

① supervised learning 的算法中, 各算法比较相似, 但 Data/Amount 等因素起了决定作用.
Feature 选择

Regularizing 过程



Machine Learning

Support Vector Machines

Optimization objective

② 但 SVM 非常 powerful 在 both 工业界和学术界

③ SVM 与神经网络相比 cleaner, 与 Logistic Regression 相比 powerful

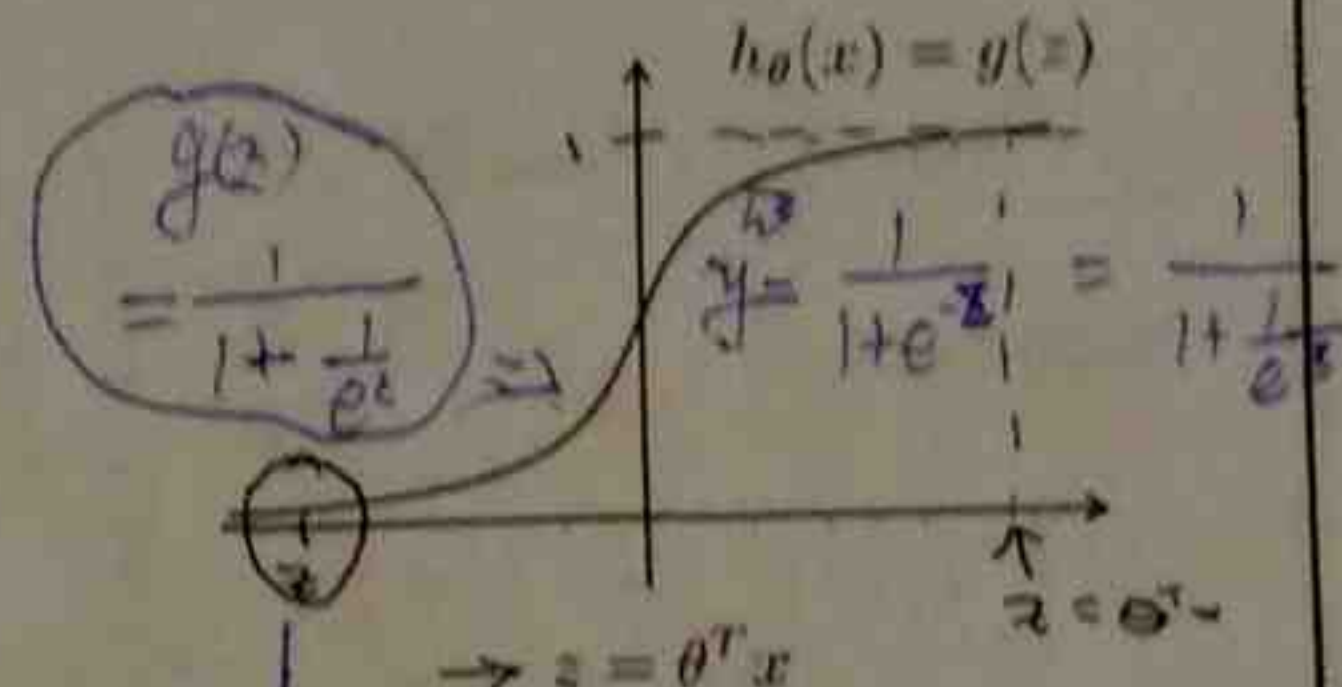
从 LR 开始, 再讲 SVM 修改, 引出 SVM

Alternative view of logistic regression

从 LR 开始

hypothesis function

$$h_{\theta}(x) = \frac{1}{1 + e^{-\theta^T x}}$$



If $y = 1$, we want $h_{\theta}(x) \approx 1$, $\theta^T x \gg 0 \rightarrow +\infty$
If $y = 0$, we want $h_{\theta}(x) \approx 0$, $\theta^T x \ll 0 \rightarrow -\infty$
 $z = \theta^T x$

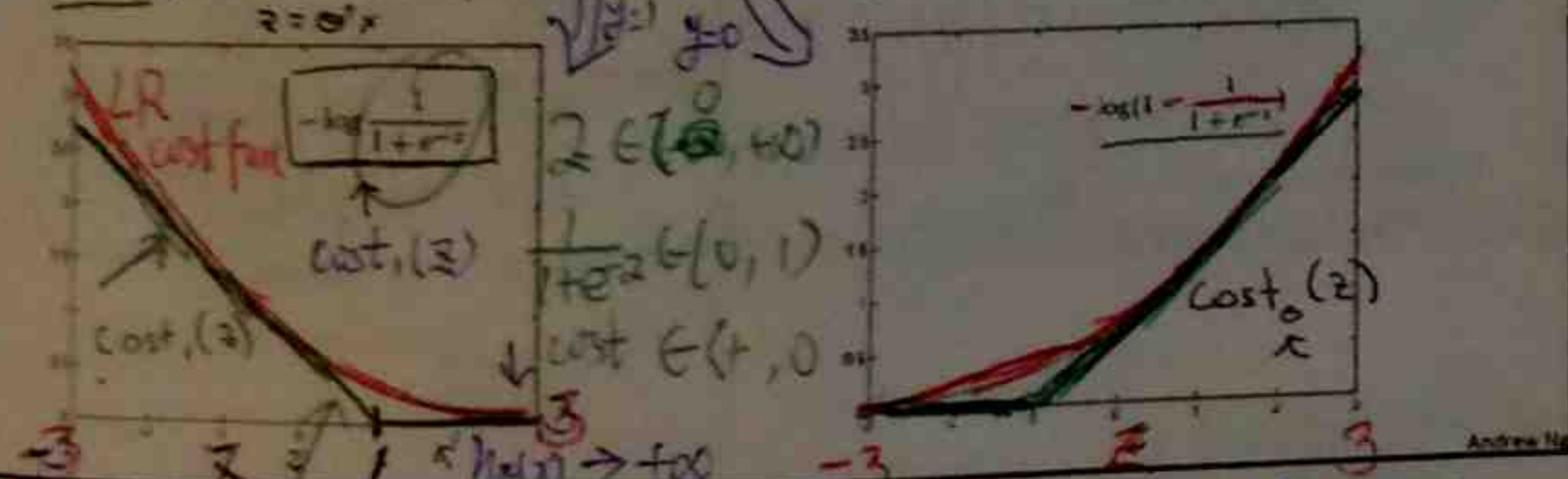
Alternative view of logistic regression

Cost of example: $-(y \log h_{\theta}(x) + (1 - y) \log(1 - h_{\theta}(x)))$ ← cost function

$$= -y \log \frac{1}{1 + e^{-\theta^T x}} - (1 - y) \log \left(1 - \frac{1}{1 + e^{-\theta^T x}} \right)$$

If $y = 1$ (want $\theta^T x \gg 0$):

If $y = 0$ (want $\theta^T x \ll 0$):



Support vector machine

Logistic regression: 最小化 cost func

$$\min_{\theta} \frac{1}{m} \left[\sum_{i=1}^m y^{(i)} \left(-\log h_{\theta}(x^{(i)}) \right) + (1 - y^{(i)}) \left(-\log(1 - h_{\theta}(x^{(i)})) \right) \right] + \frac{\lambda}{2m} \sum_{j=1}^n \theta_j^2$$

Support vector machine:

不会改变训练出的 θ

$$\min_{\theta} (12(-5)^2 + 1) \rightarrow \theta = 5$$

$$\min_{\theta} (10(-5)^2 + 10) \rightarrow \theta = 5$$

没有 $\frac{1}{m}$ 不改变最优 θ

$$\min_{\theta} C \sum_{i=1}^m \left[y^{(i)} \text{cost}_1(\theta^T x^{(i)}) + (1 - y^{(i)}) \text{cost}_0(\theta^T x^{(i)}) \right] + \frac{1}{2} \sum_{j=1}^n \theta_j^2$$

把括号外的 $\frac{1}{m}$ 移到括号内

用 $A + B$ 来控制 θ

$C = \frac{1}{m}$ (没有 $\frac{1}{m}$ 类似 $\frac{1}{m}$, 让 $C = \frac{1}{m}$)

