

Recursion

induction with finite data

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February 25, 2019

Recursion

Recursion: What is it?



Figure 1: Matryoshka dolls

Recursion: The elements

- acts on finite *data*
- define your solution in terms of a simpler solution
- MUST define a solution for all base cases

Recursion: So what?

- separation of concerns: state $(k + 1)^{th}$ solution in terms of k^{th}
- assumes k^{th} solution is correct
- explicitly specify base case solution
- MUST converge toward base cases
- parallels to induction going in reverse direction

Induction: Example

Prove that $1 + 2 + 3 + \dots + n = n(n + 1)/2$

Step 1 - Show this holds for base case ($n = 1$): $1 = 1(1 + 1)/2 = 1$

Step 2a - Assume this holds for k^{th} case: $1 + \dots + k = k(k + 1)/2$

Step 2b - From this show that the $(k + 1)^{th}$ case holds:

$$1 + \dots + k + (k + 1) = (k(k + 1)/2) + (k + 1)$$

$$\implies (k(k + 1) + 2(k + 1))/2 = (k^2 + k + 2k + 2)/2$$

$$\implies (k^2 + 3k + 2)/2 = ((k + 1)(k + 2))/2$$

This is equivalent to substituting $(k + 1)$ as n in the original statement. \square

Recursion Example

```
1  data Nat
2    = Zero
3    | Succ Nat
4
5  natToInteger :: Nat -> Integer
6  natToInteger Zero = 0
7  natToInteger (Succ k) = 1 + natToInteger k
8
9  sumOfConsecutiveNats :: Nat -> Integer
10 sumOfConsecutiveNats Zero = 0
11 sumOfConsecutiveNats (Succ k)
12   = ((natToInteger k) + 1) + sumOfConsecutiveNats k
```


What now?

- live coding examples and exercises in `src/Effpee/Many.hs`