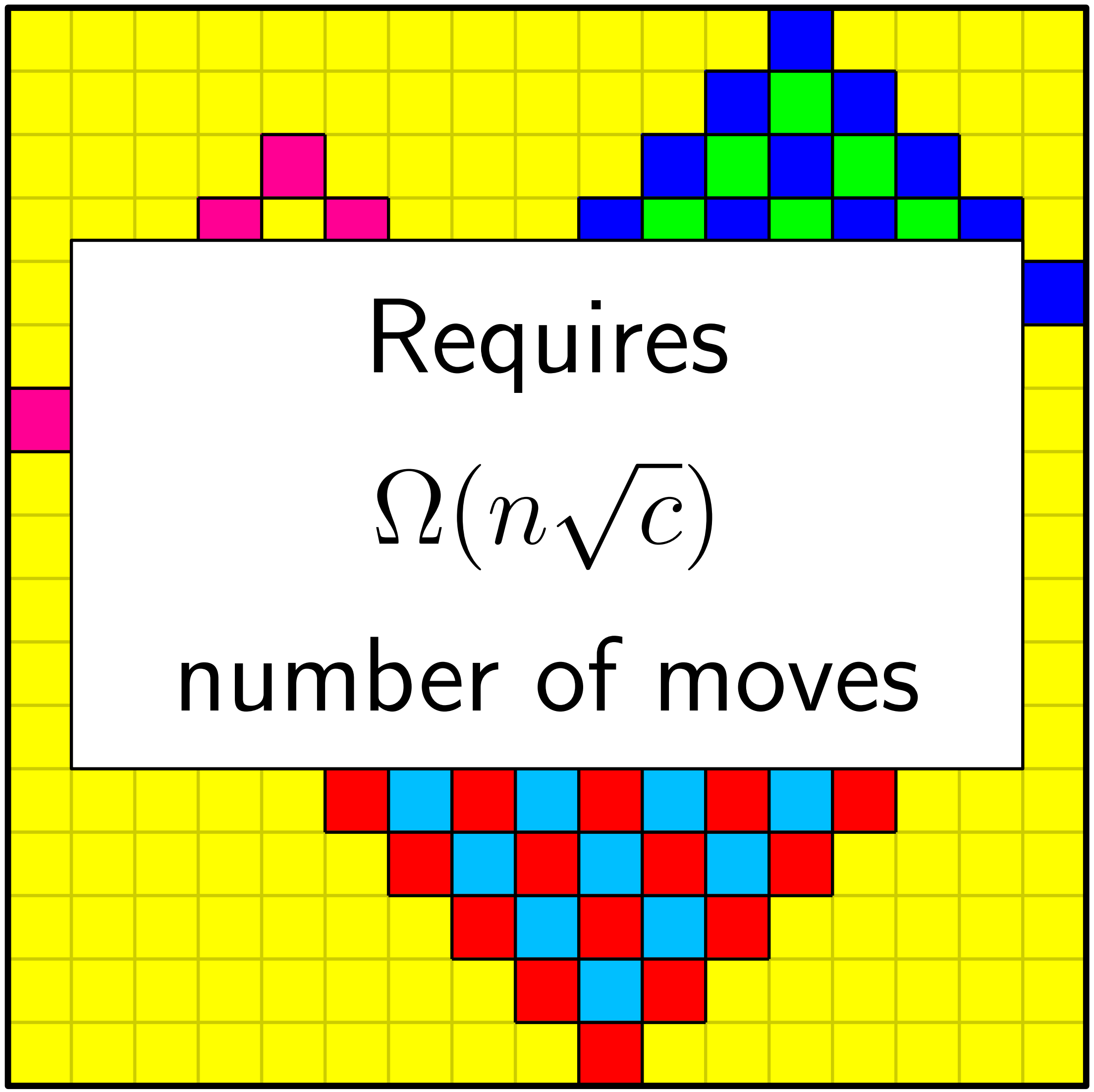
Bad Boards



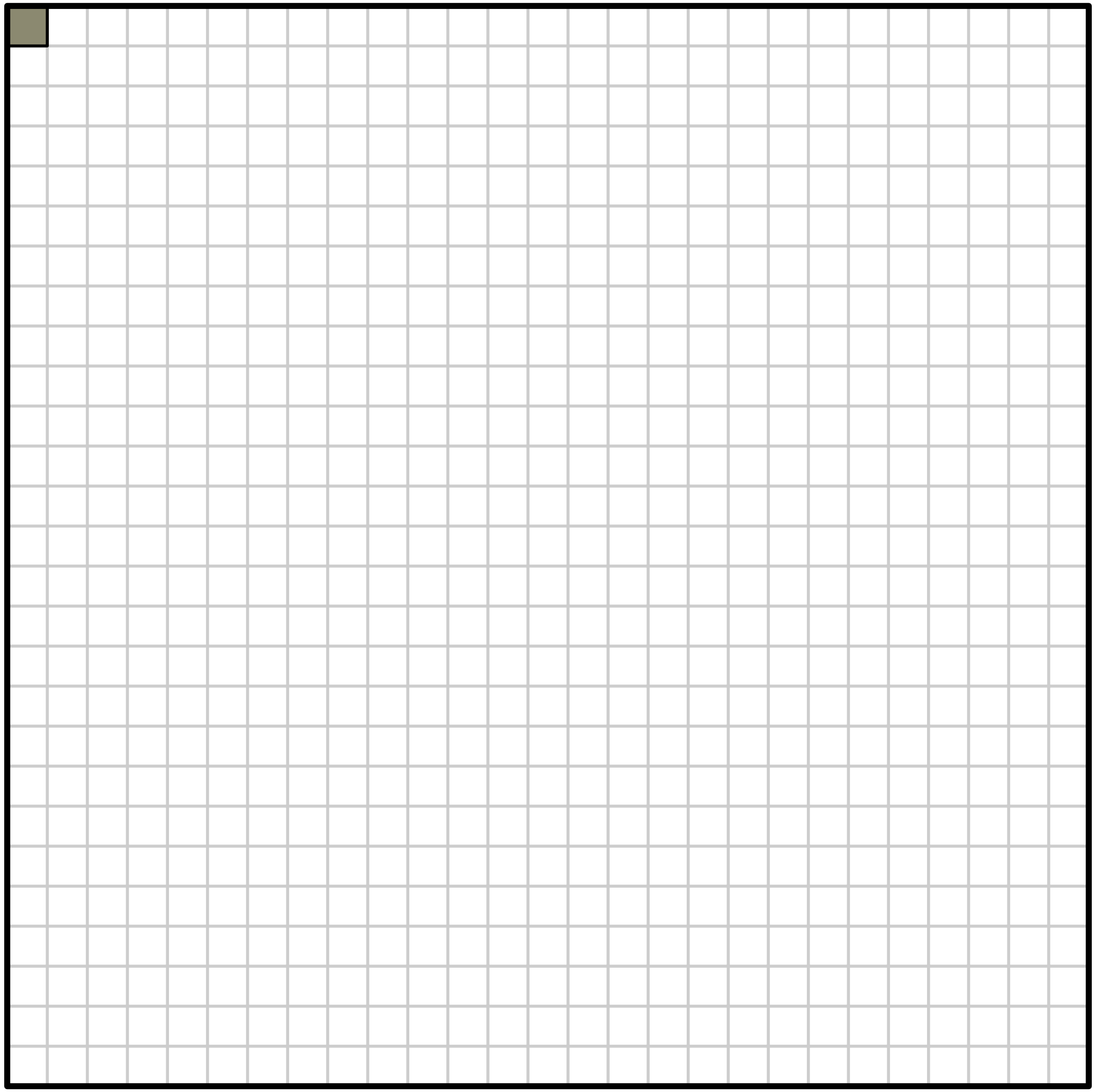
Bad Boards



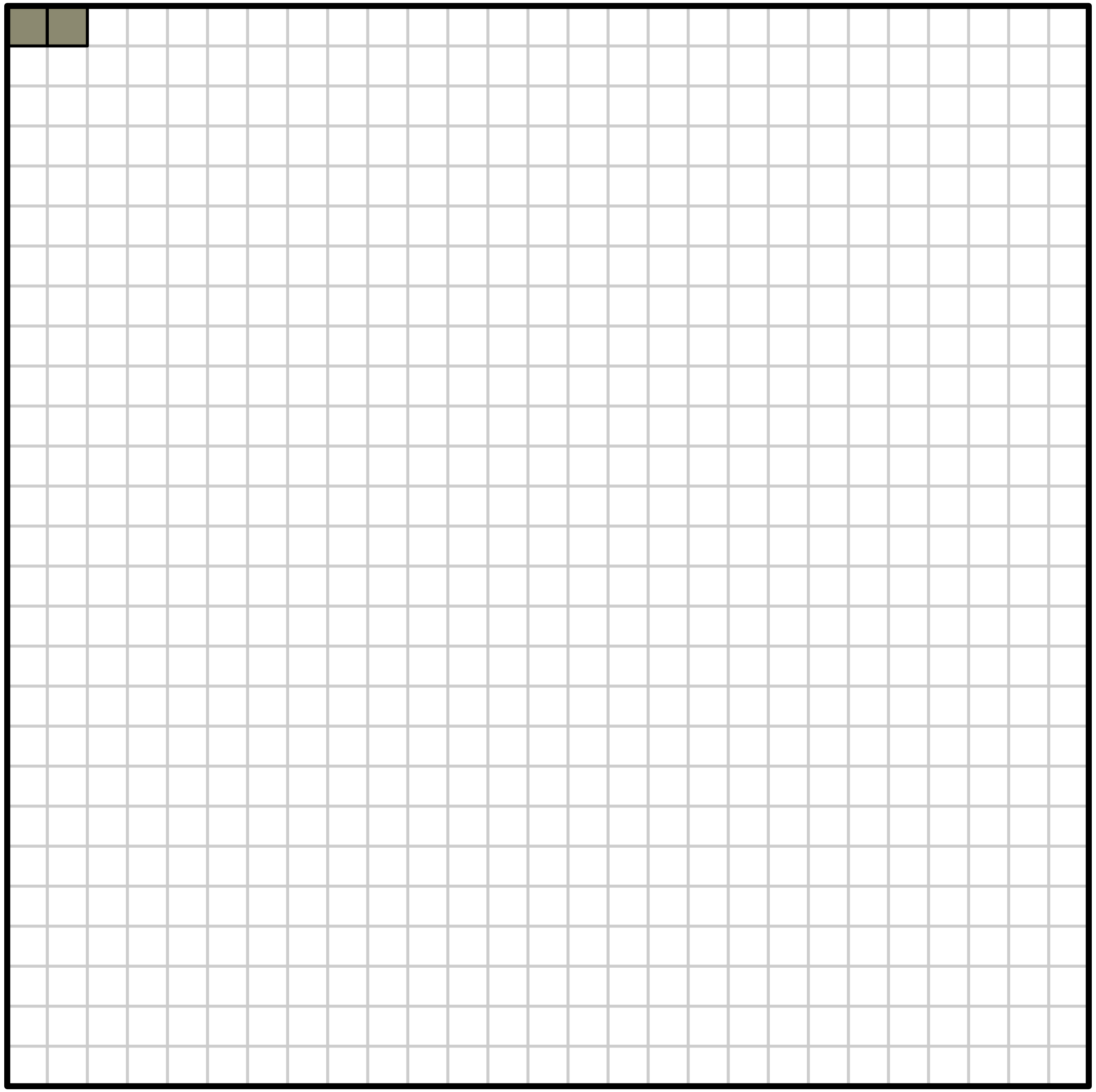
Bad Boards



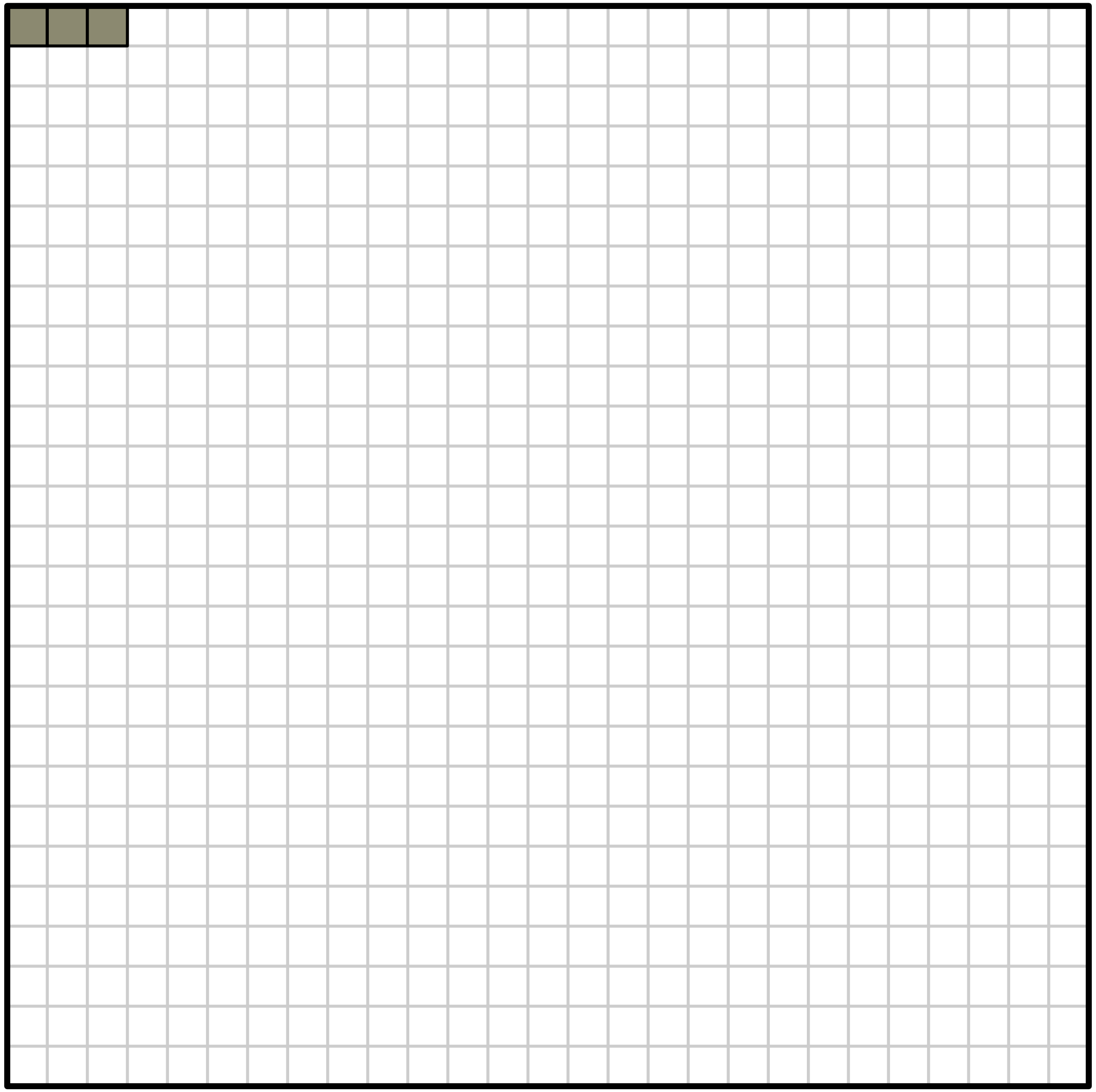
Upper Bound



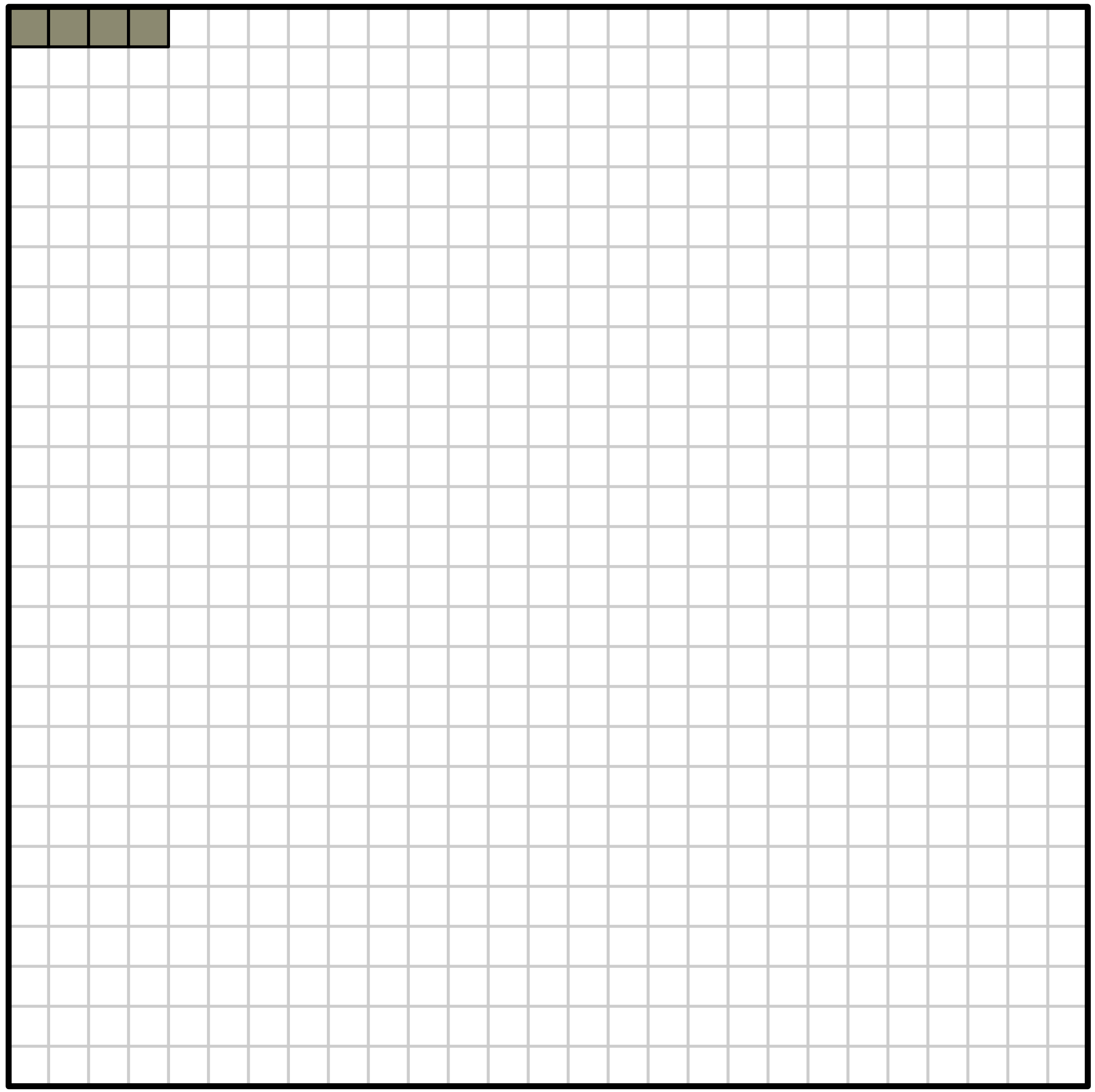
Upper Bound



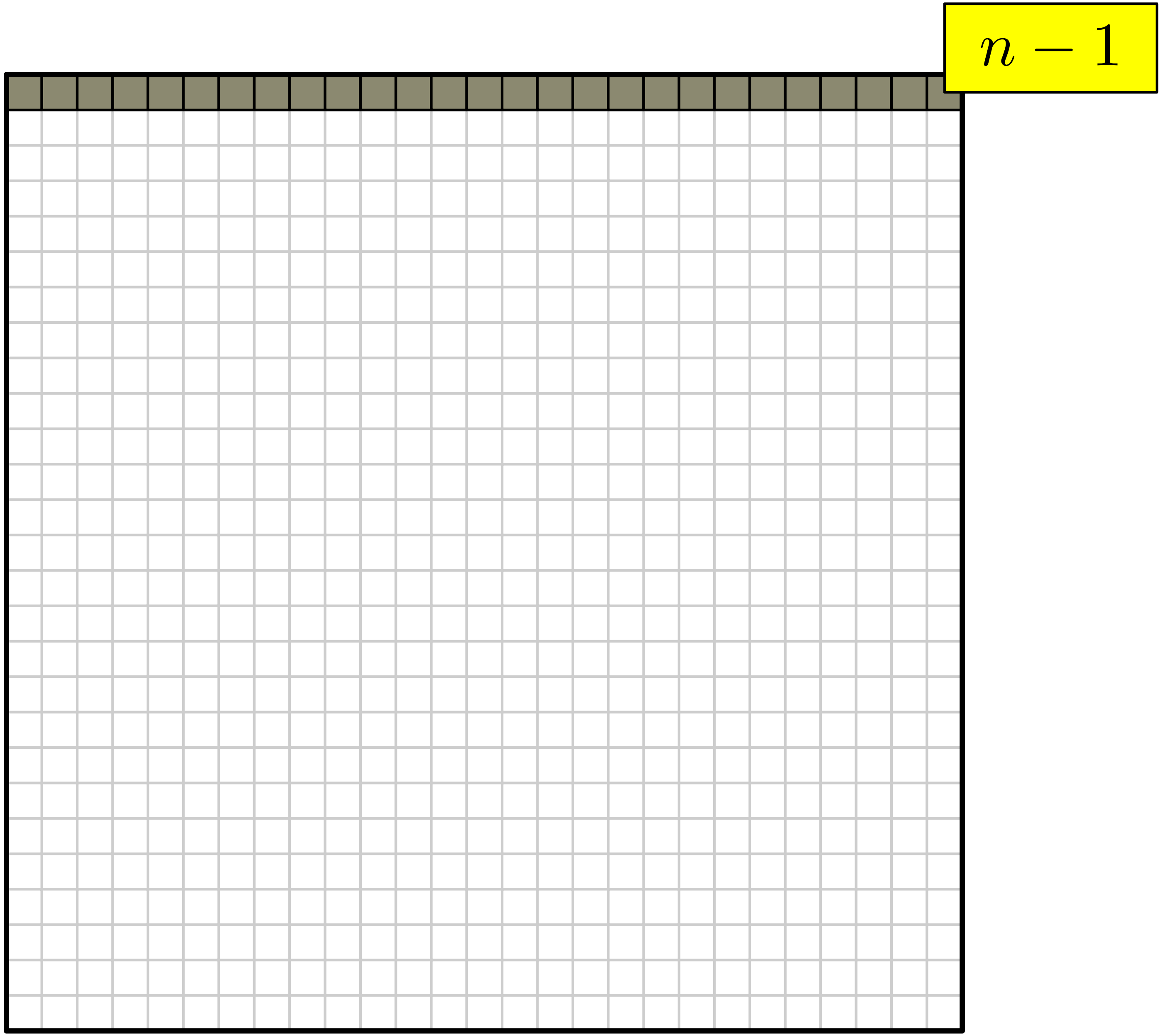
Upper Bound



Upper Bound



Upper Bound



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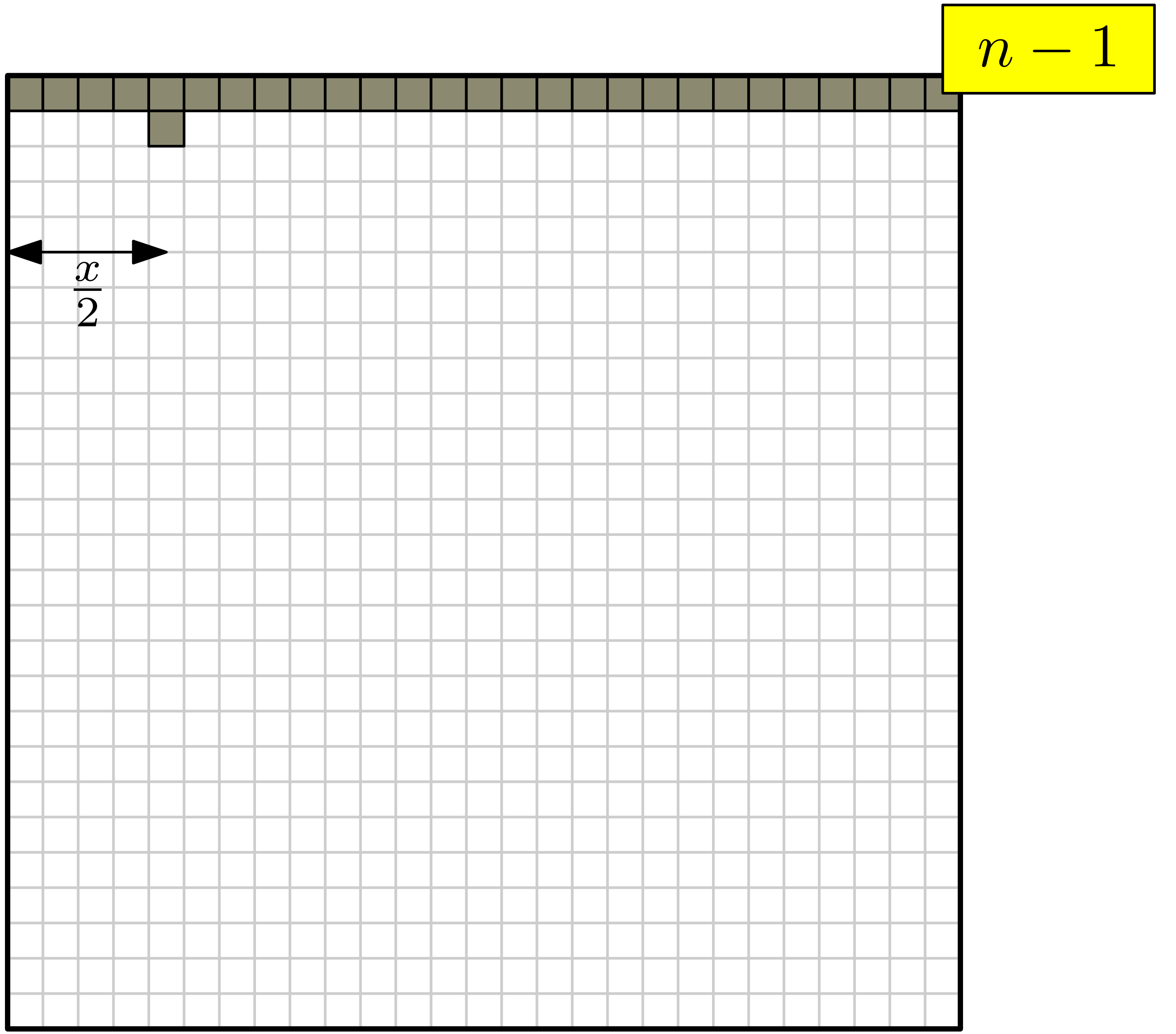