用 $CA + B$ 来控制 θ

没有 $\frac{1}{m}$ 也行

$$CA + B = A + B$$

只是提了一个

Regularization Term

λ 入换为 C

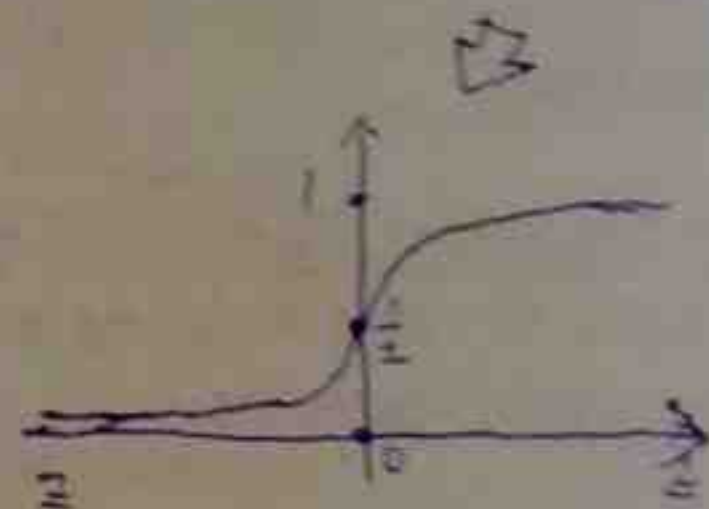
修改 cost function
 $z = \theta^T x \rightarrow +\infty$ cost(z) $\rightarrow 0$
 $\frac{1}{1+e^z} \rightarrow 1$
 $z = -\log h$

$z = h_{\theta}(x) \rightarrow -\infty$
cost(z) $\rightarrow 0$
 $1 - z = \frac{1}{1+e^z} \rightarrow 0$
 $z = 1 - \frac{1}{1+e^z} \rightarrow 1$

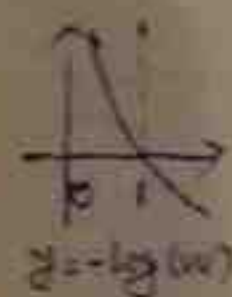
$\log(1 - \frac{1}{1+e^z}) \rightarrow 0$
cost(z) $\rightarrow 0$

hypothesis function of LR:

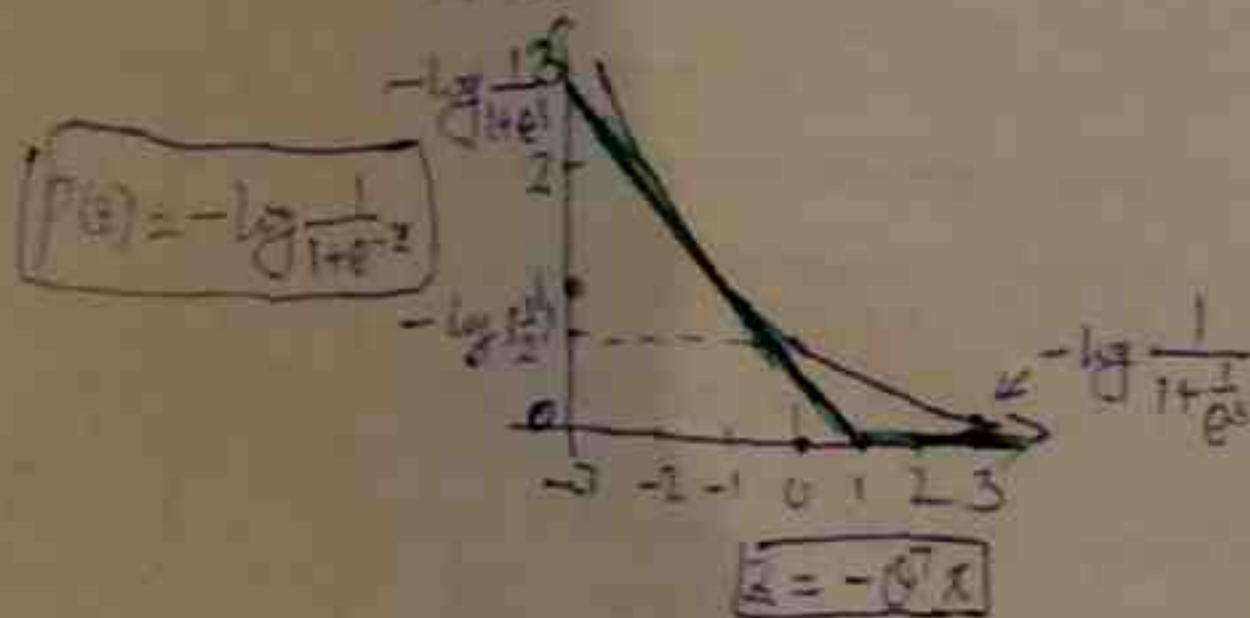
$$\boxed{\nearrow} h_0(x) = \frac{1}{1 + e^{-\theta^T x}} \Leftrightarrow g(z) = \frac{1}{1 + \frac{1}{e^z}}$$



$\boxed{\searrow}$ $y=1$ 时, $z \rightarrow +\infty$, $g(z) \rightarrow 1$, $-\log(g(z)) \rightarrow 0$
 虽然 $g(z) > \frac{1}{2}$, $z > 0$, 但假设 $z \rightarrow -\infty$ 时:
 $z \rightarrow -\infty$, $g(z) \rightarrow 0$, $-\log(g(z)) \rightarrow +\infty$



因此



同理可推 $y=0$ 时.

对正例, 预测得越准, cost 越小 (20)
 对正例, 预测得越不准, cost 越大 (20)

SVM: cost function 作此修改 同理可推 $y=0$ 时
 分为两段直线

$z > 1$ 时, —

$z < 1$ 时, \

SVM hypothesis

$$\min_{\theta} C \sum_{i=1}^m [y^{(i)} \text{cost}_1(\theta^T x^{(i)}) + (1 - y^{(i)}) \text{cost}_0(\theta^T x^{(i)})] + \frac{1}{2} \sum_{j=1}^n \theta_j^2$$

Hypothesis:

$$h_{\theta}(x) = \begin{cases} 1 & \text{if } \theta^T x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

不输出概率, 直接判断

LR不同: $z \geq 0$

hold $z \geq 0.5$ if $\theta^T x \geq 0$ 期望 $\theta^T x \rightarrow 100$
 < 0.5 otherwise 期望 $\theta^T x \rightarrow 0$

200



Machine Learning

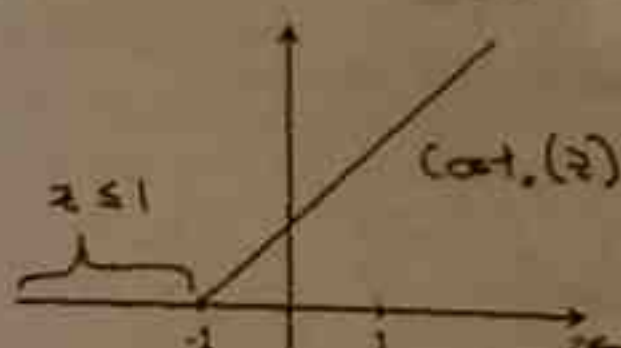
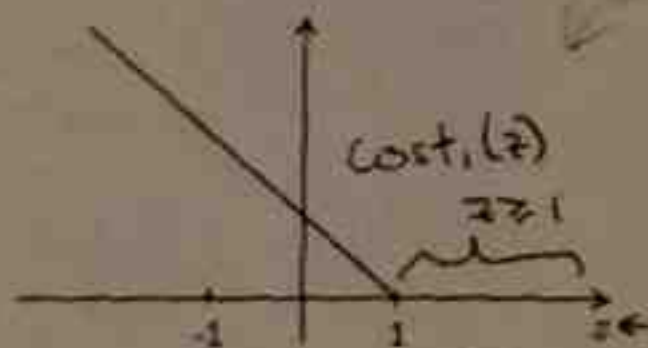
Support Vector Machines

