

**RANCANG BANGUN APLIKASI JEJARING SOSIAL UNTUK  
PEMBELAJARAN PESERTA DIDIK BERBASIS WEB**

**SKRIPSI**

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**JURUSAN TEKNIK INFORMATIKA  
FAKULTAS SAINS DAN TEKNOLOGI  
UNIVERSITAS ISLAM NEGERI MAULANA MALIK IBRAHIM  
MALANG  
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**Universitas Islam Negeri Maulana Malik Ibrahim Malang**

**Untuk Memenuhi Salah Satu Persyaratan dalam**

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Menyatakan dengan sebenarnya bahwa hasil penelitian saya ini tidak terdapat unsur-unsur penjiplakan karya penelitian atau karya ilmiah yang pernah dilakukan atau dibuat orang lain, kecuali yang secara tertulis dikutip dalam naskah ini atau disebutkan dalam sumber kutipan dan daftar pustaka.

Apabila ternyata hasil penelitian ini terbukti terdapat unsur-unsur jiplakan maka saya bersedia untuk mempertanggung jawabkan, serta diproses sesuai peraturan yang berlaku.

Malang,  
Yang Membuat Pernyataan,

**HARIS RIDWAN ZAIN**  
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## MOTTO

*Man Jadda Wa Jada.*

Siapa yang bersungguh-sungguh akan berhasil.

*Man shabaro zafiro*

Siapa yang bersabar akan beruntung

*Man saaro 'alaa darbi washola*

Siapa yang berjalan di jalur-Nya akan sampai

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yang selalu sabar memberikan bimbingan dan nasehat kepadaku serta  
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motivasi dan doa dalam tercapainya skripsi.

Tak lupa semua pihak yang turut serta membantu dalam penyelesaian skripsi ini,  
terima kasih atas semuanya

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Malang, 3 Juli 2013

Penulis

## DAFTAR ISI

HALAMAN JUDUL.....	i
HALAMAN PENGAJUAN.....	ii
HALAMAN PERSETUJUAN.....	iii
HALAMAN PENGESAHAN.....	iv
HALAMAN PERNYATAAN .....	v
MOTTO .....	vi
HALAMAN PERSEMBAHAN .....	vii
KATA PENGANTAR .....	viii
DAFTAR ISI .....	x
DAFTAR TABEL.....	xiii
DAFTAR GAMBAR .....	xiv
ABSTRAK .....	xvi

### BAB I PENDAHULUAN

1.1 Latar Belakang .....	1
1.2 Rumusan Masalah .....	5
1.3 Batasan Masalah .....	5
1.4 Tujuan Penelitian .....	5
1.5 Manfaat Penelitian .....	5
1.6. Metode Penelitian .....	6
1.7 Sistematika Penulisan .....	8

### BAB II TINJAUAN PUSTAKA

2.1 Pembelajaran .....	10
2.1.1 Definisi .....	10
2.1.2 Prinsip dalam pemahaman pembelajaran .....	14
2.2 Model-model pembelajaran .....	16
2.2.1 Defisi .....	16
2.2.2 Ciri-ciri pembelajaran .....	18
2.2.3 Jenis-jenis model pembelajaran .....	18
2.3 <i>E-learning</i> .....	20
2.3 1 Definisi .....	20
2.4 Jejaring Sosial .....	30

2.4.1 Definisi .....	30
2.5 <i>Learning Management System</i> .....	34
2.5.1 Definisi .....	34
2.5.2 Edmodo dan Moodle .....	35

### BAB III DESAIN DAN SISTEM

3.1 <i>Problem Statement</i> .....	40
3.1.1 Deskripsi Sistem .....	40
3.1.2 Keadaan <i>Current System</i> .....	40
3.1.3 Lingkup Proyek .....	41
3.2 Identifikasi dan Analisi Proses Bisnis .....	41
3.2.1 Identifikasi Proses Bisnis .....	41
3.2.2 Analisis Proses Bisnis .....	42
3.2.3 Permodelan Proses Bisnis .....	44
3.3 Identifikasi dan Analisi Kebutuhan .....	45
3.3.1 Identifikasi Kebutuhan Fungsional .....	45
3.3.2 Analisi Kebutuhan Fungsional .....	47
3.3.3 Dokumen yang terkait dengan sistem .....	48
3.3.4 Identifikasi dan Analisi Kebutuhan Non Fungsional .....	49
3.4 Identifikasi dan Alternatif Solusi .....	50
3.5 Identifikasi dan Desain Output .....	53
3.5.1 Identifikasi Output .....	55
3.5.2 Desai Output	
3.6 Identifikasi dan Desain Input .....	60
3.6.1 Identifikasi Input .....	60
3.6.2 Desain Input .....	62
3.7 Desain Perancangan Sistem .....	63
3.7.1 <i>Sitemap</i> .....	63
3.7.2 Konteks Diagram .....	63
3.7.3 <i>Data Flow Diagram</i> .....	66
3.7.4 <i>Entity Relation Diagram</i> .....	73
3.7.5 Struktur Data .....	75

### BAB IV HASIL DAN PEMBAHASAN

4.1 Prosedur Pengujian .....	80
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4.2 Pengujian Hasil dan Pembahasan .....	81
4.2.1 Pengujian login user .....	81
4.2.2 Pengujian Halaman <i>User</i> Dosen .....	83
4.2.3 Pengujian Halaman User Mahasiswa .....	92
4.2.4 Pengujian Halaman Admin .....	99

## BAB V PENUTUP

5.1 Kesimpulan .....	105
5.2 Saran .....	105

## DAFTAR PUSTAKA

**DAFTAR TABEL**

Tabel 3.1 Analisi Proses Bisnis .....	42
Tabel 3.2 Proses Bisnis Pendaftaran .....	45
Tabel 3.3 Proses Bisnis Kelas Ajar .....	45
Tabel 3.4 Proses Bisnis <i>Chat Group</i> .....	46
Tabel 3.5 Analisis Kebutuhan Fungsional .....	47
Tabel 3.6 Analisis Kebutuhan Non Fungsional .....	49
Tabel 3.7 Identifikasi dan Alternatif Solusi .....	50
Tabel 3.8 Identifikasi Output .....	53
Tabel 3.9 Identifikasi Input .....	60
Tabel 3.10 <i>User</i> .....	75
Tabel 3.11 Group study .....	75
Tabel 3.12 <i>Chat</i> .....	76
Tabel 3.13 File .....	76
Tabel 3.14 Jawab .....	76
Tabel 3.15 Mapel .....	77
Tabel 3.16 Materi .....	77
Tabel 3.17 Nilai .....	77
Tabel 3.18 Respon .....	78
Tabel 3.19 Soal .....	78
Tabel 3.20 <i>User chat</i> .....	78
Tabel 3.21 Wall .....	79
Tabel 4.1 Rekapitulasi Responden .....	104

## DAFTAR GAMBAR

Gambar 3.1 <i>Workflow</i> Pendaftaran <i>Study Group</i> .....	44
Gamabr 3.2 <i>Workflow</i> Gabung Kelas .....	44
Gambar 3.3 <i>Workflow</i> Distribusi Soal .....	45
Gambar 3.4 Desain halaman utama .....	55
Gambar 3.5 Desain halaman admin .....	55
Gambar 3.6 Desain output laporan kelas mata kuliah .....	56
Gambar 3.7 Desain output materi kuliah .....	56
Gambar 3.8 Desain output laporan file materi .....	57
Gambar 3.9 Desain output laporan <i>wall mind</i> .....	57
Gambar 3.10 Desain menu respon pada halaman admin .....	58
Gambar 3.11 Desain home mahasiswa .....	58
Gambar 3.12 Desain Ruang kelas mahasiswa .....	59
Gambar 3.13 Desain <i>Chat</i> mahasiswa .....	59
Gambar 3.14Desain input registrasi dosen .....	62
Gambar 3.15 Desain input registrasi mahasiswa .....	62
Gambar 3.16 Sitemap .....	63
Gambar 3.17 Context diagram .....	64
Gambar 3.18 DFD level 1 admin .....	67
Gambar 3.19 DFD level 1 mahasiswa .....	68
Gambar 3.20 DFD level 1 dosen .....	69
Gambar 3.21 DFD level 2 admin .....	70
Gambar 3.22 DFD level 2 mahasiswa .....	71
Gambar 3.23 DFD level 2 dosen .....	72
Gambar 3.24 ERD .....	74
Gambar 4.1 Halaman utama jejaring sosial untuk peserta didik .....	82
Gambar 4.2 Peringatan ketika pengguna lupa memasukkan <i>username</i> .....	82
Gambar 4.3 Halaman utama <i>user</i> dosen .....	84
Gambar 4.4 Membuat kelas .....	85
Gambar 4.5 Daftar mahasiswa yang ikut dalam kelas .....	85
Gambar 4.6 Data info profil mahasiswa .....	86
Gambar 4.7 Submenu materi.....	87
Gambar 4.8 Submenu file .....	87
Gambar 4.9 <i>Interface upload</i> soal .....	88

Gambar 4.10 <i>Report</i> .....	89
Gambar 4.11 <i>Wall mind</i> .....	90
Gambar 4.12 <i>Chat group</i> .....	91
Gambar 4.13 Halaman utama user mahasiswa .....	92
Gambar 4.14 Gabung kelas .....	92
Gambar 4.15 View siswa .....	93
Gambar 4.16 View materi .....	94
Gambar 4.17 View isi materi .....	94
Gambar 4.18 View File Materi pada submenu File .....	95
Gambar 4.19 <i>Interface disable</i> jawaban .....	96
Gambar 4.20 Lembar jawaban pada soal singkat .....	96
Gambar 4.21 Lembar jawaban pada soal uraian .....	97
Gambar 4.22 Pemberitahuan soal baru .....	98
Gambar 4.27 Report nilai mahasiswa .....	98
Gambar 4.28 Tampilan menu home admin .....	99
Gambar 4.29 Tampilan menu data submenu <i>user</i> .....	100
Gambar 4.30 Input <i>user</i> melalui admin .....	100
Gambar 4.31 Tampilan menu data submenu kelas .....	101
Gambar 4.32 Tampilan menu data submenu materi .....	101
Gambar 4.33 Tampilan view materi melalui admin halaman admin .....	102
Gambar 4.34 Tampilan menu data submenu file .....	102
Gambar 4.35 Tampilan menu data <i>wall</i> .....	103
Gambar 4.36 Tampilan menu respon .....	103
Gambar 4.37 View respon melalui admin .....	103

## ABSTRAK

Zain,Haris Ridwan. 2013. **Rancang Bangun Aplikasi Jejaring Sosial Untuk Pembelajaran Peserta Didik Berbasis Web.** Skripsi. Jurusan Teknik Informatika, Fakultas Sains dan Teknologi, Universitas Islam Negeri Maulana Malik Ibrahim Malang. Pembimbing:(I) DR. Suhartono,M. Kom. (II) A'la Sauqi,M.Kom.

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**Kata Kunci:** Pembelajaran, Jejaring sosial, edukatif, *e-learning*

Proses pembelajaran aktivitasnya dalam bentuk interaksi belajar mengajar dalam suasana interaksi edukatif, yaitu interaksi yang sadar akan tujuan, artinya interaksi yang telah dicanangkan untuk suatu tujuan tertentu setidaknya adalah pencapaian tujuan instruksional atau tujuan pembelajaran yang telah dirumuskan pada satuan mata kuliah dan kurikulum pendidikan. Beban materi kurikulum yang diimplementasikan dan seringnya terjadi pergantian membuat mahasiswa semakin enggan dan berat dalam mengikuti kegiatan belajar mengajar.

Dampak yang terjadi mahasiswa lebih banyak menghabiskan waktunya untuk diluar dari pada menghabiskan waktu belajar di kampus terlebih lagi dengan perkembangan teknologi internet mereka lebih tertarik untuk menghabiskan waktu dengan online di sosial media dikenal dengan jejaring sosial seperti *twitter*, *facebook*, media sosial lain nya dimana jumlah masyarakat Asia terutama dari tingkat pelajar hampir mencapai 87 juta pengguna menghabiskan waktunya dengan jejaring sosial.

Banyak cara yang ditempuh oleh seorang pendidik agar dapat memaksimalkan proses pemahaman terhadap suatu materi pada satuan mata kuliah yang telah dibebankan dalam kurikulum pendidikan. Adanya sebuah aplikasi pembelajaran dengan konsep *e-learning* dan jejaring sosial diharapkan dapat lebih mengoptimalkan kegiatan pembelajaran diluar jam kuliah serta solusi yang cukup bagus dibandingkan metode *e-learning* pada umumnya. Aplikasi jejaring sosial untuk pembelajaran peserta didik berupa website yang ditujukan untuk pendidikan dan aplikasi ini juga dapat digunakan sebagai media pembelajaran jarak jauh dimana tidak terpaku pada ruang kelas saja.

## ABSTRACT

Zain,Haris Ridwan. 2013. **Social Networking Application Form Design for Web-Based Learner.** Thesis. Informatics Department, Faculty of Science and Technology, The State Islamic University Maulana Malik Ibrahim of Malang. Advisor:(I) Dr. Suhartono,M. Kom. (II) A'la Sauqi,M.Kom.

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**Keywords:** Learning terms, social networking, educational, e-learning

The process of learning activities in form of educative learning interaction, an interaction which is aware of the purpose, meaning interaction applied for a specific purpose to achieve at least for an instructional purpose or an educative purpose which have been formulated in every course unit and every curriculum material. The obligated curriculum implemented and the frequent curriculum changed cause the students to feel more reluctant and heavy following the teaching and learning activities.

The impacts are more students prefer spending their time outside to spending the time studying on campus. Besides that, the development of internet technology made them more interested spending the time to go online in social media known as social networks like twitter, facebook, and other social media communities. The number of users in Asia, especially from student level is reaching almost 87 million users spending their time by social networking.

Many ways are done by a teacher in order to maximize the understanding of the subject matter in each course unit that has been obligated according to the education curriculum. The existence of a learning application with e-learning and social networking concepts are expected to be a further optimization in terms of learning activities outside the school time as well as a pretty good solution compared to the method of e-learning in used to use. These social networking applications in form of a website are not only devoted for classroom education but also can be used as a medium for a long distance learning which is not glued to the classroom only.

## الملخص

زين، حارس رضوان .٢٠١٣٠ .تصميم بناء تطبيقات الشبكات الاجتماعية للتعلم الطلاب المستندة إلى ويب .البحث .قسم المعلوماتية، كلية العلوم والتكنولوجيا، جامعة الإسلامية مولانا مالك إبراهيم مالانج .المشرف (١) :سوهارتونو، (٢) علاء شيقى .

كلمات البحث :التعلم، والشبكات الاجتماعية، والتعليمية، والتعلم الإلكتروني

عملية لأنشطة في شكل من أشكال التفاعل والتعلم في جو من التفاعل التربوي التعلم، والتفاعل هو على بينة من الغرض، وهذا يعني تم تخصيص التفاعل لغرض محدد هو تحقيق على الأقل تم صياغة الأهداف التعليمية أو التعلم في دورات وحدة ومواد المناهج الدراسية ويتم تنفيذ المناهج والتغيير المتكرر جعل الطلاب أكثر ترددًا والثقيلة في أنشطة التعليم والتعلم التالية.

التأثيرات التي تحدث المزيد من الطلاب يقضون وقتهم خارج من الوقت الذي يقضيه الذين يدرسون في الحرم الجامعي خصوصا مع تطور تكنولوجيا الإنترن特 هم أكثر اهتماما في وإضاعة الوقت على الإنترن特 في وسائل الإعلام الاجتماعية مع الشبكات الاجتماعية المعروفة مثل توينر والفيسبوك، والمجتمعات وسائل الإعلام الاجتماعية الأخرى حيث بلغ عدد لها آسيا، وخصوصا من مستوى طالب تقريبا وصلت ٨٧ مليون مستخدمي قضاء بعض الوقت مع الشبكات الاجتماعية.

وتتخذ العديد من الطرق من قبل مربينا من أجل تحقيق أقصى قدر من الفهم للموضوع في الوحدة ومن المتوقع أن زيادة الاستخدام الأمثل لأنشطة التعلم خارج .التي تم وضعها في المناهج التعليمية ساعات المدرسة وكذلك حل جيد جدا بالمقارنة مع أسلوب التعلم الإلكتروني بشكل عام وجود يمكن أيضا تطبيقات الشبكات .تطبيق التعلم مع مفهوم التعلم الإلكتروني والشبكات الاجتماعية الاجتماعية لتعليم الطلاب في شكل موقع على شبكة الإنترن特 المخصصة للتعليم وتطبيق استخدامها كوسيلة للتعلم عن بعد الذي لا لصقها على الفصول الدراسية وحدتها

## BAB I

### PENDAHULUAN

#### 1.1 Latar Belakang

Pengaruh yang relatif permanen atas perilaku, pengetahuan, dan keterampilan berpikir yang diperoleh melalui pengalaman atau yang lebih dikenal dengan pembelajaran, merupakan suatu komponen aktivitas yang tak pernah lepas dalam kehidupan sehari-hari. Proses pembelajaran itu sendiri aktivitasnya dalam bentuk interaksi belajar mengajar dalam suasana interaksi edukatif, yaitu interaksi yang sadar akan tujuan, artinya interaksi yang telah dicanangkan untuk suatu tujuan tertentu setidaknya adalah pencapaian tujuan instruksional atau tujuan pembelajaran yang telah dirumuskan pada satuan mata kuliah dalam hal ini proses pembelajaran secara formal yaitu kegiatan pembelajaran di kampus maupun non formal atau di luar kampus. Perintah belajar pun telah dijelaskan dalam Al Qur'an surat Al Ala'q ayat 1-5 yang berbunyi:

أَقْرَأَ بِاسْمِ رَبِّكَ الَّذِي خَلَقَ ۝ أَقْرَأَ وَرَبُّكَ الْأَكْرَمُ ۝ الَّذِي عَلَّمَ  
بِالْقَلْمَنْ ۝ عَلَّمَ الْإِنْسَنَ مَا لَمْ يَعْلَمْ ۝

*"Bacalah dengan (menyebut) nama Tuhanmu yang Menciptakan. Dia Telah menciptakan manusia dari segumpal darah. Bacalah, dan Tuhanmulah yang Maha pemurah,Yang mengajar (manusia) dengan perantaran kalam(Allah mengajar manusia dengan perantaraan tulis baca) . Dia mengajar kepada manusia apa yang tidak diketahuinya"(QS. Al Ala'q 1-5).*

Dari Ayat di atas pun jelas disebutkan bahwa dalam proses belajar Allah mengajarkan manusia dengan perantara guru sebagai pengajarnya melalui baca dan tulis. Kemampuan baca dan tulis diperoleh dan dipelajari dalam sebuah

majelis ilmu. Beragam majelis ilmu seperti sekolah, pegajian-pengajian, ataupun lainnya mempunyai tujuan yang sama, yaitu mempelajari suatu ilmu. Sedangkan dalam lingkup sekolah beragam konsep pembelajaran diterapkan oleh dinas pendidikan yang dengan tujuan agar seorang siswa mampu memahami rumusan pada satuan pelajaran. akan tetapi dewasa ini dengan sering terjadi perubahan konsep kegiatan belajar mengajar menjadi polemik yang cukup mengkhawatirkan baik di kalangan akademisi, intelektual maupun masyarakat (*tribunnews.com*).

Beban akan materi kurikulum yang diimplementasikan dan seringnya terjadi pergantian membuat mahasiswa semakin enggan dan berat dalam mengikuti kegiatan belajar mengajar. Dampak yang terjadi mahasiswa lebih banyak menghabiskan waktunya untuk di luar dari pada menghabiskan waktu belajar di kampus terlebih lagi dengan perkembangan teknologi internet mereka lebih tertarik untuk menghabiskan waktu dengan online di sosial media kenal dengan jejaring sosial seperti *twitter*, *facebook*, media sosial lainnya.

Jumlah masyarakat Asia terutama dari tingkat pelajar, *Friendster* memiliki lebih dari 32 juta pengguna, *MySpace* memiliki lebih dari 15 juta pengguna, dan *Facebook* memiliki lebih dari 14 juta pengguna per bulannya. Pada *Facebook* terdapat penambahan 200 ribu *account* baru perharinya. Lebih dari 25 juta user aktif menggunakan *Facebook* setiap harinya. Rata-rata user menghabiskan waktu sekitar 19 menit perhari untuk melakukan berbagai aktifitas di *Facebook* (*kompas.com*)

Kesemuanya sangat memungkinkan user untuk melakukan berbagai aktifitas yang didukung oleh fasilitas-fasilitas yang seperti mengabadikan sebuah

peristiwa, mencari teman baru tanpa harus bertemu terlebih dahulu, dan masih banyak contoh lainnya. Banyaknya fasilitas yang diberikan dalam sebuah jejaring sosial juga menimbulkan banyak dampak negatif, salah satu dampak negatif yang cukup mengkhawatirkan yaitu menurunnya kemauan seseorang untuk menuntut ilmu. Dalam hadits Rosulullah dijelaskan bahwa:

عَنْ أَنَسِ بْنِ مَالِكٍ رضيَ اللَّهُ عَنْهُ قَالَ: قَالَ رَسُولُ اللَّهِ صَلَّى اللَّهُ عَلَيْهِ وَسَلَّمَ  
طَلَبُ الْعِلْمِ فَرِيضَةٌ عَلَى كُلِّ مُسْلِمٍ.

*“Dari Annas bin Malik R.A Rosulullah bersabda: menuntut ilmu adalah kewajiban bagi setiap muslim”(HR.Ibnu Majjah).*

Menuntut ilmu adalah kewajiban dan ketika seseorang melakukan banyak hal tanpa adanya proses menuntut ilmu, maka dia telah melupakan kewajibannya sebagaimana yang dijelaskan pada hadits Rosulullah di atas.

Banyak cara yang ditempuh oleh seorang pendidik agar dapat memaksimalkan proses pemahaman terhadap suatu materi pada satuan mata kuliah yang telah dibebankan dalam kurikulum pendidikan. “sistem *e-learning* menjadi sangat potensial sebab peluang *mentee* untuk berinteraksi dengan *pemendor*, antar *mentee*, maupun bahan belajarnya terbuka lebih luas dan kegiatan mentoring lebih fleksibel karena dapat disesuaikan dengan ketersediaan waktu para *metee*”. ( Muhammad Guruh Wicaksana, 2001:1).

Infrastruktur *e-Learning* pun dapat berupa *personal computer* (PC), jaringan komputer, internet dan perlengkapan multimedia. Termasuk didalamnya peralatan *teleconference* apabila kita memberikan layanan *synchronous learning* melalui *teleconference*. Terminology *e-learning* sendiri dapat mengacu pada

semua kegiatan pelatihan yang menggunakan media elektronik atau teknologi informasi dimana mempunyai kelebihan yaitu segi biaya mampu mengurangi biaya pelatihan, fleksibelitas waktu, fleksibelitas tempat, fleksibelitas kecepatan pembelajaran, standarisasi pengajaran, efektivitas pengajaran, kecepatan distribusi, ketersediaan *On-Demand*, dan juga otomatisasi proses administrasi. walaupun *e-learning* menawarkan banyak keuntungan, *e-learning* juga mempunyai beberapa keterbatasan, diantaranya dari segi faktor budaya, investasi, teknologi, infrastruktur, serta materi.

Dengan mempertimbangkan kelebihan dan kekurangan yang didapat dalam sebuah aplikasi jejaring sosial dan sistem *e-learning*, maka dengan adanya sebuah aplikasi jejaring sosial pembelajaran diharapkan dapat lebih mengoptimalkan kegiatan pembelajaran di luar jam kuliah serta solusi yang cukup bagus dibandingkan metode *e-learning* pada umumnya dimana ditawarkan beberapa fasilitas yang diyakini akan menarik kemauan mahasiswa atau dosen untuk bergabung konten dan bahan ajar yang ada bisa diakses oleh mahasiswa kapanpun dan dimanapun, adanya fasilitas yang menampilkan teman yang *online*, aktivitas terbaru dari user lain, mempermudah pendistribusian materi dan soal-soal, memberikan ruang diskusi yang lebih fleksibel sehingga mempermudah seorang dosen dan mahasiswa, atau rekan untuk berkomunikasi dengan satu sama lain secara cepat, akurat, dan efisien.

## 1.2 Rumusan Masalah

Berdasarkan latar belakang di atas dapat ditarik rumusan masalah sebagai berikut: Bagaimana membuat aplikasi pembelajaran menggunakan jejaring sosial?

## 1.3 Batasan Masalah

Agar dalam penelitian tidak keluar dari pokok masalah, maka batasan masalah dalam penelitian ini yaitu:

1. Obyek yang digunakan dalam skripsi ini adalah mahasiswa Teknik Informatika Universitas Islam Negeri Maulana Malik Ibrahim Malang
2. Aplikasi jejaring sosial yang dibangun untuk pembelajaran berbasis web.
3. Aplikasi jejaring sosial untuk pembelajaran berbasis web yang dibuat bersifat *synchronous training*.

## 1.4 Tujuan Penelitian

Adapun tujuan dari penelitian ini yaitu merancang dan membuat sebuah aplikasi jejaring sosial untuk pembelajaran berbasis web yang merupakan pengembangan dari jejaring *social network education* di luar jam kuliah.

## 1.5 Manfaat Penelitian

Dari hasil penelitian ini diharapkan dapat:

1. Dapat mempermudah mahasiswa untuk bersosialisasi dan membangun jaringan belajar di luar jam kuliah.

2. Dapat mempermudah dosen dalam memantau perkembangan mahasiswa di luar jam kuliah.
3. Teridentifikasinya kebutuhan dan permasalahan mahasiswa berkaitan dengan forum *online* di kampus. Karena banyak mahasiswa tidak berani mengungkapkan sesuatu yang berlainan dengan apa yang diinginkan secara langsung sehingga aplikasi ini sangatlah tepat untuk digunakan.

## 1.6 Metode Penelitian

Dalam perancangan aplikasi ini, digunakan beberapa metode dalam pelaksananya. Adapun metode-metode lainnya adalah sebagai berikut:

1. Perencanaan
  - a. Metode Penentuan Ruang Lingkup Penelitian.
  - b. Menentukan Lingkup penelitian pada lingkungan yang akan menggunakan sistem ini. lingkup penelitian yaitu mahasiswa Teknik Informatika Universitas Islam Negeri Maulana Malik Ibrahim Malang.
2. Metode Pengumpulan Data (*Data Gathering*)

Mencari dan mengumpulkan data-data yang dibutuhkan dan berkaitan dengan pembuatan Aplikasi jejaring sosial *study group* akan dilaksanakan di jurusan Teknik Informatika Universitas Islam Negeri Maulana Malik Ibrahim Malang.

### 3. Metode observasi dan Wawancara

Dalam melakukan penelitian, selain pengumpulan data juga akan dilakukan wawancara kepada mahasiswa dan dosen Teknik Informatika Universitas Islam Negeri Maulana Malik Ibrahim Malang.

### 4. Studi Kepustakaan

Studi kepustakaan seperti mempelajari buku-buku referensi yang berhubungan dengan sosial media dan *e-learning* untuk membantu dalam pembuatan aplikasi jejaring sosial *study group*.

### 5. Fase Perancangan Sistem

Membuat desain jejaring sosial untuk pembelajaran peserta didik dengan *Entity Relation Diagram(ERD),Diagram Context, Data Flow Diagram(DFD)*

### 6. Fase Pembuatan Program

Membangun Jejaring sosial untuk pembelajaran peserta didik berbasis web dengan menggunakan *Apache, Php, dan MySql*.

### 7. Fase Uji Coba

Jejaring sosial untuk pembelajaran peserta didik diuji coba pada sistem, untuk menemukan kekurangan atau kesalahan yang mungkin timbul.

### 8. Fase Revisi Program

Dilakukan setelah uji coba, berfungsi untuk memperbaiki kesalahan-kesalahan dalam kode program maupun menambah kekurangan dari program yang dikerjakan.

## 9. Dokumentasi

Dokumentasi dilakukan untuk jangka waktu sekarang dan yang akan datang agar memudahkan *maintenance* jika terjadi kesalahan program lagi akibat ketidakstabilan perangkat atau gangguan teknis lainnya.

## 1.7 Sistematika Penulisan

Untuk memperoleh gambaran yang mudah dimengerti dan komprehensif mengenai isi dalam penulisan skripsi ini, secara keseluruhan dapat dilihat dari sistematika pembahasan skripsi dibawah ini:

### 1. BAB I PENDAHULUAN

Bab ini merupakan bab pendahuluan yang didalamnya berisi tentang latar belakang, rumusan masalah, batasan masalah, tujuan dan manfaat penelitian, metode penelitian dan sistematika penulisan.

### 2. BAB II TINJAUAN PUSTAKA

Bab ini menjelaskan tentang sistem, sosial media, model-model pembelajaran, unsur-unsur *e-learning*, jejaring sosial Edmodo dan moodle.

### 3. BAB III ANALISIS DAN DESAIN SISTEM

Bab ini menjelaskan tentang pembuatan desain jejaring sosial untuk pembelajaran peserta didik berbasis web yang meliputi *Statement of purpose (STP)*, daftar kejadian (*Event list*), *context diagram (CD)*, *data flow diagram (DFD)*, *entity relationship diagram (ERD)*, dan rancangan database.

#### 4. BAB IV HASIL DAN PEMBAHASAN

Bab ini berisi tentang pembahasan dari pengujian implementasi jejaring sosial untuk pembelajaran peserta didik dan sekaligus menerangkan cara pemakaian program dan menjelaskan proses aplikasi yang terjadi pada jejaring sosial untuk pembelajaran peserta didik.

#### 5. BAB V PENUTUP

Bab ini merupakan penutup, yang didalamnya berisi kesimpulan dan rangkuman dari pembahasan skripsi, serta berisi saran yang diharapkan dapat bermanfaat untuk pengembangan pembuatan program aplikasi selanjutnya.

## **BAB II**

### **TINJAUAN PUSTAKA**

#### **2.1 Pembelajaran**

##### **2.1.1 Definisi**

Pembelajaran berasal dari kata dasar "ajar" yang bermakna petunjuk yang diberikan kepada orang supaya diketahui (diturut). Sedangkan kata "pembelajaran" sendiri memiliki arti proses, perbuatan, cara mengajar agar peserta didik mau belajar. Pengertian pembelajaran adalah suatu proses yang muncul akibat komunikasi dan hubungan pendidik dengan anak didik. Jadi proses ini melibatkan interaksi kedua pihak sehingga memunculkan hubungan timbal balik. Adapun definisi dari pembelajaran menurut beberapa ahli:

1. Knirk & Gustafson

Pembelajaran merupakan suatu proses yang sistematis melalui tahap rancangan, pelaksanaan dan evaluasi

2. Knowles

Pembelajaran adalah cara pengorganisasian peserta didik untuk mencapai tujuan pendidikan.

3. Slavin

Pembelajaran didefinisikan sebagai perubahan tingkah laku individu yang disebabkan oleh pengalaman.

4. Woolfolk

Pembelajaran berlaku apabila sesuatu pengalaman secara relatifnya menghasilkan perubahan kekal dalam pengetahuan dan tingkah laku.

5. Crow & Crow

Pembelajaran adalah pemerolehan tabiat, pengetahuan dan sikap.

6. Rahil Mahyuddin

Pembelajaran adalah perubahan tingkah laku yang melibatkan ketrampilan kognitif yaitu penguasaan ilmu dan perkembangan kemahiran intelek.

7. Achjar Chalil

Pembelajaran adalah proses interaksi peserta didik dengan pendidik dan sumber belajar pada suatu lingkungan belajar.

8. Corey

Pembelajaran adalah suatu proses dimana lingkungan seseorang secara disengaja dikelola untuk memungkinkan ia turut serta dalam tingkah laku tertentu dalam kondisi-kondisi khusus.

9. G. A. Kimble

Pembelajaran merupakan perubahan kekal secara relatif dalam keupayaan kelakuan akibat latihan yang diperkujuh.

10. Munif Chatib

Pembelajaran adalah proses transfer ilmu dua arah, antara guru sebagai pemberi informasi dan siswa sebagai penerima informasi.

Dari definisi-definisi di atas diperoleh pengertian bahwa pembelajaran dimaksudkan bertujuan untuk mempermudah anak didik dalam belajar dengan

metode transfer dan aplikasi ilmu dari pendidik. Ilmu yang diberikan tidak hanya materi dalam buku pelajaran, namun dapat berupa pemahaman ilmu pengetahuan, nilai-nilai kehidupan, konsep-konsep penting, bahkan keterampilan dan kecakapan diri untuk bekal hidup bermasyarakat.

Kegiatan pembelajaran merupakan kegiatan interaksi secara dua arah sehingga seluruh komponen terkait harus memiliki hubungan timbal balik yang baik pula. Komponen-komponen dalam memahami pengertian pembelajaran ialah:

1. Guru atau pendidik

Pendidik atau guru dikatakan baik jika memiliki kemampuan dan kapasitas dalam memotivasi peserta didik agar melaksanakan belajar dengan kesenangan. Dengan motivasi yang diberikan, diharapkan peserta didik dapat menerima materi pembelajaran sesuai dengan kemampuan yang akan diaplikasikan pada kehidupan masyarakat. Jika pendidik dan peserta didik dapat memahami pengertian pembelajaran dengan baik dan benar, tujuan pembelajaran dalam memperoleh ketuntasan produk belajar akan tercapai, begitu pula kompetensi pembelajaran akan terurai dan teraplikasikan dengan baik.

Pendidik memegang peranan penting dalam mengatur proses belajar mengajar sehingga makna dari definisi pembelajaran terakumulasi dalam sistematika belajar integral. Tindakan pendidik dalam wujud pengertian pembelajaran diantaranya memberikan materi, semangat, aplikasi diri yang baik, refleksi pembelajaran, memberikan proses belajar mengajar yang

kondusif dan menyenangkan. Pendidik yang baik memiliki kemampuan dan kapasitas dalam menangani suatu situasi dan kondisi. Dengan kata lain, dapat mengintegrasikan kegiatan belajar mengajar dengan metode pembelajaran yang seimbang dan dinamis.

## 2. Peserta Didik

Peranan peserta didik dalam proses pembelajaran guna memahami pengertian pembelajaran adalah sebagai penerima materi dan belajar. Seorang peserta didik memiliki hak untuk menanyakan apa yang tidak ia ketahui pada pendidik, ini merupakan acuan sederhana dalam kegiatan belajar mengajar. Di samping itu, peserta didik harus mampu memberikan kontribusi yang sama dalam pencapaian kompetensi belajar dan ketuntasan produk. Diharapkan peserta didik dapat menyeimbangkan atau bersinergi dalam menciptakan kegiatan belajar mengajar yang baik.

## 3. Media

Media dalam pemahaman pengertian pembelajaran adalah perantara yang digunakan saat proses pembelajaran berlangsung. Media dapat berupa alat komunikasi maupun teknologi, seperti komputer, televisi, dan radio. Selain itu, internet juga dapat dimanfaatkan sebagai media pembelajaran. Penggunaan media tersebut bertujuan untuk mempermudah proses pembelajaran, sehingga materi dari pendidik dapat tersampaikan dan diterima dengan baik. Sekali lagi pemahaman pengertian pembelajaran dapat terjadi disebabkan komunikasi dua arah antar pendidik dan peserta didik.

#### 4. Materi Pembelajaran

Materi yang diberikan mencakup tiga aspek, yaitu aspek kognitif, aspek afektif, dan psikomotorik. Dengan ketiga aspek itu, hasil proses pembelajaran dengan tujuan pemahaman pengertian pembelajaran dapat dioptimalkan. Sementara itu, materi pembelajaran yang diberikan dapat berupa konsep-konsep yang ditanamkan pada peserta didik.

##### 2.1.2 Prinsip dalam pemahaman pembelajaran

Dalam keseluruhan proses pendidikan di sekolah, pembelajaran merupakan aktivitas yang paling utama. Ini berarti bahwa keberhasilan pencapaian tujuan pendidikan banyak bergantung pada bagaimana proses pembelajaran dapat berlangsung secara efektif. Pemahaman seorang guru terhadap pengertian pembelajaran akan sangat mempengaruhi cara guru itu mengajar. Komponen dalam pemahaman pembelajaran pun perlu diketahui sehingga proses tujuan yang diharapkan dapat tercapai. Adapun komponen tersebut yaitu:

###### 1. Motivasi

Walaupun motivasi dapat timbul secara langsung dalam diri maupun terlebih dahulu harus diberikan oleh orang lain, seperti guru, orangtua, maupun teman. Proses pembelajaran yang baik akan membangkitkan motivasi peserta didik. Motivasi itu sendiri terkait dengan minat, dalam proses pembelajaran guna mencapai pemahaman pengertian pembelajaran. Motivasi berfungsi sebagai pemompa semangat anak didik agar terpacu untuk belajar. Anak didik dengan motivasi belajar yang tinggi akan terus

bersemangat untuk menyelidiki apa yang disukai sehingga ia akan belajar dengan lebih giat.

## 2. Keaktifan

Proses pembelajaran guna pemahaman pengertian pembelajaran yang pasif tentunya tidak akan efektif. Sebisa mungkin peserta didik harus didorong untuk aktif, baik bertanya maupun aktif menyampaikan aspirasi dan keinginanannya.

## 3. Terlibat langsung

Proses pembelajaran guna pemahaman pengertian pembelajaran dapat teraplikasi secara baik apabila suatu tindakan belajar dilakukan sendiri oleh peserta didik, bukan pembelajaran yang diwakilkan. Peserta didik mengalami sendiri proses belajar dan mengamati setiap detailnya sehingga ia mampu mendeskripsikan apa yang dialami karena kita belajar dengan menyerap 10% dari yang kita baca, 20% dari yang kita dengar, 30% dari apa yang kita lihat, 50% dari apa yang kita lihat dan dengar, 70% dari apa yang kita katakan, dan 90% dari apa yang kita katakan dan lakukan.

## 4. Pengulangan

Ibarat pisau yang diasah berkali-kali akan menjadi tajam, pengulangan materi pelajaran yang telah didapat juga dapat mempertajam ingatan. Pengulangan dapat dilakukan dengan berbagai cara, menuliskannya kembali setelah membaca, membuat catatan kecil atau melakukan tanya jawab dengan teman mengenai materi yang sudah dipelajari.

## 5. Tantangan

Belajar dapat diumpamakan sebagai media yang sulit, sehingga untuk mengatasinya diperlukan kemampuan untuk menakhlukkan medan tersebut. Caranya, peserta didik diberi motivasi untuk mengubah tantangan yang sulit tadi menjadi sesuatu yang menyenangkan dengan caranya sendiri. Semangat untuk menyelesaikannya pun akan timbul. Jika tantangan dapat diatasi, tujuan belajar pun tercapai. Dalam pencapaiaannya, peserta didik dipahamkan akan pengertian pembelajaran melalui tantangan nyata secara materi ataupun psikologi.

## 6. Perbedaan karakter individu

Masing-masing orang memiliki karakter berbeda. Sikap dan caranya dalam menjawab tantangan pun berbeda-beda. Pendidik harus dapat memberikan perlakuan sesuai karakteristik yang dimiliki anak didiknya. Metode pengajaran dan pengertian pembelajaran yang terarah dengan baik secara simultan, pada akhirnya dapat dipahami peserta didik, namun proses pemahamannya akan mengalami perbedaan. Pendidik harus pandai-pandai dalam memberikan perhatian khusus sehingga perbedaan karakter tersebut tidak menjadikan masalah dalam proses pemahaman pembelajaran.

### **2.2 Model- model pembelajaran**

#### **2.2.1 Definisi**

Model pembelajaran dapat didefinisikan sebagai suatu pola mengajar yang menerangkan proses menyebutkan dan menghasilkan situasi lingkungan

tertentu yang menyebabkan para siswa berinteraksi dengan cara terjadinya perubahan khusus pada tingkah laku mereka dengan kata lain, penciptaan suatu situasi lingkungan yang memungkinkan terjadinya proses belajar. Sedangkan Joyce & Weil (1980) mendefinisikan model pembelajaran sebagai kerangka konseptual yang digunakan sebagai pedoman dalam melakukan pembelajaran. Dengan demikian, model pembelajaran merupakan kerangka konseptual yang melukiskan prosedur yang sistematis dalam mengorganisasikan pengalaman belajar untuk mencapai tujuan belajar. Jadi model pembelajaran cenderung preskriptif, yang relatif sulit dibedakan dengan strategi pembelajaran. Adapun lima unsur dasar secara rasional teoretik, tujuan, dan hasil yang ingin dicapai (Joyce & Weil (1980), yaitu :

1. *Syntax*, yaitu langkah-langkah operasional pembelajaran.
2. *Social system*, adalah suasana dan norma yang berlaku dalam pembelajaran.
3. *Principles of reaction*, menggambarkan bagaimana seharusnya guru memandang, memperlakukan, dan merespon siswa.
4. *Support system*, segala sarana, bahan, alat, atau lingkungan belajar yang mendukung pembelajaran.
5. *Instructional dan nurturant effects*—hasil belajar yang diperoleh langsung berdasarkan tujuan yang disasar (instructional effects) dan hasil belajar di luar yang disasar (nurturant effects).

### 2.2.2 Ciri- ciri model pembelajaran

Adapun ciri-ciri dari suatu model pembelajaran yaitu:

1. Memiliki prosedur ilmiah tertentu (*Scientific Procedure*)
2. Memiliki perincian dari hasil belajar (*specification of learning outcome*)
3. Menyebutkan lingkungan belajar yang diharapkan (*specification of environment*)
4. Kriteria penampilan (*criterion of performance* )
5. Cara-cara pelaksanaannya (*specification of operation* )

### 2.2.3 Jenis- jenis Model Pembelajaran

Sugiyanto (2008) mengemukakan bahwa ada banyak model pembelajaran yang dikembangkan oleh para ahli dalam usaha mengoptimalkan hasil belajar siswa. Model pembelajaran tersebut antara lain terdiri dari:

1. Model Pembelajaran Kontekstual

Model pembelajaran kontekstual merupakan konsep belajar yang mendorong guru untuk menghubungkan antara materi yang diajarkan dengan situasi dunia nyata siswa. Pembelajaran ini juga mendorong siswa membuat hubungan antara pengetahuan yang dimilikinya dan penerapannya dalam kehidupan mereka sehari-hari. Pengetahuan dan keterampilan siswa diperoleh dari usaha siswa mengkonstruksi sendiri pengetahuan dan keterampilan baru ketika siswa belajar.

## 2. Model Pembelajaran Kooperatif

Model pembelajaran kooperatif merupakan pendekatan pembelajaran yang berfokus pada penggunaan kelompok kecil siswa untuk bekerja sama dalam memaksimalkan kondisi belajar untuk mencapai tujuan belajar.

## 3. Model Pembelajaran Kuantum

Model pembelajaran kuantum merupakan rakitan dari berbagai teori atau pandangan psikologi kognitif dan pemrograman neurologi yang jauh sebelumnya sudah ada.

## 4. Model Pembelajaran Terpadu

Model pembelajaran terpadu merupakan pembelajaran yang memungkinkan siswa baik secara individual maupun kelompok aktif mencari, menggali, dan menemukan konsep serta prinsip secara holistik. Pembelajaran ini merupakan model yang mencoba memadukan beberapa pokok bahasan.

## 5. Model Pembelajaran Berbasis Masalah (*Problem Based Learning – PBL*)

Model pembelajaran berbasis masalah (*Problem Based Learning – PBL*) merupakan pembelajaran yang mengambil psikologi kognitif sebagai dukungan teoritisnya. Fokusnya tidak banyak pada apa yang sedang dikerjakan siswa tetapi pada apa yang siswa pikirkan selama mereka mengerjakannya. Guru memfungsikan diri sebagai pembimbing dan fasilitator sehingga siswa dapat belajar untuk berfikir dan menyelesaikan masalahnya sendiri.

## 2.3 E- Learning

### 2.3.1 Definisi

Sistem pembelajaran elektronik atau e-pembelajaran (Inggris: *Electronic learning* disingkat *E-learning*) adalah cara baru dalam proses belajar mengajar. *E-learning* merupakan dasar dan konsekuensi logis dari perkembangan teknologi informasi dan komunikasi. Dengan *e-learning*, peserta ajar (*learner* atau murid) tidak perlu duduk dengan manis di ruang kelas untuk menyimak setiap ucapan dari seorang guru secara langsung. Atau *E-Learning* merupakan suatu jenis belajar mengajar yang memungkinkan tersampaiannya bahan ajar ke siswa dengan menggunakan media internet, intranet, atau media jaringan computer lain. Terminology *e-learning* sendiri dapat mengacu pada semua kegiatan pelatihan yang menggunakan media elektronik atau teknologi informasi. Karena ada bermacam penggunaan *e-learning* saat ini, maka ada pembagian atau perbedaan *e-learning*. Pada dasarnya, *e-learning* mempunyai dua tipe, yaitu *Synchronous* dan *Asynchronous*.

#### 1. *Synchronous training*

*Synchronous* berarti “pada waktu yang sama”. Jadi, *synchronous training* adalah tipe pelatihan dimana proses pembelajaran terjadi pada saat yang sama ke Bahasa Inggris pengajar sedang mengajar dan murid sedang belajar. Hal tersebut memungkinkannya interaksi langsung antara guru dan murid, baik melalui internet maupun intranet. Pelatihan *e-learning synchronous* lebih banyak digunakan seminar atau konferensi yang pesertanya berasal dari beberapa Negara. Penggunaan tersebut sering pula dinamakan *web conference*

atau *webinar*(*web seminar*) dan sering digunakan kelas atau kuliah universitas online.

*Synchronous training* mengharuskan guru dan semua murid mengakses internet bersamaan. Pengajar memberikan makalah dengan slide presentasi melalui hubungan internet. Peserta pun mengajukan pertanyaan atau komentar melalui *chat window*.

Jadi, *synchronous training* sifatnya mirip pelatihan diruang kelas. Namun, kelasnya bersifat maya(*virtual*) dan peserta tersebar diseluruh dunia dan terhubung melalui internet. Oleh karena itu, *synchronous training* sering pula dinamakan *virtual classroom*.

## 2. *Asynchronous*

*asynchronous* berarti “tidak pada waktu yang bersamaan”. Jadi seseorang dapat mengambil pelatihan pada waktu yang berbeda dengan pengajar memberikan pelatihan, pelatihan ini lebih popular didunia *e-learning* karena memberikan keuntungan lebih bagi peserta pelatihan karena dapat mengakses pelatihan kapanpun dan dimanapun. Pelatihan berupa paket pelajaran yang dapat dijalankan dikomputer manapun dan tidak melibatkan interaksi dengan pengajar maupun pelajar lain. Oleh karena itu, pelajar dapat memulai pelajaran dan meyelesaiannya setiap saat. Paket pelajaran berbentuk bacaan dengan animasi,simulasi, permainan edukatif, maupun latihan atau tes dengan jawaban.

Adapun keuntungan yang ditawarkan dalam *e-learning* yaitu:

1. Segi biaya, dimana ia mampu mengurangi biaya pelatihan.

Kelebihan pertama *e-learning* adalah ia mampu mengurangi biaya pelatihan. Dengan adanya *e-learning*, perusahaan tidak perlu mengeluarkan biaya untuk menyewa pelatih dan ruang kelas serta transportasi peserta pelatihan atau pelatih.

2. Fleksibelitas waktu.

Administrator sering mengalami kesulitan menyesuaikan waktu beberapa karyawan yang ingin dilatih. Hal ini karena untuk mengikuti pelatihan dikelas, seorang karyawan harus meninggalkan pekerjaannya selama satu atau dua hari. Dengan tuntutan kompetisi yang kian meningkat, kekurangan pegawai selama lebih dari satu hari akan sangat mengganggu produktivitas perusahaan. *E-learning* membuat karyawan atau pelajar dapat menyesuaikan waktu belajar. Mereka dapat menyisipkan waktu belajar setelah makan siang, setelah kantor selesai dan menunggu jemputan, atau ketika sedang menunggu laporan rekan dan tidak ada pekerjaan.

3. Fleksibelitas tempat.

Apabila tempat pendidikan aktif menyelenggarakan acara pelatihan, akan sulit mencari ruang kelas yang memadai dan dapat menampung sekitar 10 sampai 20 orang pelajar serta menyediakan alat-alat pembelajaran lain. Adanya *e-learning* membuat karyawan santai mengakses pelatihan *e-learning* dikantor, bahkan dimeja kerja. Selama komputer terhubung dengan komputer yang menjadi *server e-learning*, mereka dapat mengaksesnya dengan mudah.

Terlebih lagi bila *server e-learning* terhubung dengan internet, maka karyawan dapat mengakses pelajaran dari rumah. sedangkan disekolah-sekolah, para pelajar tidak perlu pergi jauh keruang kelas lain ( misal tempat bimbingan belajar). Mereka hanya perlu ke laboratorium komputer sekolah, dimana *e-learning* tersebut diinstall untuk mengikuti tambahan pelajaran.

#### 4. Fleksibelitas kecepatan pembelajaran

Pelajar memiliki gaya yang berbeda-beda. Oleh karena itu, wajar bila dalam suatu kelas ada siswa yang mengerti dengan cepat dan ada yang harus mengulang pelajaran untuk memahaminya. Akan tetapi, karena pelatih atau guru dikelas mengajar dengan kecepatan sama atau semua siswa, maka siswa akan lebih lambat akan sulit memahami. Terlebih lagi guru sering tidak memiliki waktu menjawab pertanyaan siswa atau berdiskusi setelah waktu menjawab pertanyaan siswa atau berdiskusi setelah waktu pelajaran dikelas habis. Siswa menjadi frustasi. Siswa yang lebih menginginkan lebih banyak materi, sedangkan siswa yang lebih lambat menginginkan pengulangan pelajaran. *E-learning* dapat disesuaikan dengan kecepatan belajar masing-masing siswa. Siswa mengatur sendiri kecepatan belajar yang diikuti. Apabila belum mengerti, ia dapat tetap mempelajari modul tertentu dan mengulanginya nanti. Apabila seorang siswa mengerti dengan cepat, ia dapat menyelesaikan pelajaran lebih cepat dan mengisi waktu dengan belajar topik lain. Hal ini berbeda sekali dengan pelatihan di kelas karena semua pelajaran mulai dan berhenti diwaktu yang sama.

### 5. Standarisasi pengajaran.

Perbedaan antara guru atau pelataih dari segi kepopuleran sangat berpengaruh dimana hal tersebut disebabkan perbedaan kemampuan dan metode pengajaran yang diterapkan guru. Perbedaan tersebut menyebabkan kualitas pengajaran sulit dijaga karena guru favorit atau guru yang lebih populer tidak mungkin diminta mengajar seluruh mata pelajaran. Terlebih lagi guru favorit pun dapat merasa tidak fit saat mengajar sehingga kualitas pengajaran menurun. Dengan *e-learning* dapat menghapuskan perbedaan tersebut. Pelajaran *e-learning* selalu memiliki kualitas sama setiap kali diakses dan tidak tergantung suasana hati pengajar.

### 6. Efektivitas pengajaran.

Karena *e-learning* merupakan teknologi baru, pelajar dapat tertarik dan mencobanya sehingga jumlah peserta pelatihan meningkat.

### 7. Kecepatan distribusi.

Kemajuan teknologi yang pesat menuntut suatu pelatihan teknologi baru dilaksanakan secepatnya menjangkau area luas secara singkat. Melihat geografis Indonesia, bukan hal aneh bila suatu perusahaan dapat menerapkan teknologi canggih dikantor pusat di Jakarta semisalnya, sedangkan kantor cabang yang terpencil tidak mengetahui teknologi tersebut. Kebijakan baru perusahaan yang ditetapkan oleh kantor pusat kadang memerlukan pelatihan agar karyawan dapat memahaminya. Apabila pelatihan dilaksanakan diruang kelas dan kantor cabang yang terlibat sangat banyak, anda dapat membayangkan waktu yang diperlukan untuk membuat seluruh peserta

mengikuti kelas. Sebelum seluruh peserta selesai mengikuti pelatihan, teknologi atau kebijakan baru mungkin muncul. *E-learning* dapat cepat menjangkau karyawan yang berada diluar wilayah pusat. Tim desain pelatihan hanya perlu mempersiapkan bahan pelatihan secepatnya da menginstall hasil server pusat *e-learning*. Jadi, semua komputer yang terhubung ke server dapat langsung mengakses. Apabila terdapat cabang yang tidak memiliki sambungan *network* server, pelajaran hanya perlu disimpan di *compact disk* (CD) dan dikirim melalui pos. dengan *e-learning*, suatu pelatihan baru dapat langsung diterima oleh semua karyawan lebih cepat. Apabila ada perubahan materi pelatihan, administrator hanya perlu mengubah di server *e-learning*, tanpa mendatangi semua kantor cabang.

#### 8. Ketersediaan On-Demand.

Karena *e-learning* dapat sewaktu-waktu diakses, anda dapat menganggap sebagai “buku saku” yang membantu pekerjaan setiap saat. Sebagai contoh, bila anda harus membuat grafik diprogram microsoft excel dan tidak tahu caranya, anda dapat langsung memasuki program *e-learning* yang mengajarkan aplikasi microsoft excel dan langsung kebagian tentang grafik. Dengan begitu, dalam waktu 15 menit anda dapat segera mempraktikkan pelajaran dan menyelesaikan tugas dengan baik.

#### 9. Otomatisasi proses administrasi.

*E-learning* menggunakan suatu *Learning Management System*(LMS) yang berfungsi sebagai *platform* pelajaran- pelajaran *e-learning*. LMS berfungsi pula menyimpan data-data pelajar, pelajaran, dan proses

pembelajaran yang langsung. LMS yang baik dapat menyimpan dan membuat laporan tentang kegiatan belajar seorang siswa, mulai dari tentang kegiatan belajar seorang siswa, mulai dari pelajaran yang telah diambil, tanggal akses, berapa persen pelajaran diselesaikan, berapa lama pelajaran diikuti, sampai berapa hasil tes akhir diambil. Dengan adanya laporan didalam sistem, administrator pelatihan sangat terbantu. Waktu dan proses menyelesaikan tugas administrasi pelapor, pelapor akan lebih singkat dan mudah. Pelatih atau guru yang memiliki akses ke LSM dapat setiap saat mencetak sendiri laporan dengan otomatis untuk memonitor kemajuan belajar siswa, tanpa harus menunggu administrator.

Walaupun *e-learning* menawarkan banyak keuntungan bagi organisasi, praktik ini juga memiliki beberapa keterbatasan yang harus diwaspadai oleh pengelola pelatihan sebelum memutuskan menggunakan *e-learning*, diantaranya adalah:

#### 1. Budaya

Beberapa orang merasa tidak nyaman mengikuti pelatihan melalui komputer. pengguna *e-learning* menuntut budaya *self-learning*, dimana seseorang memotivasi diri sendiri agar mau belajar. Sebaliknya, pada sebagian besar budaya pelatihan di Indonesia, motivasi belajar lebih banyak tergantung pada pengajar. Apabila pengajarnya terasa cocok dan menyenangkan, motivasi pelajar bertambah, begitu pula sebaliknya. Dalam pelatihan di ruang kelas, 60% energi dari pengajar, sedangkan pelajar hanya mendengar dan mencatat. Namun, dalam *e-learning*, 100% energi dari pelajar. Oleh karena itu, beberapa

orang masih merasa segan berpindah dari pelatihan dikelas ke pelatihan *e-learning*. Adapun harus melihat budaya dan kebiasaan pengguna teknologi pelajar. Apabila mereka tidak terbiasa menggunakan komputer, implementasi *e-learning* akan memakan waktu yang lebih lama. Terlebih lagi bila budaya pelatihan sudah terbiasa dengan interaksi yang ketat antar sesama pelajar, maka praktik *e-learning* akan sulit diterima.

## 2. Investasi

Walaupun *e-learning* menghemat banyak biaya, tetapi suatu organisasi harus mengeluarkan investasi awal cukup besar untuk mulai mengimplementasikan *e-learning*. Investasi dapat berupa biaya desain dan pembuatan program *learning management system*, paket pelajaran dan biaya-biaya lain, seperti promosi dan *change management system*. Apabila infrastruktur yang dimiliki belum memadai, organisasi harus mengeluarkan sejumlah dana untuk membeli komputer, jaringan, server, dan lain sebagainya.

## 3. Teknologi

Karena teknologi yang digunakan beragam, ada kemungkinan teknologi tersebut tidak sejalan dengan yang sudah ada dan terjadi konflik teknologi sehingga *e-learning* tidak berjalan baik. sebagai contoh ada beberapa paket pelajaran *e-learning* yang hanya dapat dijalankan dibrowser explorer. Oleh karena itu, kompatibelitas teknologi yang digunakan harus teliti sebelum memutuskan menggunakan suatu paket *e-learning*.

#### 4. Infrastruktur

Internet belum menjangkau semua kota di Indonesia. Layanan *broadband* baru ada dikota-kota besar. akibatnya, belum semua orang atau wilayah belum dapat merasakan *e-learning* dengan internet.

#### 5. Materi

Walaupun *e-learning* menawarkan berbagai fungsi, ada beberapa materi yang tidak dapat diajarkan melalui *e-learning*. Pelatihan yang memerlukan banyak kegiatan fisik, seperti olahraga dan instrumen musik, sulit disampaikan melalui *e-learning* secara sempurna. Akan tetapi, *e-learning* dapat digunakan untuk memberikan dasar-dasar pelatihan sebelum masuk ke praktik. Banyak desain pelajaran *e-learning* tidak tepat sehingga pelajar bosan dan tidak mau menyelesaikan pelajaran. Kebanyakan pelajaran *e-learning* di Indonesia masih berbentuk seperti buku teks di sekolah dengan tambahan sedikit animasi. Hal tersebut tentu tidak banyak memberikan nilai tambah untuk pelajar sehingga ia merasa semua pelajaran *e-learning* membosankan. Oleh karena itu, suatu paket *e-learning* harus didesain sedemikian rupa sehingga teratur menurut keingintahuan dan minat belajar siswa. Masalah keterbatasan budaya ditempatkan diurutan pertama karena paling sulit diatasi. Meskipun masalah investasi atau biaya yang tinggi sering dianggap hambatan nomor satu, investasi tinggi masih dapat diatasi. Apabila pihak manajemen sudah melihat biaya yang dapat ditekan dengan penggunaan *e-learning*, biaya investasi yang tinggi dapat disetujui. Akan tetapi, masalah budaya memakan

waktu lama untuk diubah karena menyangkut kebiasaan organisasi yang turun temurun.

Dalam pendidikan konvensional fungsi *e-learning* bukan untuk mengganti, melainkan memperkuat model pembelajaran konvensional.

Dalam hal ini Cisco (2001) menjelaskan filosofis *e-learning* sebagai berikut:

1. *E-learning* merupakan penyampaian informasi, komunikasi, pendidikan, pelatihan secara on-line maupun offline.
2. *E-learning* menyediakan seperangkat alat yang dapat memperkaya nilai belajar secara konvensional (model belajar konvensional, kajian terhadap buku teks, CD-ROM, dan pelatihan berbasis komputer) sehingga dapat menjawab tantangan perkembangan globalisasi.
3. *E-learning* tidak berarti menggantikan model belajar konvensional di dalam kelas, tetapi memperkuat model belajar tersebut melalui pengayaan *content* dan pengembangan teknologi pendidikan.
4. Kapasitas siswa amat bervariasi tergantung pada bentuk isi dan cara penyampaiannya. Makin baik keselarasan antar *content* dan alat penyampai dengan gaya belajar, maka akan lebih baik kapasitas siswa yang pada gilirannya akan memberi hasil yang lebih baik

Dari definisi-definisi di atas dapat disimpulkan bahwa *e-learning* merupakan pemanfaatan teknologi informasi dan komunikasi untuk melakukan transfer ilmu pengetahuan,bukan hanya meliputi *online learning*, *virtual learning*, *web-based learning* melainkan juga termasuk di dalamnya pembelajaran yang menggunakan teknologi komputer baik secara online maupun offline.

## 1.4 Jejaring Sosial

### 1.4.1 Definisi

Jejaring sosial adalah suatu struktur sosial yang dibentuk dari simpul-simpul (yang umumnya adalah individu atau organisasi) yang dijalin dengan satu atau lebih tipe relasi spesifik seperti nilai, visi, ide, teman, keturunan, dll. Analisis jaringan jejaring sosial memandang hubungan sosial sebagai simpul dan ikatan. Simpul adalah aktor individu di dalam jaringan, sedangkan ikatan adalah hubungan antar aktor tersebut. Bisa terdapat banyak jenis ikatan antar simpul.

Definisi lain dari jejaring sosial adalah layanan dalam jaringan, platform, atau situs yang bertujuan memfasilitasi pembangunan jaringan sosial atau hubungan sosial di antara orang-orang yang memiliki ketertarikan, aktivitas, latar belakang, atau hubungan dunia nyata yang sama. Suatu layanan jejaring sosial terdiri dari perwakilan masing-masing pengguna (biasanya berupa profil), hubungan sosialnya, dan berbagai layanan tambahan. Kebanyakan layanan ini berbasis web dan penggunanya berinteraksi melalui Internet, seperti surat elektronik dan pesan instan. Layanan komunitas dalam jaringan kadang dianggap sebagai layanan jejaring sosial, meski dalam artian yang lebih luas layanan jejaring sosial bersifat terpusat pada individu, sementara layanan komunitas daring bersifat terpusat pada grup. Situs-situs jejaring sosial memungkinkan pengguna berbagi ide, aktivitas, acara, dan ketertarikan di dalam jaringan individunya masing-masing.

Penelitian dalam berbagai bidang akademik telah menunjukkan bahwa jaringan jejaring sosial beroperasi pada banyak tingkatan, mulai dari keluarga

hingga negara, dan memegang peranan penting dalam menentukan cara memecahkan masalah, menjalankan organisasi, serta derajat keberhasilan seorang individu dalam mencapai tujuannya. Dalam bentuk yang paling sederhana, suatu jaringan jejaring sosial adalah peta semua ikatan yang relevan antar simpul yang dikaji. Jaringan tersebut dapat pula digunakan untuk menentukan modal sosial aktor individu. Konsep ini sering digambarkan dalam diagram jaringan sosial yang mewujudkan simpul sebagai titik dan ikatan sebagai garis penghubungnya.

Situs jejaring sosial diawali oleh Classmates.com pada tahun 1995 yang berfokus pada hubungan antar mantan teman sekolah dan SixDegrees.com pada tahun 1997 yang membuat ikatan tidak langsung. Dua model berbeda dari jejaring sosial yang lahir sekitar pada tahun 1999 adalah berbasiskan kepercayaan yang dikembangkan oleh Epinions.com, dan jejaring sosial yang berbasiskan pertemanan seperti yang dikembangkan oleh Uskup Jonathan yang kemudian dipakai pada beberapa situs UK regional di antara 1999 dan 2001. Inovasi meliputi tidak hanya memperlihatkan siapa berteman dengan siapa, tetapi memberikan pengguna kontrol yang lebih akan isi dan hubungan. Pada tahun 2005, suatu layanan jejaring sosial *MySpace*, dilaporkan lebih banyak diakses dibandingkan Google dengan *Facebook*, pesaing yang tumbuh dengan cepat. Jejaring sosial mulai menjadi bagian dari strategi internet bisnis sekitar tahun 2005 ketika Yahoo meluncurkan Yahoo! 360°. Pada bulan juli 2005 *News Corporation* membeli *MySpace*, diikuti oleh ITV (UK) membeli *Friends Reunited* pada Desember 2005. Diperkirakan ada lebih dari 200 situs jejaring sosial menggunakan model jejaring sosial ini.

Adapun kelebihan jejaring sosial yaitu:

1. Anak dan remaja dapat belajar mengembangkan ketrampilan teknis dan sosial yang sangat dibutuhkan di era digital seperti sekarang ini. Mereka akan belajar beradaptasi, bersosialisasi dengan publik dan mengelola jaringan pertemanan.
2. Memperluas jaringan pertemanan. Berkat situs jejaring sosial ini anak menjadi lebih mudah berteman dengan orang lain di seluruh dunia. Meskipun sebagian besar diantaranya tidak pernah mereka temui secara langsung.
3. Anak dan remaja akan termotivasi untuk belajar mengembangkan diri melalui teman-teman yang mereka jumpai secara online, karena mereka berinteraksi dan menerima umpan balik satu sama lain.
4. Situs jejaring sosial membuat anak dan remaja menjadi lebih bersahabat, perhatian dan empati. Misalnya memberikan perhatian saat ada teman mereka berulang tahun, mengomentari foto, video dan status teman mereka, menjaga hubungan persahabatan meski tidak dapat bertemu secara fisik.
5. membantu menjaga hubungan yang baik dengan Sesama pengguna Jejaring Sosial terutama teman.
6. mempererat tali persaudaraan
7. memudahkan mencari pengetahuan
8. memudahkan mencari informasi tentang kejadian di dunia luar
9. dapat berbagi pengalaman yang dapat dipelajari pembaca
10. Dapat berinteraksi dengan teman lama yang jaraknya cukup jauh dan bisa menjalin tali silaturahmi.

11. Bisa mendapatkan informasi yang aktual seperti arus lalu lintas, diskon-diskon belanjaan, reuni sekolah/ kuliah, tempat makanan yang enak dan lain-lain.
12. Bisa mengecek keberadaan teman kita yang menggunakan jejaring sosial tersebut.
13. Bisa menjadi ajang promosi.

Selain terdapat kelebihan yang dimiliki oleh jejaring sosial, berdasarkan survei jejaring sosial juga memiliki kelemahan diantaranya adalah (*Kompas.com*):

1. Situs jejaring sosial akan membuat anak dan remaja lebih mementingkan diri sendiri. Mereka menjadi tidak sadar akan lingkungan di sekitar mereka, karena kebanyakan menghabiskan waktu di internet. Hal ini dapat mengakibatkan menjadi kurang berempati di dunia nyata.
2. Bagi anak dan remaja, tidak ada aturan ejaan dan tata bahasa di situs jejaring sosial. Hal ini membuat mereka semakin sulit untuk membedakan antara berkomunikasi di situs jejaring sosial dan di dunia nyata.
3. Situs jejaring sosial adalah lahan yang subur bagi *predator* untuk melakukan kejahatan. Kita tidak akan pernah tahu apakah seseorang yang baru kita dikenal anak kita di internet menggunakan jati diri yang sesungguhnya atau tidak.
4. Lebih banyak menghabiskan waktu untuk mampir di jejaring sosial dari pada untuk belajar atau aktif di dalam sekolah.
5. Para siswa mulai malas dengan rutinitas belajar dan akan mementingkan untuk bermain di jejaring sosial yang ada.

6. banyak terjadi kejanggalan dalam jejaring sosial, seperti penculikan anak remaja melalui "jejaring sosial" seperti *facebook, twitter* dsb.
7. Jika tidak dapat mengatur waktu yang baik, hal ini dapat menghabiskan berjam-jam di depan computer .Oleh karena itu harus dapat membatasi waktu yang dihabiskan setiap saat di jejaring Sosial.
8. Anak dan remaja menjadi malas belajar berkomunikasi di dunia nyata. Tingkat pemahaman bahasa pun menjadi terganggu. Jika anak terlalu banyak berkomunikasi di dunia maya.
9. Begitu bebas mengumbar status atau informasi data pribadi. Padahal data-data tersebut bisa didapatkan dengan mudah si penjahat menggunakan data-data tersebut untuk melakukan kejahatan.
10. Banyaknya perdagangan bebas seperti menjual jasa yg aneh-aneh, penipuan dengan menjual barang-barang seperti mobil, *handphone*, motor, dll.
11. Tidak adanya kontrol yang ketat terhadap penggunaan situs jejaring sosial.
12. Banyak provokator yang ada untuk memberikan isu-isu panas dan berita yang tidak jelas sumbernya.

## 1.5 Learning Management System

### 1.5.1 Definisi

LMS atau yang lebih dikenal dengan *Learning Management System* adalah suatu perangkat lunak atau software untuk keperluan administrasi, dokumentasi, laporan sebuah kegiatan, kegiatan belajar mengajar dan kegiatan secara online

(terhubung ke internet), *E-learning* dan materi-materi pelatihan. Dan semua itu dilakukan dengan online. (ellis 2009).

*Learning Management System* (LMS) adalah perangkat lunak yang digunakan untuk membuat materi perkuliahan *on-line* berbasiskan web dan mengelola kegiatan pembelajaran serta hasil-hasilnya. Di dalam LMS juga terdapat fitur-fitur yang dapat memenuhi semua kebutuhan dari pengguna dalam hal pembelajaran. Saat ini ada banyak jenis LMS yang ditawarkan, setiap jenis LMS memiliki fitur-fiturnya masing-masing yang digunakan dapat berbeda fiturnya.

### 1.5.2 Edmodo dan Moodle

Edmodo adalah *platform* media sosial yang sering digambarkan sebagai *facebook* untuk sekolah dan dapat berfungsi lebih banyak lagi sesuai kebutuhan. Edmodo merupakan aplikasi menarik bagi guru dan siswa dengan elemen sosial yang menyerupai *facebook*, tetapi terdapat nilai besar dalam aplikasi edukasi berbasis jejaring sosial. Edmodo yang juga berbasis *cloud* kolaborasi merupakan aplikasi yang cukup aman digunakan oleh guru dan siswa karena dapat dengan mudah dikelola sebagai sistem yang menyediakan fitur terbaik dan praktis yang dapat menghilangkan kecemasan kita terhadap aktivitas yang biasa siswa lakukan dengan internet khususnya *facebook*. Dengan *platform* edmodo akan lebih mudah mengontrol interaksi antar siswa dalam *edmodolearning environment*. Tidak ada yang bisa masuk ke group (kelas) apabila kode group tidak diketahui siswa, dan siswa tidak dapat menggunakannya untuk berhubungan dengan orang asing

seperti yang terjadi di Facebook. Kita akan mudah mengetahui jika ada pelanggar maupun orang asing yang terdaptar di grup (kelas) yang telah dikelola dengan edmodo. Selain itu, Edmodo juga menyediakan subdomain yang bisa digunakan untuk membuat subdomain sekolah/daerah (*school/district*) dimana kita bernaung/tinggal.