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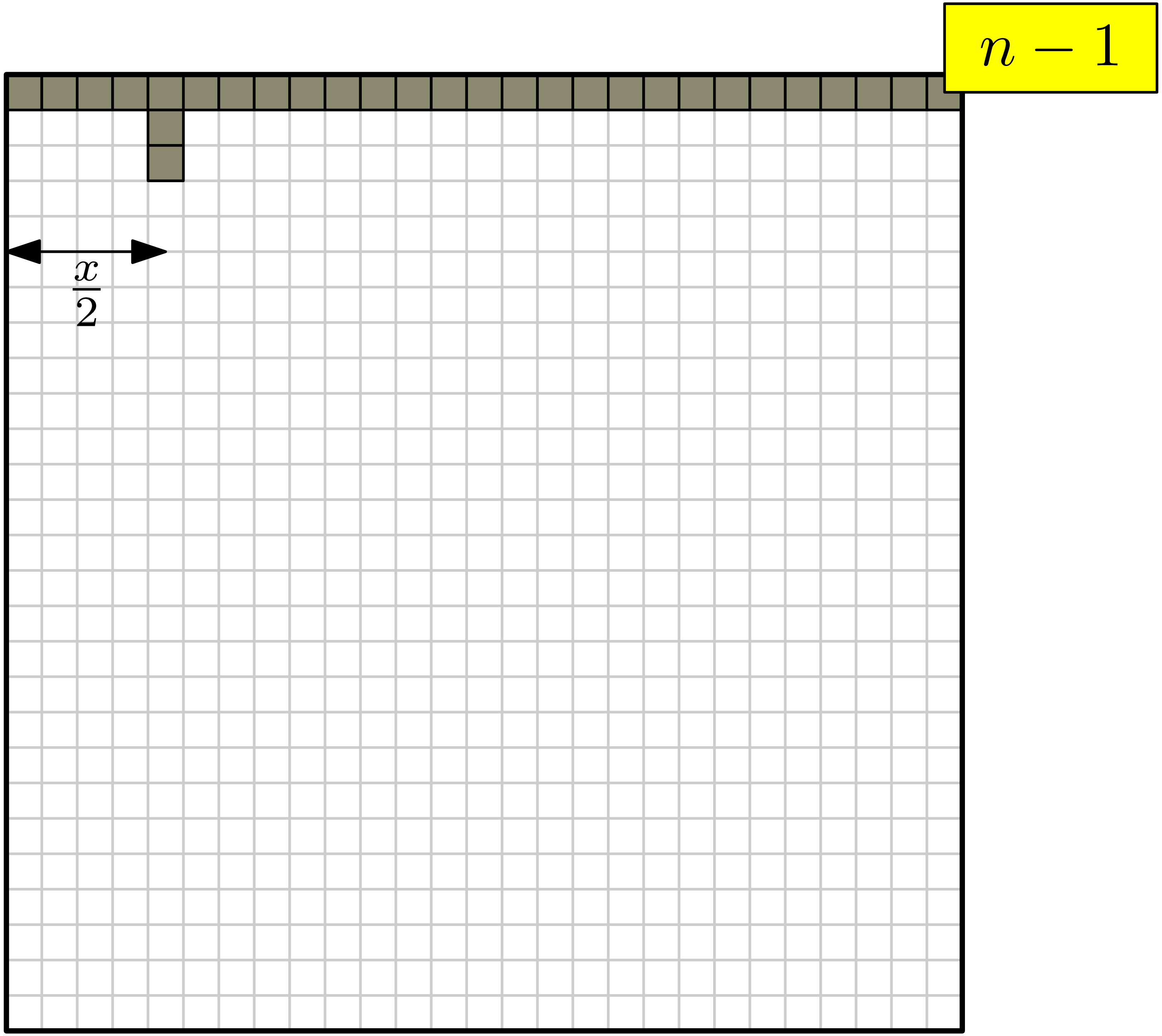

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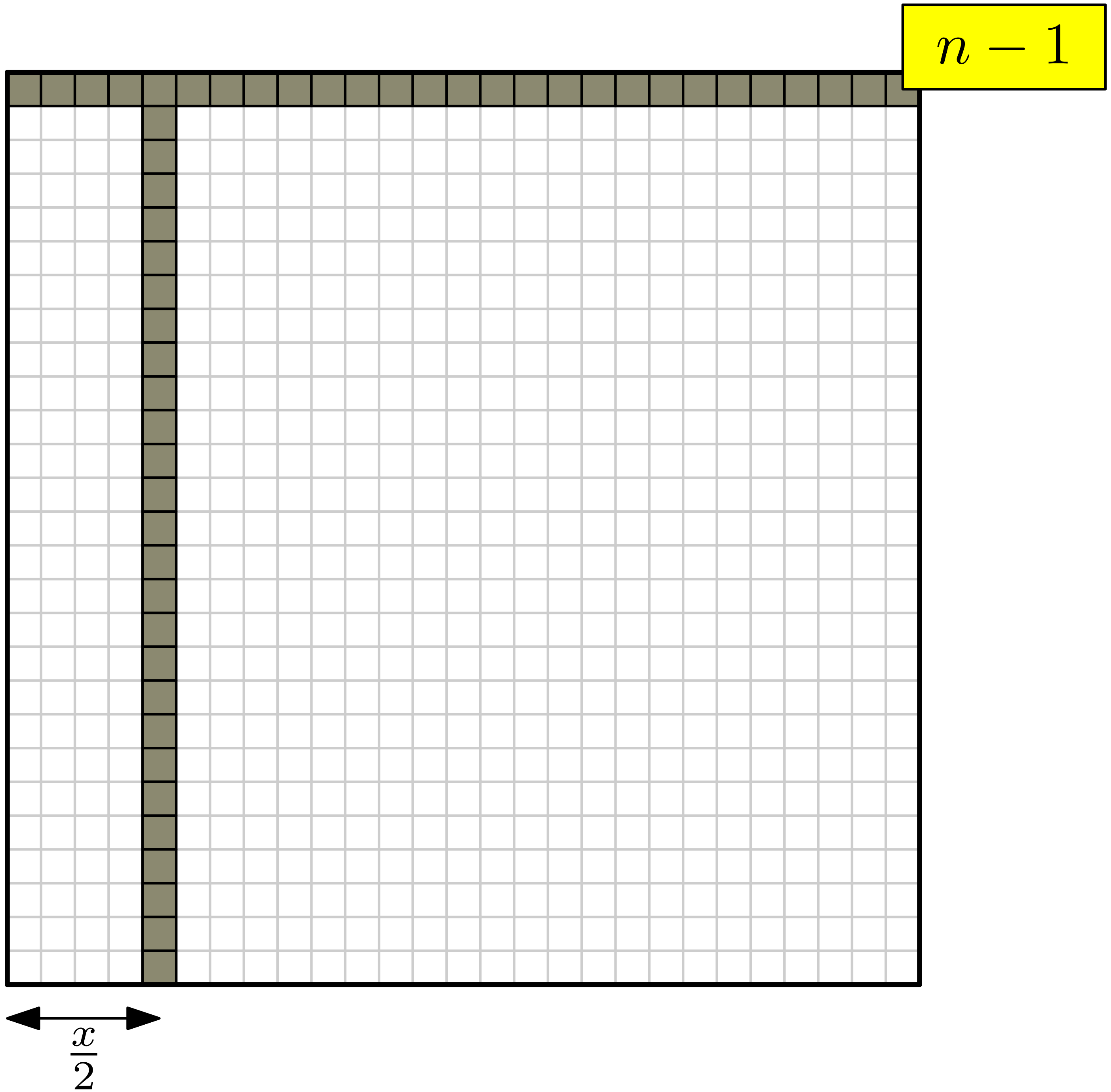

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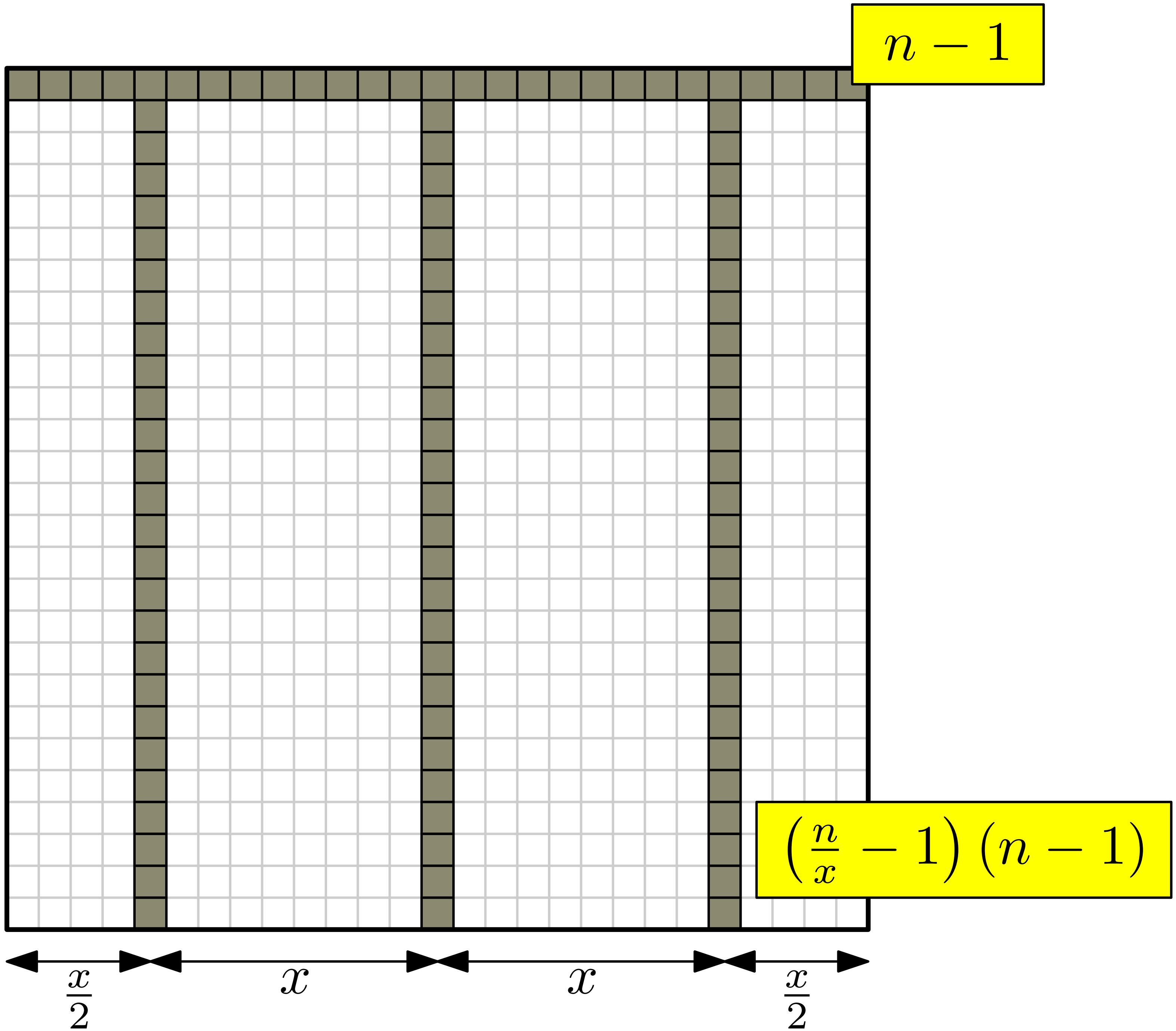

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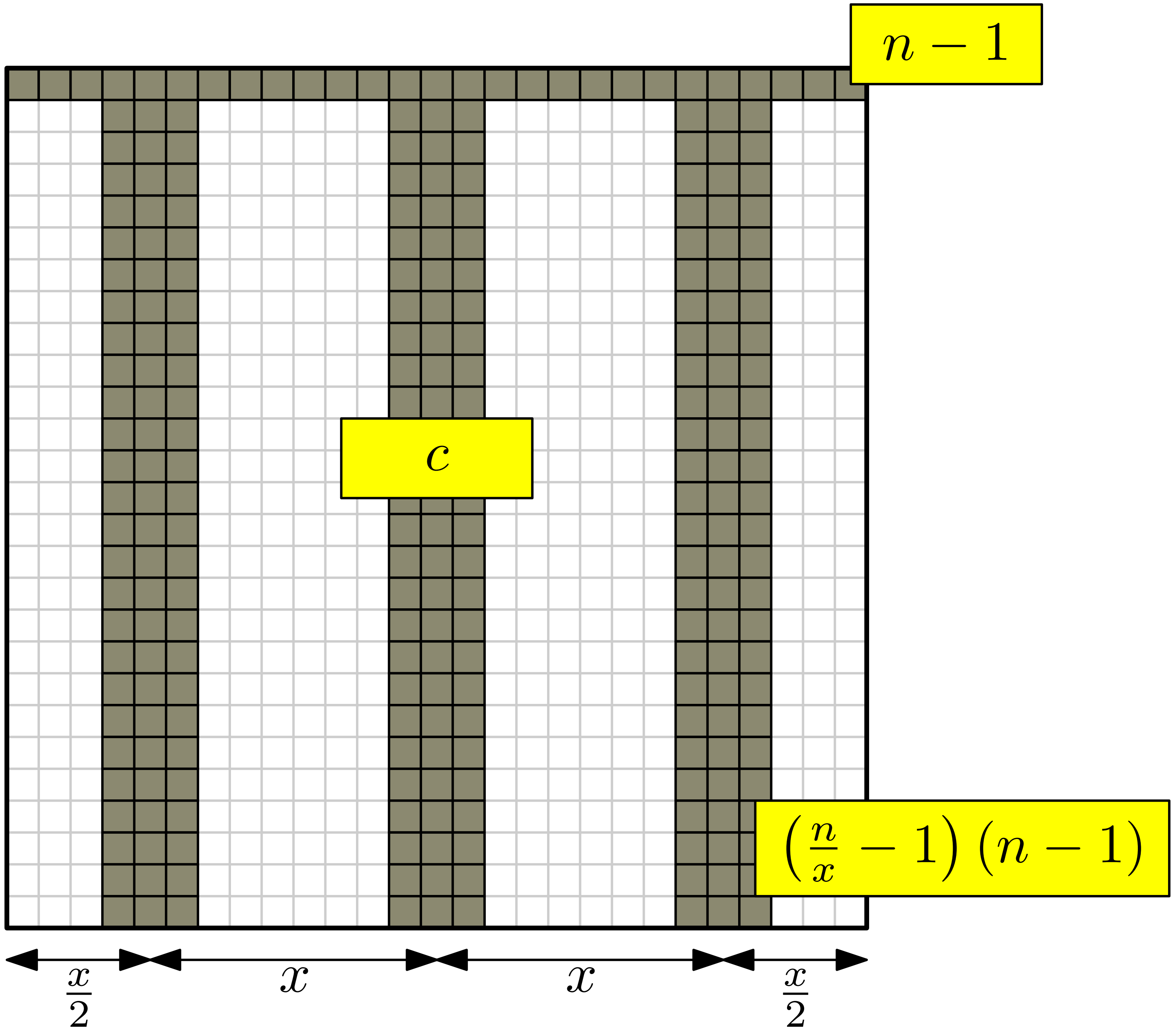
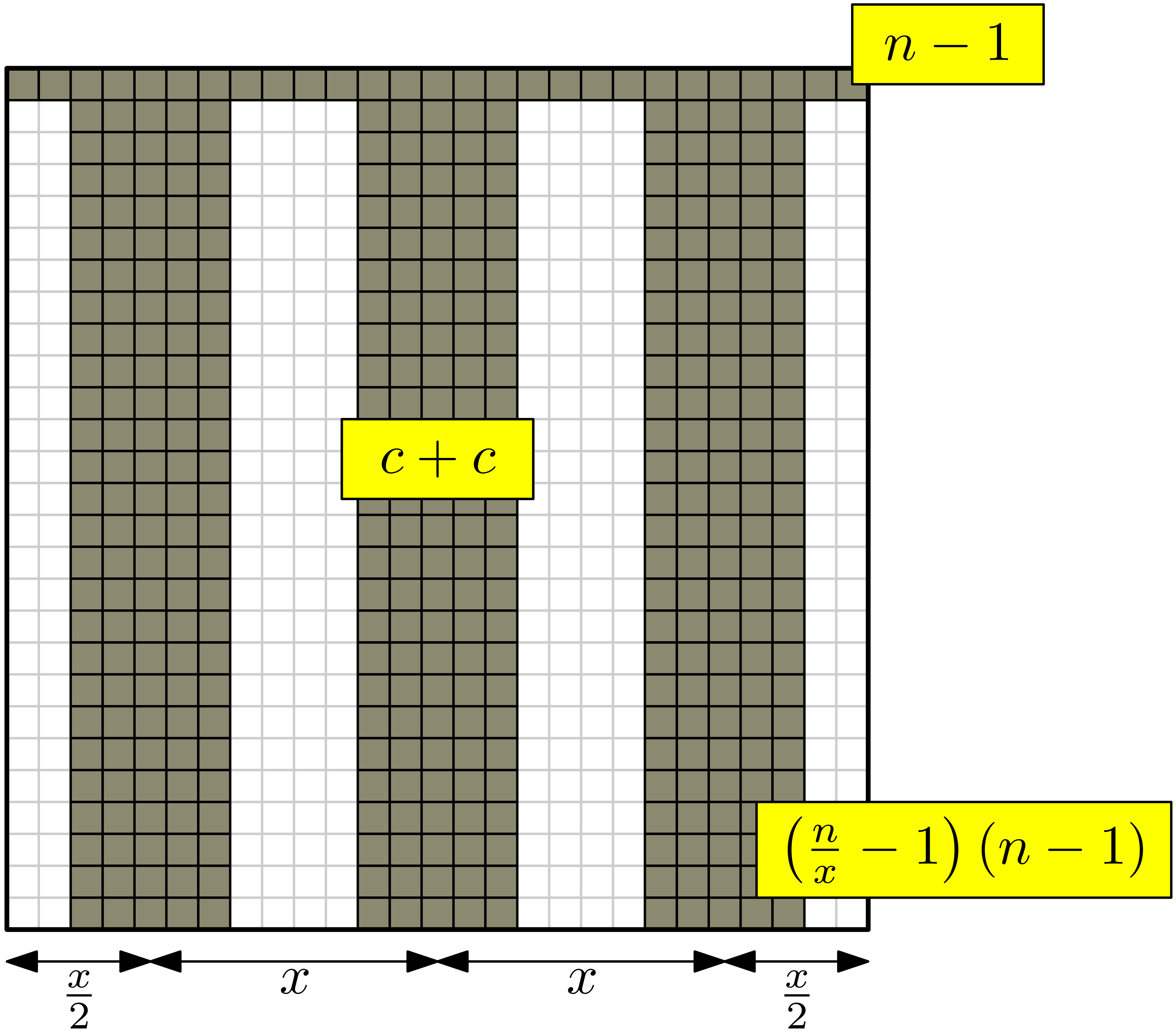
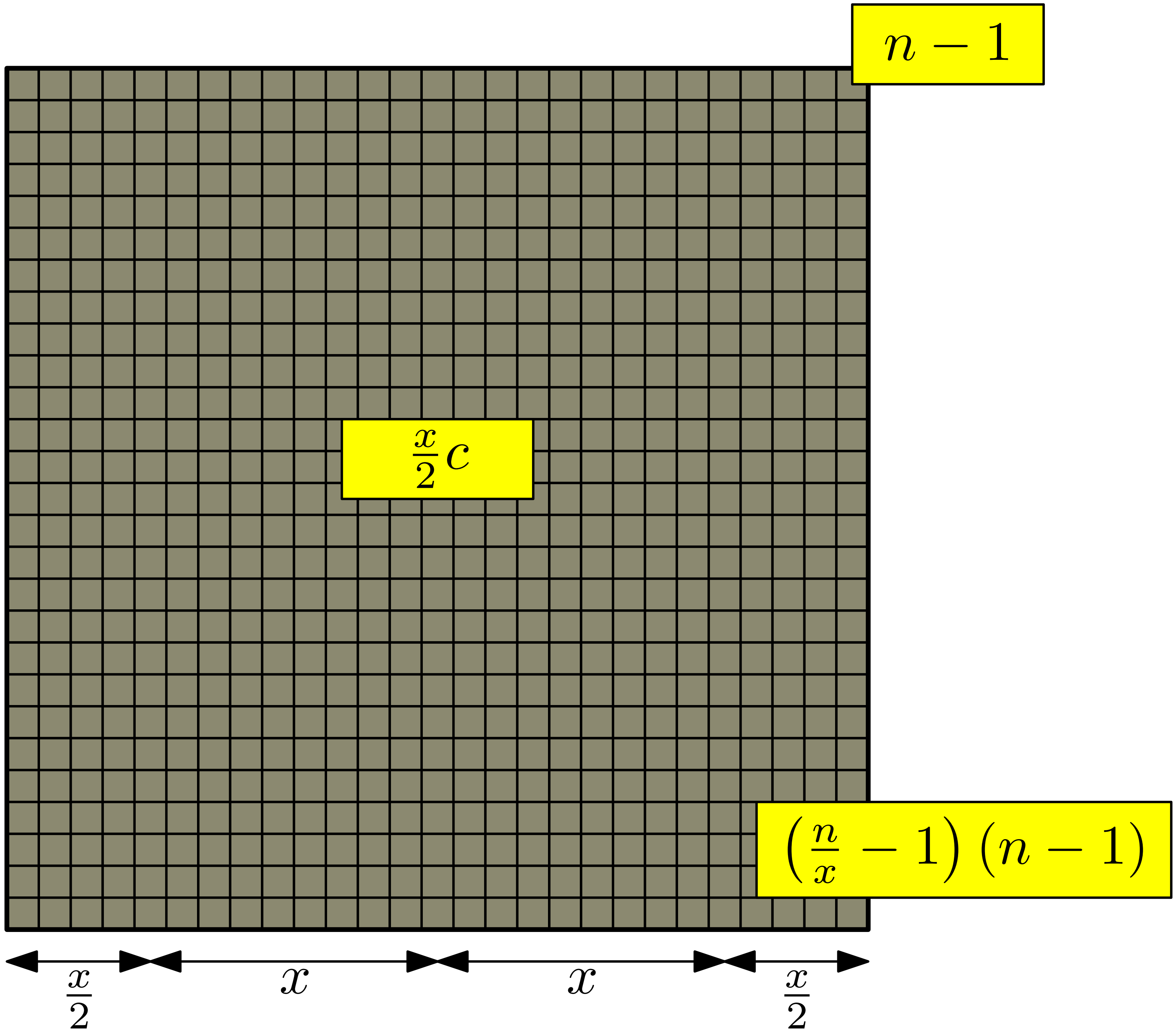