Large Margin Intuition

SVM . Large Margin Classification

Support Vector Machine

$$\rightarrow \min_{\theta} C \sum_{i=1}^m [y^{(i)} \text{cost}_1(\theta^T x^{(i)}) + (1 - y^{(i)}) \text{cost}_0(\theta^T x^{(i)})] + \frac{1}{2} \sum_{j=1}^n \theta_j^2$$



\rightarrow If $y = 1$, we want $\theta^T x \geq 1$ (not just ≥ 0)

\rightarrow If $y = 0$, we want $\theta^T x \leq -1$ (not just < 0)

$C = 100,000$

当C非常大时

不幸在此中, 是希望 $z \geq 1$ 来作为判断依据

但SVM不同, 是以 $z \geq 1, \leq -1$ 来作为判断依据

(Large Margin)

spatial margin factor

SVM Decision Boundary

当C非常大时, 这部分被乘数大于0

$$\min_{\theta} C \sum_{i=1}^m [y^{(i)} \text{cost}_1(\theta^T x^{(i)}) + (1 - y^{(i)}) \text{cost}_0(\theta^T x^{(i)})] + \frac{1}{2} \sum_{j=1}^n \theta_j^2$$

Whenever $y^{(i)} = 1$:

$$\theta^T x^{(i)} \geq 1$$

Whenever $y^{(i)} = 0$:

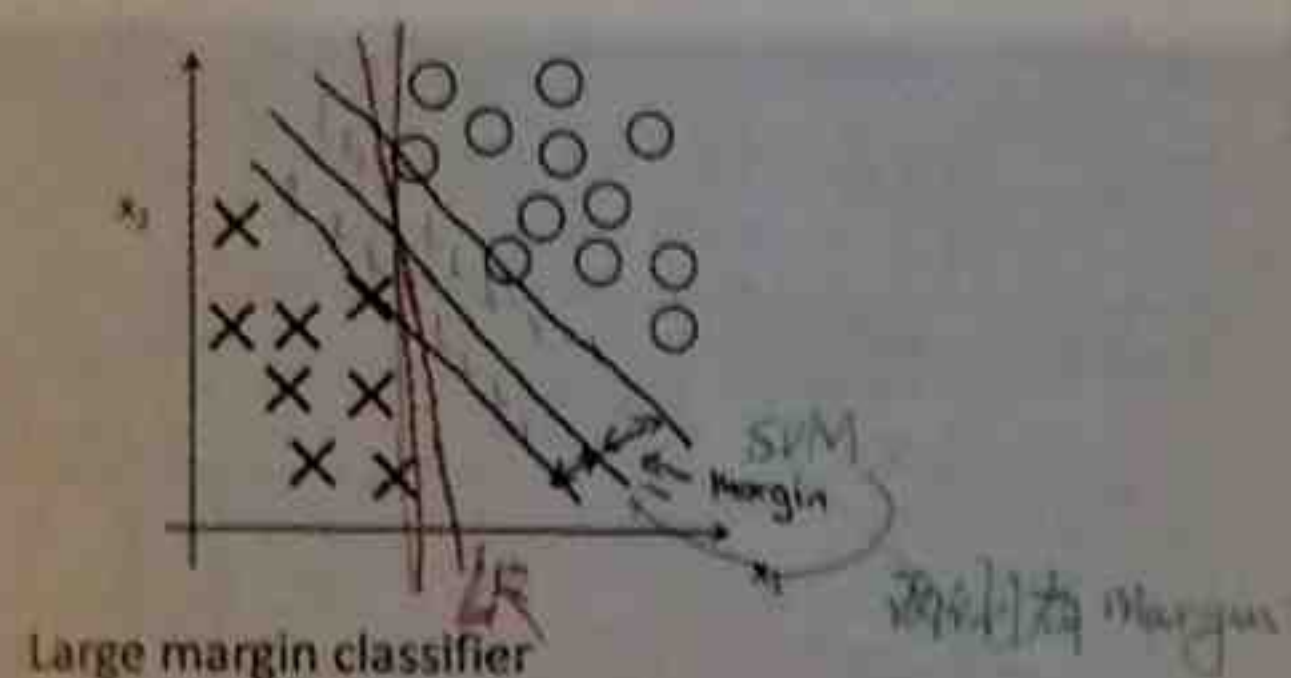
$$\theta^T x^{(i)} \leq -1$$

$$\min_{\theta} C \sum_{i=1}^m [y^{(i)} \text{cost}_1(\theta^T x^{(i)}) + (1 - y^{(i)}) \text{cost}_0(\theta^T x^{(i)})] + \frac{1}{2} \sum_{j=1}^n \theta_j^2$$

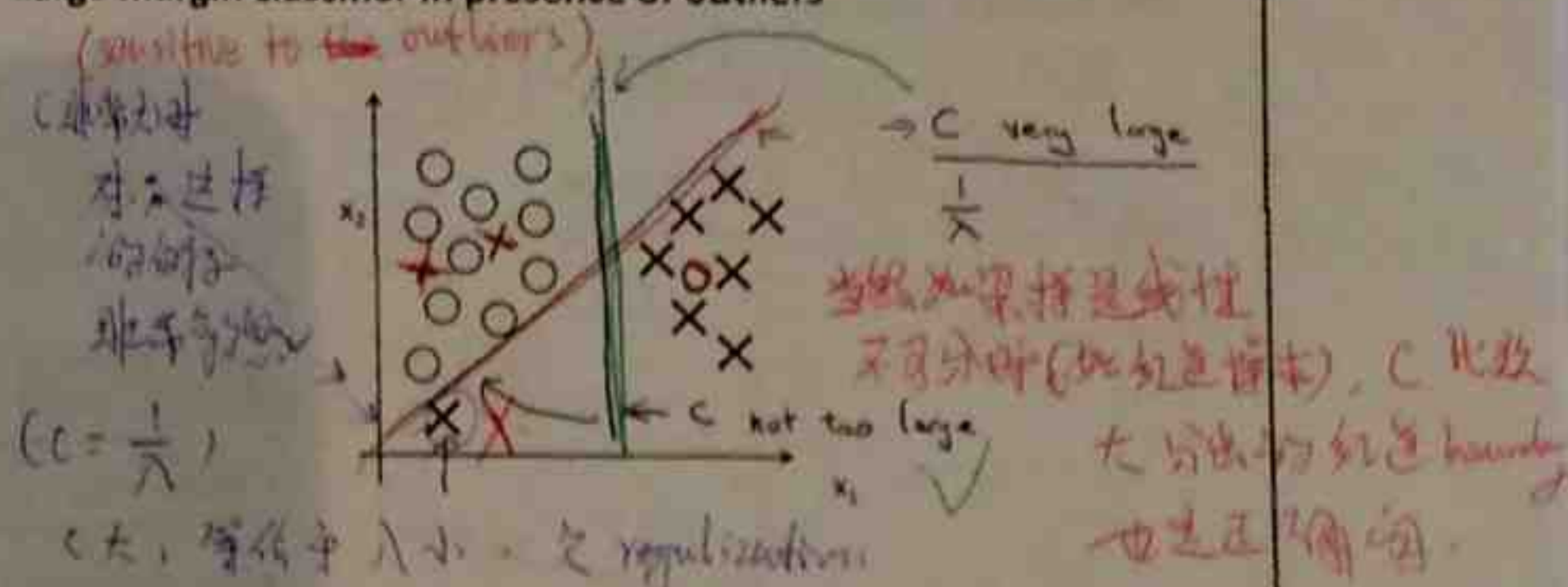
$$\text{s.t. } \theta^T x^{(i)} \geq 1 \quad \text{if } y^{(i)} = 1$$

