Activity - Lagrange Polynomial Approximation

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Problem

Given Values (Using Calculator)

x	$\tan(x)$	$\log_{10}(\tan(x))$
1.00	1.5574	0.1924
1.05	1.7415	0.2414
1.10	1.9640	0.2933
1.15	2.2945	0.3492

Formula

$$P(x) = \sum_{i=0}^{3} f(x_i) \cdot L_i(x), \quad P_3(x) = \sum_{i=0}^{3} y_i \cdot L_i(x)$$

Lagrange Basis Polynomials

$$L_0(x) = \frac{(x-1.05)(x-1.10)(x-1.15)}{(1.00-1.05)(1.00-1.10)(1.00-1.15)} = \frac{(x-2.15x+1.155)(x-1.15)}{0.00075}$$

$$= -1,333.33(x^3 - 3.3x^2 + 3.6275x - 1.32825)$$

$$L_1(x) = \frac{(x-1.00)(x-1.10)(x-1.15)}{(1.05-1.00)(1.05-1.10)(1.05-1.15)} = \frac{(x^2-2.1x+1.15)(x-1.15)}{0.000125}$$

$$= 4000(x^3 - 3.25x^2 + 3.515x - 1.265)$$

$$L_2(x) = \frac{(x-1.00)(x-1.05)(x-1.15)}{(1.10-1.00)(1.10-1.05)(1.10-1.15)} = \frac{(x^2-2.05x+1.05)(x-1.15)}{0.00025}$$

$$= -4000(x^3 - 3.2x^2 + 3.4075x - 1.2075)$$

$$L_3(x) = \frac{(x-1.00)(x-1.05)(x-1.10)}{(1.15-1.00)(1.15-1.05)(1.15-1.10)} = \frac{(x^2-2.05x+1.05)(x-1.10)}{0.000375}$$

$$= 1,333.33(x^3 - 3.15x^2 + 3.305x - 1.155)$$

Interpolated Polynomial $P(x)$

$$\begin{aligned} P(x) = {} & [-(0.1924)\cdot(133.33)]\cdot(x^3 - 3.3x^2 + 3.6275x - 1.32825) \\ & + [0.2414 \cdot 4000]\cdot(x^3 - 3.25x^2 + 3.515x - 1.265) \\ & + [-(0.2933)\cdot(4000)]\cdot(x^3 - 3.2x^2 + 3.4075x - 1.2075) \\ & + [0.3492 \cdot 1833.33]\cdot(x^3 - 3.15x^2 + 3.305x - 1.155) \end{aligned}$$

$$P(x) = 1.4641x^3 - 4.0384x^2 + 4.6368x - 1.8721$$

Evaluation at $x = 1.09$

$$\begin{aligned} & P(1.09) \approx 0.2826352729 \\ & f(1.09) = \log_{10}(\tan(1.09)) \approx 0.2826429145 \\ \Rightarrow\ & P(1.09) \approx f(1.09) \approx \boxed{0.2826} \end{aligned}$$