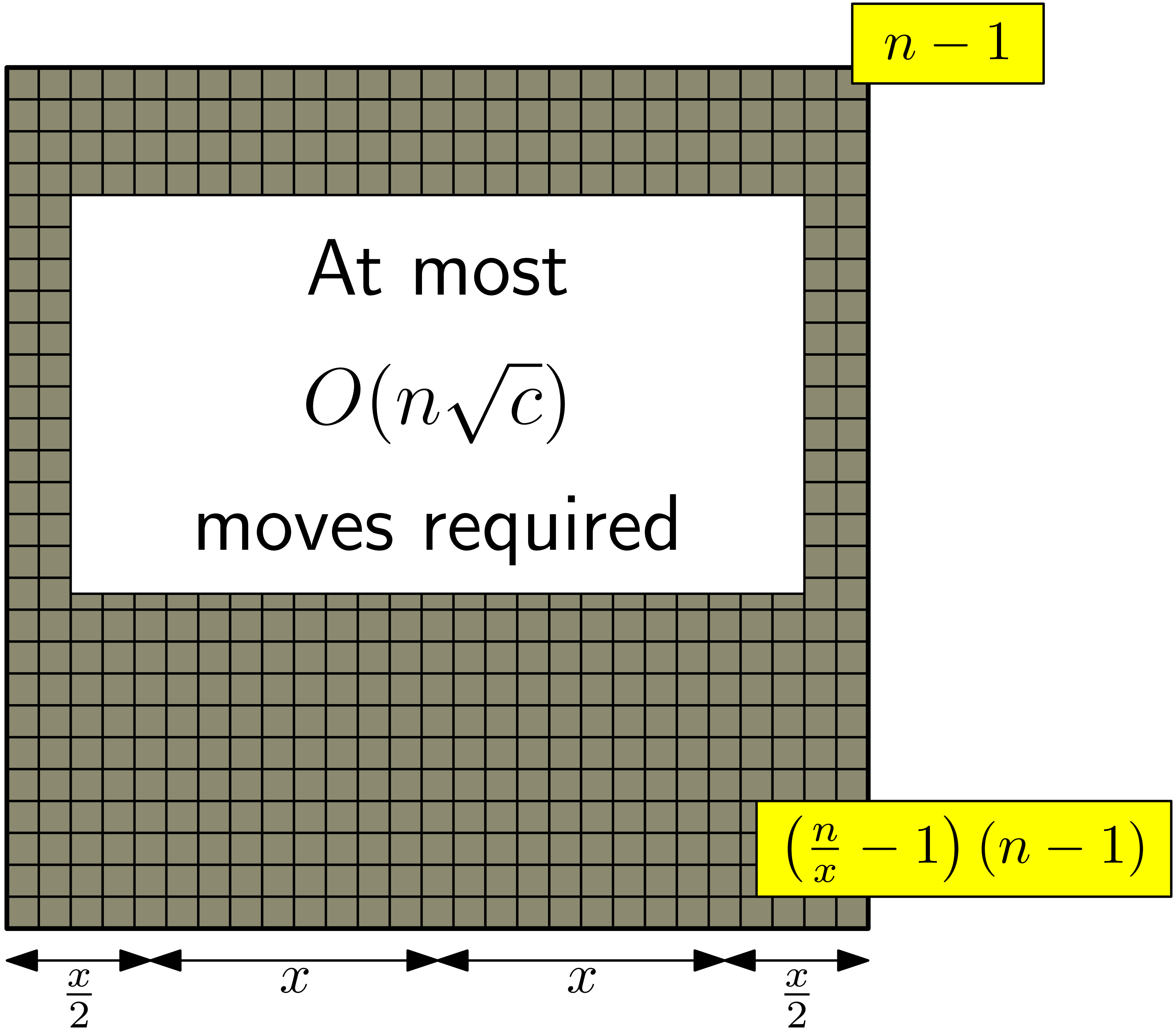
Upper Bound



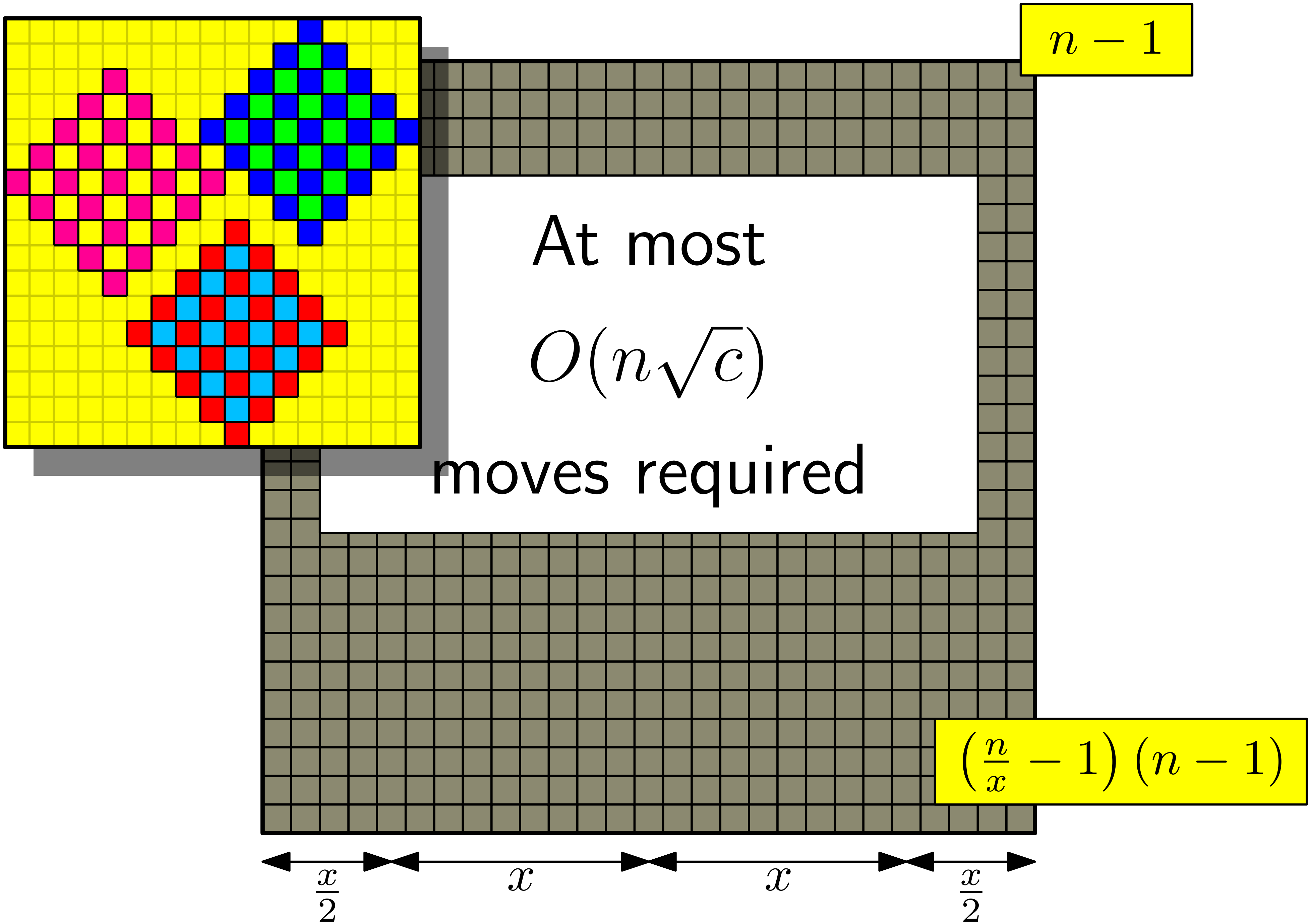
Upper Bound



Upper Bound

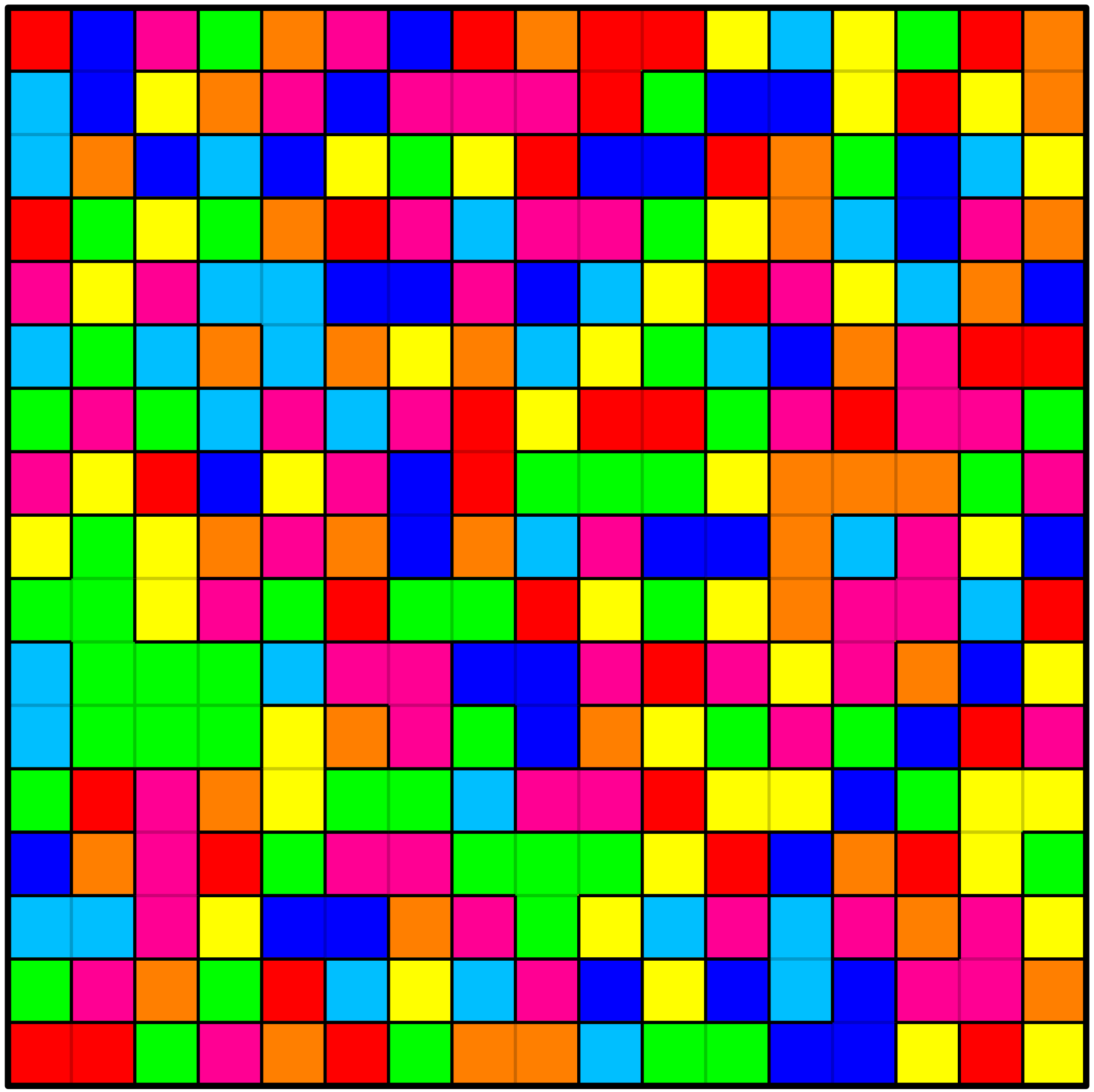

$n-1$
At most
$O(n\sqrt{c})$
moves required
$\left(\frac{n}{x}-1\right)(n-1)$
$\frac{x}{2}$
x
x
$\frac{x}{2}$

Upper Bound

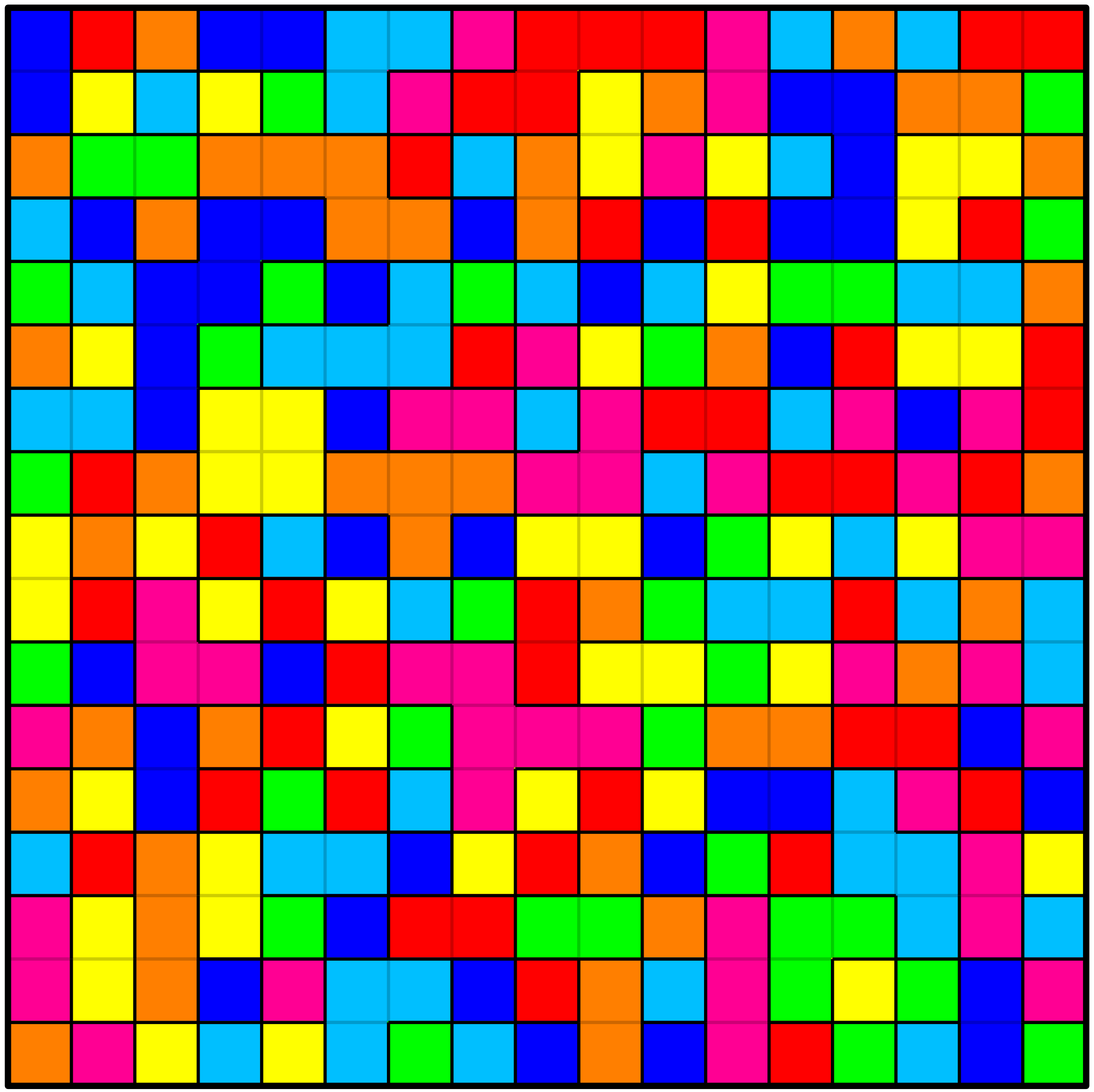


$n-1$
At most
$O(n\sqrt{c})$
moves required
$\left(\frac{n}{x}-1\right)(n-1)$
$\frac{x}{2}$
x
x
$\frac{x}{2}$