$$\theta^T x^{(i)} \leq -1 \quad \text{if } y^{(i)} = 0$$

SVM Decision Boundary: Linearly separable case



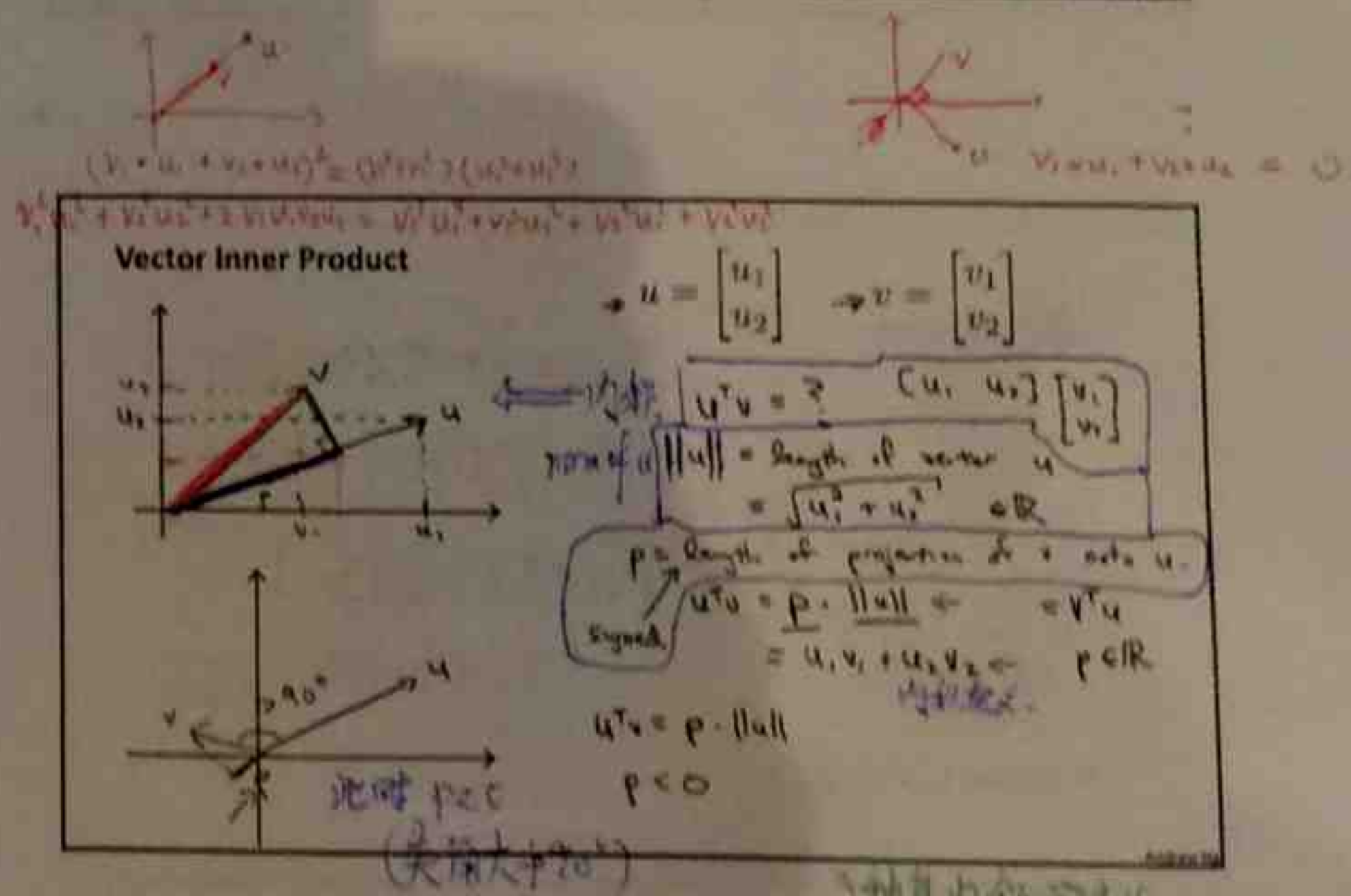
Large margin classifier in presence of outliers



Machine Learning

Support Vector Machines

The mathematics behind large margin classification (optional)



$$||u||^2 = u_1^2 + u_2^2$$

$$= (p_1^2 + p_2^2) \Rightarrow (u_1^2 + u_2^2) \text{ 在"内积"的几何意义}$$

3种表示内积的方式

$$u^T v = u_1 v_1 + u_2 v_2$$

$$||u|| ||v|| \cos \theta$$

$$p \cdot ||u||$$

$\theta_0 = 0$; $n=3$ 维空间有 2 个 feature

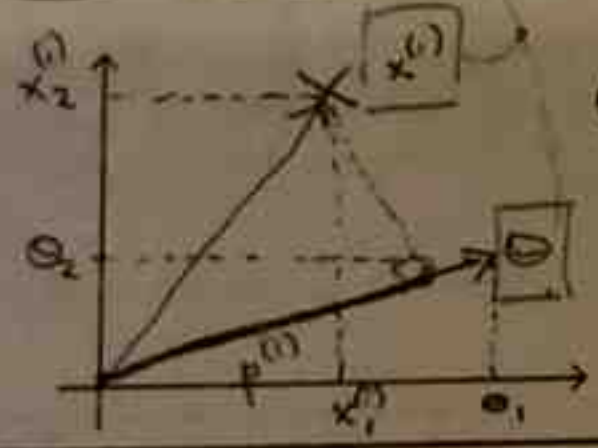
SVM Decision Boundary

$$\min_{\theta} \frac{1}{2} \sum_{j=1}^n \theta_j^2 = \frac{1}{2} (\theta_1^2 + \theta_2^2) = \frac{1}{2} (\theta_1^2 + \theta_2^2) = \frac{1}{2} \|\theta\|^2$$

s.t. $\theta^T x^{(i)} \geq 1$ if $y^{(i)} = 1$
 $\theta^T x^{(i)} \leq -1$ if $y^{(i)} = 0$

Simplification: $\theta_0 = 0$, $n=2$

$\theta^T x^{(i)} = ?$
 $\uparrow \uparrow$
 $u^T v$



$$\theta^T x^{(i)} = p^{(i)} \|\theta\|$$

$$= \theta_1 x_1 + \theta_2 x_2$$

因此 $\theta^T x^{(i)} \geq 1$
 $\theta^T x^{(i)} \leq -1$

可以转换为
 $p^{(i)} \cdot \|\theta\| \geq 1$
 $p^{(i)} \cdot \|\theta\| \leq -1$

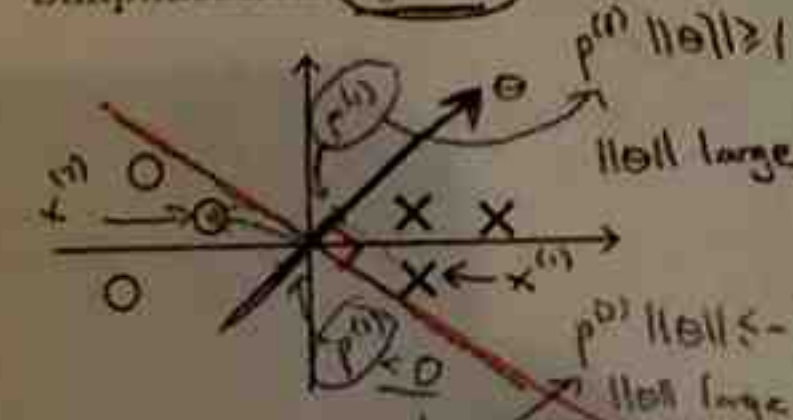
SVM Decision Boundary

$$\min_{\theta} \frac{1}{2} \sum_{j=1}^n \theta_j^2 = \frac{1}{2} \|\theta\|^2$$

s.t. $p^{(i)} \cdot \|\theta\| \geq 1$ if $y^{(i)} = 1$
 $p^{(i)} \cdot \|\theta\| \leq -1$ if $y^{(i)} = 0$

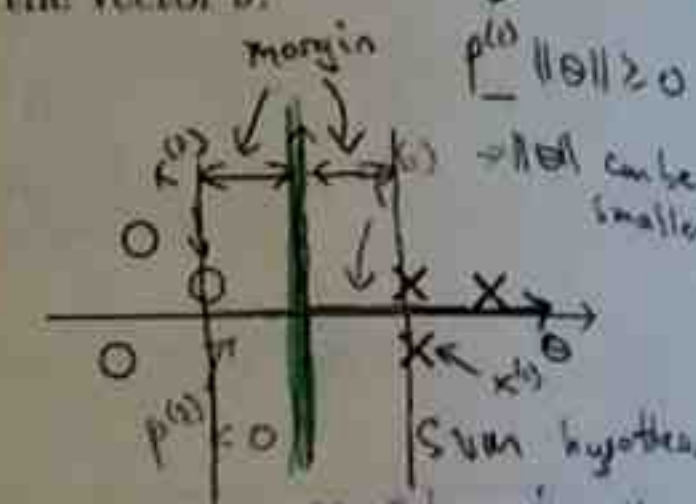
where $p^{(i)}$ is the projection of $x^{(i)}$ onto the vector θ .