Implikasi edmodo untuk pembelajaran yaitu, edmodo seperti alat bantu belajar lainnya yang merupakan sebuah *platform online* untuk mendorong pembelajaran guru, atau dapat menjadi cara yang lebih kreatif untuk melibatkan para siswa dalam pembelajaran kolaboratif. Edmodo merupakan jalur siswa untuk berinteraksi dengan rekan-rekan mereka dan guru dalam suasana akademis, siswa dapat melakukan aktifitas seperti *download* materi pelajaran, berdiskusi antar siswa dan guru, menjawab tugas atau kuis tanpa adanya tatap muka langsung dengan guru yang dapat menghasilkan kegembiraan proses belajar yang dapat mengatasi materi baru dan sulit akan meningkat. Penggunaan edmodo ini juga dapat mengajarkan siswa untuk bagaimana berprilaku secara *online* dan bertanggung jawab mengatur kegiatan belajar mereka dengan sistem yang keamananya terjamin. Pada hakikatnya edmodo merupakan *platform* yang mudah dipelajari dan mudah digunakan terutama bagi para guru yang menganggap dirinya berada di luar basis pengetahuan teknologi yang berkembang saat ini. Edmodo menyediakan cara yang aman dan mudah bagi kelas untuk terhubung dan berkolaborasi, berbagi konten dan akses pekerjaan.

Dalam mendukung proses pembelajaran, edmodo dilengkapi dengan beberapa aktivitas pembelajaran, seperti *Quiz*, *Assignment* dan *Poll*. Sedangkan

untuk *Resources* (bahan ajar), Edmodo mendukung bahan ajar berupa file dan link (*URL/Embed* media). Kelebihan dari edmodo ini adalah, dengan adanya akun *parent* dalam edmodo, ini memberi ruang bagi para orang tua untuk dapat mengontrol hasil pekerjaan anaknya. Edmodo bukanlah jawaban untuk setiap kelas tetapi yang terpenting adalah platform ini memberikan aspek penting dari sebuah lingkungan belajar yang positif. *Platform* ini memberikan siswa jalur untuk berinteraksi dengan rekan-rekan mereka dan guru mereka dalam suasana akademis. Lebih jauh lagi penggunaan *platform* ini dapat mengajarkan siswa untuk bagaimana berperilaku secara online dan bertanggung jawab dalam mengatur kegiatan belajar mereka dengan sistem yang keamanannya terjamin.

Sedangkan Moodle dapat juga dikatakan sebagai CMS (*Course Management System*), umumnya CMS dibangun berbasis web, yang akan berjalan pada sebuah web server dan dapat diakses oleh pesertanya melalui web browser (web client). Server biasanya di tempatkan di sekolah, universitas/lembaga lainnya, yang dapat di akses dimanapun oleh pesertanya, dengan memanfaatkan koneksi internet. Moodle atau kepanjangan dari *Modular Object Oriented Dynamic Learning Environment* adalah suatu *course content management* (CMS), yang diperkenalkan pertama kali oleh Martin Dougiamas, seorang *computer scientist* dan *educator*. Nama moodle memberikan suatu inspirasi bagi pengembangan *e-learning*. Menurut *Official Moodle Documentation* menjelaskan moodle merupakan sebuah CMS berbasis *open source*. Moodle dapat digunakan untuk keperluan administrasi, dokumentasi, laporan sebuah kegiatan,

kegiatan belajar mengajar dan kegiatan secara online, *E-learning* dan materi-materi pelatihan yang dilakukan dengan online.

Dari penjabaran pengertian edmodo dan moodle yang tertera diatas, dapat diketahui persamaan dan perbedaan edmodo dengan LMS moodle adalah sebagai berikut:

1. Edmodo dengan moodle sama-sama merupakan aplikasi *learning management system* yang digunakan untuk keperluan kegiatan belajar mengajar yang berjalan di jaringan internet. Namun fitur-fitur pada edmodo sudah ditentukan sesuai dengan yang disediakan oleh pengembang, dan tidak bisa dikembangkan atau dimodifikasi sesuai dengan keinginan dari sekolah, universitas atau lembaga yang menggunakannya. Sedangkan moodle dapat dikembangkan atau dimodifikasi sesuai dengan kebutuhan dari siswa, universitas atau lembaga yang menggunakannya karena moodle merupakan suatu CMS yang bersifat *free* dan *open source*.
2. Bahasa pengantar yang terdapat pada edmodo hanya dapat menggunakan bahasa inggris, sedangkan pada moodle banyak bahasa yang dapat digunakan, penuh dalam 45 bahasa termasuk bahasa Indonesia. Ini dapat memudahkan pengguna untuk beradaptasi sesuai dengan bahasa yang telah dikuasai.
3. Pada edmodo tidak terdapat fitur *live chat*, namun pada moodle dapat ditambahkan fitur *live chat* oleh lembaga yang menggunakannya.
4. Edmodo memiliki *interface* dan penataan *tools* dan *content* mirip dengan *facebook* yang dapat memudahkan pengguna untuk mengoperasikannya, karena sebagian besar orang sudah mengetahui dan menggunakan *facebook*.

Sedangkan moodle memiliki penataan *tools* dan *content* sesuai yang telah disediakan oleh CMS tersebut.

5. Edmodo tidak memerlukan server di sekolah, universitas/lembaga lain karena server sudah di sediakan oleh pengembang edmodo sendiri. Sedangkan moodle harus memiliki server dari sekolah, universitas/lembaga yang menggunakan.
6. Untuk bisa bergabung di grup atau kelas pada edmodo harus memiliki *group code* yang telah ditentukan dan dibuat, sedangkan pada moodle terdapat aktivasi dari admin apabila kita ingin bergabung pada LMS moodle tersebut.

## BAB III

### DESAIN DAN SISTEM

#### 3.1 *Problem Statement*

##### 3.1.1 Deskripsi Sistem

Sistem yang dibangun ini adalah sebuah aplikasi jejaring sosial berbasis web yang dibangun berdasarkan rujukan pada jejaring sosial pendidikan yang telah ada yang kemudian dibangun dalam lingkup internal kampus dengan subyek mahasiswa pada jurusan Teknik Informatika Universitas Islam Negeri Malang dengan fungsi sebagai media penyampaian kegiatan belajar mengajar oleh mahasiswa dan dosen diluar jam mata kuliah, baik diskusi ataupun penyampaian materi yang telah disampaikan pada jam kuliah.

##### 3.1.2 Keadaan *Current System*

Aplikasi jejaring sosial yang bersifat internal dijurusan informatika belum begitu terealisasi dengan baik. Dosen maupun mahasiswa masih banyak yang menggunakan jejaring sosial seperti *facebook* untuk penyampaian materi maupun pemberian pengumuman ujian serta informasi yang berhubungan dengan mata kuliah pada sebuah group dimana orang asing selain mahasiswa Teknik Informatika Universitas Islam Negeri Malang dapat dengan mudah ikut bergabung dengan group sehingga tingkat *privasi* group belajar yang berhubungan dengan sebuah kelas menjadi terganggu.

### 3.1.3 Lingkup Proyek

Berdasarkan keadaan current sistem tersebut diatas, maka lingkup kegiatan akan dikerjakan dalam proyek ini adalah :

1. Tahap perencanaan
2. Tahap analisis tahap desain
3. Tahap konstruksi
4. Tahap uji coba
5. Tahap implementasi

## 3.2 Identifikasi dan Analisis Proses Bisnis

### 3.2.1 Identifikasi Proses Bisnis

Dalam aplikasi jejaring sosial untuk pembelajaran peserta didik ini yang terjadi yaitu:

1. Pendaftaran study group.
2. Kelas ajar
3. Pendistribusian materi
4. Pendistribusian File
5. Pendistribusian soal
6. Report
7. *Chat group*

### 3.2.2 Analisis proses bisnis

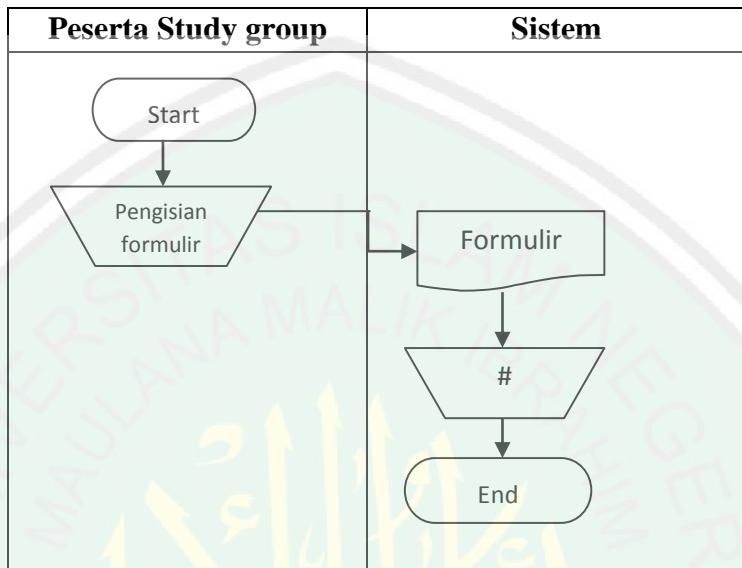
Tabel 3.1 Analisis Proses Bisnis

Nama Proses Bisnis	Siapa Saja Yang Terlibat	Di Mana Proses Bisnis Terjadi	Kapan Proses Bisnis Terjadi	Bagaimana Proses Bisnis Dijalankan	Dokumen Yang Terkait Dengan Proses Bisnis
Pendaftaran study group	Dosen, mahasiswa, admin	Jurusan	setiap hari	Pendaftaran dilakukan dengan cara memilih form yang telah disediakan diweb sesuai dengan jenis pengguna	biodata: nama, email, no telp, alamat
Kelas ajar	Dosen, mahasiswa	Jurusan	setiap hari	kelas ajar ditentukan dosen pengampu kemudian didistribusikan pada mahasiswa yang akan bergabung dalam kelas.	Kode kelas
Pendistribusian materi	Dosen, mahasiswa	Jurusan	setiap hari	Dosen menyampaikan materi kuliah yang akan dilihat oleh mahasiswa	data materi
Pendistribusian File	Dosen, mahasiswa	Jurusan	setiap hari	Dosen akan melakukan upload file materi, kemudian diunduh oleh mahasiswa.	data file upload dari dosen
Pendistribusian soal	Dosen, mahasiswa	Jurusan	setiap hari	Soal diberikan oleh dosen sesuai dengan jenis soal dan waktu pengerjaan. jawaban soal yang diinputkan oleh mahasiswa berdasarkan jenis soal.	Data soal mata kuliah
Report	Dosen, mahasiswa	Jurusan	setiap hari	Nilai diberikan dosen. mahasiswa dapat melihat nilai dan grafik perkembangan nilai dari beberapa soal yang telah dikerjakan.	Daftar nilai mahasiswa

Nama Proses Bisnis	Siapa Saja Yang Terlibat	Di Mana Proses Bisnis Terjadi	Kapan Proses Bisnis Terjadi	Bagaimana Proses Bisnis Dijalankan	Dokumen Yang Terkait Dengan Proses Bisnis
				dosen dapat melihat grafik perkembangan mahasiswa terhadap soal yang diberikan melalui nilai keseluruhan	
<i>Chat group</i>	Dosen mahasiswa	Jurusan	setiap hari	Mahasiswa dan dosen yang telah melakukan login, kemudian masuk kelas ajar, dapat mengakses chat group yang ada di kelas ajar.	history record browser

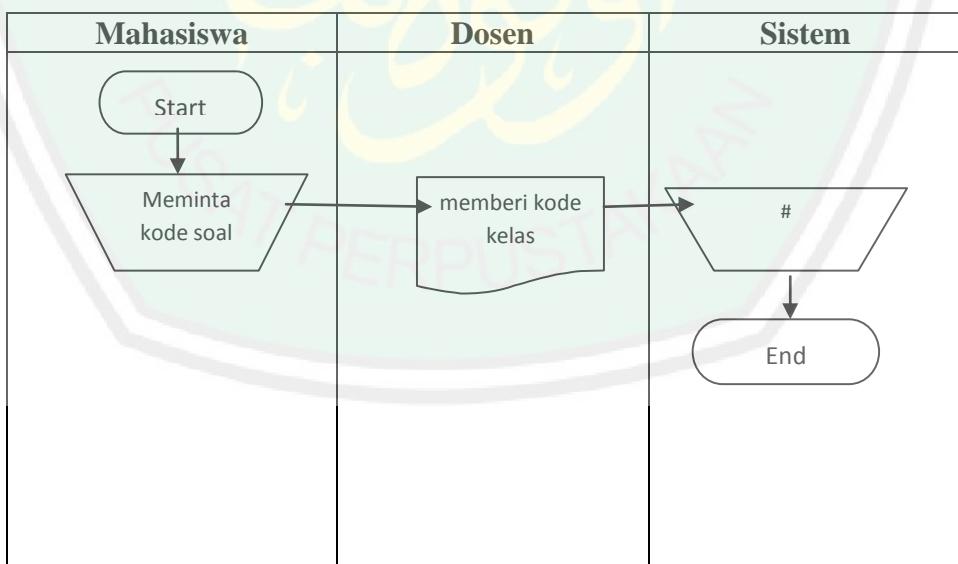
### 3.2.3 Permodelan proses bisnis

#### 1. Pendaftaran study group



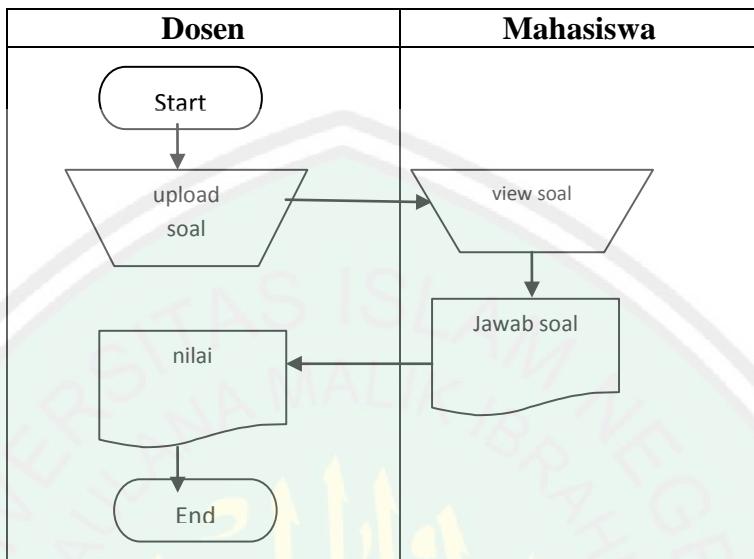
Gambar 3.1 *Workflow* pendaftaran *study group*

#### 2. Kelas ajar



Gambar 3.2 *Workflow* gabung kelas

### 3. Proses distribusi soal



Gambar 3.3 workflow distribusi soal

### 3.3 Identifikasi dan analisis kebutuhan

#### 3.3.1 Identifikasi kebutuhan fungsional

##### 1. Proses bisnis pendaftaran

Tabel 3.2 Proses bisnis pendaftaran

Calon Pendaftar	Operator	Kebutuhan fungsional Sistem Informasi
Mengisi formulir yang tersedia di web	-	pengisian formulir registrasi
mengisi biodata yang dibutuhkan diweb	-	kelengkapan biodata

##### 2. Proses bisnis kegiatan kelas ajar

Tabel 3.3 Proses bisnis kegiatan kelas ajar

Mahasiswa	Dosen	Kebutuhan fungsional Sistem Informasi
Gabung kelas	Memberi kode kelas	Mata Kuliah
Melihat materi yang di upload dosen	upload materi	Materi mata kuliah

Mahasiswa	Dosen	Kebutuhan fungsional Sistem Informasi
Mengunduh file materi	upload file materi	file materi
melihat dan menjawab soal	upload soal	data soal
melihat report nilai sendiri	melihat report nilai keseluruhan peserta	data nilai

3. Proses bisnis *chat group*

Tabel 3.4 Proses bisnis *chat group*

Mahasiswa	Dosen	Kebutuhan fungsional Sistem Informasi
Input komentar	Input komentar	history record user
View komentar	view komentar	data komentar pada <i>user chat</i>

### 3.3.2 Analisis kebutuhan fungsional

Tabel 3.5 Analisis kebutuhan fungsional

Nama kegiatan sistem	Siapa saja yang terlibat	Di mana kegiatan sistem dilakukan	Kapan kegiatan sistem terjadi	Bagaimana kegiatan sistem dijalankan	Dokumen yang terkait dengan kegiatan sistem
Sistem pendaftaran study group	Mahasiswa, dosen, admin	jurusan	setiap hari	Mahasiswa dan dosen melakukan pendaftaran berdasarkan jabatan kemudian mengisi biodata yang telah disediakan diweb	-
Sistem kelas mata kuliah	Mahasiswa, dosen	jurusan	setiap hari	Mahasiswa akan bergabung dikelas mata kuliah setelah diberi kode mata kuliah oleh dosen yang bersangkutan	-
Sistem report	Mahasiswa, dosen	jurusan	setiap hari	Pemrosesan jawaban soal ini dilakukan oleh dosen. Dan hasil nilai akan ditampilkan di web pada menu report	daftar nilai

### 3.3.3 Dokumen terkait dengan sistem



#### DAFTAR NILAI Jurusan Teknik Informatika

Nama Mahasiswa	Nilai									
	soal 1	soal 2	soal 3	soal 4	soal 5	soal 6	soal 7	soal 8	soal 9	soal 10

### 3.3.4 Identifikasi dan analisis kebutuhan non fungsional

Tabel 3.6 Identifikasi dan analisis kebutuhan non fungsional

Komponen sistem informasi	Spesifikasi	Siapa yang mengadakan	Kapan harus diadakan	Di mana harus diadakan	Bagaimana pengadaannya
<b>Hardware</b>					
Server	Processor intel i7 HDD 320 Gb RAM 4 Gb Monitor LCD flat 14" Keyboard Mouse	Jurusan Teknik Informatika	Setiap hari	dimana saja	Jurusan Teknik Informatika menyediakan PC server yang digunakan untuk menampung database Study group
<b>Software</b>					
Sistem operasi	windows server	Jurusan Teknik Informatika	Setiap hari	dimana saja	servernya menggunakan sistem windows server
<b>Web server</b>	Xammp	Jurusan Teknik Informatika	Setiap hari	dimana saja	Pada sistem ini web servernya menggunakan Xammp
<b>Network</b>	Internet	Jurusan Teknik Informatika	Setiap hari	dimana saja	Jurusan mempublikasikan alamat web kepada mahasiswa

<b>Data, informasi, dan pengetahuan</b>	<p>Data:</p> <ul style="list-style-type: none"> <li>- Biodata mahasiswa</li> <li>- Biodata dosen</li> <li>- Update status user</li> <li>- Materi</li> <li>- file materi</li> <li>- soal-soal</li> <li>- report nilai</li> </ul> <p>Pengetahuan :</p> <ul style="list-style-type: none"> <li>- Pengetahuan mata kuliah</li> </ul>	Mahasiswa dan dosen Teknik Informatika	Setiap hari	dimana saja	<p>data berupa data para mahasiswa dan dosen, update status user, data materi, data file materi, data soal-soal, dan report nilai.</p> <p>Sedangkan pengetahuan yang diperoleh adalah pengetahuan yang berhubungan dengan mata kuliah yang diambil</p>
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### 3.4 Identifikasi dan alternatif solusi

Identifikasi dan analisis alternatif solusi yang perlu yang dikembangkan:

Tabel 3.7 Identifikasi dan alternatif solusi

Karakteristik	Alternatif 1 Seluruh fasilitas jurusan	Alternatif 2 Sebagian besar fasilitas jurusan	Alternatif 3 Sebagian kecil fasilitas jurusan
<b>Bagian sistem yang terkomputerisasi</b>	Proses belajar mengajar antara mahasiswa dan dosen lebih tersosialisasi dengan baik diluar maupun didalam kampus	Memfokuskan komputerisasi pada penyampaian materi dan soal	Memfokuskan komputerisasi pada penyampaian materi dan soal

Karakteristik	Alternatif 1 Seluruh fasilitas jurusan	Alternatif 2 Sebagian besar fasilitas jurusan	Alternatif 3 Sebagian kecil fasilitas jurusan
<b>Keuntungan</b>	Memudahkan dalam proses sosialisasi materi dan soal-soal	Mengurangi limbah print out yang terpakai untuk lembar soal dan lembar jawaban	Memudahkan mahasiswa dalam mempelajari materi dan menjawab soal-soal
<b>Server dan workstation</b>	Menggunakan Operating system windows server sebagai servernya, sedangkan seluruh workstation menggunakan windows desktop.	Menggunakan ubuntu server sebagai servernya dan ubuntu desktop sebagai workstationnya. Dengan begitu akan sangat menghemat biaya karena linux merupakan OS open source.	Menggunakan operating sistem linux.
<b>Alat perangkat lunak yang dibutuhkan</b>	Kami menggunakan windows untuk memudahkan operasi, data base kami berencana menggunakan my sql, sedangkan bahasa pemrograman kami menggunakan bahasa php.	Menggunakan Ubuntu sebagai sistem operasinya, sedangkan bahasa pemrograman kami tetap menggunakan bahasa php. Ketika kami kesulitan dalam pengoperasian perangkat lunak di linux, kami akan menggunakan emulator wine.	Menggunakan OS linux dengan software open source.
<b>Perangkat lunak aplikasi</b>	Menggunakan OS Ubuntu untuk mengurangi pengeluaran biaya. Dan perangkat lunak yang lain, kita akan usahakan untuk menggunakan perangkat lunak open source juga.	Menggunakan OS Windows Xp dengan perangkat lunak yang free.	Menggunakan software open source untuk mengurangi biaya pajak.

Karakteristik	Alternatif 1 Seluruh fasilitas jurusan	Alternatif 2 Sebagian besar fasilitas jurusan	Alternatif 3 Sebagian kecil fasilitas jurusan
Metode pemrosesan data	Di dalam jejaring sosial study group ini menggunakan metode pemrosesan data real time, dimana informasi yang tidak penting dapat dihapus oleh admin yang telah masuk ke dalam data base.	Menggunakan metode pemrosesan data real time kemudian metode yang bertumpuk.	Menggunakan metode pemrosesan data secara real time saja.
Alat output	Alat output yang digunakan adalah layar monitor serta bentuk print out bila diinginkan, seperti melakukan print out materi atau report nilai.	Alat output yang digunakan hanya layar monitor dan printer.	Alat output yang digunakan adalah printer.
Alat input	Penginputan dilakukan oleh user dengan menggunakan keyboard, mouse, dll secara online. Sedangkan waktu dalam penginputan data adalah setiap saat	Alat input yang digunakan berupa keyboard, mouse.	Alat input yang digunakan berupa keyboard, mouse.
Alat penyimpan data	Data yang perlu disimpan dan akan memiliki kapasitas yang cukup besar adalah data peng upload an file materi. Data-data ini akan di simpan di data base server jurusan dan kira-kira akan membutuhkan free space 320 Gb. file-file yang telah di upload	Sama-sama menggunakan hard disk tetapi disini menggunakan kapasitas hard disk yang tidak terlalu besar.	Sama-sama menggunakan hard disk tetapi disini menggunakan kapasitas hard disk yang tidak terlalu besar.

Karakteristik	Alternatif 1 Seluruh fasilitas jurusan	Alternatif 2 Sebagian besar fasilitas jurusan	Alternatif 3 Sebagian kecil fasilitas jurusan
	dosen.		

### 3.5 Identifikasi dan desain output

#### 3.5.1 Identifikasi output

Tabel 3.8 Identifikasi output

NO	Nama Laporan	Bentuk Laporan	Periode Laporan	Alat untuk Menampilkan Laporan	Pembuat Laporan	Penerima Laporan	Data/Informasi yang ditampilkan	Deskripsi Laporan
1	Laporan kelas mata kuliah	Tabel	3 bulan setelah mid semester	Monitor, printer	admin	Eksekutif	Menampilkan kelas mata kuliah yang ada dalam sistem	Menampilkan data kelas dan jumlah kelas yang ada dalam sistem
2	Laporan materi mata kuliah	Tabel	3 bulan setelah mid semester	Monitor, printer	admin	eksekutif	Menampilkan materi mata kuliah beserta nama kelas mata kuliahnya	Laporan ini berisi data-data materi beserta kelas mata kuliahnya

3.	laporan file materi	Table	3 bulan setelah mid semester	Monitor, printer	admin	Eksekutif	Menampilkan file materi beserta kelas mata kuliahnya	Laporan ini berisi nama file materi dengan kelas mata kuliahnya serta nama title dari file.
4.	Laporan wall mind	Tabel	3 bulan setelah mid semester	Monitor, printer	admin	eksekutif	Menampilkan data wall mind	Laporan ini berisi wall mind disertai nama user dan kelas mata kuliah
5	Laporan respon	tabel	3 bulan setelah mid semester	Monitor, printer	admin	eksekutif	Menampilkan data wall mind	Laporan ini berisi respon pada wall mind

### 3.5.2 Desain output

1. Desain output halaman utama



Gambar 3.4 Desain halaman utama

- a) Desain output halaman admin



Gambar 3.5 Desain halaman admin

- b) Desain Output laporan kelas mata kuliah

The screenshot shows a web interface titled "Study Group" at the top, with "Menu" and "Logout" links. Below the title, it says "Kelas Room". A table is displayed with columns: No, Kode, Nama, and Aksi. There are two rows of data: Row 1 has "1" in the No column; Row 2 has "2" in the No column.

No	Kode	Nama	Aksi
1			
2			

Gambar 3.6 Desain Output laporan kelas mata kuliah

- c) Desain output Laporan materi mata kuliah

The screenshot shows a web interface titled "Study Group" at the top, with "Menu" and "Logout" links. Below the title, it says "Materi". A table is displayed with columns: No, Kode, Nama, Judul, and Aksi. There are two rows of data: Row 1 has "1" in the No column; Row 2 has "2" in the No column.

No	Kode	Nama	Judul	Aksi
1				
2				

Gambar 3.7 Desain output laporan materi mata kuliah

- d) Desain output laporan file materi



No	Nama	Judul	Path	Aksi
1				
2				

Gambar 3.8 Desain output laporan file materi

- e) Desain output laporan *wall mind*



No	Nama	Materi	Isi	Aksi
1				
2				

Gambar 3.9 Desain output laporan *wall mind*

- f) Desain menu respon pada halaman admin

No	Nama	Wall	Respon	Aksi
1				
2				

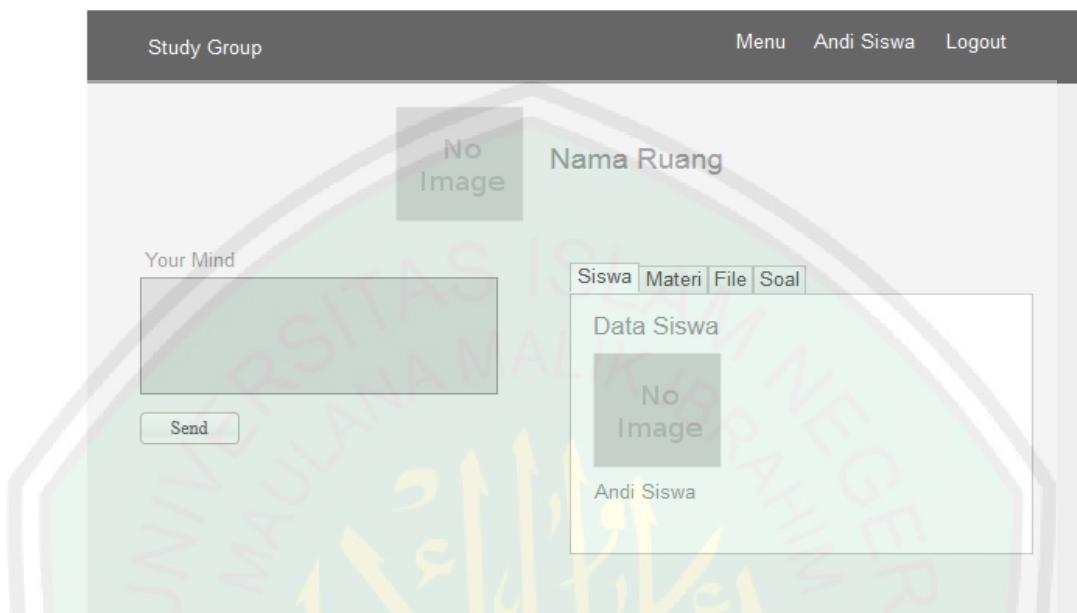
Gambar 3.10 Desain menu respon pada halaman admin

- g) Desain output home pada halaman mahasiswa



Gambar 3.11 Desain *home* mahasiswa

- h) Desain output ruang kelas pada halaman mahasiswa



Gambar 3.12 Desain Ruang kelas mahasiswa

- i) Desain output chat pada halaman *chat group*



Gambar 3.13 Desain *Chat Group*

### 3.6 Identifikasi dan desain input

#### 3.6.1 Identifikasi input

Tabel 3.9 Identifikasi output

No	Nama Input	Alat Untuk mengentri data	Bentuk input	Menyediakan Data	Mengentri Data	Periode Input	Deskripsi Input	Data/ informasi yang dientrikan
1	Formulir pendaftaran	Keyboard, scanner	Text, gambar	mahasiswa, dosen	mahasiswa, dosen	ketika pertama kali ingin mengakses web	Pengisian formulir pendaftaran user dientrikan melalui keyboard oleh mahasiswa dan dosen demikian juga untuk gambar avatar diinputkan oleh mahasiswa	nama, email, password, kode ruang kelas ajar
2	form isi materi	Keyboard	Text	dosen	dosen	ketika materi sebelumnya telah selesai disampaikan	Materi ini akan diinputkan oleh dosen sesuai dengan silabus yang ada.	data materi
3	wall mind	Keyboard	Text	mahasiswa, dosen	mahasiswa, dosen	setiap saat	wall mind diinputkan sesuai kehendak user.	-
4	soal-soal	Keyboard	Text	mahasiswa, dosen	mahasiswa, dosen	setiap saat	soal-soal ini diinputkan oleh dosen sesuai jenis soal dan jawaban akan diinputkan oleh mahasiswa	data soal dan jawaban

No	Nama Input	Alat Untuk mengentri data	Bentuk input	Menyediakan Data	Mengentri Data	Periode Input	Deskripsi Input	Data/ informasi yang dientrikan
5	nilai	Keyboard	Text	dosen	dosen	setiap saat	nilai diinputkan oleh dosen sesuai dengan hasil jawaban	data nilai



### 3.6.2 Desain Input

Desain input merupakan desain *interface* input pada program dengan alur sistematika dari *user* menuju sistem. Adapun desain input yang dibuat yaitu:

1. Desain input Registrasi dosen

The screenshot shows a registration form for faculty members. The title bar includes 'Study Group' and 'Registrasi Login'. The main form is titled 'Form Pendaftaran Dosen'. It contains four input fields: 'Nama Dosen' (Name), 'Email', 'Password' (with masked input), and 'Kode ruangan' (Room Code). Below these fields is a 'Proses' (Process) button.

Gambar 3.14 Desain input registrasi dosen

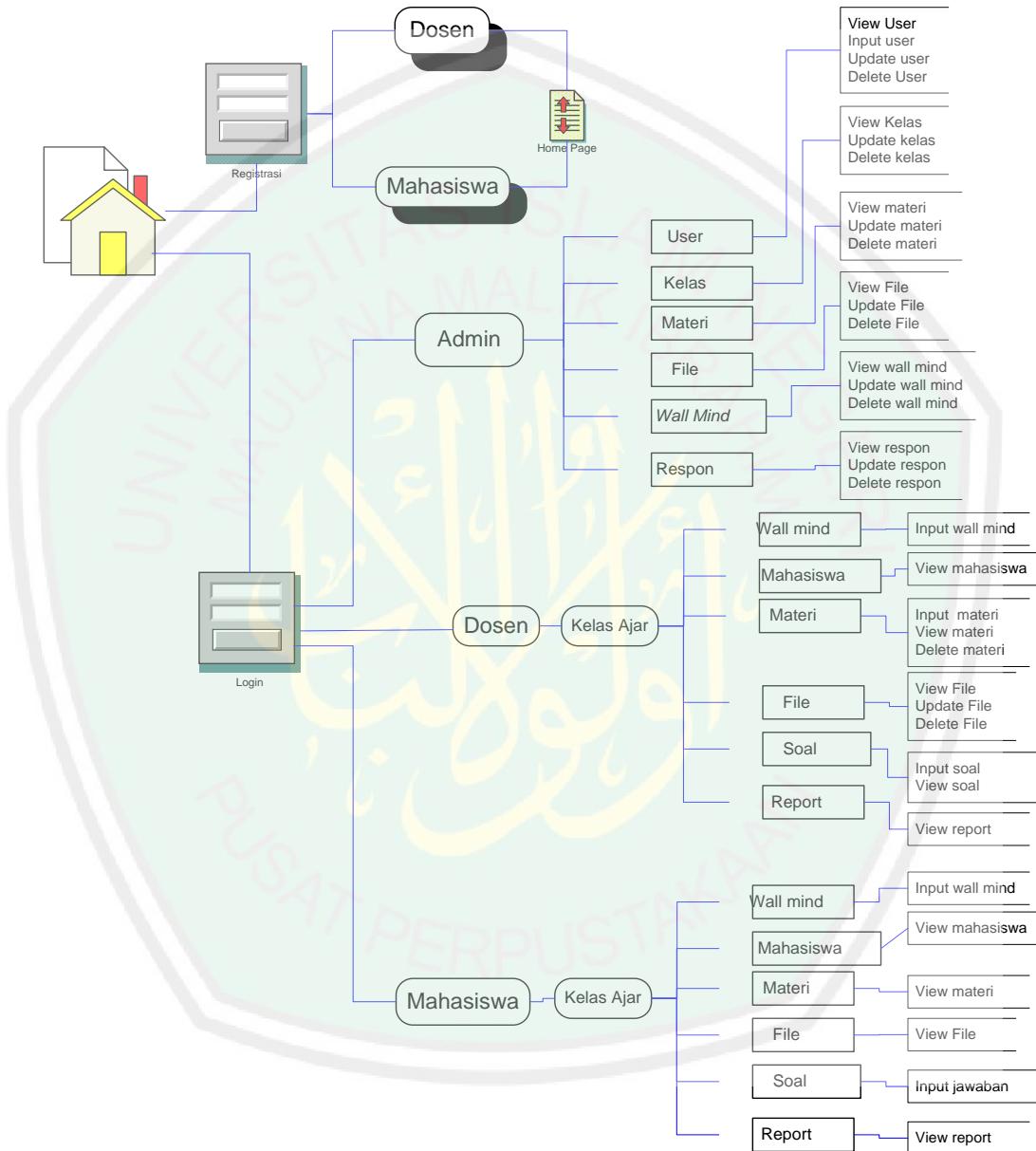
2. Desain input Registrasi Mahasiswa

The screenshot shows a registration form for students. The title bar includes 'Study Group' and 'Registrasi Login'. The main form is titled 'Form Pendaftaran Mahasiswa'. It contains three input fields: 'Nama' (Name), 'Email', and 'Password' (with masked input). Below these fields is a 'Proses' (Process) button.

Gambar 3.15 Desain input registrasi mahasiswa

### 3.7 Desain Perancangan Sistem

#### 3.7.1 Sitemap

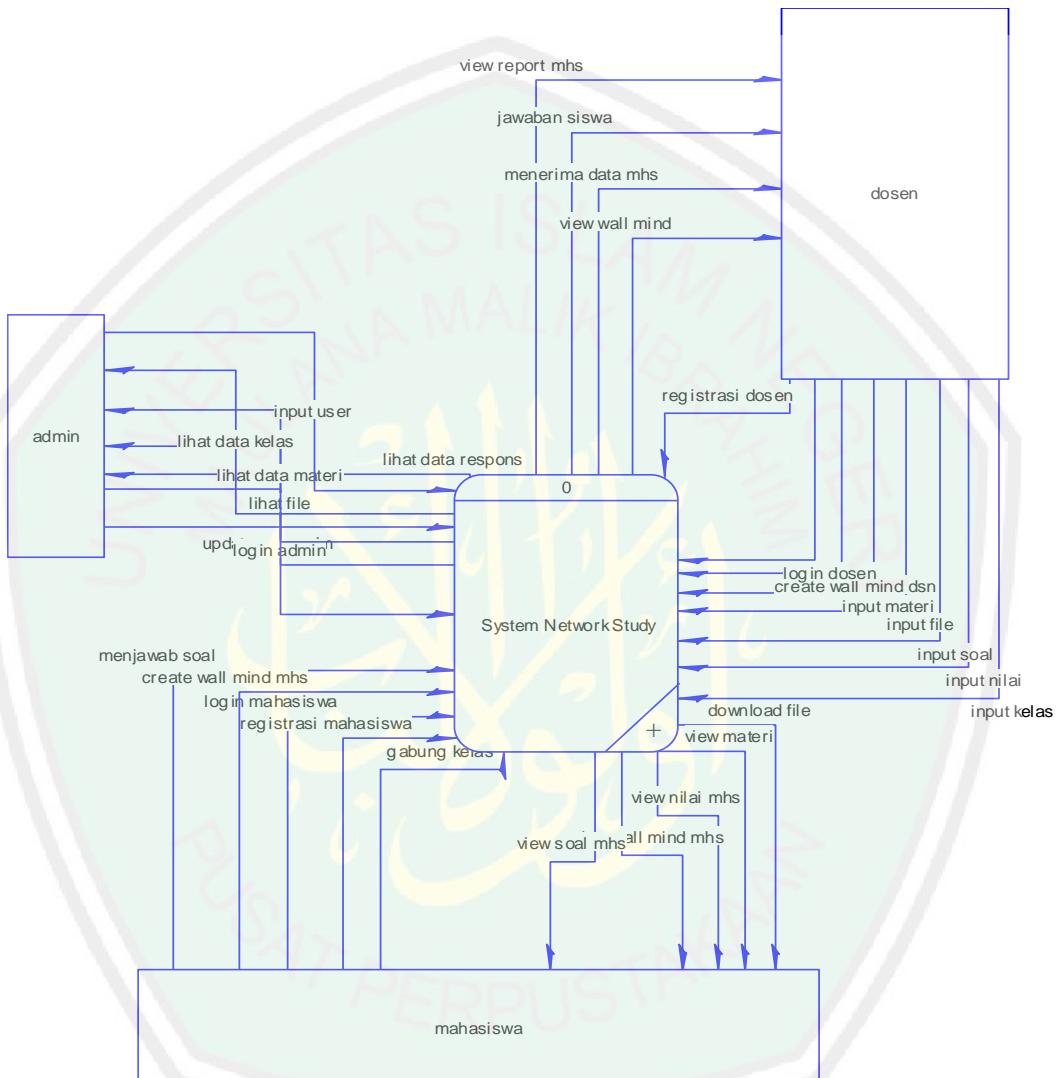


Gambar 3.16 Sitemap

#### 3.7.2 Konteks Diagram atau *Context Diagram*

*Context Diagram* adalah aliran yang memodelkan hubungan antara sistem dengan entitas dan menggambarkan ruang lingkup suatu sistem secara garis

besarnya. Aliran dalam diagram konteks memodelkan masukan ke dalam sistem dan keluaran dari sistem.



Gambar 3.17 Context diagram

Lihat Gambar 3.17 data *context diagram* dalam aplikasi Jejaring sosial *study group* sebagai media belajar mahasiswa. *Context Diagram* menerangkan bahwa arus data dalam aplikasi Jejaring sosial *study group* sebagai media belajar mahasiswa melibatkan tiga buah entitas, yaitu :

1. Admin adalah user yang mengelola sistem aplikasi Jejaring sosial *study group*. Pada entitas admin terdapat lima belas aliran data menuju ke

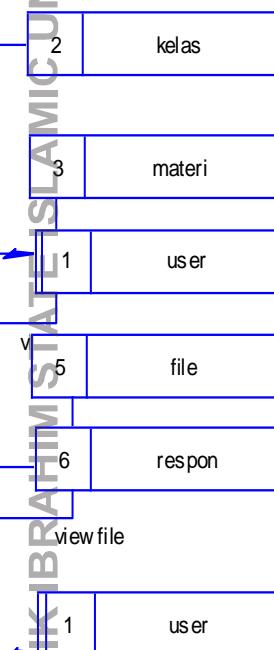
sistem, yaitu login, admin, input *user*, perbarui *user*, hapus *user*, perbarui kelas, hapus kelas, perbarui materi, hapus materi, perbarui file, *hapus* file, perbarui *wall*, hapus *wall*, perbarui respon, hapus respon. kemudian ada tujuh aliran data dari dari sistem menuju admin, yaitu lihat *user*, lihat kelas, lihat materi, data *file*, lihat soal, lihat *wall mind*, data respon.

2. *User* Dosen merupakan pengguna dari aplikasi jejaring sosial *study group*. Dalam entitas *user* dosen terdapat delapan belas aliran data, dimana sebelas aliran data dari *user* menuju ke sistem, yaitu daftar, login dosen, membuat kelas, input *wall mind*, input respon, input materi kuliah, input soal, *upload* file materi, memberi nilai, *edit* biodata dosen, input *chat*. Sedangkan delapan aliran data dari sistem menuju ke user dosen yaitu lihat *wall mind*, lihat anggota mahasiswa, lihat materi, lihat file, lihat soal, lihat *report*, lihat respon *chat*.
3. User mahasiswa adalah pengguna dari aplikasi Jejaring sosial *study group*. Dalam entitas user mahasiswa sama seperti entitas user dosen terdapat tujuh belas aliran data, dimana delapan aliran data menuju ke sistem. Yaitu login mahasiswa, edit biodata mahasiswa, gabung kelas, input *wall mind*, jawab soal, input *chat*, input respon *wall mind*, input *chat*. Sedangkan sembilan aliran data dari sistem menuju ke *user* mahasiswa yaitu lihat anggota kelas, lihat materi, download file materi, lihat soal, lihat *report*, respon *chat*, respon *wall mind*, lihat biodata dosen, lihat biodata anggota.

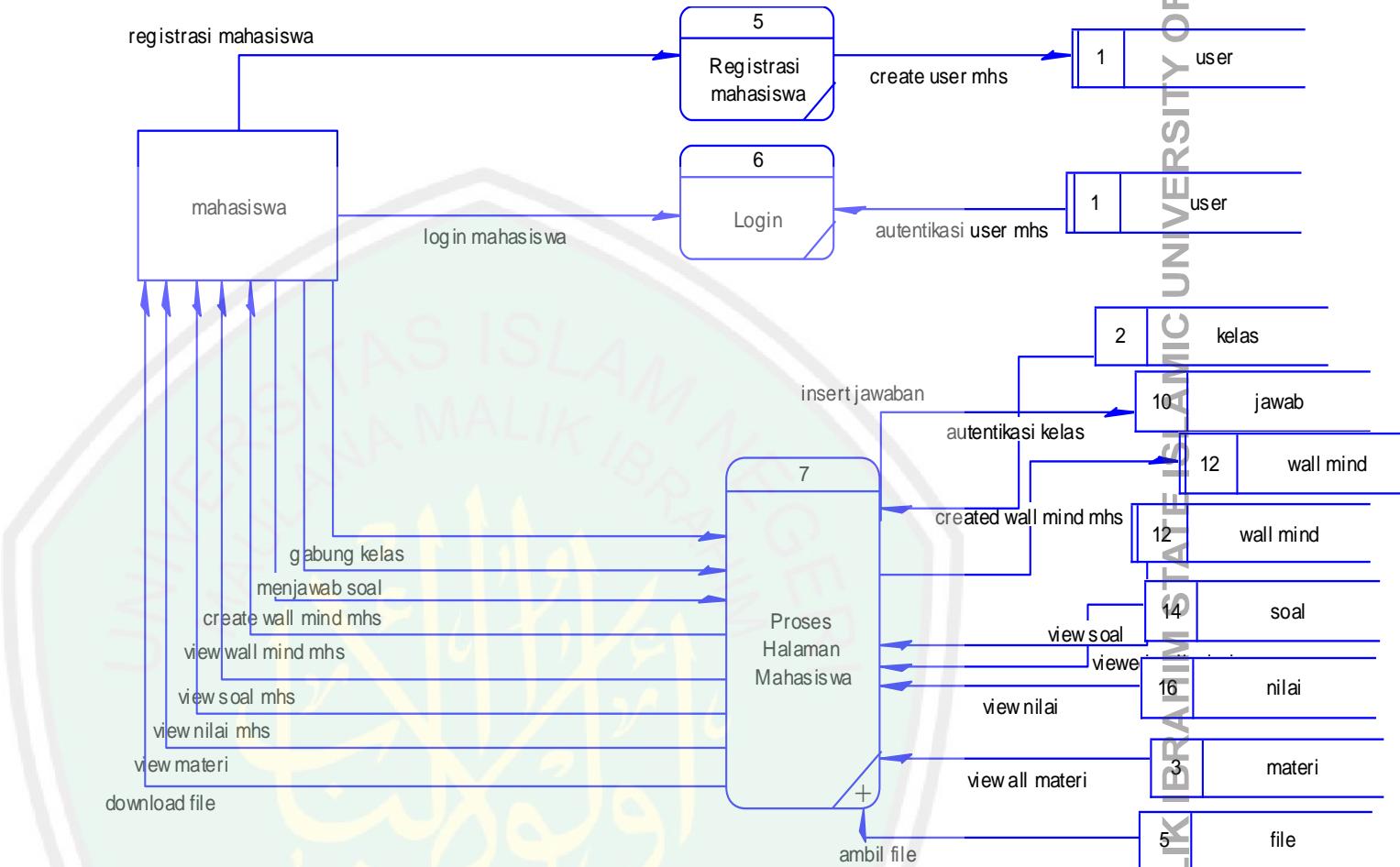
### 3.7.3 **Data Flow Diagram (DFD)**

Data Flow Diagram adalah suatu model logika data atau proses yang dibuat untuk menggambarkan asal data dan tujuan data yang keluar dari sistem, tempat penyimpan data, proses apa yang menghasilkan data tersebut, serta interaksi antara data yang tersimpan dan proses yang dikenakan pada data tersebut (Kusrini, 2007: 41).

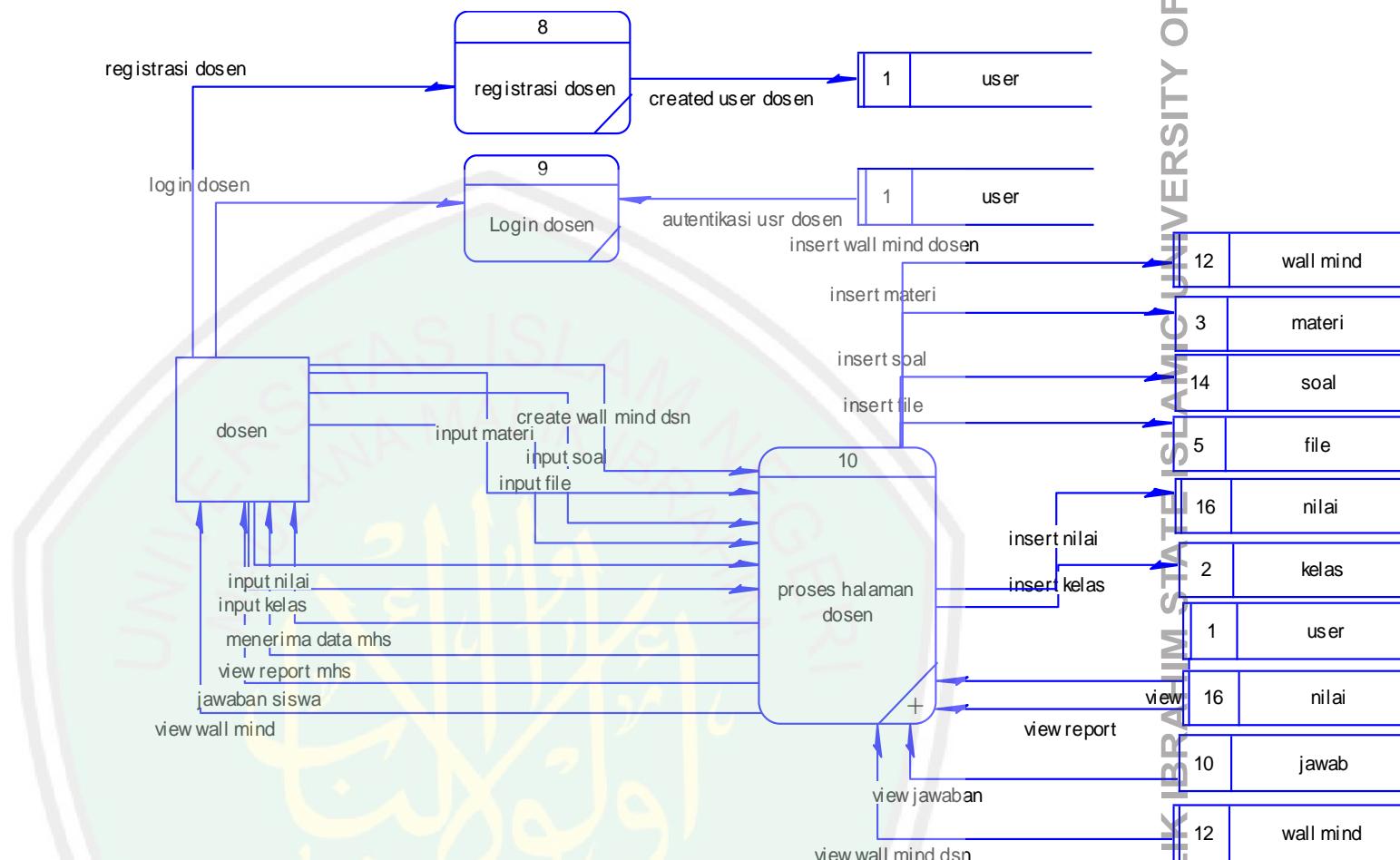
Data Flow Diagram (DFD) ini digunakan untuk menjelaskan proses yang ada pada aplikasi Jejaring sosial *study group*. Dalam aplikasi ini level DFD yang digunakan dalam menjelaskan prosesnya meliputi DFD level 1 dan DFD level 2.



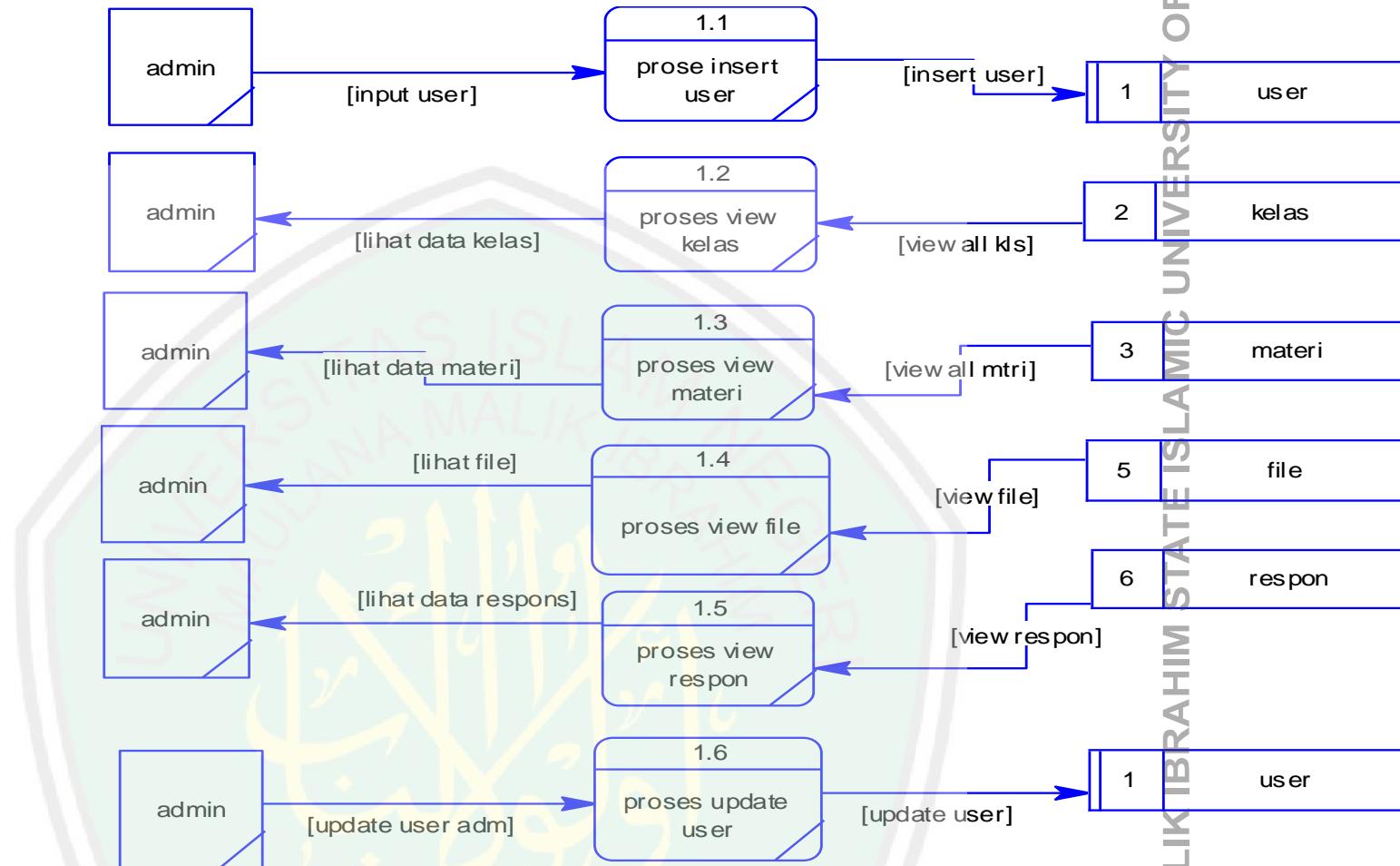
Gambar 3.18 DFD level 1 admin



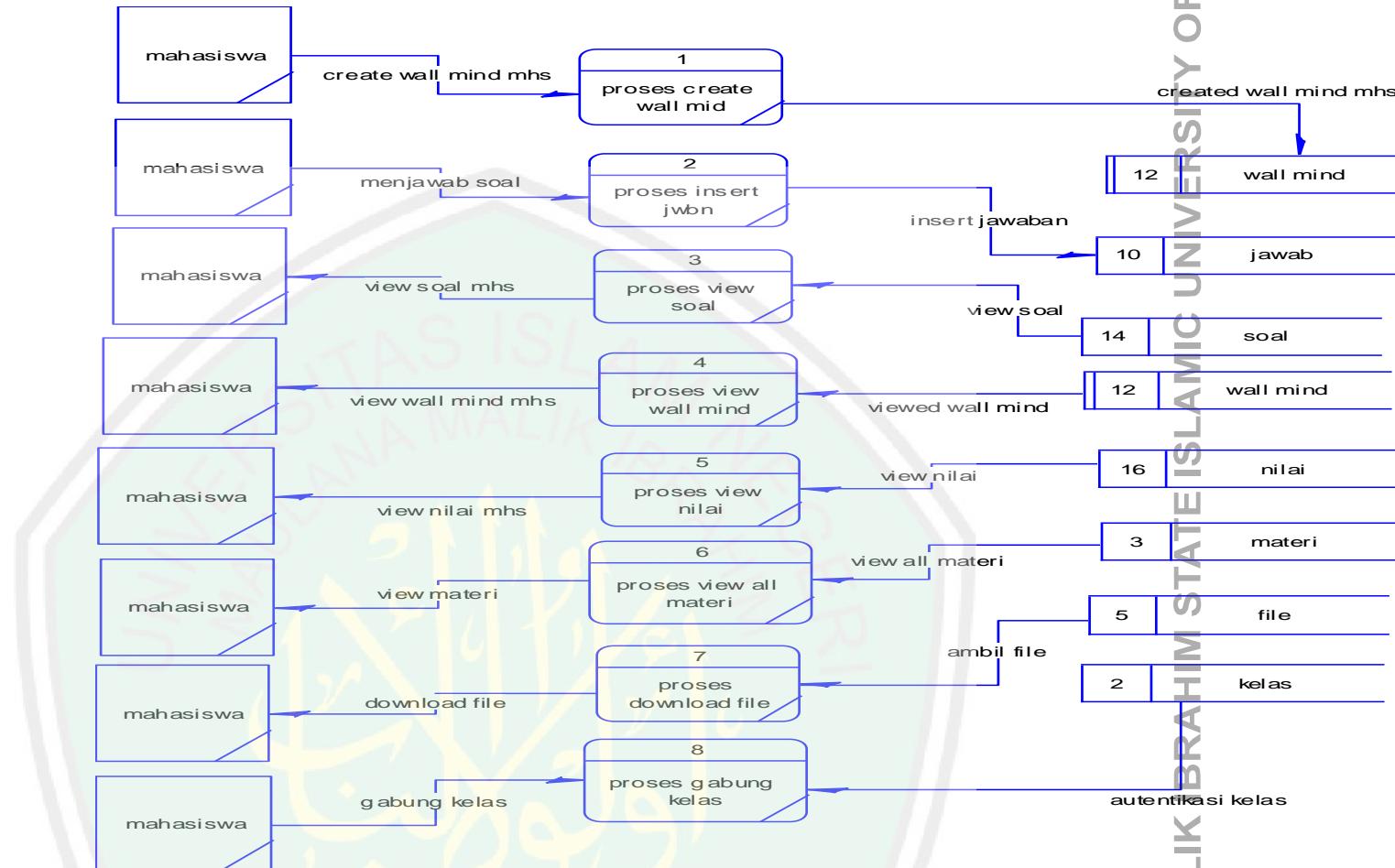
Gambar 3.19 DFD level 1 mahasiswa



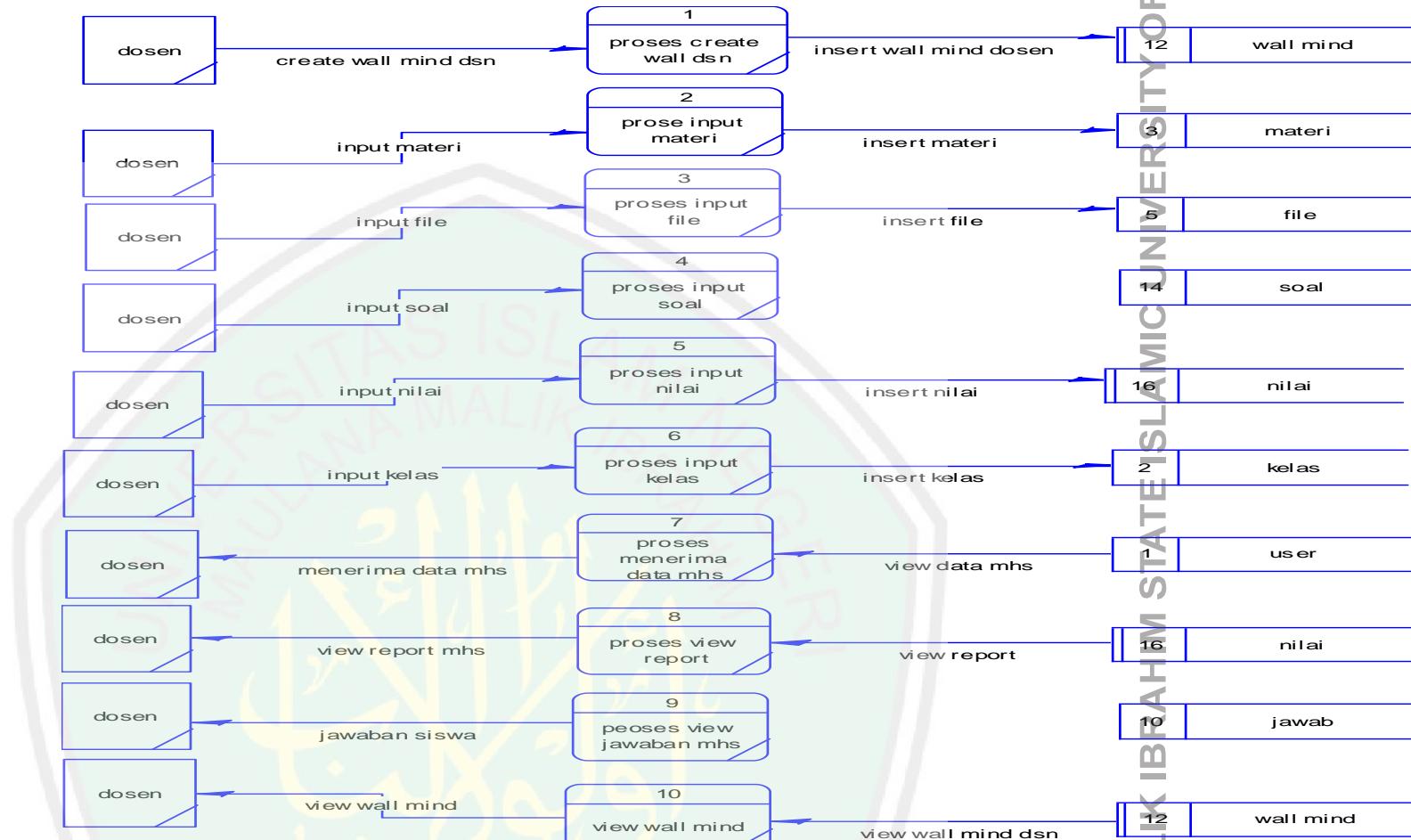
Gambar 3.20 DFD level 1 dosen



Gambar 3.21. DFD level 2 Admin



Gambar 3.22. DFD level 2 Mahasiswa



Gambar 3.23. DFD level 2 Dosen

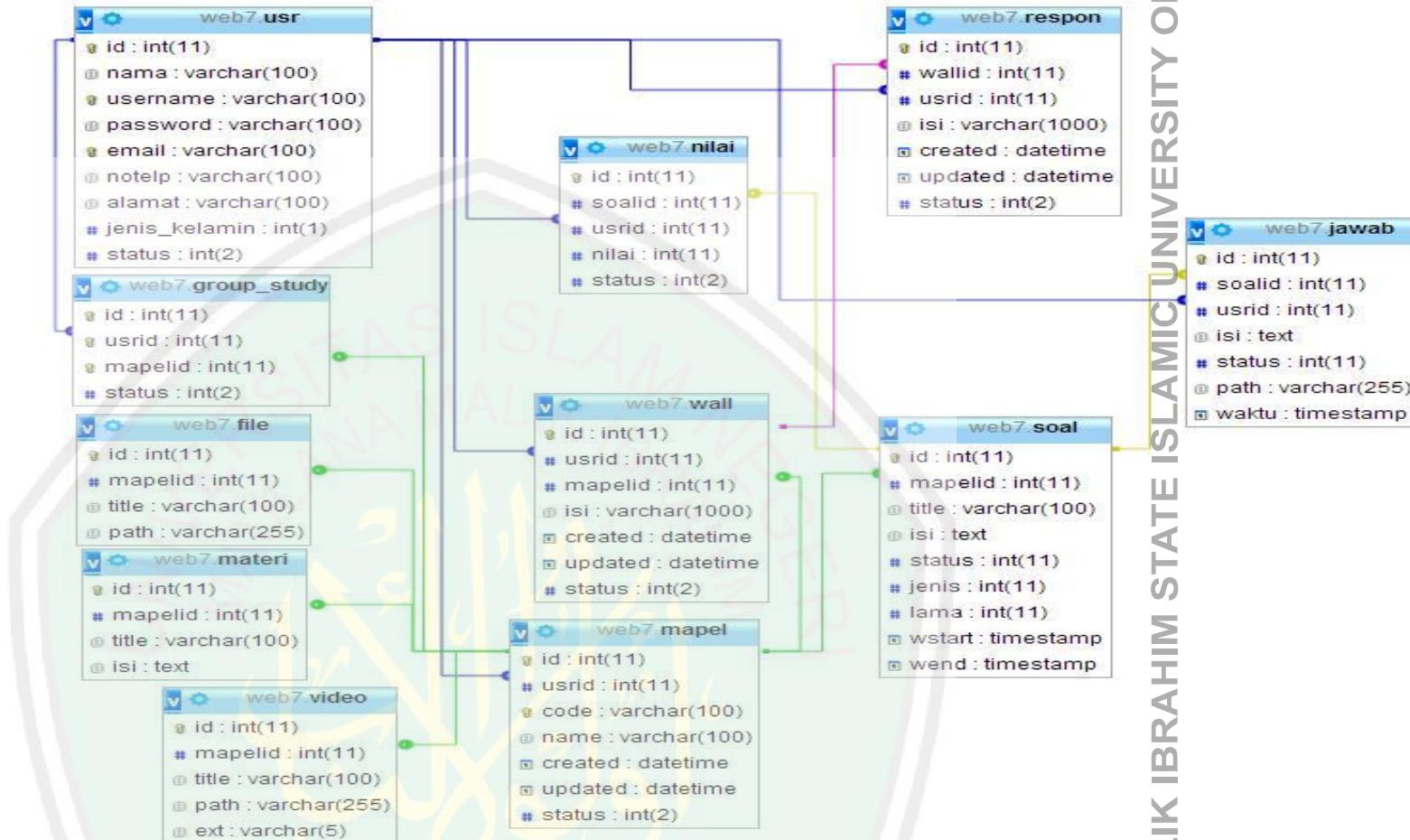
### 3.7.4 Entity Relation Diagram (ERD)

Perancangan data pada aplikasi ini digambarkan dengan ER-Diagram(*Entity Relation Diagram*). ER-Diagram adalah sebuah diagram yang menggambarkan hubungan/relasi antar Entitas (*entity*), dan setiap *Entity* terdiri atas satu atau lebih Attribut yang mempresentasikan seluruh kondisi atau fakta dari “Dunia Nyata” yang ditinjau. (Winarko,Edi, 2006: 13)

ER-Diagram juga didefinisikan suatu diagram yang digunakan untuk menggambarkan sistem database yang digunakan. ER-Diagram terdiri dari tabel, view, dan relasi. ERD digunakan untuk menunjukkan hubungan antara entity dengan database dan obyek–obyek (himpunan entitas) apa saja yang ingin dilibatkan dalam sebuah basis data dan bagaimana hubungan yang terjadi diantara objek-objek tersebut.

ERD yang berisi komponen-komponen himpunan entitas dan himpunan relasi yang masing-masing dilengkapi dengan beberapa atribut yang mempersentasikan seluruh fakta yang ditinjau dari keadaan yang nyata. Sehingga dapat digambarkan secara lebih sistematis dengan menggunakan ERD.

Gambar berikut menunjukkan hubungan-hubungan antara tabel database dengan relationship entitas yang lainnya, dimana terdapat dua puluh relasi



Gambar 3.24 ERD

### 3.7.5 Struktur Data

Basis data adalah suatu pengorganisasian sekumpulan data yang saling terkait sehingga memudahkan aktivitas untuk memperoleh informasi. (Kadir, 2003: 254).

Berikut ini adalah penjelasan mengenai struktur basis data dari file yang terdapat pada Entity Relationship Diagram. Tabel-tabel yang digunakan dalam

aplikasi ini antara lain :

1. Nama Tabel : User

Fungsi : menyimpan data user

Tabel 3.10 Tabel user

Fieldname	Datatype	Length	Description
Id	int	11	Primary key
Nama	varchar	100	
Username	varchar	100	
Password	varchar	100	
email	varchar	100	
notelp	varchar	100	
alamat	varchar	100	
jenis_kelamin	int	1	
status	int	2	

2. Nama Tabel : Group\_study

Fungsi : menyimpan data kelas

Tabel 3.11 Tabel group study

Fieldname	Datatype	Length	Description
Id	int	11	Primary key
userId	int	11	
mapelid	int	11	
status	int	2	

3. Nama Tabel : chat

Fungsi : menyimpan data chat

Tabel 3.12 Tabel chat

Fieldname	Datatype	Length	Description
Id	int	10	Primary key
author	varchar	100	
gravatar	varchar	32	
text	varchar	255	
ts	timestamp		

4. Nama Tabel : file

Fungsi : menyimpan data materi dalam bentuk file

Tabel 3.13 Tabel file

Fieldname	Datatype	Length	Description
Id	int	11	Primary key
mapelid	int	11	
title	varchar	100	
Path	varchar	100	

5. Nama Tabel : jawab

Fungsi : menyimpan data jawaban soal

Tabel 3.14 Tabel jawab

Fieldname	Datatype	Length	Description
Id	int	11	Primary key
soalid	int	11	
Userid	int	11	
isi	text		
status	int	11	
path	varchar	225	
waktu	timestamp		

6. Nama Tabel : mapel

Fungsi : menyimpan data mata kuliah

Tabel 3.15 Tabel mapel

Fieldname	Datatype	Length	Description
Id	int	11	Primary key
usrid	int	100	
code	varchar	100	
name	varchar	100	
created	datetime		
perbarui	datetime		
status	int	2	

7. Nama Tabel : materi

Fungsi : menyimpan data materi

Tabel 3.16 Tabel materi

Fieldname	Datatype	Length	Description
Id	int	11	Primary key
mapelid	int	11	
title	varchar	100	
isi	text		

8. Nama Tabel : nilai

Fungsi : menyimpan data nilai

Tabel 3.17 Tabel nilai

Fieldname	Datatype	Length	Description
Id	int	11	Primary key
soalid	int	11	
Userid	int	11	
nilai	int	11	
status	int	2	

9. Nama Tabel : respon

Fungsi : menyimpan data respon

Tabel 3.18 Tabel respon

Fieldname	Datatype	Length	Description
Id	int	11	Primary key
wallid	int	11	
Userid	int	11	
isi	varchar	1000	
created	datetime		
perbaruid	datetime		
status	int	2	

10. Nama Tabel : soal

Fungsi : menyimpan data soal

Tabel 3.19 Tabel soal

Fieldname	Datatype	Length	Description
Id	int	11	Primary key
mapelid	int	11	
title	varchar	100	
isi	text		
status	int	11	
jenis	int	11	
lama	int	11	
wstart	timestamp	1	
wend	timestamp	2	

11. Nama Tabel : user\_chat

Fungsi : menyimpan data user chat

Tabel 3.20 Tabel user chat

Fieldname	Datatype	Length	Description
Id	int	11	Primary key
Name	varchar	100	
gravatar	varchar	32	
last_activity	timestamp		

12. Nama Tabel : wall

Fungsi :menyimpan data pada wall

Tabel 3.21 Tabel wall

Fieldname	Datatype	Length	Description
Id	int	11	Primary key
userid	int	11	
mapelid	int	11	
isi	varchar	1000	
created	datetime		
perbarui	datetime		
status	int	2	

13. Nama Tabel : video

Fungsi :menyimpan data video

Tabel 3.22 Tabel video

Fieldname	Datatype	Length	Description
Id	int	11	Primary key
mapelid	int	11	
title	varchar	100	
path	varchar	225	
ext	varchar	5	

## **BAB IV**

### **HASIL DAN PEMBAHASAN**

Bab ini akan membahas tentang hasil pengujian dan analisis hasil program yang telah dibuat. Tujuan dari pengujian ini adalah untuk mengetahui apakah aplikasi yang telah dibuat sesuai dengan perancangannya. Selain itu juga untuk mengetahui detail jalannya aplikasi.