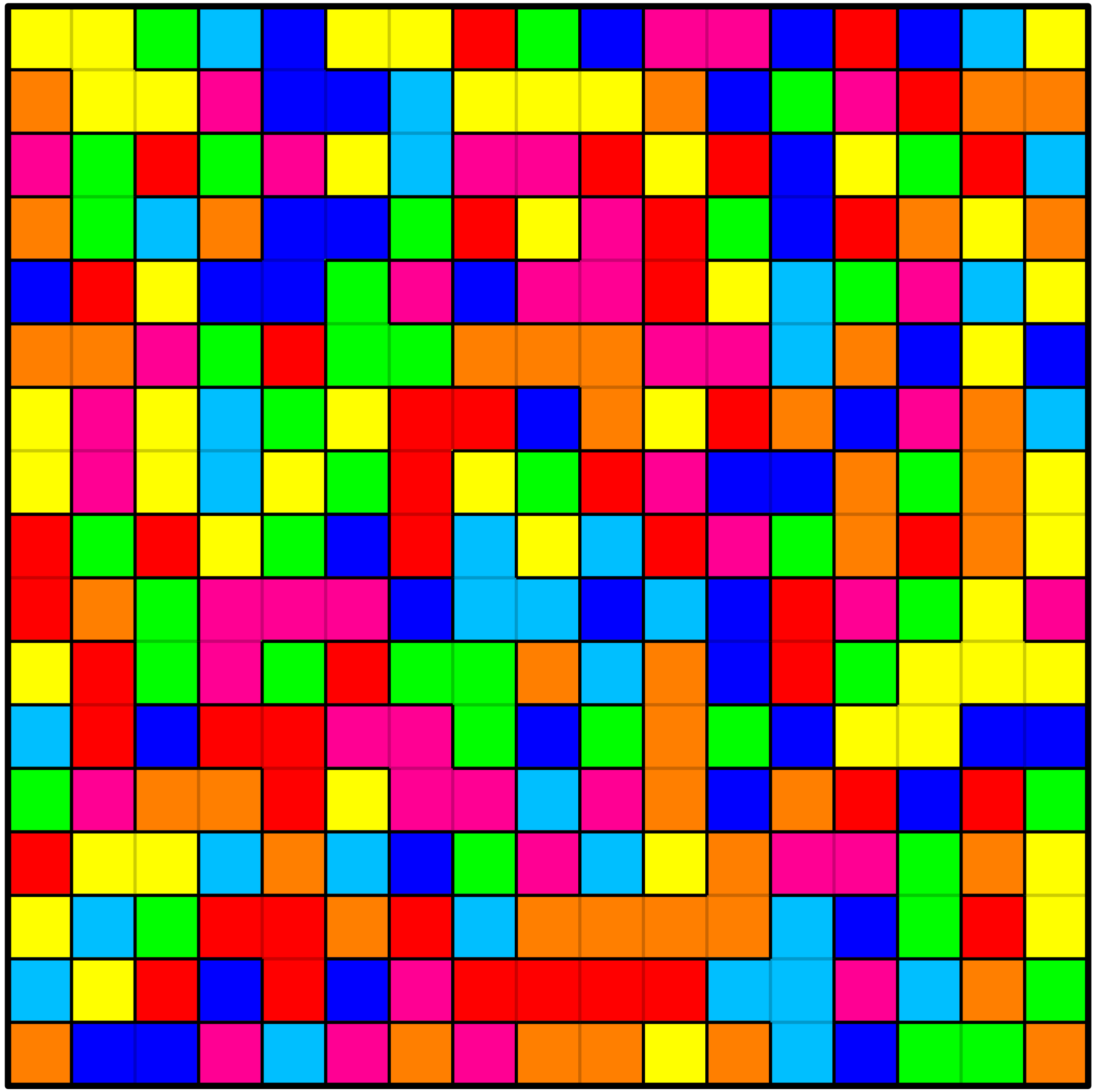
Random Boards



Random Boards

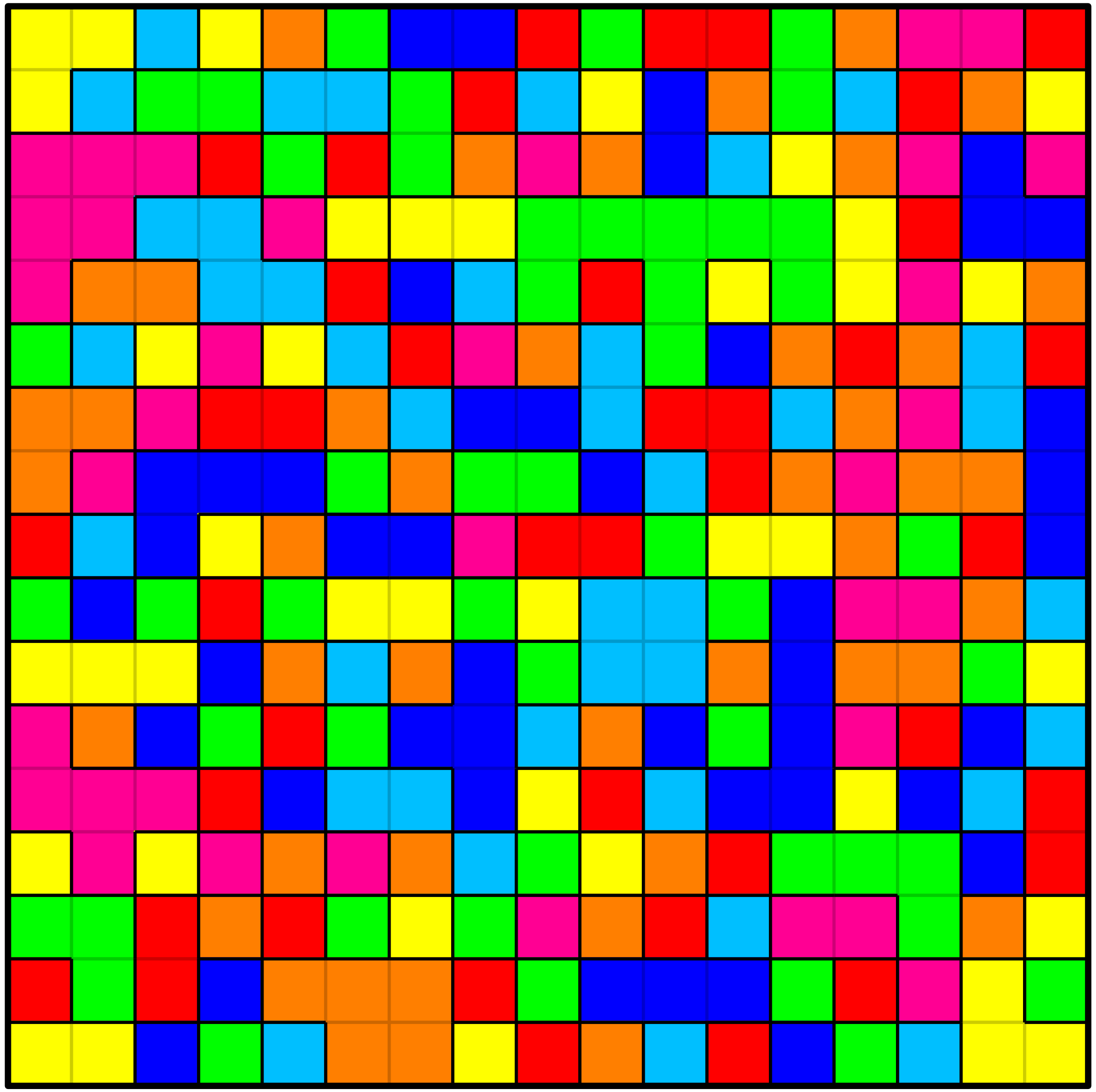


Random Boards



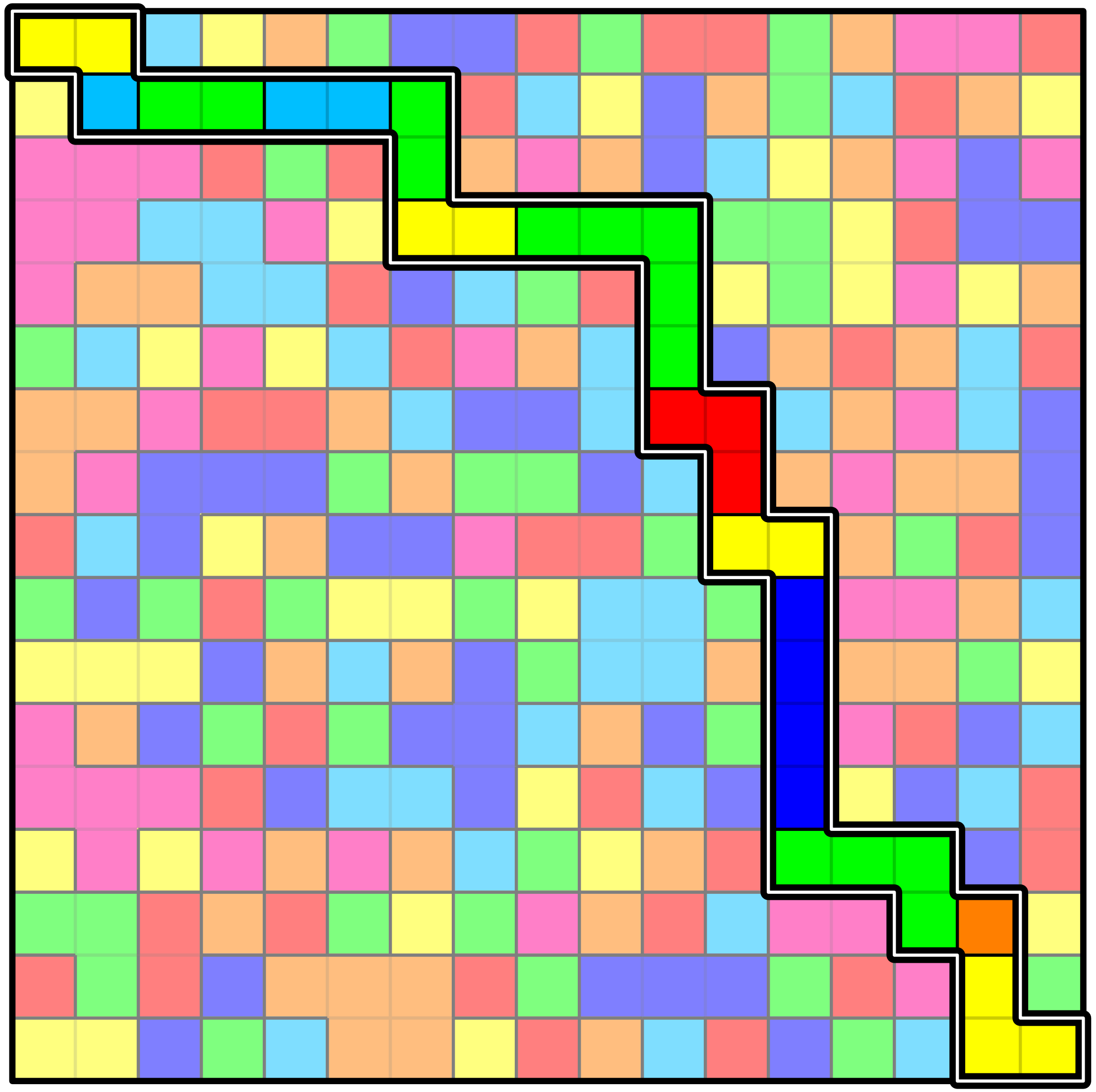
Random Boards

m moves to flood the board



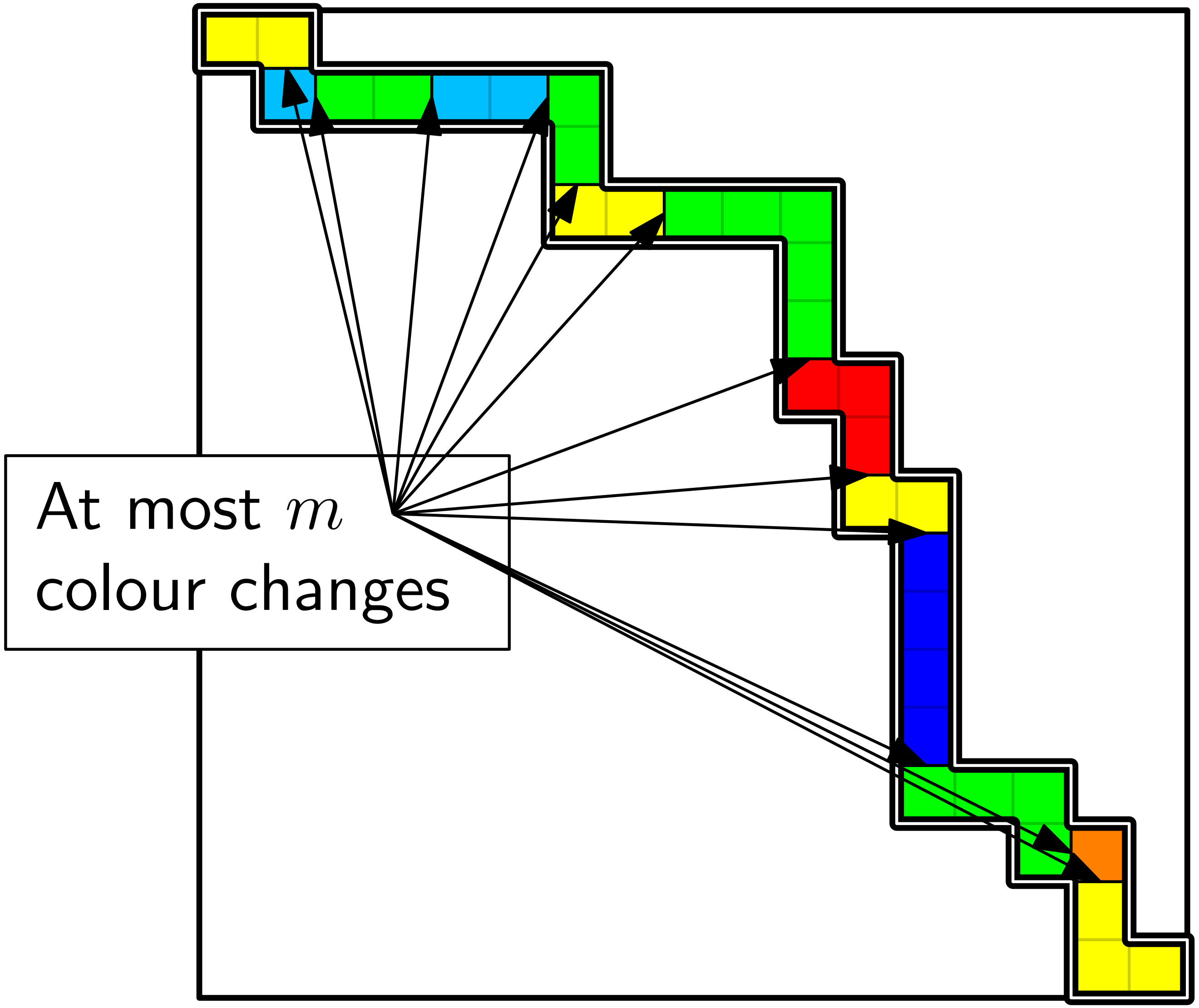
Random Boards

m moves to flood the board



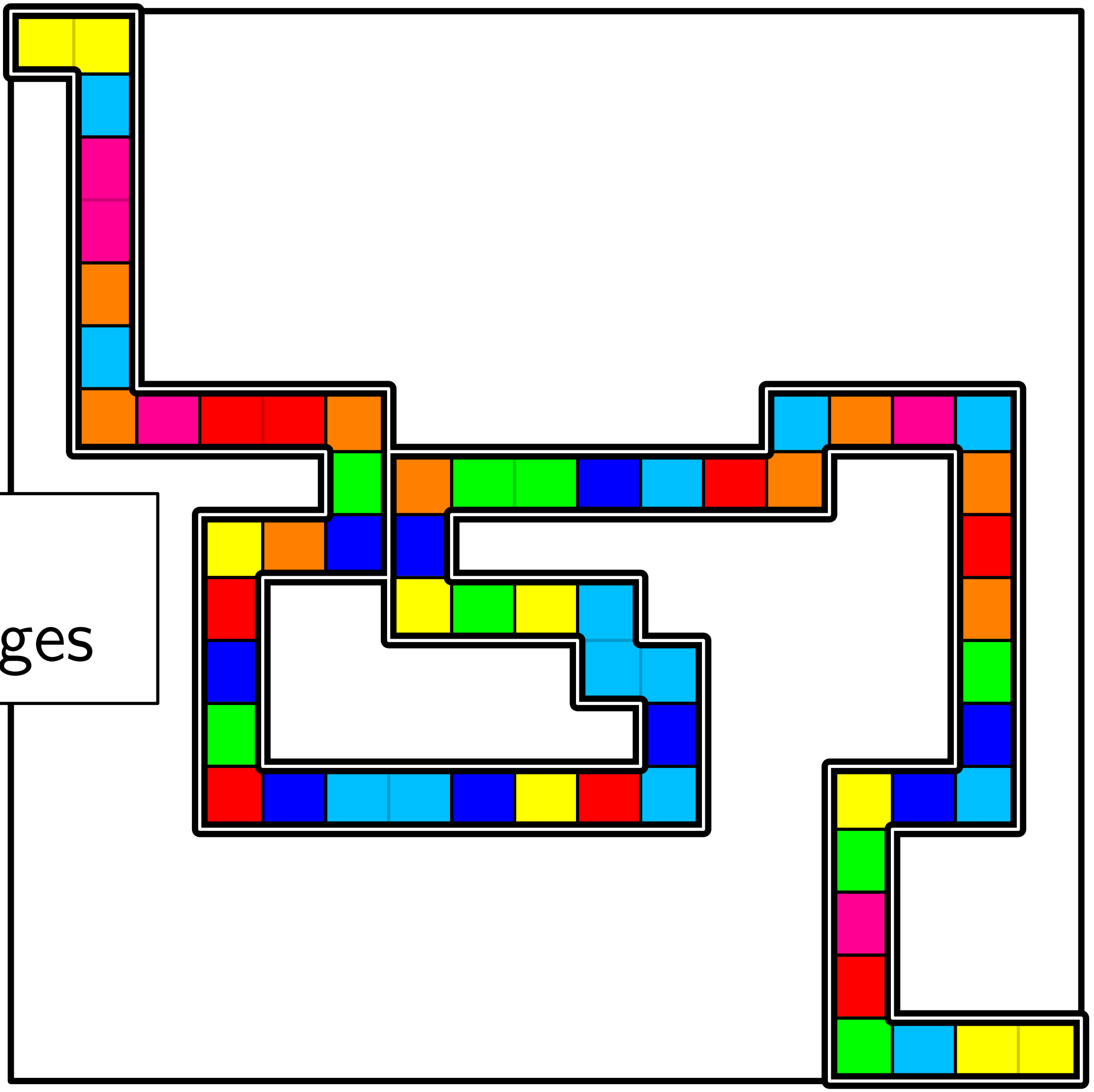
Random Boards

m moves to flood the board



Random Boards

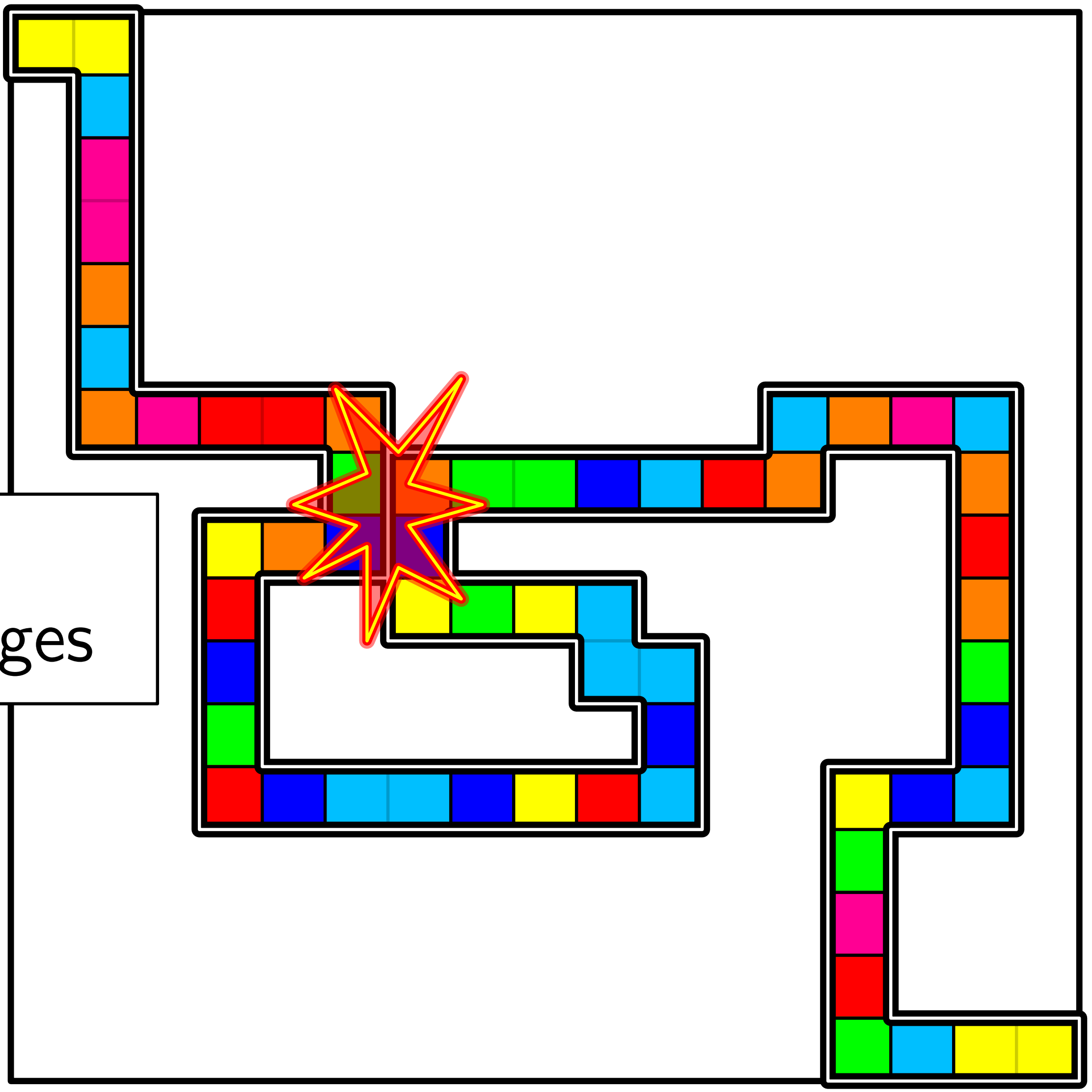
m moves to flood the board



At most m colour changes

Random Boards

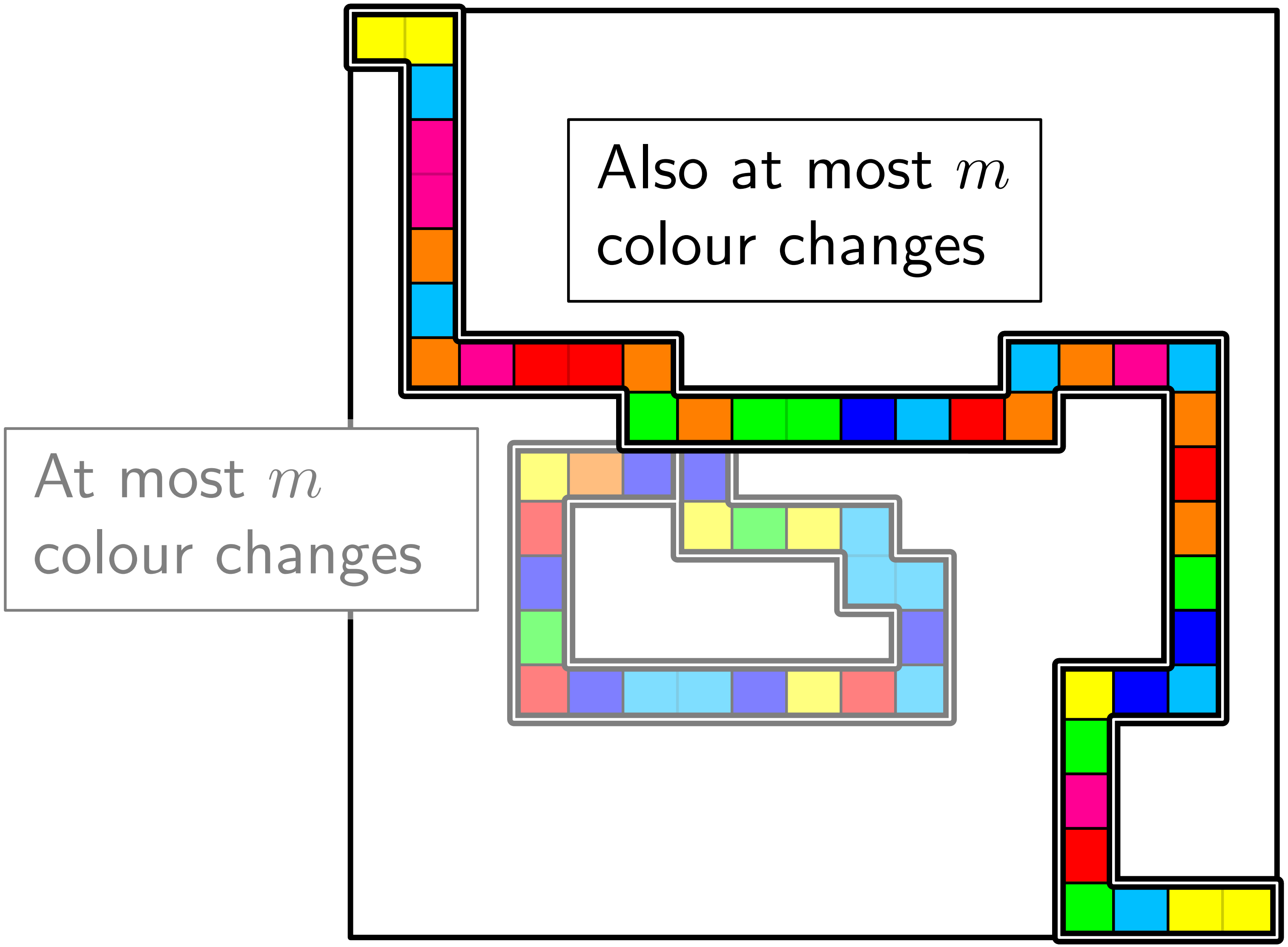
m moves to flood the board



At most m colour changes

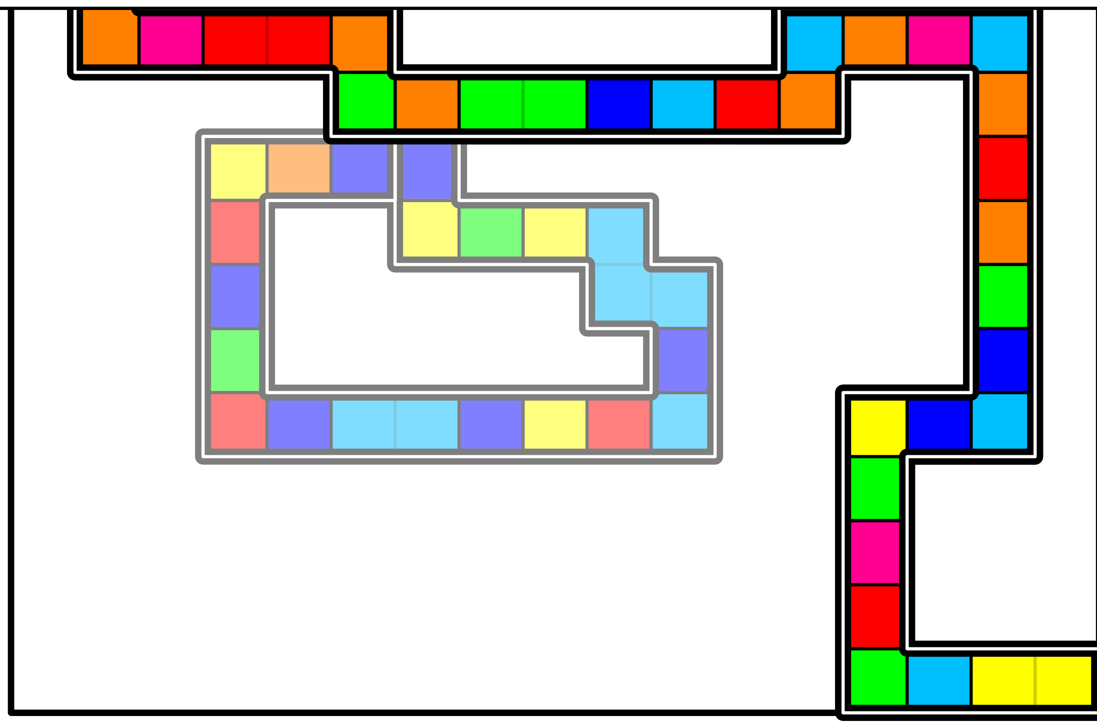
Random Boards

m moves to flood the board



Random Boards

We derive an upper bound on the probability that an arbitrary non-touching path from the top left to the bottom right tile has at most k colours changes. The bound depends on k, number of colours c and the length of the path.



Random Boards

We derive an upper bound on the probability that an arbitrary non-touching path from the top left to the bottom right tile has at most k colours changes. The bound depends on k, number of colours c and the length of the path.

The probability that there exists *any* non-touching path from the top left to the bottom right tile with at most k colour changes is upper bounded by the union bound over *all* non-touching paths from the top left to the bottom right tile.

Random Boards

We derive an upper bound on the probability that an arbitrary non-touching path from the top left to the bottom right tile has at most k colours changes. The bound depends on k, number of colours c and the length of the path.

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This union bound is an upper bound on the probability that the board is flooded within k moves.

Random Boards

We derive an upper bound on the probability that an arbitrary non-touching path from the top left to the bottom right tile has at most k colours changes. The bound depends on k, number of colours c and the length of the path.

k is order n

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Random Boards

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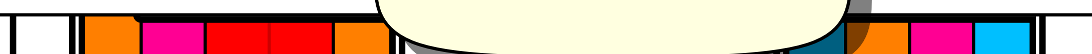
k is order n



The probability that there exists *any* non-touching path from the top left to the bottom right tile with at most k colour changes is upper bounded by the union bound over *all* non-touching paths from the top left to the bottom right tile.

Probability less than $e^{-\Omega(n)}$
(for 3 or more colours)

This union bound is an upper bound on the probability that the board is flooded within k moves.

Random Boards

We derive an upper bound on the probability that an arbitrary non-touching path from the top left to the bottom right tile has at most k colours changes. The bound depends on k, number of colours c and the

Conclusion

The number of moves required to flood a random board is $\Omega(n)$ with high probability.

the bottom right tile.

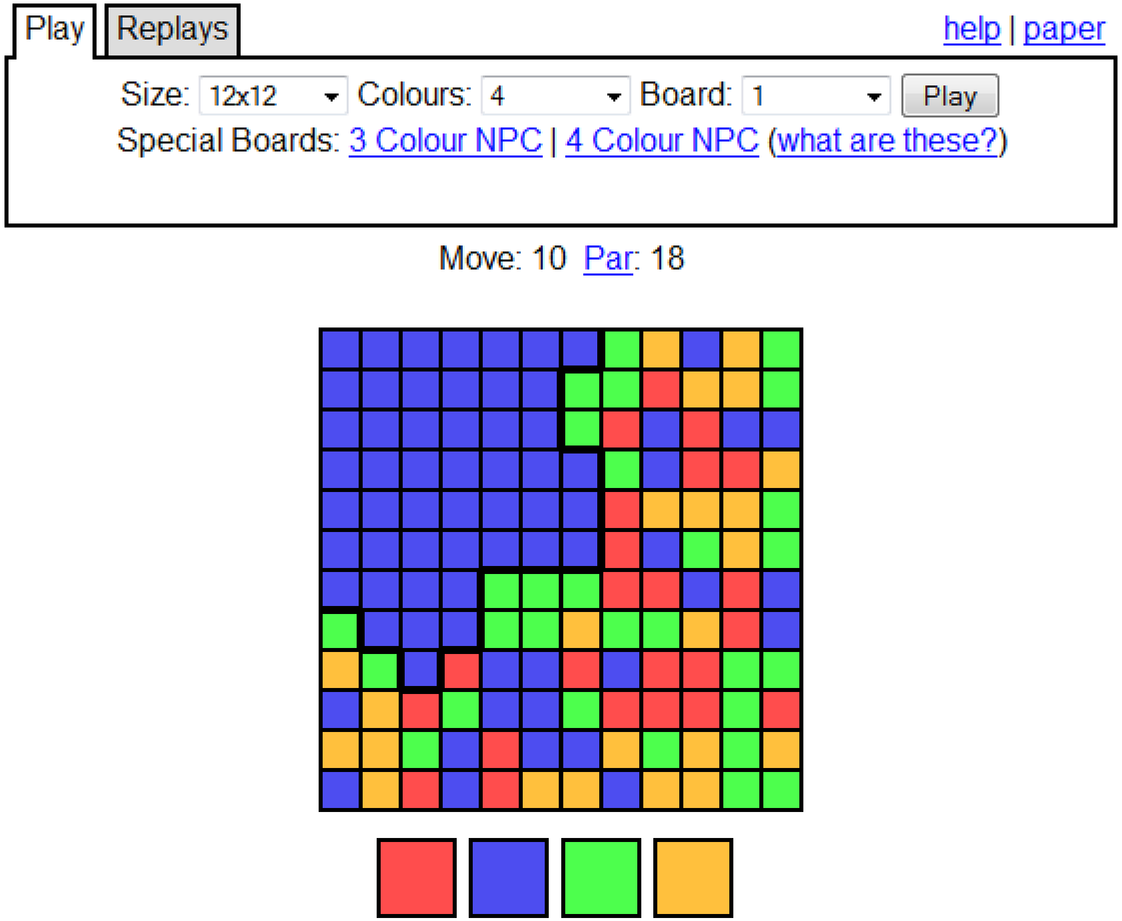
Probability less than $e^{-\Omega(n)}$
(for 3 or more colours)

This union bound is an upper bound on the probability that the board is flooded within k moves.