Simplification: $\theta_0 = 0$



每个 x, c 在 θ 上投影

$p^{(i)} > 0$, $p^{(i)} < 0$
 都非零, 使得 $\|\theta\|$ 非常大,
 但我们 objective 是让它小



若 x, c 在 θ 上投影
 $p^{(i)} > 0$, $p^{(i)} < 0$
 都非零, 使得 $\|\theta\|$ 非常大,
 但我们 objective 是让它小

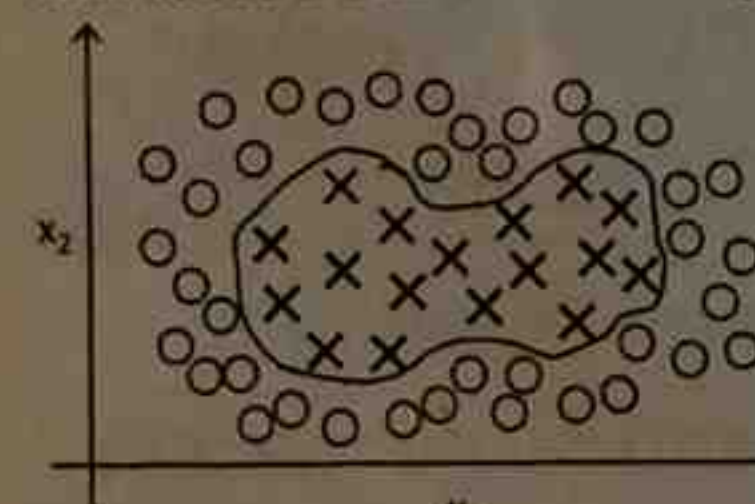


Machine Learning

Support Vector Machines Kernels I

处理复杂的非线性分类

Non-linear Decision Boundary



Predict $y = 1$ if

$$\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1 x_2 + \theta_4 x_1^2 + \theta_5 x_2^2 + \dots > 0$$

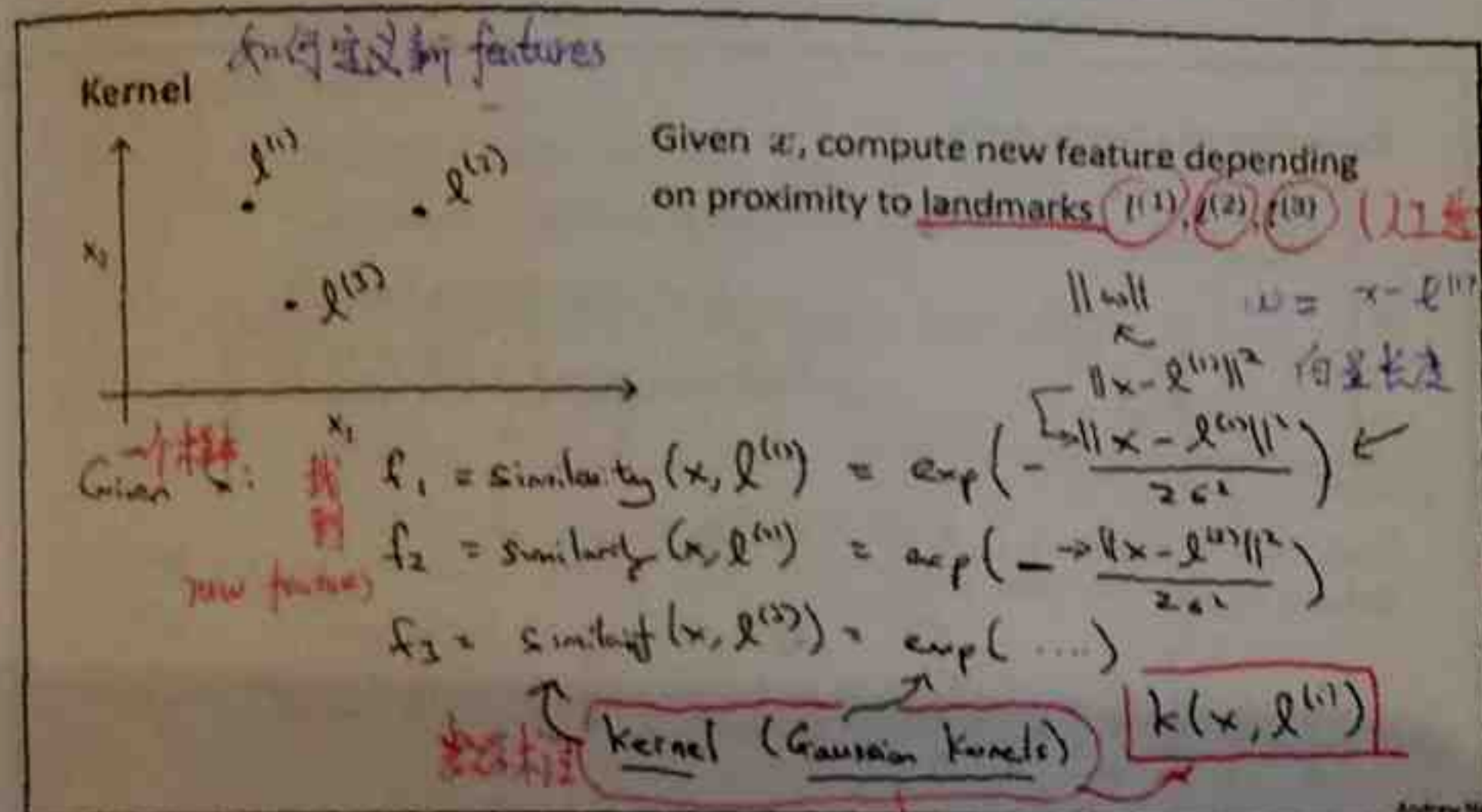
$$h_{\theta}(x) = \begin{cases} 1 & \text{if } \theta_0 + \theta_1 x_1 + \dots > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\rightarrow \theta_0 + \theta_1 f_1 + \theta_2 f_2 + \theta_3 f_3 + \dots$$