#### **4.1 Prosedur Pengujian**

Pengujian dilakukan untuk mengetahui apakah tiap-tiap fungsi yang telah dibuat bisa berjalan dengan baik. Pengujian ini dilakukan untuk mengetahui kelemahan-kelemahan dan kekurangan dari aplikasi. Pengujian yang dilakukan meliputi:

1. Pengujian halaman utama
  - a. Halaman *Login*
  - b. Halaman *Registrasi* user
2. Pengujian halaman administrator
  - a. Halaman *home* admin
  - b. Halaman managemen user
  - c. Halaman managemen ruang kelas
  - d. Halaman managemen materi
  - e. Halaman managemen File
  - f. Halaman managemen *wall mind*
  - g. Halaman managemen Respon

3. Pengujian halaman user dosen

a. Halaman *home dosen*

b. Profil

c. Kelas yang diajar

1) Mahasiswa

2) Materi

3) File

4) Report

5) Chat group

4. Pengujian halaman user mahasiswa:

a. Halaman *home* mahasiswa

b. Profil

c. Kelas yang diikuti

1) Mahasiswa

2) Materi

3) File

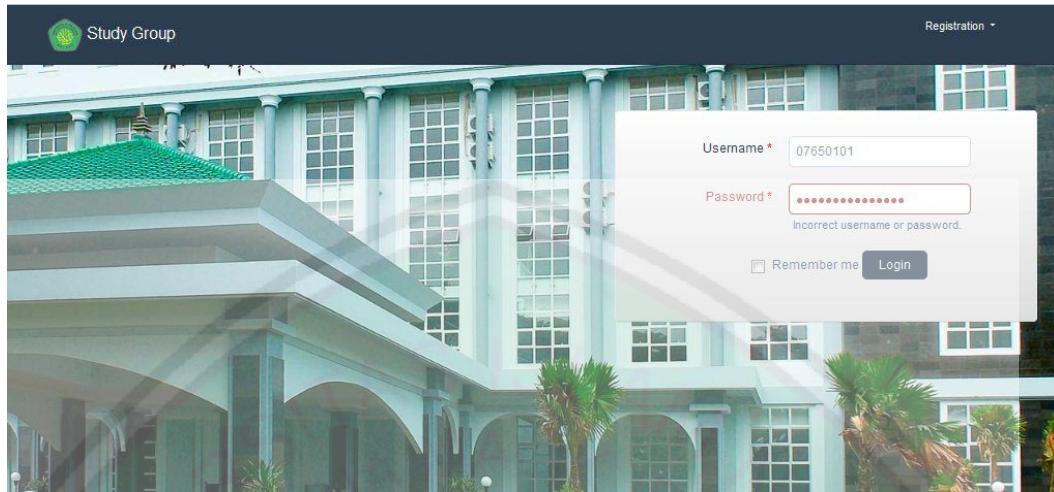
4) Report

5) Chat group

## 4.2 Pengujian Hasil dan Pembahasan

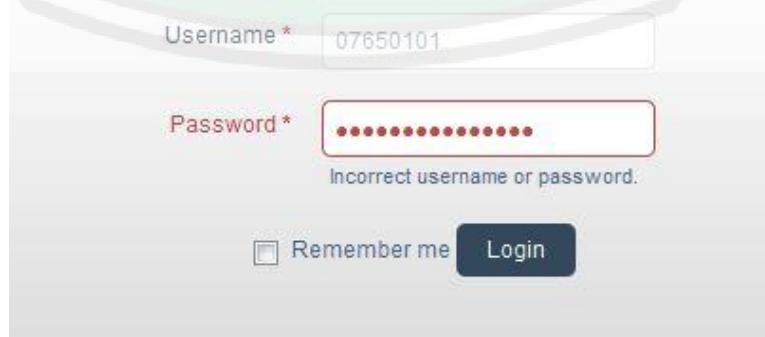
### 4.2.1 Pengujian *login* user

Pengguna saat akan menggunakan jejaring sosial untuk pembelajaran peserta didik ini , harus melakukan proses login. Berikut tampilan tatap muka halaman login:



Gambar 4.1. halaman utama jejaring sosial untuk pmbelajaran peserta didik.

Pada halaman login terdapat bagian *header* kanan yang bertuliskan “Study Group”. Kemudian di bagian *header* sebelah kiri terdapat menu registrasi yang merupakan menu yang berisi form pendaftaran user bagi pengguna *website* yang belum memiliki *account*. Sedangkan pada halaman *home* terdapat form login. Bagi pengguna yang melakukan login harus memasukkan *username* dan *password* dan tidak boleh salah satunya kosong. Jika terjadi pengguna salah memasukkan salah satu *username* atau *password* dan menekan enter atau tombol *login*, maka sistem akan memberikan peringatan sebagaimana gambar berikut:



Gambar 4.2 Peringatan ketika pengguna salah memasukkan *username* atau

*password*

Apabila data *username* dan *password* yang dimasukkan oleh pengguna benar dan cocok dengan data pengguna yang ada dalam *database* maka sistem akan langsung mengarahkan pengguna ke halaman utama berdasarkan jenis user, tetapi jika data yang dimasukkan tidak cocok dengan data yang ada dalam *database* maka pengguna akan diarahkan kembali ke halaman *login*.

Adapun *script* yang digunakan halaman admin ini:

```
public function authenticate($attribute,$params)
{
    if (!$this->hasErrors())
    {
        $this->_identity=new UserIdentity($this->username,
                                            $this->password);
        if (!$this->_identity->authenticate())
            $this->addError('password','Incorrect username or
                                password.');
    }
}
```

#### 4.2.2 Pengujian Halaman User dosen

Pengguna yang berhasil melakukan proses login akan langsung diarahkan oleh sistem untuk memasuki *home* sesuai dengan jenis user. Adapun menu-menu yang ada pada halaman utama untuk detail tampilan dan pengujinya adalah sebagai berikut:

- Halaman *home dosen*

Halaman *home dosen* berisi data diri dosen atau profil dosen serta kelas apa saja yang diajar.adapaun *interfacenya* adalah sebagai berikut:



Gambar 4.3 Halaman utama user dosen

Halaman diatas berisi tampilan profile dosen yang meliputi nama, *username*, Email, nomer telp, dan alamat lengkap dosen disertai dengan avatar dosen. kemudian disebelah kanan berisi daftar kelas yang diajar. Jika ingin memasuki kelas ajar, maka user dosen mengklik *button* masuk pada daftar kelas ajar yang dikehendaki.

#### b. Halaman kelas ajar

Halaman ini berfungsi untuk mengkomunikasikan kegiatan belajar mahasiswa dengan dosen. Melalui hak akses sebagai dosen, dosen berhak menentukan kode kelas ajar kemudian diberikan kepada mahasiswa yang ingin bergabung dalam kelas sehingga bagi user asing yang tidak mengetahui kode kelas, tidak akan dapat bergabung dengan kelas yang sudah dibuat dan kode kelas yang dibuat tidak akan sama.Gambar 4.4

### Mengikuti Group Study

Code \*  "ggggg" tidak terdaftar.

Gambar 4.4 Membuat kelas

Adapun *Script* yang digunakan dalam membuat kelas ini:

```
public function codeuniq($attribute, $params) {
    if (! $this->hasErrors ()) {
        $count = Mapel::model ()->count ( array(
            'condition' => 'code=:code',
            'params' => array (
                ':code' => strtoupper ($this->code)
            ) ) );
        if ($count > 0) {
            $this->addError ( 'code', 'Code
ruangan already exists.' );
        }
    }
}
```

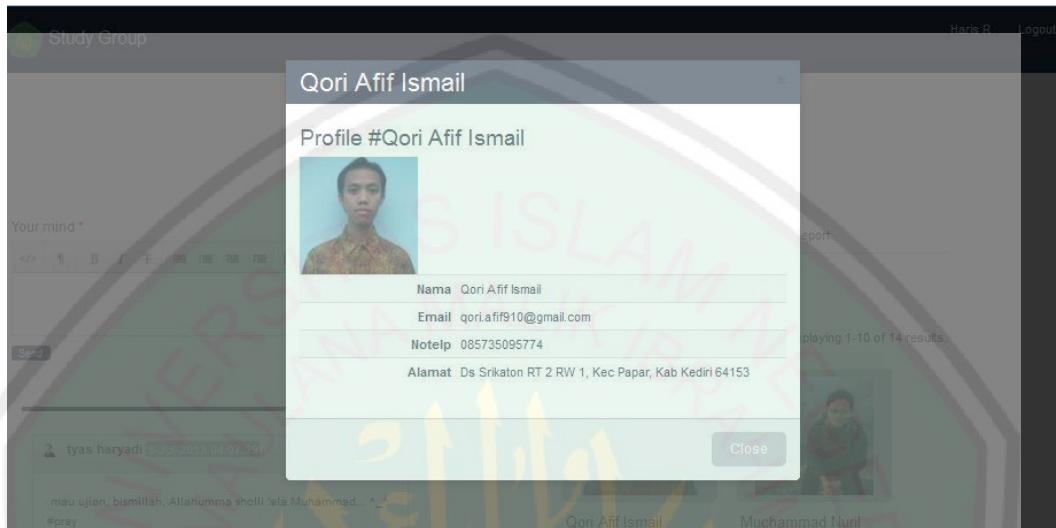
a) Submenu Mahasiswa

Pada halaman ini, dosen dapat melihat mahasiswa yang ikut dalam kelas tersebut:

The screenshot shows a web interface for a 'Study Group'. At the top, there's a navigation bar with 'Study Group', a user profile icon, 'Haris R', and 'Logout'. Below the navigation bar, there's a banner featuring a suit icon and the text 'Etika Profesi'. On the left side, there's a text input field with a toolbar above it, labeled 'Your mind \*'. Below the text input is a 'Send' button. In the center, there's a section titled 'Data Mahasiswa' with the sub-label 'Displaying 1-10 of 14 results.'. Two student profiles are visible: 'Qori Afif Ismail' and 'Muhammad Nuril', each with a small profile picture and their names below it.

Gambar 4.5 Daftar mahasiswa yang ikut dalam kelas

ketika salah satu *icon* data mahasiswa yang ikut dalam kelas diklik,maka akan tampil informasi yang berhubungan dengan data diri mahasiswa. Gambar 4.6



Gambar 4.6 Data informasi profil mahasiswa

Adapun *Script* yang digunakan dalam menampilkan informasi profil diatas:

```
<div id="modal<?php echo $this->page->m->usrid; ?>">
    class="modal hide fade" tabindex="-1" role="dialog"
    aria-labelledby="myModalLabel" aria-hidden="true">
        <div class="modal-header">
            <button type="button" class="close" data-
                dismiss="modal" aria-
                hidden="true">&times;</button>
        <h3 id="myModalLabel"><?php echo $ug->nama; ?></h3>
        </div>
        <div class="modal-body">
            <?php $this->renderPartial('lihat', array('model' =>
                $ug)); ?>
        </div>
        <div class="modal-footer">
            <button class="btn" data-
                dismiss="modal">Close</button>
        </div>
    </div>
```

b) Submenu Materi

Memberi materi baik berupa teks tunggal, seperti pada gambar 4.7

The screenshot shows a web-based study group interface. At the top, there's a header with a user icon, 'Study Group', 'Haris R', and a 'Logout' button. Below the header, there's a large text input area with a toolbar above it. To the right of the text area, there's a section titled 'Etika Profesi' with a small profile picture of a person in a suit. Below this, there's a navigation bar with tabs: 'Mahasiswa', 'Materi' (which is currently selected), 'File', 'Soal', and 'Report'. Under the 'Materi' tab, there's a heading 'Data Materi' with a '+' button. A message 'Displaying 1-6 of 6 results.' is shown above a list of three items: 1. ETIKA, 2. ETIKA KOMPUTER, and 3. PROFESI & PROFESIONAL. Each item has a small edit and delete icon next to it.

Gambar 4.7 Submenu materi

c) Submenu file

Pada submenu ini, dosen mengupload materi berupa file dalam bentuk Pdf, doc, docx, ppt,xls. Gambar 4.8

Tambah File

This is a screenshot of a 'Tambah File' (Add File) form. The title 'Tambah File' is at the top. Below it is a 'Title \*' field with a placeholder 'Title \*'. Underneath is a 'File' input field with a 'Browse...' button to its right. At the bottom of the form are two buttons: 'Proses' (Process) in a dark blue box and 'Cancel' in a light blue box.

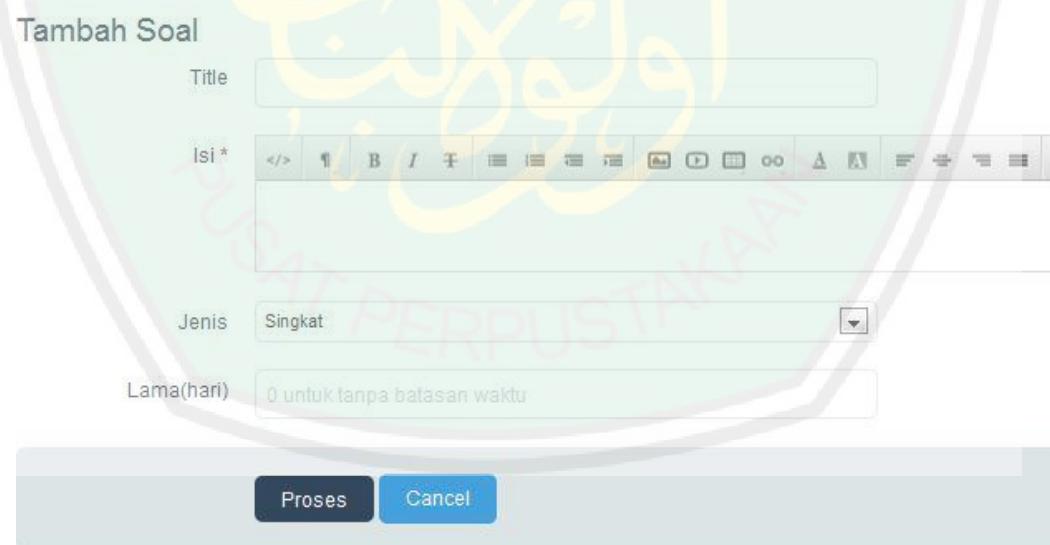
Gambar 4.8 Submenu file

Adapun *Script* yang digunakan dalam memproses file upload:

```
class FileForm extends CFormModel {
    public $title;
    public $file;
    public function rules() {
        return array (
            array ('title', 'required' ) ,
            array('file', 'file',
                'types'=>'doc,docx,pdf,ppt,xls'),
        );
    }
}
```

d) Submenu Soal

Pada menu ini dosen memberikan soal-soal dalam hal ini sudah ditentukan jenis soal maupun batas waktu penggerjaan( dalam hari). Adapun interfacenya pada gambar 4.9



Gambar 4.9 Interface upload soal

e) Submenu *Report*

Report dihasilkan dari proses penilaian dosen dari jawaban-jawaban soal yang telah diberikan sebelumnya. adapun proses pemberian nilai secara manual yang

diinputkan kedalam sistem. Report hasil penggeraan soal berupa nilai yang akan tampil dalam *submenu report*. Tampil pada user dosen berdasarkan nilai keseluruhan mahasiswa.



Gambar 4.10 Report

Adapun *Script* yang digunakan untuk menampilkan data diatas:

```

foreach ($ns as $v) {
    if ($i1!=0) {
        $a.=',';
        $b.=',';
    }
    $a.= "'". $v->soal->title."'";
    $b.=$v->nilai;
    $i1++;
}
$c.='{
    fillColor : "rgba('.(140+$i0).',187,205,0.5)",
    strokeColor : "rgba('.(140+$i0).',187,205,1)",
    pointColor : "rgba('.(140+$i0).',187,205,1)",
    pointStrokeColor : "#fff",
    data : ['. $b.']
}';

}
else{
    $ns=Nilai::model()->with(array('soal'=>array('condition
        '=>'mapelid=:s','params'=>array(':s'=>$m-
>id)))->findAllByAttributes(array('usrid'=>$v-
>usrid));
    $b.='';
}

```

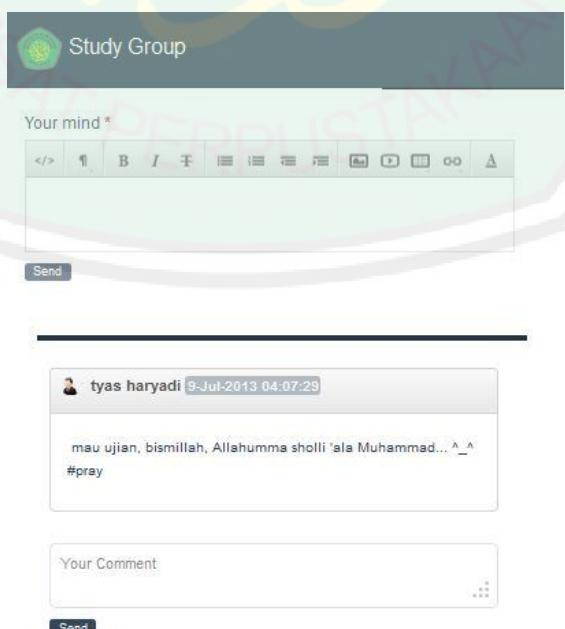
```

$il=0;
foreach ($ns as $v) {
    if ($il!=0) {
        $b.=',';
    }
    $b.=$v->nilai;
    $il++;
}

```

f) *Wall mind*

*Wall mind* merupakan fitur yang dibuat sebagai media mengirim dan merespon kiriman orang lain melalui halaman dinding kelas ajar yang dapat dilihat user lain yang tergabung dalam kelas ajar dan user lain dapat merespon kiriman pesan tersebut dengan komentar baik tema diskusi atau hanya sekedar menuangkan isi pikiran. *Wall mind* dapat *mereload* otomatis ketika user *update* status sehingga ketika halaman user lain yang sedang *login* dalam grup kelas, maka akan otomatis *refresh* dan langsung tampil status yang telah dibuat oleh user lain tersebut. Gambar 4.11



Gambar 4.11 *Wall mind*

### g) Chat

Chat disini merupakan submenu yang dibuat untuk komunikasi langsung pada saat user online dalam kelas ajar. Ketika user dalam keadaan online kemudian ada user lain yang online, user akan dapat berkomunikasi lebih cepat baik itu berupa diskusi langsung ataupun hanya sekedar menyapa. Gambar 4.12



Gambar 4.12 *Chat group*

Adapun *Script* yang digunakan dalam menjalankan fitur ini:

```
public function actionGetUsers($id) {
    $idUsr=Yii::app()->user->getState('id');
    if ($idUsr!=null) {
        $name=Yii::app()->user->getState('nama');

        ChatFunction::updateUserChat($name,$idUsr);
        ChatFunction::delActivity();
        //return ChatFunction::getUsrActiveChat();
        echo
    json_encode(ChatFunction::getUsrActiveChat());
    }
}
```

```
public static function delActivity(){
// del data 30 menit
$command = Yii:: app()->db ->createCommand("DELETE
FROM chat WHERE ts < SUBTIME(NOW(), '0:30:0')");
$command->execute();
// del data user 30 second
```

```

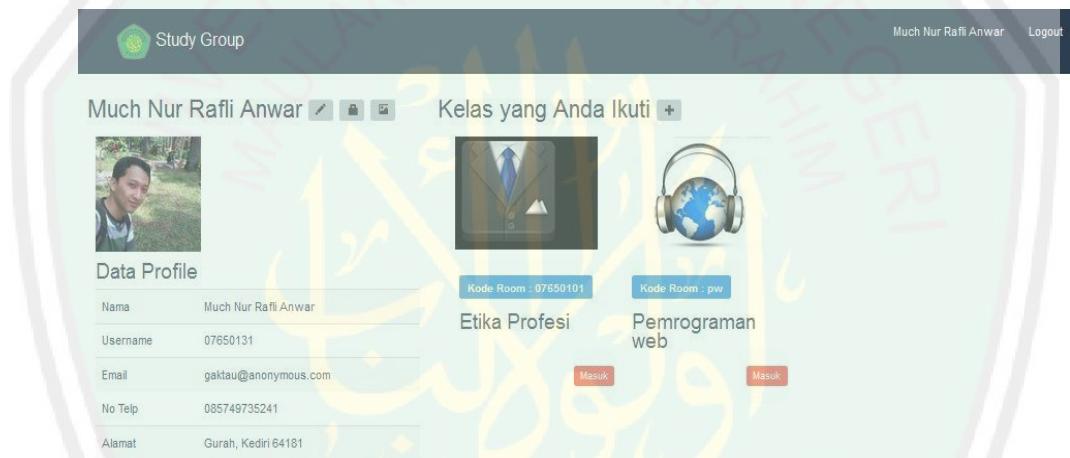
$command2 = Yii::app()->db->createCommand("DELETE
    FROM usrchat WHERE last_activity <
    SUBTIME(NOW(),'0:0:30')");
$command2->execute();
}

```

#### 4.2.3 Pengujian Halaman User Mahasiswa

##### a. Halaman *home* mahasiswa

Halaman *home* mahasiswa berisi data diri mahasiswa serta kelas apa saja yang diikuti. adapun *interfacenya* adalah sebagai berikut:



Gambar 4.13 Halaman utama user mahasiswa

##### b. Halaman kelas yang diikuti

Halaman ini berfungsi untuk mengkomunikasikan kegiatan belajar mahasiswa dengan dosen. Melalui hak akses sebagai mahasiswa, mahasiswa hanya berhak mengikuti kelas yang sudah diberikan kode kelas ajar oleh dosen. Gambar 4.14

#### Mengikuti Group Study

Code \*  "ggggg" tidak terdaftar.

Gambar 4.14 Gabung kelas

Ketika kode kelas diinputkan dan tidak terdaftar dalam database, maka akan muncul peringatan seperti pada gambar 4.14. Adapun *Script* yang digunakan dalam menjalankan peringatan tersebut:

```
public function ckcode($attribute,$params) {
    if (!$this->hasErrors()) {
        $c=Mapel::model()->
        countByAttributes(array('code'=>strtoupper($this->code)));
        if ($c==0) {
            $this->addError($attribute,"'$this->code.' tidak terdaftar.");
        }
    }
}
```

Setelah mahasiswa bergabung dengan kelas, mahasiswa tidak sepenuhnya diberikan hak akses seperti halnya dosen yaitu:

- Submenu Mahasiswa

Mahasiswa dapat melihat anggota yang telah bergabung dalam kelas seperti pada user dosen. Gambar 4.15

Gambar 4.15 View siswa

### b) Submenu Materi

Pada submenu materi, mahasiswa diberikan hak akses untuk melihat saja materi yang telah diupload oleh dosen. adapun tampilannya pada gambar 4.16 dan Gambar 4.17

The screenshot shows a web-based study group interface. At the top, there's a header with a logo, the text "Study Group", and user information "Haris R". On the right side of the header are "Logout" and other navigation links. Below the header, there's a title "Etika Profesi" with a small icon of a suit and tie. To the left of the title is a text editor window with the placeholder "Your mind \*". Below the text editor is a message from a user named "tyas haryadi" dated "3-Jul-2013 04:07:29" with the content "mau ujian, bismillah, Allahumma sholli 'ala Muhammed... ^\_^ #pray". On the right side, there's a navigation bar with tabs "Mahasiswa", "Materi" (which is currently selected), "File", "Soal", and "Report". Below the navigation bar, there's a section titled "Data Materi" with a plus sign. It displays a list of 6 results, with the first three items shown:

- 1 ETIKA
- 2 ETIKA KOMPUTER
- 3 PROFESI & PROFESIONAL

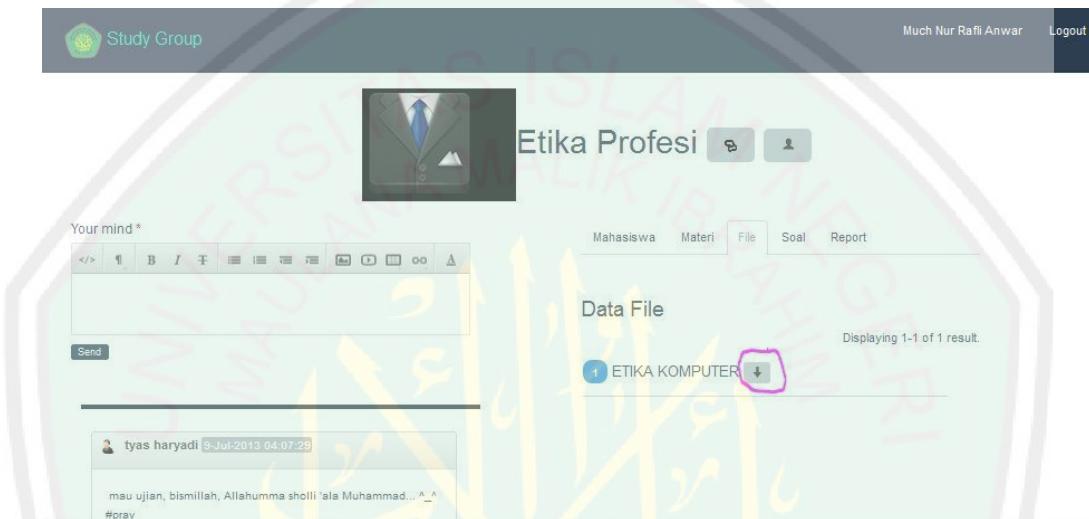
Gambar 4.16 View materi

This screenshot shows a detailed view of a material within a study group. At the top, it has the "Study Group" logo and user information "Much Nur Rafi Anwar" and "Logout". Below the header, there's a section titled "PROFESI & PROFESIONAL". Under this section, there are two main headings: "Profesi dan Profesional" and "Manusia dan Kebutuhannya". Under "Profesi dan Profesional", there's a quote by Abdulkadir Muhammad: "Abdulkadir Muhammad (2001) Mengklasifikasikan kebutuhan manusia sebagai berikut :  
a. Kebutuhan ekonomi ( material )  
b. Kebutuhan psikis ( non-materi )  
c. Kebutuhan biologis ( proses regenerasi )  
d. Kebutuhan pekerjaan ( kebutuhan akan status dan derajat)". Under "Manusia dan Kebutuhannya", there's a quote by Thomas Aquinas: "- Thomas Aquinas seperti dikutip Sumaryono ( 1995 ) mengatakan bahwa wujud kerja memiliki tujuan :  
a. Memenuhi kebutuhan hidup  
b. Mengurangi tingkat pengangguran dan kriminalitas  
c. Melayani sesama  
d. Mengontrol gaya hidup".

Gambar 4.17 View isi materi

c) Submenu File

Sedangkan dalam menu file,mahasiswa dapat mendownload materi dalam bentuk file yang telah diupload oleh dosen baik dalam format doc, docx, pdf, ppt, maupun format xls.



Gambar 4.18 View File Materi pada submenu File

Pada menu ini seperti halnya dosen, mahasiswa dapat melihat atau mengunduh file materi dengan mengklik button *download* pada samping kanan nama file yang tersimpan didalam *database* sistem.

d) Submenu Soal

Pada sub menu soal, mahasiswa dapat menjawab soal sesuai dengan jenis soal, jenis soal singkat dan uraian sesuai dengan waktu penggerjaan yang telah diberikan dosen. akan tetapi, dengan tujuan agar mahasiswa aktif dalam kelas ajar yang ada pada sistem, batas waktu penggerjaan tidak ditampilkan supaya mahasiswa tidak menunda waktu penggerjaan soal, dan apabila waktu penggerjaan belum selesai, mahasiswa masih diberikan hak akses untuk *update* jawaban karena Ketika waktu

pengerjaan habis, hak akses untuk menjawab atau *update* jawaban akan ditutup secara otomatis oleh sistem. Gambar 4.19

#### Jawab Soal

soal etika I

Jenis Singkat

1. jelaskan definisi dari etika?

2. sebutkan peranan etika dalam kehidupan sehari-hari ditinjau dari bidang pekerjaan yang dijalani dan sebutkan masing-masing 1 contoh dalam setiap bidang tersebut?

(Jawaban sudah tidak dapat di edit lagi)

Proses Cancel

Gambar 4.19 *Interface disable* jawaban

Terdapat 2 jenis soal yang diberikan, yaitu soal singkat dan soal uraian. Soal singkat ini merupakan jenis soal yang diberikan dosen dimana mahasiswa tidak perlu menyisipkan file untuk jawaban soal sehingga mahasiswa bisa langsung menjawab di text area yang telah disediakan sistem. Sedangkan jenis soal uraian merupakan jenis soal dimana untuk jawaban soal tersebut membutuhkan file sisipan ataupun jawaban dari soal itu harus dikerjakan dimedia lain, contohnya dalam bentuk makalah, file zip, dan lain-lainnya.

#### Jawab Soal

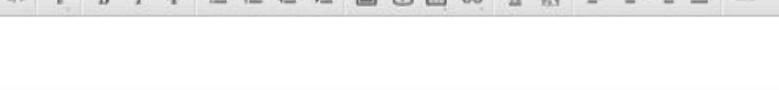
soal 1

Jenis Singkat

1. Apakah fungsi Router?

2. sebutkan komponen apa saja yang dapat digunakan didalam jaringan serta jelaskan fungsi masing-masing?

Isi

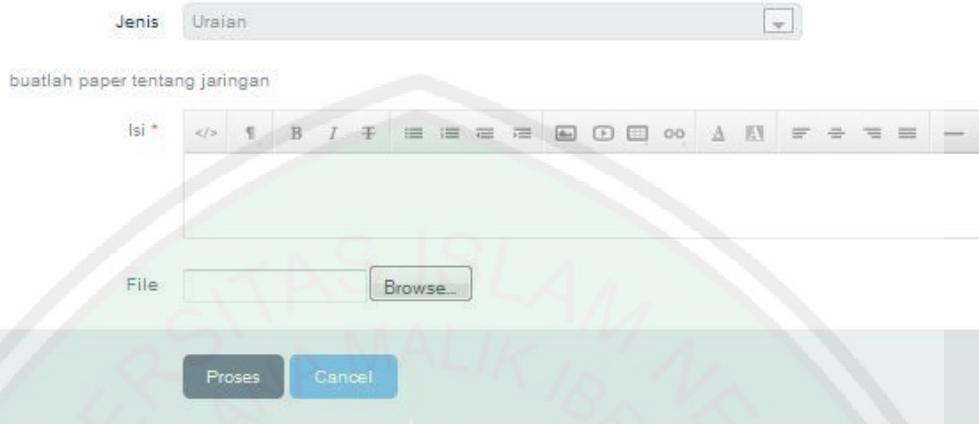


Proses Cancel

Gambar 4.20 Lembar jawaban pada soal singkat

### Jawab Soal

soal 2



Gambar 4.21 Lembar jawaban pada soal uraian

Adapun *Script* yang digunakan untuk menampilkan *interface* diatas:

```
<?php
if ($soal->jenis==2) {
    // soal + file
    if ($boleh) {
        echo $form-
>fileFieldRow($model,'file',array('class'=>'span5'));
    }
    else{
        echo $form-
            >fileFieldRow($model,'file',array('class'=>'span5',
                'disabled'=>'disabled'));
    }
}
?>
```

pemberitahuan tentang adanya soal baru yang harus dikerjakan akan ditampilkan pada home mahasiswa pada bagian daftar kelas yang diikuti sehingga mempermudah mahasiswa untuk mengetahui adanya moment penting berhubungan dengan soal yang mempengaruhi nilai mahasiswa, tanpa harus masuk satu per satu *home* kelas ajar yang diikuti. adapun tampilan nya pada gambar 4.22



Gambar 4.22 Pemberitahuan soal baru

e) Submenu *Report*

*Report* dihasilkan dari proses penilaian dosen dari jawaban-jawaban mahasiswa untuk soal-soal yang telah diberikan sebelumnya oleh dosen. adapun proses pemberian nilai oleh dosen *report* hasil penggerjaan soal berupa nilai yang akan tampil dalam *submenu report*. Tampilan pada halaman user mahasiswa tidak didasarkan pada keseluruhan nilai mahasiswa, akan tetapi didasarkan pada presentase nilai dari mahasiswa itu sendiri.



Gambar 4.23 *Report* nilai mahasiswa

#### 4.2.4 Pengujian Halaman Admin

Untuk masuk halaman admin, user langsung masuk melalui home utama dengan memasukkan *username* dan password yang telah terdaftar dan sistem akan otomatis mendeteksi jenis user.

a. Menu *Home* admin

Pada awal masuk halaman admin, pengguna akan diarahkan ke menu *home* yang berisi ucapan selamat datang.



Gambar 4.24 Tampilan menu *home* admin.

b. Menu pada admin

Menu ini berisi data-data master yang diolah pada halaman utama.

a) Submenu managemen user

Submenu ini berisi pengelolaan (*input, update, delete*) data master user.

ID	Nama	Username	Email	Notelp
1	Administrator	admin	admin@gmail.com	085649789900
3	Haris R	07650101	hrz.scorpion@gmail.com	085649789900
8	DR.Suhartono,M.Kom	super.admin@gmail.com	super.admin@gmail.com	
9	Qori Afif Ismail	07660113	qori.afif910@gmail.com	085735095774

Gambar 4.25 Tampilan menu data submenu user.

Pada submenu managemen user, administrator diberikan hak untuk input user baru apabila user yang ingin melakukan registrasi mengalami kesulitan pada halaman registrasi. Gambar 4.26

Create Usr

Fields with \* are required.

Nama \*

Username \*

Password \*

Email \*

Notelp

Alamat

Jenis Kelamin Pilih Jenis Kelamin

Create

Gambar 4.26 Input user melalui admin

#### b) Submenu kelas

Submenu ini berisi pengelolaan (*view, update, delete*) data kelas.



ID	Code	Name	
1	1	Sistem Operasi	 
2	07650101	Etika Profesi	 
3	STD	Struktur data	 
4	JAR	Jaringan	 
5	PW	Pemrograman web	 

Gambar 4.27 Tampilan menu data submenu kelas.

c) Submenu materi

Submenu ini berisi pengelolaan (*view, update, delete*) data materi.



ID	Code	Name	Title	
2	07650101	Etika Profesi	ETIKA	 
3	07650101	Etika Profesi	ETIKA KOMPUTER	 
4	07650101	Etika Profesi	PROFESI & PROFESIONAL	 
5	07650101	Etika Profesi	PROFESI DIBIDANG IT	 
6	07650101	Etika Profesi	CYBERLAW	 

Gambar 4.28 Tampilan menu data submenu materi.

Pada halaman admin ini, admin dapat melihat materi yang diinputkan oleh dosen.

Gambar 4.29

The screenshot shows a web application interface. At the top, there is a header with a logo, the text "Study Group", "Menu", and "Logout". Below the header, the title "Lihat Materi #2" is displayed. Underneath the title, there are fields for "ID" (2), "Name" (Etika Profesi), and "Title" (ETIKA). A sidebar on the right contains "Menu", "Edit Materi", "Delete Materi", and "Data Materi". The main content area contains text and a list:

**Isi**: Menurut Kamus Besar Bahasa Indonesia etika adalah:

- a. Ilmu tentang apa yang baik dan yang buruk, tentang hak dan kewajiban moral.
- b. Kumpulan asas / nilai yang berkenaan dengan akhlak.
- c. Nilai mengenai yang benar dan salah yang dianut masyarakat

Dari asal usul kata, Etika berasal dari bahasa Yunani "ethos" yang berarti adat istiadat / kebiasaan yang baik. Perkembangan etika : studi tentang kebiasaan manusia berdasarkan kesepakatan, menurut ruang dan waktu yang berbeda, yang menggambarkan perangai manusia dalam kehidupan pada umumnya. Secara etimologi etika dapat disamakan dengan moral. Moral berasal dari bahasa latin "mos" yang berarti adat kebiasaan.

Moral adalah nilai-nilai atau norma-norma yang menjadi pegangan seseorang atau suatu kelompok dalam mengatur tingkah-lakunya dalam bermasyarakat. Sebagai contoh: "Kepala Proyek Pengembangan TI di perusahaan ini tidak bermoral....." (melanggar norma-norma etis yang berlaku dalam kelompok atau organisasi). Menurut Frans Magnus Susemc (1987), "moral adalah nilai-nilai yang mengandung peraturan, perintah dan lain sebagainya yang terbentuk secara turun temurun melalui suatu budaya tertentu tentang bagaimana manusia harus hidup dengan baik". Kesimpulannya Etika = moral adalah pegangan tingkah laku didalam bermasyarakat.

Perbedaan moral dan etika:

- Moral menekankan pada cara melakukan sesuatu.

Gambar 4.29 Tampilan view materi melalui Admin halaman admin

d) Submenu file

Submenu ini berisi pengelolaan (*view, update, delete*) data file.

The screenshot shows a web application interface. At the top, there is a header with a logo, the text "Study Group", "Menu", and "Logout". Below the header, the title "Files" is displayed. Underneath the title, there is a table listing files. A sidebar on the right contains "Menu" and "Data File". The table has columns for ID, Name, Title, and Path. The data is as follows:

ID	Name	Title	Path
1	Etika Profesi	ETIKA KOMPUTER	/folder/materi/c4ca4238a0b923820d09a6f75849b.pdf
3	Pemrograman web	PHP	/folder/materi/eccbc87e4b5ce2fe28308fd9f2a7baf3.pdf
4	Pemrograman web	HTML,CSS	/folder/materi/a87ff679a2f3e71d9181a67b7542122c.pdf
5	Pemrograman web	Java Script, Frame work, Js	/folder/materi/e4da3b7fb0e2345d7772b0674a318d5.pdf
6	Pemrograman web	Modul Praktikum web	/folder/materi/1679091c5a880faf0fb5e6087eb1b2dc.pptx
7	Pemrograman web	Phpmysql,MySQL,DML	/folder/materi/8f14e45f0cea167a5a36dedd4bea2543.pptx

Displaying 1-7 of 7 results.

Gambar 4.30 Tampilan menu data submenu file

e) Submenu *wall mind*

Submenu ini berisi pengelolaan (*view, update, delete*) data *wall mind*

Study Group

Wall

Displaying 1-2 of 2 results.

ID	Nama	Name	Isi
1	Haris R	Etika Profesi	testing progres telah dilakukan..lanjutkan...!!
2	tyas haryadi	Etika Profesi	mau ujian, bismillah, Allahumma sholli 'ala Muhammad... ^_^ #pray

Gambar 4.31 Tampilan menu data *wall mind*.

f) Submenu respon

Submenu ini berisi pengelolaan respon yang diberikan pada *wall mind*.

Study Group

Wall

Displaying 1-2 of 2 results.

ID	Nama	Name	Isi
1	Haris R	Etika Profesi	testing progres telah dilakukan..lanjutkan...!!
2	tyas haryadi	Etika Profesi	mau ujian, bismillah, Allahumma sholli 'ala Muhammad... ^_^ #pray

Gambar 4.32 Tampilan menu respon.

Halaman admin dapat melihat respon yang diberikan user pada *wall mind* untuk mengatur ataupun *filter* tiap respon yang masuk. Gambar 4.34

Study Group

Lihat Wall #1

ID	1
Nama	Haris R
Group Belajar	Etika Profesi
Isi	testing progres telah dilakukan..lanjutkan...!!

Gambar 4.34 View respon melalui admin

Tabel 4.1 Rekapitulasi Responden terhadap program

No	Pertanyaan	Nilai
1	Kemudahan dalam mengakses jejaring sosial untuk pendidikan ini	8
2	Interface Aplikasi	7
3	Pemahaman terhadap fitur yang ada pada aplikasi	7
4	Aksesibilitas Grup belajar	8
5	Pemahaman dan kemudahan dalam penyampaian materi	7
6	Proses penyampaian soal dan jawaban	8
7	Report terhadap hasil tanya jawab pd soal	7
8	Aksesibilitas chat dalam grup	8
9	Kemudahan Aplikasi terhadap proses belajar	7
10	Kepuasan terhadap aplikasi jejaring sosial untuk pendidikan peserta didik ini	7

Dari hasil rekapitulasi yang dapat dilihat pada tabel diatas, didapatkan hasil bahwa kemudahan penggunaan, pemahaman fitur, manfaat sistem serta kepuasan pengguna pada aplikasi aplikasi jejaring sosial *group study* termasuk dalam kategori baik dan membantu dalam proses belajar mengajar. Dalam pemahaman tentang fitur-fitur yang disediakan aplikasi ini 8 dari responden menjawab mudah dan 7 responden menjawab biasa. Tetapi pada poin tingkat kepuasan terdapat 8 dari 15 responden menyatakan biasa. Hal itu karena tampilan program aplikasi terlalu sederhana.

Pada tahap penyampaian materi dan soal 11 dari 15 responden menunjukkan hasil bahwa program sangat membantu dalam proses belajar mengajar sehingga tidak perlu repot-repot lagi mencetak dalam bentuk *hard copy*. Akan tetapi tampilan yang sederhana kurang begitu menarik user sehingga perlu mendesain tampilan agar lebih menarik dan penambahan menu maupun fitur yang lebih lengkap sehingga user lebih tertarik lagi menggunakan aplikasi ini.

## BAB V

### PENUTUP

#### 5.1 Kesimpulan

Dari keseluruhan penulisan ini dibuat diperoleh beberapa kesimpulan yaitu:

1. Aplikasi jejaring sosial untuk pembelajaran peserta didik merupakan website pengembangan jejaring sosial untuk pendidikan yang bersifat lokal.
2. Aplikasi ini hanya bisa diakses oleh dosen dan mahasiswa yang telah melakukan registrasi dalam program aplikasi jejaring sosial *study group* saja.
3. Selain untuk menjalin komunikasi antar user dalam sebuah kelas belajar, aplikasi ini juga dapat digunakan sebagai media pembelajaran jarak jauh. mahasiswa dapat berdiskusi secara langsung dengan mahasiswa lain yang telah bergabung dalam grup pada keadaan online *live chat group* yang telah disediakan ataupun melalui *wall mind*.
4. Keunggulan aplikasi ini adalah proses pembelajaran dapat dilakukan jarak jauh, selain itu dalam aplikasi ini mahasiswa yang bersangkutan dapat melihat materi-materi dan mengerjakan soal-soal yang diberikan dosen secara langsung.

#### 5.2 Saran

Kekurangan itu milik makhluk, karena kesempurnaan hanya milik Allah SWT. Saran untuk pengembangan lebih lanjut terhadap sistem ini di masa yang akan datang, yaitu:

1. Penambahan *live streaming* memungkinkan penggunaan aplikasi lebih nyata.
2. Desain tampilan dibuat lebih menarik lagi agar mahasiswa dan dosen lebih tertarik menggunakan aplikasi ini.
3. Penambahan ujian online dan penilaian otomatis pada ujian online memungkinkan dosen memperoleh kemudahan yang lebih dalam realisasi soal ujian.

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# Engineering Analysis

using Scilab and C

Version 0.2 (August 2006)

by Hugh Jack  
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# Engineer On a Disk

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1. PREFACE .....	12
1.1 Todo -	12
1.2 Problems Not Sorted -	14
2. NUMERICAL VALUES AND UNITS .....	16
2.1 Introduction -	16
2.2 Numerical Values -	16
Constants and Other Stuff -	16
Factorial -	18
Significant Figures -	18
Scientific and Engineering Notations -	19
2.3 Complex Numbers -	20
2.4 Units and Conversions -	21
How to Use Units -	21
SI Units -	22
A Table -	23
2.5 Problems -	27
3. ALGEBRA.....	29
3.1 The Fundamentals -	29
Basic Operations -	29
Exponents -	30
Basic Polynomials -	31
3.2 Special Forms -	33
Completing the Square -	33
Newton-Raphson to Find Roots -	34
3.3 Complex Numbers -	35
3.4 Equality and Inequality -	38
3.5 Functions -	40
3.6 Special Functions -	41
Logarithms -	41
3.7 Solving Systems of Linear Equations -	42
Substitution -	43
Addition -	43
3.8 Simplifying Polynomial Expressions -	44
Partial Fractions -	44
Summation and Series -	47
3.9 Limits -	49
3.10 Problems -	49
3.11 Challenge Problems -	53
4. TRIGONOMETRY .....	55
4.1 Introduction -	55
Functions -	56
Inverse Functions -	59
Triangles -	60
Relationships -	61
Hyperbolic Functions -	63
Special Relationships -	64

Planes, Lines, etc.	65
4.2 Coordinate Systems	67
Cylindrical Coordinates	67
Spherical Coordinates	68
4.3 Problems	69
5. VECTORS	73
5.1 Introduction	73
Dot (Scalar) Product	74
Cross Product	79
5.2 Problems	81
6. MATRICES	82
6.1 Introduction	82
Basic Matrix Operations	83
Determinants	85
Transpose	86
Adjoint Matrices	86
Inverse Matrices	87
Identity Matrix	88
Eigenvalues	88
Eigenvectors	88
6.2 Matrix Applications	88
Solving Linear Equations with Matrices	88
Gauss-Jordan Row Reduction	91
Cramer's Rule	92
Triple Product	94
Gauss-Siedel	94
6.3 Problems	94
7. GRAPHING	97
7.1 Introduction	97
7.2 Graphing Functions	98
7.3 LOG Plots	99
7.4 Multiple Plots	99
7.5 Other Items of Interest	100
7.6 Problems	100
7.7 Challenge Problem	101
8. PROGRAMMING	102
8.1 Overview	102
8.2 Introduction	102
8.3 Examples	105
8.4 Summary	109
8.5 Problems	109
8.6 Challenge Problems	109
9. PERMUTATIONS AND COMBINATIONS	110
9.1 Introduction	110
9.2 Permutations	110
9.3 Combinations	111

9.4 Probability - - - - -	112
9.5 Problems - - - - -	114
10. STATISTICS .....	117
10.1 Introduction - - - - -	117
10.2 Data Distributions - - - - -	118
Histograms- - - - -	119
10.3 Discrete Distributions - - - - -	121
Normal Distribution - - - - -	121
Hypergeometric Distribution - - - - -	121
Binomial Distribution- - - - -	122
Poisson Distribution - - - - -	125
10.4 Other Distributions - - - - -	127
Polynomial Expansions-	127
Discrete and Continuous Probability Distributions - - - - -	127
10.5 Continuous Distributions - - - - -	128
Describing Distribution Centers With Numbers - - - - -	129
Dispersion As A Measure of Distribution - - - - -	130
The Shape of the Distribution - - - - -	132
Kurtosis - - - - -	133
Generalizing From a Few to Many - - - - -	134
The Normal Curve - - - - -	135
Probability plots - - - - -	136
10.6 Problems - - - - -	138
10.7 Challeneg Problem - - - - -	139
11. RELIABILITY .....	140
11.1 Introduction - - - - -	140
11.2 Component Failure Rates - - - - -	141
11.3 Serial System Reliabilty - - - - -	147
11.4 Parallel System Reliability - - - - -	148
11.5 Formal Analysis Techniques - - - - -	150
Failure Modes and Effects Analysis (FMEA) - - - - -	150
11.6 References and Bibliography - - - - -	152
11.7 Problems - - - - -	152
12. CALCULUS .....	154
12.1 Introduction - - - - -	154
12.2 Derivatives - - - - -	154
12.3 Integrals - - - - -	158
Integration Examples - - - - -	163
12.4 Vectors - - - - -	164
12.5 Numerical Tools - - - - -	166
Approximation of Integrals and Derivatives from Sampled Data - - - - -	166
Centroids and Moments of Inertia - - - - -	167
12.6 Problems - - - - -	167
13. ANALYSIS OF DIFFERENTIAL EQUATIONS .....	172
13.1 Introduction - - - - -	172
13.2 Explicit Solutions - - - - -	173

13.3 Responses - - - - -	187
First-order - - - - -	188
Second-order - - - - -	194
Other Responses - - - - -	199
13.4 Response Analysis - - - - -	202
13.5 Non-Linear Systems - - - - -	204
Non-Linear Differential Equations - - - - -	205
Non-Linear Equation Terms - - - - -	209
Changing Systems - - - - -	212
13.6 Case Study - - - - -	218
13.7 Summary - - - - -	222
13.8 Problems - - - - -	222
13.9 Problems Solutions - - - - -	227
13.10 Challenge Problems - - - - -	233
14. ELECTRICAL AND COMPUTER ENGINEERING REVIEW .....	234
14.1 Introductions - - - - -	234
14.2 Examples - - - - -	234
14.3 Summary - - - - -	234
14.4 References/Bibliography - - - - -	234
14.5 Problems - - - - -	235
14.6 Challenge Problems - - - - -	235
15. MECHANICAL ENGINEERING REVIEW .....	237
15.1 Introduction - - - - -	237
15.2 Examples - - - - -	237
15.3 Summary - - - - -	237
15.4 References/Bibliography - - - - -	237
15.5 Problems - - - - -	237
15.6 Challenge Problems - - - - -	238
16. INDUSTRIAL ENGINEERING OVERVIEW .....	240
16.1 Introduction - - - - -	240
16.2 Examples - - - - -	240
16.3 Summary - - - - -	240
16.4 References/Bibliography - - - - -	240
16.5 Problems - - - - -	240
17. PRODUCT DESIGN AND MANUFACTURING REVIEW .....	242
17.1 Introduction - - - - -	242
17.2 Examples - - - - -	242
17.3 Summary - - - - -	242
17.4 References/Bibliography - - - - -	242
17.5 Problems - - - - -	243
17.6 Challenge Problems - - - - -	243
18. BOOLEAN LOGIC DESIGN .....	245
18.1 Introduction - - - - -	245
18.2 Boolean Algebra - - - - -	245
18.3 Logic Design - - - - -	250
Boolean Algebra Techniques - - - - -	256

18.4 Common Logic Forms - - - - -	257
Complex Gate Forms - - - - -	257
Multiplexers - - - - -	258
18.5 Simple Design Cases - - - - -	259
Basic Logic Functions - - - - -	259
Car Safety System - - - - -	260
Motor Forward/Reverse- - - - -	261
A Burglar Alarm - - - - -	261
18.6 Summary - - - - -	265
18.7 Problems - - - - -	266
18.8 Problem Solutions - - - - -	271
18.9 Challenge Problems - - - - -	278
19. NUMBERS AND DATA .....	279
19.1 Introduction - - - - -	279
19.2 Numerical Values - - - - -	280
Binary - - - - -	280
Boolean Operations	283
Binary Mathematics	284
Other Base Number Systems - - - - -	288
BCD (Binary Coded Decimal) - - - - -	289
19.3 Data Characterization - - - - -	289
ASCII (American Standard Code for Information Interchange) - - - - -	289
Parity - - - - -	292
Checksums- - - - -	293
Gray Code - - - - -	294
19.4 Summary - - - - -	295
19.5 Problems - - - - -	295
19.6 Problems Solutions - - - - -	298
19.7 Challenge Problems - - - - -	301
20. TRANSFORMS .....	302
20.1 Laplace Transforms - - - - -	302
Laplace Transform Tables - - - - -	303
20.2 z-Transforms - - - - -	305
20.3 Fourier Series - - - - -	308
20.4 Problems - - - - -	308
20.5 Challenge Problems - - - - -	309
21. GEOMETRY .....	310
21.1 Introduction - - - - -	310
Inertia - - - - -	310
21.2 Problems - - - - -	329
Challenge Problems - - - - -	329
22. FINANCIAL .....	330
22.1 Introduction - - - - -	330
22.2 Problems - - - - -	337
22.3 Challenge Problems - - - - -	338
23. OPTIMIZATION.....	339

23.1	Introduction	339
23.2	System Modeling and Variable Identification	339
23.3	Cost Functions and Constraints	340
23.4	Single Variable Searches	344
	Example Problem	344
23.5	Multivariable Searches	350
	Algorithms	353
	Random Walk	353
	Gradient Decent	354
	Simplex	354
23.6	Summary	354
23.7	Problems	354
23.8	Challenge Problems	354
24.	PROJECT SCHEDULING	356
24.1	Introduction	356
24.2	Gantt Charts	356
24.3	Critical Path Method (CPM)	357
24.4	Program Evaluation and Review Technique (PERT)	359
24.5	Problems	360
24.6	Challenge Problems	361
25.	DIRECTED GRAPHS	362
25.1	Introduction	362
25.2	Data Structures	363
25.3	Applications	367
	Precedence Identification	367
	Graph Searching	368
25.4	Other Topics	369
25.5	Summary	370
25.6	Problems	370
25.7	Challenge Problems	370
26.	TREE DATA STRUCTURES	371
26.1	Introduction	371
26.2	Data Structures	372
26.3	Applications	374
	Precedence Identification	374
	Tree Searching	375
26.4	Summary	377
26.5	Problems	377
26.6	Challenge Problems	377

# EGR 600 – Analysis

## Summer 2006

**Description:** Undergraduate mathematics topics are reviewed and then extended to solve advanced engineering problems. The course will focus on solving problems such as project management, economic justification, modeling random processes, risk analysis, and system modeling. The course will make extensive use of computers in an active learning environment.

**Pre-requisite:** Admission to MSE or permission of instructor

**Texts:** *Jack, H.*, Engineering Analysis, 2006.

**Software:** Scilab ([www.scilab.org](http://www.scilab.org)) or Matlab

C/C++ compiler

**Instructor:** Dr. Hugh Jack

**Office:** EC 718

**E-mail addresses:** jackh@gvsu.edu

**Phone:** 331-6755

**Office hours:** 5:00-6:00pm Mondays

**Class time:** 6:00-8:50pm Mondays

**Instruction Methods:** Lecture, discussion, assignments.

**Homework:** Weekly homework problems will be assigned to reinforce concepts. In some cases students will be assigned alternate homework problems specific to their discipline.

**Quizzes:** Quizzes will be given frequently to assess pre-lecture reading and topic mastery.

**Exams:** A midterm and comprehensive final will be used to assess student learning.

**Semester Project:** Students will work individually on a semester project that explores an advanced area of engineering analysis. The project report will emphasize the need for the clear communication of mathematical concepts including a written report and an oral presentation.

**Computer Use:** Students will be expected to use computers to analyze engineering problems. Scilab will be the preferred computational platform, however Matlab may also be used. Students may be required to write programs in C or C++.

**Grading Policies:**

Homework and Quizzes	50%
Midterm Exam(s)	15%
Final Exam	35%

**Note:** Students must earn passing grades in all components of the course in order to receive a passing grade for the course.

***Grading Scale:***

- A 80 – 100
- B 70 - 80
- C 60 - 70
- D 55 - 60
- F 0 – 55

**Disability Services:** If there is any student in this class who has special needs because of a learning, physical, or other disability, please contact me of the Office of Academic Support (OAS) at 331.2490.

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**Tentative Topics:**

1. Mathematics Review and introduction to Scilab
  - Scilab Tutorial
  - Chapter - Numerical Values and Units
  - Chapter - Algebra
  - Chapter - Trigonometry
2. Programming and Data Presentation
  - Chapter - Graphing
  - Chapter - (PROGRAMMING??)
3. Probability and statistics applications
  - Chapter - Permutations and Combinations
  - Chapter - Probability
4. Calculus applications
  - Chapter - Calculus

5. Differential equations  
    Chapter - Differential Equations
6. Economics  
    Chapter - Financial
7. General emphasis specific review  
    Chapter - (by discipline)  
    Chapter - Boolean  
    Chapter - Transforms
8. Optimization  
    Chapter - Optimization
9. System reliability  
    Chapter - Reliability
10. Project scheduling  
    Chapter - Projects
11. Directed graphs  
    Chapter - Graphs  
    Chapter - Trees

## 27. PREFACE

- the book is designed for use for entry level graduate students, or working professionals looking for review/topical introduction.
- computer based solutions are used liberally throughout the book with a bias towards practical problems.
- basic book design - short chapters, easily used as one or more modules per class
- review modules are marked with the title review. This allows review material to be reviewed without having it dominate the book
- programs are written in C and Scilab. Scilab has been used because it is available freely, but is very similar to matlab
- example programs are boxed
- sample problem solutions are boxed
- asides and notes are given as appropriate
- the text is written to be very direct and student friendly in tone.
- written from an engineers perspective - using mathematics as a tool
- chapters begin and end with summaries
- in some cases topics are presented out of order. Although this would be a major problem if it was a first introduction, as a review it is done in the interests of brevity.

### 27.1 Todo

- create a map of techniques for solving specific problems.
- add a symbolic algebra package

Topics to Add/expand ??-----

	analysis of constrained path systems
	Discrete math
Algebra	
	power series and Taylor series
Data	
	graphing data
	error estimation
	linear regression
	curve fitting with splines
	data representation graphs, tables and accuracy

statistical analysis of a system

Random Processes

- random number generation
- prediction of process outcome
- Quality control

System Modeling

- (ME) state space analysis
- (ME) Eigenvalues and vectors

Advanced Calculus

- (ME) Partial differential equations
- (ECE) Laplace transforms
- Convolution
- Fast Fourier transforms
- differential equations, state space and numerical integration

Optimization

- (PDM) Searching convex/concave functions
- (MO) Searching convex/concave functions
- (MO) Gradient descent
- (MO) Cost and penalty functions

Programming

- programs - basic structure and execution
- program entry points
- syntax
- Variable names
- variable/data types
- input
- outputs
- logical expression
- conditional execution with ifs
- for loops and conditions
- functions/subroutines
  - input/output argument lists
  - defining a function
  - calling a function and dealing with return values
  - the difference between calling and defining a function

Discipline Specific

ECE

- Frequency domain - Fourier, Bessel
- EM Theory review
- Feedback Control Theory
- 3 phase electrical theory

## 27.2 Problems Not Sorted

12a. Find  $y(t)$ .

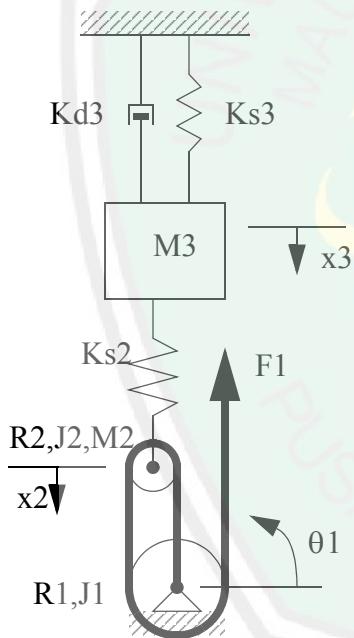
$$\frac{y(s)}{x(s)} = \frac{s^2 + 4s}{s^2 + 6s + 9} \quad x(t) = 5$$

12b. Find  $x(t)$ .

$$\ddot{x} + 4\dot{x} + 2x = 3 \quad x(0) = 1 \quad \dot{x}(0) = 0$$

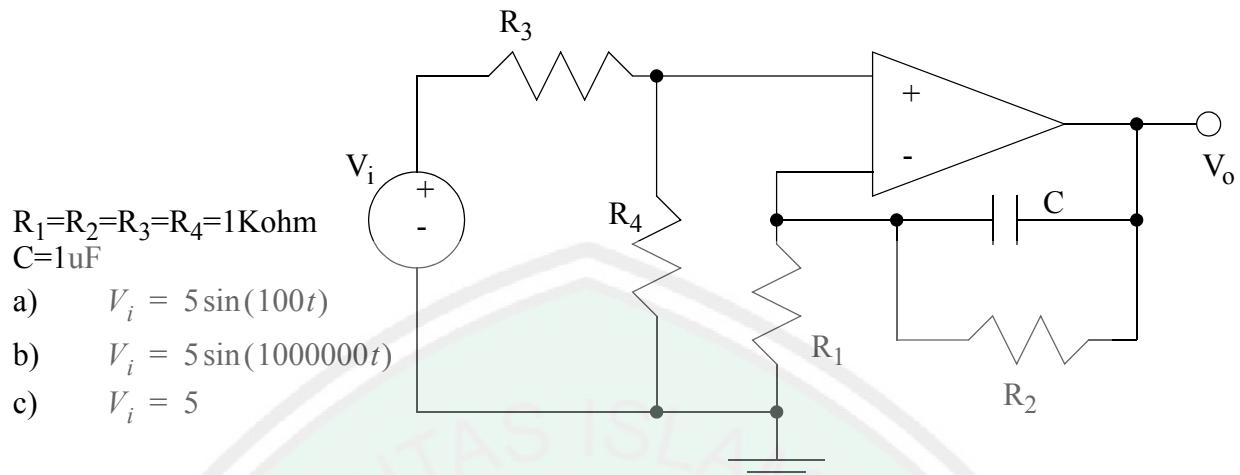
12c. An existing manual production line costs \$100,000 to operate per year. A new piece of automated equipment is being considered to replace the manual production line. The new equipment costs \$150,000 and requires \$30,000 to operate. The decision to purchase the new machine will be based on a 3 year period with a 25% interest rate. Compare the present value of the two options.

14. For the system pictured below put the equations in state variable form and simulate the system using numerical integration. Assume values to test your program.



A pulley system has the bottom pulley anchored. A mass is hung in the middle of the arrangement with springs and dampers on either side. Assume that the cable is always tight.

15. Find the steady state outputs for the system using the given input functions.



## 28. NUMERICAL VALUES AND UNITS

Topics:

- Numbers, Constants, Units

Objectives:

- To review the use of basic numbers, including significant figures
- To review the use of units to keep track of numerical magnitudes

### 28.1 Introduction

### 28.2 Numerical Values

#### 28.2.1 Constants and Other Stuff

- Some basic definitions,  
    numeric - a literal numerical value  
    variable - a symbol used to represent a quantity that will change, often represented with a lower case symbol  
    constant - a value that will not change, often represented with an upper case symbol  
    subscripts - letters or numbers below a variable to create new (related) variables.
- greek letters are often used for variables and constants

lower case	upper case	name
$\alpha$	A	alpha
$\beta$	B	beta
$\gamma$	$\Gamma$	gamma
$\delta$	$\Delta$	delta
$\varepsilon$	E	epsilon
$\zeta$	Z	zeta
$\eta$	H	eta
$\theta$	$\Theta$	theta
$\iota$	I	iota
$\kappa$	K	kappa
$\lambda$	$\Lambda$	lambda
$\mu$	M	mu
$\nu$	N	nu
$\xi$	[E]	xi
$\omicron$	O	omicron
$\pi$	$\Pi$	pi
$\rho$	R	rho
$\sigma$	$\Sigma$	sigma
$\tau$	T	tau
$\upsilon$	Y	upsilon
$\phi$	$\Phi$	phi
$\chi$	X	chi
$\psi$	$\Psi$	psi
$\omega$	$\Omega$	omega

- The constants listed are some of the main ones, other values can be derived through calculation.

$$e = 2.7182818\dots = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = \text{natural logarithm base}$$

$$\pi = 3.1415927\dots = \text{pi}$$

$$\gamma = 0.57721566 = \text{Eulers constant}$$

$$1 \text{ radian} = 57.29578^\circ$$

- In Scilab

```
// basic variables and constants
a = 5; // define a variable with a value of 5
b = %pi; // the value for pi is assigned to b
c = %e; // the natural number
d = %inf; // infinity
e = %nan; // not a number
m = %t; // a logical true
n = %f; // a logical false
p = %i; // the imaginary number
q = eps; // a very small positive number, or 0+

abs(x); // returns the magnitude of x
int(x); // converts a real to an integer value

// getting information
who // print all variables
help sin // open a help window for the sin function
help + // get help on basic operators
apropos imaginary // look for functions on imaginary numbers
```

## 28.2.2 Factorial

- A compact representation of a series of increasing multiples.

$$n! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot \dots \cdot n$$

$$0! = 1$$

## 28.2.3 Significant Figures

- Sig figures rules,

- leading zeroes do not count as significant figures.
- trailing zeros will count as significant figures.
- when doing multiplying the results should will (generally) have the same number of significant figures as the least significant number.
- when adding, the least accurate number determines the accuracy of the result.

123.456	6 significant figures
123.45600	8 significant figures
0.000345	3 significant figures
$123 \cdot 456789 = 56185047 = 56.2 \times 10^3$	3 significant figures
$0.12(3456) = 414.72 = 0.41 \times 10^3$	2 significant figures
$34 + 56.789 = 90.789 = 91$	2 significant figures

- In computation the standard is to keep all of the digits, but the final answer should be rounded to the correct number of significant figures
- Based upon the accuracy of most measuring instruments, and the ability to specify components, most engineering calculations will have 3-6 significant figures. Do not use all of the digits produced by computer/calculator unless all of the digits can be justified.

#### 28.2.4 Scientific and Engineering Notations

- In scientific notation one digit is ahead of the decimal, and all other values follow the decimal. The exponent is adjusted accordingly.

$$a = 1234.56789 = 1.23456789 \times 10^3 = 1.23456789e3$$

- Scilab

```
a = 1234.5678912345678;
a // by default 8 digits will be printed
format('v', 20) // set the number of displayed characters to 20
a // prints 1234.567891234567892
format('v', 5)
a // prints 1234.
format('e', 8) // set the display to exponent notation
a // prints 1.2D+03
a = 1.23456789123456789e3; // the same value of a entered in exponent format
```

- Engineering notation is similar to scientific notation, but the exponent is always a multiple of 3 so that it corresponds to magnitude multipliers (i.e., micro, milli, kilo, mega).

$$a = 12345.6789 = 12.3456789e3$$

$$b = 0.000123456789 = 123.456789e - 6$$

- The current version of Scilab does not seem to support engineering notation.

### 28.3 Complex Numbers

- ‘j’ will be the preferred notation for the complex number, this is to help minimize confusion with the ‘i’ used for current in electrical engineering.
- The basic algebraic properties of these numbers are,

The Complex (imaginary) Number:

$$j = \sqrt{-1} \quad j^2 = -1$$

- Scilab,

```
j = sqrt(-1); // define j as the imaginary number  
A = 5 + 3*j; // define a complex number  
B = 7 + 9j; // define another  
A / B // a complex operation
```

## 28.4 Units and Conversions

- Units are essential when describing real things.
- Good engineering practice demands that each number should always be accompanied with a unit.

### 28.4.1 How to Use Units

- This section does not give an exhaustive list of conversion factors, but instead a minimal (but fairly complete) set is given. From the values below most conversion values can be derived.
- A simple example of unit conversion is given below,

\*\*a simple unit conversion example:

Given,

$$d_x = 10m \quad d_y = 5ft$$

Find the distance 'd',

$$d = \sqrt{d_x^2 + d_y^2}$$

keep the units in the equation

$$\therefore d = \sqrt{(10m)^2 + (5ft)^2}$$

multiply by 1

$$\therefore d = \sqrt{100m^2 + 25ft^2}$$

$$\therefore d = \sqrt{100m^2 + 25ft^2 \left(\frac{0.3048m}{1ft}\right)^2}$$

From the tables

$$1ft = 0.3048m$$

$$\therefore 1 = \frac{0.3048m}{1ft}$$

$$\therefore d = \sqrt{100m^2 + 25ft^2 (0.092903) \frac{m^2}{ft^2}}$$

cancel out units

$$\therefore d = \sqrt{100m^2 + 25(0.092903)m^2}$$

$$\therefore d = \sqrt{102.32m^2} = 10.12m$$

## 28.4.2 SI Units

1. Beware upper/lower case letter in many cases they can change meanings.  
e.g. m = milli or mega?
2. Try to move prefixes out of the denominator of the units.  
e.g., N/cm or KN/m
3. Use a slash or exponents.  
e.g.,  $(kg \cdot m/s^2)$  or  $(kg \cdot m \cdot s^{-2})$  or  $(kg \cdot m \cdot s^{-2})$  or  $(kg \cdot m \cdot s^{-2})$
4. Use a dot in compound units when possible.  
e.g., N•m
5. Use spaces to divide digits when there are more than 5 figures, commas are avoided because their use is equivalent to decimal points in many places (e.g., Europe).

- Base and derived units

Base units

 $m$  = length $kg$  = mass $s$  = time $A$  = current $K$  = temerature $mol$  = chemical quantity $cd$  = candela

Derived unit examples

$$N = \frac{kg \cdot m}{s^2}$$

$$F = \frac{C}{V}$$

$$J = N \cdot m$$

$$\Omega = \frac{V}{A}$$

$$Pa = \frac{N}{m^2}$$

$$Wb = V \cdot s$$

$$W = \frac{J}{s}$$

$$T = \frac{Wb}{m^2}$$

$$V = \frac{W}{A}$$

$$H = \frac{Wb}{A}$$

- In some cases units are non-standard. There are two major variations US units are marked with 'US' and Imperial units (aka English and inch based) are marked with 'IMP'.