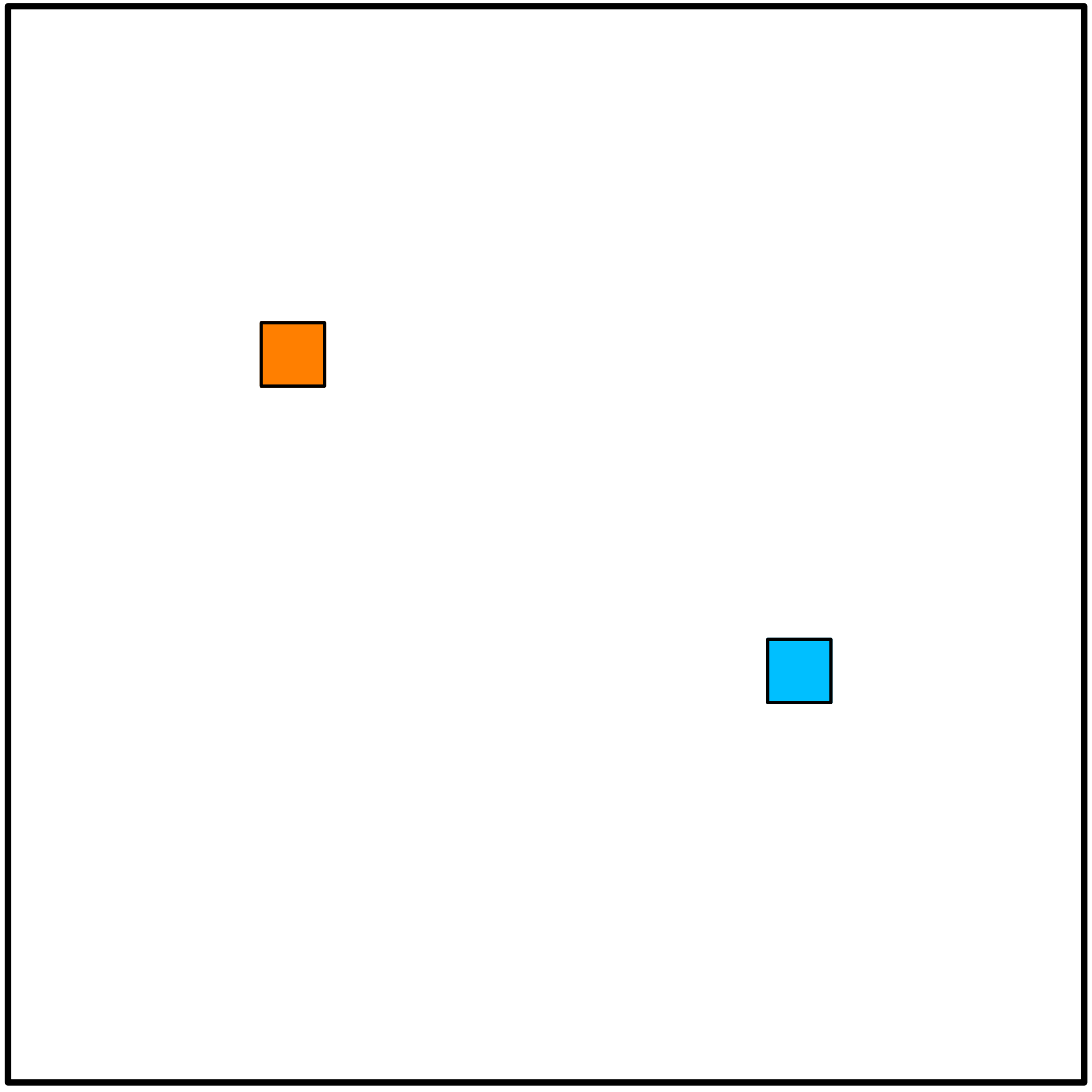
Thank You!

Don't forget our website:

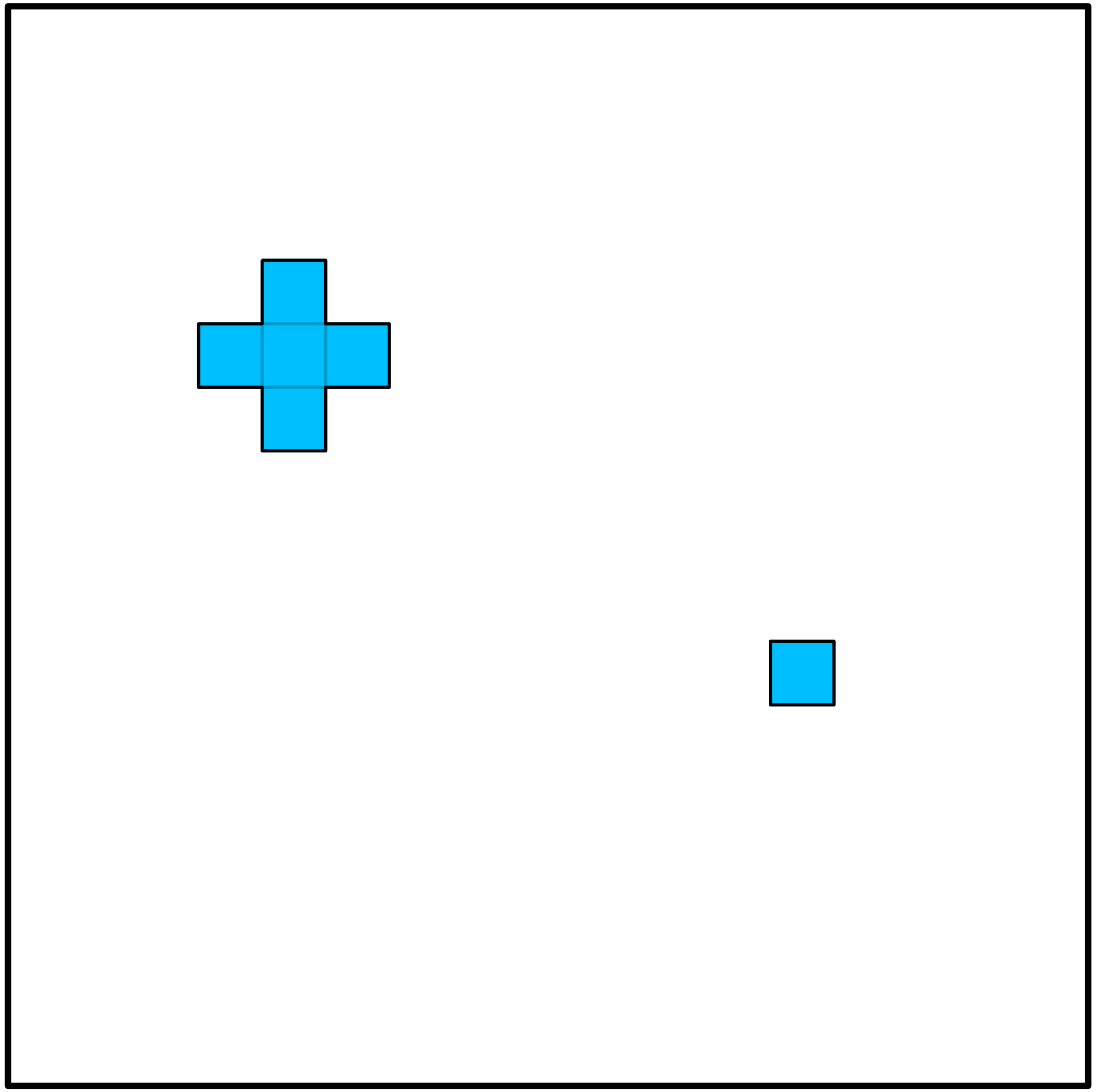
`http://floodit.cs.bris.ac.uk/`



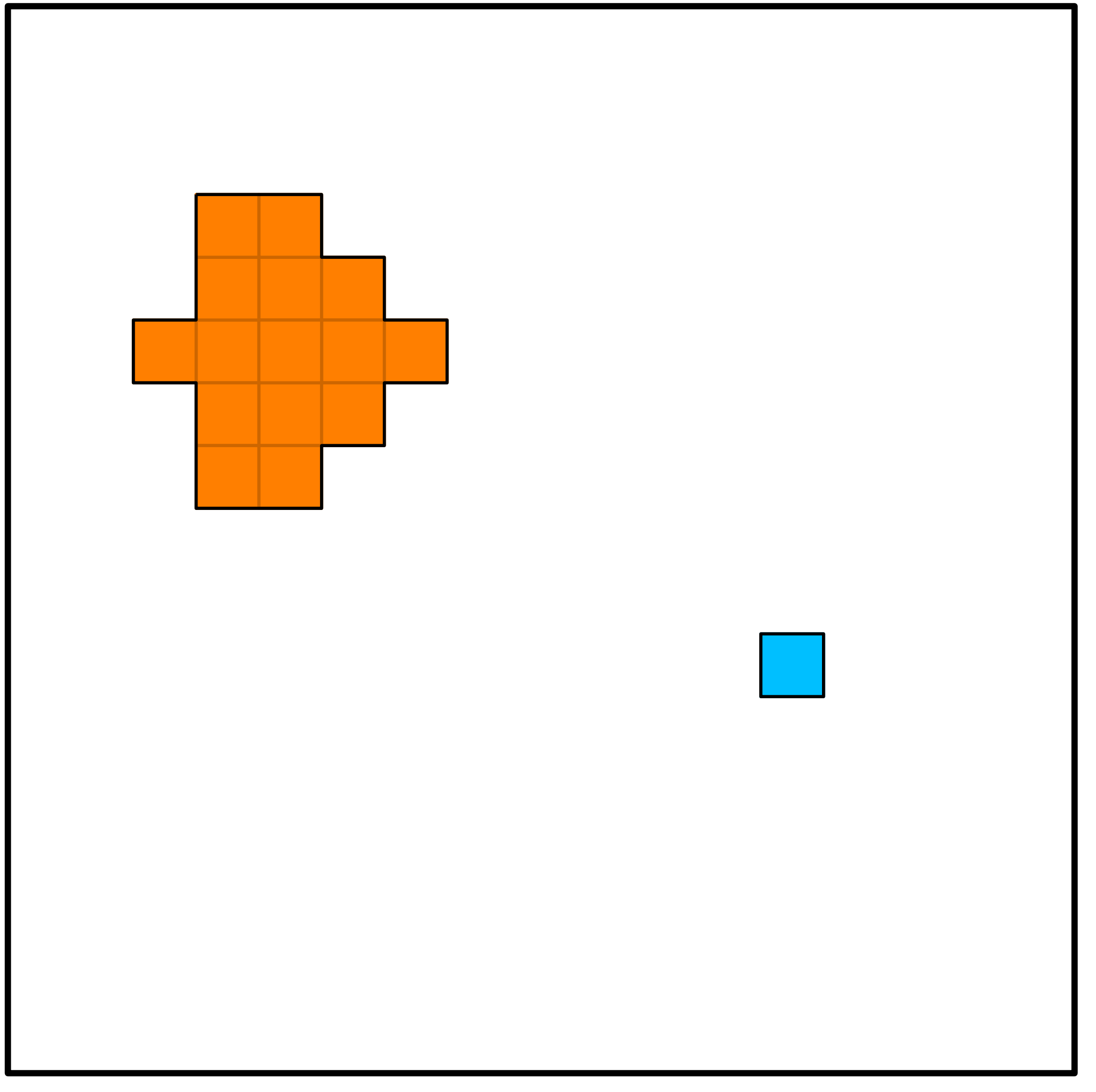
2 Colours (Free)



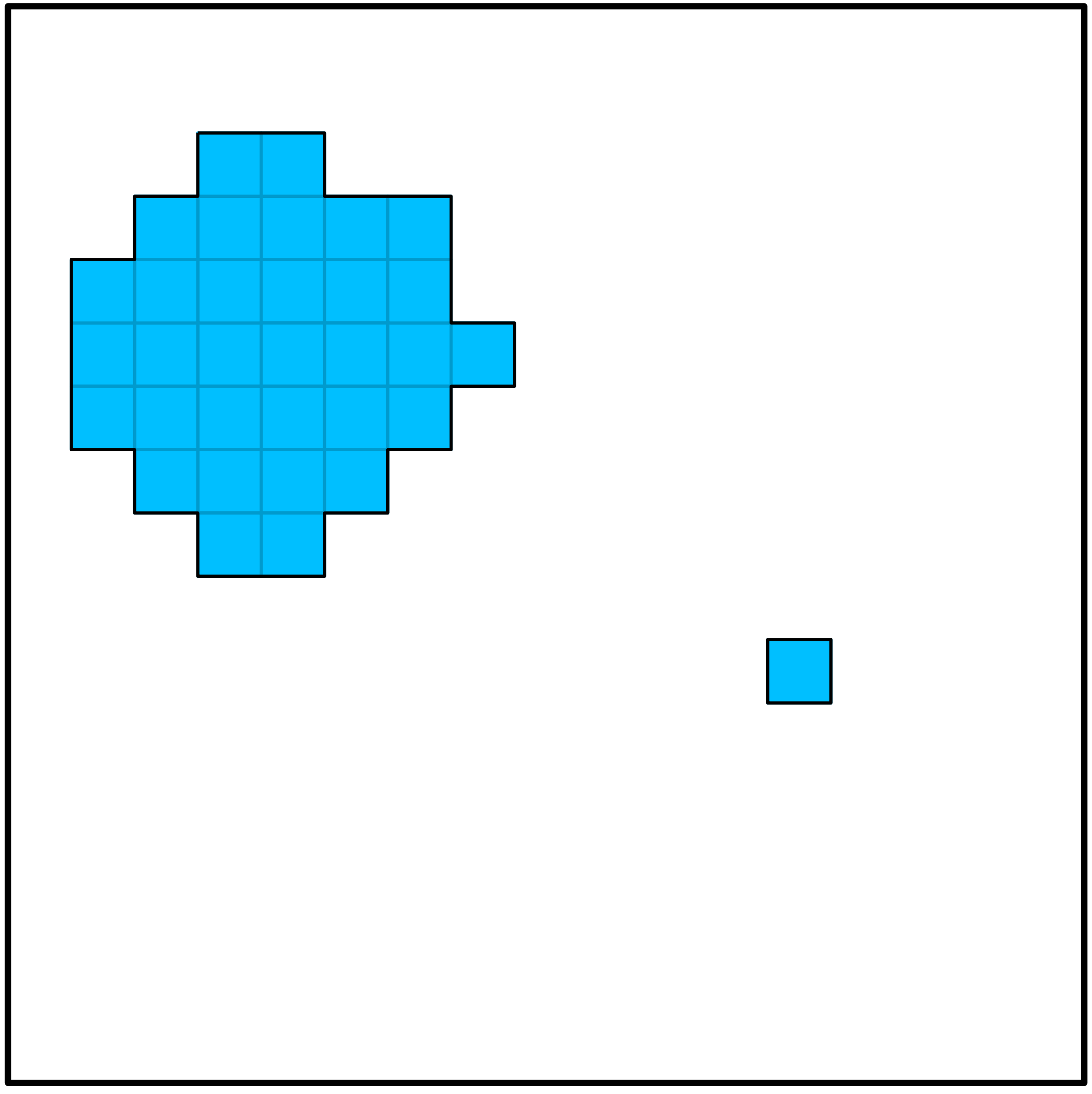
2 Colours (Free)



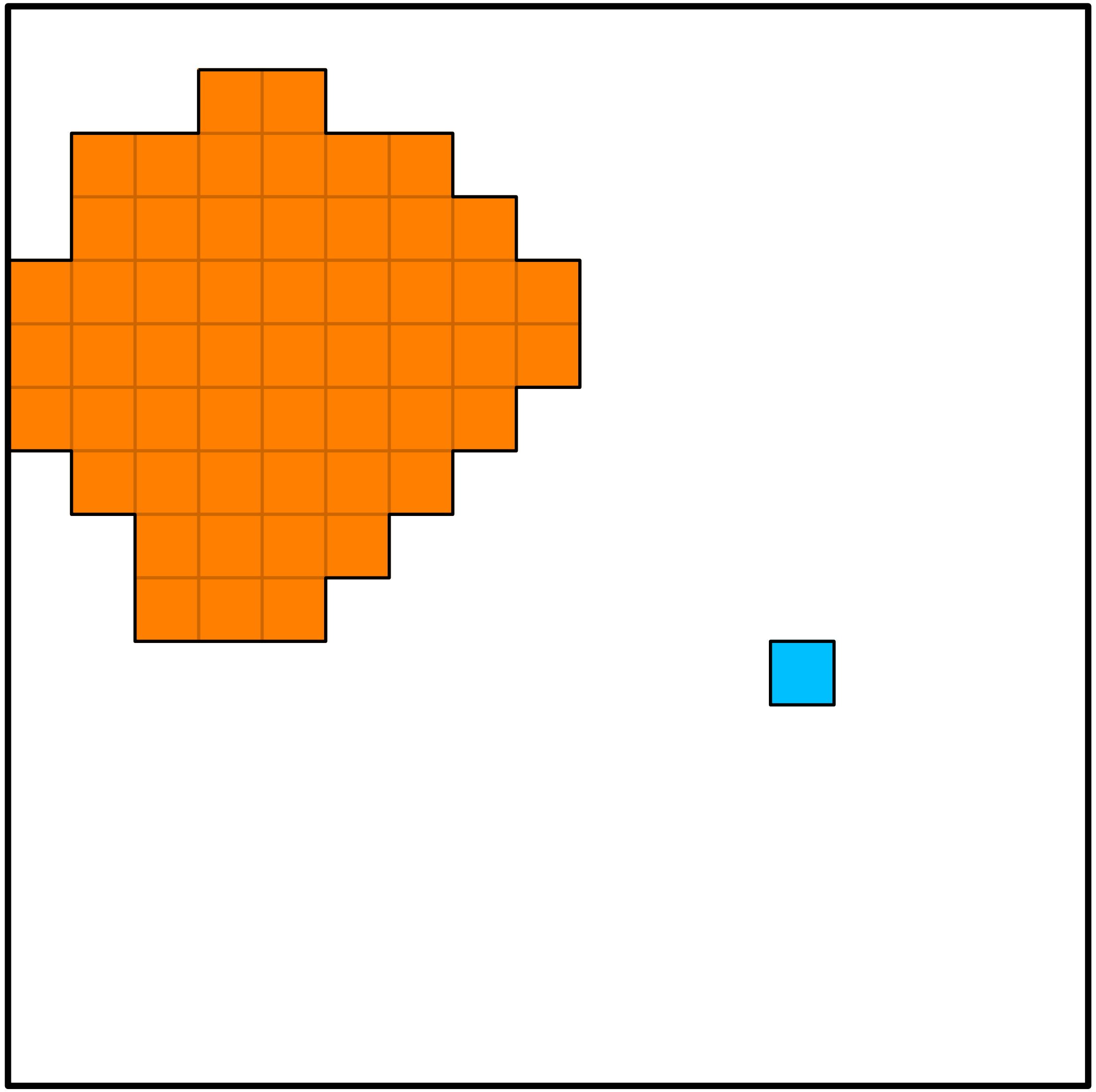
2 Colours (Free)



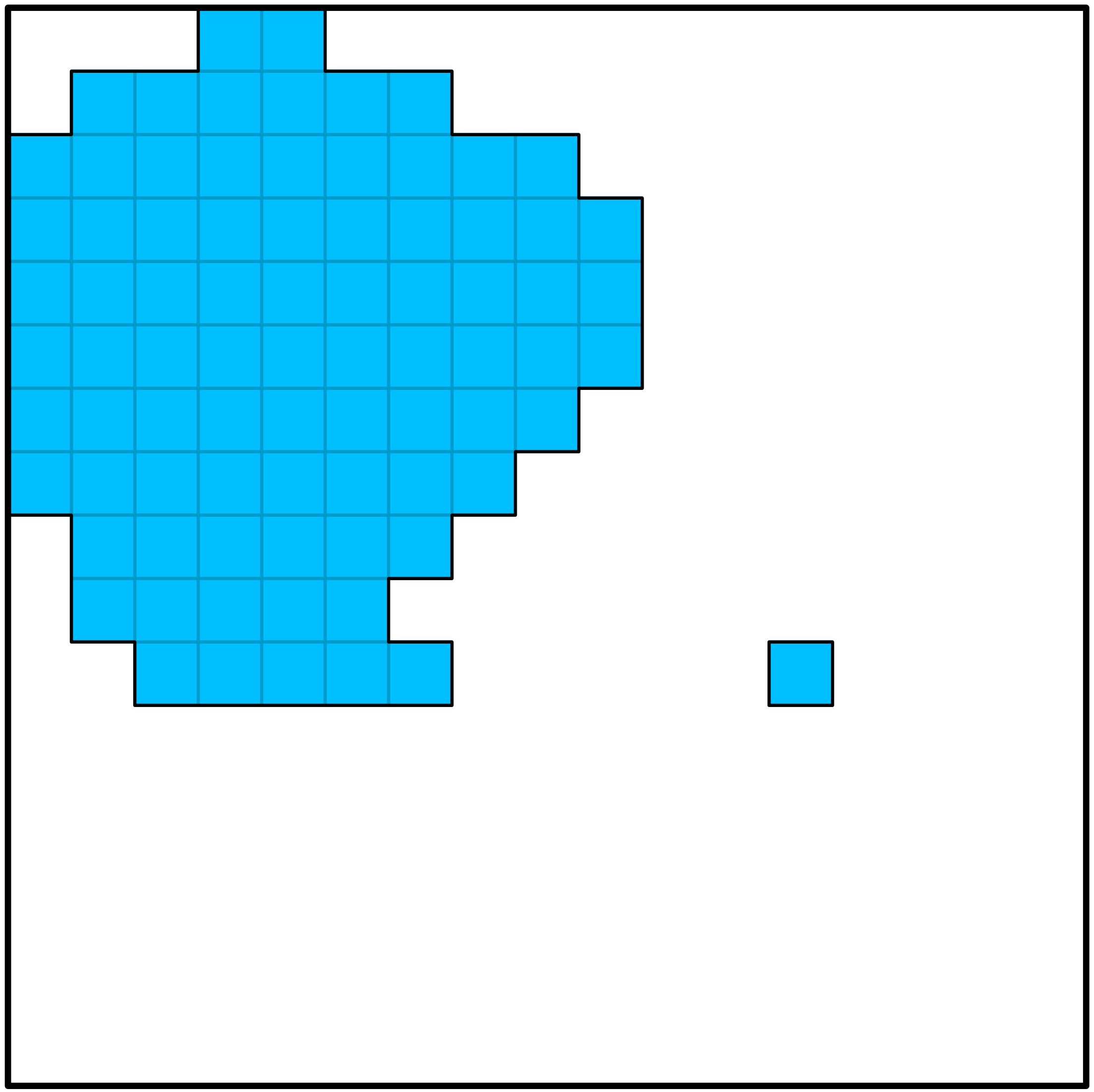
2 Colours (Free)



