Activity - Interpolation

May 3, 2025

Construct a divided difference table for the data below. Is smoothing with a low-order polynomial appropriate? If so, choose an appropriate polynomial and fit using the least square criterion of best fit. Analyze the goodness of fit and graph the model and the data points in one graph.

$$\begin{array}{rccccc} x_i: & 0 & 2 & 4 & 6 & 8 \\ y_i: & 0 & 4 & 16 & 36 & 64 \end{array}$$

$$f(x) = a + bx + cx^2 \quad ; \quad \frac{\partial S}{\partial a} = \frac{\partial S}{\partial b} = \frac{\partial S}{\partial c} = 0$$

$$na + \left(\sum x_i\right) b + \left(\sum x_i^2\right) c = \sum y_i$$
$$\left(\sum x_i\right) a + \left(\sum x_i^2\right) b + \left(\sum x_i^3\right) c = \sum x_i y_i$$
$$\left(\sum x_i^2\right) a + \left(\sum x_i^3\right) b + \left(\sum x_i^4\right) c = \sum x_i^2 y_i$$

Normal Equations:

$$5a + 20b + 120c = 120 \tag{1}$$
$$20a + 120b + 800c = 800 \tag{2}$$
$$120a + 800b + 5760c = 5760 \tag{3}$$

Solving for a, b, c:
From (eq. 1):

$$5a + 20b + 120c = 120 \Rightarrow a + 4b + 24c = 24 \Rightarrow -4b - 24c = -24$$

Substitute into (eq. 2) and (eq. 3):

$$120a + 800b + 5760c = 5760 \tag{4}$$
$$120a + 720b + 4800c = 4800 \tag{5}$$
$$\Rightarrow 80b + 960c = 960 \tag{6}$$

Solving:

$$80b + 960c = 960 \Rightarrow b + 12c = 12$$

From earlier:

$$-4b - 24c = -24$$

Multiply last equation by -1 and add to above:

$$4b + 48c = 48$$

$$(-4b - 24c) + (4b + 48c) = -24 + 48 \Rightarrow 24c = 24 \Rightarrow c = 1$$

Substitute $c = 1$ into $b + 12c = 12$:

$$b + 12 = 12 \Rightarrow b = 0$$

Then from eq. 1:

$$5a + 20(0) + 120(1) = 120 \Rightarrow 5a = 0 \Rightarrow a = 0$$

Therefore, the best fit polynomial is:

$$f(x) = x^2$$

Divided Difference Table

x_i	y_i	Δ	Δ^2	Δ^3	Δ^4
0	0				
		2			
2	4	4	2		
		6	2	0	
4	16	10	2	0	
		10	0	0	
6	36	14			
		14			
8	64				

Since a non-zero result showed up at the third divided difference, the data can be modeled as a ***quadratic polynomial****.*

Least Squares Method Table

x_i	y_i	x_i^2	x_i^3	x_i^4	$x_i y_i$	$x_i^2 y_i$
0	0	0	0	0	0	0
2	4	4	8	16	8	16
4	16	16	64	256	64	256
6	36	36	216	1296	216	1296
8	64	64	512	4096	512	4096
$\sum$	20	120	800	5760	800	5760

$$\text{From previous:} \quad -4b - 80c = -80 \Rightarrow -4b = 0 \Rightarrow b = 0$$
$$\text{Then:} \quad 20a + 800(0) = 800 \Rightarrow 20a = 800 \Rightarrow a = 0$$

Thus: $a = 0$, $b = 0$, and $c = 1$

Final Model

The function

$$f(x) = a + bx + cx^2 \Rightarrow f(x) = x^2$$

or

$$y = x^2$$

Since the error is zero, **the model is a good fit**.

Plot of the Model $y = x^2$