### 28.4.3 A Table

#### Distance

1 ft. (feet) = 12 in. (inches) = 0.3048 m (meter)

1 mile = 1760 yards = 5280 ft = 1.609km

1 in.(inch) = 2.540 cm

1 yd (yard) = 3 ft.

1 nautical mile = 6080 ft. = 1852 m = 1.150782 mi

1 micron =  $10^{-6}$  m

1 angstrom =  $10^{-10}$  m

$1 \text{ mil} = 10^{-3} \text{ in}$   
 $1 \text{ acre} = 43,560 \text{ ft.} = 0.4047 \text{ hectares}$   
 $1 \text{ furlong} = 660 \text{ ft}$   
 $1 \text{ lightyear} = 9.460528\text{e}15 \text{ m}$   
 $1 \text{ parsec} = 3.085678\text{e}16 \text{ m}$

**Area**

$1 \text{ acre} = 43,559.66 \text{ ft}^2$   
 $1 \text{ Hectare (ha)} = 10,000 \text{ m}^2$   
 $1 \text{ Hectare (ha)} = 10,000 \text{ m}^2$   
 $1 \text{ Hectare (ha)} = 10,000 \text{ m}^2$   
 $1 \text{ Hectare (ha)} = 10,000 \text{ m}^2$

**Velocity**

$1 \text{ mph} = 0.8689762 \text{ knot}$

**Angle**

$1 \text{ rev} = 2\pi \text{ radians} = 360 \text{ degrees} = 400 \text{ gradians}$   
 $1 \text{ degree} = 60 \text{ minutes}$   
 $1 \text{ minute} = 60 \text{ seconds}$

**Volume**

$1 \text{ US gallon} = 231 \text{ in}^3$   
 $1 \text{ CC} = 1 \text{ cm}^3$   
 $1 \text{ IMP gallon} = 277.274 \text{ in}^3$   
 $1 \text{ barrel} = 31 \text{ IMP gal.} = 31.5 \text{ US gal.}$   
 $1 \text{ US gal.} = 3.785 \text{ l} = 4 \text{ quarts} = 8 \text{ pints} = 16 \text{ cups}$   
 $1 \text{ liter (l)} = 0.001 \text{ m}^3 = 2.1 \text{ pints (pt)} = 1.06 \text{ quarts (qt)} = 0.26 \text{ gallons (gal)}$   
 $1 \text{ qt (quart)} = 0.9464 \text{ l}$   
 $1 \text{ cup (c)} = 0.2365882 \text{ l} = 8 \text{ US oz}$   
 $1 \text{ US oz} = 8 \text{ US drams} = 456.0129 \text{ drops} = 480 \text{ US minim} = 1.0408 \text{ IMP oz}$   
 $= 2 \text{ tablespoons} = 6 \text{ teaspoons}$   
 $1 \text{ IMP gal.} = 1.201 \text{ U.S. gal.}$   
 $1 \text{ US pint} = 16 \text{ US oz}$   
 $1 \text{ IMP pint} = 20 \text{ IMP oz}$   
 $1 \text{ tablespoon} = 0.5 \text{ oz.}$   
 $1 \text{ bushel} = 32 \text{ quarts}$   
 $1 \text{ peck} = 8 \text{ quarts}$

**Force/Mass**

$1 \text{ N (newton)} = 1 \text{ kg}\cdot\text{m/s}^2 = 100,000 \text{ dyne}$   
 $1 \text{ dyne} = 2.248\cdot 10^{-6} \text{ lb. (pound)}$   
 $1 \text{ kg} = 9.81 \text{ N (on earth surface)} = 2.2046 \text{ lb}$

1lbf = 16 oz. (ounce) = 4.448N  
 1 oz. = 28.35 g (gram) = 0.2780N  
 1 lb = 0.03108 slug  
 1 kip = 1000 lb.  
 1 slug = 14.59 kg  
 1 imperial ton = 2000 lb = 907.2 kg  
 1 metric tonne = 1000 kg  
 1 troy oz = 480 grain (gr)  
 1 g = 5 carat  
 1 pennyweight = 24 grain  
 1 stone = 14 lb  
 1 long ton = 2240 lb  
 1 short ton = 2000 lb

#### Pressure

1 Pascal (Pa) = 1 N/m<sup>2</sup> = 6.895 kPa  
 1 atm (metric atmos.) = 760 mmHg at 0°C = 14.223 lb/in<sup>2</sup> = 1.0132\*10<sup>5</sup> N/m<sup>2</sup>  
 1 psi = 2.0355 in. Hg at 32F = 2.0416 in. Hg at 62F  
 1 microbar = 0.1 N/m<sup>2</sup>

#### Scale/Magnitude

atto (a) = 10<sup>-18</sup>  
 femto (f) = 10<sup>-15</sup>  
 pico (p) = 10<sup>-12</sup>  
 nano (n) = 10<sup>-9</sup>  
 micro ( $\mu$ ) = 10<sup>-6</sup>  
 milli (m) = 10<sup>-3</sup>  
 centi (c) 10<sup>-2</sup>  
 deci (d) = 10<sup>-1</sup>  
 deka (da) = 10  
 hecto (h) = 10<sup>2</sup>  
 kilo (K) = 10<sup>3</sup>  
 mega (M) = 10<sup>6</sup>  
 giga (G) = 10<sup>9</sup>  
 tera (T) = 10<sup>12</sup>  
 peta (P) = 10<sup>15</sup>  
 exa (E) = 10<sup>18</sup>

#### Power

1 h.p. (horsepower) = 745.7 W (watts) = 2.545 BTU/hr. = 550 ft.lb./sec.  
 1 ft•lb/s = 1.356 W  
 1 J (joule) = 1 N•m = 10<sup>7</sup> ergs = 0.2389 cal.

$1 \text{ W} = 1 \text{ J/s}$

$1 \text{ ev} = 1.60219 \times 10^{-19} \text{ J}$

$1 \text{ erg} = 10^{-7} \text{ J}$

#### Temperature

$^{\circ}\text{F} = [(\text{ }^{\circ}\text{C} \times 9)/5] + 32$ ,  $\text{ }^{\circ}\text{C}$  = Celsius (Centigrade),  $\text{F}$  = Fahrenheit

$\text{K}$  = Kelvin

Rankine ( $\text{R}$ ) =  $\text{F} - 459.666$

0.252 calories = 1 BTU (British Thermal Unit)

-273.2  $\text{ }^{\circ}\text{C}$  = -459.7  $^{\circ}\text{F}$  = 0 K = 0 R = absolute zero

0  $\text{ }^{\circ}\text{C}$  = 32  $^{\circ}\text{F}$  = 273.3 K = 491.7 R = Water Freezes

100  $\text{ }^{\circ}\text{C}$  = 212  $^{\circ}\text{F}$  = 373.3 K = 671.7 R = Water Boils (1 atm. pressure)

1 therm = 100,000 BTU

#### Mathematical

$\pi$  radians = 3.1416 radians = 180 degrees = 0.5 cycles

1 Hz = 1 cycle/sec.

1 rpm (revolutions per minute) = 60 RPS (Revolutions per second) = 60Hz

1 fps (foot per second) = 1 ft/sec

1 mph (miles per hour) = 1 mi./hr.

1 cfm (cubic foot per minute) = 1 ft<sup>3</sup>/min.

e = 2.718

#### Time

1 Hz (hertz) = 1 s<sup>-1</sup>

1 year = 365 days = 52 weeks = 12 months

1 leap year = 366 days

1 day = 24 hours

1 fortnight = 14 days

1 hour = 60 min.

1 min = 60 seconds

1 millennium = 1000 years

1 century = 100 years

1 decade = 10 years

#### Physical Constants

R = 1.987 cal/mole K = ideal gas law constant

K = Boltzmann's constant =  $1.3 \times 10^{-16}$  erg/K =  $1.3 \times 10^{-23}$  J/K

h = Planck's constant =  $6.62 \times 10^{-34}$  erg-sec =  $6.62 \times 10^{-34}$  J.sec

Avagadro's number =  $6.02 \times 10^{23}$  atoms/atomic weight

density of water = 1 g/cm<sup>3</sup>

electron charge =  $1.60 \times 10^{-19}$  coul.

electron rest mass =  $9.11 \times 10^{-31}$  Kg

proton rest mass =  $1.67 \times 10^{-27}$  Kg  
 speed of light (c) =  $3.00 \times 10^{10}$  cm/sec  
 speed of sound in dry air 25 C = 331 m/s  
 gravitational constant =  $6.67 \times 10^{-11}$  Nm $^2$ /Kg $^2$   
 permittivity of free space =  $8.85 \times 10^{-12}$  farad/m  
 permeability of free space =  $1.26 \times 10^{-6}$  henry/m  
 mean radius of earth = 6370 Km  
 mass of earth =  $5.98 \times 10^{24}$  Kg

### Electromagnetic

magnetic flux = weber (We) =  $10^{-8}$  maxwell  
 inductance = henry  
 magnetic flux density = tesla (T) =  $10^{-4}$  gauss  
 magnetic intensity = ampere/m =  $0.004\pi$  oersted  
 electric flux density = coulomb/m $^2$   
 capacitance = farad  
 permeability = henry/m  
 electric field strength = V/m  
 luminous flux = lumen  
 luminance = candela/m $^2$   
 1 flame = 4 foot candles = 43.05564 lux = 43.05564 meter-candles  
 illumination = lux  
 resistance = ohm

### 28.5 Problems

1. Show the units for Joules in base units. (ans. kg m $^2$  / s $^2$ )
2. How many kips are in 2.00 metric tonnes. (ans. 4.41 kip)
3. Do the following calculation using significant figures.

$$0.010 \times 1.2345 + 0.0234567 \quad (\text{ans. } 35\text{e-}3)$$

2. Simplify the following expressions.

a)  $(6 + 8j)^2$  ans.  $-28 + 96j$

b)  $\frac{8j + 6}{(4j + 3)^2}$  ans.  $\frac{6}{25} - \frac{8}{25}j$

c)  $2 + 5!$  ans. 122

## 29. ALGEBRA

Topics:

- 

Objectives:

- 

### 29.1 The Fundamentals

#### 29.1.1 Basic Operations

commutative

$$a + b = b + a$$

$$a + b = c + d$$

$$\therefore a = c + d - b$$

distributive/collective  $a(b + c) = ab + ac$ 

associative

$$a(bc) = (ab)c$$

$$a + (b + c) = (a + b) + c$$

$$ab = cd$$

$$\therefore a = \frac{cd}{b}$$

silly mistakes

$$\frac{1}{a+b} \neq \frac{1}{a} + \frac{1}{b}$$

- Scilab example,

```
b = 5;
c = 6;
d = 7;
a = c * d / b
a = c + d - b
```

### 29.1.2 Exponents

- The basic properties of exponents are so important they demand some sort of mention

$$(x^n)(x^m) = x^{n+m} \quad x^0 = 1, \text{ if } x \text{ is not } 0 \quad x^{\frac{1}{n}} = \sqrt[n]{x} \text{ (nth root)}$$

$$\frac{(x^n)}{(x^m)} = x^{n-m} = \frac{1}{x^{m-n}}$$

$$(x^n)^m = x^{n \cdot m}$$

$$x^{-p} = \frac{1}{x^p}$$

$$(xy)^n = (x^n)(y^n)$$

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

$$x^{\frac{m}{n}} = \sqrt[n]{x^m}$$

$$\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$$

- Scilab example,

```
x = 5;
n = 2;
m = 4;
y = 6;
x^n * x^m
x^(n + m)
x**(n+m)
```

### 29.1.3 Basic Polynomials

- The quadratic equation.

$$ax^2 + bx + c = 0 = a(x - r_1)(x - r_2)$$

$$r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- e.g.,

$$2x^2 + 4x + 2 = 0$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(2)(2)}}{2(2)} = -1 \pm \sqrt{0} = -1, -1$$

$\therefore (x+1)(x+1) = 0$  note the signs

- Complex roots will occur when,

$$b^2 - 4ac < 0$$

- Cubic equations can be solved explicitly, although this is too much for common memorization.

$$x^3 + ax^2 + bx + c = 0 = (x - r_1)(x - r_2)(x - r_3)$$

First, calculate,

$$Q = \frac{3b - a^2}{9} \quad R = \frac{9ab - 27c - 2a^3}{54} \quad S = \sqrt[3]{R + \sqrt{Q^3 + R^2}} \quad T = \sqrt[3]{R - \sqrt{Q^3 + R^2}}$$

Then the roots,

$$r_1 = S + T - \frac{a}{3} \quad r_2 = \frac{S + T}{2} - \frac{a}{3} + \frac{j\sqrt{3}}{2}(S - T) \quad r_3 = \frac{S + T}{2} - \frac{a}{3} - \frac{j\sqrt{3}}{2}(S - T)$$

- On a few occasions a quartic (4th order) equation will also have to be solved. This can be done by first reducing the equation to a quadratic,

$$x^4 + ax^3 + bx^2 + cx + d = 0 = (x - r_1)(x - r_2)(x - r_3)(x - r_4)$$

First, solve the equation below to get a real root (call it 'y'),

$$y^3 - by^2 + (ac - 4d)y + (4bd - c^2 - a^2 d) = 0$$

Next, find the roots of the 2 equations below,

$$r_1, r_2 = z^2 + \left( \frac{a + \sqrt{a^2 - 4b + 4y}}{2} \right) z + \left( \frac{y + \sqrt{y^2 - 4d}}{2} \right) = 0$$

$$r_3, r_4 = z^2 + \left( \frac{a - \sqrt{a^2 - 4b + 4y}}{2} \right) z + \left( \frac{y - \sqrt{y^2 - 4d}}{2} \right) = 0$$

- In Scilab,

```
x = poly(0, 'x');
roots(3 * x^2 + 4 * x + 2)

q = [2, 4, 3];
p = poly(q, 'x', 'coeff'); // defines a polynomial using a vector
roots(p)

derivative(p, 'x'); // finds the derivative of the polynomial
horner(p, 5); // evaluates the polynomial at 5
s = (x + 1) / p; // an algebra operation
```

## 29.2 Special Forms

### 29.2.1 Completing the Square

$$\begin{aligned}x^2 + Ax + B &= (x + C)^2 + D \\&= x^2 + 2Cx + (C^2 + D)\end{aligned}$$

$$\begin{array}{ll}A = 2C & B = C^2 + D \\C = \frac{A}{2} & D = B - C^2\end{array}$$

for example, given,

$$\begin{aligned}5x^2 + 50x + 10 &= 5(x^2 + 10x + 2) \\&= 5((x + 5)^2 - 23)\end{aligned}$$

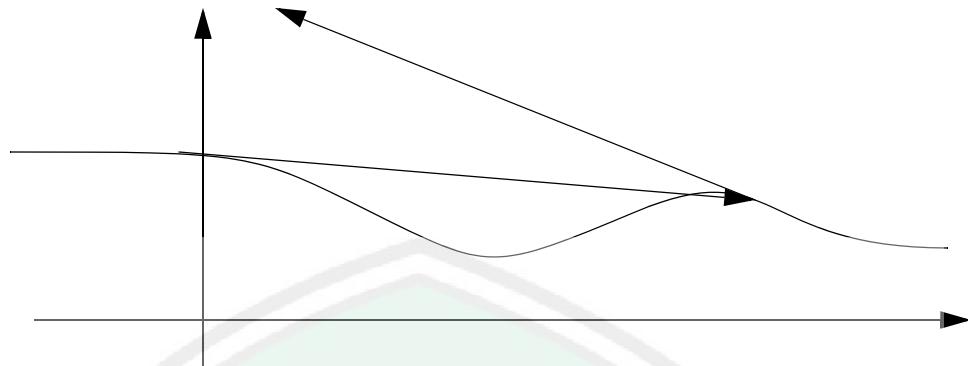
### 29.2.2 Newton-Raphson to Find Roots

- When given an equation where an algebraic solution is not feasible, a numerical solution may be required. One simple technique uses an instantaneous slope of the function, and takes iterative steps towards a solution.

$$x_{i+1} = x_i - \frac{f(x_i)}{\left(\frac{d}{dx}f(x_i)\right)}$$

- The function  $f(x)$  is supplied by the user along with an initial guess.
- This method can become divergent if the function has an inflection point near the root.

- The technique is also sensitive to the initial guess.



- This calculation should be repeated until the final solution is found.
- Scilab example,

NEWTON RAPHSON ROOT EXAMPLE

### 29.3 Complex Numbers

- Complex values

The Complex (imaginary) Number:

$$j = \sqrt{-1} \quad j^2 = -1$$

Complex Numbers:

$$a + bj \quad \text{where, } a \text{ and } b \text{ are both real numbers}$$

Complex Conjugates (denoted by adding an asterisk '\*' to the variable):

$$N = a + bj \quad N^* = a - bj$$

Basic Properties:

$$(a + bj) + (c + dj) = (a + c) + (b + d)j$$

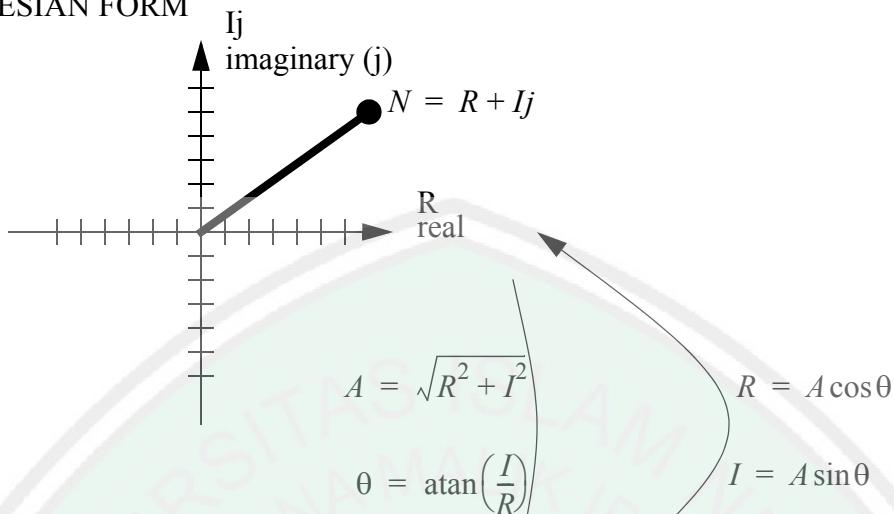
$$(a + bj) - (c + dj) = (a - c) + (b - d)j$$

$$(a + bj) \cdot (c + dj) = (ac - bd) + (ad + bc)j$$

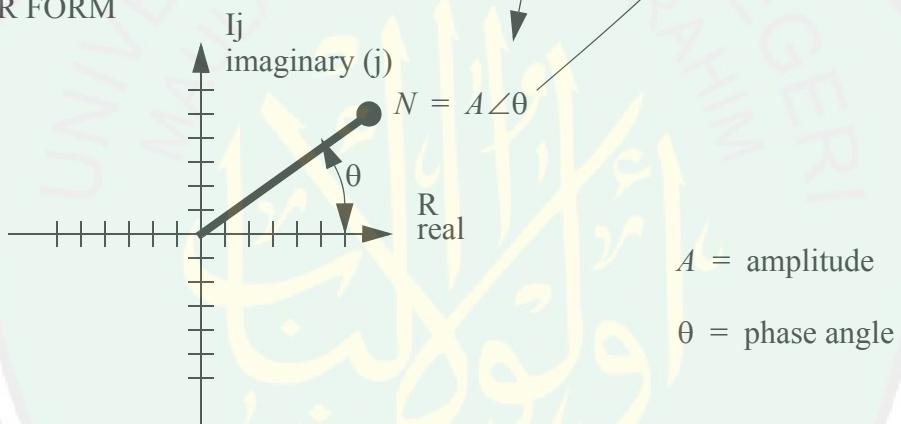
$$\frac{N}{M} = \frac{a + bj}{c + dj} = \frac{N(N^*)}{M(N^*)} = \left(\frac{a + bj}{c + dj}\right)\left(\frac{c - dj}{c - dj}\right) = \frac{ac + bd}{c^2 + d^2} + \left(\frac{bc - ad}{c^2 + d^2}\right)j$$

- We can also show complex numbers graphically. These representations lead to alternative representations. If it is not obvious above, please consider the notation uses a cartesian notation, but a polar notation can also be very useful when doing large calculations.

## CARTESIAN FORM



## POLAR FORM



- We can also do calculations using polar notation (this is well suited to multiplication and division, whereas cartesian notation is easier for addition and subtraction),

$$A \angle \theta = A(\cos \theta + j \sin \theta) = Ae^{j\theta}$$

$$e^{A+jB} = e^A e^{jB} = e^A (\cos \theta + j \sin \theta)$$

$$(A_1 \angle \theta_1)(A_2 \angle \theta_2) = (A_1 A_2) \angle (\theta_1 + \theta_2)$$

$$\frac{(A_1 \angle \theta_1)}{(A_2 \angle \theta_2)} = \left(\frac{A_1}{A_2}\right) \angle (\theta_1 - \theta_2)$$

$$(A \angle \theta)^n = (A^n) \angle (n\theta) \quad (\text{DeMoivre's theorem})$$

- Note that DeMoivre's theorem can be used to find exponents (including roots) of complex numbers
- Euler's formula:  $e^{j\theta} = \cos\theta + j\sin\theta$

Note: for  $0 + 1j = \cos\theta + j\sin\theta$

$$\theta = \frac{\pi}{2}$$

$$e^{\frac{j\pi}{2}} = j$$

- From the above, the following useful identities arise:

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

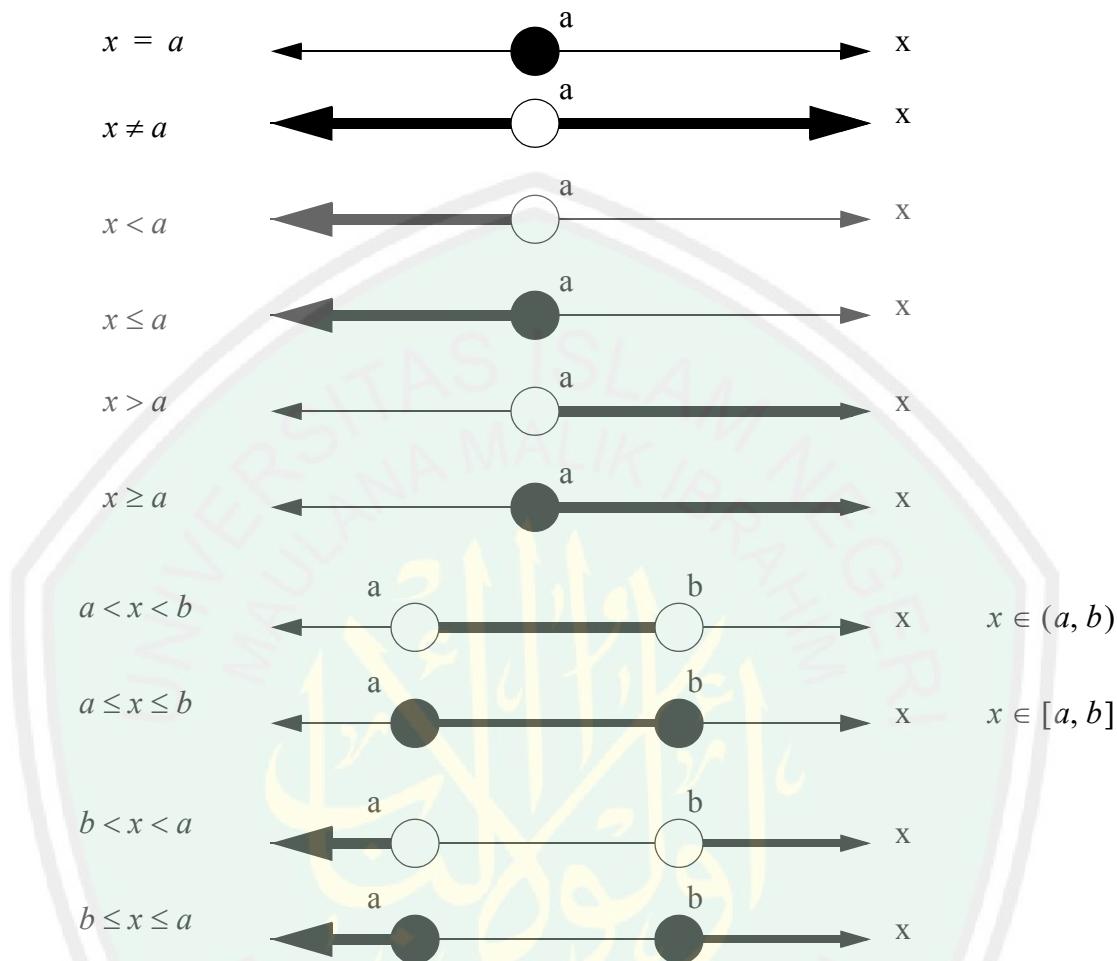
- In Scilab

```
j = sqrt(-1);
A = (1 + 2 * j) / (3 + 4 * j);
A
[mag, theta] = polar(A);
mag, theta // the magnitude and angle of the complex angle A
abs(A); // the magnitude of the complex number
real(A); // the real part of the complex number
imag(A); // the imaginary part of the complex number
conj(A); // the conjugate of the complex number
atan(imag(A), real(A)); // the angle of the complex number
```

## 29.4 Equality and Inequality

- Some basic relationships are illustrated below with number lines. The shaded dots indicate that

the values include the point. Unshaded dots indicate that the values approach but do not equal the value.



- rearranging equations with inequalities

$$a + b < 5 \quad \therefore a < 5 - b$$

- When doing calculations there is some roundoff error that result in a number that should be zero but has a finite value. Consider single precision floating point numbers with 7 digits, or double precision with 14, the last digit is equality in numerical calculations using a tolerance. In these

cases a tolerance is used between the target and actual values to determine equality.

$$\varepsilon = |val| - \text{target}$$

$$if(\varepsilon < 0.000001) then \varepsilon = 0$$

- In Scilab,

```
if ( x > A ) | ( x < B ) then
    // program statement
else if x == A then
    // more program statements
end
```

- Another Scilab example,

$$\begin{aligned} F(x) &= 5 & 0 < x \leq 5 \\ &= 0 & x \leq 0, x > 5 \end{aligned}$$

```
function foo = F(x)
    if (x > 0) & (x <= 5) then
        foo = 5;
    else
        foo = 0;
    end
end_function
```

## 29.5 Functions

- Functions help to identify and organize self contained expressions.
- defining functions encourages reuse of mathematical expresions or program code
- terminology,

$$f(x) \quad (\text{say } 'f \text{ of } x')$$

- an example function is,

$$f(x) = x^2 + 3$$

$$f(2) = 2^2 + 3 = 7$$

- In Scilab the function becomes,

```
function foo = f(x)
    foo = x^2 + 3;
endfunction

mprintf("%f\n", f(2));
```

Note: Use indents when writing programs to ensure structure. While these are not necessary they will make debugging much easier.

Note: Using functions is not necessary in simpler programs, but larger programs will become very difficult to work with if functions are not used.

## 29.6 Special Functions

### 29.6.1 Logarithms

- Logarithms also have a few basic properties of use,

The basic base 10 logarithm:

$$\log x = y \quad x = 10^y$$

The basic base n logarithm:

$$\log_n x = y \quad x = n^y$$

The basic natural logarithm (e is a constant with a value found near the start of this section):

$$\ln x = \log_e x = y \quad x = e^y$$

- All logarithms observe a basic set of rules for their application,

$$\log_n(xy) = \log_n(x) + \log_n(y)$$

$$\log_n(n) = 1$$

$$\log_n\left(\frac{x}{y}\right) = \log_n(x) - \log_n(y)$$

$$\log_n(1) = 0$$

$$\log_n(x^y) = y \log_n(x)$$

$$\log_n(x) = \frac{\log_m(x)}{\log_m(n)}$$

$$\ln(A \angle \theta) = \ln(A) + (\theta + 2\pi k)j \quad k \in I$$

e.g.,

$$\text{solve} \quad 5 = 10^n$$

$$\log(5) = n \log(10)$$

$$n = \frac{\log(5)}{\log(10)}$$

- Note: most computers use the 'log' function for the natural logarithm. The base 10 logarithm is normally 'log10'.
- In Scilab this is

`log10(3)` - base 10 log

`log(3)` - natural log

`log(exp(10))`

## 29.7 Solving Systems of Linear Equations

- Systems of linear equations are of the form below,

$$x + y = 5$$

$$2x + 3y = 8$$

- In general there must be the same number of equations and unknowns to solve the equations.
- In some cases the equations will not be solvable. In this case we say the equations are singular.

### 29.7.1 Substitution

- Substitution is the most fundamental method for solving linear equations, but it is the least routine,

Given,

$$x + y = 5$$

$$2x + 3y = 8$$

To solve for y, we substitute to eliminate x.

$$\therefore x = 5 - y$$

$$\therefore 2(5 - y) + 3y = 8$$

$$\therefore y = -2$$

To solve for x, we substitute the value for y into an earlier equation.

$$\therefore x = 5 - (-2) = 7$$

- In Scilab this is

EXAMPLE ????????????

### 29.7.2 Addition

- Polynomials can be added to eliminate variables (in a matrix for this is referred to as the Gauss-Jordan row reduction method).

Given,

$$x + y = 5$$

$$2x + 3y = 8$$

multiply and add the equations,

$$(-2)(x + y = 5)$$

$$2x + 3y = 8$$

$$\begin{array}{r} 0x + 1y = -2 \\ \hline \end{array}$$

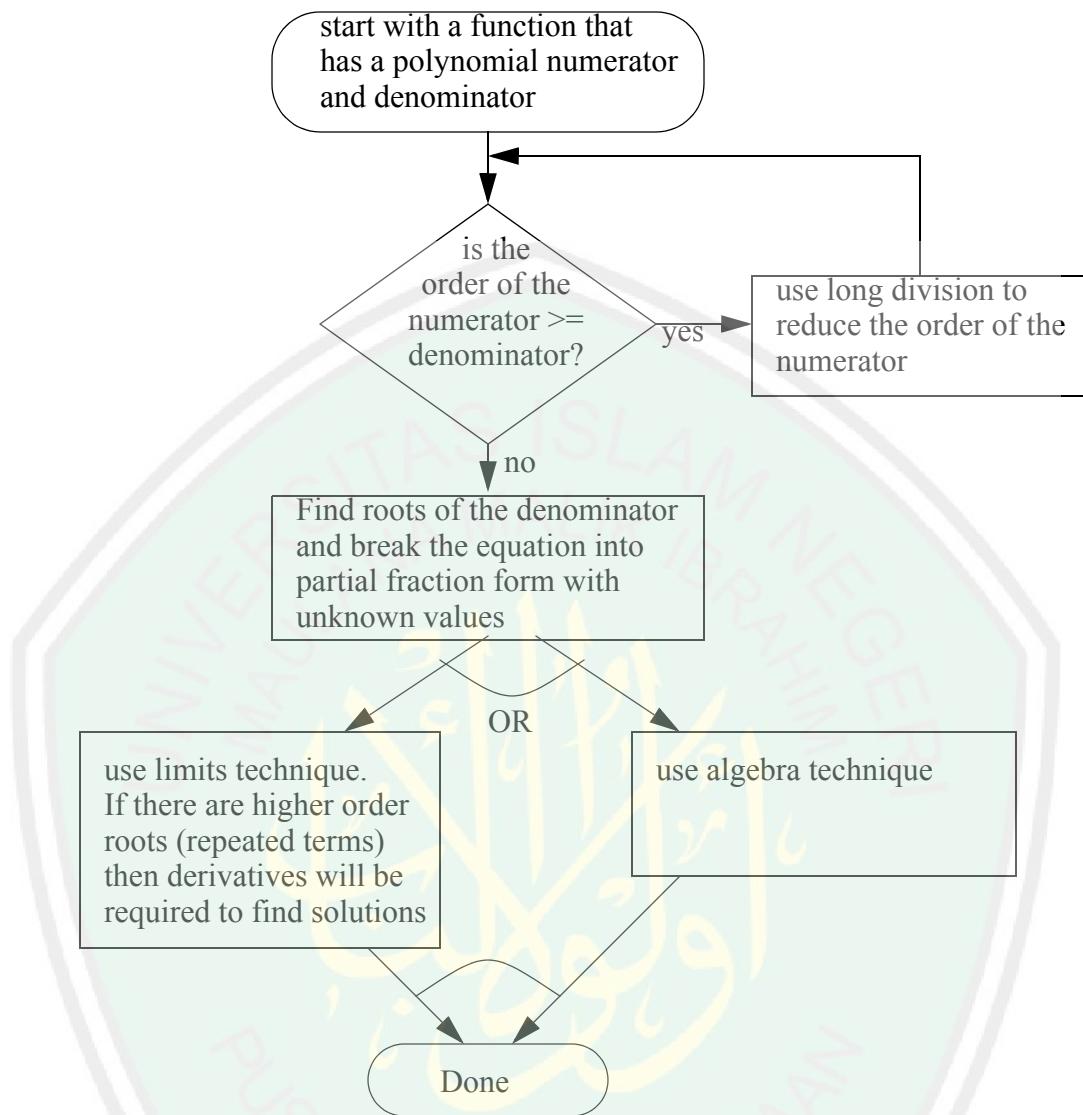
$$\therefore y = -2$$

$$x = 5 - (-2) = 7$$

## 29.8 Simplifying Polynomial Expressions

### 29.8.1 Partial Fractions

- The next is a flowchart for partial fraction expansions.



- The partial fraction expansion for,

$$x(s) = \frac{1}{s^2(s+1)} = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s+1}$$

$$C = \lim_{s \rightarrow -1} \left[ (s+1) \left( \frac{1}{s^2(s+1)} \right) \right] = 1$$

$$A = \lim_{s \rightarrow 0} \left[ s^2 \left( \frac{1}{s^2(s+1)} \right) \right] = \lim_{s \rightarrow 0} \left[ \frac{1}{s+1} \right] = 1$$

$$B = \lim_{s \rightarrow 0} \left[ \frac{d}{ds} \left[ s^2 \left( \frac{1}{s^2(s+1)} \right) \right] \right] = \lim_{s \rightarrow 0} \left[ \frac{d}{ds} \left( \frac{1}{s+1} \right) \right] = \lim_{s \rightarrow 0} [-(s+1)^{-2}] = -1$$

- Consider the example below where the order of the numerator is larger than the denominator.

$$x(s) = \frac{5s^3 + 3s^2 + 8s + 6}{s^2 + 4}$$

This cannot be solved using partial fractions because the numerator is 3rd order and the denominator is only 2nd order. Therefore long division can be used to reduce the order of the equation.

$$\begin{array}{r} 5s + 3 \\ \hline s^2 + 4 \left| \begin{array}{r} 5s^3 + 3s^2 + 8s + 6 \\ 5s^3 + 20s \\ \hline 3s^2 - 12s + 6 \\ 3s^2 + 12 \\ \hline -12s - 6 \end{array} \right. \end{array}$$

This can now be used to write a new function that has a reduced portion that can be solved with partial fractions.

$$x(s) = 5s + 3 + \frac{-12s - 6}{s^2 + 4} \quad \text{solve} \quad \frac{-12s - 6}{s^2 + 4} = \frac{A}{s+2j} + \frac{B}{s-2j}$$

- When the order of the denominator terms is greater than 1 it requires an expanded partial fraction form, as shown below.

$$F(s) = \frac{5}{s^2(s+1)^3}$$

$$\frac{5}{s^2(s+1)^3} = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{(s+1)^3} + \frac{D}{(s+1)^2} + \frac{E}{(s+1)}$$

- We can solve the previous problem using the algebra technique.

$$\begin{aligned}\frac{5}{s^2(s+1)^3} &= \frac{A}{s^2} + \frac{B}{s} + \frac{C}{(s+1)^3} + \frac{D}{(s+1)^2} + \frac{E}{(s+1)} \\ &= \frac{A(s+1)^3 + Bs(s+1)^3 + Cs^2 + Ds^2(s+1) + Es^2(s+1)^2}{s^2(s+1)^3} \\ &= \frac{s^4(B+E) + s^3(A+3B+D+2E) + s^2(3A+3B+C+D+E) + s(3A+B) + (A)}{s^2(s+1)^3}\end{aligned}$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 3 & 0 & 1 & 2 \\ 3 & 3 & 1 & 1 & 1 \\ 3 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \\ E \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 5 \end{bmatrix} \quad \begin{bmatrix} A \\ B \\ C \\ D \\ E \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 3 & 0 & 1 & 2 \\ 3 & 3 & 1 & 1 & 1 \\ 3 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 5 \\ -15 \\ 5 \\ 10 \\ 15 \end{bmatrix}$$

$$\frac{5}{s^2(s+1)^3} = \frac{5}{s^2} + \frac{-15}{s} + \frac{5}{(s+1)^3} + \frac{10}{(s+1)^2} + \frac{15}{(s+1)}$$

## 29.8.2 Summation and Series

b

- The notation  $\sum_{i=a}^b x_i$  is equivalent to  $x_a + x_{a+1} + x_{a+2} + \dots + x_b$  assuming  $a$  and  $b$  are integers and  $b \geq a$ . The index variable  $i$  is an index often replaced with  $j, k, m$ , and  $n$ .
- Operations on summations:

$$\begin{aligned}
 \sum_{\substack{i=a \\ b}}^b x_i &= \sum_{\substack{i=b \\ i=a}}^a x_i \\
 \sum_{\substack{i=a \\ b}}^b \alpha x_i &= \alpha \sum_{\substack{i=a \\ b}}^b x_i \\
 \sum_{\substack{i=a \\ b}}^b x_i + \sum_{\substack{j=a \\ c}}^c y_j &= \sum_{\substack{i=a \\ c}}^c (x_i + y_j) \\
 \sum_{\substack{i=a \\ b}}^b x_i + \sum_{\substack{i=b+1 \\ d}}^d x_i &= \sum_{\substack{i=a \\ d}}^d x_i \\
 \left( \sum_{i=a}^b x_i \right) \left( \sum_{j=c}^d y_j \right) &= \sum_{i=a}^b \sum_{j=c}^d x_i y_j
 \end{aligned}$$

- Some common summations:

$$\sum_{i=1}^N i = \frac{1}{2}N(N+1)$$

$$\sum_{i=0}^{N-1} \alpha^i = \begin{cases} \frac{1-\alpha^N}{1-\alpha}, & \alpha \neq 1 \\ N, & \alpha = 1 \end{cases} \text{ for both real and complex } \alpha .$$

$\sum_{i=0}^{\infty} \alpha^i = \frac{1}{1-\alpha}$ ,  $|\alpha| < 1$  for both real and complex  $\alpha$ . For  $|\alpha| \geq 1$ , the summation does not converge.

- In Scilab

### LOOP EXAMPLE

## 29.9 Limits

- Limits stuff goes here.....

## 29.10 Problems

1. Rearrange the following equation so that only 'y' is on the left hand side.

$$\frac{y+x}{y+z} = x+2 \quad \text{ans.} \quad y = \frac{-x + xz + 2z}{-x - 1}$$

2. Solve the following equation to find 'x'.

$$2x^2 + 8x = -8 \quad \text{ans. } x = -2, -2$$

3. Solve the following system of equations using substitution.

$$x + 2y + 3z = 5$$

$$\text{ans. } x=-7, y=18.75, z=-8.5$$

$$x + 4y + 8z = 0$$

$$4x + 2y + z = 1$$

4. Simplify the following expressions.

a)  $n \log(x) + m \log(y) - \log(z)$

ans.

$$\log\left(\frac{x^n y^m}{z}\right)$$

5. Simplify,

$$(5x + 3y) - (2x - 7y)$$

$$(5x^2 + 3y) - (2x - 7y)$$

$$\begin{array}{r} x^2 + 5x + 6 \\ \hline x^2 + 7x + 12 \end{array}$$

$$\frac{5}{x} + \frac{7}{3x}$$

$$\frac{5}{x} - \frac{7}{3x}$$

$$\begin{array}{r} 5 \\ 3 - \frac{7}{x} \end{array}$$

$$5(x+3) - 9(x-6)$$

$$\frac{5}{x-2} = \frac{6}{x+3}$$

ans.  $3x + 10y$

$$5x^2 - 2x + 4y$$

$$\frac{x+2}{x+4}$$

$$\frac{22}{3x}$$

$$\frac{8}{3x}$$

$$\frac{5x}{3x-7}$$

$$-4x + 69$$

$$x = 27$$

6. Multiply,

$$(2x + 3)(5x + 4)$$

ans.  $10x^2 + 23x + 12$

$$(2x^2 + 3x + 4)(5x + 4)$$

$$10x^3 + 23x^2 + 32x + 16$$

7. Factor,

$$x^2 - 4$$

$$x^2 - 4y^2$$

$$x^2 - 4x + 4$$

$$x^2 + 5x + 6$$

ans.

$$(x + 2)(x - 2)$$

$$(x + 2y)(x - 2y)$$

$$(x - 2)(x - 2)$$

$$(x + 2)(x + 3)$$

10. solve for x, y,

$$5x + 3y = 10$$

$$x - y = 0$$

$$\text{ans. } x=y=1.25$$

11. Find roots,

$$x^2 + 2x + 100 = 0$$

$$x^4 + 2x^2 + 100 = 0$$

ans.

$$-1 \pm \frac{\sqrt{-396}}{4}$$

$$x = \pm \sqrt{-1 \pm \frac{\sqrt{-396}}{4}}$$

13. Convert the following log to base 10,

$$\log_6 3$$

$$\text{ans. } 0.613$$

14. Find x,

$$x^{12} = 1372$$

$$\text{ans. } x = 1.826$$

$$5^{12+x} = 1372$$

15. Simplify,

$$\frac{\log A + \log B}{C \log D}$$

ans.

$$\frac{1}{C} \log(AB - D)$$

16. Find the polar form for,

$$5 + 6j$$

ans.  $7.81 \angle 0.876\text{rad}$

17. Find the cartesian complex value for,

$$10 \angle 0.13\text{rad}$$

ans.  $9.92 + 1.30j$

18. Find the cartesian complex value for,

$$(10 \angle 0.13\text{rad})^3$$

ans.  $924.9 + 380.2j$

19. Solve,

$$-5x - 2 > 3x + 4$$

ans.  $x < -0.75$

$$x^2 - 36 > 0$$

$$|x| > 6$$

$$x^2 + 36 > -12x$$

$$x >$$

20. Solve,

$$(8 - 3j) - (12 + 4j)$$

ans.  $-4 - 7j$

$$(4 + 5j)^2$$

$$-9 - 40j$$

$$\left(\frac{8 - 3j}{4 + 5j}\right)$$

$$0.415 - 1.268j$$

$$(x - 6 - 4j)(x - 6 + 4j)$$

$$x^2 - 12x + 52$$

21. Find the limits below.

a)  $\lim_{t \rightarrow 0} \left( \frac{t^3 + 5}{5t^3 + 1} \right)$  ans. 5

b)  $\lim_{t \rightarrow \infty} \left( \frac{t^3 + 5}{5t^3 + 1} \right)$  ans. 1/5

22. Evaluate the limits,

$$\lim_{t \rightarrow \infty} \frac{\sin(t)}{t}$$

$$\lim_{t \rightarrow 0} \frac{\sin(t)}{t}$$

$$\lim_{t \rightarrow \infty} \sin(t)$$

$$\lim_{t \rightarrow 0} \sin(t)$$

$$\lim_{t \rightarrow \infty} te^{-t}$$

$$\lim_{t \rightarrow 0} te^{-t}$$

ans.

23. Reduce the following expression to partial fraction form.

$$\frac{x^2 + 4}{x^3 + 6x^2 + 9x}$$

ans.  $\frac{4}{9x} - \frac{13}{3(x+3)^2} + \frac{5}{9(x+3)}$

## 29.11 Challenge Problems

1. Write a program (in Scilab, C, etc.) that will perform the following calculation.

$$y(x) = \sum_{i=1}^n (x + i)^2$$

2. Develop a program to fit a polynomial to set set of points. In the case where the order of the polynomial does not allow an exact fit, a least squares method should be used to obtain the best fit.
3. Write a program in Scilab (ask if you would prefer to use another platform). The program should use the Newton-Raphson method to find the zeros of an arbitrary function. The functions below should be used for testing the program. The final program will be tested for robustness. The final program should be structured.

$$x^2 + x + 5 = 0$$

$$-5x^3 + x^2 + 10 = 0$$

## 30. TRIGONOMETRY

Topics:

- 

Objectives:

- 

### 30.1 Introduction

- Angles degrees and radians

$$360^\circ = 2\pi \text{ radians}$$

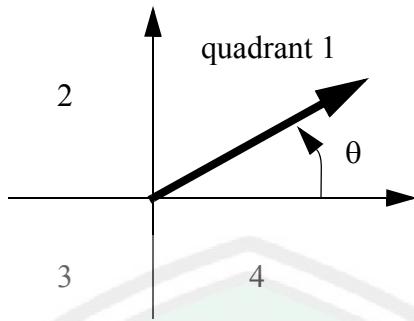
$$380^\circ = 20^\circ = -340^\circ$$

$$60 \text{ minutes} = 60' = 1^\circ$$

$$60 \text{ seconds} = 60'' = 1'$$

- Most computers do calculations in radians

- Angle quadrants,



### 30.1.1 Functions

- The basic trigonometry functions are,

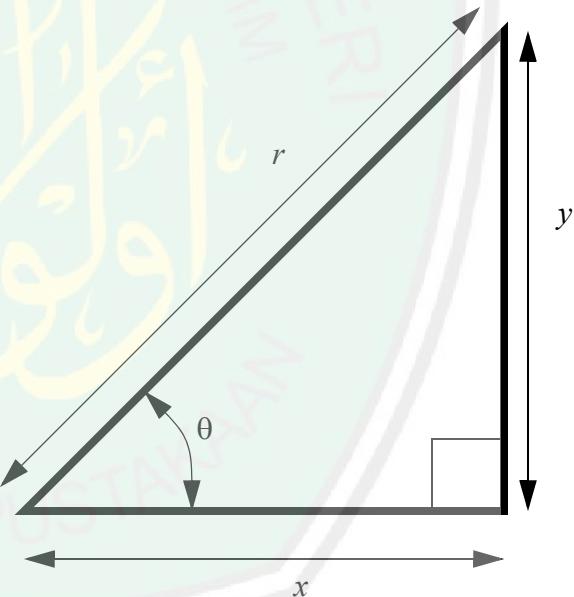
$$\sin \theta = \frac{y}{r} = \frac{1}{\csc \theta}$$

$$\cos \theta = \frac{x}{r} = \frac{1}{\sec \theta}$$

$$\tan \theta = \frac{y}{x} = \frac{1}{\cot \theta} = \frac{\sin \theta}{\cos \theta}$$

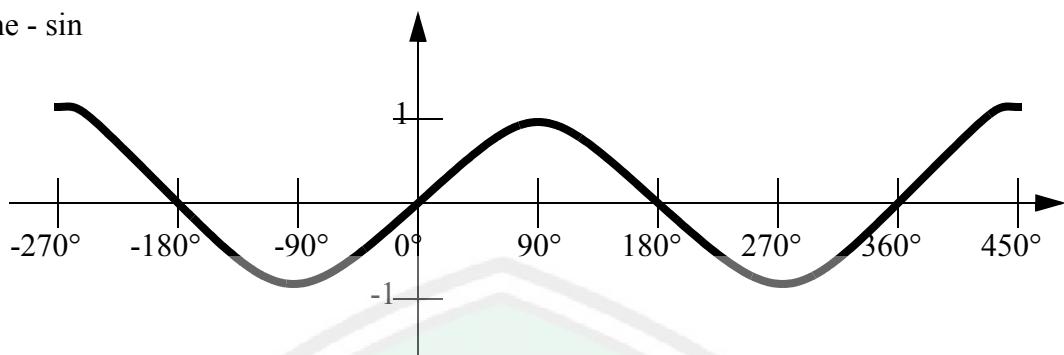
Pythagorean Formula:

$$r^2 = x^2 + y^2$$

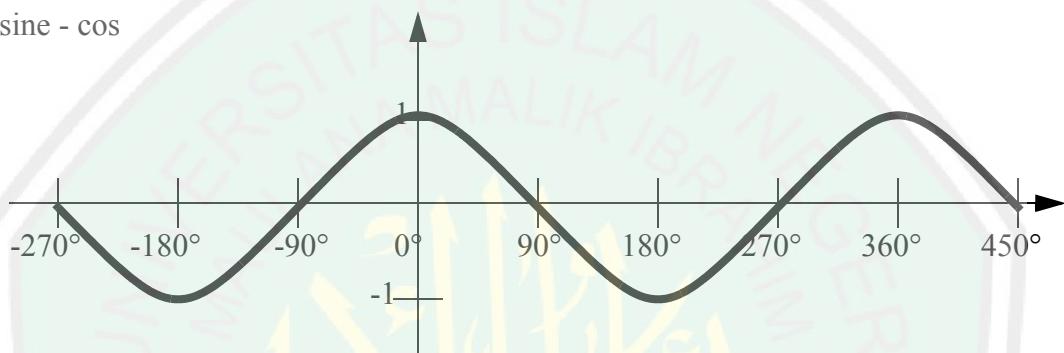


- Graphs of these functions are given below,

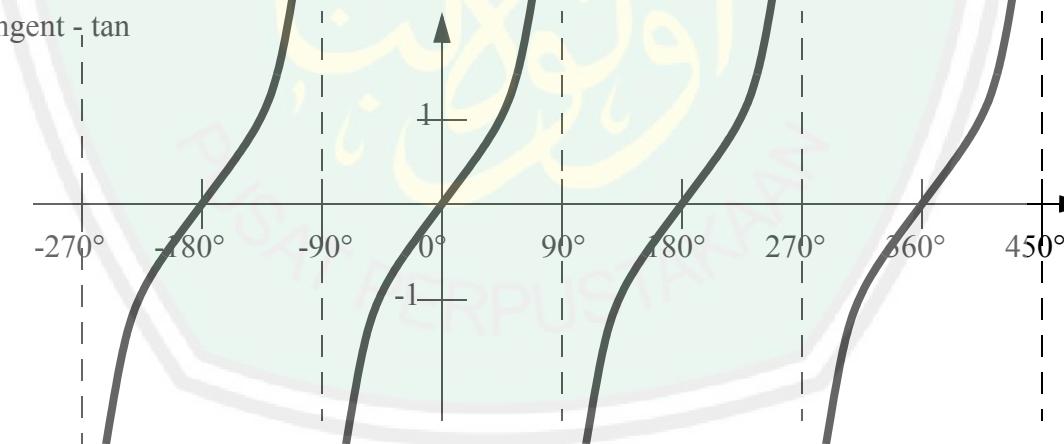
Sine - sin

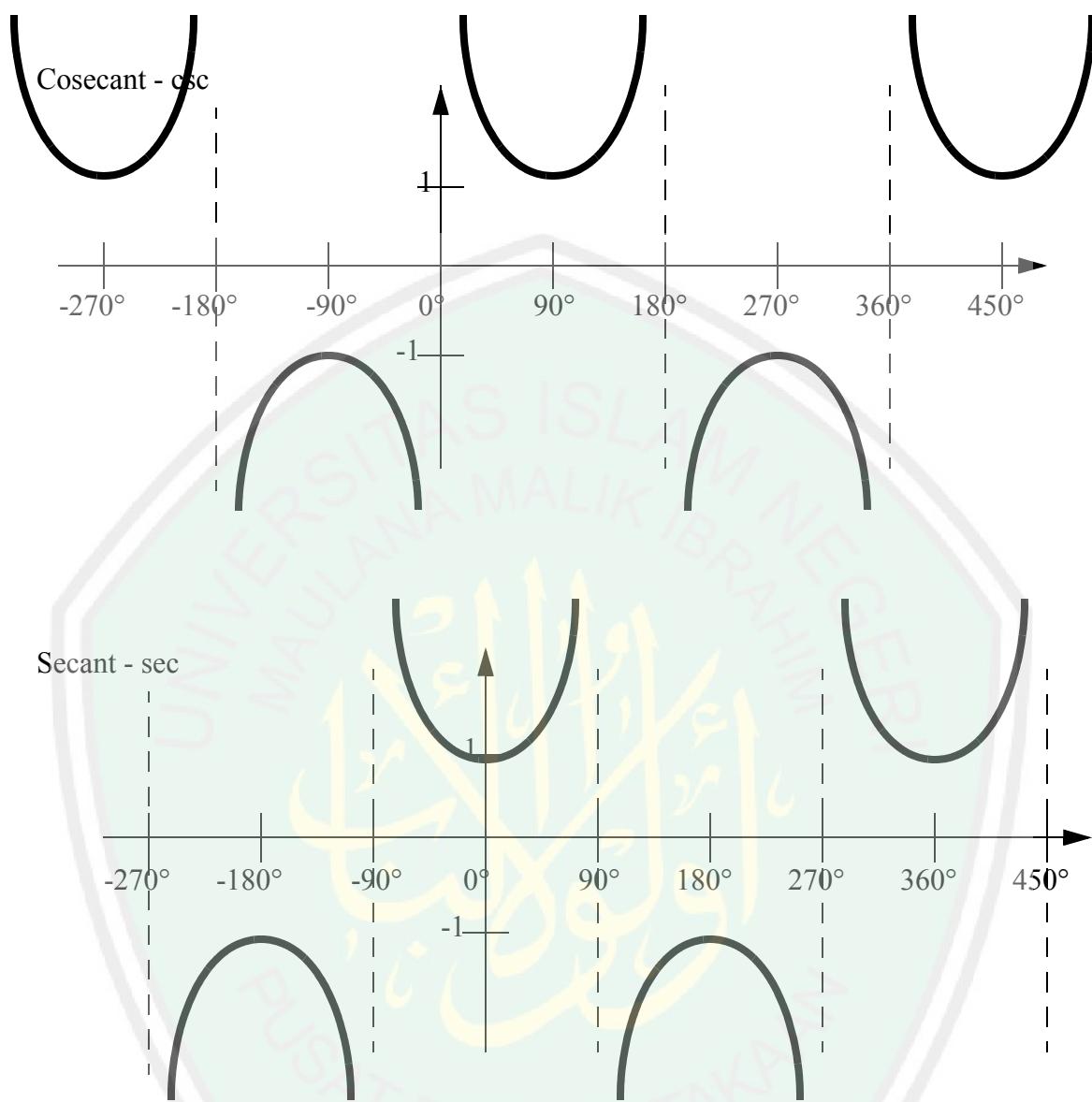


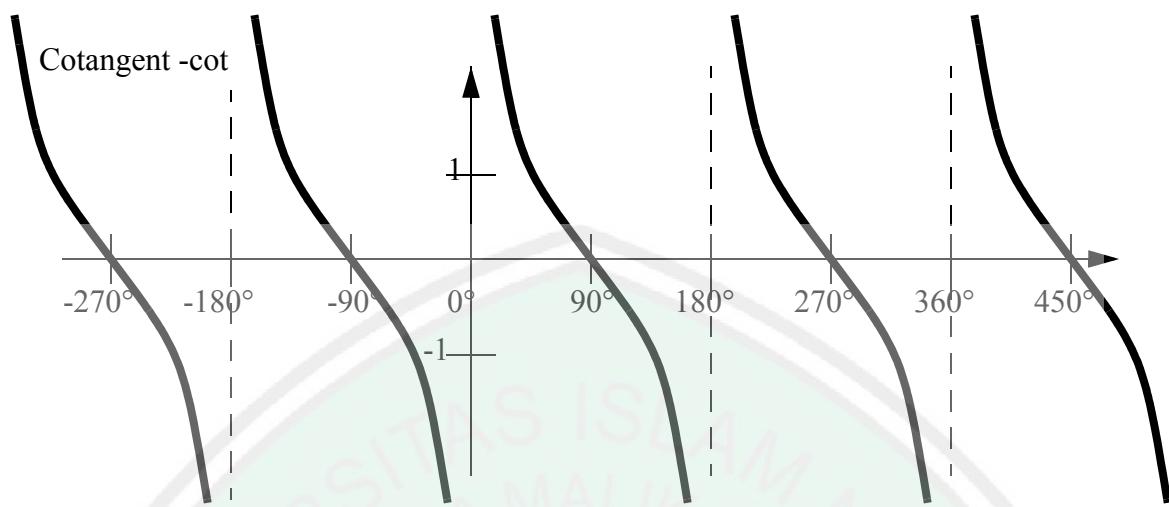
Cosine - cos



Tangent - tan







### 30.1.2 Inverse Functions

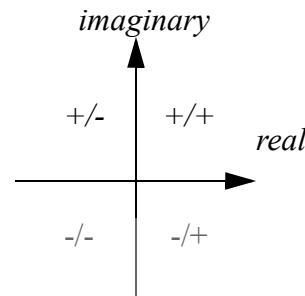
- Inverse Functions

$$\tan^{-1}\left(\frac{y}{x}\right) = \text{atan}\left(\frac{y}{x}\right) = \theta$$

$$\sin^{-1}\left(\frac{y}{r}\right) = \text{asin}\left(\frac{y}{r}\right) = \theta$$

$$\cos^{-1}\left(\frac{x}{r}\right) = \text{acos}\left(\frac{x}{r}\right) = \theta$$

Note: recall that  $\tan\theta = \frac{Re}{Im}$  but the  $\text{atan}\theta$  function in calculators and software only returns values between -90 to 90 degrees. To compensate for this the sign of the real and imaginary components must be considered to determine where the angle lies. If it lies beyond the -90 to 90 degree range the correct angle can be obtained by adding or subtracting 180 degrees.



- Note: trig calculations can take a while and should be minimized or avoided in programs.
- Scilab example,

```
sin(3.14159)
asin(0.5)
cos(3.14159)
acos(0.5)
tan(3.14159)
atan(0.5)
atan(1.0, 0.5)
```

### 30.1.3 Triangles

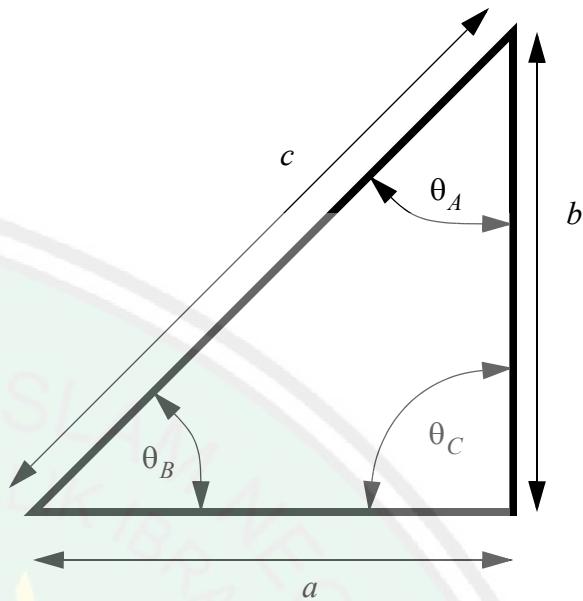
- NOTE: Keep in mind when finding these trig values, that any value that does not lie in the right hand quadrants of cartesian space, may need additions of  $\pm 90^\circ$  or  $\pm 180^\circ$ .

Cosine Law:

$$c^2 = a^2 + b^2 - 2ab \cos \theta_c$$

Sine Law:

$$\frac{a}{\sin \theta_A} = \frac{b}{\sin \theta_B} = \frac{c}{\sin \theta_C}$$



### 30.1.4 Relationships

- Now a group of trigonometric relationships will be given. These are often best used when attempting to manipulate equations.

$$\sin(\theta) = \sin(\theta \pm 360n) \quad n \in I$$

$$\sin(-\theta) = -\sin\theta \quad \cos(-\theta) = \cos\theta \quad \tan(-\theta) = -\tan\theta$$

$$\sin\theta = \cos(\theta - 90^\circ) = \cos(90^\circ - \theta) = \text{etc.}$$

$$\sin(\theta_1 \pm \theta_2) = \sin\theta_1 \cos\theta_2 \pm \cos\theta_1 \sin\theta_2 \quad \text{OR} \quad \sin(2\theta) = 2 \sin\theta \cos\theta$$

$$\cos(\theta_1 \pm \theta_2) = \cos\theta_1 \cos\theta_2 \mp \sin\theta_1 \sin\theta_2 \quad \text{OR} \quad \cos(2\theta) = (\cos\theta)^2 - (\sin\theta)^2$$

$$\tan(\theta_1 \pm \theta_2) = \frac{\tan\theta_1 \pm \tan\theta_2}{1 \mp \tan\theta_1 \tan\theta_2} \quad 1 + (\tan\theta)^2 = (\sec\theta)^2$$

$$\cot(\theta_1 \pm \theta_2) = \frac{\cot\theta_1 \cot\theta_2 \mp 1}{\tan\theta_2 \pm \tan\theta_1} \quad 1 + (\cot\theta)^2 = (\csc\theta)^2$$

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos\theta}{2}}$$

-ve if in left hand quadrants

$$\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos\theta}{2}}$$

$$\tan \frac{\theta}{2} = \frac{\sin\theta}{1 + \cos\theta} = \frac{1 - \cos\theta}{\sin\theta}$$

$$(\cos\theta)^2 + (\sin\theta)^2 = 1$$

- Scilab for trig identities,

#### EXAMPLES OF TRIG IDENTITIES

- These can also be related to complex exponents,

$$\cos \theta = \frac{e^{j\theta} + e^{-j\theta}}{2} \quad \sin \theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

### 30.1.5 Hyperbolic Functions

- The basic definitions are given below,

$$\sinh(x) = \frac{e^x - e^{-x}}{2} = \text{hyperbolic sine of } x$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2} = \text{hyperbolic cosine of } x$$

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \text{hyperbolic tangent of } x$$

$$\operatorname{csch}(x) = \frac{1}{\sinh(x)} = \frac{2}{e^x - e^{-x}} = \text{hyperbolic cosecant of } x$$

$$\operatorname{sech}(x) = \frac{1}{\cosh(x)} = \frac{2}{e^x + e^{-x}} = \text{hyperbolic secant of } x$$

$$\operatorname{coth}(x) = \frac{\cosh(x)}{\sinh(x)} = \frac{e^x + e^{-x}}{e^x - e^{-x}} = \text{hyperbolic cotangent of } x$$

- some of the basic relationships are,

$$\sinh(-x) = -\sinh(x)$$

$$\cosh(-x) = \cosh(x)$$

$$\tanh(-x) = -\tanh(x)$$

$$\operatorname{csch}(-x) = -\operatorname{csch}(x)$$

$$\operatorname{sech}(-x) = \operatorname{sech}(x)$$

$$\operatorname{coth}(-x) = -\operatorname{coth}(x)$$

- Some of the more advanced relationships are,

$$(\cosh x)^2 - (\sinh x)^2 = (\operatorname{sech} x)^2 + (\tanh x)^2 = (\operatorname{coth} x)^2 - (\operatorname{csch} x)^2 = 1$$

$$\sinh(x \pm y) = \sinh(x)\cosh(y) \pm \cosh(x)\sinh(y)$$

$$\cosh(x \pm y) = \cosh(x)\cosh(y) \pm \sinh(x)\sinh(y)$$

$$\tanh(x \pm y) = \frac{\tanh(x) \pm \tanh(y)}{1 \pm \tanh(x)\tanh(y)}$$

- Some of the relationships between the hyperbolic, and normal trigonometry functions are,

$$\sin(jx) = j \sinh(x)$$

$$j \sin(x) = \sinh(jx)$$

$$\cos(jx) = \cosh(x)$$

$$\cos(x) = \cosh(jx)$$

$$\tan(jx) = j \tanh(x)$$

$$j \tan(x) = \tanh(jx)$$

### 30.1.6 Special Relationships

- The Small Angle Approximation

$$\sin \theta = \theta \quad \text{when} \quad \theta \approx 0$$

### 30.1.7 Planes, Lines, etc.

- The most fundamental mathematical geometry is a line. The basic relationships are given below,

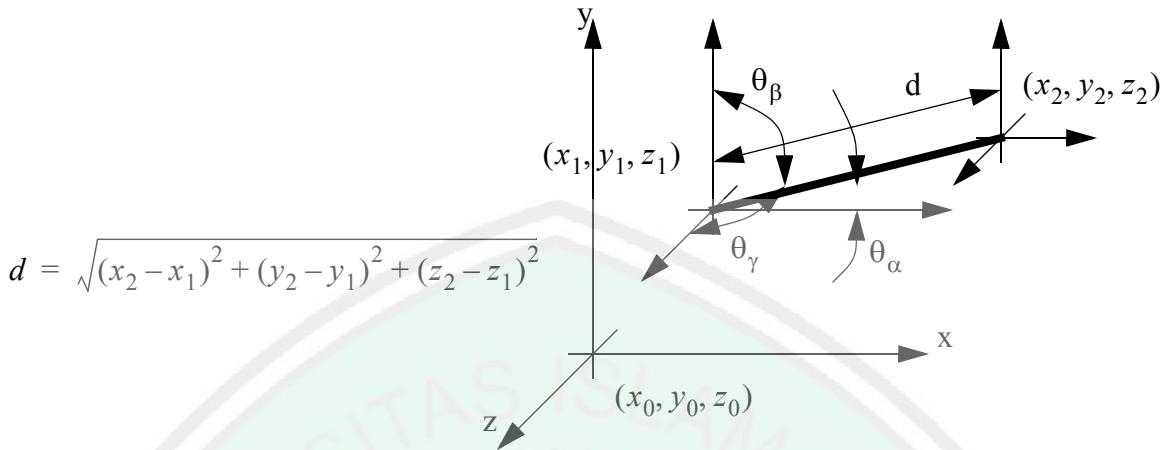
$$y = mx + b \quad \text{defined with a slope and intercept}$$

$$m_{\text{perpendicular}} = \frac{1}{m} \quad \text{a slope perpendicular to a line}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{the slope using two points}$$

$$\frac{x}{a} + \frac{y}{b} = 1 \quad \text{as defined by two intercepts}$$

- If we assume a line is between two points in space, and that at one end we have a local reference frame, there are some basic relationships that can be derived.