2 Colours (Free)



2 Colours (Free)



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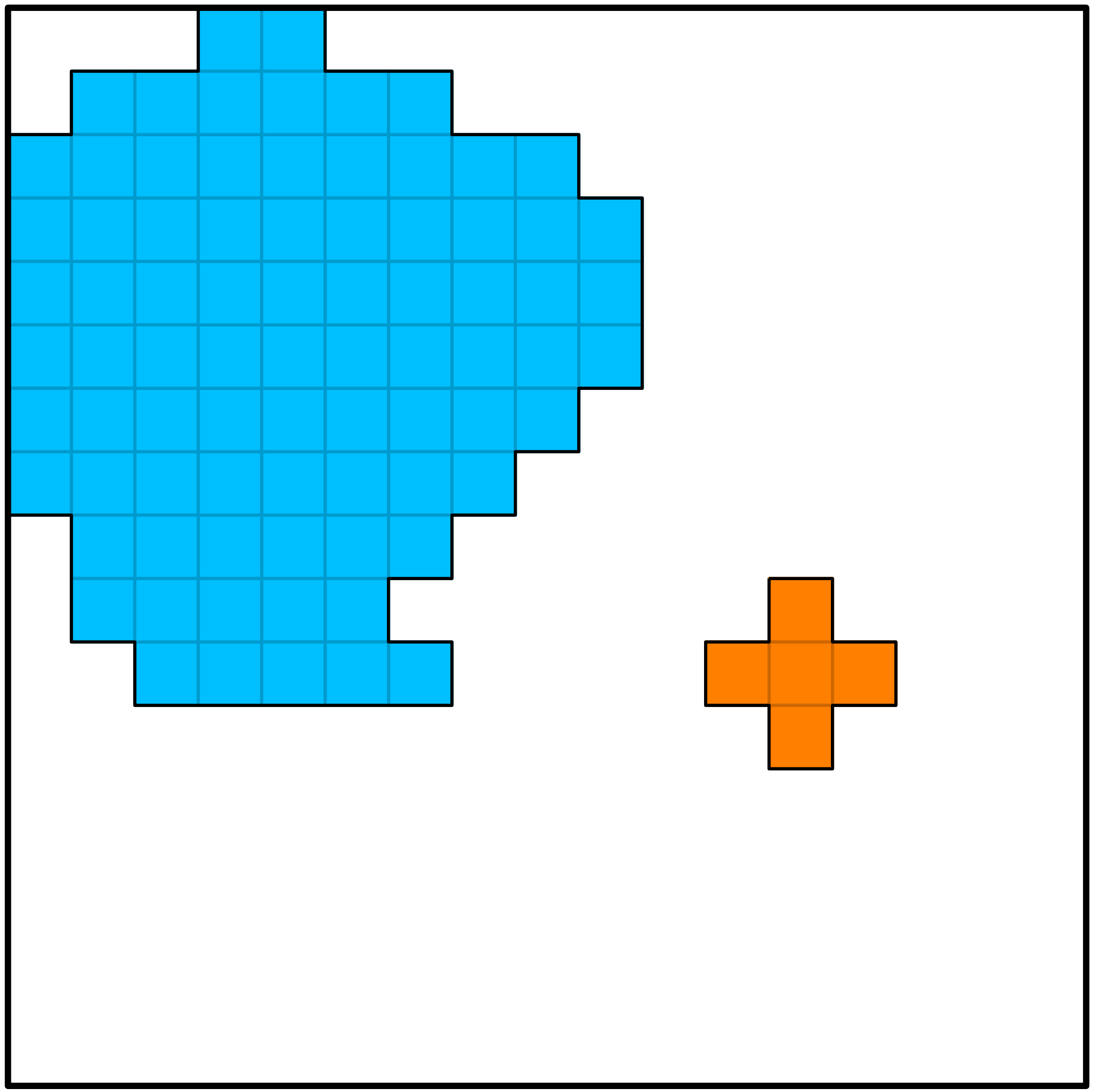

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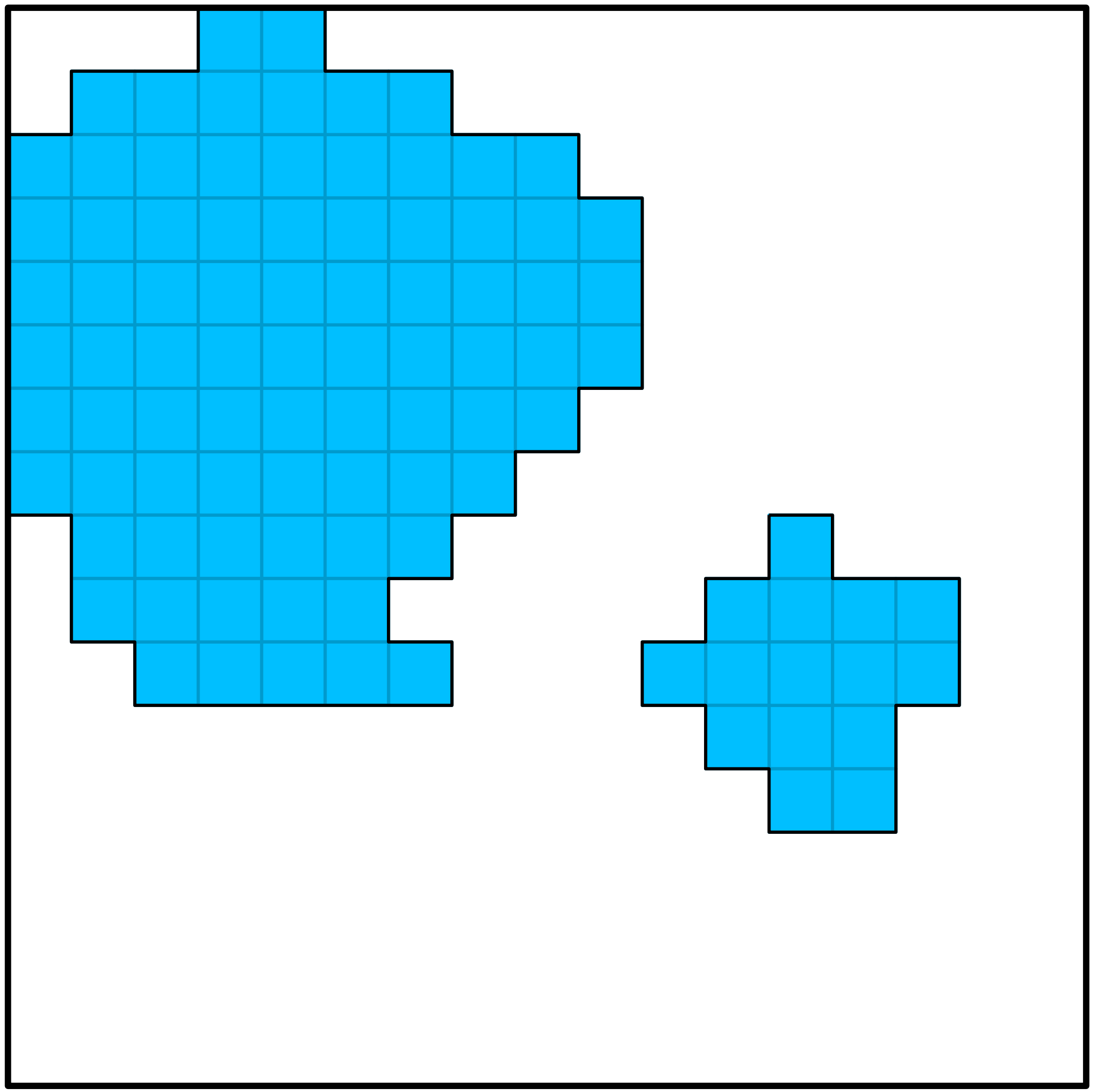

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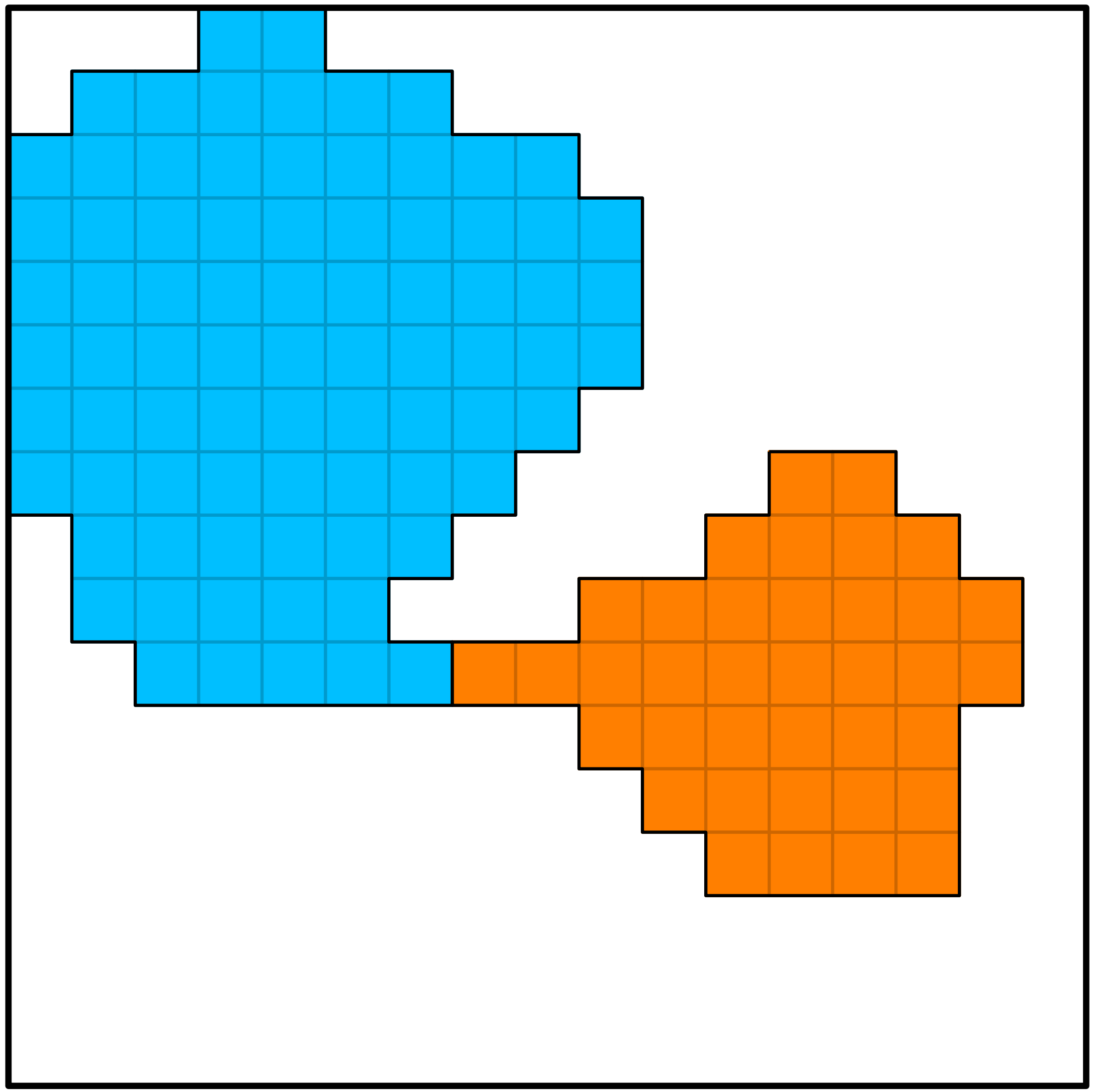

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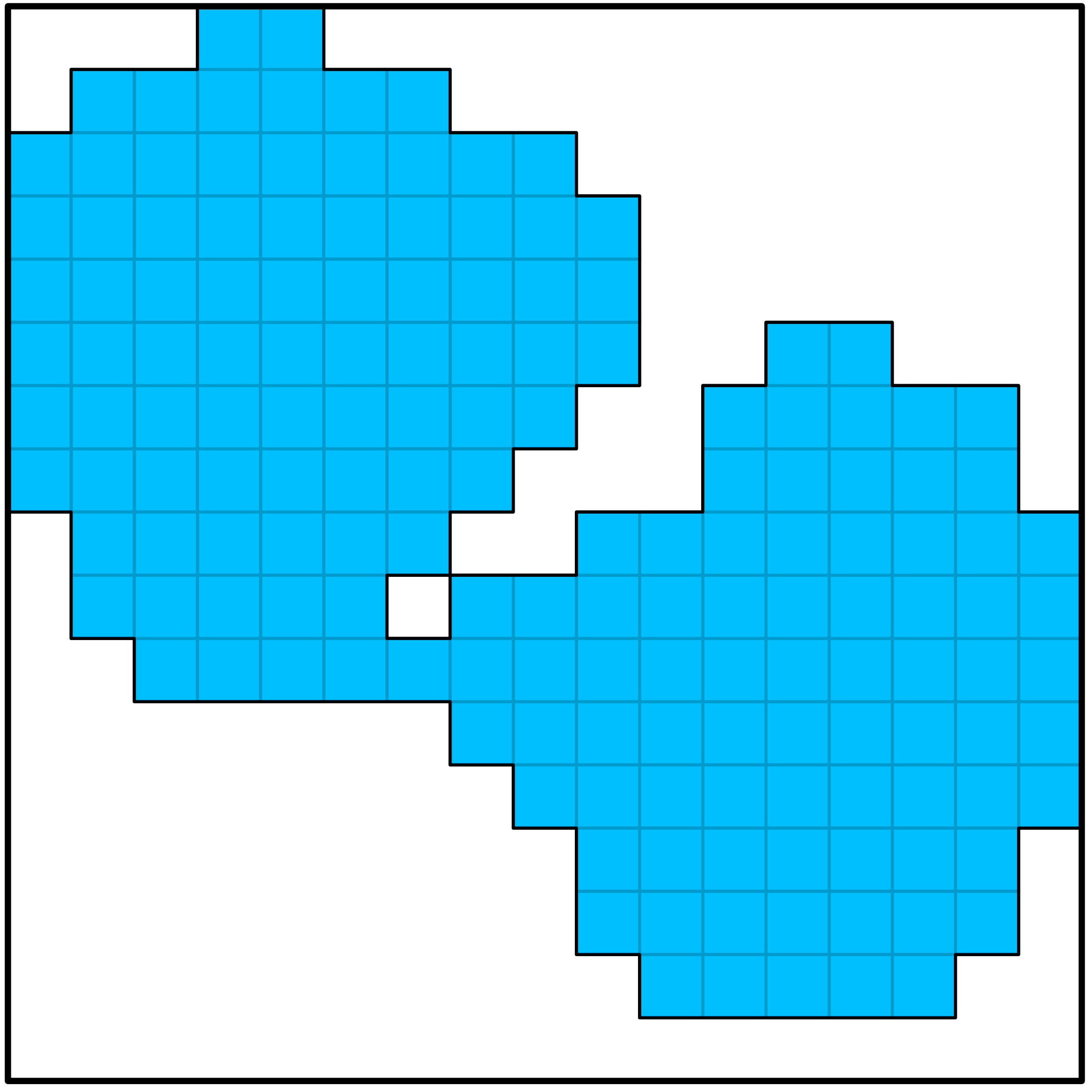
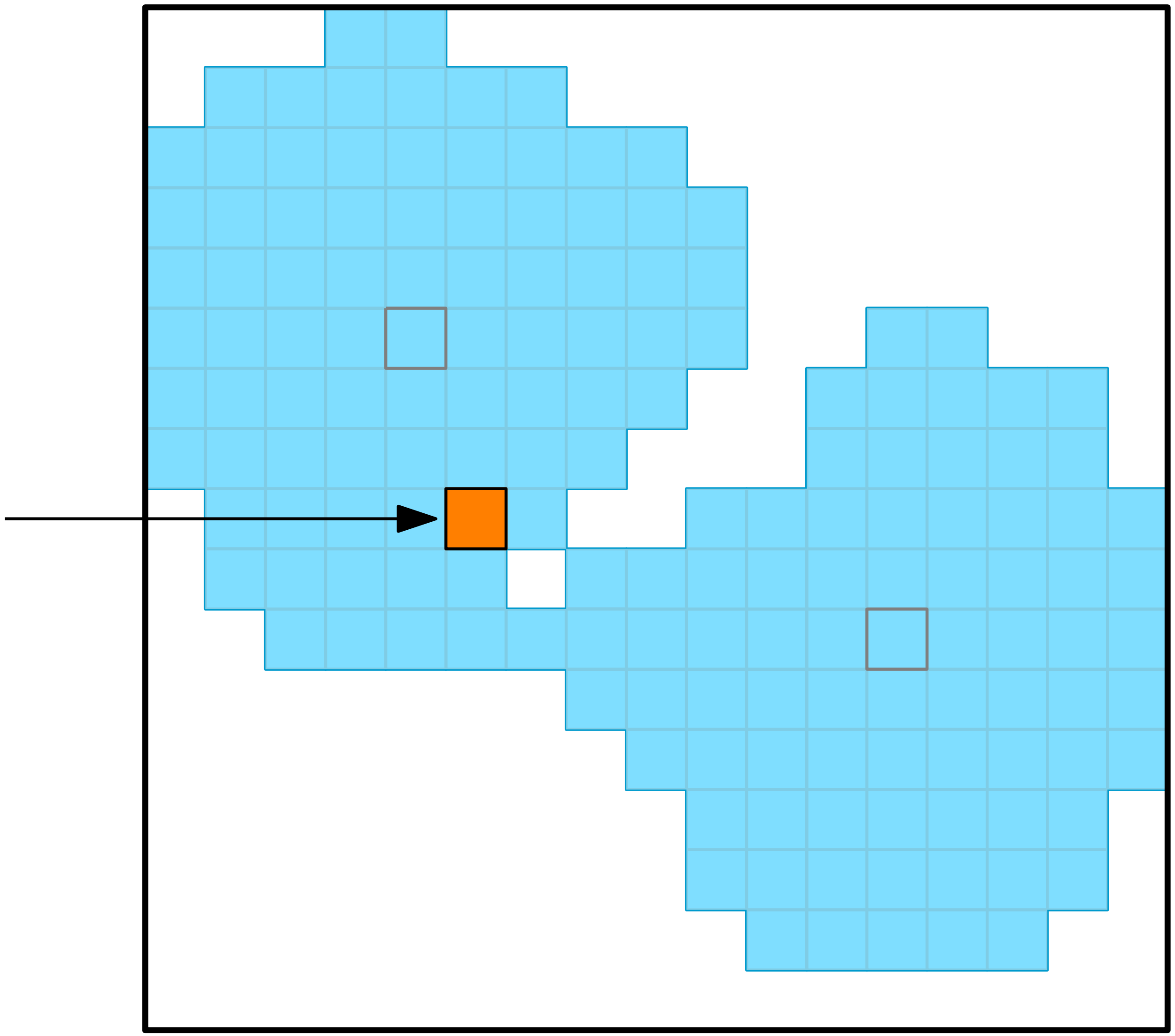


2 Colours (Free)



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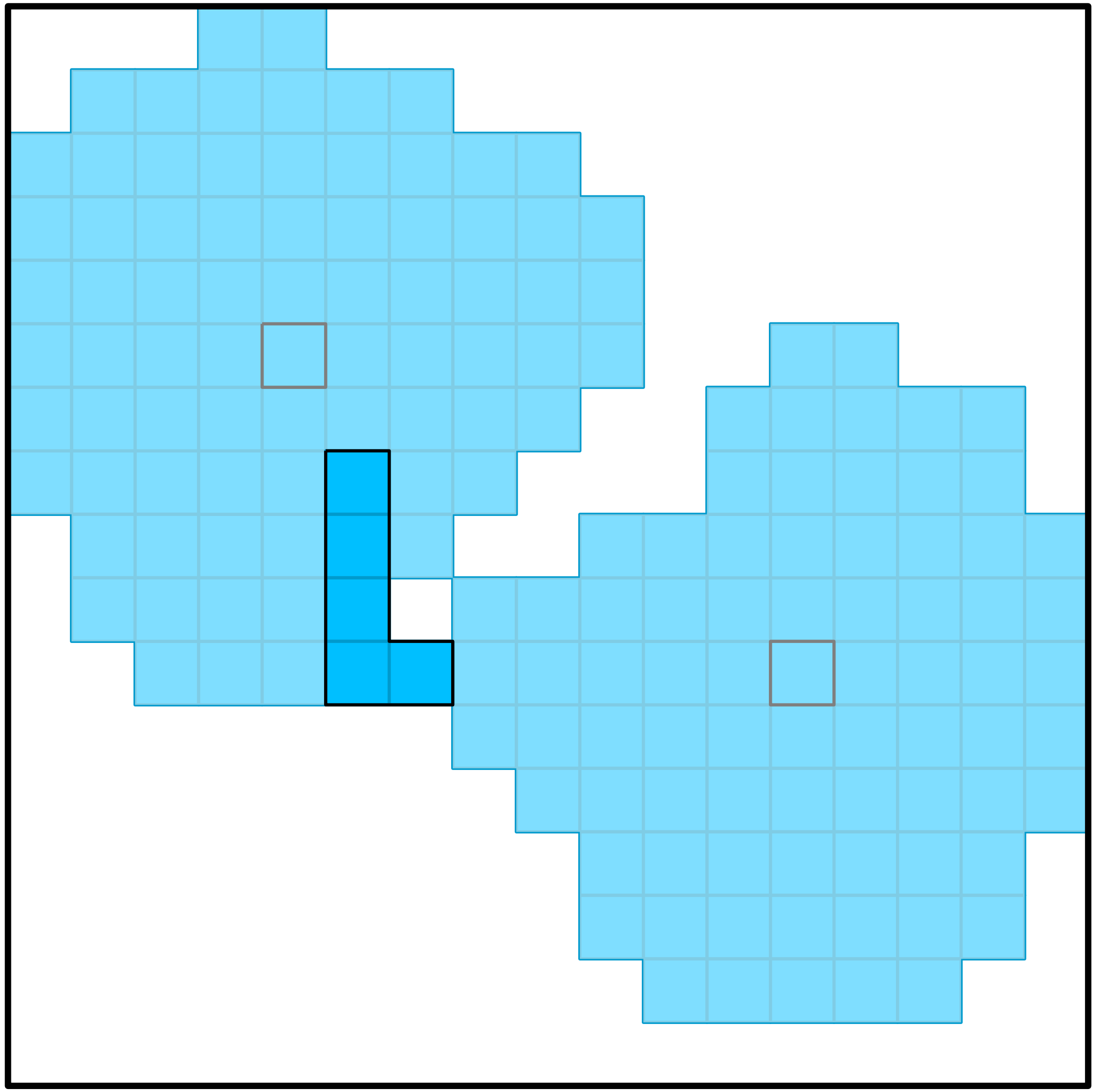

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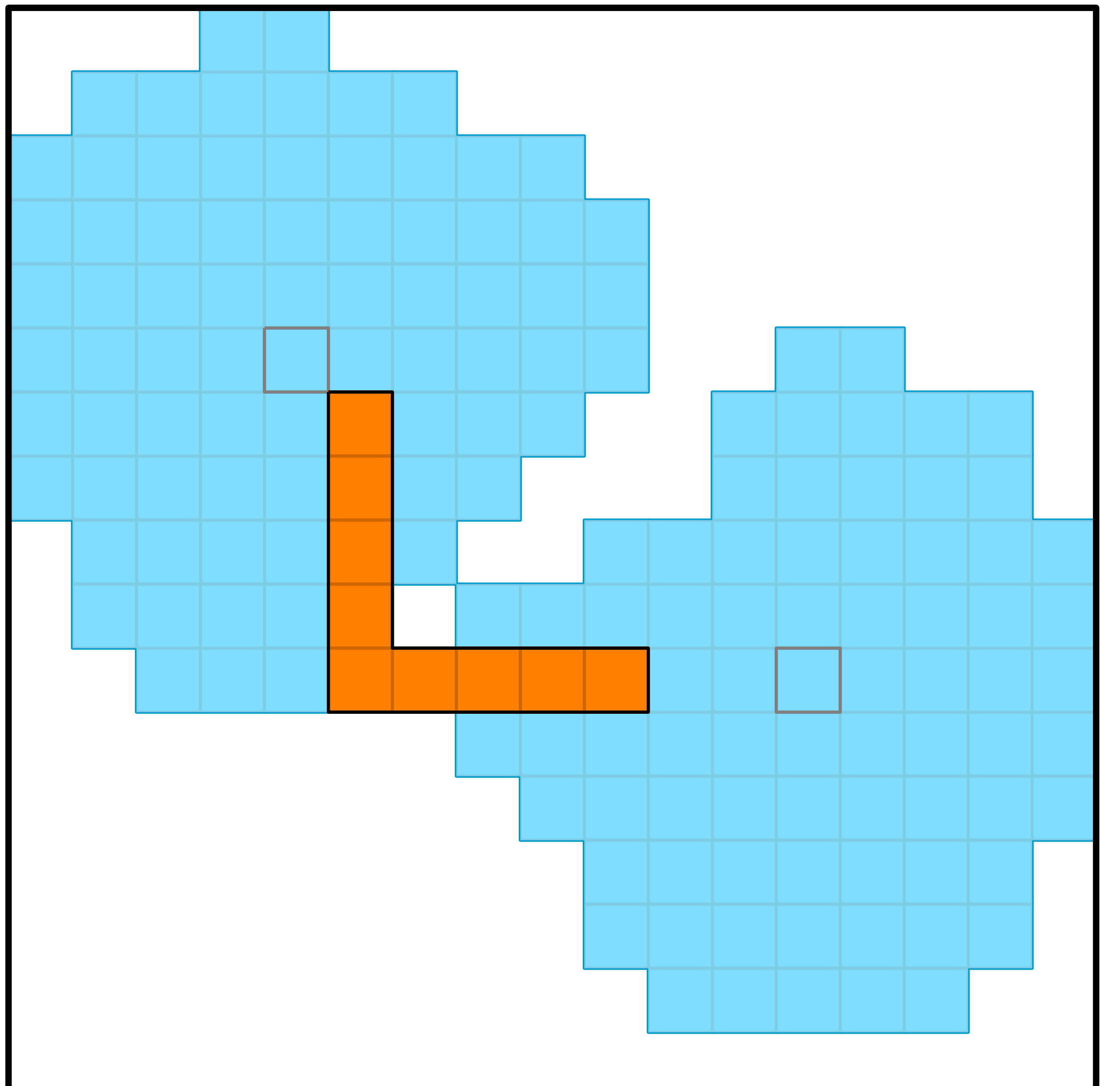

