# Case Study Modelling an Electronic Component

Azure Hutchings Jean-Luc Danoy
Faris Saad S Alsubaie

28 October 2019

## Executive Summary

Write Abstract Here

# Contents

# 1 Introduction

## 1.1 Purpose of the Report

The following report investigates the steady-state heat distribution in a newly designed component.

The report will discuss the mathematical model of the heat distribution in the component and the numerical methods used to solve it in MATLAB.

## 1.2 The Maths of the Problem

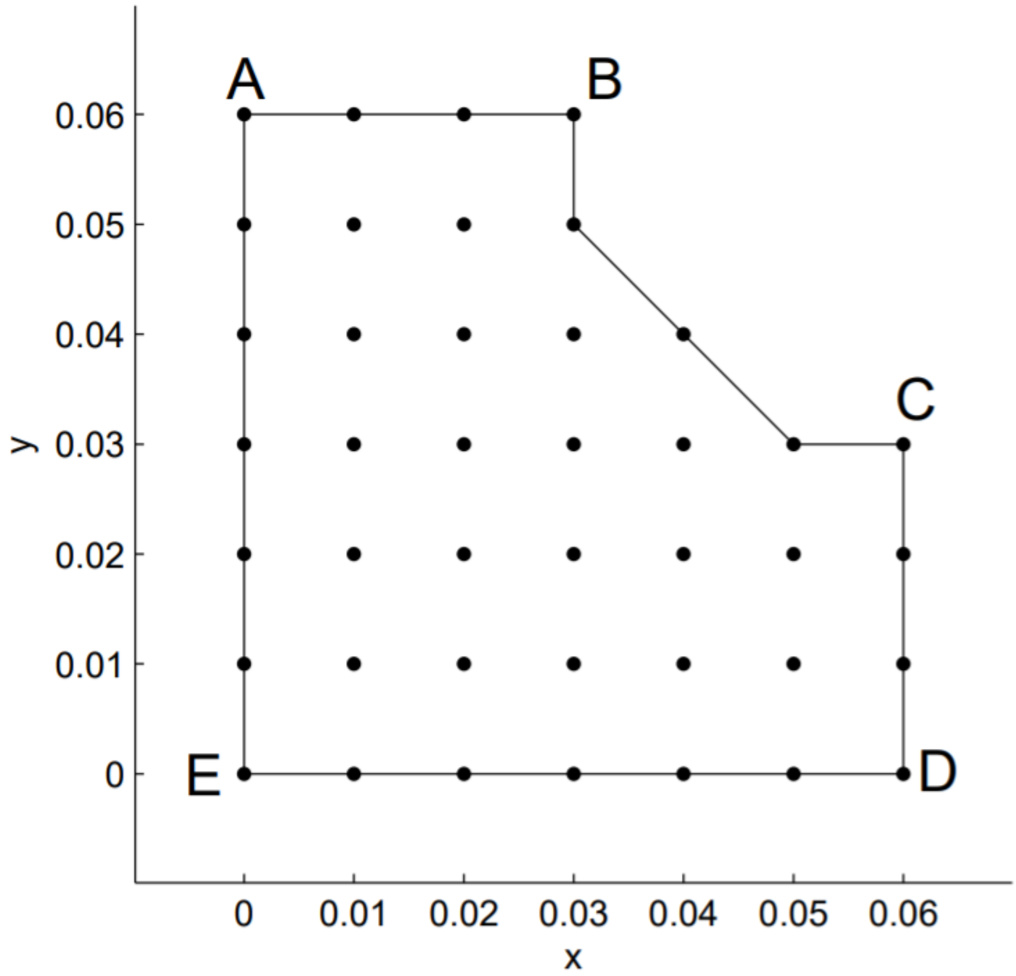


Figure 1: Schematic of electronic component.

The component schematic is shown in Figure 1. The location of the component within the device means it's subject to different temperature condition along it's boundaries. The boundary A-B is in perfect thermal contact with another component which the temperature is known to 70°C. The boundary C-D is also in perfect thermal contact with another component which the temperature is known to be 40°C. The boundary A-E-D is thermally insulated and the boundary B-C is exposed to the air at ambient temperature.