The direction cosines of the angles are,

$$\theta_\alpha = \cos\left(\frac{x_2 - x_1}{d}\right) \quad \theta_\beta = \cos\left(\frac{y_2 - y_1}{d}\right) \quad \theta_\gamma = \cos\left(\frac{z_2 - z_1}{d}\right)$$

$$(\cos\theta_\alpha)^2 + (\cos\theta_\beta)^2 + (\cos\theta_\gamma)^2 = 1$$

The equation of the line is,

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

Explicit

$$(x, y, z) = (x_1, y_1, z_1) + t((x_2, y_2, z_2) - (x_1, y_1, z_1))$$

Parametric  $t=[0,1]$

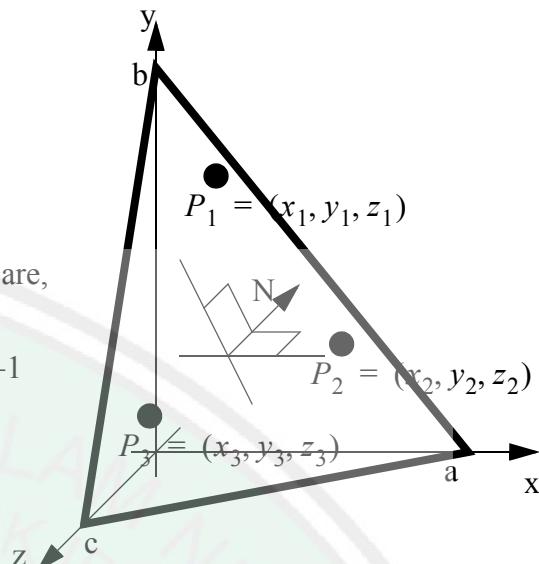
- The relationships for a plane are,

The explicit equation for a plane is,

$$Ax + By + Cy + D = 0$$

where the coefficients defined by the intercepts are,

$$A = \frac{1}{a} \quad B = \frac{1}{b} \quad C = \frac{1}{c} \quad D = -1$$



The determinant can also be used,

$$\det \begin{bmatrix} x - x_1 & y - y_1 & z - z_1 \\ x - x_2 & y - y_2 & z - z_2 \\ x - x_3 & y - y_3 & z - z_3 \end{bmatrix} = 0$$

$$\therefore \det \begin{bmatrix} y_2 - y_1 & z_2 - z_1 \\ y_3 - y_1 & z_3 - z_1 \end{bmatrix} (x - x_1) + \det \begin{bmatrix} z_2 - z_1 & x_2 - x_1 \\ z_3 - z_1 & x_3 - x_1 \end{bmatrix} (y - y_1)$$

$$+ \det \begin{bmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{bmatrix} (z - z_1) = 0$$

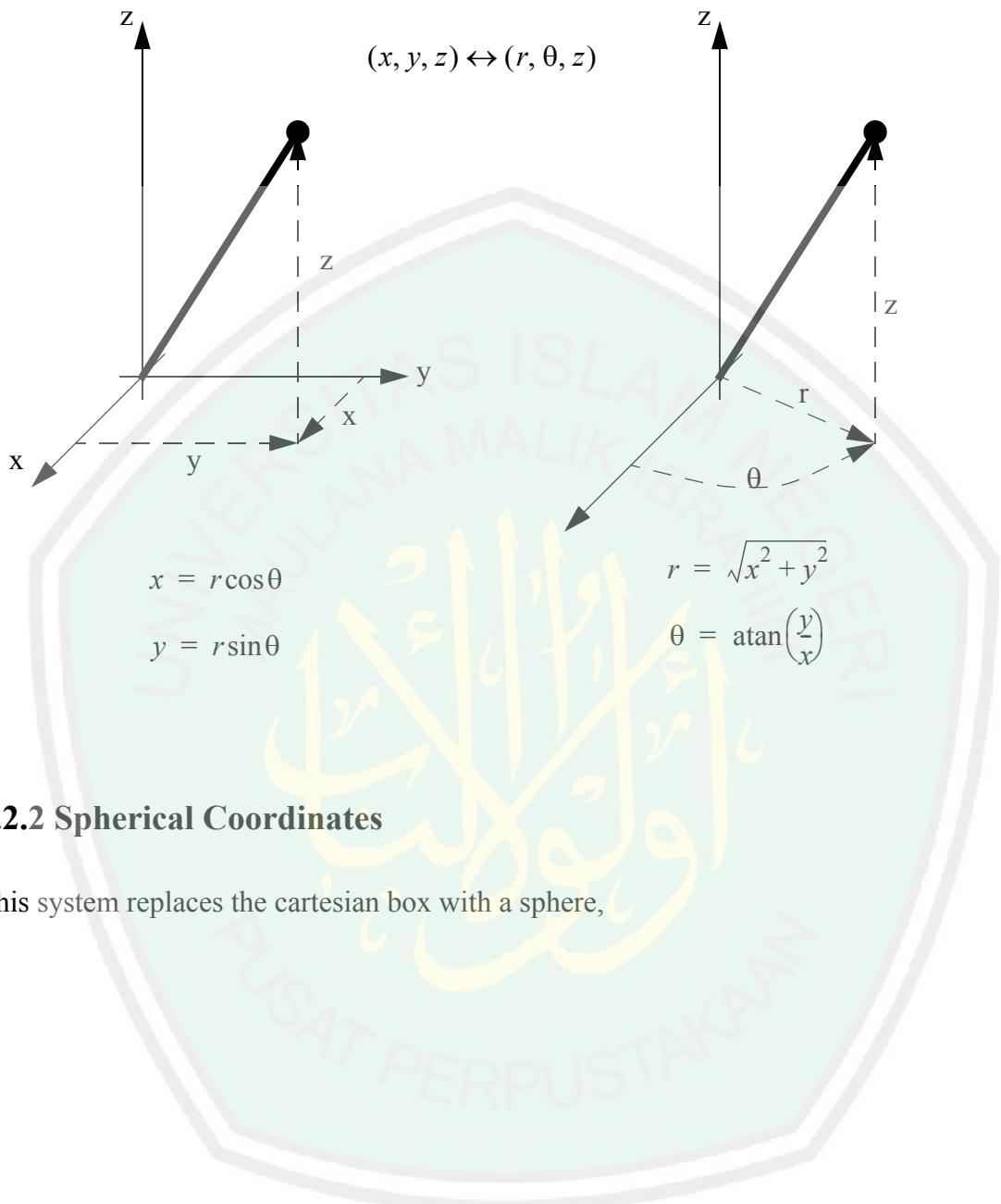
The normal to the plane (through the origin) is,

$$(x, y, z) = t(A, B, C)$$

## 30.2 Coordinate Systems

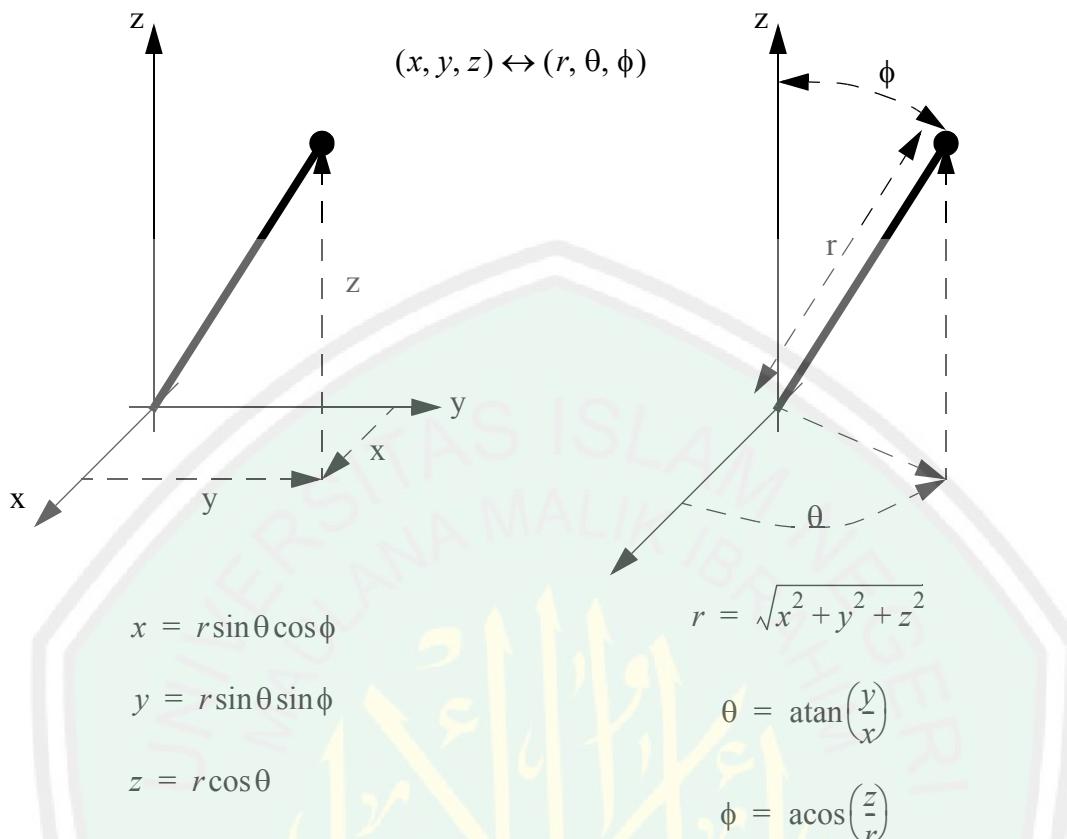
### 30.2.1 Cylindrical Coordinates

- Basically, these coordinates appear as if the cartesian box has been replaced with a cylinder,



### 30.2.2 Spherical Coordinates

- This system replaces the cartesian box with a sphere,



### 30.3 Problems

1. For the following angles, i) indicate the quadrants, ii) write the sine, cosine, and tangent values, iii) write an expression for all equivalent angles.

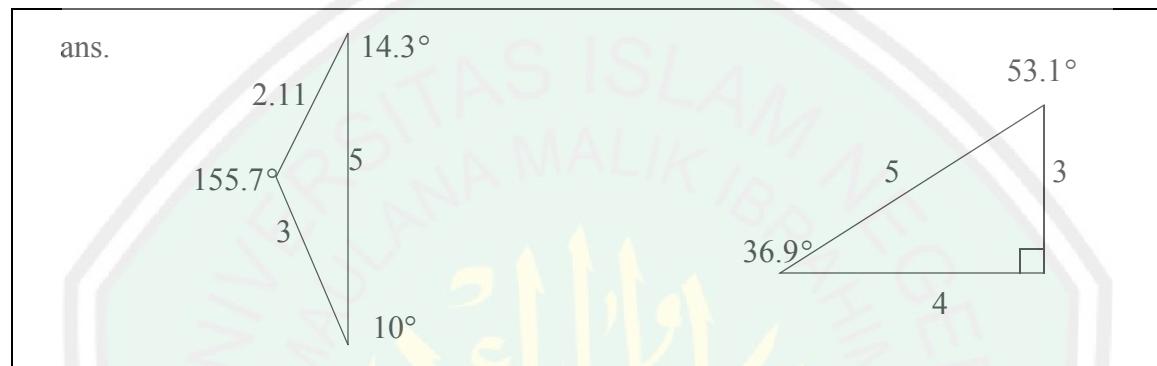
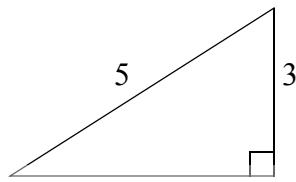
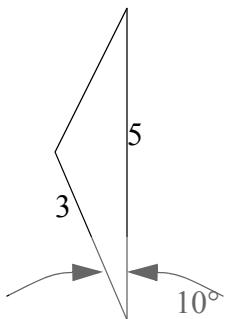
a)  $20^\circ$

b)  $2\text{rad}$

c)  $-2\text{rad}$

d)  $2000^\circ$

2. Find all of the missing side lengths and corner angles on the two triangles below,



3. A line that passes through the point (1, 2) and has a slope of 2. Find the equation for the line, and for a line perpendicular to it passing through the given point. (ans.  $y = 2x + a$ ,  $y = -0.5x+b$ )

4. Convert the following angles to/from degrees or radians.

to rad.:  $30^\circ, 280^\circ, 192^\circ 5' 30''$

to deg.:  $\frac{5\pi}{36} \text{ rad}, \frac{7\pi}{12} \text{ rad}$

ans.  $\frac{\pi}{6} \text{ rad}, \frac{14}{9}\pi \text{ rad}, 1.067\pi \text{ rad}$   
 $25^\circ, 105^\circ$

5. On a circle with a diameter of 0.5in. what is,

- a) the arc length for a 1 rad angle
- b) the arc length for a 20 degree angle
- c) the circumference.
- d) the angle resulting in a 0.5m arc.

(ans. 0.25in,  $\pi/36$  in, 0.5 PI in., 78.7rad.)

6. If a 30cm radius rotating mass has a rotational rate of 200rpm, how fast is a point at the outside moving? What angular velocity is required for an outside speed of 2m/s? (ans. 6.28m/s, 6.67 rad/s)

7. Given the (x, y) points below, find the angle

a) (3, 4)

(ans.  $53^\circ$ )

b) (-3, 4)

$126^\circ$

c) (3, -4)

$-53^\circ$

d) (-3, -4)

$-126^\circ$


8. Convert the following to a function with the smallest positive angle possible

a)  $\sin(200^\circ)$

(ans.  $-\sin(20^\circ)$ )

b)  $\tan(-170^\circ)$

$\tan(10^\circ)$

c)  $\cos(325^\circ)$

$\cos(35^\circ)$

d)  $\cos(3.5\pi \text{rad})$

$\cos(90^\circ)$  or  $\sin(0^\circ)$


9. Given a triangle ABC find the missing side lengths or angles

$\theta_A$	$\theta_A$	$\theta_A$	$L_A$	$L_B$	$L_C$

10. Simplify the following expressions.

$$\sin 2\theta \left( \frac{(\cos 2\theta)^2}{\sin 2\theta} + \sin 2\theta \right) \quad (\text{ans. } = 1)$$

11. Prove the following

a)  $\frac{\sin \theta \cos \theta}{\tan \theta} = \frac{(\cos \theta)^2 - (\sin \theta)^2}{1 - (\tan \theta)^2}$

b)  $\sin \theta = 1 - \frac{(\cos \theta)^2}{1 + (\sin \theta)}$

c)  $(1 + (\tan \theta)^2)(1 - (\sin \theta)^2) = 1$

d)  $\sin(\theta_1 + \theta_2) - \sin(\theta_1 - \theta_2) = 2 \cos \theta_1 \sin \theta_2$

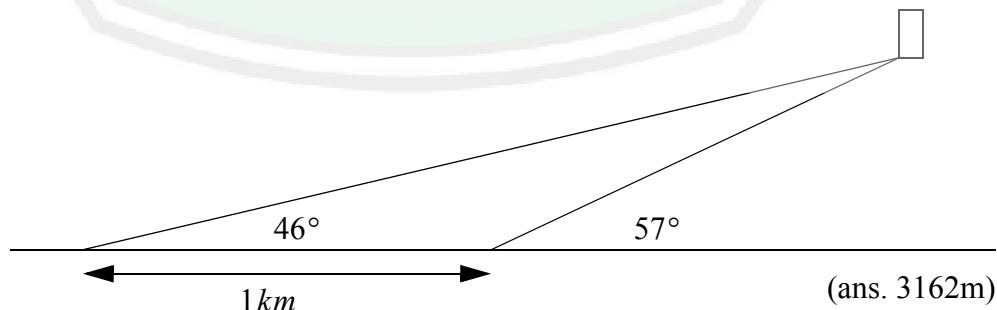
e)  $\sin(\theta_1 + \theta_2)(-\sin(\theta_1 - \theta_2)) = (\sin(\theta_1))^2 - (\sin(\theta_2))^2$

f)  $\tan \theta \sin 2\theta = 2(\sin \theta)^2$

12. Solve the equation,

$$(\cos \theta)^2 + 2 \sin \theta - 2 = 0 \quad (\text{ans. } \theta =$$

13. Two observers measure the height of a rocket by angle. Calculate the height of the rocket.



## 31. VECTORS

Topics:

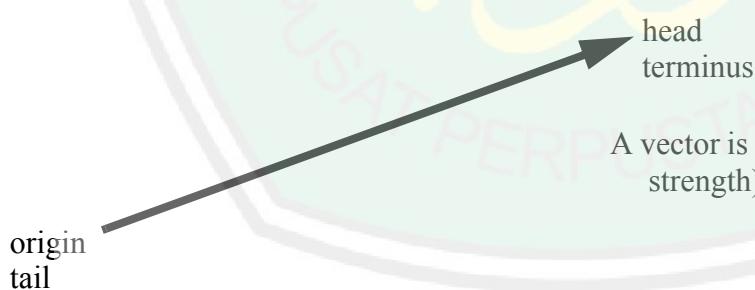
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Objectives:

- 

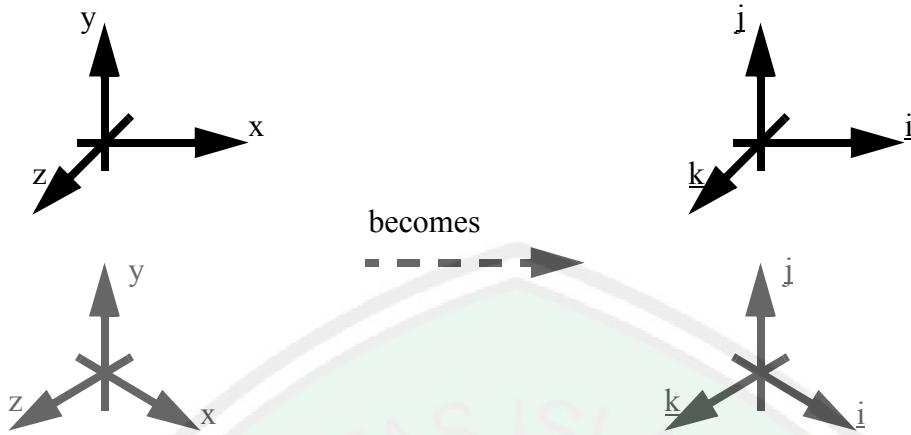
### 31.1 Introduction

- Vectors are often drawn with arrows, as shown below,



A vector is said to have magnitude (length or strength) and direction.

- Cartesian notation is also a common form of usage.



- Vectors can be added and subtracted, numerically and graphically,

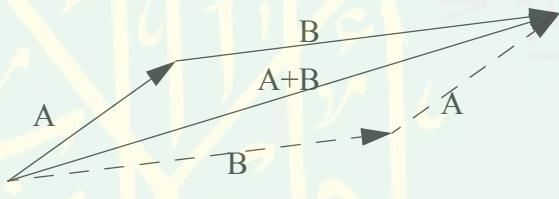
$$A = (2, 3, 4)$$

$$B = (7, 8, 9)$$

$$A + B = (2 + 7, 3 + 8, 4 + 9)$$

$$A - B = (2 - 7, 3 - 8, 4 - 9)$$

Parallelogram Law



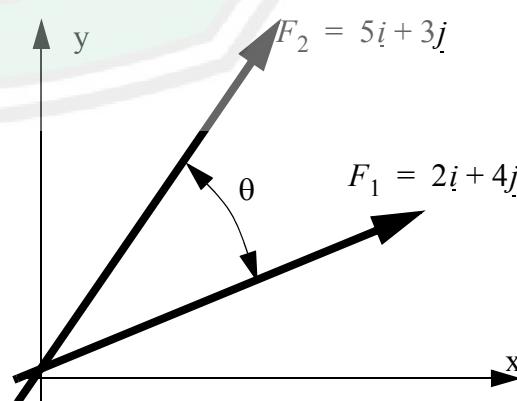
### 31.1.1 Dot (Scalar) Product

- We can use a dot product to find the angle between two vectors

$$\cos \theta = \frac{F_1 \cdot F_2}{|F_1||F_2|}$$

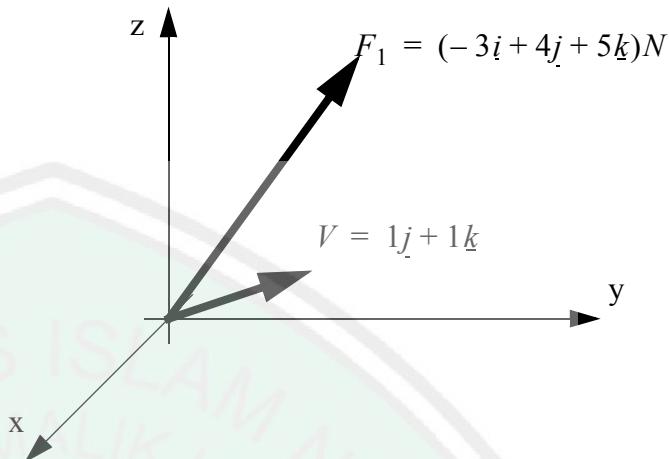
$$\therefore \theta = \arccos \left( \frac{(2)(5) + (4)(3)}{\sqrt{2^2 + 4^2} \sqrt{5^2 + 3^2}} \right)$$

$$\therefore \theta = \arccos \left( \frac{22}{\sqrt{20}(6)} \right) = 32.5^\circ$$



- We can use a dot product to project one vector onto another vector.

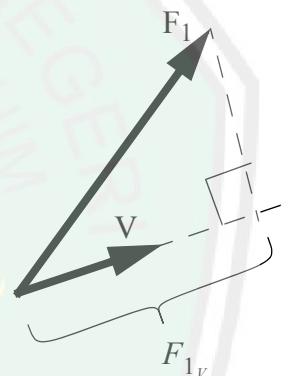
We want to find the component of force  $F_1$  that projects onto the vector  $V$ . To do this we first convert  $V$  to a unit vector, if we do not, the component we find will be multiplied by the magnitude of  $V$ .



$$\lambda_V = \frac{V}{|V|} = \frac{1j + 1k}{\sqrt{1^2 + 1^2}} = 0.707j + 0.707k$$

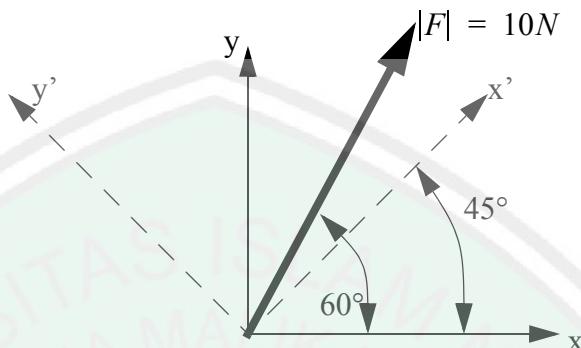
$$F_{1_V} = \lambda_V \bullet F_1 = (0.707j + 0.707k) \bullet (-3i + 4j + 5k)N$$

$$\therefore F_{1_V} = (0)(-3) + (0.707)(4) + (0.707)(5) = 6N$$



- We can consider the basic properties of the dot product and units vectors.

Unit vectors are useful when breaking up vector magnitudes and direction. As an example consider the vector, and the displaced x-y axes shown below as  $x'$ - $y'$ .



We could write out 5 vectors here, relative to the x-y axis,

$$x \text{ axis} = 2i$$

$$y \text{ axis} = 3j$$

$$x' \text{ axis} = 1i + 1j$$

$$y' \text{ axis} = -1i + 1j$$

$$F = 10N\angle 60^\circ = (10\cos 60^\circ)i + (10\sin 60^\circ)j$$

None of these vectors has a magnitude of 1, and hence they are not unit vectors. But, if we find the equivalent vectors with a magnitude of one we can simplify many tasks. In particular if we want to find the x and y components of  $F$  relative to the x-y axis we can use the dot product.

$$\lambda_x = 1i + 0j \quad (\text{unit vector for the x-axis})$$

$$F_x = \lambda_x \bullet F = (1i + 0j) \bullet [(10\cos 60^\circ)i + (10\sin 60^\circ)j]$$

$$\therefore = (1)(10\cos 60^\circ) + (0)(10\sin 60^\circ) = 10N\cos 60^\circ$$

This result is obvious, but consider the other obvious case where we want to project a vector onto itself,

$$\lambda_F = \frac{F}{|F|} = \frac{10\cos 60^\circ i + 10\sin 60^\circ j}{10} = \cos 60^\circ i + \sin 60^\circ j$$

Incorrect - Not using a unit vector

$$\begin{aligned} F_F &= F \bullet F \\ &= ((10\cos 60^\circ i + 10\sin 60^\circ j) \bullet ((10\cos 60^\circ i + 10\sin 60^\circ j)) \\ &= (10\cos 60^\circ)(10\cos 60^\circ) + (10\sin 60^\circ)(10\sin 60^\circ) \\ &= 100((\cos 60^\circ)^2 + (\sin 60^\circ)^2) = \cancel{100} \end{aligned}$$

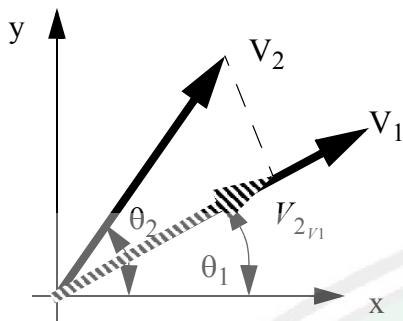
Using a unit vector

$$\begin{aligned} F_F &= F \bullet \lambda_F \\ &= ((10\cos 60^\circ i + 10\sin 60^\circ j) \bullet ((\cos 60^\circ i + \sin 60^\circ j)) \\ &= (10\cos 60^\circ)(\cos 60^\circ) + (10\sin 60^\circ)(\sin 60^\circ) \\ &= 10((\cos 60^\circ)^2 + (\sin 60^\circ)^2) = 10 \quad \text{Correct} \end{aligned}$$

Now consider the case where we find the component of  $F$  in the  $x'$  direction. Again, this can be done using the dot product to project  $F$  onto a unit vector.

$$\begin{aligned} u_{x'} &= \cos 45^\circ i + \sin 45^\circ j \\ F_{x'} &= F \bullet \lambda_{x'} = ((10\cos 60^\circ i + 10\sin 60^\circ j) \bullet ((\cos 45^\circ i + \sin 45^\circ j)) \\ &= (10\cos 60^\circ)(\cos 45^\circ) + (10\sin 60^\circ)(\sin 45^\circ) \\ &= 10(\cos 60^\circ \cos 45^\circ + \sin 60^\circ \sin 45^\circ) = 10(\cos(60^\circ - 45^\circ)) \end{aligned}$$

Here we see a few cases where the dot product has been applied to find the vector projected onto a unit vector. Now finally consider the more general case,



First, by inspection, we can see that the component of  $V_2$  (projected) in the direction of  $V_1$  will be,

$$|V_{2_{V1}}| = |V_2| \cos(\theta_2 - \theta_1)$$

Next, we can manipulate this expression into the dot product form,

$$\begin{aligned} &= |V_2| (\cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2) \\ &= |V_2| [(\cos \theta_1 i + \sin \theta_1 j) \bullet (\cos \theta_2 i + \sin \theta_2 j)] \\ &= |V_2| \left[ \frac{V_1}{|V_1|} \bullet \frac{V_2}{|V_2|} \right] = |V_2| \left[ \frac{V_1 \bullet V_2}{|V_1||V_2|} \right] = \frac{V_1 \bullet V_2}{|V_1|} = V_2 \bullet \lambda_{V_1} \end{aligned}$$

Or more generally,

$$|V_{2_{V1}}| = |V_2| \cos(\theta_2 - \theta_1) = |V_2| \left[ \frac{V_1 \bullet V_2}{|V_1||V_2|} \right]$$

$$\therefore |V_2| \cos(\theta_2 - \theta_1) = |V_2| \left[ \frac{V_1 \bullet V_2}{|V_1||V_2|} \right]$$

$$\therefore \cos(\theta_2 - \theta_1) = \left[ \frac{V_1 \bullet V_2}{|V_1||V_2|} \right]$$

\*Note that the dot product also works in 3D, and similar proofs are used.

- In Scilab,

$$\begin{aligned} \mathbf{A} &= [1 \ 2 \ 3]; \\ \mathbf{B} &= [4 \ 5 \ 6]; \\ \text{dot} &= \mathbf{A}' * \mathbf{B}; \end{aligned}$$

### 31.1.2 Cross Product

- First, consider an example,

$$\mathbf{F} = (-6.43\mathbf{i} + 7.66\mathbf{j} + 0\mathbf{k})N$$

$$\mathbf{d} = (2\mathbf{i} + 0\mathbf{j} + 0\mathbf{k})m$$

$$\mathbf{M} = \mathbf{d} \times \mathbf{F} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2m & 0m & 0m \\ -6.43N & 7.66N & 0N \end{bmatrix}$$

NOTE: note that the cross product here is for the right hand rule coordinates. If the left handed coordinate system is used  $\mathbf{F}$  and  $\mathbf{d}$  should be reversed.

$$\therefore \mathbf{M} = (0m0N - 0m(7.66N))\mathbf{i} - (2m0N - 0m(-6.43N))\mathbf{j} + (2m(7.66N) - 0m(-6.43N))\mathbf{k} = 15.3\mathbf{k}(mN)$$

NOTE: there are two things to note about the solution. First, it is a vector. Here there is only a z component because this vector points out of the page, and a rotation about this vector would rotate on the plane of the page. Second, this result is positive, because the positive sense is defined by the vector system. In this right handed system find the positive rotation by pointing your right hand thumb towards the positive axis (the 'k' means that the vector is about the z-axis here), and curl your fingers, that is the positive direction.

- The basic properties of the cross product are,

The cross (or vector) product of two vectors will yield a new vector perpendicular to both vectors, with a magnitude that is a product of the two magnitudes.

$$V_1 \times V_2 = (x_1\hat{i} + y_1\hat{j} + z_1\hat{k}) \times (x_2\hat{i} + y_2\hat{j} + z_2\hat{k})$$

$$V_1 \times V_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

$$V_1 \times V_2 = (y_1z_2 - z_1y_2)\hat{i} + (z_1x_2 - x_1z_2)\hat{j} + (x_1y_2 - y_1x_2)\hat{k}$$

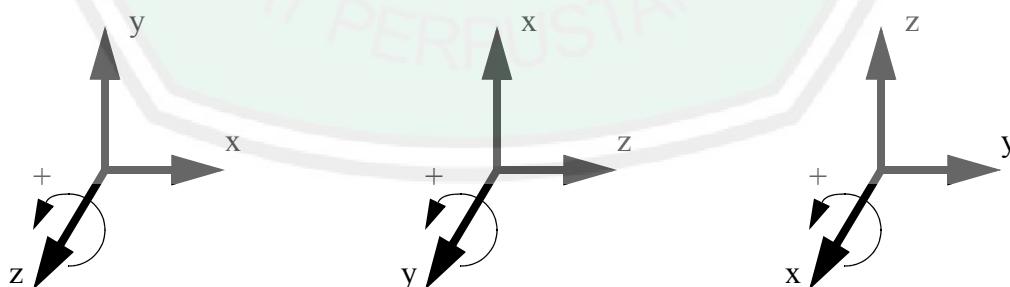

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We can also find a unit vector normal 'n' to the vectors 'V1' and 'V2' using a cross product, divided by the magnitude.

$$\lambda_n = \frac{V_1 \times V_2}{|V_1 \times V_2|}$$

- When using a left/right handed coordinate system,

The positive orientation of angles and moments about an axis can be determined by pointing the thumb of the right hand along the axis of rotation. The fingers curl in the positive direction.



- The properties of the cross products are,

The cross product is distributive, but not associative. This allows us to collect terms in a cross product operation, but we cannot change the order of the cross product.

$$r_1 \times F + r_2 \times F = (r_1 + r_2) \times F \quad \text{DISTRIBUTIVE}$$

---

$r \times F \neq F \times r$ but $r \times F = -(F \times r)$	NOT ASSOCIATIVE
---	-----------------

- In Scilab,

```
function val=crossproduct(A, B) // No function is defined so use the following
    val = [A(2) * B(3) - A(3) * B(2) ;
           A(3) * B(1) - A(1) * B(3)
           A(1) * B(2) - A(2) * B(1)];
endfunction
```

## 31.2 Problems

1.

## 32. MATRICES

Topics:

- 

Objectives:

- 

### 32.1 Introduction

- Matrices allow simple equations that drive a large number of repetitive calculations - as a result they are found in many computer applications.
- A matrix has the form seen below,

$$\begin{bmatrix} a_{11} & a_{21} & \dots & a_{n1} \\ a_{12} & a_{22} & \dots & a_{n2} \\ \dots & \dots & \dots & \dots \\ a_{1m} & a_{2m} & \dots & a_{nm} \end{bmatrix}$$

n columns      m rows

If  $n=m$  then the matrix is said to be square.  
 Many applications require square matrices.  
 We may also represent a matrix as a 1-by-3 for a vector.

- In Scilab,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

```

A = [ 1 2 3 ; 4 5 6 ; 7 8 9 ]; // ; means next row
A(2, 3) // , means next element
A(1:2, 2:3) // : means a range
A(:, 2)
A(2, :)
A(2, $) // means last row or column
A($, 2)
// other commands - please use help to explore
// these
size(A); // returns the rows, columns, etc of A
ones(A); // fills the matrix with 1s
zeros(A); // fills the matrix with 0s
eye(5, 5); // creates a 5x5 identity matrix
diag(A); // gets the diagonal of matrix A
rand()
max()
rank()
cond()
spec()
trace()
// also try
B = [ 1 2 3 ];
C = [ 1 ; 2 ; 3 ];
D = 1:0.1:3;
E = [ B 4 ]; // adds a column to B to make E
F = [ C ; 4 ]; // add a row to C to make F
length(C); // the rows in C
G = A(2:3 , 1:2); // extracts a 2x2 from A
H = B. * C; // element wise operation, a 3x3 results
J = B^2; // same as B*B
K = B.^2; // each element is squared

```

### 32.1.1 Basic Matrix Operations

- Matrix operations are available for many of the basic algebraic expressions, examples are given

below. There are also many restrictions - many of these are indicated.

$$A = 2 \quad B = \begin{bmatrix} 3 & 4 & 5 \\ 6 & 7 & 8 \\ 9 & 10 & 11 \end{bmatrix} \quad C = \begin{bmatrix} 12 & 13 & 14 \\ 15 & 16 & 17 \\ 18 & 19 & 20 \end{bmatrix} \quad D = \begin{bmatrix} 21 \\ 22 \\ 23 \end{bmatrix} \quad E = \begin{bmatrix} 24 & 25 & 26 \end{bmatrix}$$


---

Addition/Subtraction

$$A + B = \begin{bmatrix} 3+2 & 4+2 & 5+2 \\ 6+2 & 7+2 & 8+2 \\ 9+2 & 10+2 & 11+2 \end{bmatrix} \quad B + C = \begin{bmatrix} 3+12 & 4+13 & 5+14 \\ 6+15 & 7+16 & 8+17 \\ 9+18 & 10+19 & 11+20 \end{bmatrix}$$

$B + D$  = not valid

$$B - A = \begin{bmatrix} 3-2 & 4-2 & 5-2 \\ 6-2 & 7-2 & 8-2 \\ 9-2 & 10-2 & 11-2 \end{bmatrix} \quad B - C = \begin{bmatrix} 3-12 & 4-13 & 5-14 \\ 6-15 & 7-16 & 8-17 \\ 9-18 & 10-19 & 11-20 \end{bmatrix}$$

$B - D$  = not valid

$$A \cdot B = \begin{bmatrix} 3(2) & 4(2) & 5(2) \\ 6(2) & 7(2) & 8(2) \\ 9(2) & 10(2) & 11(2) \end{bmatrix} \quad \frac{B}{A} = \begin{bmatrix} \frac{3}{2} & \frac{4}{2} & \frac{5}{2} \\ \frac{6}{2} & \frac{7}{2} & \frac{8}{2} \\ \frac{9}{2} & \frac{10}{2} & \frac{11}{2} \end{bmatrix}$$

$$B \cdot D = \begin{bmatrix} (3 \cdot 21 + 4 \cdot 22 + 5 \cdot 23) \\ (6 \cdot 21 + 7 \cdot 22 + 8 \cdot 23) \\ (9 \cdot 21 + 10 \cdot 22 + 11 \cdot 23) \end{bmatrix} \quad D \cdot E = 21 \cdot 24 + 22 \cdot 25 + 23 \cdot 26$$

$$B \cdot C = \begin{bmatrix} (3 \cdot 12 + 4 \cdot 15 + 5 \cdot 18) & (3 \cdot 13 + 4 \cdot 16 + 5 \cdot 19) & (3 \cdot 14 + 4 \cdot 17 + 5 \cdot 20) \\ (6 \cdot 12 + 7 \cdot 15 + 8 \cdot 18) & (6 \cdot 13 + 7 \cdot 16 + 8 \cdot 19) & (6 \cdot 14 + 7 \cdot 17 + 8 \cdot 20) \\ (9 \cdot 12 + 10 \cdot 15 + 11 \cdot 18) & (9 \cdot 13 + 10 \cdot 16 + 11 \cdot 19) & (9 \cdot 14 + 10 \cdot 17 + 11 \cdot 20) \end{bmatrix}$$

$\frac{B}{C}, \frac{B}{D}, \frac{D}{B}$ , etc = not allowed (see inverse)

Note: To multiply matrices, the first matrix must have the same number of columns as the second matrix has rows.

### 32.1.2 Determinants

- Determinants give a 'magnitude product' of a matrix. This can be thought of as a general magnitude of the matrix.
- To find a determinant the matrix must be square.
- 

- For a 2 by 2 matrix.

Determinant

$$|B| = 3 \cdot \begin{vmatrix} 7 & 8 \\ 10 & 11 \end{vmatrix} - 4 \cdot \begin{vmatrix} 6 & 8 \\ 9 & 11 \end{vmatrix} + 5 \cdot \begin{vmatrix} 6 & 7 \\ 9 & 10 \end{vmatrix} = 3 \cdot (-3) - 4 \cdot (-6) + 5 \cdot (-3) = 0$$

$$\begin{vmatrix} 7 & 8 \\ 10 & 11 \end{vmatrix} = (7 \cdot 11) - (8 \cdot 10) = -3$$

$$\begin{vmatrix} 6 & 8 \\ 9 & 11 \end{vmatrix} = (6 \cdot 11) - (8 \cdot 9) = -6$$

$$\begin{vmatrix} 6 & 7 \\ 9 & 10 \end{vmatrix} = (6 \cdot 10) - (7 \cdot 9) = -3$$

$|D|, |E|$  = not valid (matrices not square)

- For a 3 by 3 matrix

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$= a(ei - fh) - b(di - fg) + c(dh - eg)$$

- Higher order matrices follow a similar pattern. For example a 4th order matrix has the pattern,

$$\begin{vmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{vmatrix} = a \begin{vmatrix} f & g & h \\ j & k & l \\ n & o & p \end{vmatrix} - b \begin{vmatrix} e & g & h \\ i & k & l \\ m & o & p \end{vmatrix} + c \begin{vmatrix} e & f & h \\ i & j & l \\ m & n & p \end{vmatrix} - d \begin{vmatrix} e & f & g \\ i & j & k \\ m & n & o \end{vmatrix}$$

### 32.1.3 Transpose

$$B^T = \begin{bmatrix} 3 & 6 & 9 \\ 4 & 7 & 10 \\ 5 & 8 & 11 \end{bmatrix} \quad D^T = \begin{bmatrix} 21 & 22 & 23 \end{bmatrix} \quad E^T = \begin{bmatrix} 24 \\ 25 \\ 26 \end{bmatrix}$$

### 32.1.4 Adjoint Matrices

$$\|B\| = \left[ \begin{array}{cc|cc|cc} 7 & 8 & - & 6 & 8 & 6 & 7 \\ 10 & 11 & & 10 & 11 & 9 & 10 \\ \hline - & 4 & 5 & 3 & 5 & - & 3 & 4 \\ & 10 & 11 & 9 & 11 & & 9 & 10 \\ \hline & 4 & 5 & - & 3 & 5 & 3 & 4 \\ & 7 & 8 & & 6 & 8 & 6 & 7 \end{array} \right]^T$$

The matrix of determinant to the left is made up by getting rid of the row and column of the element, and then finding the determinant of what is left. Note the sign changes on alternating elements.

$$\|D\| = \text{invalid (must be square)}$$

### 32.1.5 Inverse Matrices

$$D = B \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

To solve this equation for x,y,z we need to move B to the left hand side. To do this we use the inverse.

$$B^{-1}D = B^{-1} \cdot B \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$B^{-1}D = I \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$B^{-1} = \frac{\|B\|}{|B|}$$

In this case B is singular, so the inverse is undetermined, and the matrix is indeterminate.

$$D^{-1} = \text{invalid (must be square)}$$

- Some Scilab,

```
A = [ 1 2 3 ; 4 5 6 ; 7 8 9 ];
B = [ 10 ; 11 ; 12 ];
A' // transpose
det(A) // determinant
inv(A) // inverse
A^-1 // also inverse
spec(A)
[D, X] = bdiag(A)
linsolve(A, B)
A * B
B * A
A + B // will not work
```

### 32.1.6 Identity Matrix

This is a square matrix that is the matrix equivalent to ‘1’.

$$B \cdot I = I \cdot B = B$$

$$D \cdot I = I \cdot D = D$$

$$B^{-1} \cdot B = I$$

$$\begin{bmatrix} 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \text{etc}=I$$

### 32.1.7 Eigenvalues

- The eigenvalue of a matrix is found using,

$$|A - \lambda I| = 0$$

### 32.1.8 Eigenvectors

## 32.2 Matrix Applications

### 32.2.1 Solving Linear Equations with Matrices

- Note: if the determinant of a matrix is 0, the matrix is singular and there is no solution for the linear equations

Given,

$$x + y = 5$$

$$2x + 3y = 8$$

The equations can be written in matrix form,

$$\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

The solution is found using the inverse,

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 5 \\ 8 \end{bmatrix} = \frac{\begin{bmatrix} 3 & -1 \\ -2 & 1 \\ 1 & 1 \\ 2 & 3 \end{bmatrix}}{\begin{bmatrix} 8 \end{bmatrix}} \begin{bmatrix} 5 \\ 8 \end{bmatrix} = \frac{\begin{bmatrix} 15 - 8 \\ -10 + 8 \\ 3 - 2 \end{bmatrix}}{\begin{bmatrix} 8 \end{bmatrix}} = \begin{bmatrix} 7 \\ -2 \end{bmatrix}$$

- In Scilab,

```
A = [1 1 ; 2 3];
B = [5; 8];
X = inv(A) * B
X = A \ B; // another way to solve linear equations
r = B - A * x; // The residual should be 0
```

- We can solve systems of equations using the inverse matrix,

Given,

$$2 \cdot x + 4 \cdot y + 3 \cdot z = 5$$

$$9 \cdot x + 6 \cdot y + 8 \cdot z = 7$$

$$11 \cdot x + 13 \cdot y + 10 \cdot z = 12$$

Write down the matrix, then rearrange, and solve.

$$\begin{bmatrix} 2 & 4 & 3 \\ 9 & 6 & 8 \\ 11 & 13 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ 12 \end{bmatrix} \quad \therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & 4 & 3 \\ 9 & 6 & 8 \\ 11 & 13 & 10 \end{bmatrix}^{-1} \begin{bmatrix} 5 \\ 7 \\ 12 \end{bmatrix} = \boxed{\boxed{\boxed{\quad}}}$$

- We can solve systems of equations using Cramer's rule (with determinants),

Given,

$$2 \cdot x + 4 \cdot y + 3 \cdot z = 5$$

$$9 \cdot x + 6 \cdot y + 8 \cdot z = 7$$

$$11 \cdot x + 13 \cdot y + 10 \cdot z = 12$$

Write down the coefficient and parameter matrices,

$$A = \begin{bmatrix} 2 & 4 & 3 \\ 9 & 6 & 8 \\ 11 & 13 & 10 \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ 7 \\ 12 \end{bmatrix}$$

Calculate the determinant for A (this will be reused), and calculate the determinants for matrices below. Note: when trying to find the first parameter 'x' we replace the first column in A with B.

$$x = \frac{\begin{vmatrix} 5 & 4 & 3 \\ 7 & 6 & 8 \\ 12 & 13 & 10 \end{vmatrix}}{|A|} =$$

$$y = \frac{\begin{vmatrix} 2 & 5 & 3 \\ 9 & 7 & 8 \\ 11 & 12 & 10 \end{vmatrix}}{|A|} =$$

$$z = \frac{\begin{vmatrix} 2 & 4 & 5 \\ 9 & 6 & 7 \\ 11 & 13 & 12 \end{vmatrix}}{|A|} =$$

### 32.2.2 Gauss-Jordan Row Reduction

- In many ways Gauss-Jordan is a form of substitution based upon rearranging equations into an upper-right triangular form.
- The general method works top to bottom .....

Given,

$$x + y = 5$$

$$2x + 3y = 8$$

The equations can be written in matrix form,

$$\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

eliminate values in the first column by multiplying by a factor and subtracting from the first row,

$$-\frac{1}{2} \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 1 - \frac{1}{2}2 & 1 - \frac{1}{2}3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 5 - \frac{1}{2}8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & -0.5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

Finally, solve for the values starting with the last row and work towards the top row.

$$-0.5y = 1 \quad y = -2$$

$$1x + 1(-2) = 5 \quad x = 7$$

### 32.2.3 Cramer's Rule

Given,

$$x + y = 5$$

$$2x + 3y = 8$$

The equations can be written in matrix form,

$$\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

To find x,

$$x = \frac{\begin{vmatrix} 5 & 1 \\ 8 & 3 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}} = \frac{15 - 8}{3 - 2} = 7$$

To find y,

$$y = \frac{\begin{vmatrix} 1 & 5 \\ 2 & 8 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}} = \frac{8 - 10}{3 - 2} = -2$$

$$x = \det([5 \ 1 ; 8 \ 3]) / \det([1 \ 1 ; 2 \ 3])$$

$$y = \det([1 \ 5 ; 2 \ 8]) / \det([1 \ 1 ; 2 \ 3])$$

### 32.2.4 Triple Product

When we want to do a cross product, followed by a dot product (called the mixed triple product), we can do both steps in one operation by finding the determinant of the following. An example of a problem that would use this shortcut is when a moment is found about one point on a pipe, and then the moment component twisting the pipe is found using the dot product.

$$(d \times F) \bullet u = \begin{vmatrix} u_x & u_y & u_z \\ d_x & d_y & d_z \\ F_x & F_y & F_z \end{vmatrix}$$

### 32.2.5 Gauss-Siedel

### 32.3 Problems

1. Perform the vector operations below,

$$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad B = \begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix}$$

Cross Product

$$A \times B =$$

Dot Product

$$A \bullet B =$$

ANS.

$$A \times B = (-4, 17, -10)$$

$$A \bullet B = 13$$

2. Perform the following matrix calculations.

a)

$$\begin{bmatrix} a & b & c \end{bmatrix}^T \begin{bmatrix} d & e & f \\ g & h & k \\ m & n & p \end{bmatrix}$$

ans.

not solvable

b)

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$ad - bc$$

c)

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1}$$

$$\frac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}{ad - bc}$$

3. Perform the matrix operations below.

$$\begin{vmatrix} 2 & 0 \\ -6 & 3 \end{vmatrix}$$

$$\begin{vmatrix} 2 & 1 \\ -6 & 3 \end{vmatrix}$$

$$\begin{vmatrix} 2 & -1 \\ -6 & 3 \end{vmatrix}$$

ans. =6

=12

=0

Multiply

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 10 \\ 11 \\ 12 \end{bmatrix} =$$

Determinant

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 2 & 6 \\ 7 & 8 & 9 \end{vmatrix} =$$

Inverse

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 2 & 6 \\ 7 & 8 & 9 \end{bmatrix}^{-1} =$$

ANS.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 10 \\ 11 \\ 12 \end{bmatrix} = \begin{bmatrix} 68 \\ 167 \\ 266 \end{bmatrix}$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 2 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 36$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 2 & 6 \\ 7 & 8 & 9 \end{bmatrix}^{-1} = \begin{bmatrix} -0.833 & 0.167 & 0.167 \\ 0.167 & -0.333 & 0.167 \\ 0.5 & 0.167 & -0.167 \end{bmatrix}$$

4. Solve the following equations using matrices,

$$5x - 2y + 4z = -1$$

$$6x + 7y + 5z = -2$$

$$2x - 3y + 6z = -3$$

ANS.

x= 0.273

y= -0.072

z= -0.627

5. Solve the following system of equations using a) substitution, b) matrices.

$$x + 2y + 3z = 5$$

$$\text{ans. } x = -7$$

$$x + 4y + 8z = 0$$

$$y = 18.75$$

$$4x + 2y + z = 1$$

$$z = -8.5$$

6. Find the dot product, and the cross product, of the vectors A and B below.

$$A = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$B = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

$$\text{ans. } A \bullet B = xp + yq + zr$$

$$A \times B = \begin{bmatrix} yr - qz \\ zp - xr \\ xq - py \end{bmatrix}$$

## 33. GRAPHING

Topics:

- 

Objectives:

- 

### 33.1 Introduction

- coordinates
- types - line, bar, scatter, x-y, chart
- curves vs. data
- SCILAB examples

- With Scilab,

```
t = [ 1 2 3 4 5 ];
f = [ 1 4 9 16 25 ];
plot2d(t, f);
// pause here and look at the graph
xbasc(); // clears the screen
plot2d(t, f, style=-1, leg='test graph')
xtitle('time (min)');
```

- Scilab graph styles,

-1	+	-8	
-2		-9	
-3		-10	
-4		-11	
-5		-12	
-6		-13	
-7		-14	

### 33.2 Graphing Functions

- In Scilab,

```
deff('[y]=f(t)', [y=%e^(-5*t)]);
f(2)
t = (0:0.1:10);
fplot2d(t, f);
```

// OR

```
t = (0:0.1:10);
y=%e^(-5*t);
plot2d(t, f);
```

### 33.3 LOG Plots

- In Scilab,

```
f = [1, 10, 100, 1000, 10000];
G = [10, 10, 20, 100, 1000];
plot2d(log10(f), log10(G), style=-1);
// OR
plot2d(f, G, style=-1, logflag='11');
```

### 33.4 Multiple Plots

- In Scilab,

```
t = [0: 0.1: 6];  
y = sin(t) + sin(3 * t);  
z = sin(t) + cos(t);  
xset('window', 0); plot(t, y);  
xset('window', 1); plot(t, z);
```

### 33.5 Other Items of Interest

- In Scilab,

```
xbasimg(0, 'filename.eps');
```

### 33.6 Problems

1. Plot the following function using 20 datapoints in Scilab from -10 to 10.

$$y(t) = e^{-5t} \sin(3t)$$

2. Draw a scatter plot in Scilab for the following (x, y) data.

(1,1) (2, 3) (2, 5) (1, 7) (5, 8)

### 33.7 Challenge Problem

1. Draw a histogram (bar chart) for the following raw data.

1, 4, 7, 3, 5, 2, 4, 3, 7, 9, 7, 3, 4, 1, 7, 5, 2, 4, 3, 6, 9, 5, 1, 5, 2, 3, 8

## 34. PROGRAMMING

Topics:

- 

Objectives:

- 

### 34.1 Overview

- brief summary of topic(s)
- category: review / fundamental / application / research

### 34.2 Introduction

- Some useful program elements
  - 'clear' empties the screen
  - 'exec' executes a file as a script.
  - 'chdir' changes a working directory
  - 'getf('functionfile') - gets a function from a working directory
  - pwd - returns the current working directory
  - 'xbasc()' - clears a graphics window

- For-loops in Scilab,

```

for i=0:0.1:6 // 0 to 6 in 0.1 steps
    // loop code goes here
end

// OR using vectors
V=[0:0.1:6];
for i=V
    // loop code goes here
end

```

- While-loops in Scilab,

```

i = 0;
while(i <= 5),
    i = i + 1;
    i
end
i

```

- If-then in Scilab,

```

if(__logical__),
    // some code
else
    // other code
end

// >, >=, <, <=, ==, ~= Basic comparisons
// &, |, ~ Basic Boolean operators

```

- Functions in Scilab,

```
function returnval = functionname(arg1, arg2)
    returnval = arg1 + arg2;
    if(returnval < 0),
        returnval = 0;
        return
    end
endfunction

functionname(5, 3)
```

- Note that when a function is defined in Scilab it is not actually run. However when the function is called it will be interpreted.
- Cases in Scilab,

```
select val,
    case 0,
        // code
    case 1,
        // more code
    else
        break
end
```

- Printing in Scilab,

```
mprintf("format string goes here %d %f \n", arg_int, arg_float)

// %d - integer
// %f - float
// \n - end of line
```

- Inputting values in Scilab,

```
a = input("input a value for a");
```

### 34.3 Examples

- A simple Scilab Program,

```

//  

// test.sce  

//  

// A simple program to integrate an accelerating mass  

//  

// To run this in Scilab use 'File' then 'Exec'.  

//  

// by: H. Jack Aug 27, 2002  

//  

// Set the time length and step size  

steps = 100;  

t_end = 10;  

delta_t = t_end / steps;  

// Initial conditions for the motion  

pos=[0];  

vel=[0];  

t=[0];  

acc=9.81;  

// Loop to integrate the motion  

for i=1:steps,  

    t = [t; i * delta_t];  

    vel = [vel; vel($) + acc];  

    pos = [pos; pos($) + vel($) * delta_t + 0.5 * acc * delta_t^2];  

end  

// Dump the values to the screen  

//[t vel pos]  

// Graph the values  

plot2d(t, [pos vel], [-2, -5], leg="position@velocity");  

    // leg - the legend titles  

    // style - draw lines with marks  

    // nax - grid lines for the graph  

xtitle('Time (s)');  

// Write values to a file (delete the existing file first)  

unix("del c:\temp\data.txt");  

write("c:\temp\data.txt", [t vel pos]);

```

- A simple numerical integration of the equations of motion (ME/PDM),

```

// first_order.sce
// A first order integration of an accelerating mass
// To run this in Scilab use 'File' then 'Exec'.
// by: H. Jack Sept., 16, 2002

// System component values
mass = 10;
force = 100;

x0 = 8;           // initial conditions
v0 = 12;
X=[x0, v0];

// define the state matrix function
// the values returned are [x, v]
function foo=f(state,t)
    foo = [ state($, 2), force/mass]; // d/dt x = v, d/dt v = F/M
endfunction

// Set the time length and step size for the integration
steps = 100;
t_start = 1;
t_end = 100;
h = (t_end - t_start) / steps;

// Loop for integration
for i=1:steps,
    X = [X ; X($,:)+h*f(X, i*h)];
end
printf("The value at the end of first order integration is (x, v) = (%f, %f)\n", ...
    X($,1), ...
    X($,2));

// Explicit equation
function x=position(x0, v0, a0, t)
    x = (0.5 * a0 * t^2) + (v0 * t) + x0;
endfunction

function v=velocity(v0, a0, t)
    v = (a0 * t) + v0;
endfunction

```

- Integration continued,

```
printf("The value with integration is (x, v) = (%f, %f)\n", ...  
      position(x0, v0, force/mass, t_end), ...  
      velocity(v0, force/mass, t_end));  
  
//  
// The results should be  
//   first order integration = (49710, 1002)  
//   explicit           = (51208, 1012)  
//  
// The difference is 1498 for position and 10 for velocity. This is relatively small, but  
// shows a clear case of the innaccuracy of the numerical solutions.  
//  
// Note: increasing the number of steps increases the accuracy  
//
```

- A simple Bode plot example Program (for electricals),

```

steps_per_dec = 6;
decades = 6;
start_freq = 0.1;

// The transfer function
function foo=G(w)
    D = %i * w;
    foo = (D + 5) / (D^2 + 100*D + 10000);
endfunction

fd = mopen("data.txt", "w");
for step = 0:(steps_per_dec * decades),
    f = start_freq * 10 ^ (step/steps_per_dec); // calculate the next frequency
    w = f * (2 * %pi); // convert the frequency to radians
    [gain, phase] = polar(G(w)); // get the result and convert to magnitude/angle
    gaindb = 20*log10(gain); // convert to dB
    phasedeg = 180 * phase / %pi; // convert to degrees
    mfprintf(fd, "%f, %f, %f\n", f, gaindb, phasedeg);
end
mclose(fd);

// To graph it directly
D=poly(0,'D');
h=syslin('c', (D + 5) / (D^2 + 100*D + 10000));
bode(h, 0.1, 1000, 'Sample Transfer Function');

```

## 34.4 Summary

## 34.5 Problems

1. Write a program that chooses a random number between 1 and 100. A user will then have to guess the value. Each guess will be given a hint for higher/lower. At the end the game should report the number of guesses required.

## 34.6 Challenge Problems

## 35. PERMUTATIONS AND COMBINATIONS

Topics:

- 

Objectives:

- 

### 35.1 Introduction

- Permutations - a count of the variety of sequences
- Combinations - a count of the number of selections
- Probability - The chance something will happen.

### 35.2 Permutations

- the possible arrangements of objects given,
  - $n$  = number of source objects
  - $r$  = number of arranged objects
- Permutations - for exact definitions of not only possibilities, but also order.

$$P_r^n = \frac{n!}{(n-r)!}$$

e.g. How many permutations picking 2 cards off a deck (52)

$$\frac{(52)!}{(52-2)!} = \frac{52 \times 51 \times 50 \times \dots}{50 \times 49 \times \dots} = 52 \times 51$$

- permutations for n unique objects arranged into r spots

$$\frac{n!}{(n-r)!}$$

For example 5 objects in 3 spots

$$\frac{5!}{(5-3)!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = 60$$

- permutations for non-unique objects (using all)

For example consider 3 pairs of objects that will fill 6 spots

$$\frac{n!}{d_1!d_2!d_3!} = \frac{6!}{2!2!2!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{8} = 90$$

### 35.3 Combinations

- Combinations - similar to before except order does not matter.

$$C_r^n = \frac{n!}{r!(n-r)!} \quad (\text{n choose r})$$

e.g. How many combinations of the first 2 cards can be picked off the deck

$$\frac{(52)!}{2!(52-2)!} = \frac{52 \times 51 \times 50 \times \dots}{(2 \times 1)(50 \times 49 \times \dots)} = \frac{52 \times 51}{2}$$

- Possible outcomes if 'r' objects are taken from a group of 'n'.

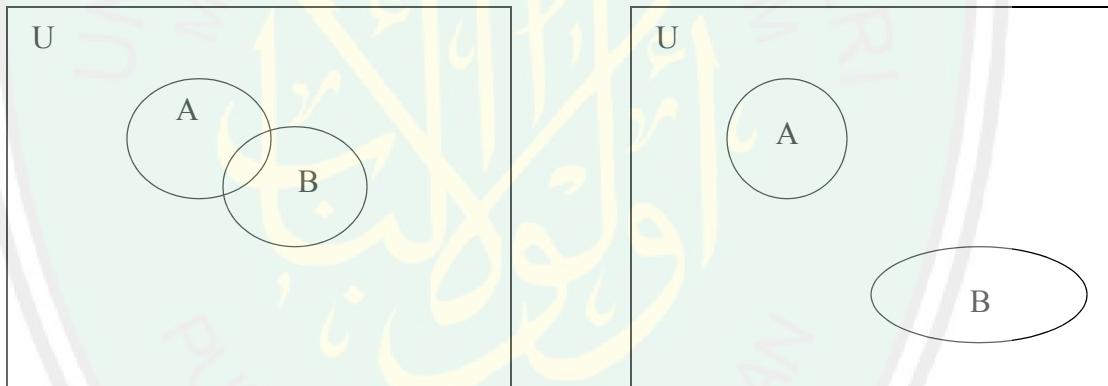
$$nCr = \frac{nPr}{r!} = \frac{n(n-1)(n-2)\dots(n-r+1)}{r!}$$

For example consider 2 cars chosen from a set of 4

$$\frac{4 \cdot 3}{2 \cdot 1} = 6$$

### 35.4 Probability

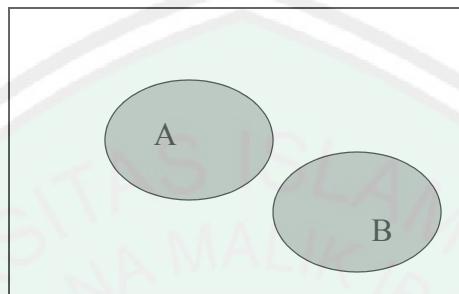
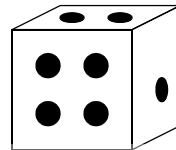
- The chance 'P(A)' some event 'A' will happen.
- A way to figure out how chances interact
- Venn diagrams can be useful for describing interactions,



- Mutually Exclusive - Probable events can only happen as one or the other.

e.g. Only one number from 1 to 6 will come up when rolling a die.

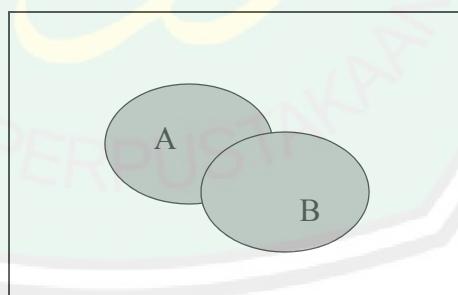
$$P(A) + P(B) + \dots = 1$$



- Not Mutually Exclusive - Probable events can occur simultaneously

e.g. Two dice are rolled to get a number from 2 to 12, the chance that a 4 will come up is,

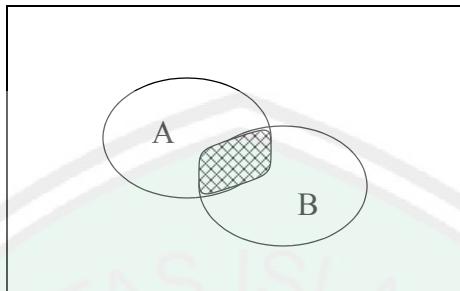
$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = 1/6 + 1/6 - 1/36 \quad P(A \cup B)$$



- Independent Probabilities - Events will happen separately

e.g. The chance of rolling a number  $\geq 3$  then  $< 3$  on a 6 sided die.

$$P(A \text{ and } B) = P(A) * P(B) = (4/6)*(2/6) = 8/36 \quad P(A \cap B)$$



- Dependant Probabilities - The outcome of one event effects the outcome of another event

e.g. The chance a student will pass an exam if they show up (99/100) and they know the material (7/10)

$$P(A \text{ and } B) = P(A) * P(B \text{ when } A) = (99/100)*(7/10)$$

- empirical probability - experimentally determine the probability with,

$$P = \frac{\# \text{ of outcomes}}{\# \text{ of trials}}$$

- Probability Density Function

$$P(x_1 \leq x \leq x_2) = \int_{x_1}^{x_2} f(x) dx$$

## 35.5 Problems

1. The marketing department has asked how many product permutations are possible for a new product. The product will hold three colored balls. In total there are 6 ball colors available. If balls must be used in all spots, and ball colors can be used more than once how many permutations are possible? (ans. 216)

2. A worker is packing small cereal boxes into a larger box. If the worker is picking from a set of 10 different cereal boxes, but can only place 8 in the larger box, how many combinations are possible? (ans. 45)

3. An assembly operation places two parts a board. There is a 3% chance that part A is bad, and a 2% chance that part B is bad. What is the chance the board contains a bad part? (ans. 4.94%)

4. How many ways could 5 operators be assigned to 5 workstations? (ans. 120)

5. How many ways could 8 operators be assigned to 5 workstations? (ans. 6720)

6. There are nine product, each a different color. They are to be put into 3 boxes each holding 3 products. a) How many unique package arrangements are possible? b) How many different combinations of packaged products are possible if the position does not matter? (ans. a) 362,880 b) 1680)

7. An electronics company will assemble circuit boards with interchangeable components. There are 3 places to mount the components and there are 5 types of components. Each component type may only be used once. How many different outcomes are possible? (ans. 60)

8. A toy is being manufactured to have prizes in 2 of 5 slots. How many prize layouts are possible? (ans. 10)

9. Calculate the following values.

a)  ${}_7C_3 =$

(ans.  $= 35$ )

b)  ${}_4C_2 =$

$= 6$

c)  ${}_7C_r = 21$

$r = 2$

10. There are 8 machines (A to H) waiting to be shipped. 3 of these will be tested.

- a) How many combinations are possible?
- b) How many of those combinations contain machine C?
- c) How many combinations contain A or H, but not both?
- d) If 2 trucks are loaded with 4 machines, how many distributions are possible?
- e) Resolve part d) if machine A and B are in the same truck.  
(ans. a) 56, b) 21, c) 30, d) 70, e) 15)

11. A carton contains 12 parts, 4 are red and 8 are green.

- a) Find the probability that the first part removed is green.
- b) If 3 parts are removed what is the probability that all are red?
- c) Repeat b) for all green.
- d) Repeat b) for one red and 2 green.  
(ans. a) 2/3, b) 1/55, c) 14/55, d) 28/165)

12. Write routines to implement basic functions.



## 36. STATISTICS

Topics:

- 

Objectives:

- 

### 36.1 Introduction

- Descriptive vs. predictive statistics
- Why?
  - Because we can't sample every part.
  - Because no matter what we do, no two parts will be the same.
  - Because we define a product to meet specifications and we can measure how well it conforms.
  - Because differences between parts are hard (assignable causes) or impossible (chance causes) to predict.
  - Because we can sample a few and draw conclusions about the whole group.
- What is the objective?
  - We want to sample as little data as possible to draw the most accurate conclusions about the distribution of the values.
- Two type of statistics
  - Inductive - Try to get overall variance within group. i.e. assume all of group should conform \*\*\*WE USE THIS TYPE
  - Deductive - Attempt to classify differences that exist within a group (i.e. election polls).

## 36.2 Data Distributions

- Showing Differences
  - consider Dominos pizza that delivered < 30minutes by tolerance

Delivery times for one night					
GROUPED to nearest value	20	23	27	23	26
	16	26	17	29	26
	28	26	23	26	31
	21	27	29	17	27

because data is grouped, 20.7 minutes becomes 21

Pizzas with black olives for 20 days					
UNGROUPED data	1	4	5	2	1
	2	4	4	7	1
	8	2	3	3	6
	3	7	9	7	7

We can draw all with a tally sheet

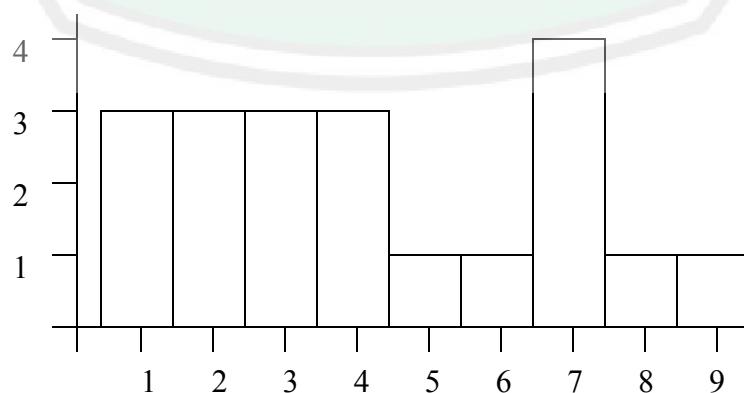
#	count	freq.
1	III	3
2	III	3
3	III	3
4	III	3
5	I	1
6	I	1
7	IIII	4
8	I	1
9	I	1

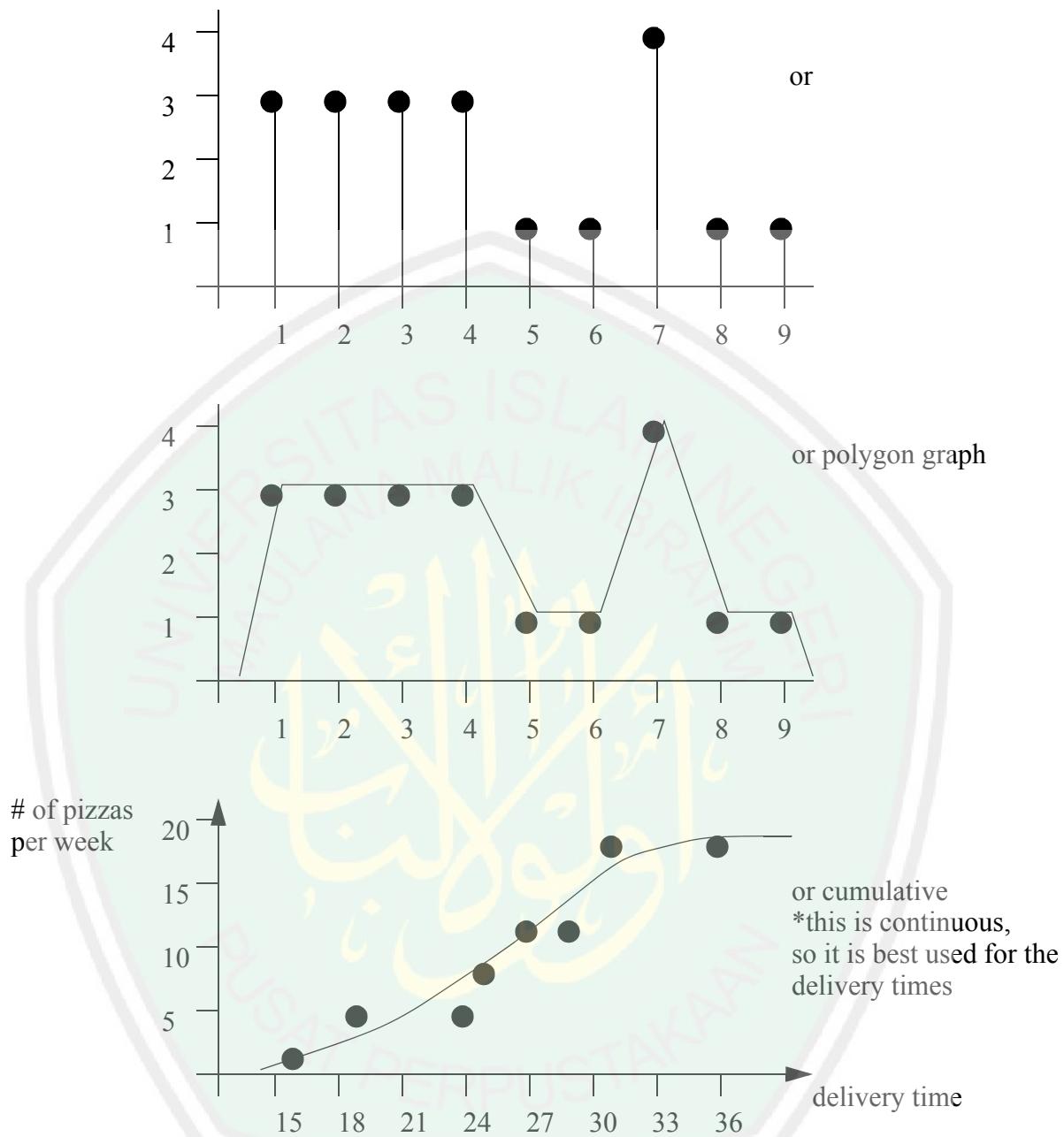
- In Scilab,

```
D = [ 3 ; 7 ; 8 ; 2 ; 5 ; 6 ; 3 ; 2 ; 9 ; 4 ; 5 ; 8 ]; // a data set  
min(D); // the minimum value in D  
max(D); // the maximum value in D  
median(D); // the median value in D  
mean(D); // the average value for the samples  
find(...); // a function for looking for logical conditions  
sum(D); // the sum of all elements in D
```

### 36.2.1 Histograms

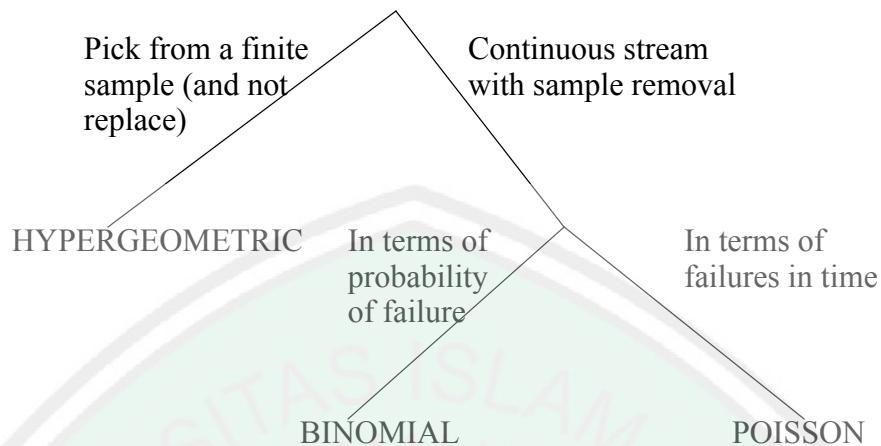
- Histograms can be used to show these distributions graphically.





- Cumulative distributions can be used to estimate the probability of an event. For example, if in the graph above we want to know how many pizzas are delivered within 25 minutes, we could read 10 (approx.) every week off the graph.
- there are typically 10 to 20 divisions in a histogram
- percentages can be used on the right axis, in place of counts.

### 36.3 Discrete Distributions



#### 36.3.1 Normal Distribution

- Symbolically

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$P(x) =$$

#### 36.3.2 Hypergeometric Distribution

Hypergeometric

$$P(d) = \frac{C_d^D C_{n-d}^{N-D}}{C_n^N} \quad (\text{probability of } d \text{ nonconforming in sample } n)$$

basic combinatorial formula

N - number in lot

n - number in sample

D - number nonconforming in lot

d - number nonconforming in sample

### 36.3.3 Binomial Distribution

Binomial

$$P(d) = \frac{n!}{d!(n-d)!} (p_0)^d (q_0)^{n-d} \quad (\text{probability of } d \text{ nonconforming})$$

n - number in sample

d - number nonconforming in sample

p<sub>0</sub> - fraction nonconforming in sample

q<sub>0</sub> - (1 - p<sub>0</sub>)

The binomial distribution is based on a simple principle of probability.