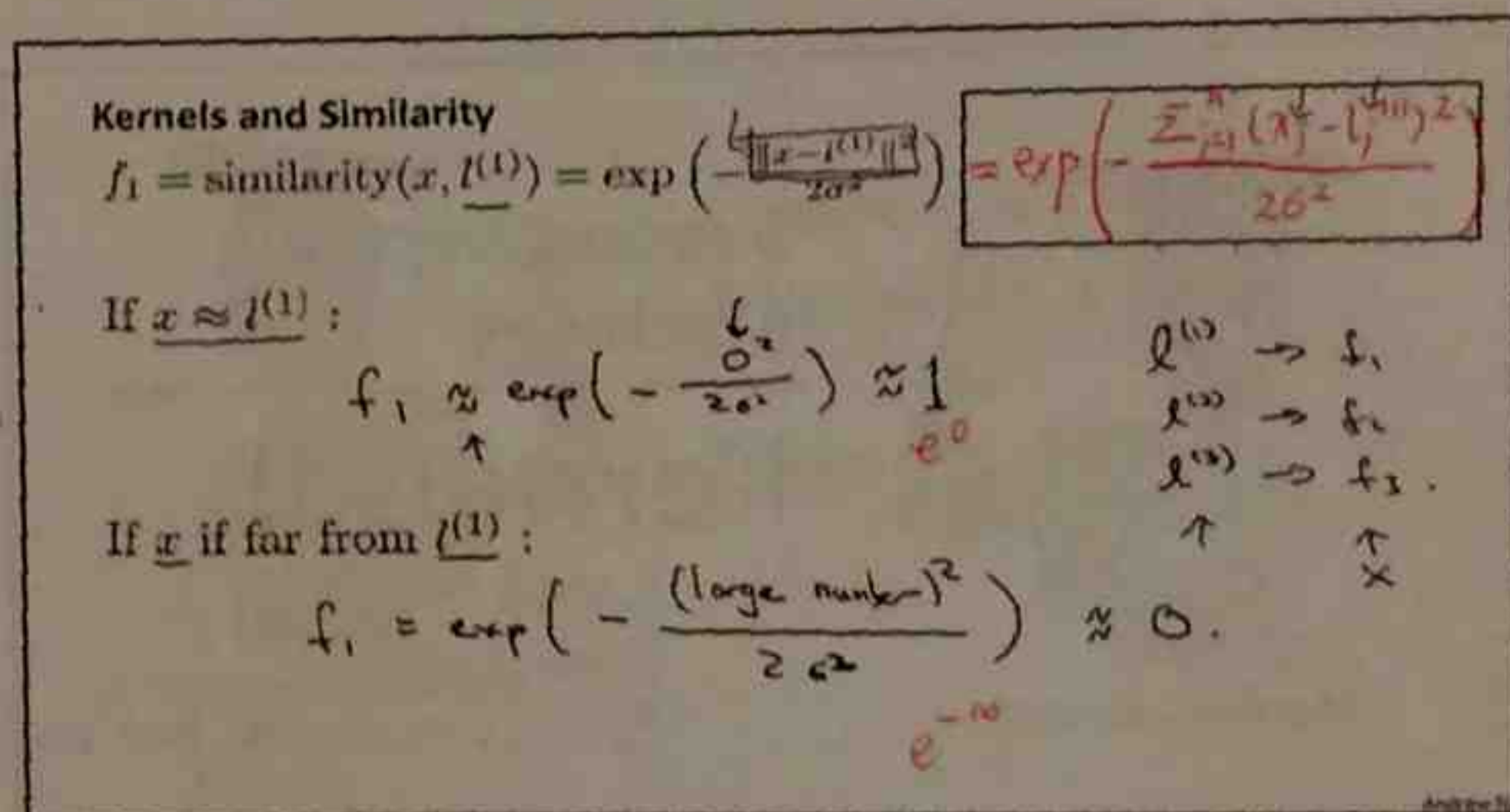
$$f_1 = x_1, f_2 = x_2, f_3 = x_1 x_2, f_4 = x_1^2, f_5 = x_2^2, \dots$$

Is there a different / better choice of the features f_1, f_2, f_3, \dots ?

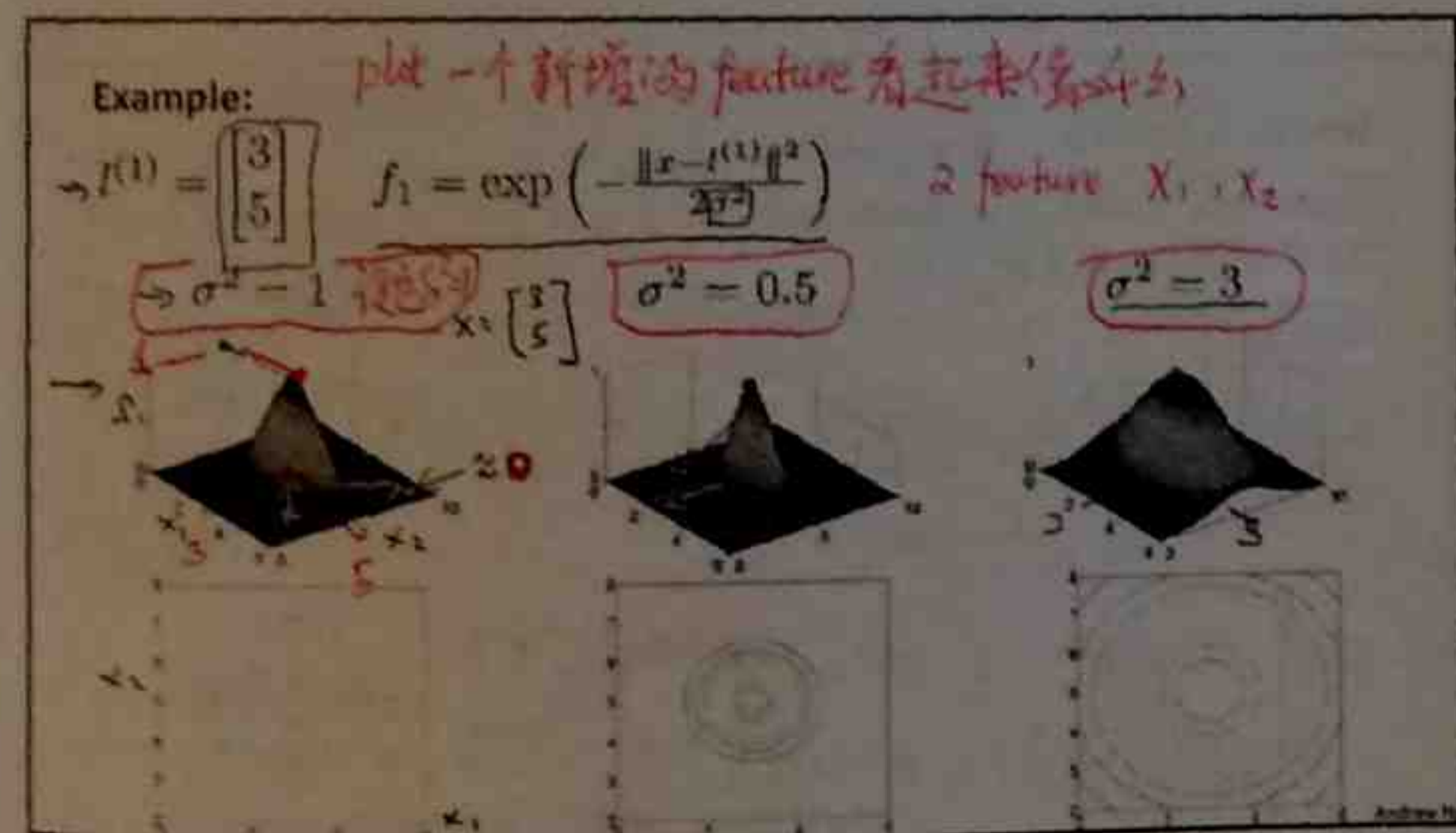
我们不知道这些 f_i 是否有用
 并且 expensive



这个叫高斯(kernels)
还有其它 kernel



忽略 x_0
(是 x_0)