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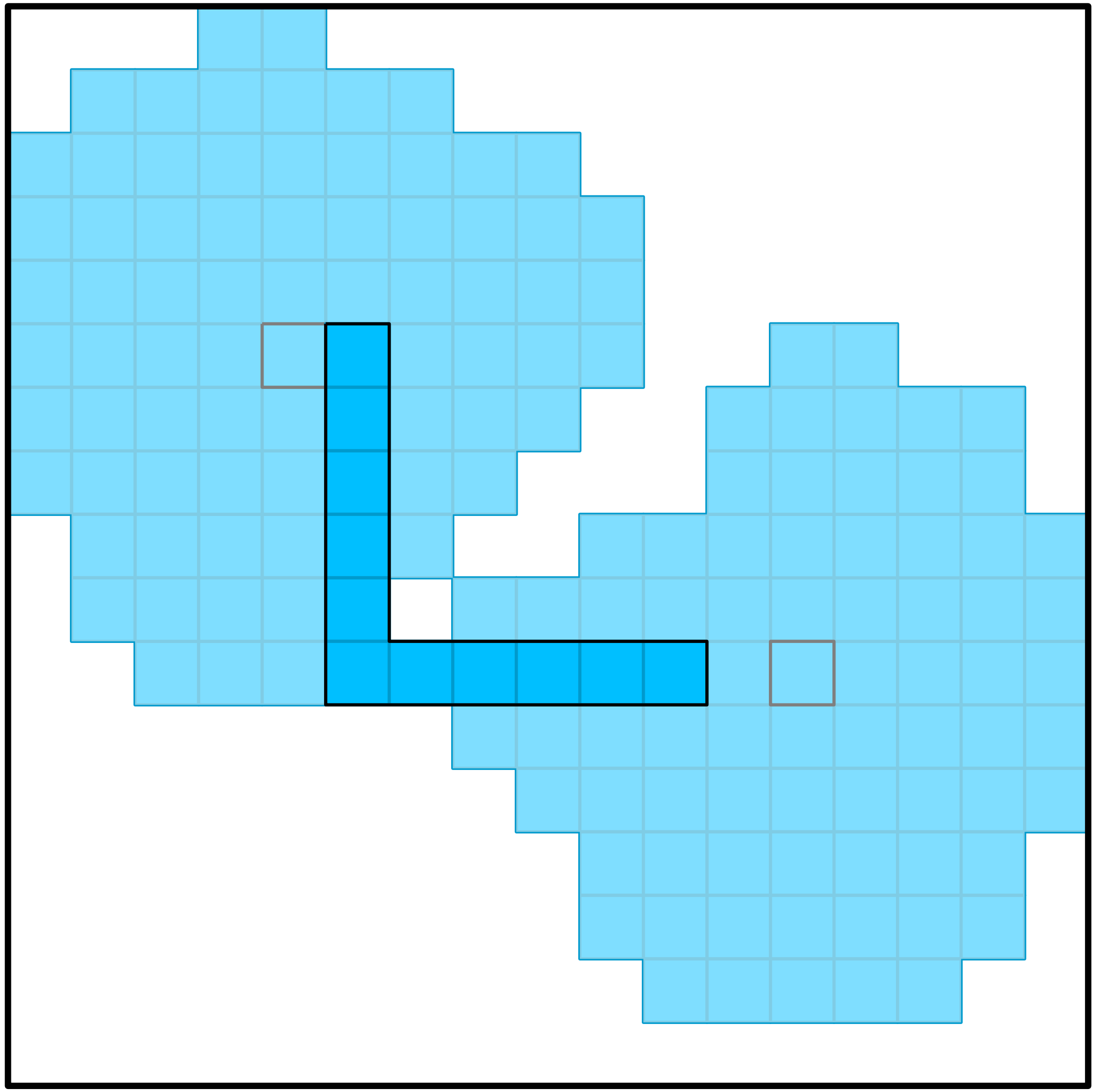
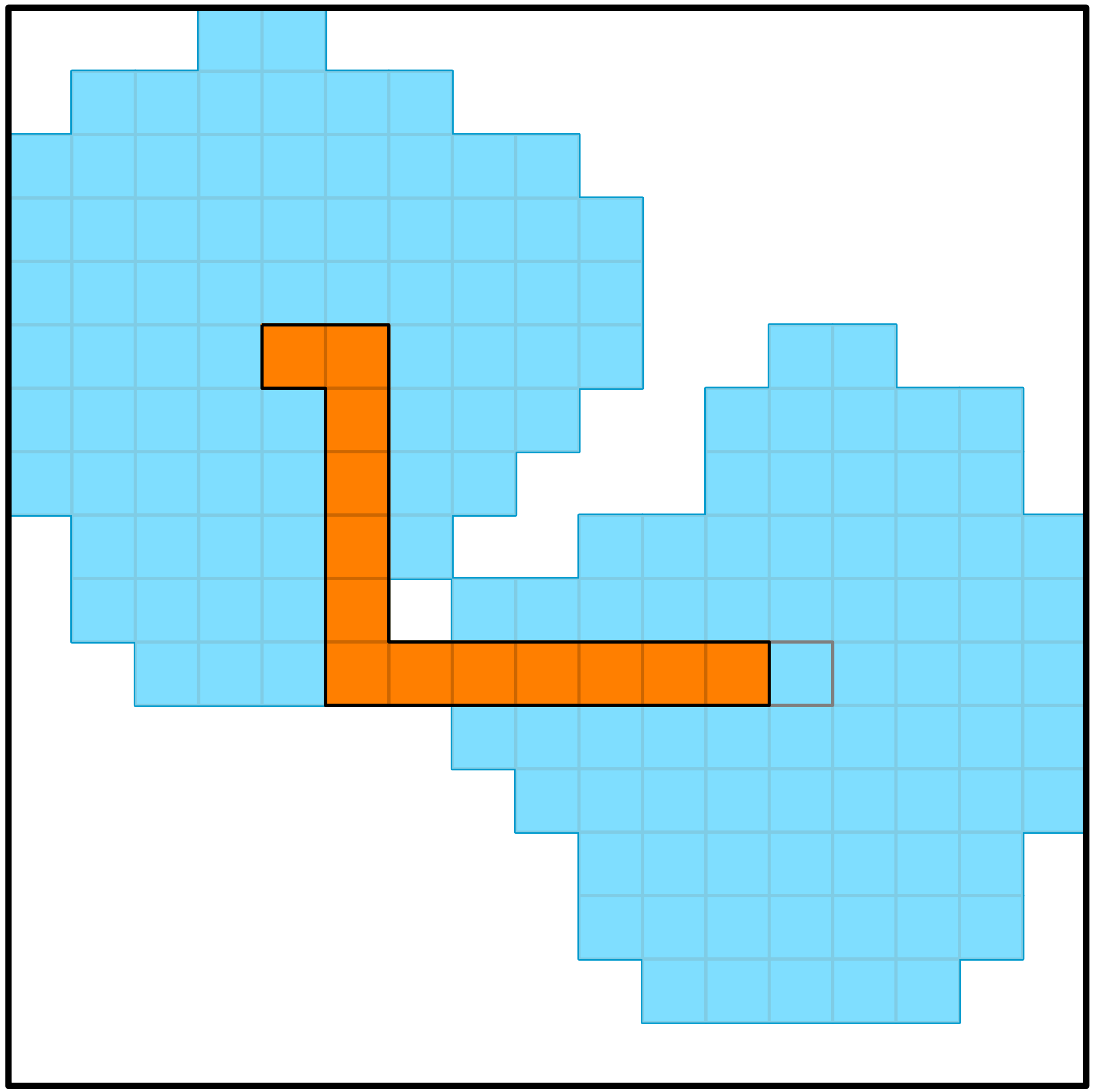
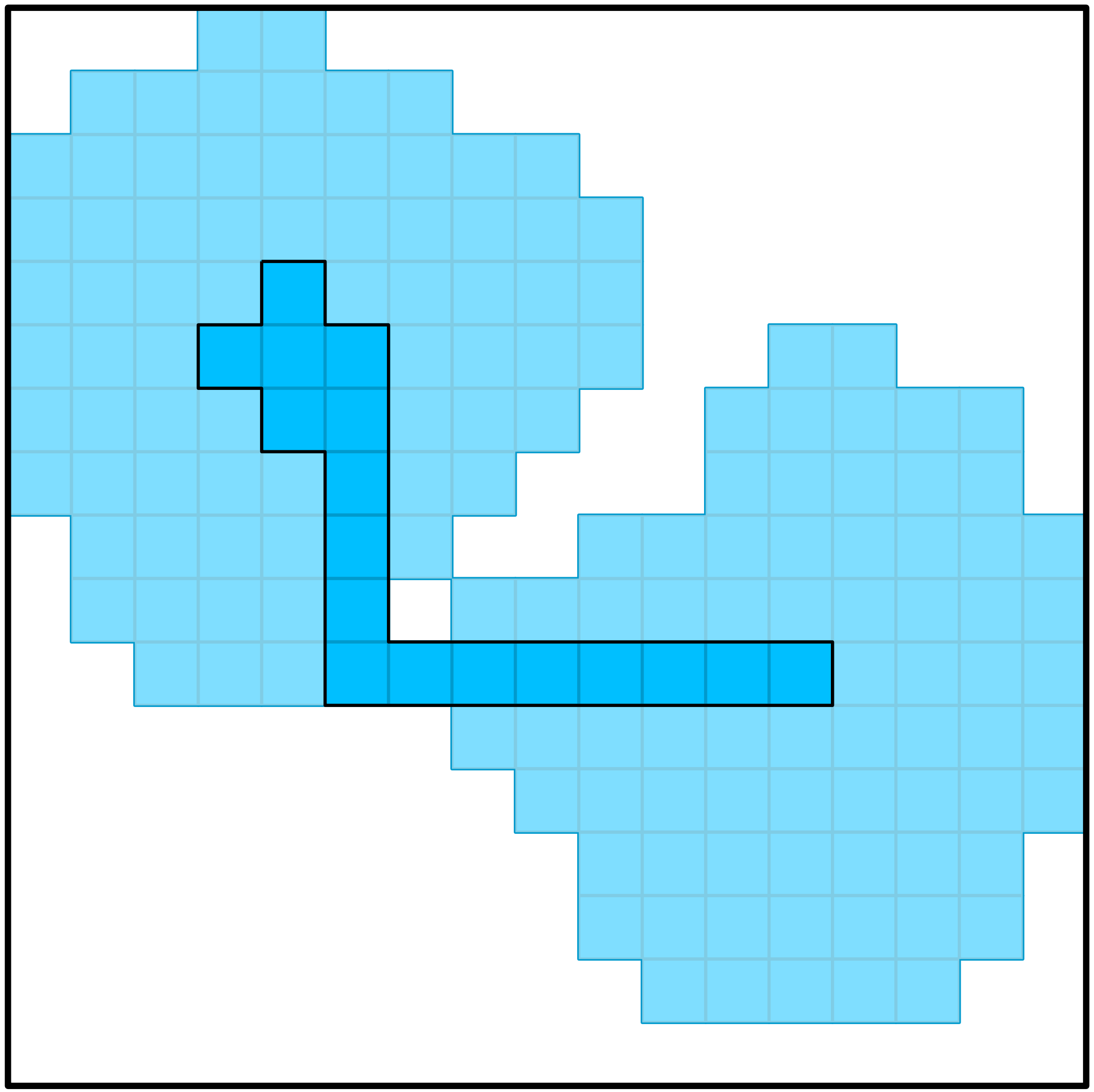


2 Colours (Free)



2 Colours (Free)



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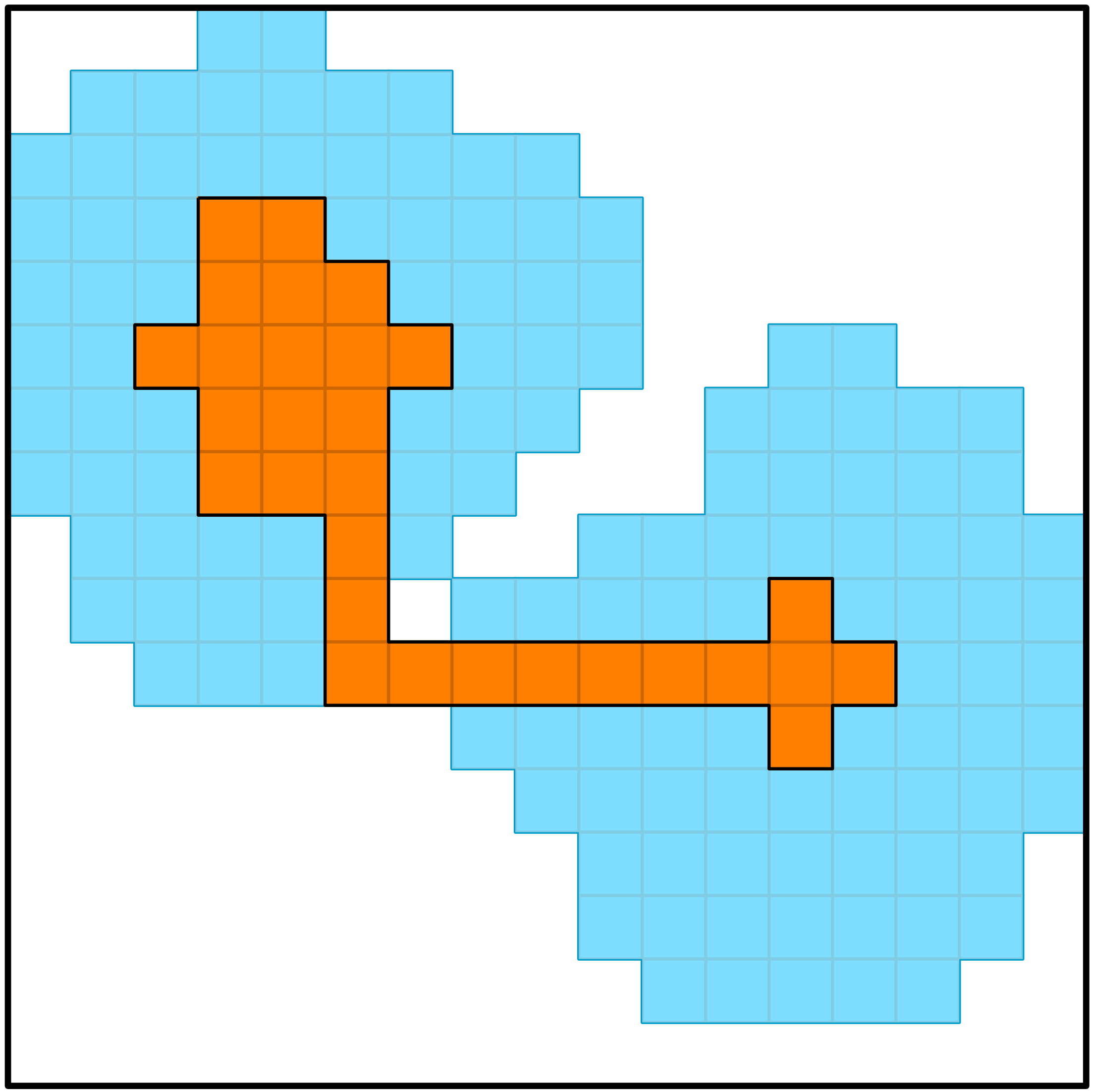

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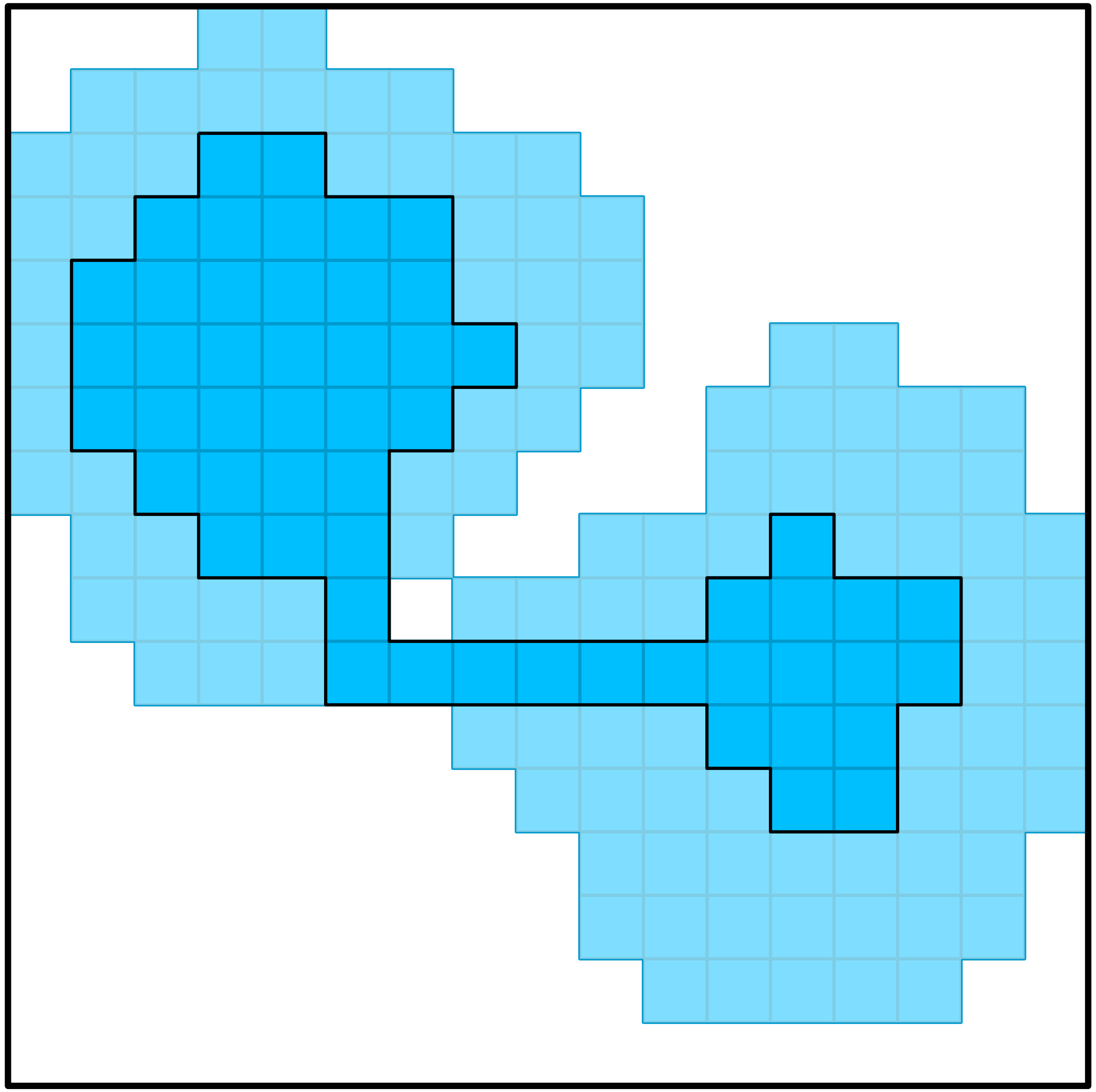
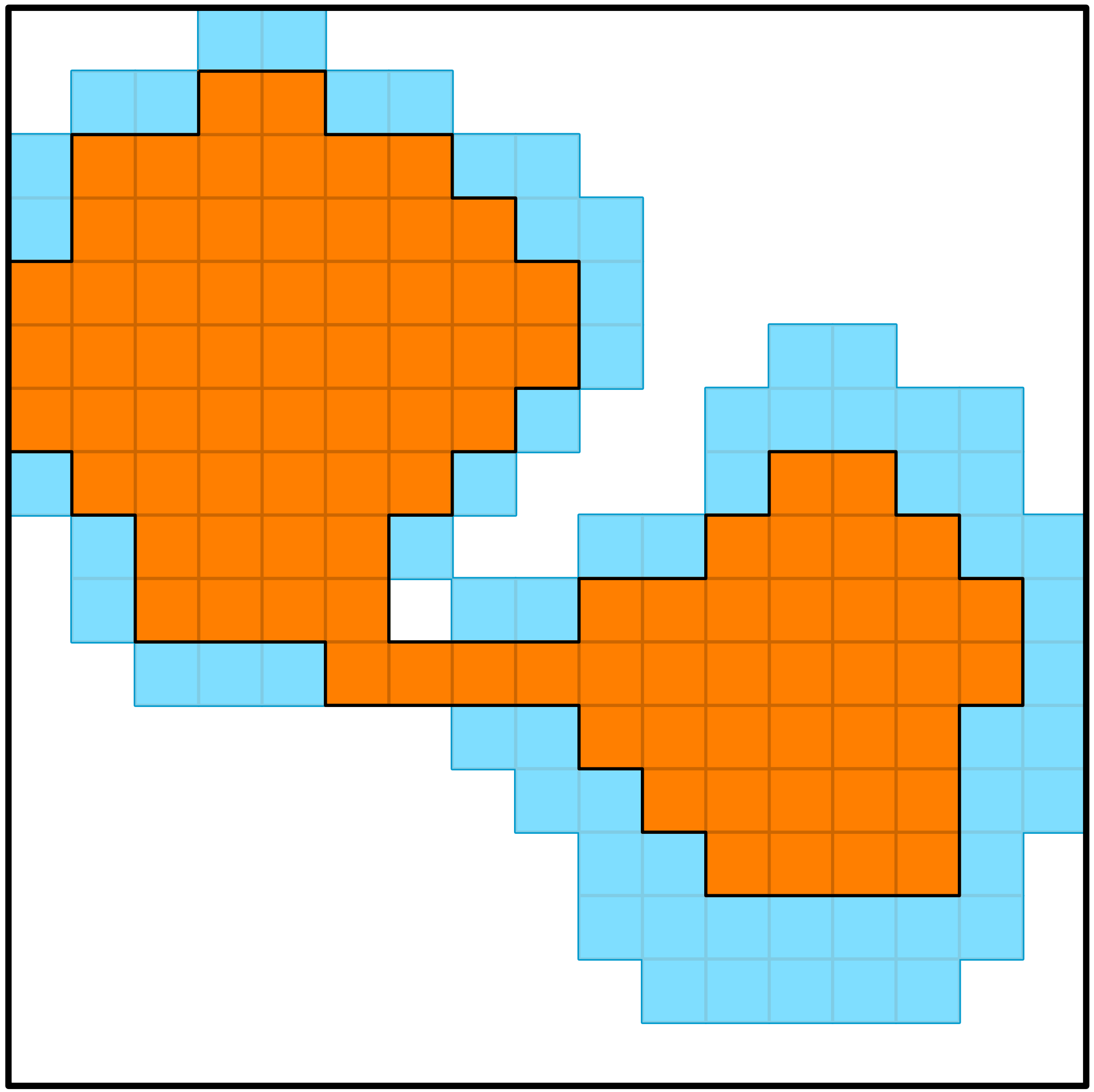