If we consider that there is a probability  $p_0$  that something will fail, there is a probability  $q_0 = 1 - p_0$  that it will pass. In statistical terms, we are looking at all possible mutually exclusive outcomes (add the probabilities) of the independent probabilities (multiply). Therefore, for a sample size of 1 (=n),

outcome #	possible outcome	probability
1	failure	$p_0$
2	pass	$q_0$

$$\therefore \sum P = p_0 + q_0 = (p_0 + q_0)^1 = p_0 + (1 - p_0) = 1$$

To take this a step farther, if there are 2 (=n) samples,

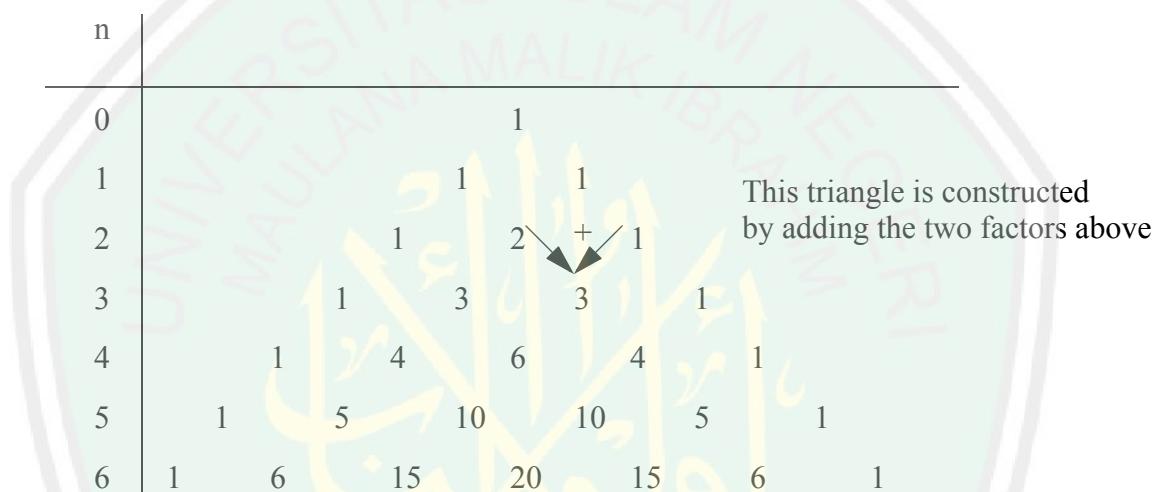
outcome #	possible outcome	probability
1	both fail	$p_0 * p_0 = p_0^2$
2	first fails, second passes	$p_0 * q_0$
3	first passes, second fails	$q_0 * p_0$
4	both pass	$q_0 * q_0 = q_0^2$

$$\therefore \sum P = p_0^2 + 2p_0q_0 + q_0^2 = (p_0 + q_0)^2 = (p_0 + (1 - p_0))^2 = (1)^2 = 1$$

We can continue in this way, and it will eventually show a pattern emerge in the form.  
This can be written as a general equation.

$$(p_0 + q_0)^n = p_0^n + np_0^{n-1}q_0 + \frac{n(n-1)}{2!}p_0^{n-2}q_0^2 + \frac{n(n-1)(n-2)}{3!}p_0^{n-3}q_0^3 + \dots + q_0^n$$

The coefficients for this method can also be found using pascals triangle. These coefficients replace the terms in front of the  $p_0$  and  $q_0$  terms.



We can then use this chart to construct a binomial equation. Say we have a sequence of 4 events, therefor n=4

$$1 = (1)p_0^4 + (4)p_0^3q_0 + (6)p_0^2q_0^2 + (4)p_0q_0^3 + (1)q_0^4$$

Now that we have the binomial function, we can estimate the probability of various outcomes. A table is given for clarity below,

term #	meaning	probability
0	all four of the cases fail	$(1)p_0^4$
1	three of the cases fail	$(4)p_0^3q_0$
2	two of the cases fail	$(6)p_0^2q_0^2$
3	one of the cases fail	$(4)p_0q_0^3$
4	none of the cases fail	$(1)q_0^4$

### 36.3.4 Poisson Distribution

Poisson

$$P(c) = \frac{(np_0)^c}{c!} e^{-np_0} \quad (\text{probability of } d \text{ nonconforming})$$

c - count of nonconforming

np<sub>0</sub> - total count in sample

e = 2.718281828....

Poisson's distribution can be used to predict non-related events.

It's general form is,

$$\sum P = 1 = e^m \times e^{-m}$$

We can use a power factor expansion of one of the terms,

$$e^m = 1 + m + \frac{m^2}{2!} + \frac{m^3}{3!} + \frac{m^4}{4!} + \frac{m^5}{5!} + \dots$$

Substitution leads to the final distribution function,

$$\therefore 1 = e^{-m} \left( 1 + m + \frac{m^2}{2!} + \frac{m^3}{3!} + \frac{m^4}{4!} + \frac{m^5}{5!} + \dots \right)$$

$$\therefore 1 = e^{-m} + me^{-m} + \frac{m^2}{2!}e^{-m} + \frac{m^3}{3!}e^{-m} + \frac{m^4}{4!}e^{-m} + \frac{m^5}{5!}e^{-m} + \dots$$

We can now relate this back to the probability discussed before, and into a general form.

$$\therefore \sum P = 1 = P(0) + P(1) + P(2) + P(3) + \dots$$

$$\therefore P(n) = \frac{m^n}{n!} e^{-m}$$

where,

$P(n)$  = the probability of 'n' occurrences

n = the number of occurrences being considered for a probability

m = the expected number of occurrences

Next, let us consider an example of a soft drink manufacturer that inspects outgoing bottles to see if the labels are right side up. If the typical number found is 2 per hour ( $m=2$ ), what is the probability that less than 4 will occur in one hour ( $n<4$ )?

$$P(n < 4) = P(0) + P(1) + P(2) + P(3)$$

$$\therefore P(n < 4) = e^{-2} + 2e^{-2} + \frac{2^2}{2!}e^{-2} + \frac{2^3}{3!}e^{-2} =$$

## 36.4 Other Distributions

### 36.4.1 Polynomial Expansions

- Binomial expansion for polynomials,

$$(a+x)^n = a^n + na^{n-1}x + \frac{n(n-1)}{2!}a^{n-2}x^2 + \dots + x^n$$

### 36.4.2 Discrete and Continuous Probability Distributions

- The Binomial distribution is,

$$P(m) = \sum_{t \leq m} \binom{n}{t} p^t q^{n-t} \quad q = 1 - p \quad q, p \in [0, 1]$$

- The Poisson distribution is,

$$P(m) = \sum_{t \leq m} \frac{\lambda^t e^{-\lambda}}{t!} \quad \lambda > 0$$

- The Hypergeometric distribution is,

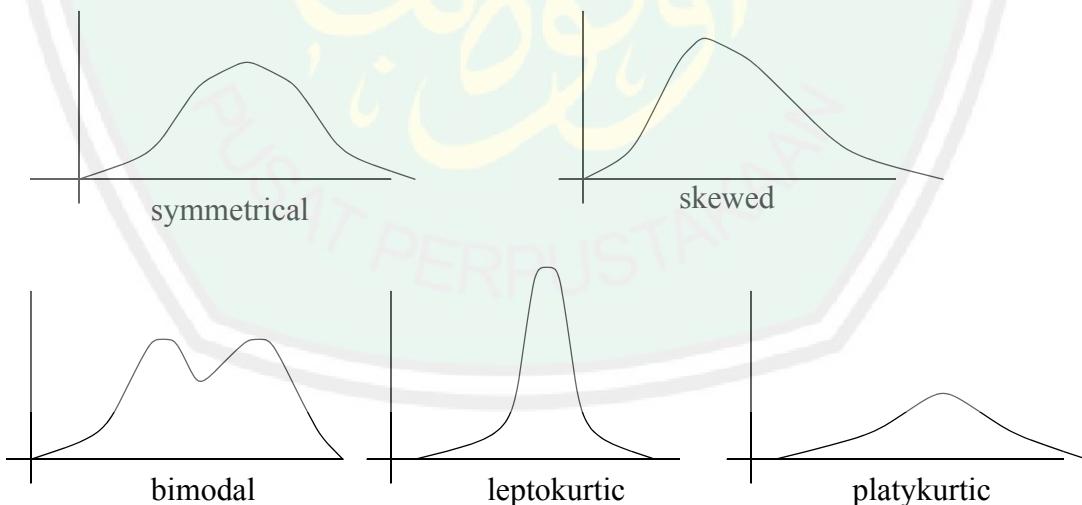
$$P(m) = \sum_{t \leq m} \frac{\binom{r}{t} \binom{s}{n-t}}{\binom{r+s}{n}}$$

- The Normal distribution is,

$$P(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-t^2/2} dt$$

### 36.5 Continuous Distributions

- Histograms are useful for grouped data, but in the cases where the data is continuous, we use distributions of probability.
- In general
  - the area under the graph = 1.00000.....
  - the graphs often stretch (asymptotically) to infinity
- In specific, some of the distribution properties are,



- In addition the centre of the distribution can vary (i.e. the average or mean)
- More on distribution later

### 36.5.1 Describing Distribution Centers With Numbers

- The best known method is the average. This gives the centre of a distribution

$$\sum_{i=1}^n X_i$$

for ungrouped numbers       $\bar{X} = \frac{\sum_{i=1}^n X_i}{n}$

where

$\bar{X}$  = average  
 $X_i$  = a data point  
 $n$  = number of points

for grouped numbers

$$\bar{X} = \sum_{i=1}^h \frac{f_i X_i}{n}$$

where

$\bar{X}$  = average  
 $f_i$  = the number of matches in range i  
 $X_i$  = the central value for the range i  
 $h$  = the number of ranges  
 $n$  = the number of matches for all ranges

e.g. ungrouped

1, 2, 3, 4

$$\bar{X} = \frac{1+2+3+4}{4} = 2.5$$

e.g. grouped

central value	freq.
1	2
3	1
5	5
	8

$$\bar{X} = \frac{2(1) + 1(3) + 5(5)}{8} = \frac{30}{8} = 3.75$$

- Another good measure is the median

- If odd number of samples it is the middle number
- if an even number of samples, it is the average of the left and right bounding numbers

e.g. odd

1  
2  
3

even

1  
2  
4  
7

$$\text{median} = \frac{2+4}{2} = 3$$

- If the numbers are grouped the median becomes

$$M_d = L_m + \left( \frac{\frac{n}{2} - cf_m}{f_m} \right) i$$

$M_d$  = median

$L_m$  = lower bound of the range of the median

$n$  = number of samples overall

$cf_m$  = cumulative frequency of all cells below  $L_m$

$f_m$  = frequency of median cell

$i$  = cell interval

- Mode can be useful for identifying repeated patterns
  - a mode is a repeated value that occurs the most. Multiple modes are possible.

e.g. 1 3 2 9 3 7 6 3 4 4 5  
mode = 3

### 36.5.2 Dispersion As A Measure of Distribution

- The range of values covered by a distribution are important
  - Range is the difference between the highest and lowest numbers.

e.g

$$\begin{array}{ccccccc} 7 & 8 & 3 & 1 & 5 & 6 & 2 \\ \text{range} = 8 - 1 = 7 \end{array}$$

- Standard deviation is a classical measure of grouping (for normal distributions???????????)

- the equation is,

$$s = \sqrt{\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}}$$

$s$  = standard deviation  
 $X_i$  = observed value  
 $\bar{X}$  = average  
 $n$  = number of values

e.g. 1 2 7 6 5 0 7

$$\bar{X} = \frac{1 + 2 + 7 + 6 + 5 + 0 + 7}{7} = \frac{27}{7} = 4$$

$$\begin{aligned} s &= \sqrt{\frac{(1-4)^2 + (2-4)^2 + (7-4)^2 + (6-4)^2 + (5-4)^2 + (0-4)^2 + (7-4)^2}{7-1}} \\ &= \sqrt{8.666} = 2.8 \end{aligned}$$

- When we use a standard deviation, we can estimate the distribution of the samples.

e.g

$$\bar{X} = 4$$

$$s = 2.8$$

therefore, the range  $4-2.8$  to  $4+2.8$  (1.2 to 6.8) will contain  
68.26% of the samples

- By adding standard deviations to increase the range size, the percentage of samples included are,

$$\begin{aligned}\bar{X} \pm s &\rightarrow 68.26\% \\ \bar{X} \pm 2s &\rightarrow 95.46\% \\ \bar{X} \pm 3s &\rightarrow 99.73\% \\ \text{etc....} &\end{aligned}$$

Assuming a normal distribution

- Other formulas for standard deviation are,

$$s = \sqrt{\frac{n \sum_{i=1}^n X_i^2 - \left( \sum_{i=1}^n X_i \right)^2}{n(n-1)}}$$

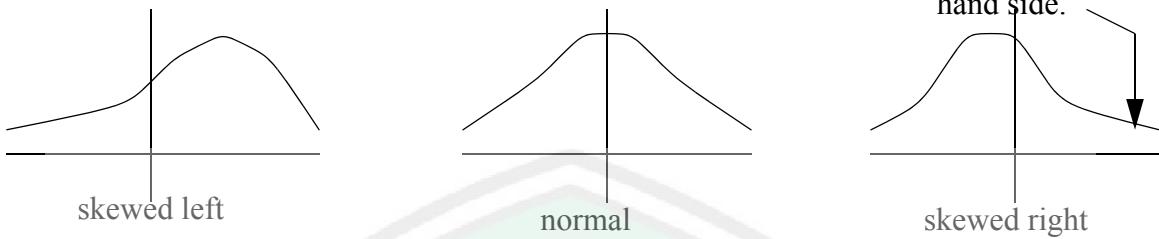
$$s = \sqrt{\frac{h \sum_{i=1}^h (f_i X_i^2) - \left( \sum_{i=1}^h f_i X_i \right)^2}{n(n-1)}}$$

$h$  = number of cells

$f_i$  = frequency of values in a cell

### 36.5.3 The Shape of the Distribution

- Skewed functions



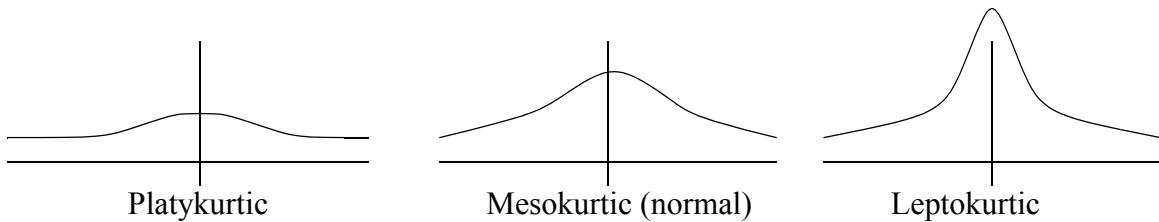
- this lack of symmetry tends to indicate a bias in the data (and hence in the real world)
- a skew factor can be calculated

$$a_3 = \frac{\sum_{i=1}^n \frac{f_i(X_i - \bar{X})^3}{n}}{s^3}$$

if  $a_3 = 0$  then the distribution is symmetrical (normal)  
 $a_3 > 0$  then skewed to the right  
 $a_3 < 0$  then skewed to the left  
 $a_3$  is not 0 indicates the distribution is not normal.

#### 36.5.4 Kurtosis

- This is a peaking in the data

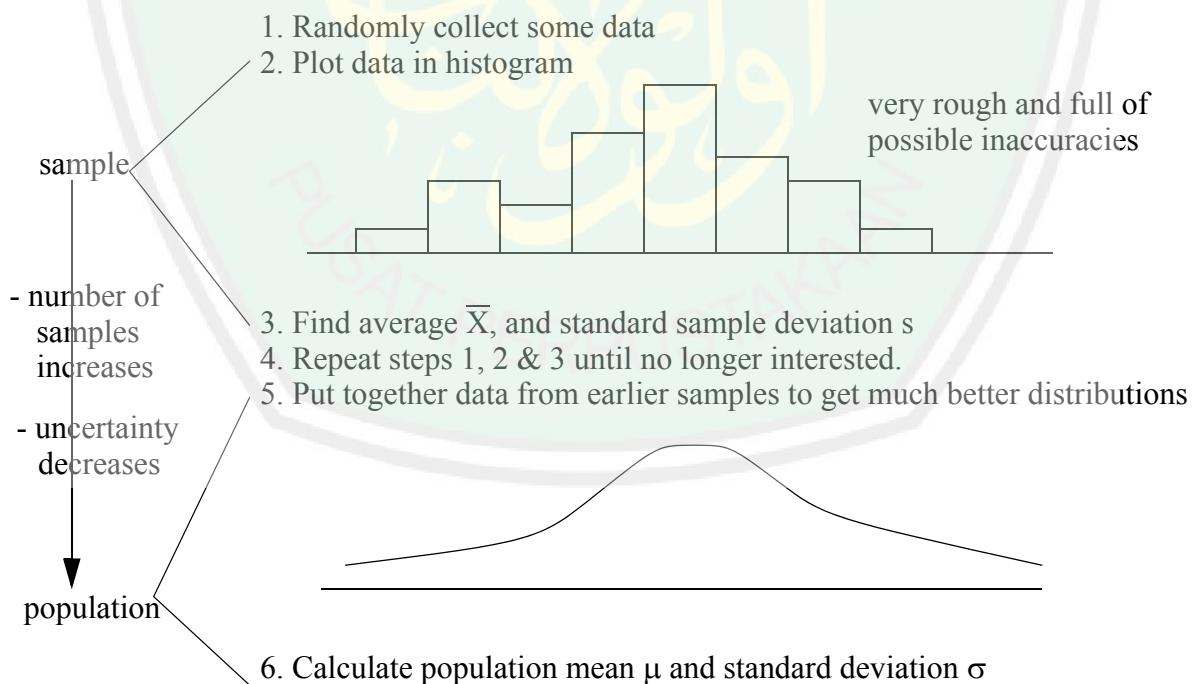


$$a_4 = \frac{\sum_{i=1}^h \frac{f_i(X_i - \bar{X})^4}{n}}{S^4}$$

- $a_4 = 3$  mesokurtic
- $a_4 > 3$  leptokurtic
- $a_4 < 3$  platykurtic

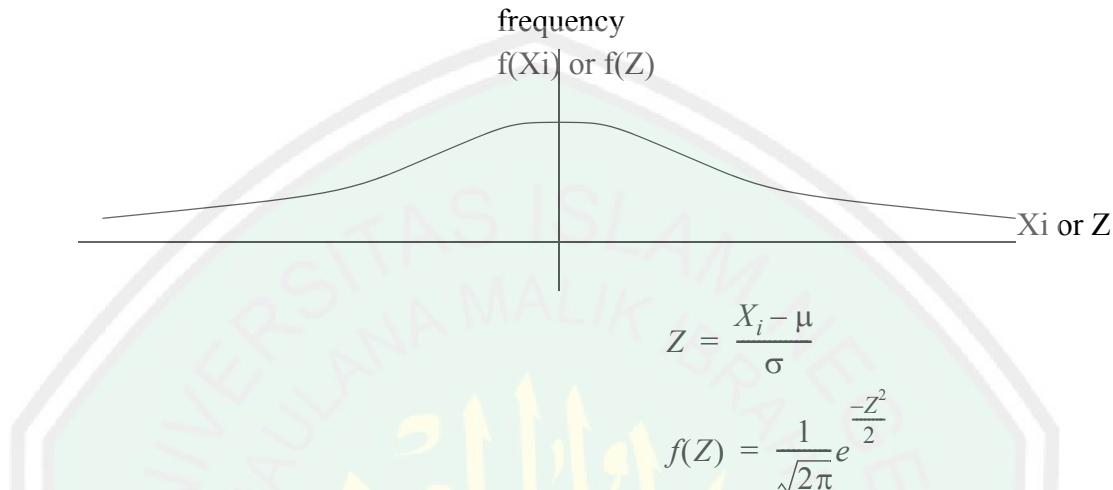
- This is best used for comparison to other values. i.e. you can watch the trends in the values of  $a_4$ .

### 36.5.5 Generalizing From a Few to Many

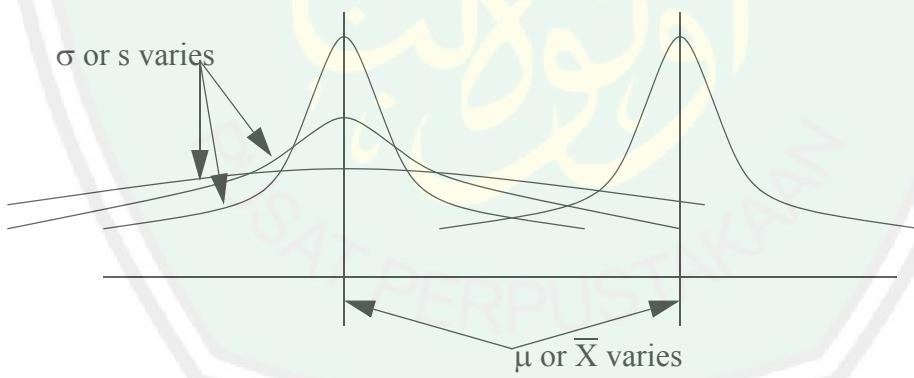


### 36.5.6 The Normal Curve

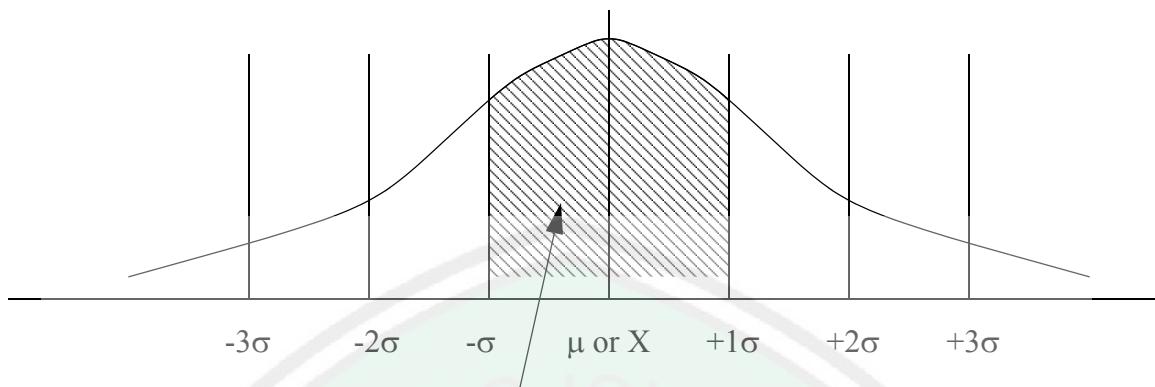
- this is a good curve that tends to represent distributions of things in nature (also called Gaussian)
- This distribution can be fitted for populations ( $\mu, \sigma$ ), or for samples ( $\bar{X}, s$ )



- The area under the curve is 1, and therefore will enclose 100% of the population.
- the parameters vary the shape of the distribution



- The area under the curve indicates the cumulative probability of some event



the area bounded by range indicates the cumulative probability. In this case the area = .6826

$\mu \pm 1\sigma = 68.26\%$  of the values bounded

$\mu \pm 2\sigma = 95.46\%$  of the values bounded

between  $\mu + 1\sigma$  and  $\mu - 2\sigma$   $\frac{68.26}{2} + \frac{95.46}{2} = 81.86$  of the values are found.

\*Table A in text gives integrated area under the curve so that probability may be found.

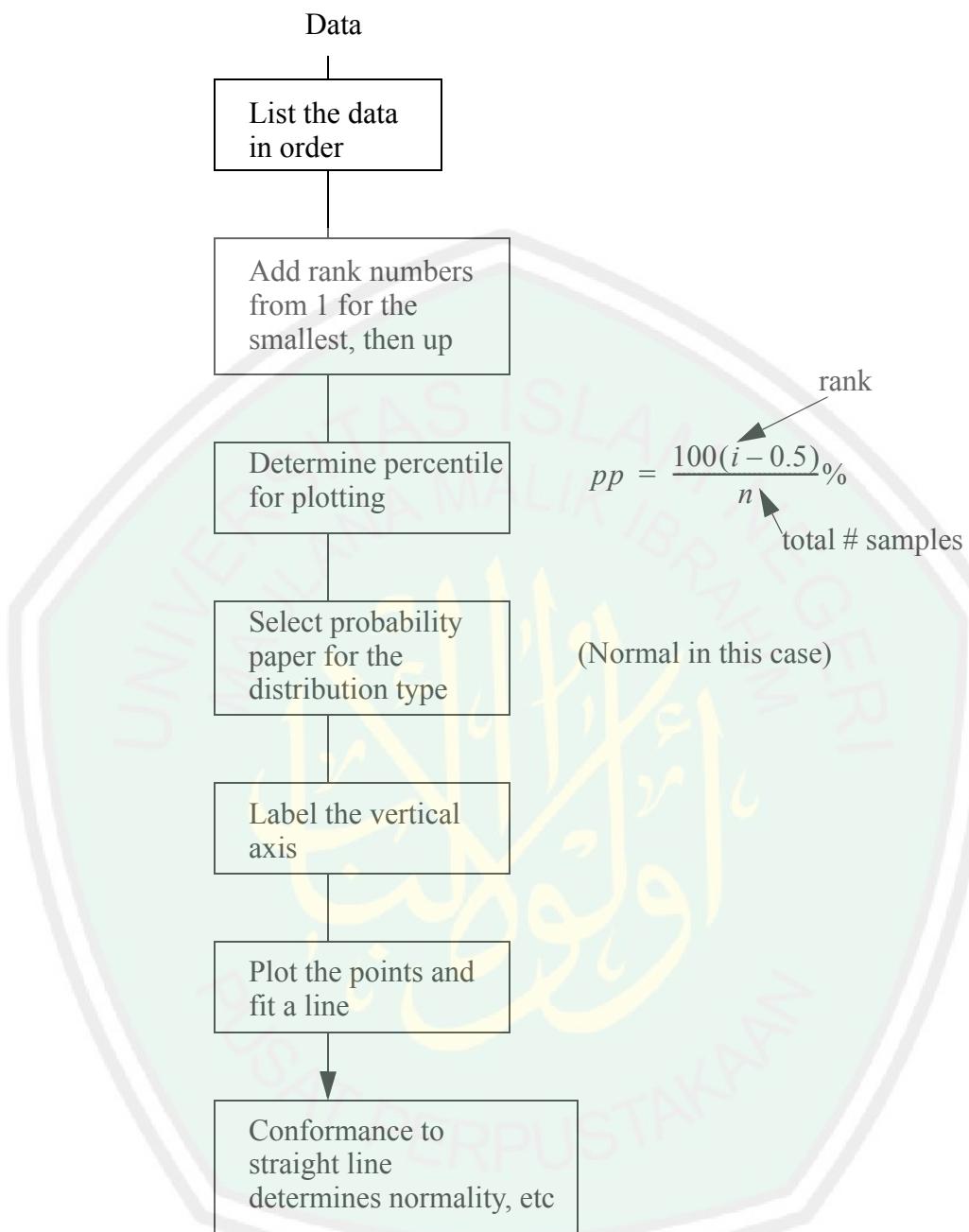
\*\*\*\*\* ADD IN MORE TO FIND AREA UNDER DISTRIBUTION

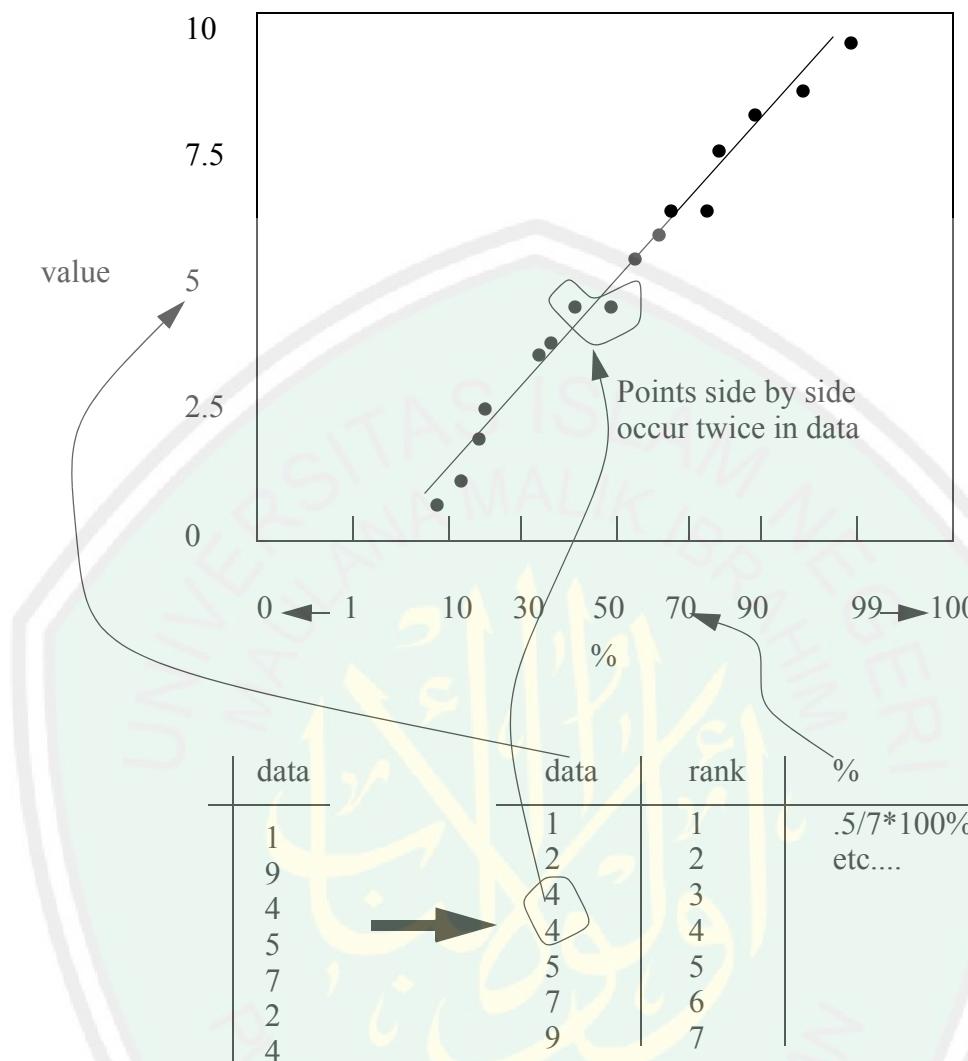
- When applied to quality  $\pm 3\sigma$  are used to define a typical “process variability” for the product. This is also known as the upper and lower natural limits (UNL & LNL)

\*\*\*\*\* LOOK INTO USE OF SYMBOLS, and UNL, LNL, UCL, LCL, etc.

### 36.5.7 Probability plots

- Procedure





### 36.6 Problems

1. Write a Scilab program to compute the mean and standard deviation for the data below. It should then check to see if the values 1.0, 2.0, 3.0 are within  $\pm 3$  standard deviations.

x

1.4  
2.5  
1.1  
0.6  
1.9  
1.0  
1.5  
0.9  
1.6  
1.1  
1.7  
2.0  
1.2  
1.4  
1.8

### 36.7 Challenge Problems

1. Write a function that generates random numbers that follow a gaussian distribution for an arbitrary mean and standard deviation. Verify the routine.
2. Use the subroutine to simulate the system described below. A production process consists of two machines. The first machine takes 3 minutes for an operation with a standard deviation of 10 seconds. The second machine takes 4 minutes with a standard deviation of 15 seconds. If the first machine finishes before the second machine, it will have to wait for the second machine to finish before the job moves on. Use Monte Carlo simulation to estimate the average cycle time.

## 37. RELIABILITY

Topics:

- Reliability of series and parallel system components
- MTBF, MTTF, MTTR, system availability
- FMEA

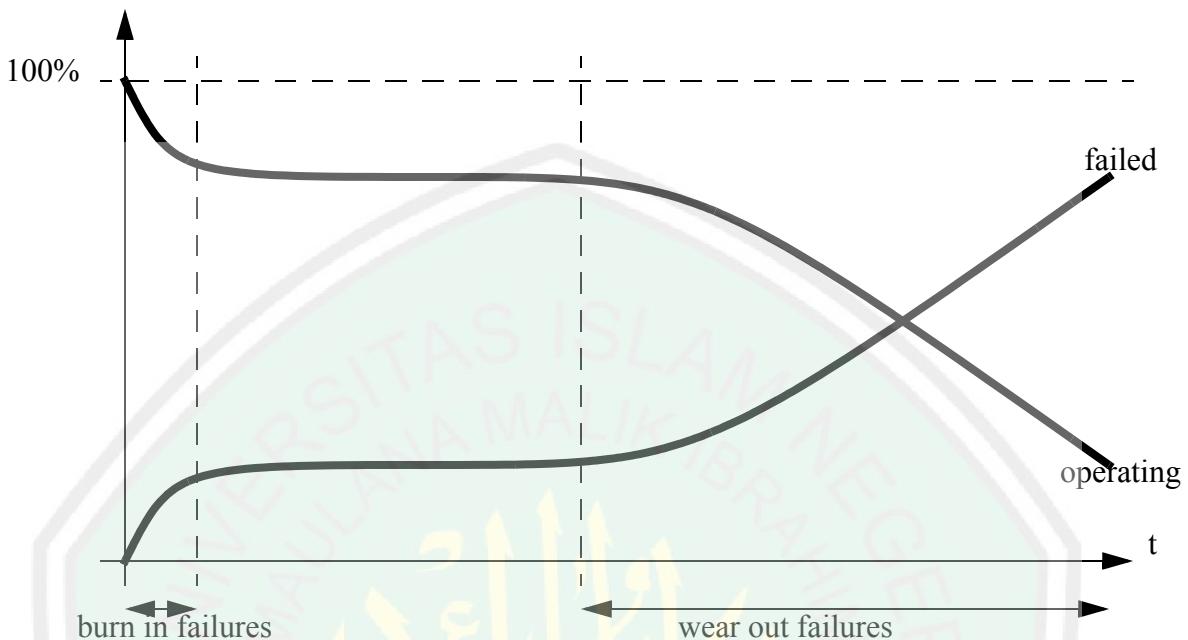
Objectives:

- To be able to estimate the reliability of a multicomponent system.

### 37.1 Introduction

- This chapter will assume that a system or component is functional, or not. In practice systems fail slowly, but we will consider failure to be when they cease being functional.
- No system is perfect and will fail eventually, being able to predict this allows us to determine the useable life.
- Dependability is a combination of,
  - reliability - the probability that a system operates through a given operation specification.
  - availability - the probability that the system will be available at any instant required.

- Typical failures follow the following curve.



### 37.2 Component Failure Rates

- Failure rate is the expected number of failures per unit time, and is shown with the constant ( $\lambda$ ), with the units of failures per hour.
- Basically,

$$R(t) = \frac{N(t)}{N(0)}$$

where,

$N(t)$  = the number operating at time t

$R(t)$  = the reliability, the portion surviving over the time  $[t_0, t]$

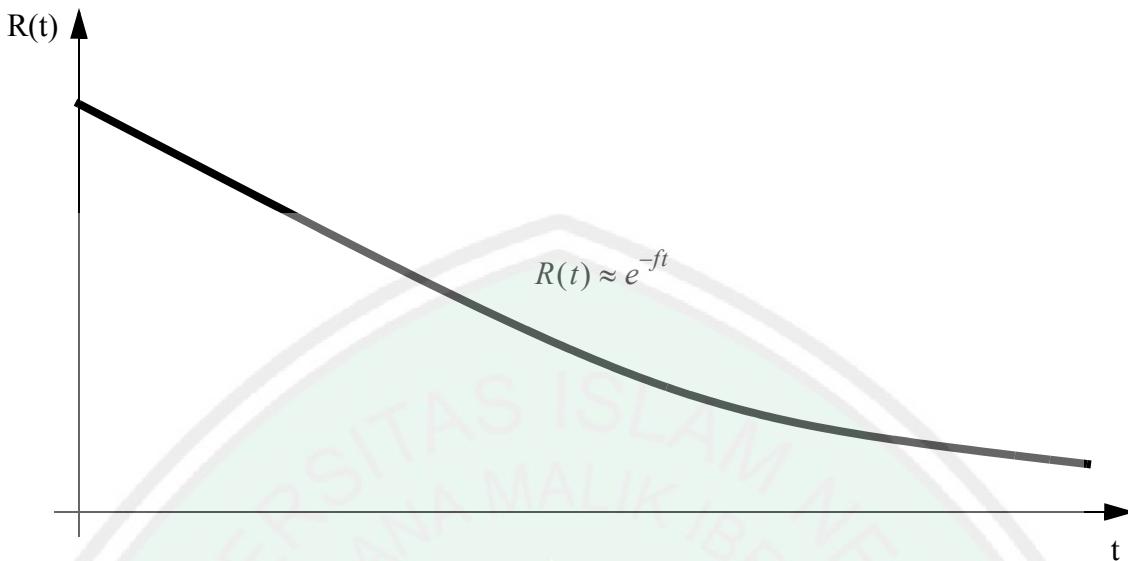
$$Q(t) = 1 - R(t)$$

where,

$Q(t)$  = unreliability

- Failures tend (but do not have to) follow exponential failure rate curves. This also suggests a

failure function.



Assume a function,  $R(t) = e^{-ft}$

$$\frac{d}{dt}R(t) = -fe^{-ft}$$

$$\therefore \frac{d}{dt}\frac{N(t)}{dtN(0)} = -fR(t)$$

$$\therefore \frac{d}{dt}\frac{N(t)}{dtN(0)} = -f\frac{N(t)}{N(0)}$$

$$\therefore \frac{d}{dt}N(t) = -fN(t)$$

$$\therefore f(t) = \frac{-N(t)}{\frac{d}{dt}N(t)}$$

where,

$f(t)$  = the failure rate

- A constant failure rate is the most commonly assumed. When this is the case the failure rate is also the Mean Time Before Failure (MTBF). Note that the reliability is also the probability

that the system will be operational.

$$f(t) = \frac{1}{\lambda} = \frac{1}{MTBF}$$

$$R(t) = e^{-\frac{t}{MTBF}}$$

where,

$MTBF$  = Mean Time Before Failure

- Even when the e-to-the-t function is a good model for system failure a system MTBF can be varied during system life by variations in product usage. Examples of these include,

- high heat levels
- large loads
- excessive stresses
- "pot holes"
- etc.

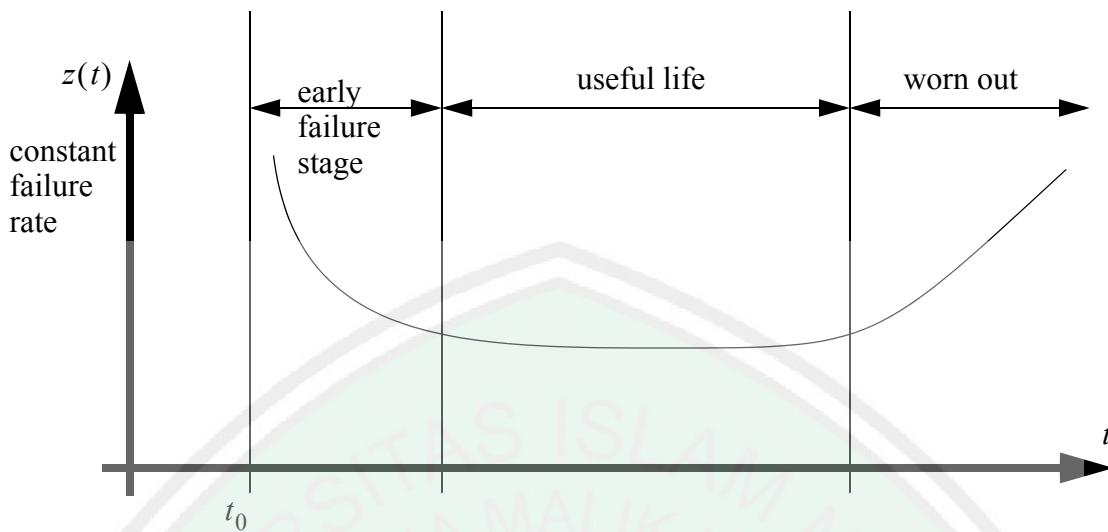
$$f(t) = -\frac{\frac{d}{dt}R(t)}{R(t)} = \frac{\frac{d}{dt}Q(t)}{1-Q(t)}$$

$$z(t) = -\frac{\frac{d}{dt}R(t)}{R(t)} = \frac{\frac{d}{dt}Q(t)}{1-Q(t)}$$

where,

$z(t)$  = the failure rate

- The bathtub curve shows typical values for the failure rate.



- The basic reliability equation can be rearranged, eventually leading to a compact expression,

$$z(t) = -\frac{d}{dt}R(t)$$

$$\therefore \frac{d}{dt}R(t) = -z(t)R(t)$$

$$\therefore R(t) = e^{-\int z(t)dt}$$

During the useful life of the product, we can approximate the failure rate as linear, as reflected by the relation below,

$$\int z(t)dt \approx \lambda t$$

$$\therefore R(t) = e^{-\lambda t} = \text{The Exponential Failure Law}$$

- MTTF (Mean Time To Failure) - this is the expected time before a failure.

$$E[X] = \int_{-\infty}^{\infty} xf(x)dx$$

where,

$X$  = a random variable

$E[X]$  = expected value

$f(x)$  = a probability density function

$$MTTF = \int_0^{\infty} tf(t)dt$$

Given the probability density function, and using integration by parts we can find the relationship between the MTTF, and reliability.

$$f(t) = \frac{d}{dt}Q(t)$$

$$\therefore MTTF = \int_0^{\infty} t \frac{d}{dt}Q(t)dt = -\int_0^{\infty} t \frac{d}{dt}R(t)dt = [-tR(t) + \int R(t)dt] \Big|_0^{\infty} = \int_0^{\infty} R(t)dt$$

- The MTTR (Mean Time To Repair) for a system is the average time to repair a system. This is not simple to determine and often is based on experimental estimates.

$$MTTR = \frac{1}{\mu}$$

where,

$$\mu = \text{the repair rate} = \frac{\text{number of repairs}}{\text{time period for all repairs}}$$

- The MTTF and MTTR both measure the time that the system is running between repairs, and the time the system is down for repairs. But, they must be combined for the more useful measure MTBF (Mean Time Before Failure),

$$MTBF = MTTF + MTTR$$

- The difference between MTBF and MTTR is often small, but when critical the difference must be observed.

- availability is the chance that at any time a system will be operational. This can be determined experimentally, or estimated. For a system that is into its useful lifetime, this can be a good measure. Note that at the beginning, and end of its life, this value will be changing, and will not be reliable.

$$A(t) = \frac{t_o}{t_o + t_r} = \frac{MTTF}{MTTF + MTTR} = \frac{MTTF}{MTBF} = \frac{1}{1 + \frac{\lambda}{\mu}}$$

where,

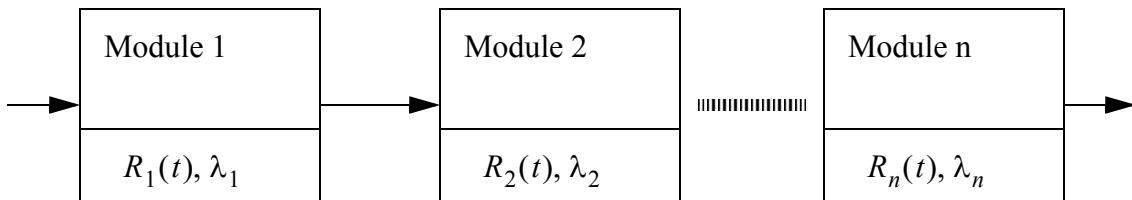
$A(t)$  = probability that a system will be available at any time

$t_o$  = hours in operation over a time period

$t_r$  = hours in repair over a time period

### 37.3 Serial System Reliability

- Fault Coverage is the probability that a system will recover from a failure. This can be derived approximately by examining the design, and making reliable estimates. This number will be difficult to determine exactly because it is based on real, and often unpredictable phenomenon.
- Reliability can be determined with individual system components as a function of probabilities. The two main categories of systems are series, and parallel (redundant). In the best case a high reliability system would have many parallel systems in series.
- In terms of design, a system designer must have an intuitive understanding of the concept of series/parallel functions.
- We can consider a series system where if any of the units fails, then the system becomes inoperative. Here the reliabilities of each of the system components is chained (ANDed) together.



$$R_s(t) = (R_1(t))(R_2(t)) \dots (R_n(t)) = \prod_{i=1}^n R_i(t)$$

where,

$R_s(t)$  = the reliability of a series system at time t

$R_i(t)$  = the reliability of a unit at time t

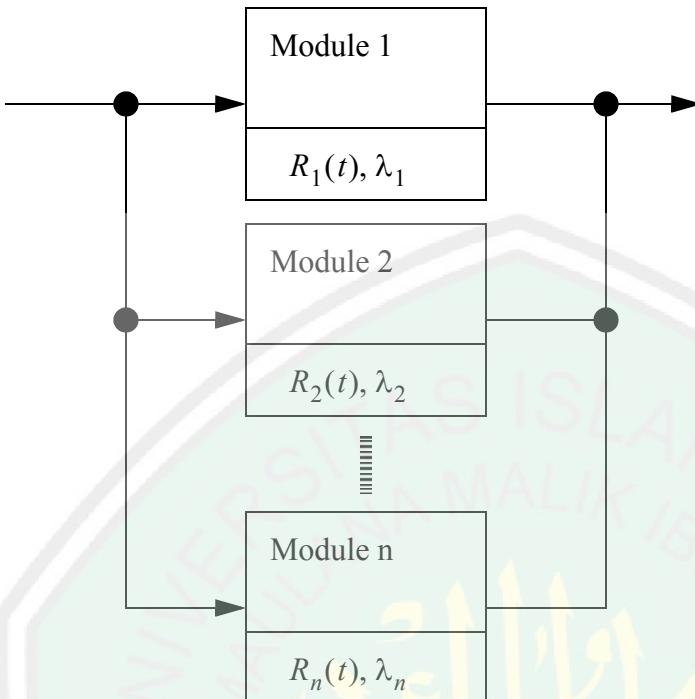
Now, consider the exponential failure law presented before. If each element in a system observes this law, then we can get an exact value of reliability.

$$R_s(t) = (e^{-\lambda_1 t})(e^{-\lambda_2 t}) \dots (e^{-\lambda_n t}) = \prod_{i=1}^n e^{-\lambda_i t} = e^{-\sum_{i=1}^n \lambda_i t}$$

Note: this form  
is very nice.

### 37.4 Parallel System Reliability

- When a 'parallel' component fails the reliability of the overall system is reduced, but the system remains completely or partially functional.
- This type of reliability adds cost, so it is normally only used in critical systems where failure is not acceptable.
- Examples of systems using parallel reliability include,
  - brakes on a car - 4 brakes
  - electronic brakes, also have mechanical backups
  - lights - in dark places multiple bulbs are used so a failed bulb does not leave it dark.
- If any of the units fails the system will continue to operate. Failure will only come when all of the modules fail. Here we are concerned with complements of the chained unreliabilities.



$$Q_p(t) = (Q_1(t))(Q_2(t)) \dots (Q_n(t)) = \prod_{i=1}^n Q_i(t)$$

$$R_p(t) = 1 - Q_p(t) = 1 - \prod_{i=1}^n (1 - R_i(t))$$

where,

$Q_s(t)$  = the unreliability of a parallel system at time t

$Q_i(t)$  = the unreliability of a module at time t

$R_p(t)$  = the reliability of a parallel system at time t

$R_i(t)$  = the unreliability of a module at time t

Note: The parallel form will not result in a simple closed form as it did with the series case.

- also consider the case of a parallel system that requires 'm' of 'n' identical modules to be functional, such as a hybrid system, or a voting system that needs two out of three functional units. The student will consider the binomial form of the probabilities.

$$R_{m;n}(t) = \sum_{i=0}^{n-m} \binom{n}{i} (R(t))^{(n-i)} (1-R(t))^i$$

where,

$R_{m;n}(t)$  = reliability of a system that contains m of n parallel modules

$R(t)$  = the reliability of the modules at time t

$\binom{n}{i} = \frac{n!}{(n-i)! i!}$  = the binomial operator (we can also use Pascal's triangle)

- keep in mind that many systems are a combination of series and parallel units, to find the total reliability, calculate the reliability of the parallel units first, and then calculate the series reliability, replacing the parallel units with their grouped reliability.

## 37.5 Formal Analysis Techniques

### 37.5.1 Failure Modes and Effects Analysis (FMEA)

- Estimates overall reliability of a detailed or existing product design in terms of probability of failure
- basically, each component is examined for failure modes, and the effects of each failure is considered. In turn, the effects of these failures on other parts of the system is considered.
- the following is a reasonable FMEA chart.

Critical Components	Failure Probability	Failure Mode	Number of Failures by Mode	EFFECTS	
				Critical	Non critical
car brakes (car in motion)	$10^{-4}$	disengage engage weaken	10 5 85	$1 \times 10^{-5}$ $5 \times 10^{-6}$	X
car brakes (car parked)	$10^{-6}$	disengage engage weaken	40 30 30	$4 \times 10^{-7}$	X X

- the basic steps to filling one out is,

1. consider all critical components in a system. These are listed in the critical items column.
2. If a component has more than one operation mode, each of these should be considered individually.
3. estimate failure probability based on sources such as those listed below. Error bounds may also be included in the FMEA figures when numbers are unsure. These figures are entered in the “Failure Probability” column.
  - historical data for similar components in similar conditions
  - published values
  - experienced estimates
  - testing
  - etc.
4. The failures in a particular operation mode can take a number of forms. Therefore, each mode of failure for a system is considered and its % of total failures is broken down.
5. In this case the table shows failures divided into critical/non-critical (others are possible). The effects are considered, and in the event of critical failures the probabilities are listed and combined to get the overall system reliability.

- Suitable applications include,

- analyze single units or failures to target reliability problems.
- identify,
  - redundant and fail-safe design requirements

- single item failure modes
- inspection and maintenance requirements
- components for redesign

- This technique is very complete, but also time consuming.
- not suited to complex systems where cascaded errors may occur.

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## 37.7 Problems

1. How are series and parallel reliability different?

ans. Series reliability means that a failure of any unit will cause the entire group to fail. Parallel reliability means that there is some redundancy.

2. A set of 4 production machines are running in parallel. The first two machines have a MTBF of 100 hours and a MTTR of 3 hours. The second two machines have a MTBF of 150 hours and a MTTR of 15 hours. What is the total MTTF for the system?

3. Write a program that will accept test data to determine the Exponential Failure Equation coefficient. This should then be used to calculate the MTTF.

4. Write a program that will accept MTTF for multiple components in series, or parallel, and then calculate the combined MTTF for the system.



## 38. CALCULUS

Topics:

- 

Objectives:

- 

### 38.1 Introduction

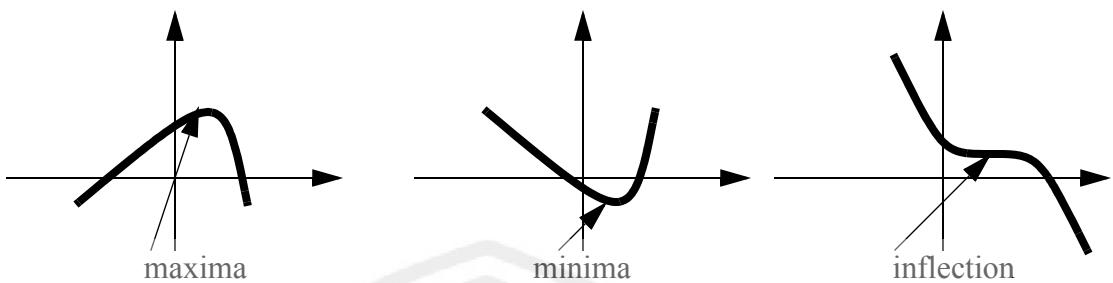
### 38.2 Derivatives

- The basic definition of a derivative is,

$$\frac{d}{dt}f(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

- First derivatives are often used to get the slope of a function. When this is zero the function may

be at a minimum/maximum.



- Notations,

$$\frac{d}{dx}y = y'$$

$$\frac{d}{dt}y = \dot{y}$$

- The basic principles of differentiation are,

Both u, v and w are functions of x, but this is not shown for brevity.

Also note that C is used as a constant, and all angles are in radians.

---

$$\frac{d}{dx}(C) = 0$$

$$\frac{d}{dx}(Cu) = (C)\frac{d}{dx}(u)$$

$$\frac{d}{dx}(u + v + \dots) = \frac{d}{dx}(u) + \frac{d}{dx}(v) + \dots$$

$$\frac{d}{dx}(u^n) = (nu^{n-1})\frac{d}{dx}(u)$$

$$\frac{d}{dx}(uv) = (u)\frac{d}{dx}(v) + (v)\frac{d}{dx}(u) \quad \text{product rule}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \left(\frac{v}{v^2}\right)\frac{d}{dx}(u) - \left(\frac{u}{v^2}\right)\frac{d}{dx}(v) = \frac{v\frac{d}{dx}(u) - u\frac{d}{dx}(v)}{v^2}$$

$$\frac{d}{dx}(uvw) = (uv)\frac{d}{dx}(w) + (uw)\frac{d}{dx}(v) + (vw)\frac{d}{dx}(u)$$

$$\frac{d}{dx}(y) = \frac{d}{du}(y)\frac{d}{dx}(u) \quad \text{chain rule}$$

$$\frac{d}{dx}(u) = \frac{1}{\frac{d}{du}(x)}$$

$$\frac{d}{dx}(y) = \frac{\frac{d}{du}(y)}{\frac{d}{du}(x)}$$

- Examples,

$$\frac{d}{dt}(t^2 + t + 3) = 2t + 1$$

$$\frac{d}{dt}\left(\frac{t^2 + t + 3}{t + 2}\right) = \frac{2t + 1}{t + 2} - \frac{t^2 + t + 3}{(t + 2)^2}$$

- Differentiation rules specific to basic trigonometry and logarithm functions

$$\frac{d}{dx}(\sin u) = (\cos u) \frac{d}{dx}(u)$$

$$\frac{d}{dx}(\cos u) = (-\sin u) \frac{d}{dx}(u)$$

$$\frac{d}{dx}(\tan u) = \left(\frac{1}{\cos u}\right)^2 \frac{d}{dx}(u)$$

$$\frac{d}{dx}(e^u) = (e^u) \frac{d}{dx}(u)$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\cot u) = (-\csc u)^2 \frac{d}{dx}(u)$$

$$\frac{d}{dx}(\sec u) = (\tan u \sec u) \frac{d}{dx}(u)$$

$$\frac{d}{dx}(\csc u) = (-\csc u \cot u) \frac{d}{dx}(u)$$

$$\frac{d}{dx}(\sinh u) = (\cosh u) \frac{d}{dx}(u)$$

$$\frac{d}{dx}(\cosh u) = (\sinh u) \frac{d}{dx}(u)$$

$$\frac{d}{dx}(\tanh u) = (\operatorname{sech} u)^2 \frac{d}{dx}(u)$$

- L'Hospital's rule can be used when evaluating limits that go to infinity.

$$\lim_{x \rightarrow a} \left( \frac{f(x)}{g(x)} \right) = \lim_{x \rightarrow a} \left( \frac{\left( \frac{d}{dt} \right) f(x)}{\left( \frac{d}{dt} \right) g(x)} \right) = \lim_{x \rightarrow a} \left( \frac{\left( \frac{d}{dt} \right)^2 f(x)}{\left( \frac{d}{dt} \right)^2 g(x)} \right) = \dots$$

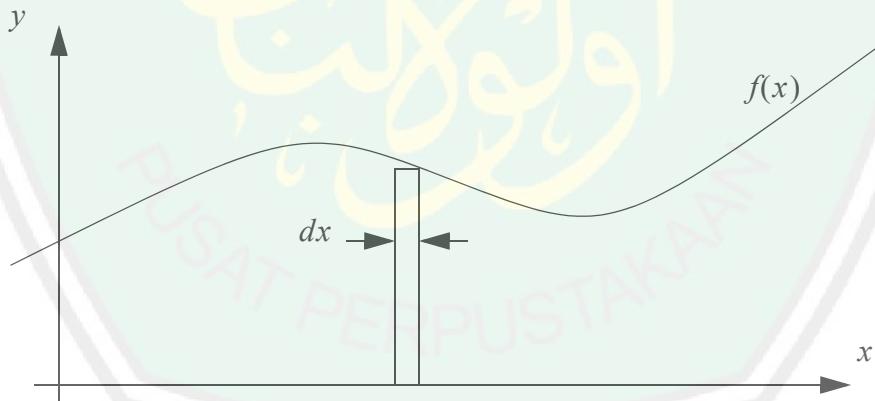
- Some techniques used for finding derivatives are,

Leibnitz's Rule, (notice the form is similar to the binomial equation) can be used for finding the derivatives of multiplied functions.

$$\begin{aligned} \left(\frac{d}{dx}\right)^n(uv) &= \left(\frac{d}{dx}\right)^0(u)\left(\frac{d}{dx}\right)^n(v) + \binom{n}{1}\left(\frac{d}{dx}\right)^1(u)\left(\frac{d}{dx}\right)^{n-1}(v) \\ &\quad \binom{n}{2}\left(\frac{d}{dx}\right)^2(u)\left(\frac{d}{dx}\right)^{n-2}(v) + \dots + \binom{n}{n}\left(\frac{d}{dx}\right)^n(u)\left(\frac{d}{dx}\right)^0(v) \end{aligned}$$

### 38.3 Integrals

- Integrals are often referred to as anti-derivatives
- definite integrals have boundaries defined. Indefinite integrals do not have boundaries defined and a constant must be added to the result.
- To set up integrals use integration elements (aka slices),



$$dA = \text{width} \cdot \text{height} = dx \cdot f(x)$$

$$A = \int f(x) dx$$

- Some basic properties of indefinite integrals (no given start and end limits) include,

In the following expressions, u, v, and w are functions of x. in addition to this, C is a constant. and all angles are radians.

$$\int Cdx = ax + C$$

$$\int Cf(x)dx = C\int f(x)dx$$

$$\int(u + v + w + \dots)dx = \int u dx + \int v dx + \int w dx + \dots$$

$$\int u dv = uv - \int v du = \text{integration by parts}$$

$$\int f(Cx)dx = \frac{1}{C}\int f(u)du \quad u = Cx$$

$$\int F(f(x))dx = \int F(u)\frac{d}{du}(x)du = \int \frac{F(u)}{f'(x)}du \quad u = f(x)$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad \int \frac{1}{x} dx = \ln|x| + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C \quad \int e^x dx = e^x + C$$

- Some of the trigonometric integrals are,

$$\int \sin x dx = -\cos x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int (\sin x)^2 dx = -\frac{\sin x \cos x + x}{2} + C$$

$$\int (\cos x)^2 dx = \frac{\sin x \cos x + x}{2} + C$$

$$\int (\sin x)^3 dx = -\frac{\cos x ((\sin x)^2 + 2)}{3} + C$$

$$\int (\cos x)^3 dx = \frac{\sin x ((\cos x)^2 + 2)}{3} + C$$

$$\int x \cos(ax) dx = \frac{\cos(ax)}{a^2} + \frac{x}{a} \sin(ax) + C$$

$$\int x^2 \cos(ax) dx = \frac{2x \cos(ax)}{a^2} + \frac{a^2 x^2 - 2}{a^3} \sin(ax) + C$$

$$\int (\cos x)^4 dx = \frac{3x}{8} + \frac{\sin 2x}{4} + \frac{\sin 4x}{32} + C$$

$$\int \cos x (\sin x)^n dx = \frac{(\sin x)^{n+1}}{n+1} + C$$

$$\int \sinh x dx = \cosh x + C$$

$$\int \cosh x dx = \sinh x + C$$

$$\int \tanh x dx = \ln(\cosh x) + C$$

- Some other integrals of use that are basically functions of x are,

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int (a+bx)^{-1} dx = \frac{\ln(a+bx)}{b} + C$$

$$\int (a+bx^2)^{-1} dx = \frac{1}{2\sqrt{(-b)a}} \ln\left(\frac{\sqrt{a}+2\sqrt{-b}}{\sqrt{a}-x\sqrt{-b}}\right) + C, a > 0, b < 0$$

$$\int x(a+bx^2)^{-1} dx = \frac{\ln(bx^2+a)}{2b} + C$$

$$\int x^2(a+bx^2)^{-1} dx = \frac{x}{b} - \frac{a}{b\sqrt{ab}} \operatorname{atan}\left(\frac{x\sqrt{ab}}{a}\right) + C$$

$$\int (a^2-x^2)^{-1} dx = \frac{1}{2a} \ln\left(\frac{a+x}{a-x}\right) + C, a^2 > x^2$$

$$\int (a+bx)^{-1} dx = \frac{2\sqrt{a+bx}}{b} + C$$

$$\int x(x^2 \pm a^2)^{-\frac{1}{2}} dx = \sqrt{x^2 \pm a^2} + C$$

$$\int (a+bx+cx^2)^{-1} dx = \frac{1}{\sqrt{c}} \ln\left[\sqrt{a+bx+cx^2} + x\sqrt{c} + \frac{b}{2\sqrt{c}}\right] + C, c > 0$$

$$\int (a+bx+cx^2)^{-1} dx = \frac{1}{\sqrt{-c}} \operatorname{asin}\left[\frac{-2cx-b}{\sqrt{b^2-4ac}}\right] + C, c < 0$$

$$\int (a + bx)^{\frac{1}{2}} dx = \frac{2}{3b} (a + bx)^{\frac{3}{2}}$$

$$\int (a + bx)^{\frac{1}{2}} dx = \frac{2}{3b} (a + bx)^{\frac{3}{2}}$$

$$\int x(a + bx)^{\frac{1}{2}} dx = -\frac{2(2a - 3bx)(a + bx)^{\frac{3}{2}}}{15b^2}$$

$$\int (1 + a^2 x^2)^{\frac{1}{2}} dx = \frac{x(1 + a^2 x^2)^{\frac{1}{2}} + \frac{\ln\left(x + \left(\frac{1}{a^2} + x^2\right)^{\frac{1}{2}}\right)}{a}}{2}$$

$$\int x(1 + a^2 x^2)^{\frac{1}{2}} dx = \frac{a\left(\frac{1}{a^2} + x^2\right)^{\frac{3}{2}}}{3}$$

$$\int x^2(1 + a^2 x^2)^{\frac{1}{2}} dx = \frac{ax\left(\frac{1}{a^2} + x^2\right)^{\frac{3}{2}}}{4} - \frac{8}{8a^2} x(1 + a^2 x^2)^{\frac{1}{2}} - \frac{\ln\left(x + \left(\frac{1}{a^2} + x^2\right)^{\frac{1}{2}}\right)}{8a^3}$$

$$\int (1 - a^2 x^2)^{\frac{1}{2}} dx = \frac{1}{2} \left[ x(1 - a^2 x^2)^{\frac{1}{2}} + \frac{\arcsin(ax)}{a} \right]$$

$$\int x(1 - a^2 x^2)^{\frac{1}{2}} dx = -\frac{a}{3} \left( \frac{1}{a^2} - x^2 \right)^{\frac{3}{2}}$$

$$\int x^2(a^2 - x^2)^{\frac{1}{2}} dx = -\frac{x}{4} (a^2 - x^2)^{\frac{3}{2}} + \frac{1}{8} \left[ x(a^2 - x^2)^{\frac{1}{2}} + a^2 \arcsin\left(\frac{x}{a}\right) \right]$$

$$\int (1 + a^2 x^2)^{-\frac{1}{2}} dx = \frac{1}{a} \ln\left(x + \left(\frac{1}{a^2} + x^2\right)^{\frac{1}{2}}\right)$$

$$\int (1 - a^2 x^2)^{-\frac{1}{2}} dx = \frac{1}{a} \arcsin(ax) = -\frac{1}{a} \arccos(ax)$$

- Integrals using the natural logarithm base ‘e’,

$$\int e^{ax} dx = \frac{e^{ax}}{a} + C$$

$$\int xe^{ax} dx = \frac{e^{ax}}{a^2}(ax - 1) + C$$

### 38.3.1 Integration Examples

- Integration by parts - It is normal to have to do the integration by parts more than once to solve a problem.

$$\int x^2 \sin 4x dx$$

$$u =$$

$$v =$$

- Substitution.

$$\int te^{t^2+1} dt$$

guess,

$$w = t^2 + 1 \quad \frac{d}{dt}w = 2t \quad dw = 2tdt$$

$$\int te^{t^2+1} dt = \int e^{t^2+1} t dt = \int e^w \frac{1}{2} dw = \frac{1}{2} e^w + C = \frac{1}{2} e^{t^2+1} + C$$

- Partial fractions can be used to reduce complex polynomials to simple to integrate forms.

$$\int \left( \frac{5x+10}{x^2+3x+2} \right) dx =$$

### 38.4 Vectors

- When dealing with large and/or time varying objects or phenomenon we must be able to describe the state at locations, and as a whole. To do this vectors are a very useful tool.
- Consider a basic function and how it may be represented with partial derivatives.

$$y = f(x, y, z)$$

We can write this in differential form, but the right hand side must contain partial derivatives. If we separate the operators from the function, we get a simpler form. We can then look at them as the result of a dot product, and divide it into two vectors.

$$(d)y = \left( \left( \frac{\partial}{\partial x} \right) f(x, y, z) \right) dx + \left( \left( \frac{\partial}{\partial y} \right) f(x, y, z) \right) dy + \left( \left( \frac{\partial}{\partial z} \right) f(x, y, z) \right) dz$$

$$(d)y = \left[ \left( \frac{\partial}{\partial x} \right) dx + \left( \frac{\partial}{\partial y} \right) dy + \left( \frac{\partial}{\partial z} \right) dz \right] f(x, y, z)$$

$$(d)y = \left[ \left( \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k \right) \bullet (dx i + dy j + dz k) \right] f(x, y, z)$$

We then replace these vectors with the operators below. In this form we can manipulate the equation easily (whereas the previous form was very awkward).

$$(d)y = [\nabla \bullet dX] f(x, y, z)$$

$$(d)y = \nabla f(x, y, z) \bullet dX$$

$$(d)y = |\nabla f(x, y, z)| |dX| \cos \theta$$

In summary,

$$\nabla = \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k$$

$\nabla \bullet F$  = the divergence of function F

$$F = F_x i + F_y j + F_z k$$

$\nabla \times F$  = the curl of function F

- Gauss's or Green's or divergence theorem is given below. Both sides give the flux across a surface, or out of a volume. This is very useful for dealing with magnetic fields.

$$\int_V (\nabla \bullet F) dV = \oint_A F dA$$

where,

$V, A$  = a volume V enclosed by a surface area A

$F$  = a field or vector value over a volume

- Stoke's theorem is given below. Both sides give the flux across a surface, or out of a volume. This is very useful for dealing with magnetic fields.

$$\int_A (\nabla \times F) dA = \oint_L F dL$$

where,

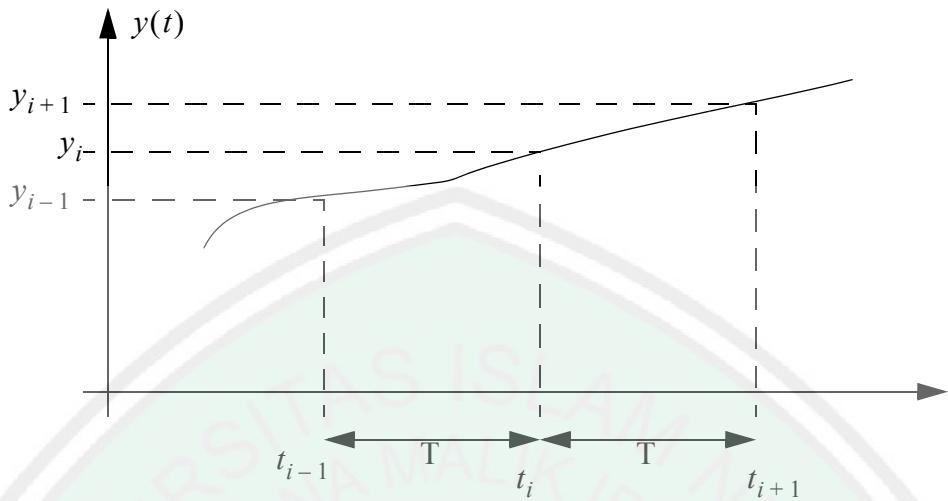
$A, L$  = A surface area  $A$ , with a bounding parameter of length  $L$

$F$  = a field or vector value over a volume

## 38.5 Numerical Tools

### 38.5.1 Approximation of Integrals and Derivatives from Sampled Data

- This form of integration is done numerically - this means by doing repeated calculations to solve the equation. Numerical techniques are not as elegant as solving differential equations, and will result in small errors. But these techniques make it possible to solve complex problems much faster.
- This method uses forward/backward differences to estimate derivatives or integrals from measured data.



$$\int_{t_{i-1}}^{t_i} y(t_i) \approx \left( \frac{y_i + y_{i-1}}{2} \right) (t_i - t_{i-1}) = \frac{T}{2} (y_i + y_{i-1})$$

$$\frac{d}{dt} y(t_i) \approx \left( \frac{y_i - y_{i-1}}{t_i - t_{i-1}} \right) = \left( \frac{y_{i+1} - y_i}{t_{i+1} - t_i} \right) = \frac{1}{T} (y_i - y_{i-1}) = \frac{1}{T} (y_{i+1} - y_i)$$

$$\left( \frac{d}{dt} \right)^2 y(t_i) \approx \frac{\frac{1}{T} (y_{i+1} - y_i) - \frac{1}{T} (y_i - y_{i-1})}{T} = \frac{-2y_i + y_{i-1} + y_{i+1}}{T^2}$$

### 38.5.2 Centroids and Moments of Inertia

## 38.6 Problems

1. Find the following derivatives.

a)  $\frac{d}{dx} \left( \frac{1}{x+1} \right)$

ans.  $\frac{-1}{(x+1)^2}$

b)  $\frac{d}{dt} (e^{-t} \sin(2t-4))$

$-e^{-t} \sin(2t-4) + 2e^{-t} \cos(2t-4)$

2. Solve the following integrals.

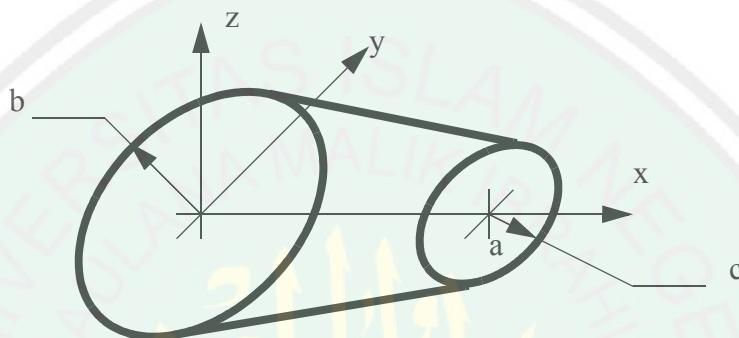
a)  $\int e^{2t} dt$

ans.  $\frac{1}{2}e^{2t} + C$

b)  $\int (\sin \theta + \cos 3\theta) d\theta$

$-\cos \theta + \frac{1}{3} \sin 3\theta + C$

3. Set up an integral and solve it to find the area inside the volume below. The shape is basically a cone with the top cut off.