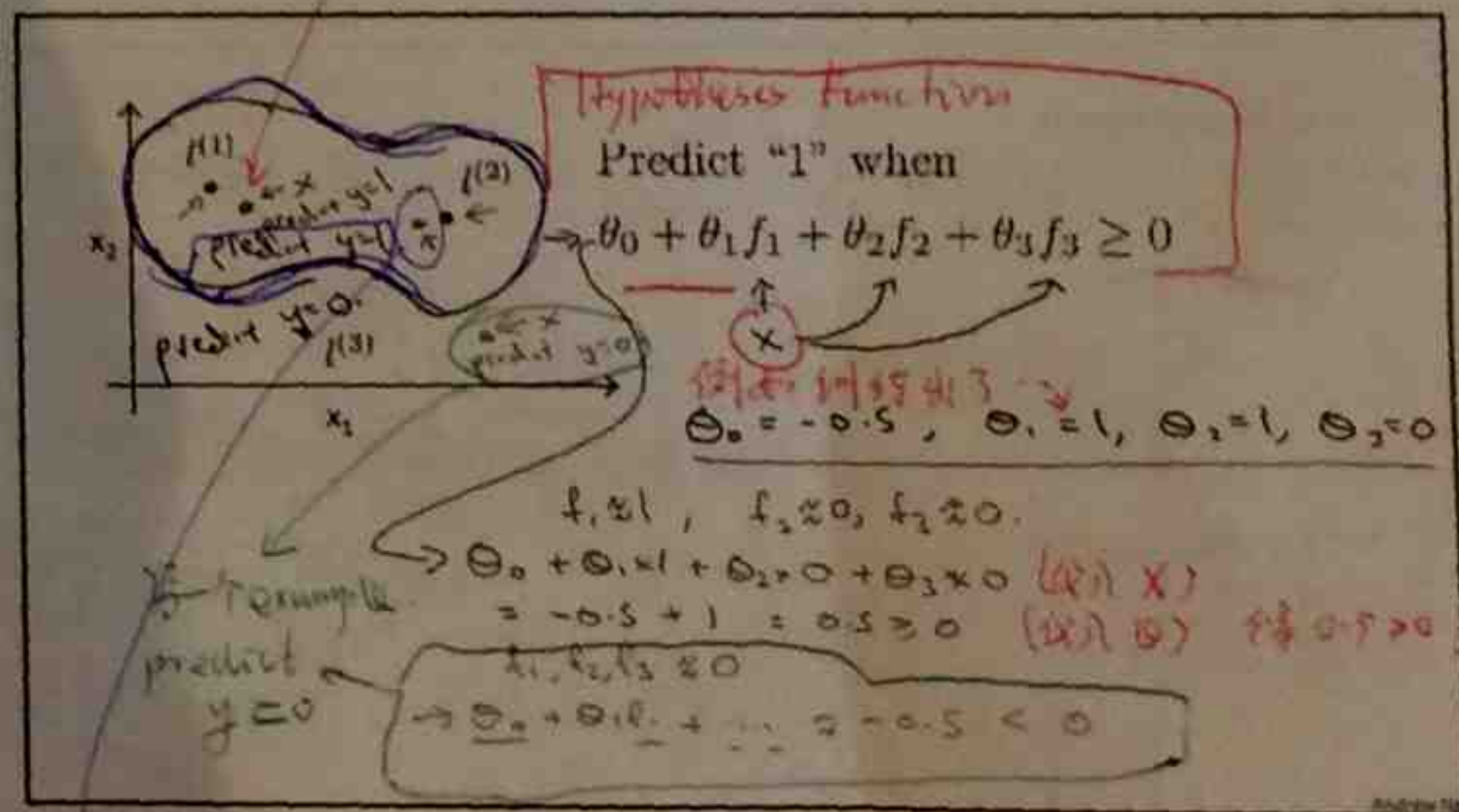


$\sigma^2 = 0.5$
land 下降到更快

$\sigma^2 = 3$
land 下降到更慢

推广到任意个
的线性组合

通过 landmark 找到 $y=1$ 的 boundary.



第3个 example, 可以预测 $y=1$, 因为发现
接近 $l^{(1)}, l^{(2)}$ 的都是 $y=1$; 远离的都是 $y=0$.



Machine Learning

Support Vector Machines

Kernels II

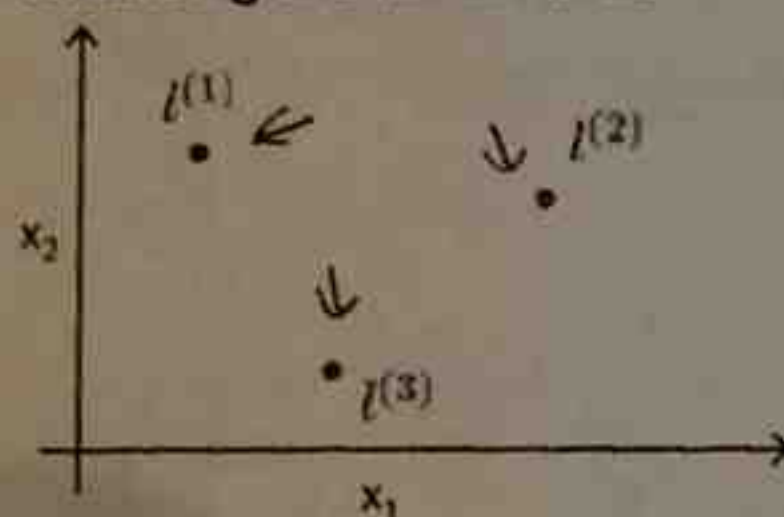
- ① detail
- ② how to use
- ③ bias-variance problems

- ① 找 landmark
- ② 用 GK 找 ... 来 build $H(x)$

How to find landmarks
 \Rightarrow 放在 m training examples 上
 $l^{(1)}, l^{(2)}, \dots, l^{(m)}$
 m 个 examples $\Rightarrow m$ 个 landmark

对于任意一个
计算
(下-到)

Choosing the landmarks



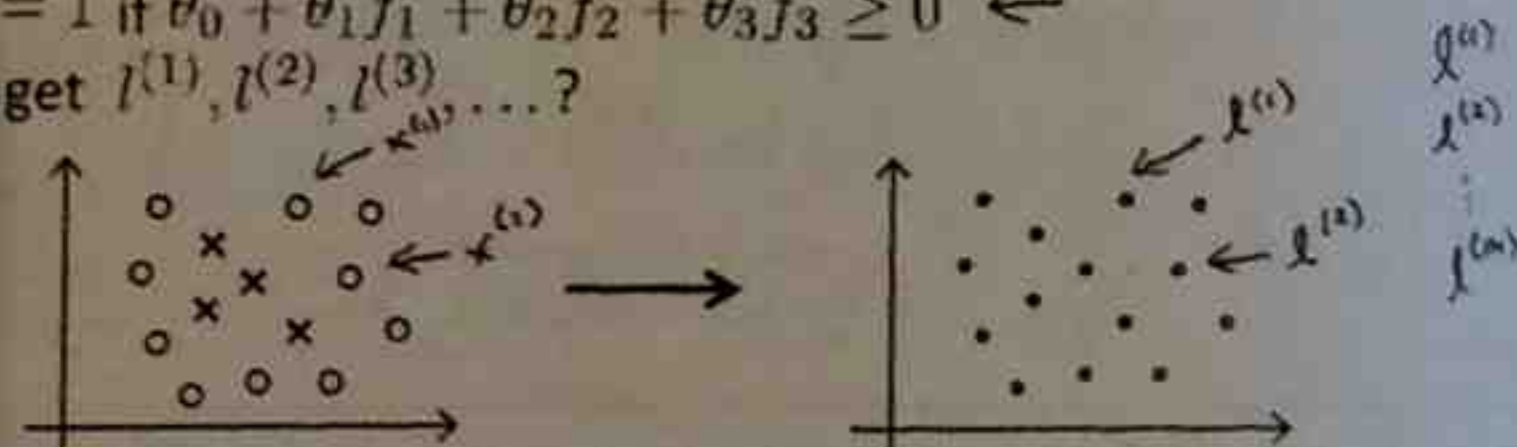
Given x :

$$f_i = \text{similarity}(x, l^{(i)})$$

$$= \exp\left(-\frac{\|x - l^{(i)}\|^2}{2\sigma^2}\right)$$

Predict $y = 1$ if $\theta_0 + \theta_1 f_1 + \theta_2 f_2 + \theta_3 f_3 \geq 0$

Where to get $l^{(1)}, l^{(2)}, l^{(3)}, \dots$?