2 Colours (Free)



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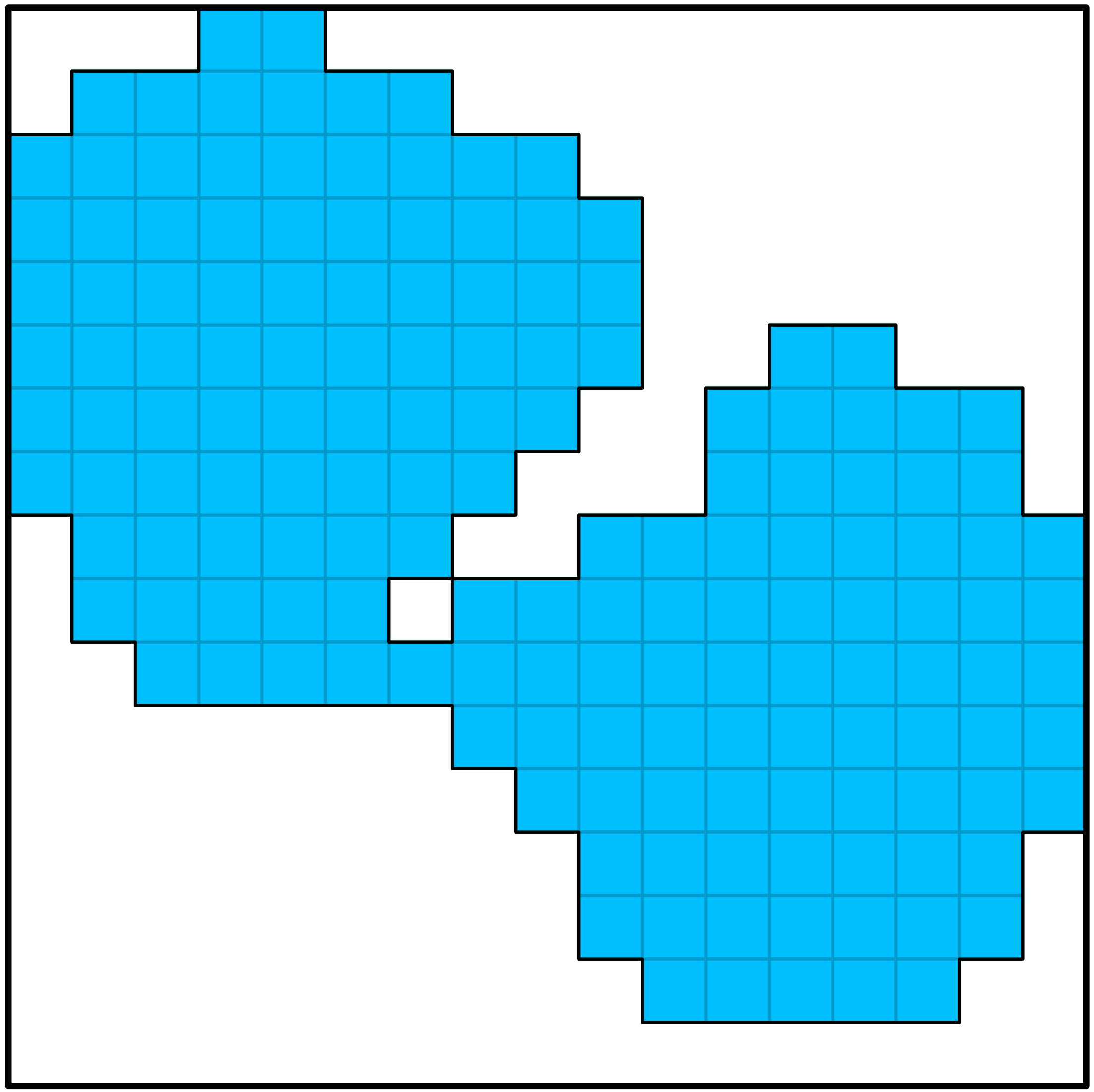
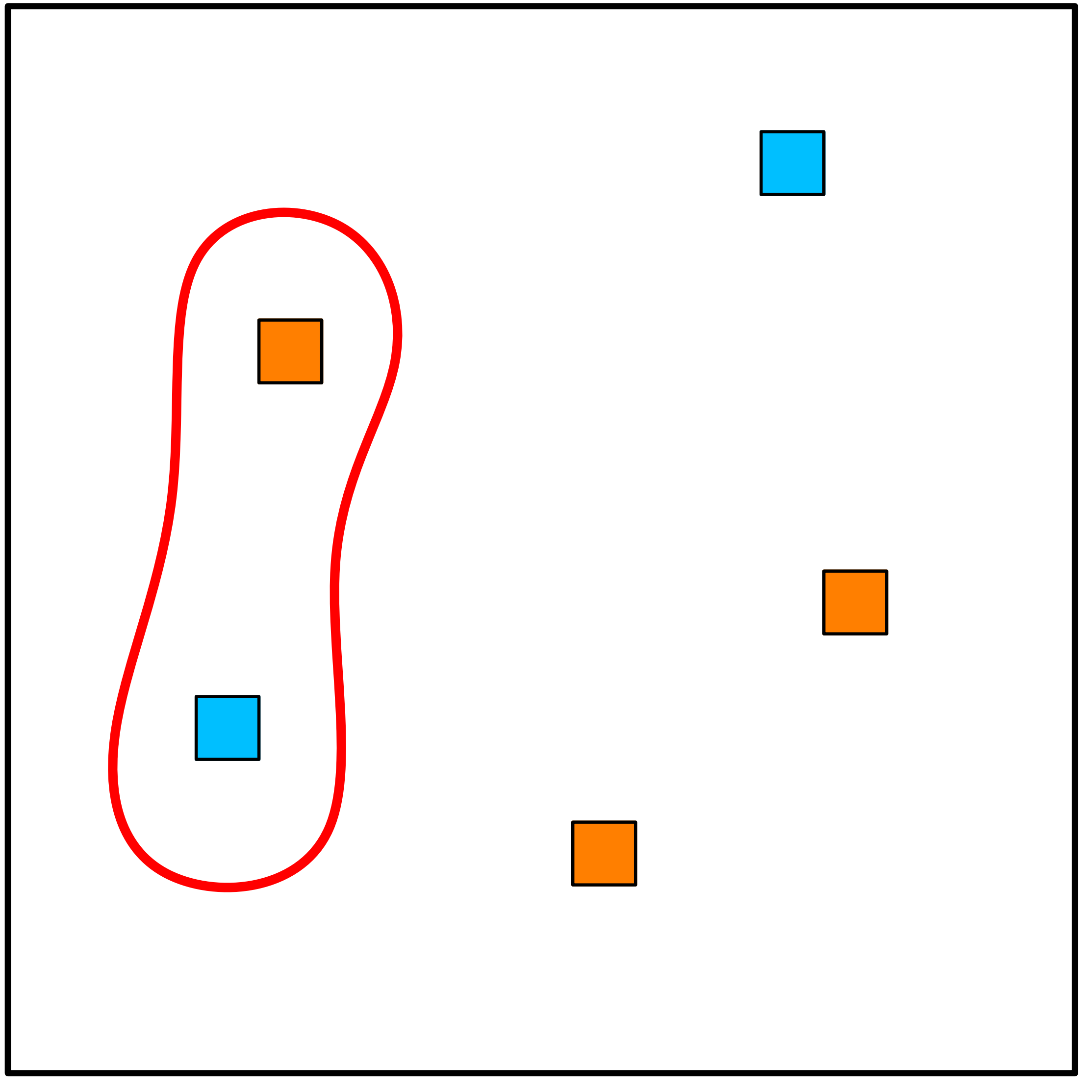
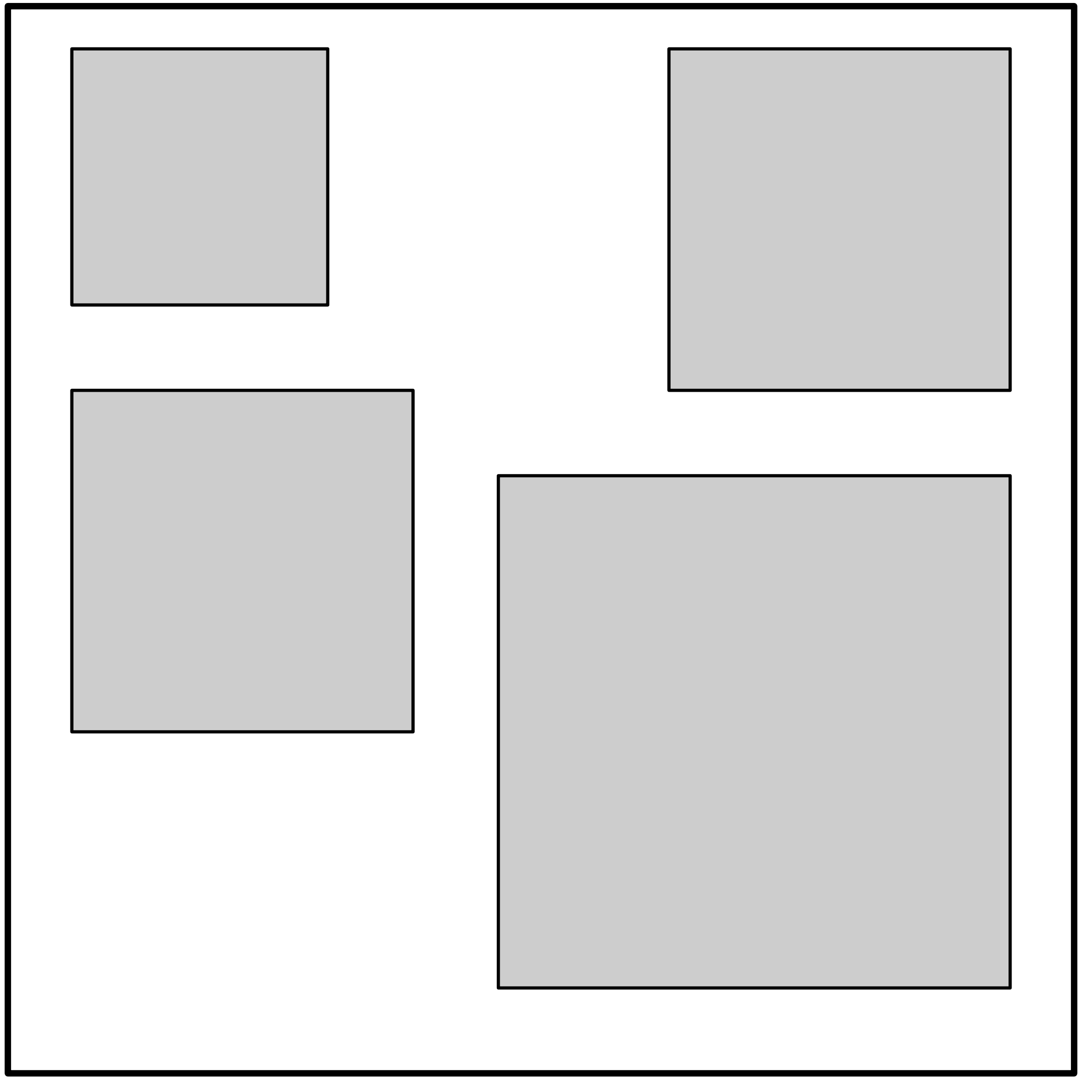


2 Colours (Free)



3 or More Colours (Free)



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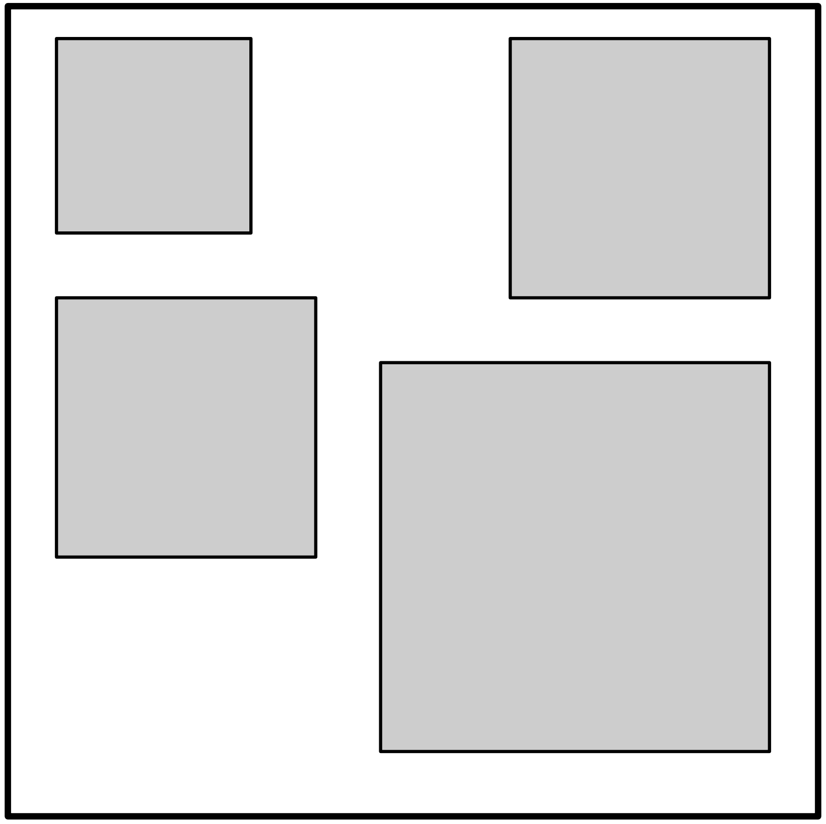

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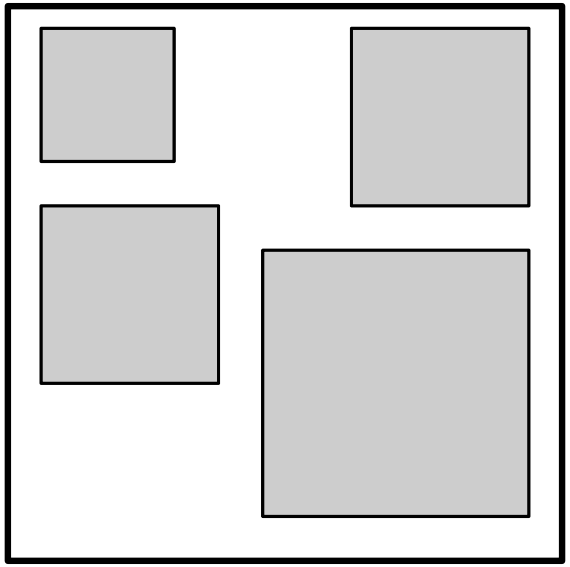

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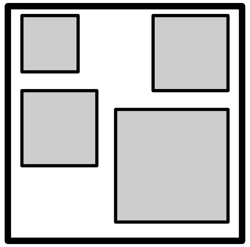

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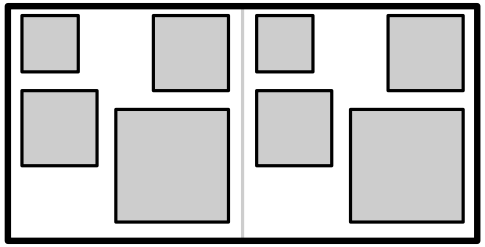

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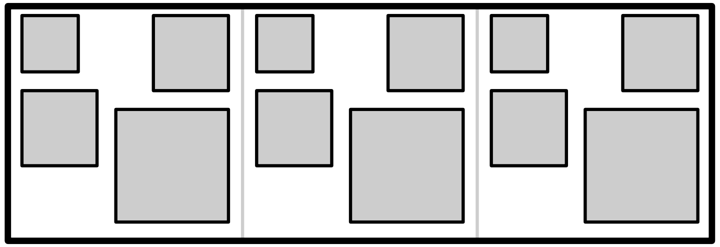

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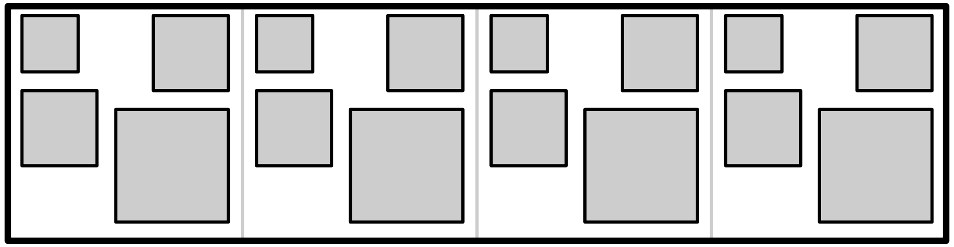

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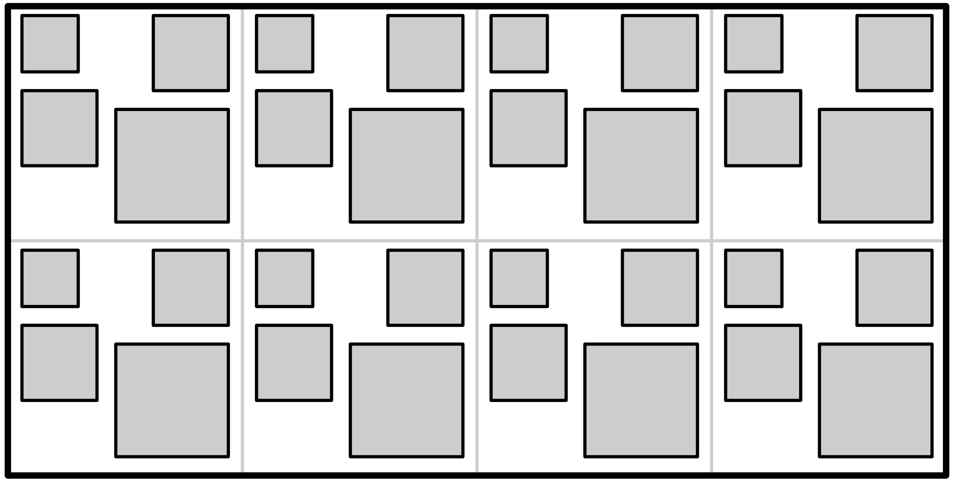

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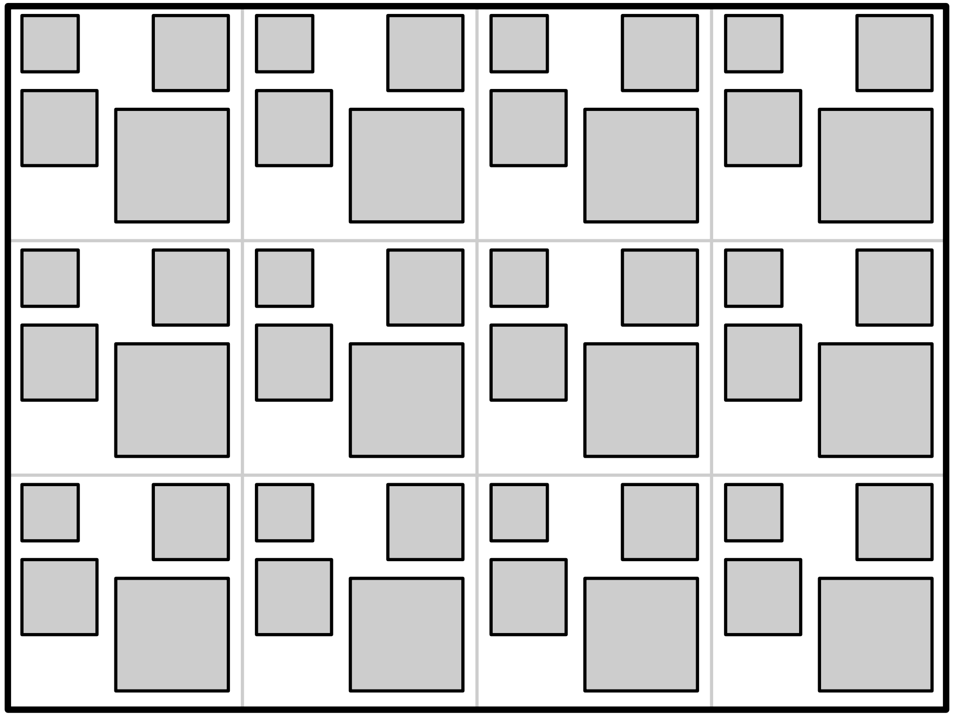

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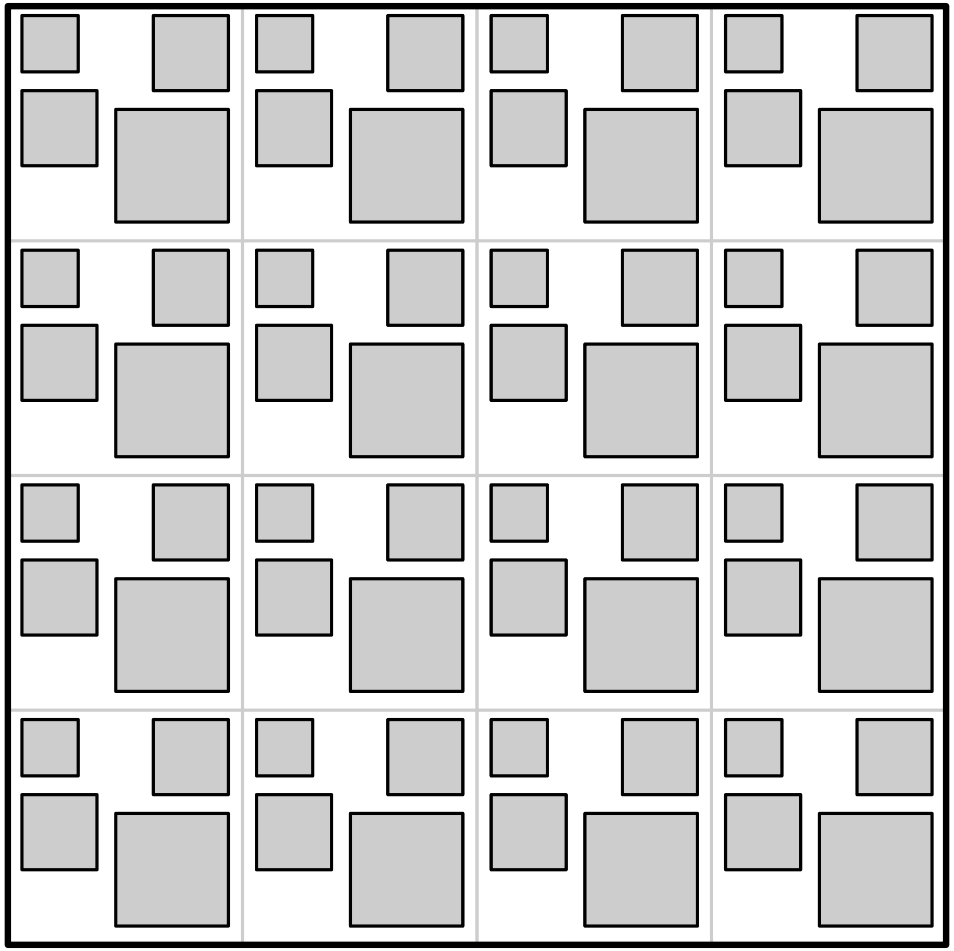

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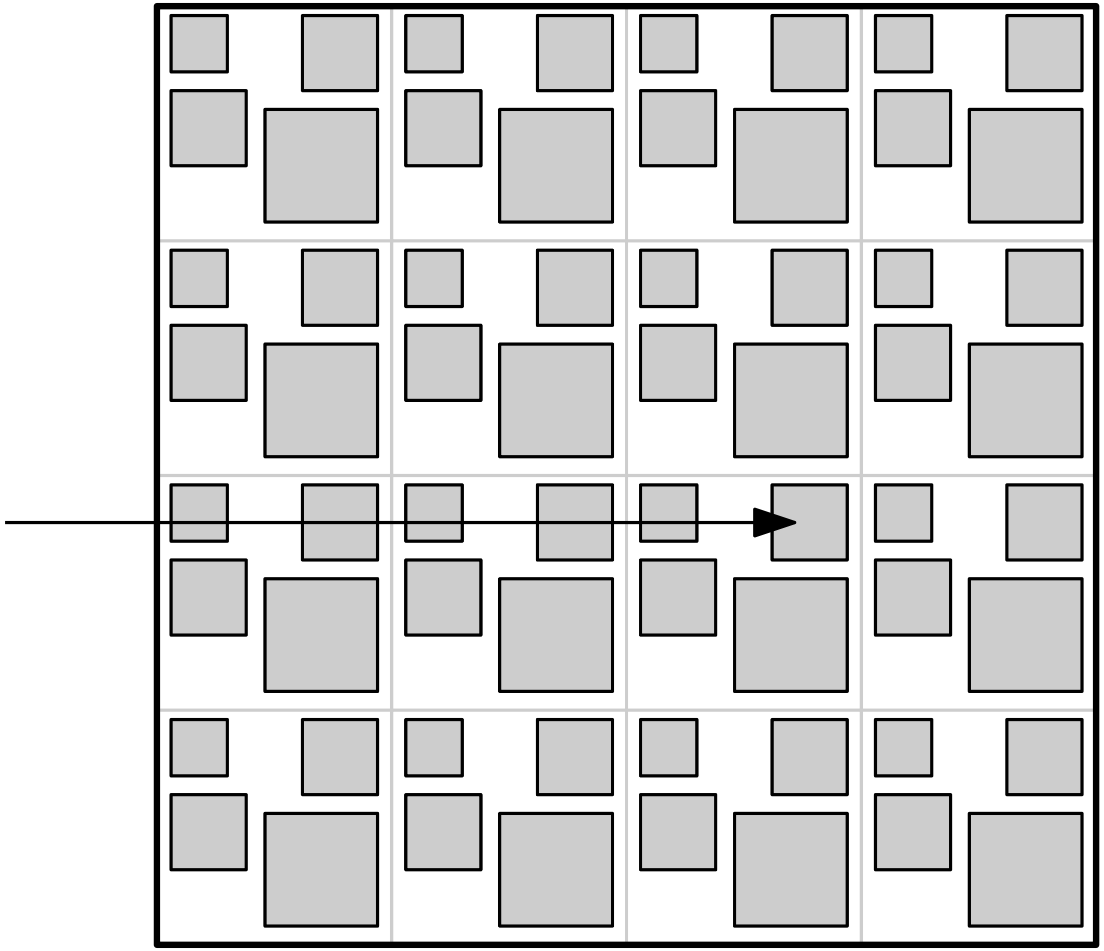

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