# Simulation Modeling

May 17, 2025

## Activity: Simulating Deterministic Behavior

1. Use Monte Carlo simulation to approximate the area under the curve $y = \sqrt{x}$, over the interval $\frac{1}{2} \leq x \leq \frac{3}{2}$. Give the actual area using the definite integral.

   **MATLAB Code:**

```
a = 0.5;
b = 1.5;
fmax = sqrt(b);
total_points = 100000;

x_rand = a + (b - a) * rand(1, N);
y_rand = fmax * rand(1, N);

under_curve = y_rand <= sqrt(x_rand);
approx_area = sum(under_curve) / total_points * (b - a) * fmax;

actual_area = (2/3) * ((b)^(3/2) - (a)^(3/2));

fprintf(actual_area);
```

   **Output:**

```
0.98904
```

   **Solution:**

$$\begin{aligned} \text{A} &= \int_{1/2}^{3/2} \sqrt{x}\, dx = \frac{2}{3} x^{3/2} \Big|_{1/2}^{3/2} \\ &= \frac{2}{3}\left(\left(\frac{3}{2}\right)^{3/2} - \left(\frac{1}{2}\right)^{3/2}\right) \\ &= \frac{2}{3}(1.8371 - 0.3536) \\ &= \frac{2}{3}(1.4835) \approx 0.9890 \end{aligned}$$

   As shown in the MATLAB simulation and in the step-by-step solution, the actual area is approximately $\boxed{0.9890}$

2. Use Monte Carlo simulation. Write an algorithm to calculate the part of the volume of an ellipsoid

$$\frac{x^2}{2} + \frac{y^2}{4} + \frac{z^2}{8} \leq 16$$

   that lies in the first octant, $x > 0, y > 0, z > 0$.

   **MATLAB Code:**

```
x_max = sqrt(32);
y_max = sqrt(64);
z_max = sqrt(128);
total_points = 100000;

x = x_max * rand(1, total_points);
y = y_max * rand(1, total_points);
z = z_max * rand(1, total_points);

inside = (x.^2)/2 + (y.^2)/4 + (z.^2)/8 <= 16;

box_volume = x_max * y_max * z_max;
volume = (sum(inside) / total_points) * box_volume;

fprintf(volume);
```

   **Output:**

```
268.07808
```

**Solution:**

$$\begin{aligned}
V &= \frac{1}{8} \cdot \frac{4}{3}\pi \cdot \sqrt{32} \cdot \sqrt{64} \cdot \sqrt{128} \\
&= \frac{1}{8} \cdot \frac{4}{3}\pi \cdot (2^{2.5})(2^{3})(2^{3.5}) \\
&= \frac{1}{8} \cdot \frac{4}{3}\pi \cdot 2^{9} \\
&= \frac{1}{8} \cdot \frac{2048}{3}\pi \\
&= \frac{256}{3}\pi \\
&\approx 268.08
\end{aligned}$$

The calculated part of the volume of the ellipsoid is approximately $\boxed{268.08}$, using MATLAB simulation and the step-by-step solution as the reference.

## Activity: Generating Random Numbers

1. Use middle square method to generate 10 random numbers using $x = 1009$.
   **MATLAB Code:**

```
x = 1009;
total_points = 10;

fprintf(' i     x_i   x_i^2\n');
fprintf('-------------------\n');

for i = 0:total_points
    x_sq_str = sprintf('%08d', x^2);
    mid = floor((length(x_sq_str) - 4) / 2) + 1;
    middle_digits = x_sq_str(mid:mid+3);

    fprintf('%2d  %5d   %s\n', i, x, middle_digits);
    x = str2double(middle_digits);
end
```

**Output:**

```
 i     x_i   x_i^2
-------------------
 0   1009   1808
 1    180   0324
 2    324   1049
 3   1049   1004
 4   1004   0080
 5     80   0064
 6     64   0040
 7     40   0016
 8     16   0002
 9      2   0000
10      0   0000
```