(ans.

$$V = \pi \left( \frac{c^2 a}{3} + \frac{b^2 a}{3} + \frac{abc}{3} \right)$$

(move later) 4. Write a program that integrates the following function using the trapezoidal rule and Simpson's rule. The period of integration should range from 0 to 10 seconds. The

program should compare the numerical results to the exact result.

$$y(t) = 5t^2 + \sin(20t)$$

```
// sample program
function foo = y(t)
    foo = 5 * t ^ 2 + sin ( 20 * t );
endfunction

sum = 0;
h = 0.1; // step size
for t = 0:h:10
    sum = sum + h * (f(t+h) + f(t)) / 2; // uses the trapezoid rule
end

t = 10;
actual = (5 / 3) * t ^ 3 - 0.05 * cos(20 * t);
mprintf("integral value numerical = %f, actual = %f\n", sum, actual);
// the value should be 1667 approximately
```

(move later - near end) 5. Write a program that finds the location of the minimum value for the function given below.

$$y(x) = \sin(\sin(5x^2) + \cos(20x) - 5x) + (x - 10)^2$$

```
// sample program
function foo = y(x)
    foo = sin(sin(5 * x * x) + cos(20 * x) - 5*x) + (x - 10) ^ 2;
endfunction

y_min = 100000000000; // something big to start
x_min = 0; // this value doesn't matter
h = 0.001; // step size - also the accuracy
for x = -100:h:100
    if (y(x) < y_min), // look for the smallest value
        x_min = x;
        y_min = y(x);
    end
end
x_min, y_min
```

## 8. Differentiate.

a)  $(x^3 + 4)^4$

(ans.  $4(3)x^2(x^3 + 4)^3$ )

b)  $(x^3 + 4)^{-4}$

$-4(3)x^2(x^3 + 4)^{-5}$

c)  $\ln(x)$

$\frac{1}{x}$

d)  $\ln(x + x^3)$

$\frac{3x^2 + 1}{x^3 + x}$

e)  $e^x$

$e^x$

f)  $e^{x+x^2-x^3}$

$(-3x^2 + 2x + 1)e^{x+x^2-x^3}$

g)  $\frac{x}{x^2 + 5}$

$\frac{-x^2 + 5}{(x^2 + 5)^2}$

h)  $\frac{x^2}{x^3 + 5x}$

$\frac{-x^2 + 5}{(x^2 + 5)^2}$

i)  $e^{x^2+5}$

$2xe^{x^2+5}$

j)  $\sin(x^2)$

$2x\cos x^2$

k)  $e^{x^2} \sin(x)$

$2xe^{x^2} \sin(x) + e^{x^2} \cos(x)$

l)  $\frac{\sin x}{x}$

$\frac{x\cos x - \sin x}{x^2}$

9. Integrate the following indefinite functions with respect to x.

a)  $\frac{1}{x}$

(ans)  $\ln(x) + C$

b)  $\frac{x}{x+5}$

$x - 5 \ln(x+5) + C$

c)  $\frac{1}{x^3}$

$\frac{-1}{2x^2} + C$

d)  $5x^3 + 2x^2$

$1.25x^4 + \frac{2}{3}x^3 + C$

e)  $xe^{x^2}$

$\frac{1}{2}e^{x^2} + C$

f)  $\cos x$

$\sin x + C$

g)  $(\cos x)^2$

$\frac{\sin x \cos x + x}{2} + C$

10. Solve the following integrals.

a)  $\int_0^\infty e^{-t} dt$

(ans)  $-1$

b)  $\int_0^{10} (t^2 + 5t^3) dt$

$12833.33$

c)  $\int_0^{4\pi} \cos(t) dt$

$0$

11. Write a Scilab program that numerically integrates the following functions.

a)  $\int_0^{10} (t^2 + 5t^3) dt$

b)  $\int_0^{4\pi} \cos(t) dt$

# 1. ANALYSIS OF DIFFERENTIAL EQUATIONS

Topics:

- First and second-order homogeneous differential equations
- Non-homogeneous differential equations
- First and second-order responses
- Non-linear system elements
- Design case

Objectives:

- To develop explicit equations that describe a system response.
- To recognize first and second-order equation forms.

## 1.1 Introduction

In the previous chapter we derived differential equations of motion for translating systems. These equations can be used to analyze the behavior of the system and make design decisions. The most basic method is to select a standard input type (a forcing function) and initial conditions, and then solve the differential equation. It is also possible to estimate the system response without solving the differential equation as will be discussed later.

Figure 1.1 shows an abstract description of a system. The basic concept is that the system changes the inputs to outputs. Say, for example, that the system to be analyzed is an elevator. Inputs to the system would be the mass of human riders and desired elevator height. The output response of the system would be the actual height of the elevator. For analysis, the system model could be developed using differential equations for the motor, elastic lift cable, mass of the car, etc. A basic test would involve assuming that the elevator starts at the ground floor and must travel to the top floor. Using assumed initial values and input functions the differential equation could be solved to get an explicit equation for elevator height. This output response can then be used as a guide to modify design choices (parameters). In practice, many of the assumptions and tests are mandated by law or by groups such as Underwriters Laboratories (UL), Canadian Standards Association (CSA) and the European Commission (CE).

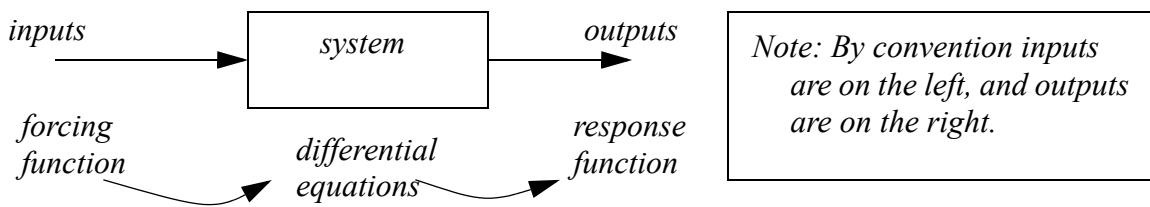


Figure 1.1 A system with and input and output response

There are several standard input types used to test a system. These are listed below in order of relative popularity with brief explanations.

- step - a sudden change of input, such as very rapidly changing a desired speed from 0Hz to 50Hz.
- ramp - a continuously increasing input, such as a motor speed that increases constantly at 10Hz per minute.
- sinusoidal - a cyclic input that varies continuously, such as wave height that is continually oscillating at 1Hz.
- parabolic - an exponentially increasing input, such as a motor speed that is 2Hz at 1 second, 4rad/s at 2 seconds, 8rad/s at 3 seconds, etc.

After the system has been modeled, an input type has been chosen, and the initial conditions have been selected, the system can be analyzed to determine its behavior. The most fundamental technique is to integrate the differential equation(s) for the system.

## 1.2 Explicit Solutions

Solving a differential equation results in an explicit solution. This equation provides the general response as a function of time, but it can also be used to find frequencies and other characteristics of interest. This section will review techniques used to integrate first and second-order homogenous differential equations. These equations correspond to systems without inputs, also called unforced systems. Non-homogeneous differential equations will also be reviewed.

The basic types of differential equations are shown in Figure 1.2. Each of these equations is linear. On the left hand side is the integration variable 'x'. If the right hand side is zero, then the equation is homogeneous. Each of these equations is linear because each of the terms on the left hand side is simply multiplied by a linear coefficient.

$A\dot{x} + Bx = 0$	<i>first-order homogeneous</i>
$A\dot{x} + Bx = Cf(t)$	<i>first-order non-homogeneous</i>
$A\ddot{x} + B\dot{x} + Cx = 0$	<i>second-order homogeneous</i>
$A\ddot{x} + B\dot{x} + Cx = Df(t)$	<i>second-order non-homogeneous</i>

Figure 1.2 Standard equation forms

A general solution for a first-order homogeneous differential equation is given in Figure 1.3. The solution begins with the solution of the homogeneous equation where a general form is 'guessed'. Substitution leads to finding the value of the coefficient 'Y'. Following this, the initial conditions for the equation are used to find the value of the coefficient 'X'. Notice that the final equation will begin at the initial displacement, but approach zero as time goes to infinity. The e-to-the-x behavior is characteristic for a first-order response.

Given the general form of a first-order homogeneous equation,

$$A\dot{x} + Bx = 0 \quad \text{and} \quad x(0) = x_0$$

Guess a solution form and solve.

$$x = Xe^{-Yt} \quad \dot{x} = -YXe^{-Yt}$$

$$A(-YXe^{-Yt}) + B(Xe^{-Yt}) = 0$$

$$A(-Y) + B = 0$$

$$Y = \frac{B}{A}$$

Therefore the general form is,

$$x_h = Xe^{-\frac{B}{A}t}$$

Next, use the initial conditions to find the remaining unknowns.

$$x_h = Xe^{-\frac{B}{A}t}$$

$$x_0 = Xe^{-\frac{B}{A}0}$$

$$x_0 = X$$

Therefore the final equation is,

$$x(t) = x_0 e^{-\frac{B}{A}t}$$

Note: The general form below is useful for finding almost all homogeneous equations

$$x_h(t) = Xe^{-Yt}$$

Figure 1.3 General solution of a first-order homogeneous equation

Solve the following differential equation given the initial condition.

$$\dot{x} + 2x = 0$$

$$x(0) = 3$$

$$\text{ans. } x(t) = 3e^{-2t}$$

Figure 1.4 Drill Problem: First order homogeneous differential equation

The general solution to a second-order homogeneous equation is shown in Figure 1.5. The solution begins with a guess of the homogeneous solution, and the solution of a quadratic equation. There are three possible cases that result from the solution of the quadratic equation: different but real roots; two identical real roots; or two complex roots. The three cases result in three different forms of solutions, as shown. The complex result is the most notable because it results in sinusoidal oscillations. It is not shown, but after the homogeneous solution has been found, the initial conditions need to be used to find the remaining coefficient values.

Given,

$$A\ddot{x} + B\dot{x} + Cx = 0 \quad x(0) = x_0 \quad \text{and} \quad \dot{x}(0) = v_0$$

Guess a general equation form and substitute it into the differential equation,

$$x_h = Xe^{Yt} \quad \dot{x}_h = YXe^{Yt} \quad \ddot{x}_h = Y^2 Xe^{Yt}$$

$$A(Y^2 Xe^{Yt}) + B(YXe^{Yt}) + C(Xe^{Yt}) = 0$$

$$A(Y^2) + B(Y) + C = 0$$

$$Y = \frac{-B \pm \sqrt{(B)^2 - 4(AC)}}{2A} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

Note: There are three possible outcomes of finding the roots of the equations: two different real roots, two identical real roots, or two complex roots. Therefore there are three fundamentally different results.

If the values for  $Y$  are both real, but different, the general form is,

$$Y = R_1, R_2 \quad x_h = X_1 e^{R_1 t} + X_2 e^{R_2 t}$$

Note: The initial conditions are then used to find the values for  $X_1$  and  $X_2$ .

If the values for  $Y$  are both real, and identical, the general form is,

$$Y = R_1, R_1 \quad x_h = X_1 e^{R_1 t} + X_2 t e^{R_1 t}$$

The initial conditions are then used to find the values for  $X_1$  and  $X_2$ .

If the values for  $Y$  are complex, the general form is,

$$Y = \sigma \pm \omega j \quad x_h = X_3 e^{\sigma t} \cos(\omega t + X_4)$$

The initial conditions are then used to find the values of  $X_3$  and  $X_4$ .

**Figure 1.5** Solution of a second-order homogeneous equation

As mentioned above, a complex solution when solving the homogeneous equation results in a sinusoidal oscillation, as proven in Figure 1.6. The most notable part of the solution is that there is both a frequency of oscillation and a phase shift. This form is very useful for analyzing the frequency response of a system, as will be seen in a later chapter.

Consider the situation where the results of a homogeneous solution are the complex conjugate pair.

$$Y = R \pm Cj$$

This gives the general result, as shown below:

$$\begin{aligned}
 x &= X_1 e^{(R+Cj)t} + X_2 e^{(R-Cj)t} \\
 x &= X_1 e^{Rt} e^{Cjt} + X_2 e^{Rt} e^{-Cjt} \\
 x &= e^{Rt} (X_1 e^{Cjt} + X_2 e^{-Cjt}) \\
 x &= e^{Rt} (X_1 (\cos(Ct) + j \sin(Ct)) + X_2 (\cos(-Ct) + j \sin(-Ct))) \\
 x &= e^{Rt} (X_1 (\cos(Ct) + j \sin(Ct)) + X_2 (\cos((Ct) - j \sin(Ct)))) \\
 x &= e^{Rt} ((X_1 + X_2) \cos(Ct) + j(X_1 - X_2) \sin(Ct)) \\
 x &= e^{Rt} \frac{((X_1 + X_2) \cos(Ct) + j(X_1 - X_2) \sin(Ct))}{\sqrt{(X_1 + X_2)^2 + j^2 (X_1 - X_2)^2}} \\
 x &= e^{Rt} \frac{\sqrt{X_1^2 + 2X_1 X_2 + X_2^2 - (X_1^2 + -2X_1 X_2 + X_2^2)}}{\sqrt{X_1^2 + 2X_1 X_2 + X_2^2 - (X_1^2 + -2X_1 X_2 + X_2^2)}} ((X_1 + X_2) \cos(Ct) + j(X_1 - X_2) \sin(Ct)) \\
 x &= e^{Rt} \frac{\sqrt{4X_1 X_2}}{\sqrt{4X_1 X_2}} ((X_1 + X_2) \cos(Ct) + j(X_1 - X_2) \sin(Ct)) \\
 x &= e^{Rt} \sqrt{4X_1 X_2} \left( \frac{(X_1 + X_2)}{\sqrt{4X_1 X_2}} \cos(Ct) + j \frac{(X_1 - X_2)}{\sqrt{4X_1 X_2}} \sin(Ct) \right) \\
 x &= e^{Rt} \sqrt{4X_1 X_2} \cos \left( Ct + \operatorname{atan} \left( \frac{(X_1 - X_2)}{(X_1 + X_2)} \right) \right) \\
 x &= e^{Rt} X_3 \cos(Ct + X_4)
 \end{aligned}$$

where,  $X_3 = \sqrt{4X_1 X_2}$   
 $X_4 = \operatorname{atan} \left( \frac{(X_1 - X_2)}{(X_1 + X_2)} \right)$

*frequency*      *phase shift*

Figure 1.6 Phase shift solution for a second-order homogeneous differential equation

*Note: Occasionally a problem solution might consist of both a sine and cosine term with the same frequency. These should normally be combined to a single term with a phase shift as shown below.*

*Recall the double angle formula,*

$$\sin(\omega t + \theta) = \sin\omega t \cos\theta + \sin\theta \cos\omega t$$

*This can be written in a more common form,*

$$A(\sin\omega t \cos\theta + \sin\theta \cos\omega t) = A \sin(\omega t + \theta)$$

$$A \cos\theta \sin\omega t + A \sin\theta \cos\omega t = A \sin(\omega t + \theta)$$

$$B \sin\omega t + C \cos\omega t = A \sin(\omega t + \theta) \quad \text{where,}$$

$$B = A \cos\theta$$

$$C = A \sin\theta$$

$$A = \frac{B}{\cos\theta} = \frac{C}{\sin\theta}$$

$$\frac{\sin\theta}{\cos\theta} = \frac{C}{B}$$

$$\theta = \tan^{-1}\left(\frac{C}{B}\right)$$

$$A = \sqrt{B^2 + C^2}$$

*Consider the example,*

$$3 \sin 5t + 4 \cos 5t = \sqrt{3^2 + 4^2} \sin\left(5t + \tan^{-1}\left(\frac{4}{3}\right)\right) = 5 \sin(5t + 0.927)$$

Figure 1.7 Phase shift solution form

Solve the following differential equation given the initial condition.

$$\ddot{x} + 2\dot{x} + x = 0$$

$$x(0) = 1 \quad \dot{x}(0) = 2$$

*ans.*  $x(t) = e^{-t} + 3te^{-t}$

Figure 1.8 Drill Problem: Second order homogeneous differential equation

The methods for solving non-homogeneous differential equations builds upon the methods used for the solution of homogeneous equations. This process adds a step to find the particular solution of the equation. An example of the solution of a first-order non-homogeneous equation is shown in Figure 1.9. To find the homogeneous solution the non-homogeneous part of the equation is set to zero. To find the particular solution the final

form must be guessed. This is then substituted into the equation, and the values of the coefficients are found. Finally the homogeneous and particular solutions are added to get the final equation. The overall response of the system can be obtained by adding the homogeneous and particular parts. This is acceptable because the equations are linear, and the principle of superposition applies. The homogeneous equation deals with the response to initial conditions, and the particular solution deals with the response to forced inputs.

*Generally,*

$$A\dot{x} + Bx = Cf(t) \quad x(0) = x_0$$

*First, find the homogeneous solution as before, in Figure 1.3.*

$$x_h = x_0 e^{-\frac{B}{A}t}$$

*Next, guess the particular solution by looking at the form of 'f(t)'. This step is highly subjective, and if an incorrect guess is made, it will be unsolvable. When this happens, just make another guess and repeat the process. An example is given below. In the case below the guess should be similar to the exponential forcing function.*

*For example, if we are given*

$$6\dot{x} + 2x = 5e^{4t}$$

*A reasonable guess for the particular solution is,*

$$x_p = C_1 e^{4t} \quad \dot{x}_p = 4C_1 e^{4t}$$

*Substitute these into the differential equation and solve for A.*

$$\begin{aligned} 6(4C_1 e^{4t}) + 2(C_1 e^{4t}) &= 5e^{4t} \\ 24C_1 + 2C_1 &= 5 \quad \therefore C_1 = \frac{5}{26} \end{aligned}$$

*Combine the particular and homogeneous solutions.*

$$x = x_p + x_h = \frac{5}{26} e^{4t} + x_0 e^{-\frac{6}{2}t}$$

*Figure 1.9* Solution of a first-order non-homogeneous equation

The method for finding a particular solution for a second-order non-homogeneous differential equation is shown in Figure 1.10. In this example the forcing function is sinusoidal, so the particular result should also be sinusoidal. The final result is converted into a phase shift form.

Generally,

$$A\ddot{x} + B\dot{x} + Cx = Df(t) \quad x(0) = x_0 \quad \text{and} \quad \dot{x}(0) = v_0$$

1. Find the homogeneous solution as before.

$$\begin{aligned} x_h &= X_3 e^{\sigma t} \cos(\omega t + X_4) \\ \text{or } x_h &= X_1 e^{\sigma_1 t} + X_2 t e^{\sigma_2 t} \\ \text{or } x_h &= X_1 e^{\sigma_1 t} + X_2 e^{\sigma_2 t} \end{aligned}$$

2. Guess the particular solution by looking at the form of ' $f(t)$ '. This step is highly subjective, and if an incorrect guess is made it will be unsolvable. When this happens, just make another guess and repeat the process. For the purpose of illustration an example is given below. In the case below it should be similar to the sine function.

For example, if we are given

$$2\ddot{x} + 6\dot{x} + 2x = 2\sin(3t + 4)$$

A reasonable guess is,

$$\begin{aligned} x_p &= A\sin(3t) + B\cos(3t) \\ \dot{x}_p &= 3A\cos(3t) - 3B\sin(3t) \\ \ddot{x}_p &= -9A\sin(3t) - 9B\cos(3t) \end{aligned}$$

Substitute these into the differential equation and solve for  $A$  and  $B$ .

$$\begin{aligned} 2(-9A\sin(3t) - 9B\cos(3t)) + 6(3A\cos(3t) - 3B\sin(3t)) + \\ 2(A\sin(3t) + B\cos(3t)) &= 2\sin(3t + 4) \\ (-18A - 18B + 2A)\sin(3t) + (-18B + 18A + 2B)\cos(3t) &= 2\sin(3t + 4) \\ (-16A - 18B)\sin(3t) + (18A - 16B)\cos(3t) &= 2(\sin 3t \cos 4 + \cos 3t \sin 4) \\ (-16A - 18B)\sin(3t) + (18A - 16B)\cos(3t) &= (2\cos 4)\sin(3t) + (2\sin 4)\cos(3t) \\ -16A - 18B &= 2\cos 4 \quad 18A - 16B = 2\sin 4 \\ \begin{bmatrix} -16 & -18 \\ 18 & -16 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} &= \begin{bmatrix} 2\cos 4 \\ 2\sin 4 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} -16 & -18 \\ 18 & -16 \end{bmatrix}^{-1} \begin{bmatrix} -1.307 \\ -1.514 \end{bmatrix} = \begin{bmatrix} -0.0109 \\ 0.0823 \end{bmatrix} \end{aligned}$$

Next, rearrange the equation to phase shift form.

$$\begin{aligned} x_p &= -0.0109\sin(3t) + 0.0823\cos(3t) \\ x_p &= \sqrt{-0.0109^2 + 0.0823^2} \sin\left(3t + \tan\left(\frac{0.0823}{-0.0109}\right) + \frac{\pi}{2}\right) \end{aligned}$$

3. Use the initial conditions to determine the coefficients in the homogeneous solution.

Figure 1.10 Solution of a second-order non-homogeneous equation

When guessing particular solutions, the forms in Figure 1.11 can be helpful.

Forcing Function	Guess
$A$	$C$
$Ax + B$	$Cx + D$
$e^{Ax}$	$Ce^{Ax}$ or $Cxe^{Ax}$
$B\sin(Ax)$ or $B\cos(Ax)$	$C\sin(Ax) + D\cos(Ax)$ or $Cx\sin(Ax) + xD\cos(Ax)$

Figure 1.11 General forms for particular solutions

Solve the following differential equation given the initial condition.

$$\ddot{x} + 2\dot{x} + x = 1$$

$$x(0) = 0 \quad \dot{x}(0) = 0$$

$$ans. \quad x(t) = -e^{-t} - te^{-t} + 1$$

Figure 1.12 Drill Problem: Second order non-homogeneous differential equation

An example of a second-order system is shown in Figure 1.13. As expected, it begins with a FBD and summation of forces. This is followed with the general solution of

the homogeneous equation. Real roots are assumed thus allowing the problem solution to continue in Figure 1.14.

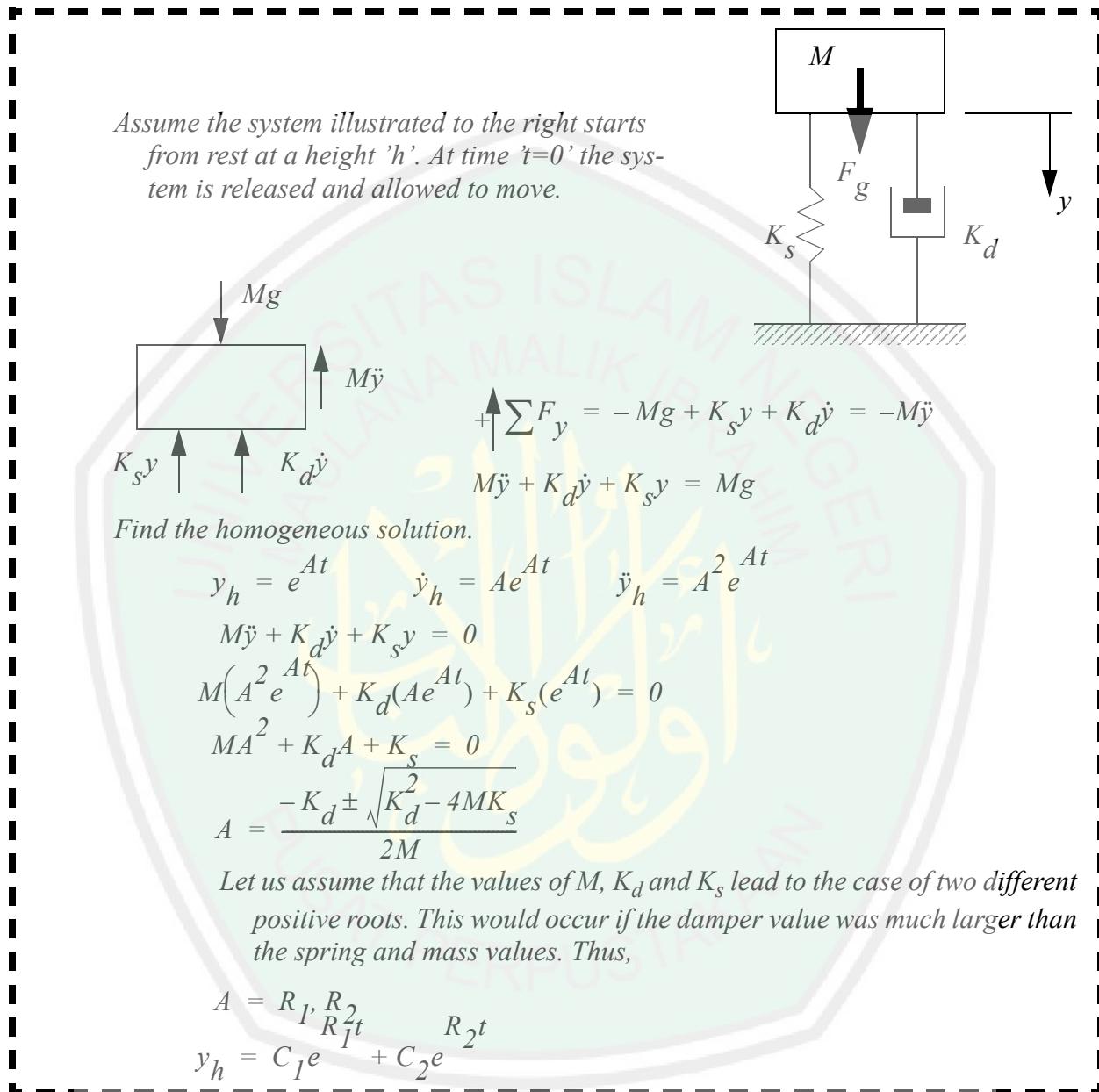


Figure 1.13 Second-order system example

The solution continues by assuming a particular solution and calculating values for the coefficients using the initial conditions in Figure 1.14. The final result is a second-order system that is overdamped, with no oscillation.

Next, find the particular solution.

$$y_p = C \quad \dot{y}_h = 0 \quad \ddot{y}_h = 0$$

$$M(0) + K_d(0) + K_s(C) = Mg$$

$$C = \frac{Mg}{K_s}$$

Now, add the homogeneous and particular solutions and solve for the unknowns using the initial conditions.

$$y(t) = y_p + y_h = \frac{Mg}{K_s} + C_1 e^{R_1 t} + C_2 e^{R_2 t}$$

$$y(0) = h \quad y'(0) = 0$$

$$h = \frac{Mg}{K_s} + C_1 e^0 + C_2 e^0$$

$$C_1 + C_2 = h - \frac{Mg}{K_s}$$

$$y'(t) = R_1 C_1 e^{R_1 t} + R_2 C_2 e^{R_2 t}$$

$$0 = R_1 C_1 e^0 + R_2 C_2 e^0$$

$$0 = R_1 C_1 + R_2 C_2 \quad C_1 = \frac{-R_2}{R_1} C_2$$

$$-\frac{R_2}{R_1} C_2 + C_2 = h - \frac{Mg}{K_s}$$

$$C_2 = \left( \frac{K_s h - Mg}{K_s} \right) \left( \frac{-R_1}{R_1 - R_2} \right) \quad C_1 = \frac{-R_2}{R_1} \left( \frac{K_s h - Mg}{K_s} \right) \left( \frac{-R_1}{R_1 - R_2} \right)$$

Now, combine the solutions and solve for the unknowns using the initial conditions.

$$y(t) = \frac{Mg}{K_s} + \left( \frac{K_s h - Mg}{K_s} \right) \left( \frac{-R_1}{R_1 - R_2} \right) e^{R_1 t} + \frac{-R_2}{R_1} \left( \frac{K_s h - Mg}{K_s} \right) \left( \frac{-R_1}{R_1 - R_2} \right) e^{R_2 t}$$

$$y(t) = \frac{Mg}{K_s} + \left( \frac{K_s h - Mg}{K_s} \right) \left( \frac{-R_1}{R_1 - R_2} \right) e^{R_1 t} + \left( \frac{K_s h - Mg}{K_s} \right) \left( \frac{R_2}{R_1 - R_2} \right) e^{R_2 t}$$

Figure 1.14 Second-order system example (continued)

Given,

$$A \sin(\omega t + \theta) \quad (\text{the desired final form})$$

$$A(\cos \omega t \sin \theta + \sin \omega t \cos \theta)$$

$$(A \sin \theta) \cos \omega t + (A \cos \theta) \sin \omega t$$

$$B \cos \omega t + C \sin \omega t \quad (\text{the form we start with})$$

where,

$$B = A \sin \theta$$

$$C = A \cos \theta$$

To find theta,

$$\frac{B}{C} = \frac{A \sin \theta}{A \cos \theta} = \tan \theta$$

$$\theta = \text{atan}\left(\frac{B}{C}\right)$$

To find A, (method #1)

$$A = \frac{B}{\sin \theta} = \frac{C}{\cos \theta}$$

To find A, (method #2)

$$A = \sqrt{B^2 + C^2}$$

For example,

$$3 \cos 5t + 4 \sin 5t \\ \sqrt{3^2 + 4^2} \sin\left(5t + \text{atan}\frac{3}{4}\right)$$

$$5 \sin(5t + 0.6435)$$

Figure 1.15 Proof for conversion to phase form

### 1.3 Responses

Solving differential equations tends to yield one of two basic equation forms. The e-to-the-negative-t forms are the first-order responses and slowly decay over time. They never naturally oscillate, and only oscillate if forced to do so. The second-order forms may

include natural oscillation.

### 1.3.1 First-order

A first-order system is described with a first-order differential equation. The response function for these systems is natural decay or growth as shown in Figure 1.16. The time constant for the system can be found directly from the differential equation. It is a measure of how quickly the system responds to a change. When an input to a system has changed, the system output will be approximately 63% of the way to its final value when the elapsed time equals the time constant. The initial and final values of the function can be determined algebraically to find the first-order response with little effort.

If we have experimental results for a system, we can calculate the time constant, initial and final values. The time constant can be found two ways, one by extending the slope of the first part of the curve until it intersects the final value line. That time at the intersection is the time constant. The other method is to look for the time when the output value has shifted 63.2% of the way from the initial to final values for the system. Assuming the change started at  $t=0$ , the time at this point corresponds to the time constant.

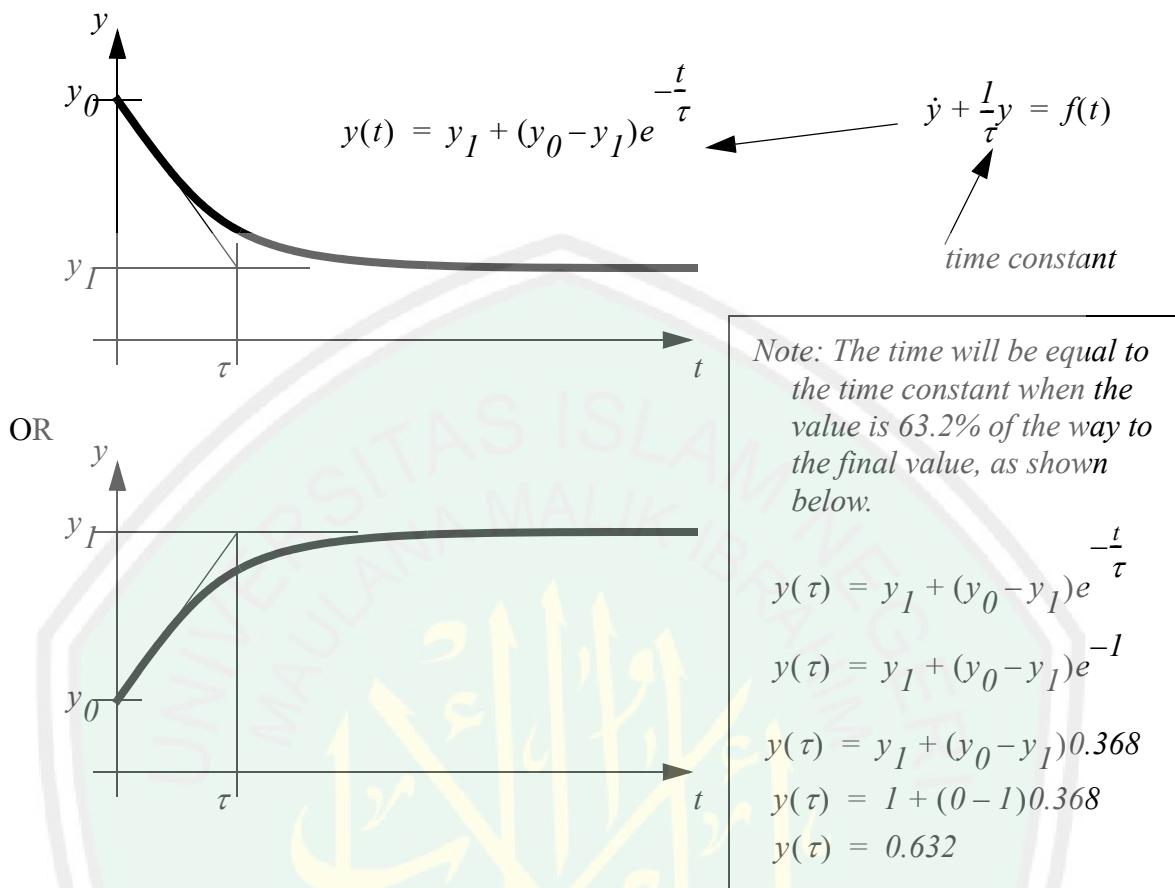
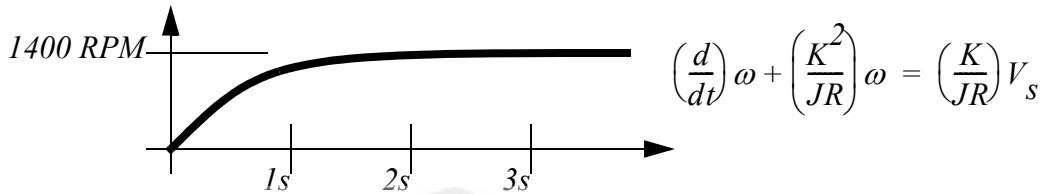


Figure 1.16 Typical first-order responses

The example in Figure 1.17 calculates the coefficients for a first-order differential equation given a graphical output response to an input. The differential equation is for a permanent magnet DC motor, and will be examined in a later chapter. If we consider the steady state when the speed is steady at 1400RPM, the first derivative will be zero. This simplifies the equation and allows us to calculate a value for the parameter K in the differential equation. The time constant can be found by drawing a line asymptotic to the start of the motor curve, and finding the point where it intercepts the steady-state value. This gives an approximate time constant of 0.8 s. This can then be used to calculate the remaining coefficient. Some additional numerical calculation leads to the final differential equation as shown.

For the motor, use the differential equation and the speed curve when  $V_s=10V$  is applied:

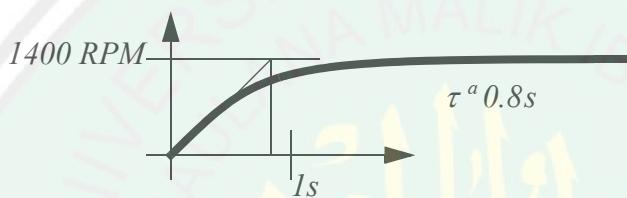


For steady-state

$$\left(\frac{d}{dt}\right)\omega = 0 \quad \omega = 1400\text{RPM} = 146.6\text{rads}^{-1}$$

$$0 + \left(\frac{K^2}{JR}\right)146.6 = \left(\frac{K}{JR}\right)10$$

$$K = 0.0682$$



$$\left(\frac{K^2}{JR}\right) = \frac{I}{0.8s}$$

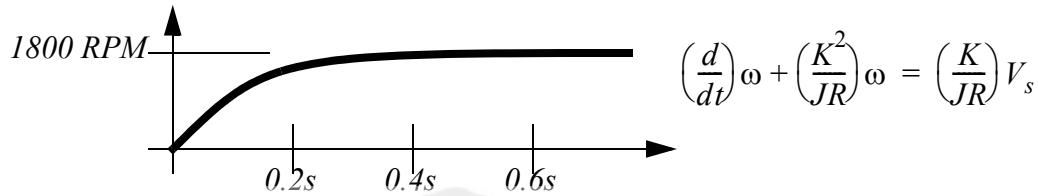
$$0.0682\left(\frac{K}{JR}\right) = \frac{I}{0.8s}$$

$$\frac{K}{JR} = 18.328$$

$$\dot{\omega} + \frac{I}{0.8}\omega = 18.328V_s$$

Figure 1.17 Finding an equation using experimental data

Find the differential equation when a step input of  $V_s=12V$  is applied:

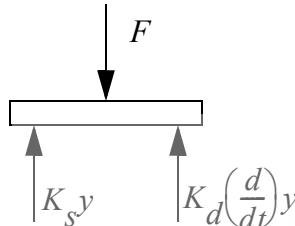


ans.  $\left(\frac{d}{dt}\right)\omega + \frac{1}{0.15}\omega = \frac{1800}{12(0.15)}V_s$

Figure 1.18 Drill problem: Find the constants for the equation

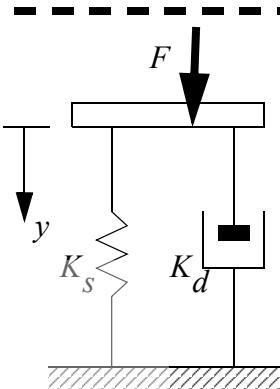
A simple mechanical example is given in Figure 1.19. The modeling starts with a FBD and a sum of forces. After this, the homogenous solution is found by setting the non-homogeneous part to zero and solving. Next, the particular solution is found, and the two solutions are combined. The initial conditions are used to find the remaining unknown coefficients.

*Find the response to the applied force if the force is applied at  $t=0s$ . Assume the system is initially deflected a height of  $h$ .*



$$\sum F_y = -F + K_s y + K_d \left( \frac{dy}{dt} \right) = 0$$

$$K_d \dot{y} + K_s y = F$$



*Find the homogeneous solution.*

$$y_h = A e^{Bt} \quad \dot{y}_h = A B e^{Bt}$$

$$K_d(A B e^{Bt}) + K_s(A e^{Bt}) = 0$$

$$K_d B + K_s = 0$$

$$B = \frac{-K_s}{K_d}$$

*Next, find the particular solution.*

$$y_p = C \quad \dot{y}_p = 0$$

$$K_d(0) + K_s(C) = F \quad \therefore C = \frac{F}{K_s}$$

*Combine the solutions, and find the remaining unknown.*

$$y(t) = y_p + y_h = A e^{\frac{-K_s}{K_d} t} + \frac{F}{K_s}$$

$$y(0) = h$$

$$h = A e^0 + \frac{F}{K_s} \quad \therefore A = h - \frac{F}{K_s}$$

*The final solution is,*

$$y(t) = \left( h - \frac{F}{K_s} \right) e^{\frac{-K_s}{K_d} t} + \frac{F}{K_s}$$

Figure 1.19 First-order system analysis example

Use the general form given below to solve the problem in Figure 1.19 without solving the differential equation. Assume the system starts at  $y=-20$ .

$$\dot{y} + 10y = 5 \longrightarrow y(t) = y_1 + (y_0 - y_1)e^{-\frac{t}{\tau}}$$

*ans.*  $y(t) = 0.5 - 19.5e^{-10t}$

Figure 1.20 Drill problem: Developing the final equation using the first-order model form

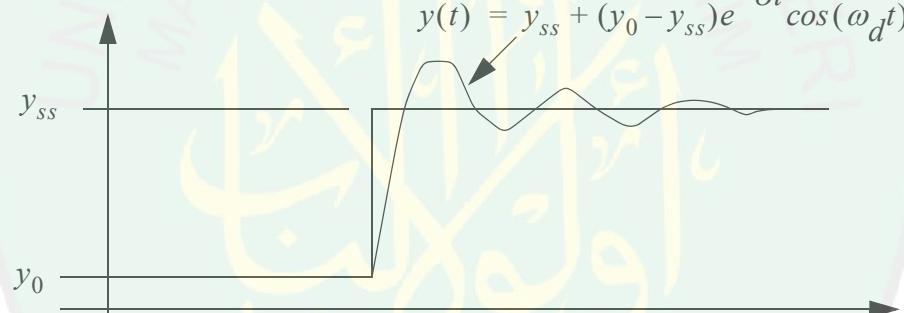
A first-order system tends to be passive, meaning it doesn't deliver energy or power. A first-order system will not oscillate unless the input forcing function is also oscillating. The output response lags the input and the delay is determined by the system's time constant.

### 1.3.2 Second-order

A second-order system response typically contains two first-order responses, or a first-order response and a sinusoidal component. A typical sinusoidal second-order response is shown in Figure 1.21. Notice that the coefficients of the differential equation include a damping coefficient and a natural frequency. These can be used to develop the final response, given the initial conditions and forcing function. Notice that the damped frequency of oscillation is the actual frequency of oscillation. The damped frequency will be lower than the natural frequency when the damping coefficient is between 0 and 1. If the damping coefficient is greater than one the damped frequency becomes negative, and the system will not oscillate because it is overdamped.

*A second-order system, and a typical response to a stepped input.*

$$\ddot{y} + 2\zeta\omega_n\dot{y} + \omega_n^2 y = f(t) \longrightarrow \sigma = \zeta\omega_n \quad \omega_d = \omega_n\sqrt{1 - \zeta^2}$$



$\omega_n$  Natural frequency of system - Approximate frequency of control system oscillations.

$\xi$  Damping factor of system - If  $< 1$  underdamped, and system will oscillate. If  $= 1$  critically damped. If  $> 1$  overdamped, and never any oscillation (more like a first-order system). As damping factor approaches 0, the first peak becomes infinite in height.

$\omega_d$  The actual frequency of oscillation - It is below the natural frequency because of the damping.

Figure 1.21 The general form for a second-order system

When only the damping coefficient is increased, the frequency of oscillation, and overall response time will slow, as seen in Figure 1.22. When the damping coefficient is 0 the system will oscillate indefinitely. Critical damping occurs when the damping coeffi-

cient is 1. At this point both roots of the differential equation are equal. The system will not oscillate if the damping coefficient is greater than or equal to 1.

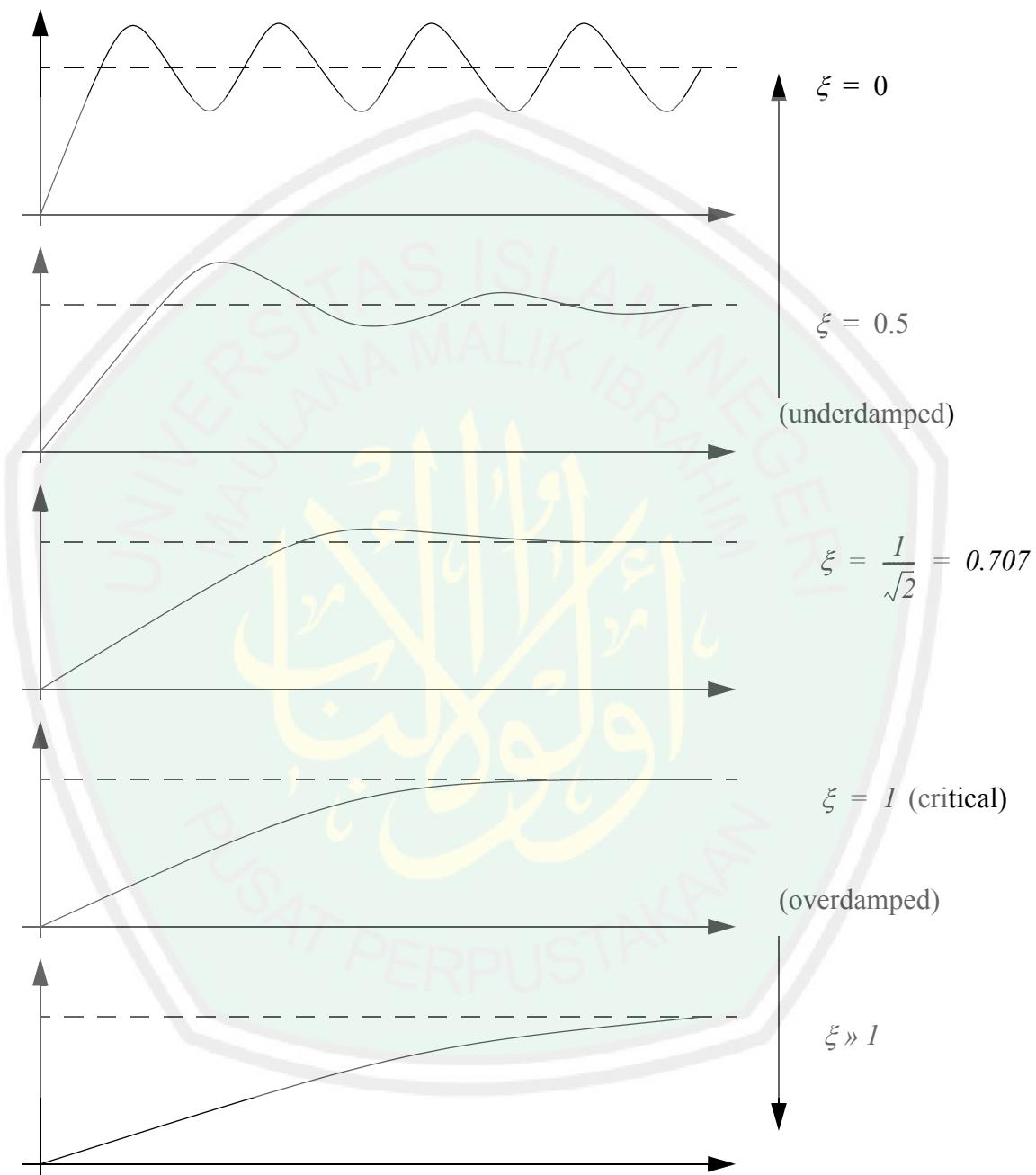


Figure 1.22 The effect of the damping coefficient

When observing second-order systems it is more common to use more direct measurements of the response. Some of these measures are shown in Figure 1.23. The rise

time is the time it takes to go from 10% to 90% of the total displacement, and is comparable to a first order time constant. The settling time indicates how long it takes for the system to pass within a tolerance band around the final value. The permissible zone shown is 2%, but if it were larger the system would have a shorter settling time. The period of oscillation can be measured directly as the time between peaks of the oscillation, the inverse is the damping frequency. (Note: don't forget to convert to radians.) The damped frequency can also be found using the time to the first peak, as half the period. The overshoot is the height of the first peak. Using the time to the first peak, and the overshoot the damping coefficient can be found.

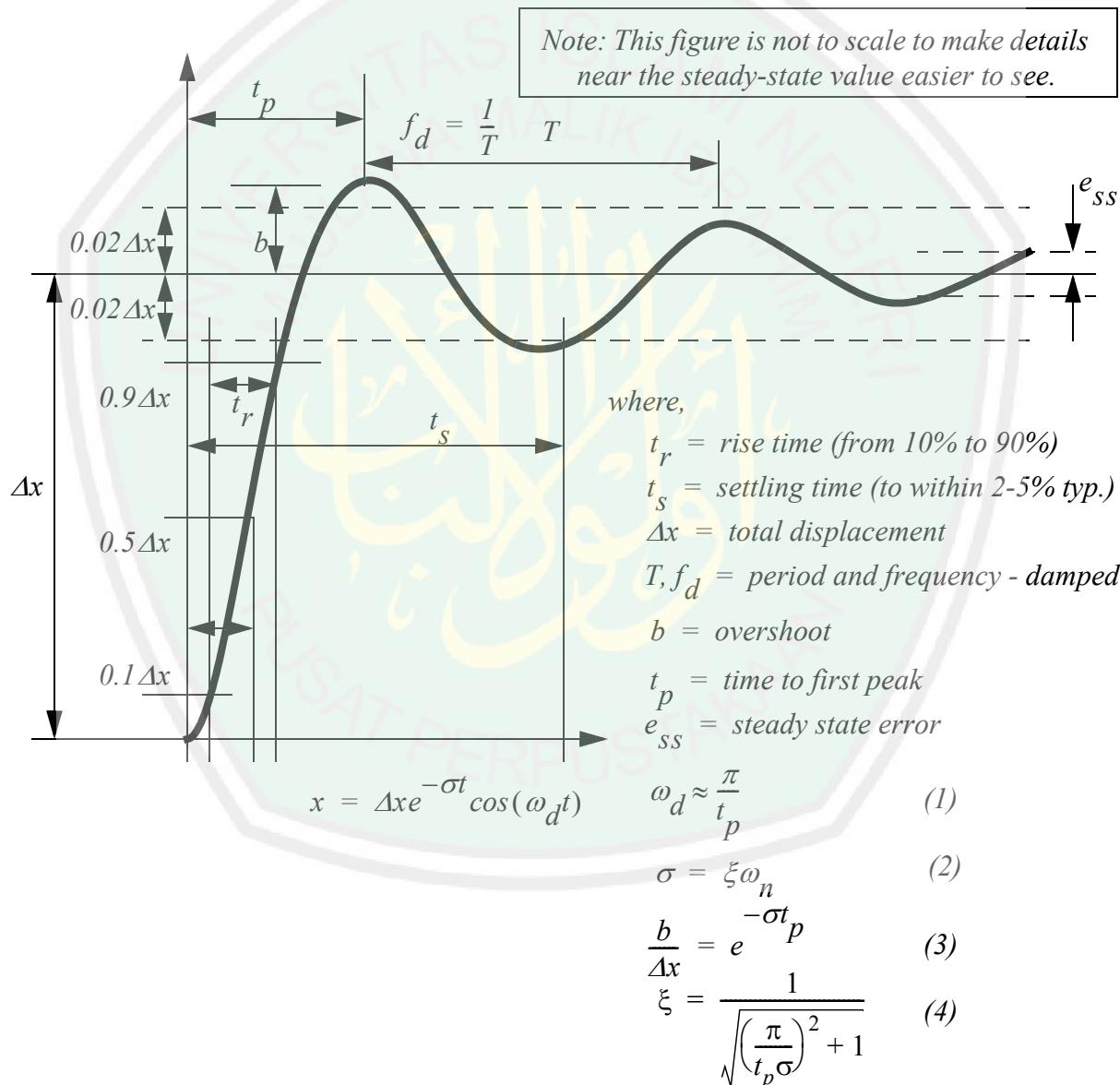


Figure 1.23 Characterizing a second-order response (not to scale)

Note: We can calculate these relationships using the complex homogenous form, and the generic second order equation form.

$$A^2 + 2\xi\omega_n A + \omega_n^2 = 0$$

$$A = \frac{-2\xi\omega_n \pm \sqrt{4\xi^2\omega_n^2 - 4\omega_n^2}}{2} = \sigma \pm j\omega_d$$

$$\frac{-2\xi\omega_n}{2} = \sigma \neq -\xi\omega_n \quad \omega_n = \frac{\sigma}{-\xi} \quad (1)$$

$$\frac{\sqrt{4\xi^2\omega_n^2 - 4\omega_n^2}}{2} = j\omega_d$$

$$4\xi^2\omega_n^2 - 4\omega_n^2 = 4(-1)\omega_d^2$$

$$\omega_n^2 - \xi^2\omega_n^2 = \omega_d^2 \quad \omega_n\sqrt{1 - \xi^2} = \omega_d \quad (2)$$

$$\frac{\sigma^2}{\xi^2} - \xi^2\frac{\sigma^2}{\xi^2} = \omega_d^2$$

$$\frac{1}{\xi^2} = \frac{\omega_d^2}{\sigma^2} + 1 \quad \xi = \frac{1}{\sqrt{\frac{\omega_d^2}{\sigma^2} + 1}} \quad (3)$$

The time to the first peak can be used to find the approximate decay constant

$$x(t) = C_1 e^{-\sigma t} \cos(\omega_d t + C_2)$$

$$\omega_d = \frac{\pi}{t_p} \quad (4)$$

$$b \approx \Delta x e^{-\sigma t_p} (1)$$

$$\sigma = -\frac{\ln\left(\frac{b}{\Delta x}\right)}{t_p} \quad (5)$$

Figure 1.24 Second order relationships between damped and natural frequency

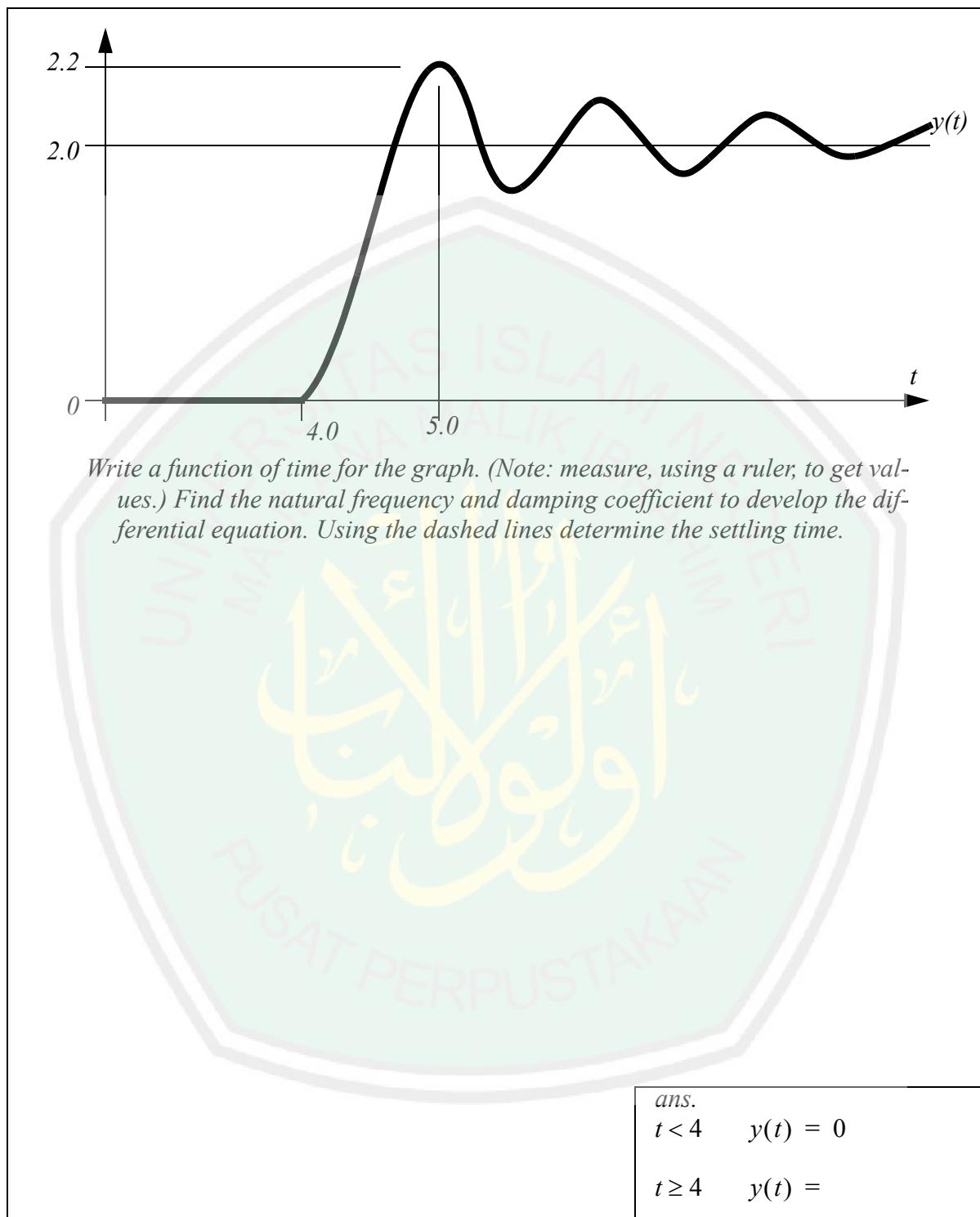


Figure 1.25 Drill problem: Find the equation given the response curve

### 1.3.3 Other Responses

First-order systems have e-to-the-t type responses. Second-order systems add another e-to-the-t response or a sinusoidal excitation. As we move to higher order linear systems we typically add more e-to-the-t terms, and/or more sinusoidal terms. A possible higher order system response is seen in Figure 1.26. The underlying function is a first-order response that drops at the beginning, but levels out. There are two sinusoidal functions superimposed, one with about one period showing, the other with a much higher frequency.



Figure 1.26 An example of a higher order system response

The basic techniques used for solving first and second-order differential equations can be applied to higher order differential equations, although the solutions will start to become complicated for systems with much higher orders. The example in Figure 1.27 shows a fourth order differential equation. In this case the resulting homogeneous solution yields four roots. The result in this case are two real roots, and a complex pair. The two real roots result in e-to-the-t terms, while the complex pair results in a damped sinusoid. The particular solution is relatively simple to find in this example because the non-homogeneous term is a constant.

*Given the homogeneous differential equation*

$$\left(\frac{d}{dt}\right)^4 x + 13\left(\frac{d}{dt}\right)^3 x + 34\left(\frac{d}{dt}\right)^2 x + 42\left(\frac{d}{dt}\right)x + 20x = 5$$

*Guess a solution for the homogeneous equation,*

$$x_h = e^{At}$$

$$\frac{d}{dt}x_h = Ae^{At} \quad \left(\frac{d}{dt}\right)^2 x_h = A^2 e^{At} \quad \left(\frac{d}{dt}\right)^3 x_h = A^3 e^{At} \quad \left(\frac{d}{dt}\right)^4 x_h = A^4 e^{At}$$

*Substitute the values into the differential equation and find a value for the unknown.*

$$A^4 e^{At} + 13A^3 e^{At} + 34A^2 e^{At} + 42Ae^{At} + 20e^{At} = 0$$

$$A^4 + 13A^3 + 34A^2 + 42A + 20 = 0$$

$$A = -1, -10, -1-j, -1+j$$

$$x_h = C_1 e^{-t} + C_2 e^{-10t} + C_3 e^{-t} \cos(t + C_4)$$

*Guess a particular solution, and the solve for the coefficient.*

$$x_p = A \quad \frac{d}{dt}x_p = 0 \quad \left(\frac{d}{dt}\right)^2 x_p = 0 \quad \left(\frac{d}{dt}\right)^3 x_p = 0 \quad \left(\frac{d}{dt}\right)^4 x_p = 0$$

$$0 + 13(0) + 34(0) + 42(0) + 20A = 5 \quad A = 0.25$$

Figure 1.27 Solution of a higher order differential equation

The example is continued in Figure 1.28 and Figure 1.29 where the initial conditions are used to find values for the coefficients in the homogeneous solution.

Solve for the unknowns, assuming the system starts at rest and undeflected.

$$x(t) = C_1 e^{-t} + C_2 e^{-10t} + C_3 e^{-t} \cos(t + C_4) + 0.25 \quad (1)$$

$$0 = C_1 + C_2 + C_3 \cos(C_4) + 0.25 \quad (1)$$

$$C_3 \cos(C_4) = -C_1 - C_2 - 0.25 \quad (2)$$

$$\frac{d}{dt} x_h(t) = -C_1 e^{-t} - 10C_2 e^{-10t} - C_3 e^{-t} \cos(t + C_4) - C_3 e^{-t} \sin(t + C_4) \quad (3)$$

$$0 = -C_1 - 10C_2 - C_3 \cos(C_4) - C_3 \sin(C_4) \quad (3)$$

Equations (1) and (3) can be added to get the simplified equation below.

$$0 = -9C_2 - C_3 \sin(C_4) + 0.25 \quad (4)$$

$$C_3 \sin(C_4) = -9C_2 + 0.25 \quad (4)$$

$$\left(\frac{d}{dt}\right)^2 x_h(t) = C_1 e^{-t} + 100C_2 e^{-10t} + C_3 e^{-t} \cos(t + C_4) + C_3 e^{-t} \sin(t + C_4) + \\ C_3 e^{-t} \sin(t + C_4) - C_3 e^{-t} \cos(t + C_4)$$

$$0 = C_1 + 100C_2 + C_3 \cos(C_4) + C_3 \sin(C_4) + C_3 \sin(C_4) - C_3 \cos(C_4)$$

$$0 = C_1 + 100C_2 + 2C_3 \sin(C_4) \quad (5)$$

Equations (4) and (5) can be combined.

$$0 = C_1 + 100C_2 + 2(-9C_2 + 0.25) \quad (6)$$

$$0 = -17C_1 + 100C_2 + 0.5 \quad (6)$$

$$\left(\frac{d}{dt}\right)^3 x_h(t) = -C_1 e^{-t} + (-1000)C_2 e^{-10t} - 2C_3 e^{-t} \sin(t + C_4) + 2C_3 e^{-t} \cos(t + C_4)$$

$$0 = -C_1 + (-1000)C_2 - 2C_3 \sin(C_4) + 2C_3 \cos(C_4) \quad (7)$$

Figure 1.28 Solution of a higher order differential equation

*Equations (2 and (4) are substituted into equation (7).*

$$\begin{aligned} 0 &= -C_1 + (-1000)C_2 - 2(-9C_2 + 0.25) + 2(-C_1 - C_2 - 0.25) \\ 0 &= -3C_1 + (-984)C_2 - 1 \\ C_1 &= \left(-\frac{984}{3}\right)C_2 - \frac{1}{3} \end{aligned} \quad (8)$$

*Equations (6) and (8) can be combined.*

$$\begin{aligned} 0 &= -17\left(\left(-\frac{984}{3}\right)C_2 - \frac{1}{3}\right) + 100C_2 + 0.5 \\ 0 &= 5676C_2 + 6.1666667 \quad C_2 = -0.00109 \\ C_1 &= \left(-\frac{984}{3}\right)(-0.00109) - \frac{1}{3} \quad C_1 = 0.0242 \end{aligned}$$

*Equations (2) and (4) can be combined.*

$$\begin{aligned} \frac{C_3 \sin(C_4)}{C_3 \cos(C_4)} &= \frac{-9C_2 + 0.25}{-C_1 - C_2 - 0.25} \\ \tan(C_4) &= \frac{-9(-0.00109) + 0.25}{-(0.0242) - (-0.00109) - 0.25} \quad C_4 = -0.760 \end{aligned}$$

*Equation (4) can be used to find the remaining unknown.*

$$C_3 \sin(-0.760) = -9(-0.00109) + 0.25 \quad C_3 = -0.377$$

*The final response function is,*

$$x(t) = 0.0242e^{-t} + (-0.00109)e^{-10t} + (-0.377)e^{-t} \cos(t - 0.760) + 0.25$$

Figure 1.29 Solution of a higher order differential equation (cont'd)

In some cases we will have systems with multiple differential equations, or non-linear terms. In these cases explicit analysis of the equations may not be feasible. In these cases we may use other techniques, such as numerical integration, which will be covered in later chapters.

## 1.4 Response Analysis

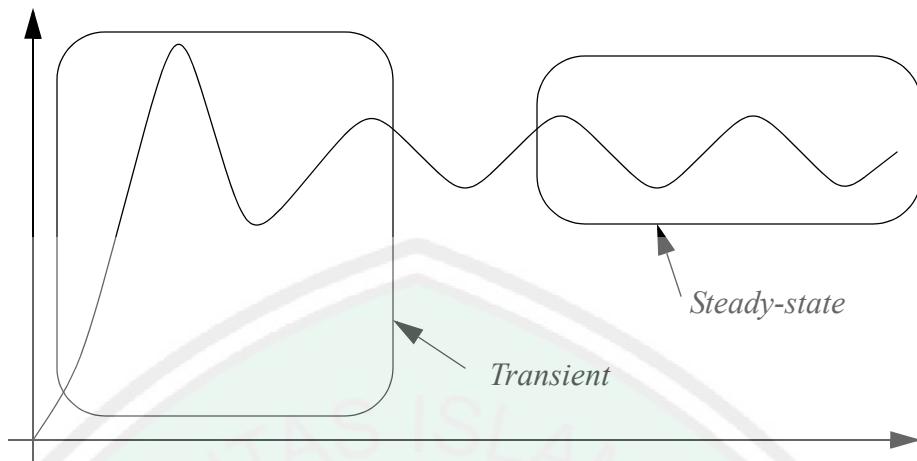
Up to this point we have mostly discussed the process of calculating the system response. As an engineer, obtaining the response is important, but evaluating the results is more important. The most critical design consideration is system stability. In most cases a system should be inherently stable in all situations, such as a car "cruise control". In other

cases an unstable system may be the objective, such as an explosive device. Simple methods for determining the stability of a system are listed below:

1. If a step input causes the system to go to infinity, it will be inherently unstable.
2. A ramp input might cause the system to go to infinity; if this is the case, the system might not respond well to constant change.
3. If the response to a sinusoidal input grows with each cycle, the system is probably resonating, and will become unstable.

Beyond establishing the stability of a system, we must also consider general performance. This includes the time constant for a first-order system, or damping coefficient and natural frequency for a second-order system. For example, assume we have designed an elevator that is a second-order system. If it is under damped the elevator will oscillate, possibly leading to motion sickness, or worse. If the elevator is over damped it will take longer to get to floors. If it is critically damped it will reach the floors quickly, without overshoot.

Engineers distinguish between initial setting effects (transient) and long term effects (steady-state). The transient effects are closely related to the homogeneous solution to the differential equations and the initial conditions. The steady-state effects occur after some period of time when the system is acting in a repeatable or non-changing form. Figure 1.30 shows a system response. The transient effects at the beginning include a quick rise time and an overshoot. The steady-state response settles down to a constant amplitude sine wave.



*Note: the transient response is predicted with the homogeneous solution. The steady state response is mainly predicted with the particular solution, although in some cases the homogeneous solution might have steady state effects, such as a non-decaying oscillation.*

Figure 1.30 A system response with transient and steady-state effects

## 1.5 Non-Linear Systems

Non-linear systems cannot be described with a linear differential equation. A basic linear differential equation has coefficients that are constant, and the derivatives are all first order. Examples of non-linear differential equations are shown in Figure 1.31.

$$\begin{aligned} \dot{x} + x^2 &= 5 \\ \dot{x}^2 + x &= 5 \\ \dot{x} + \log(x) &= 5 \\ \dot{x} + (5t)x &= 5 \end{aligned}$$

*Note: the sources of non-linearity are circled.*

Figure 1.31 Examples of non-linear differential equations

Examples of system conditions that lead to non-linear solutions are,

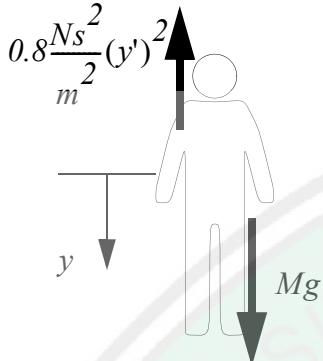
- aerodynamic drag
- forces that are a squared function of distance
- devices with non-linear responses

Explicitly solving non-linear differential equations can be difficult, and will typically involve complex solutions for simple problems.

### 1.5.1 Non-Linear Differential Equations

A non-linear differential equation is presented in Figure 1.32. It involves a person ejected from an aircraft with a drag force coefficient of 0.8. (Note: This coefficient is calculated using the drag coefficient and other properties such as the speed of sound and cross sectional area.) The FBD shows the sum of forces, and the resulting differential equation. The velocity squared term makes the equation non-linear, and so it cannot be analyzed with the previous methods. In this case the terminal velocity is calculated by setting the acceleration to zero. This results in a maximum speed of 126 kph.

Consider the differential equation for a 100kg human ejected from an airplane. The aerodynamic drag will introduce a squared variable, therefore making the equation non-linear.



$$\begin{aligned}\sum F_y &= 0.8(\dot{y})^2 - Mg = -M\ddot{y} \\ 100kg\ddot{y} + 0.8\frac{Ns^2}{m^2}(\dot{y})^2 &= 100kg9.81\frac{N}{kg} \\ 100kg\ddot{y} + 0.8\frac{Ns^2}{m^2}(\dot{y})^2 &= 981N \\ 100kg\ddot{y} + 0.8kg\frac{m}{s^2}(\dot{y})^2 &= 981kg\frac{m}{s^2} \\ 100\ddot{y} + 0.8m^{-1}(\dot{y})^2 &= 981ms^{-2} \\ \ddot{y} + 8 \times 10^{-3} m^{-1}(\dot{y})^2 &= 9.81ms^{-2}\end{aligned}$$

The terminal velocity can be found by setting the acceleration to zero.

$$\begin{aligned}(0) + 8 \times 10^{-3} m^{-1}(\dot{y})^2 &= 9.81ms^{-2} \\ \dot{y} &= \sqrt{\frac{9.81ms^{-2}}{8 \times 10^{-3} m^{-1}}} = \sqrt{\frac{9.81}{8 \times 10^{-3}} m^2 s^{-2}} = 35.0 \frac{m}{s} = 126 \frac{km}{h}\end{aligned}$$

Figure 1.32 Development of a non-linear differential equation

The equation can also be solved using explicit integration, as shown in Figure 1.33. In this case the equation is separated and rearranged to isolate the 'v' terms on the left, and time on the right. The term is then integrated in Figure 1.34 and Figure 1.35. The final form of the equation is non-trivial, but contains e-to-t terms, as we would expect.