This type of model can be described with Laplace's equation. Letting $T(x, y)$ represent the temperature of the component at point $(x, y)$, the model is as follows

$$
\begin{array}{cc}
\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 & \text{in the interior} \\
T = 70 & \text{on boundary A-B} \\
T = 40 & \text{on boundary C-D} \\
\boldsymbol{\nabla} T \cdot \hat{\mathbf{n}} = 0 & \text{on boundary A-E-D} \\
k \boldsymbol{\nabla} T \cdot \hat{\mathbf{n}} = h(T_\infty - T) & \text{on boundary B-C}
\end{array}
$$

Where the thermal conductivity is $k = 3Wm^{-1}C^{-1}$, and the heat transfer coefficient is $h = 20Wm^{-2}C^{-1}$. To begin with, we will assume the ambient termperature is $T_\infty = 20$.

# 2 Discretising the Problem

If this was left as a continuous partial differential equation, we would need to analytically solve it. However, it can be much quicker to numerically solve this problem and still retain a high degree of accuracy with the final answers. In this section we will show how we converted the analytical problem to a numerical problem, how the mesh was constructed, the node ordering used, the linear system we dervied from the mesh and discuss the matrix that was created from it.

## 2.1 Analytical approach

To build a numerical approach, we must first understand the basics of the analytical method for solving heat distributions.

The Heat Equation is a partial differential equation which states that the change in temperature over time is proportional to the double derivative of temperature with repsect to its spacial variables

$$\frac{\partial u}{\partial t} \propto \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$
$$\frac{\partial u}{\partial t} = D\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

## 2.2 Numerical approach and Finite Difference Mesh

Suppose we had a function $\phi(x, y, t)$ which gives the temperature at the point (x,y) on a 2-dimensional plane where t is the time since the start of the initial conditions. If the function has a steady-state solution, then it means at some point in time, any increase in time will not result in a change of temperature. This means that

$$\frac{d\phi}{dx}\Big|_{x_i} \approx \frac{\phi(x_i+\Delta x)-\phi(x_i)}{\Delta x},$$

where the change in $x$ along $\phi$ at the point $x_i$ is approximately the functions forward difference over the distance between nodes. We can then take the derivative of this again using backwards difference to get the second derivative centered around $x_i$.

$$\frac{d^2\phi}{dx^2}\Big|_{x_i} \approx \frac{\phi(x_i+\Delta x)-2\phi(x_i)+\phi(x_i-\Delta x)}{(\Delta x)^2},$$

If we do the same for the change in y, we arrive at an analogous equation

$$\frac{d^2\phi}{dy^2}\Big|_{x_i} \approx \frac{\phi(y_i+\Delta y)-2\phi(y_i)+\phi(y_i-\Delta y)}{(\Delta y)^2},$$

To numerically solve this problem, we must construct a finite difference mesh and create a matrix to represent this information.

By splitting the component up into intervals of 0.01 in both the x and y direction, we can split the component into a 7x7 grid. Because we know that the temperature on the upper and right hand boundaries are fixed temperatures, we can ignore them in our discretisation as nodes. They will come into effect later as the right hand side matrix. Given the previous mesh from the introduction, we can label each node $u_{i,j}$ for i,j = 0 to 6 where i, j are given by the row and column of the node starting from the bottom left hand corne. Replacing $\phi(x, y, t)$ with $u_{i,j}(t)$ and noting that $\Delta x = \Delta y$, we arrive at the set of equations for the change in $u_{i,j}(t)$.

$$u'_{i,j}(t) \approx \tfrac{D}{\Delta x^2}\big(u_{i+1,j}(t) + u_{i,j+1}(t) - 4u_{i,j}(t) + u_{i-1,j}(t) + u_{i,j-1}(t)\big)$$

There are 3 different types of mesh that need to be created. These are for the A-E-D boundary, the inside of the mesh, and on the B-C boundary.

On the interior of the mesh, each node contains 4 other nodes adjacent to it, and the change in temperature at that node is given by the equation

$$\begin{gathered}\tfrac{\partial^2 T}{\partial x^2} + \tfrac{\partial^2 T}{\partial y^2} = 0 \\ \tfrac{\partial}{\partial x}(\tfrac{\partial T}{\partial x}) + \tfrac{\partial}{\partial x}(\tfrac{\partial T}{\partial y}) = 0 \\ \tfrac{\partial T}{\partial x} = \tfrac{T_{i+1} - T_i}{\delta}\end{gathered}$$