**Solution:**

$$\begin{aligned}
&x_0 = 1009 \\
&x_0^2 = 1018081 \Rightarrow 01018081 \Rightarrow x_1 = 0180 \\
&x_1^2 = 32400 \Rightarrow 00032400 \Rightarrow x_2 = 0324 \\
&x_2^2 = 104976 \Rightarrow 00104976 \Rightarrow x_3 = 1049 \\
&x_3^2 = 1100401 \Rightarrow 01100401 \Rightarrow x_4 = 1004 \\
&x_4^2 = 1008016 \Rightarrow 01008016 \Rightarrow x_5 = 0080 \\
&x_5^2 = 6400 \Rightarrow 00006400 \Rightarrow x_6 = 0064 \\
&x_6^2 = 4096 \Rightarrow 00004096 \Rightarrow x_7 = 0040 \\
&x_7^2 = 1600 \Rightarrow 00001600 \Rightarrow x_8 = 0016 \\
&x_8^2 = 256 \Rightarrow 00000256 \Rightarrow x_9 = 0002 \\
&x_9^2 = 4 \Rightarrow 00000004 \Rightarrow x_{10} = 0000
\end{aligned}$$

2. Use the linear congruence method to generate 20 random numbers using $a = 5$, $b = 3$, and $c = 16$. Was there cycling? If so, when did it occur?

   **Solution:**

$$\begin{aligned}
x_0 &= 1 \\
x_1 &= (5 \cdot 1 + 3) \bmod 16 = 8 \\
x_2 &= (5 \cdot 8 + 3) \bmod 16 = 11 \\
x_3 &= (5 \cdot 11 + 3) \bmod 16 = 10 \\
x_4 &= (5 \cdot 10 + 3) \bmod 16 = 5 \\
x_5 &= (5 \cdot 5 + 3) \bmod 16 = 12 \\
x_6 &= (5 \cdot 12 + 3) \bmod 16 = 15 \\
x_7 &= (5 \cdot 15 + 3) \bmod 16 = 14 \\
x_8 &= (5 \cdot 14 + 3) \bmod 16 = 9 \\
x_9 &= (5 \cdot 9 + 3) \bmod 16 = 0 \\
x_{10} &= (5 \cdot 0 + 3) \bmod 16 = 3 \\
x_{11} &= (5 \cdot 3 + 3) \bmod 16 = 2 \\
x_{12} &= (5 \cdot 2 + 3) \bmod 16 = 13 \\
x_{13} &= (5 \cdot 13 + 3) \bmod 16 = 4 \\
x_{14} &= (5 \cdot 4 + 3) \bmod 16 = 7 \\
x_{15} &= (5 \cdot 7 + 3) \bmod 16 = 6 \\
x_{16} &= \boxed{(5 \cdot 6 + 3) \bmod 16 = 1}
\end{aligned}$$

   **Yes, and the cycle starts at** $x_{16}$, which repeats the initial value.

## Activity: Simulating Probabilistic Behavior

1. You arrive at the beach for a vacation and are dismayed to learn that PAGASA is predicting 50% chance of rain every day. Using Monte Carlo simulation, predict the chance that it rains three consecutive days during your vacation.

   **MATLAB Code:**

```
total_points = 100000;
count = 0;

for i = 1: total_points
    day1 = rand < 0.5;
    day2 = rand < 0.5;
    day3 = rand < 0.5;

    if day1 && day2 && day3
        count = count + 1;
    end
end

prob = count / total_points;
fprintf(total_points, prob, prob * 100);
```

   **Output:**

```
0.1249 (12.49%)
```

   **Solution:**

$$\begin{aligned}
P(\text{rain each day}) &= P(\text{rain on day 1}) \cdot P(\text{rain on day 2}) \cdot P(\text{rain on day 3}) \\
&= 0.5 \cdot 0.5 \cdot 0.5 = \boxed{0.125}
\end{aligned}$$

   As displayed in the MATLAB simulation and step-by-step solution for the probability, there is approximately $\boxed{12.50\%}$ chance that it rains for 3 consecutive days during my vacation.