SVM with Kernels

- \Rightarrow Given $(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \dots, (x^{(m)}, y^{(m)})$
- \Rightarrow choose $l^{(1)} = x^{(1)}, l^{(2)} = x^{(2)}, \dots, l^{(m)} = x^{(m)}$

Given example x :

- $\Rightarrow f_1 = \text{similarity}(x, l^{(1)})$
- $\Rightarrow f_2 = \text{similarity}(x, l^{(2)})$

$$f = \begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ \vdots \\ f_m \end{bmatrix} \quad f_0 = 1$$

For training example $(x^{(i)}, y^{(i)})$:

$$f_1^{(i)} = \sin(x^{(i)}, l^{(1)})$$

$$f_2^{(i)} = \sin(x^{(i)}, l^{(2)})$$

$$\vdots$$

$$f_m^{(i)} = \sin(x^{(i)}, l^{(m)})$$

其中有一个

landmark 是
example 的

用 feature vector
来建 training example

SVM with Kernels

Hypothesis: Given x , compute features $f \in \mathbb{R}^{m+1}$

\Rightarrow Predict "y=1" if $\theta^T f \geq 0$

Training:

$$\min_{\theta} C \sum_{i=1}^m y^{(i)} \text{cost}_1(\theta^T f^{(i)}) + (1 - y^{(i)}) \text{cost}_0(\theta^T f^{(i)}) + \frac{1}{2} \sum_{j=1}^m \theta_j^2$$

$$\theta^T f^{(i)} = \theta_0 + \theta_1 f_1^{(i)} + \dots + \theta_m f_m^{(i)}$$

$$\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \vdots \\ \theta_m \end{bmatrix}$$

$$M = \begin{bmatrix} f_1^{(1)} & \dots & f_1^{(m)} \\ \vdots & \ddots & \vdots \\ f_m^{(1)} & \dots & f_m^{(m)} \end{bmatrix}$$

$$\theta^T M \theta$$

很多 SVM 会改一下 θ 的
虽然用特都是 minimize $\| \theta \|^2$
会让 SVM 收敛的快
Kernel 用在以上会很慢

如何选参数平衡 bias/variance

SVM parameters:

$C (= \frac{1}{\lambda})$. \rightarrow Large C: Lower bias, high variance. (small λ)
 \rightarrow Small C: Higher bias, low variance. (large λ)

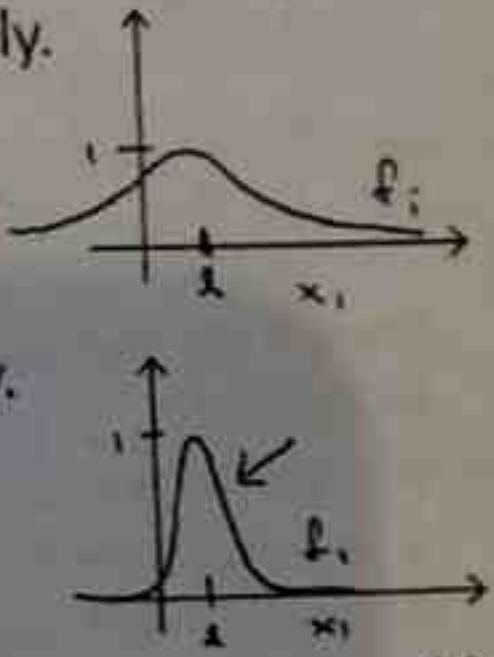
σ^2 : Large σ^2 : Features f_i vary more smoothly.
 \rightarrow Higher bias, lower variance.

$$\exp\left(-\frac{\|x - l^{(i)}\|^2}{2\sigma^2}\right)$$

正则化程度 不是正则化程度

Small σ^2 : Features f_i vary less smoothly.
 Lower bias, higher variance.

容易过拟合 不容易欠拟合



Andrew Ng



Machine Learning

Support Vector Machines

Using an SVM

Use SVM software package (e.g. liblinear, libsvm, ...) to solve for parameters θ .

那使用什么, 世界参

Need to specify:

\rightarrow Choice of parameter C .

Choice of kernel (similarity function):

E.g. No kernel ("linear kernel")
 Predict "y = 1" if $\theta^T x \geq 0$

$$\theta_0 + \theta_1 x_1 + \dots + \theta_n x_n \geq 0 \quad x \in \mathbb{R}^{n+1}$$

\rightarrow n large, m small

\rightarrow Gaussian kernel:

$$f_i = \exp\left(-\frac{\|x - l^{(i)}\|^2}{2\sigma^2}\right), \text{ where } l^{(i)} = x^{(i)}.$$

Need to choose σ^2

features, 样本多时

用kernel 替代一个复杂的非线性模型



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Kernel (similarity) functions:

function $f = \text{kernel}(x_1, x_2)$

$$f = \exp\left(-\frac{\|x_1 - x_2\|^2}{2\sigma^2}\right)$$

return

例: 高斯 kernel

\rightarrow Note: Do perform feature scaling before using the Gaussian kernel.

$$\|x - l\|^2$$

$$v = x - l$$

$$\|v\|^2 = v_1^2 + v_2^2 + \dots + v_n^2$$

$$= (x_1 - l_1)^2 + (x_2 - l_2)^2 + \dots + (x_n - l_n)^2$$

1000 feet² 1.5 bedrooms

feature scaling 看前几节

要归一到同一个区间内

有非零自己实现一个 kernel

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Other choices of kernel : Linear kernel, merging no kernel, gaussian kernel

Note: Not all similarity functions $\text{similarity}(x, l)$ make valid kernels.

→ (Need to satisfy technical condition called "Mercer's Theorem" to make sure SVM packages' optimizations run correctly, and do not diverge).

Many off-the-shelf kernels available:

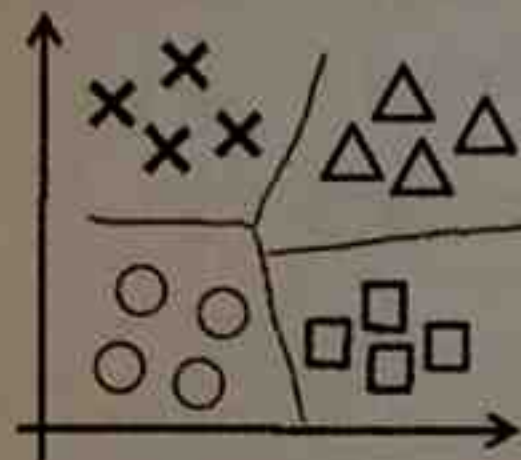
Polynomial kernel: $k(x, l) = (x^T l + \text{constant})^{\text{degree}}$

$(x^T l + 1)^2$, $(x^T l + 1)^3$, $(x^T l + 5)$

More esoteric: String kernel, chi-square kernel, histogram intersection kernel, ...

$\text{sim}(x, l)$

Multi-class classification



$$y \in \{1, 2, 3, \dots, K\}$$

Many SVM packages already have built-in multi-class classification functionality.

→ Otherwise, use one-vs.-all method. (Train K SVMs, one to distinguish $y = i$ from the rest, for $i = 1, 2, \dots, K$), get $\theta^{(1)}, \theta^{(2)}, \dots, \theta^{(K)}$
Pick class i with largest $(\theta^{(i)})^T x$

用 K 个 model 各预测一下
分到得分最高的那个类别中

Logistic regression vs. SVMs

n = number of features ($x \in \mathbb{R}^{n+1}$), m = number of training examples

→ If n is large (relative to m): (e.g. $n \geq 10,000$, $m = 10 \pm 1,000$)

→ Use logistic regression, or SVM without a kernel ("linear kernel")

→ If n is small, m is intermediate: ($n = 1-1,000$, $m = 10-10,000$)

→ Use SVM with Gaussian kernel

If n is small, m is large: ($n = 1-1,000$, $m = 50,000+$)

→ Create/add more features, then use logistic regression or SVM without a kernel

→ Neural network likely to work well for most of these settings, but may be slower to train.

特征太多了, 用 LR 就可以

特征少, 样本也少也不是太密集

样本量太少了, 还是加特征来解决 LR 特征少的问题

LR 与 SVM-NO-KERNEL 很相似

SVM: Convex optimization

no worry for local optimization problems

Neural Network:

一般也不用担心 local optimization problems

但还是比较慢