An explicit solution can begin by replacing the position variable with a velocity variable and rewriting the equation as a separable differential equation.

$$100\ddot{y} + 0.8m^{-1}(\dot{y})^2 = 981ms^{-2}$$

$$100\dot{v} + 0.8m^{-1}v^2 = 981ms^{-2}$$

$$100\frac{dv}{dt} + 0.8m^{-1}v^2 = 981ms^{-2}$$

$$100\frac{dv}{dt} = 981ms^{-2} - 0.8m^{-1}v^2$$

$$\frac{100}{981ms^{-2} - 0.8m^{-1}v^2} dv = dt$$

$$\int \frac{\frac{100}{-0.8m^{-1}}}{\frac{981}{-0.8m^{-1}}ms^{-2} + v^2} dv = \int dt$$

$$\int \frac{-125m}{v^2 - 1226.25m^2 s^{-2}} dv = t + C_1$$

$$\int \frac{-125m}{(v + 35.02\frac{m}{s})(v - 35.02\frac{m}{s})} dv = t + C_1$$

Figure 1.33 Developing an integral

This can be reduced with a partial fraction expansion.

$$\int \left[ \frac{A}{\left(v + 35.02 \frac{m}{s}\right)} + \frac{B}{\left(v - 35.02 \frac{m}{s}\right)} \right] dv = t + C_1$$

$$Av - A\left(35.02 \frac{m}{s}\right) + Bv + B\left(35.02 \frac{m}{s}\right) = -125m$$

$$v(A + B) + 35.02 \frac{m}{s}(-A + B) = -125m$$

$$A + B = 0 \quad A = -B$$

$$35.02 \frac{m}{s}(-A + B) = -125m$$

$$(-(-B) + B) = -\frac{125}{35.02}s \quad B = -1.785s$$

$$A = 1.785s$$

$$\int \left[ \frac{1.785s}{\left(v + 35.02 \frac{m}{s}\right)} + \frac{-1.785s}{\left(v - 35.02 \frac{m}{s}\right)} \right] dv = t + C_1$$

The integral can then be solved using an identity from the integral table. In this case the integration constants can be left off because they are redundant with the one on the right hand side.

$$1.785s \ln \left| v + 35.02 \frac{m}{s} \right| - 1.785s \ln \left| v - 35.02 \frac{m}{s} \right| = t + C_1$$

$$1.785s \ln \left| \frac{v + 35.02 \frac{m}{s}}{v - 35.02 \frac{m}{s}} \right|^t = t + C_1$$

$$\int (a + bx)^{-1} dx = \frac{\ln|a + bx|}{b} + C$$

$$\left| \frac{v + 35.02 \frac{m}{s}}{v - 35.02 \frac{m}{s}} \right|^t = e^{\frac{1.785s}{1.785s} + C_1}$$

$$\left| \frac{v + 35.02 \frac{m}{s}}{v - 35.02 \frac{m}{s}} \right|^t = e^{C_1} e^{\frac{1.785s}{1.785s}}$$

Figure 1.34 Solution of the integral

$$\left| \frac{v + 35.02 \frac{m}{s}}{v - 35.02 \frac{m}{s}} \right| = C_2 e^{\frac{t}{1.785s}}$$

An initial velocity of zero can be assumed to find the value of the integration constant

$$\left| \frac{0 + 35.02 \frac{m}{s}}{0 - 35.02 \frac{m}{s}} \right| = C_2 e^{\frac{0}{1.785s}} \quad I = C_2$$

This can then be simplified, and the absolute value sign eliminated.

$$\begin{aligned} \frac{v + 35.02 \frac{m}{s}}{v - 35.02 \frac{m}{s}} &= \pm e^{\frac{t}{1.785s}} \\ v + 35.02 \frac{m}{s} &= \pm v e^{\frac{t}{1.785s}} \mp 35.02 \frac{m}{s} e^{\frac{t}{1.785s}} \\ v \left( 1 \mp e^{\frac{t}{1.785s}} \right) &= \mp 35.02 \frac{m}{s} e^{\frac{t}{1.785s}} - 35.02 \frac{m}{s} \\ v &= 35.02 \frac{m}{s} \left( \frac{\mp e^{\frac{t}{1.785s}} - 1}{1 \mp e^{\frac{t}{1.785s}}} \right) \quad 0 = 35.02 \frac{m}{s} \left( \frac{\mp 1 - 1}{1 \mp 1} \right) = \left( \frac{1 - 1}{1 + 1} \right) = \frac{0}{2} \\ v &= 35.02 \frac{m}{s} \left( \frac{e^{\frac{t}{1.785s}} - 1}{1 + e^{\frac{t}{1.785s}}} \right) \end{aligned}$$

Figure 1.35 Solution of the integral and application of the initial conditions

As evident from the example, non-linear equations are involved and don't utilize routine methods. Typically the numerical methods discussed in the next chapter are preferred.

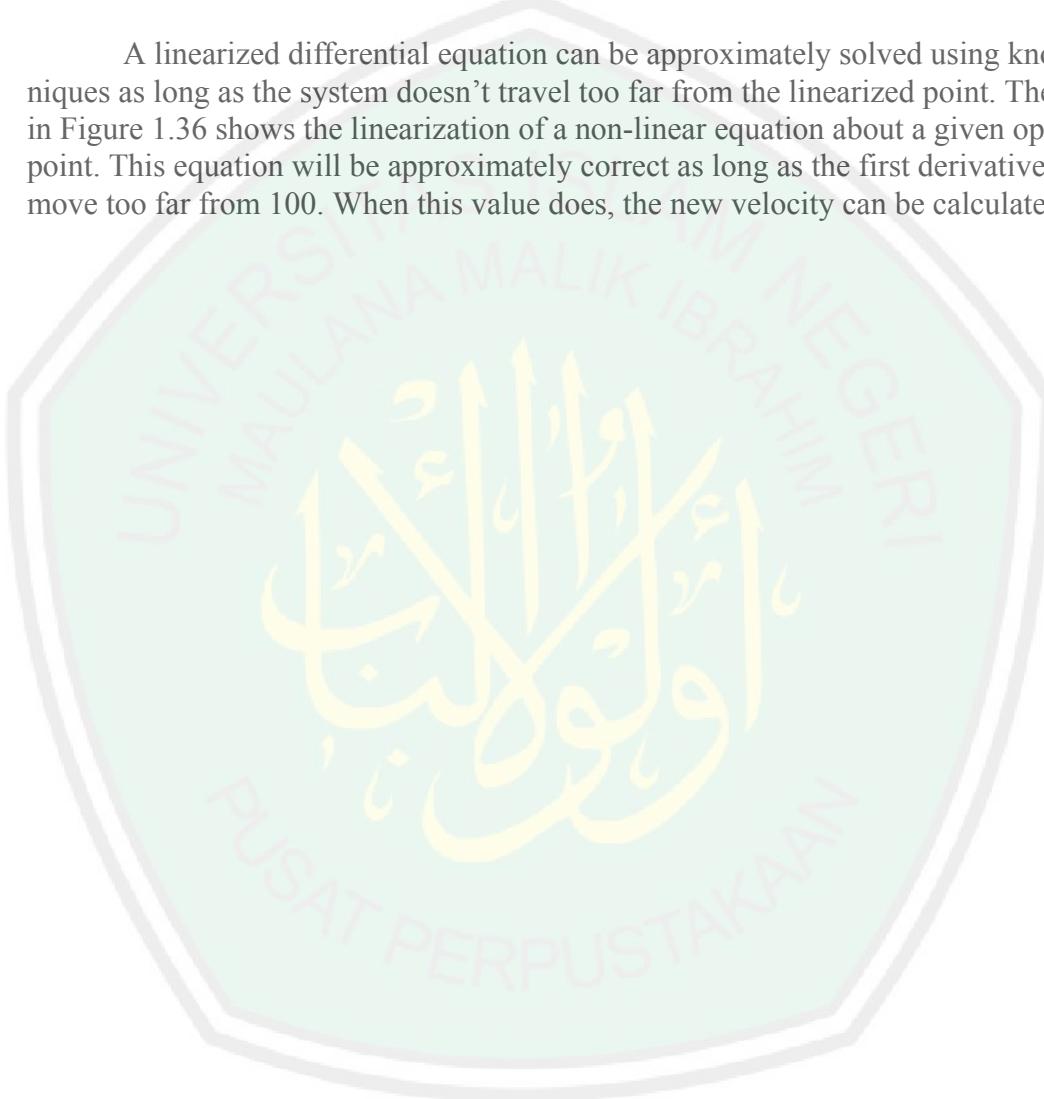
### 1.5.2 Non-Linear Equation Terms

If our models include a device that is non-linear and we want to use a linear tech-

nique to solve the equation, we will need to linearize the model before we can proceed. A non-linear system can be approximated with a linear equation using the following method.

1. Pick an operating point or range for the component.
2. Find a constant value that relates a change in the input to a change in the output.
3. Develop a linear equation.
4. Use the linear equation for the analysis.

A linearized differential equation can be approximately solved using known techniques as long as the system doesn't travel too far from the linearized point. The example in Figure 1.36 shows the linearization of a non-linear equation about a given operating point. This equation will be approximately correct as long as the first derivative doesn't move too far from 100. When this value does, the new velocity can be calculated.



Assume we have the non-linear differential equation below. It can be solved by linearizing the value about the operating point

Given,

$$\dot{y}^2 + 4y = 200 \quad y(0) = 10$$

We can make the equation linear by replacing the velocity squared term with the velocity times the actual velocity. As long as the system doesn't vary too much from the given velocity the model should be reasonably accurate.

$$\dot{y} = \pm\sqrt{200 - 4y}$$

$$\dot{y}(0) = \pm\sqrt{200 - 4(10)} = \pm12.65$$

$$12.65\dot{y} + 4y = 20$$

This system may now be solved as a linear differential equation. If the velocity (first derivative of  $y$ ) changes significantly, then the differential equation should be changed to reflect this.

Homogeneous:

$$12.65\dot{y} + 4y = 0$$

$$12.65A + 4 = 0 \quad A = -0.316$$

$$y_h = Ce^{-0.316t}$$

Particular:

$$y_p = A$$

$$12.65(0) + 4A = 200 \quad A = 50$$

Initial conditions:

$$y(t) = Ce^{-0.316t} + 50$$

$$10 = Ce^0 + 50 \quad C = -40$$

$$y(t) = -40e^{-0.316t} + 50$$

Figure 1.36 Linearizing a differential equation

If the velocity (first derivative of  $y$ ) changes significantly, then the differential equation should be changed to reflect this. For example we could decide to recalculate the equation value after 0.1s.

$$y(0.1) = -40e^{-0.316(0.1)} + 50 = 11.24$$

$$\frac{d}{dt}y(0.1) = -40(-0.316)e^{-0.316(0.1)} = 12.25 \quad \text{Note: a small change}$$

$$12.25\dot{y} + 4y = 20$$

Now recalculate the solution to the differential equation.

*Homogeneous:*

$$12.25\dot{y} + 4y = 0$$

$$12.25A + 4 = 0 \quad A = -0.327$$

$$y_h = Ce^{-0.327t}$$

*Particular:*

$$y_p = A$$

$$12.25(0) + 4A = 200 \quad A = 50$$

*Initial conditions:*

$$y(t) = Ce^{-0.327t} + 50$$

$$11.24 = Ce^{0.1} + 50 \quad C = -35.070575$$

$$y(t) = -35.07e^{-0.316t} + 50$$

Notice that the values have shifted slightly, and as the analysis progresses the equations will adjust slowly. Higher accuracy can be obtained using smaller steps in time.

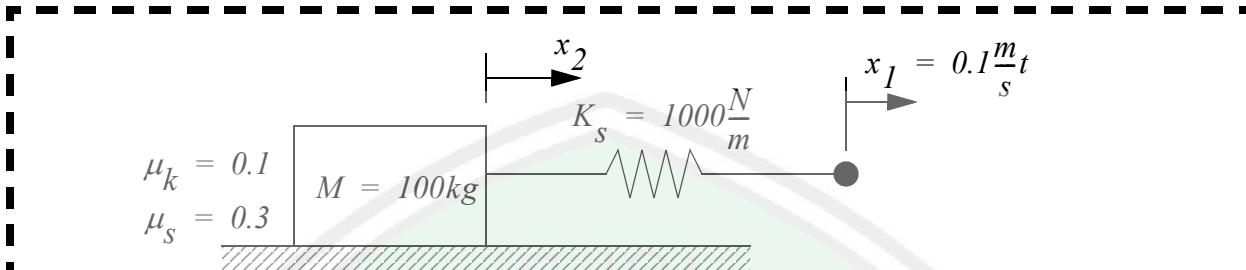
Figure 1.37 Linearizing a differential equation

### 1.5.3 Changing Systems

In practical systems, the forces at work are continually changing. For example a system often experiences a static friction force when motion is starting, but once motion starts it is replaced with a smaller kinetic friction. Another example is tension in a cable. When in tension a cable acts as a spring. But, when in compression the force goes to zero.

Consider the example in Figure 1.38. A mass is pulled by a springy cable. The

right hand side of the cable is being pulled at a constant rate, while the block is free to move, only restricted by friction forces and inertia. At the beginning all components are at rest and undeflected.



An FBD and equation can be developed for the system. The friction force will be left as a variable at this point.

For the cable/spring in tension  $x_1 - x_2 > 0$

$$\begin{aligned}
 & \text{Free Body Diagram:} \\
 & \quad \text{Mass } M = 100\text{kg} \text{ with forces } F_F \text{ (left)} \text{ and } K_s(x_1 - x_2) \text{ (right).} \\
 & \sum F_x = -F_F + K_s(x_1 - x_2) = Mx_2^{\ddot{}} \\
 & -F_F + 1000 \frac{\text{N}}{\text{m}} (0.1 \frac{\text{m}}{\text{s}} t - x_2) = 100\text{kg} x_2^{\ddot{}} \\
 & 100\text{kg} x_2^{\ddot{}} + 1000 \frac{\text{N}}{\text{m}} x_2 = 1000 \frac{\text{N}}{\text{m}} 0.1 \frac{\text{m}}{\text{s}} t - F_F \\
 & x_2^{\ddot{}} + 10 \frac{\text{N}}{\text{kgm}} x_2 = 1 \frac{\text{N}}{\text{kgs}} t - \frac{F_F}{100\text{kg}} \\
 & x_2^{\ddot{}} + 10 \text{s}^{-2} x_2 = 1 \frac{\text{m}}{\text{s}^3} t - \frac{F_F}{100\text{kg}}
 \end{aligned}$$

For the cable/spring in compression  $x_1 - x_2 < 0$

$$\begin{aligned}
 & \text{Free Body Diagram:} \\
 & \quad \text{Mass } M = 100\text{kg} with forces } F_F \text{ (left) and } -F_F \text{ (right).} \\
 & \sum F_x = -F_F = Mx_2^{\ddot{}} \\
 & -F_F = 100\text{kg} x_2^{\ddot{}} \\
 & 100\text{kg} x_2^{\ddot{}} = -F_F \\
 & x_2^{\ddot{}} = -\frac{F_F}{100\text{kg}}
 \end{aligned}$$

Figure 1.38 A differential equation for a mass pulled by a springy cable

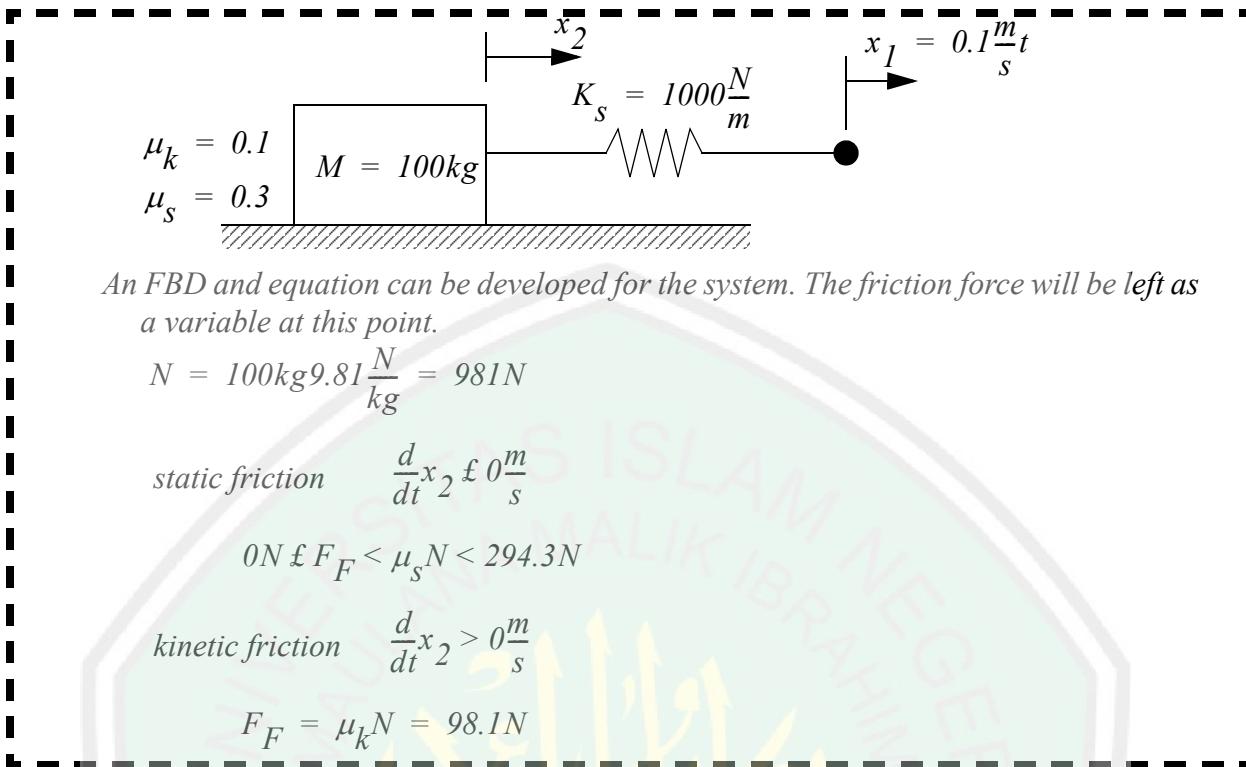


Figure 1.39 Friction forces for the mass

The analysis of the system begins by assuming the system starts at rest and undeflected. In this case the cable/spring will be undeflected with no force, and the mass will be experiencing static friction. Therefore the block will stay in place until the cable stretches enough to overcome the static friction.

$$\begin{aligned} x_2 &= 0 & \ddot{x}_2 &= 0 & F_F &= 294.3\text{N} \\ \ddot{x}_2 + 10s^{-2}x_2 &= 1 \frac{\text{m}}{\text{s}^3} - \frac{F_F}{100\text{kg}} & & & & \\ 0 + 10s^{-2}0 &= 1 \frac{\text{m}}{\text{s}^3} - \frac{294.3\text{N}}{100\text{kg}} & & & & \\ 1 \frac{\text{m}}{\text{s}^3}t &= \frac{294.3\text{kgm}}{100\text{kgs}^2} & & & & \\ t &= 2.943\text{s} & & & & \end{aligned}$$

Therefore the system is static from 0 to 2.943s

Figure 1.40 Analysis of the object before motion begins

After motion begins the object will only experience kinetic friction, and continue to accelerate until the cable/spring becomes loose in compression. This stage of motion requires the solution of a differential equation.

$$\ddot{x}_2 + 10s^{-2}x_2 = I \frac{m}{3}t - \frac{98.1N}{100kg}$$

For the homogeneous,

$$\begin{aligned}\ddot{x}_2 + 10s^{-2}x_2 &= 0 \\ A + 10s^{-2} &= 0 \quad A = \pm 3.16js^{-1} \\ x_h &= C_1 \sin(3.16t + C_2)\end{aligned}$$

For the particular,

$$\begin{aligned}x_p &= At + B \quad \dot{x}_p = A \quad \ddot{x}_p = 0 \\ 0 + 10s^{-2}(At + B) &= I \frac{m}{3}t - \frac{98.1N}{100kg} \\ 10s^{-2}A &= I \frac{m}{3} \quad A = 0.1 \frac{m}{s} \\ 10s^{-2}B &= -\frac{98.1N}{100kg} \quad B = -0.0981m\end{aligned}$$

Figure 1.41 Analysis of the object after motion begins

For the initial conditions,

$$x(2.943s) = 0m \quad \frac{d}{dt}x(2.943s) = 0\frac{m}{s}$$

$$x(t) = C_1 \sin(3.16t + C_2) + 0.1\frac{m}{s}t - 0.0981m$$

$$0 = C_1 \sin(3.16(2.943) + C_2) + 0.1\frac{m}{s}(2.943) - 0.0981m$$

$$C_1 \sin(9.29988 + C_2) = -0.1962$$

$$\frac{d}{dt}x(t) = 3.16C_1 \cos(3.16t + C_2) + 0.1\frac{m}{s}$$

$$0 = 3.16C_1 \cos(3.16(2.943) + C_2) + 0.1\frac{m}{s}$$

$$C_1 \cos(9.29988 + C_2) = -0.0316$$

$$\frac{C_1 \sin(9.29988 + C_2)}{C_1 \cos(9.29988 + C_2)} = \frac{-0.1962}{-0.0316}$$

$$\tan(9.29988 + C_2) = 6.209 \quad C_2 = (-7.889 + \pi n)rad \quad n \in I$$

$$C_1 = \frac{-0.1962}{\sin(9.29988 - 7.889)} = -0.199m$$

$$x(t) = -0.199m \sin(3.16t - 7.889rad) + 0.1\frac{m}{s}t - 0.0981m$$

$$\frac{d}{dt}x(t) = -0.199(3.16)m \cos(3.16t - 7.889rad) + 0.1\frac{m}{s}$$

Figure 1.42 Analysis of the object after motion begins

The equation of motion changes after the cable becomes slack. This point in time can be determined when the displacement of the block equals the displacement of the cable/spring end.

$$0.1 \frac{m}{s} t = -0.199 m \sin(3.16t - 7.889 \text{ rad}) + 0.1 \frac{m}{s} t - 0.0981 m$$

$$-0.199 m \sin(3.16t - 7.889 \text{ rad}) = 0.0981 m$$

$$3.16t - 7.889 + \pi n = -0.51549413 \quad t = 3.328 s$$

$$x(3.328) = 0.137 m \quad \frac{d}{dt}x(2.333) = 0.648 \frac{m}{s}$$

After this the differential equation without the cable/spring is used.

$$x_2'' = -\frac{98.1 N}{100 kg} = -0.981 \frac{m}{s^2}$$

$$\dot{x}_2 = \left(-0.981 \frac{m}{s^2}\right)t + C_1$$

$$0.648 \frac{m}{s} = \left(-0.981 \frac{m}{s^2}\right)(3.328 s) + C_1$$

$$C_1 = 3.913 \frac{m}{s}$$

$$x_2 = \left(-\frac{0.981 m}{2 s^2}\right)t^2 + 3.913 \frac{m}{s} t + C_2$$

$$0.137 m = \left(-\frac{0.981 m}{2 s^2}\right)(3.328 s)^2 + 3.913 \frac{m}{s}(3.328 s) + C_2$$

$$C_2 = -7.453 m$$

$$x_2(t) = \left(-\frac{0.981 m}{2 s^2}\right)t^2 + 3.913 \frac{m}{s} t - 7.453 m$$

This motion continues until the block stops moving.

$$0 = \left(-0.981 \frac{m}{s^2}\right)t + 3.913 \frac{m}{s}$$

$$t = 3.989 s$$

The solution can continue, considering when to switch the analysis conditions.

Figure 1.43 Determining when the cable become slack

## 1.6 Case Study

A typical vibration control system design is described in Figure 1.44.

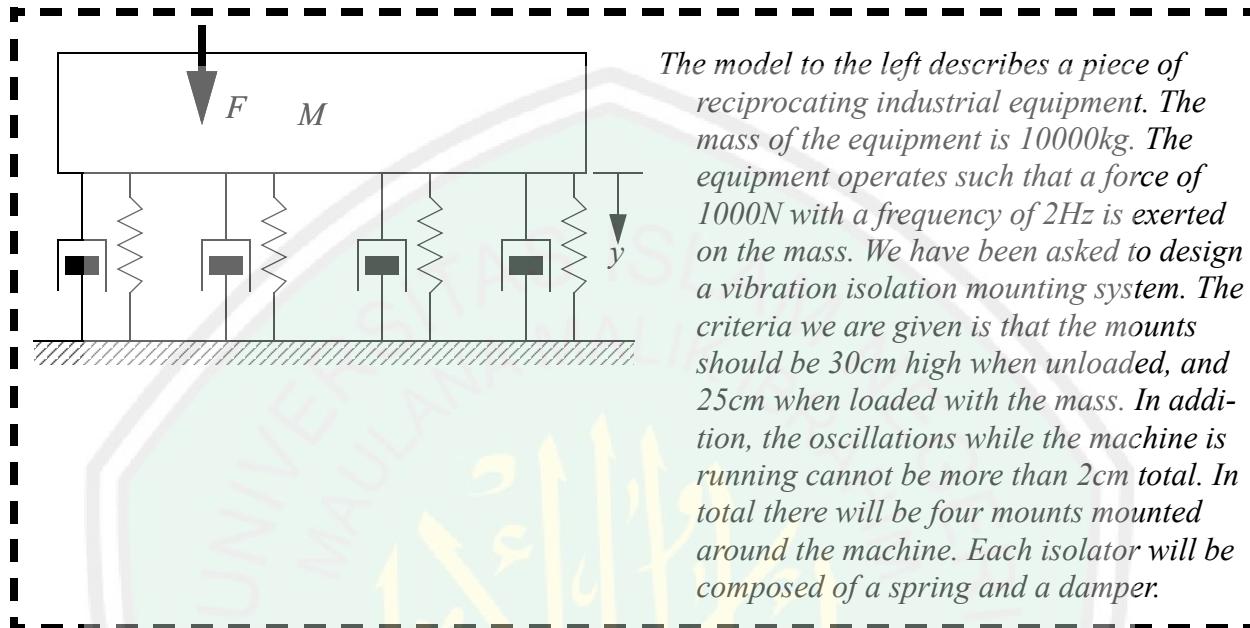


Figure 1.44 A vibration control system

There are a number of elements to the design and analysis of this system, but as usual the best place to begin is by developing a free body diagram, and a differential equation. This is done in Figure 1.45.

$$\begin{aligned}
 +\downarrow \sum F_y &= F - 4K_s y - 4K_d \dot{y} + Mg = M\ddot{y} \\
 M\ddot{y}' + 4K_d \dot{y} + 4K_s y &= F + Mg \\
 \ddot{y} + \frac{4K_d \dot{y}}{M} + \frac{4K_s y}{M} &= \frac{F}{M} + g \\
 \ddot{y} + \frac{4K_d \dot{y}}{10000Kg} + \frac{4K_s y}{10000Kg} &= \frac{1000N}{10000Kg} \sin(2(2\pi)t) + 9.81ms^{-2} \\
 \ddot{y} + 0.0004Kg^{-1} K_d \dot{y} + 0.0004Kg^{-1} K_s y &= 0.1ms^{-2} \sin(4\pi t) + 9.81ms^{-2}
 \end{aligned}$$

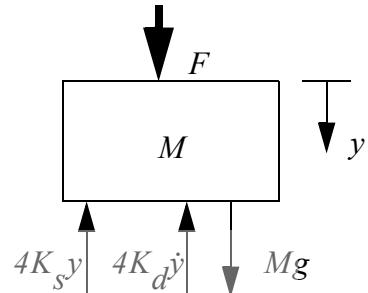


Figure 1.45 FBD and derivation of equation

Using the differential equation, the spring values can be found by assuming the machine is at rest. This is done in Figure 1.46.

*When the system is at rest the equation is simplified; the acceleration and velocity terms both become zero. In addition, we will assume that the cyclic force is not applied for the unloaded/loaded case. This simplifies the differential equation by eliminating several terms.*

$$0.0004Kg^{-1} K_s y = 9.81ms^{-2}$$

Now we can consider that when unloaded the spring is 0.30m long, and after loading the spring is 0.25m long. This will result in a downward compression of 0.05m, in the positive y direction.

$$0.0004Kg^{-1} K_s (0.05m) = 9.81ms^{-2}$$

$$K_s = \frac{9.81}{0.0004(0.05)} Kgms^{-2} m^{-1}$$

$$\therefore K_s = 491KNm^{-1}$$

Figure 1.46 Calculation of the spring coefficient

The remaining unknown is the damping coefficient. At this point we have determined the range of motion of the mass. This can be done by developing the particular

solution of the differential equation, as it will contain the steady-state oscillations caused by the forces as shown in Figure 1.47.

$$\ddot{y} + 0.0004Kg^{-1}K_d\dot{y} + 0.0004Kg^{-1}(491KNm^{-1})y = 0.1ms^{-2}\sin(4\pi t) + 9.81ms^{-2}$$

$$\ddot{y} + 0.0004Kg^{-1}K_d\dot{y} + 196s^{-2}y = 0.1ms^{-2}\sin(4\pi t) + 9.81ms^{-2}$$

*The particular solution can now be found by guessing a value, and solving for the coefficients. (Note: The units in the expression are uniform (i.e., the same in each term) and will be omitted for brevity.)*

$$y = A\sin(4\pi t) + B\cos(4\pi t) + C$$

$$y' = 4\pi A\cos(4\pi t) - 4\pi B\sin(4\pi t)$$

$$y'' = -16\pi^2 A\sin(4\pi t) - 16\pi^2 B\cos(4\pi t)$$

$$\therefore (-16\pi^2 A\sin(4\pi t) - 16\pi^2 B\cos(4\pi t)) + 0.0004K_d(4\pi A\cos(4\pi t) - 4\pi B\sin(4\pi t)) + 196(A\sin(4\pi t) + B\cos(4\pi t) + C) = 0.1\sin(4\pi t) + 9.81$$

$$-16\pi^2 B + 0.0004K_d4\pi A + 196A = 0$$

$$B = A(31.8 \times 10^{-6} K_d + 1.24)$$

$$-16\pi^2 A + 0.0004K_d(-4\pi B) + 196A = 0.1$$

$$A(-16\pi^2 + 196) + B(-5.0 \times 10^{-3} K_d) = 0.1$$

$$A(-16\pi^2 + 196) + A(31.8 \times 10^{-6} K_d + 1.24)(-5.0 \times 10^{-3} K_d) = 0.1$$

$$A = \frac{0.1}{-16\pi^2 + 196 + (31.8 \times 10^{-6} K_d + 1.24)(-5.0 \times 10^{-3} K_d)}$$

$$A = \frac{0.1}{K_d^2(-159 \times 10^{-9}) + K_d(-6.2 \times 10^{-3}) + 38.1}$$

$$B = \frac{3.18 \times 10^{-6} K_d - 0.124}{K_d^2(-159 \times 10^{-9}) + K_d(-6.2 \times 10^{-3}) + 38.1}$$

$$C = 9.81ms^{-2}$$

Figure 1.47 Particular solution of the differential equation

The particular solution can be used to find a damping coefficient that will give an overall oscillation of 0.02m, as shown in Figure 1.48. In this case Mathcad was used to find the solution, although it could have also been found by factoring out the algebra, and finding the roots of the resulting polynomial.

In the previous particular solution the values were split into cosine and sine components. The magnitude of oscillation can be calculated with the Pythagorean formula.

$$\text{magnitude} = \sqrt{A^2 + B^2}$$

$$\text{magnitude} = \frac{\sqrt{(0.1)^2 + ((3.18 \cdot 10^{-6})K_d - 0.124)^2}}{K_d^2(-159 \cdot 10^{-9}) + K_d(-(6.2 \cdot 10^{-3})) + 38.1}$$

The design requirements call for a maximum oscillation of 0.02m, or a magnitude of 0.01m.

$$0.01 = \frac{\sqrt{(0.1)^2 + ((3.18 \cdot 10^{-6})K_d - 0.124)^2}}{K_d^2(-159 \cdot 10^{-9}) + K_d(-(6.2 \cdot 10^{-3})) + 38.1}$$

A given-find block was used in Mathcad to obtain a damper value of,

$$K_d = 3411 \text{ N} \frac{\text{s}}{\text{m}}$$

Aside: the Mathcad solution

$$f(k) := \frac{\sqrt{0.01 + [(3.18 \cdot 10^{-6})k - 0.124]^2}}{[k \cdot k \cdot (-159 \cdot 10^{-9})] + k \cdot (-6.2 \cdot 10^{-3}) + 38.1}$$

$$k_d := 1$$

given

$$f(k_d) = 0.01$$

$$\text{find}(k_d) = 3.411 \times 10^3$$

Figure 1.48 Determining the damping coefficient

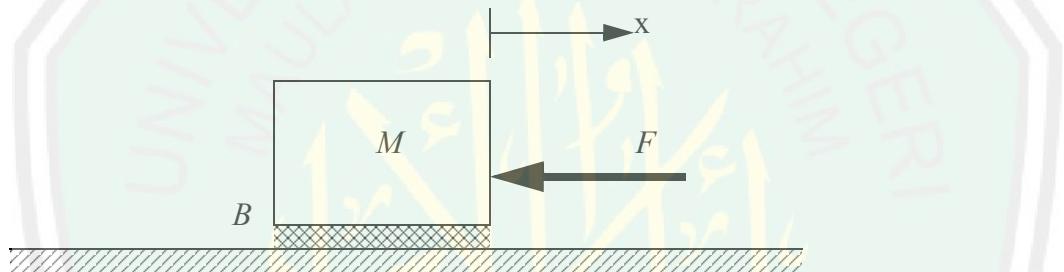
The values of the spring and damping coefficients can be used to select actual components. Some companies will design and build their own components. Components can also be acquired by searching catalogs, or requesting custom designs from other companies.

## 1.7 Summary

- First and second-order differential equations were analyzed explicitly.
- First and second-order responses were examined.
- The topic of analysis was discussed.
- A case study looked at a second-order system.
- Non-linear systems can be analyzed by making them linear.

## 1.8 Problems

1. The mass,  $M$ , illustrated below starts at rest. It can slide across a surface, but the motion is opposed by viscous friction (damping) with the coefficient  $B$ . Initially the system starts at rest, when a constant force,  $F$ , is applied. Write the differential equation for the mass, and solve the differential equation. Leave the results in variable form.



2. The following differential equation was derived for a mass suspended with a spring. At time 0s the system is released and allowed to drop. It then oscillates. Solve the differential equation to find the motion as a function of time.

FBD:  
 $\begin{array}{c} \uparrow K_s y \\ \boxed{M} \\ \downarrow Mg \end{array}$

$$\sum F_y = K_s y - Mg = -M\ddot{y}$$

$$\left(100 \frac{N}{m}\right)y - (1Kg)\left(9.81 \frac{N}{Kg}\right) = (-1Kg)\ddot{y}$$

$$\left(\frac{1Nm}{s^2}\right)\ddot{y} + \left(100 \frac{N}{m}\right)y = 9.81 N$$

$$(1Kg)\ddot{y} + \left(100 \frac{Kgm}{ms^2}\right)y = 9.81 \frac{Kgm}{s^2}$$

$$\ddot{y} + (100s^{-2})y = 9.81ms^{-2}$$

$$y_0 = 0m \quad \dot{y}_0 = 0ms^{-1}$$

3. Solve the following differential equation with the three given cases. All of the systems have a

step input 'y' and start undeflected and at rest.

$$\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2x = y \quad \text{initial conditions } \begin{aligned} \dot{x} &= 0 \\ x &= 0 \\ y &= 1 \end{aligned}$$

case 1:  $\xi = 0.5$      $\omega_n = 10$

case 2:  $\xi = 1$      $\omega_n = 10$

case 3:  $\xi = 2$      $\omega_n = 10$

4. Solve the following differential equation with the given initial conditions and draw a sketch of the first 5 seconds. The input is a step function that turns on at t=0.

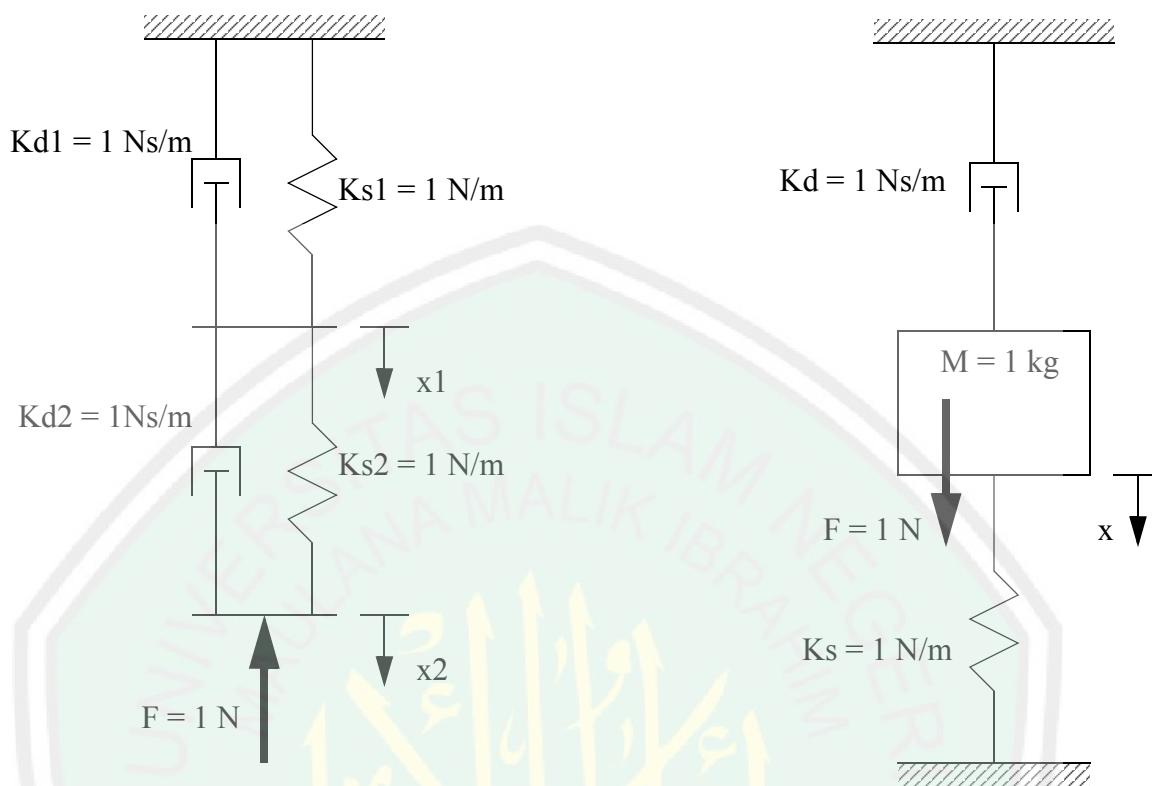
$$0.5\ddot{V}_o + 0.6\dot{V}_o + 2.1V_o = 3V_i + 2 \quad \text{initial conditions } \begin{aligned} V_i &= 5 \\ V_o &= 0 \\ \dot{V}_o &= 0 \end{aligned}$$

5. Solve the following differential equation with the given initial conditions and draw a sketch of the first 5 seconds. The input is a step function that turns on at t=0.

$$0.5\ddot{V}_o + 0.6\dot{V}_o + 2.1V_o = 3V_i + 2 \quad \text{initial conditions } \begin{aligned} V_i &= 5 \\ V_o &= 0 \\ \dot{V}_o &= 1 \end{aligned}$$

6. a) Write the differential equations for the system below. Solve the equations for x assuming that

the system is at rest and undeflected before  $t=0$ . Also assume that gravity is present.



- b) State whether each system is first or second-order. If the system if first-order find the time constant. If it is second-order find the natural frequency and damping ratio.

7. Solve the following differential equation with the three given cases. All of the systems have a sinusoidal input 'y' and start undeflected and at rest.

$$\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2x = y$$

initial conditions  $\dot{x} = 0$

$$x = 0$$

$$y = \sin(t)$$

case 1:  $\xi = 0.5$      $\omega_n = 10$

case 2:  $\xi = 1$      $\omega_n = 10$

case 3:  $\xi = 2$      $\omega_n = 10$

8. A spring damper system supports a mass of 34N. If it has a spring constant of 20.6N/cm, what is the systems natural frequency?

9. Using a standard lumped parameter model the weight is 36N, stiffness is  $2.06 \times 10^3$  N/m and damping is 100Ns/m. What are the natural frequency (Hz) and damping ratio?

10. What is the differential equation for a second-order system that responds to a step input with an overshoot of 20%, with a delay of 0.4 seconds to the first peak?
11. A system is to be approximated with a mass-spring-damper model using the following parameters: weight 28N, viscous damping 6Ns/m, and stiffness 36N/m. Calculate the undamped natural frequency (Hz) of the system, the damping ratio and describe the type of response you would expect if the mass were displaced and released. What additional damping would be required to make the system critically damped?

$$M\ddot{x} + K_d\dot{x} + K_s x = F$$

12. Solve the differential equation below using homogeneous and particular solutions. Assume the system starts undeflected and at rest.

$$\ddot{\theta} + 40\ddot{\theta} + 20\dot{\theta} + 2\theta = 4$$

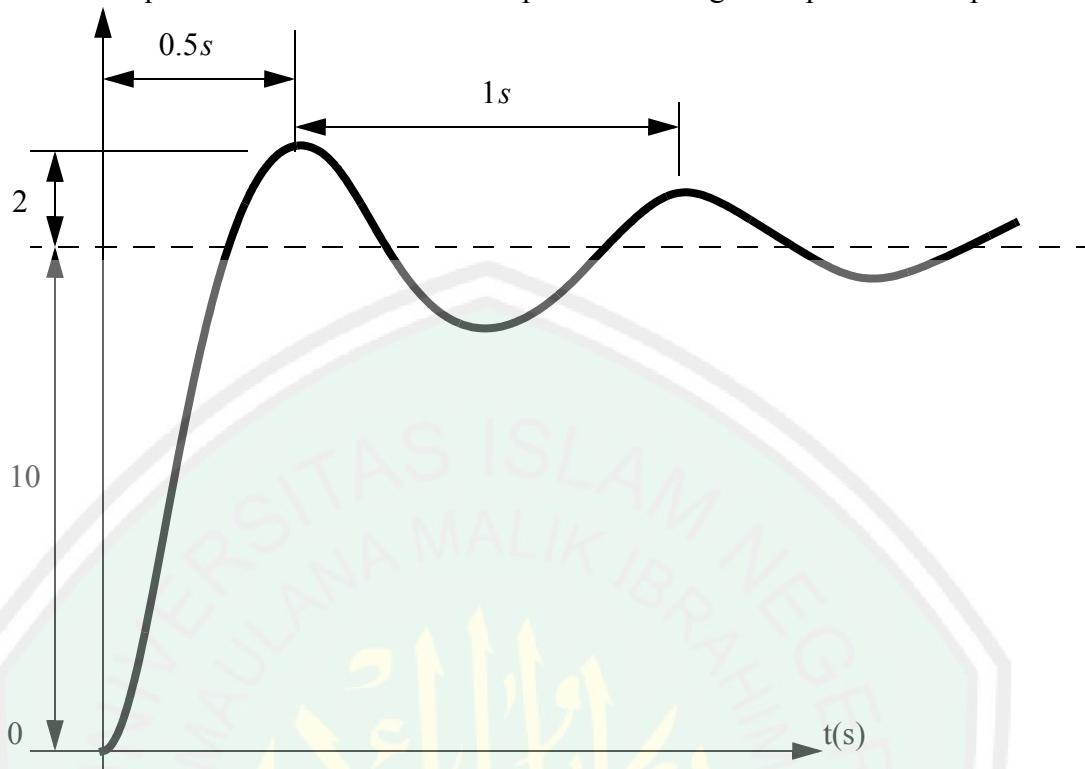
13. What would the displacement amplitude after 100ms for a system having a natural frequency of 13 rads/sec and a damping ratio of 0.20. Assume an initial displacement of 50mm, and a steady state displacement of 0mm. (Hint: Find the response as a function of time.)

14. Determine the first order differential equation given the graphical response shown below. Assume the input is a step function.

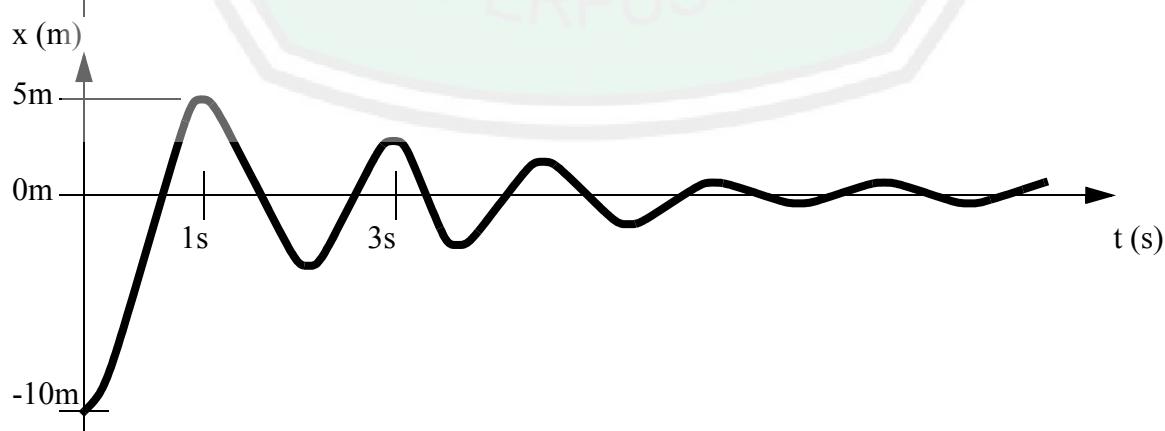
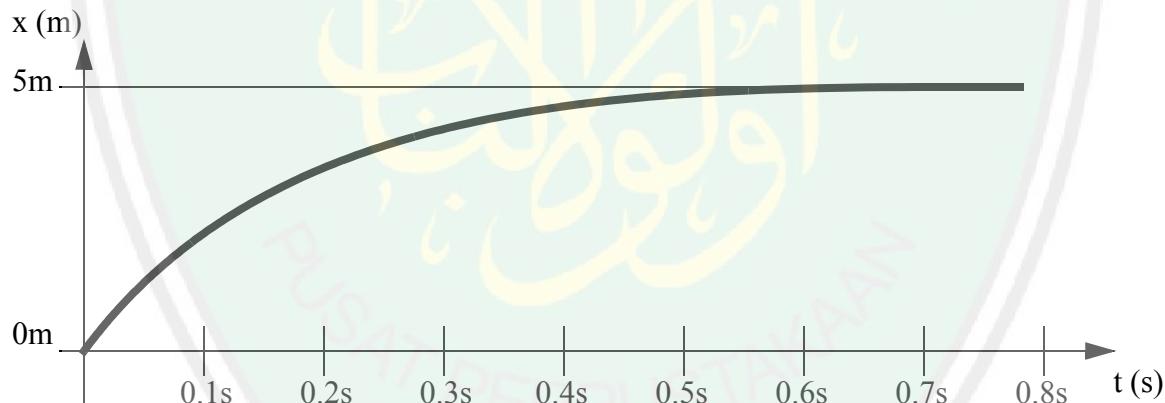


15. Explain with graphs how to develop first and second-order equations using experimental data.
16. The second order response below was obtained experimentally. Determine the parameters of

the differential equation that resulted in the response assuming the input was a step function.



17. Develop equations (function of time) for the first and second order responses shown below.



18. A mass-spring-damper system has a mass of 10Kg and a spring coefficient of 1KN/m. Select a damping coefficient so that the system will have an overshoot of 20% for a step input.

## 1.9 Problems Solutions

1.

$$x(t) = \frac{-FM}{B^2} e^{-\frac{B}{M}t} - \frac{F}{B} t + \frac{FM}{B^2}$$

2.

homogeneous: guess  $y_h = e^{At}$   $\dot{y}_h = Ae^{At}$   $\ddot{y}_h = A^2e^{At}$

$$A^2e^{At} + (100s^{-2})e^{At} = 0$$

$$A^2 = -100s^{-2} \quad A = \pm 10js^{-1}$$

$$y_h = C_1 \cos(10t + C_2)$$

particular: guess  $y_p = A$   $\dot{y}_p = 0$   $\ddot{y}_p = 0$

$$(0) + (100s^{-2})A = 9.81ms^{-2}$$

$$(100s^{-2})A = 9.81ms^{-2}$$

$$A = \frac{9.81ms^{-2}}{100s^{-2}} = 0.0981m$$

$$y_p = 0.0981m$$

initial conditions:  $y = y_h + y_p = C_1 \cos(10t + C_2) + 0.0981m$

$$y' = -10C_1 \sin(10t + C_2)$$

for  $d/dt y(0) = 0$ :

$$0 = -10C_1 \sin(10(0) + C_2) \quad C_2 = 0$$

for  $y(0) = 0$ :

$$0 = C_1 \cos(10(0) + (0)) + 0.0981m$$

$$-0.0981m = C_1 \cos(0) \quad C_1 = -0.0981m$$

$$y(t) = (-0.0981m) \cos(10t) + 0.0981m$$

3.

case 1:  $x(t) = -0.0115e^{-5t} \cos(8.66t - 0.524) + 0.010$

case 2:  $x(t) = -0.010e^{-10t} - 0.10te^{-10t} + 0.010$

case 3:  $x(t) = 775 \cdot 10^{-6}e^{-37.32t} - 0.0108e^{-2.679t} + 0.010$

4.

$$V_o(t) = -8.465e^{-0.6t} \sin(1.960t + 1.274) + 8.095$$

or

$$V_o(t) = -8.465e^{-0.6t} \cos(1.960t - 0.2971) + 8.095$$

5.

$$V_0(t) = -8.331e^{-0.6t} \cos(1.96t - 0.238) + 8.095$$

6.

a)

$$\begin{aligned}x_1(t) &= e^{-t} - 1 \\x_2(t) &= 2e^{-t} - 2 \\ \tau &= 1\end{aligned}$$

b)

$$\begin{aligned}x(t) &= -12.485e^{-0.5t} \cos(0.866t - 0.524) + 10.81 \\ \zeta &= 0.5 \quad \omega_n = 1\end{aligned}$$

7.

case 1:  $x(t) = -0.00117e^{-5t} \sin(8.66t - 1.061) + 0.0101 \sin(t - 0.101)$

case 2:  $x(t) = (1.96 \cdot 10^{-3})e^{-10t} + (9.9 \cdot 10^{-3})te^{-10t} + (9.9 \cdot 10^{-3}) \sin(t + 0.20)$

case 3:  $x(t) = (3.5 \cdot 10^{-3})e^{-2.679t} - (18 \cdot 10^{-6})e^{-37.32t} + (9.4 \cdot 10^{-3}) \sin(t + 0.382)$

8. 24.37 rad/sec

9. fn=3.77Hz, damp.=.575

10.

$$\ddot{x} + 8.048\dot{x} + 77.88x = F(t)$$

11.

Given       $K_d = 6N\frac{s}{m}$        $K_s = 36\frac{N}{m}$        $M = \frac{28N}{9.81\frac{N}{kg}} = 2.85kg$

The typical transfer function for a mass-spring-damper systems is,

$$\ddot{x} + \dot{x}\left(\frac{K_d}{M}\right) + x\left(\frac{K_s}{M}\right) = \frac{F}{M}$$

The second order parameters can be calculated from this.

$$\ddot{x} + \dot{x}(2\zeta\omega_n) + x(\omega_n^2) = y(t)$$

$$\omega_n = \sqrt{\frac{K_s}{M}} = \sqrt{\frac{36\frac{N}{m}}{2.85kg}} = \sqrt{\frac{36\frac{kgm}{ms^2}}{2.85kg}} = \sqrt{12.63s^{-2}} = 3.55\frac{rad}{s} = 0.6Hz$$

$$\xi = \frac{\left(\frac{K_d}{M}\right)}{2\omega_n} = \frac{6N\frac{s}{m}}{2(3.55)\frac{rad}{s}2.85kg} = 0.296$$

$$\omega_d = \omega_n\sqrt{1 - \xi^2} = 3.39\frac{rad}{s}$$

If pulled and released the system would have a decaying oscillation about 0.52Hz

A critically damped system would require a damping coefficient of....

$$\xi = \frac{\left(\frac{K_d}{M}\right)}{2\omega_n} = \frac{K_d}{2(3.55)\frac{rad}{s}2.85kg} = 1.00 \quad K_d = 20.2\frac{Ns}{m}$$

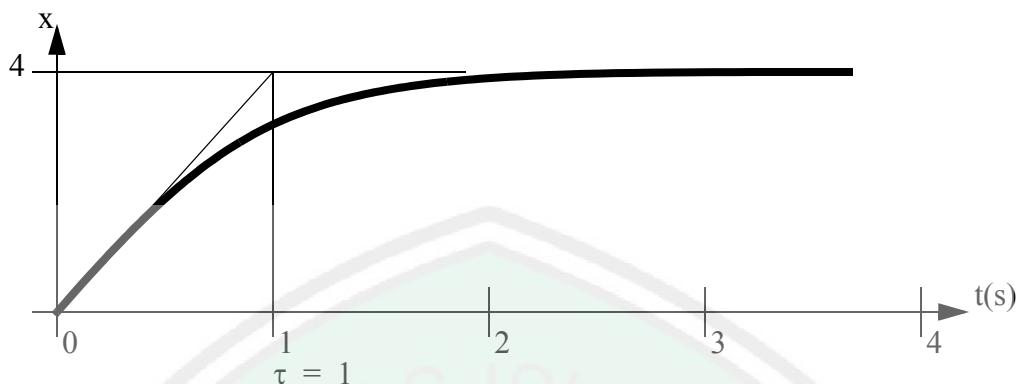
12.

$$\theta(t) = -66 \cdot 10^{-6} e^{-39.50t} - 3.216e^{0.1383t} + 1.216e^{-0.3368t} + 2.00$$

13.

$$y(t) = 0.05e^{-2.6t} \cos(12.74t) = 0.05e^{-2.6t} \sin\left(12.74t + \frac{\pi}{2}\right)$$

14.



Given the equation form,

$$\dot{x} + \frac{1}{\tau}x = A$$

The values at steady state will be

$$\dot{x} = 0 \quad x = 4$$

So the unknown 'A' can be calculated.

$$0 + \frac{1}{1}4 = A \quad A = 4$$

$$\dot{x} + \frac{1}{1}x = 4$$

$$\dot{x} + x = 4$$

15.

Key points:

First-order:

find initial final values

find time constant with 63% or by slope

use these in standard equation

Second-order:

find damped frequency from graph

find time to first peak

use these in cosine equation

16.

For the first peak:

$$\frac{b}{\Delta x} = e^{-\sigma t_p}$$

$$\frac{2}{10} = e^{-\sigma 0.5} \quad \omega_d \approx \frac{\pi}{t_p}$$

$$\ln\left(\frac{2}{10}\right) = -\sigma 0.5$$

$$\sigma = -2 \ln\left(\frac{2}{10}\right) = 3.219$$

For the damped frequency:

$$\omega_d = \frac{2\pi}{1s} = 2\pi$$

$$\xi = \frac{1}{\sqrt{\left(\frac{\pi}{t_p \sigma}\right)^2 + 1}}$$

These values can be used to find the damping coefficient and natural frequency

$$\sigma = \xi \omega_n \quad \omega_n = \frac{3.219}{\xi}$$

$$\omega_d = \omega_n \sqrt{1 - \xi^2}$$

$$2\pi = \frac{3.219}{\xi} \sqrt{1 - \xi^2}$$

$$\left(\frac{2\pi}{3.219}\right)^2 = \frac{1 - \xi^2}{\xi^2}$$

$$\left(\frac{2\pi}{3.219}\right)^2 + 1 = \frac{1}{\xi^2} \quad \xi = \sqrt{\frac{1}{\left(\frac{2\pi}{3.219}\right)^2 + 1}} = 0.4560$$

$$\omega_n = \frac{3.219}{\xi} = \frac{3.219}{0.4560} = 7.059$$

This leads to the final equation using the steady state value of 10

$$\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2 x = F$$

$$\ddot{x} + 2(0.4560)(7.059)\dot{x} + (7.059)^2 x = F$$

$$\ddot{x} + 6.438\dot{x} + 49.83x = F$$

$$(0) + 6.438(0) + 49.83(10) = F \quad F = 498.3$$

$$\ddot{x} + 6.438\dot{x} + 49.83x = 498.3$$

17.

$$x(t) = 5 \left( 1 - e^{\frac{-t}{0.18}} \right) \quad x(t) = -10e^{-0.693t} \cos(\pi t)$$

## 1.10 Challenge Problems

1. Write a Scilab program to solve first and second order differential equations for step inputs. The program should accept coefficients for the differential equations and initial conditions. It should then produce a function of time.

## 2. ELECTRICAL AND COMPUTER ENGINEERING REVIEW

Topics:

- 

Objectives:

- 

### 2.1 Introductions

### 2.2 Examples

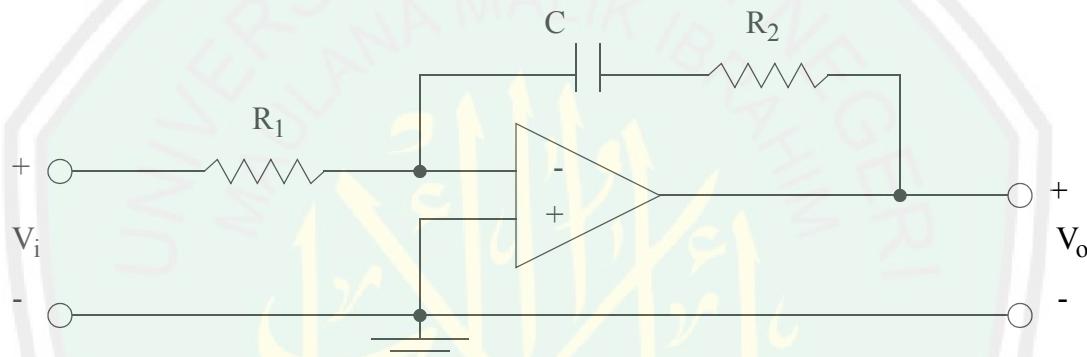
### 2.3 Summary

### 2.4 References/Bibliography

## 2.5 Problems

- F.E. Chapter 8 - Differential Equations - Problems , 6-11, 14
- F.E. Chapter 39 - Complex Numbers and Electrostatics - Problems 1-5
- F.E. Chapter 40 - Direct Current Circuits - Problems 1-32
- F.E. Chapter 41 - Alternating Current Circuits - Problems 1-17
- F.E. Chapter 42 - Three-Phase Systems and Electronics - Problems 1-13
- F.E. Chapter 43 - Computer Hardware - Problems 1-17
- F.E. Chapter 44 - Computer Software - Problems 1-32
- F.E. Chapter 46 - Controls - Problems 1-5

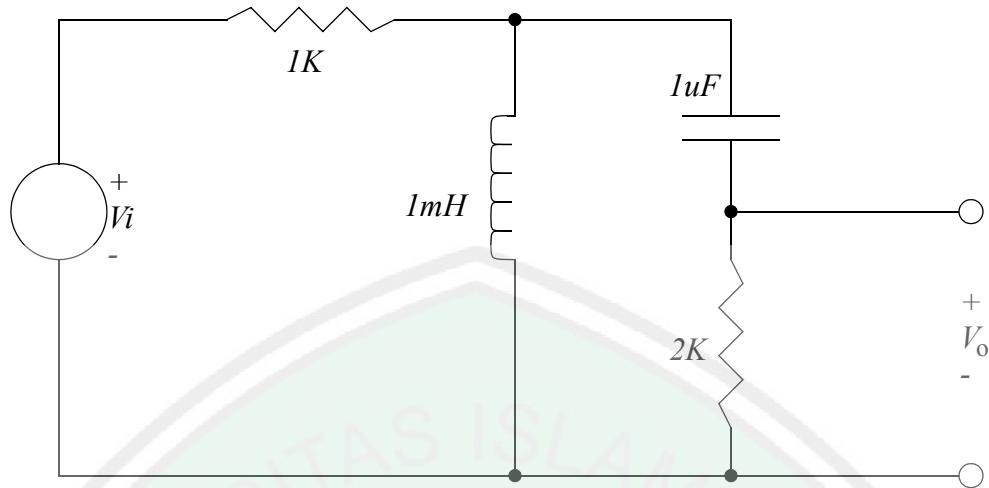
1. Use the loop or node voltage method to write equations for the circuit shown. Write a Scilab program to solve the circuit as a function of frequency.



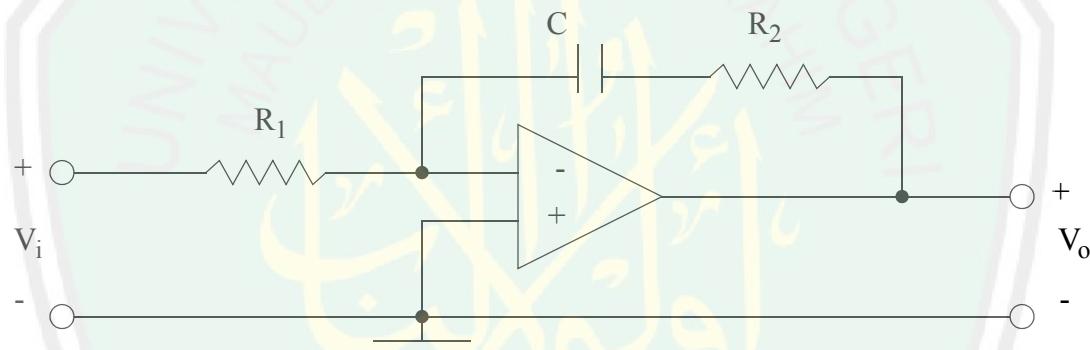
## 2.6 Challenge Problems

1. Write a program that would allow the specification of a transfer function using points on a Bode plot. The transfer function order should be adjusted to allow the number of points specified.

2. Calculate the output Voltage as a function of the input voltage using Matrices in Scilab.



3. Calculate the output Voltage as a function of the input voltage using State Equations and numerical methods in Scilab.



### 3. MECHANICAL ENGINEERING REVIEW

Topics:

- 

Objectives:

- 

#### 3.1 Introduction

#### 3.2 Examples

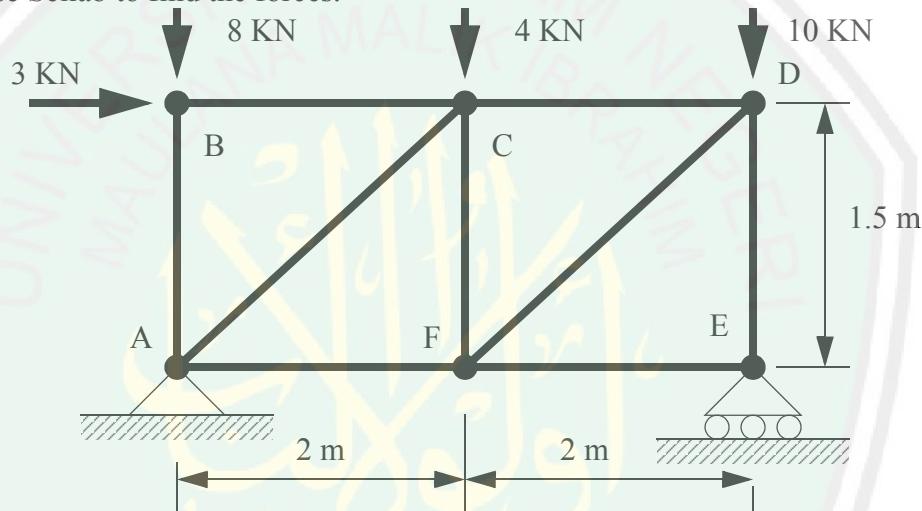
#### 3.3 Summary

#### 3.4 References/Bibliography

#### 3.5 Problems

- F.E. Chapter 10 - Systems of Forces - Problems 1-23  
 F.E. Chapter 11 - Trusses - Problems 1-10  
 F.E. Chapter 12 - Pulleys, Cables, and Friction - Problems 1-12  
 F.E. Chapter 13 - Centroids and Moments of Inertia - Problems 1-18  
 F.E. Chapter 18 - Stress and Strain - Problems 1-18  
 F.E. Chapter 19 - Thermal, Hoop and Torsional Stress - Problems 1-18  
 F.E. Chapter 20 - Beams - Problems 1-15  
 F.E. Chapter 21 - Columns - Problems 1-10  
 F.E. Chapter 36 - Crystallography and Atomic Bonding - Problems 1-8  
 F.E. Chapter 37 - Material Testing - Problems 1-11  
 F.E. Chapter 38 - Metallurgy - Problems 1-18

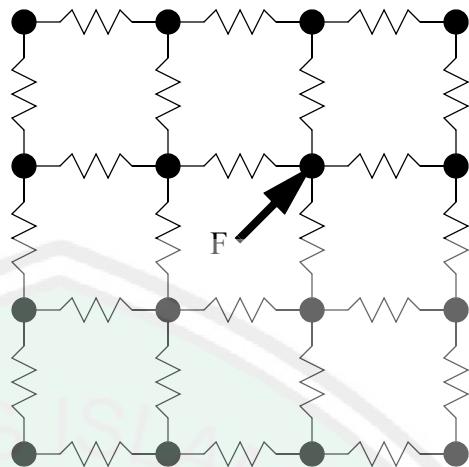
1. Write equations for the following truss using the method of joints. Enter the equations in a matrix and use Scilab to find the forces.



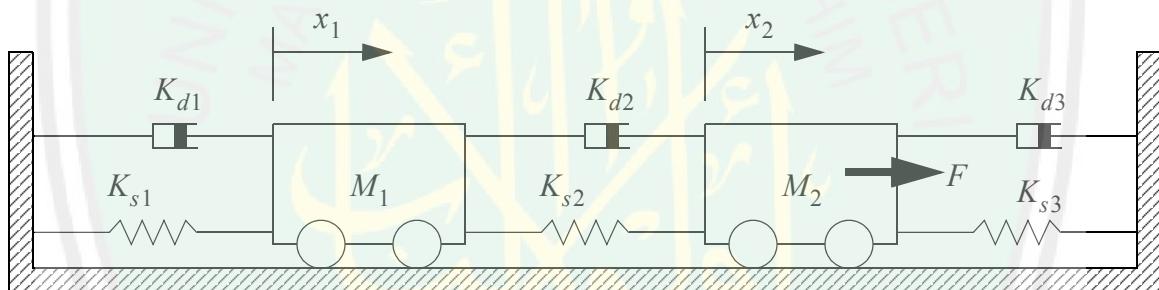
### 3.6 Challenge Problems

1. Write a program to find the deformation of an array of springs (as pictured below) with a force applied to an internal node. Use a matrix method to solve the problem. Assume that the

nodes at the four corners are fixed and unable to move.



2. Write the differential equations for one of the following systems. Simulate the system response using a numerical method, such as Runge-Kutta integration.



## 4. INDUSTRIAL ENGINEERING OVERVIEW

Topics:

- 

Objectives:

- 

### 4.1 Introduction

### 4.2 Examples

### 4.3 Summary

### 4.4 References/Bibliography

### 4.5 Problems

- F.E. Chapter 10 - Systems of Forces - Problems 1-23  
 F.E. Chapter 11 - Trusses - Problems 1-10  
 F.E. Chapter 12 - Pulleys, Cables, and Friction - Problems 1-12  
 F.E. Chapter 13 - Centroids and Moments of Inertia - Problems 1-18  
 F.E. Chapter 18 - Stress and Strain - Problems 1-18  
 F.E. Chapter 19 - Thermal, Hoop and Torsional Stress - Problems 1-18  
 F.E. Chapter 20 - Beams - Problems 1-15  
 F.E. Chapter 21 - Columns - Problems 1-10  
 F.E. Chapter 36 - Crystallography and Atomic Bonding - Problems 1-8  
 F.E. Chapter 37 - Material Testing - Problems 1-11  
 F.E. Chapter 38 - Metallurgy - Problems 1-18

1. The data set below was obtained over a two week period for a 1.000" shaft with a tolerance of +/- 0.010". Write a program to automatically update the X-bar, UCL/LCL values given new values. When a new set of values is entered the program should check to see if the process is in control.

Date	Samples			
Nov., 1, 1994	1.0034"	0.9999"	0.9923"	1.0093"
Nov., 2, 1994	0.9997"	1.0025"	0.9993"	0.9938"
Nov., 3, 1994	1.0001"	1.0009"	0.9997"	1.0079"
Nov., 4, 1994	1.0064"	0.9934"	1.0034"	1.0064"
Nov., 5, 1994	0.9982"	0.9987"	0.9990"	0.9957"
Nov., 6, 1994	0.9946"	1.0101"	1.0000"	0.9974"
Nov., 7, 1994	1.0033"	1.0011"	1.0031"	0.9935"
Nov., 8, 1994	1.0086"	0.9945"	1.0045"	1.0034"
Nov., 9, 1994	0.9997"	0.9969"	1.0067"	0.9972"
Nov., 10, 1994	0.9912"	1.0011"	0.9998"	0.9986"
Nov., 11, 1994	1.0013"	1.0031"	0.9992"	1.0054"
Nov., 12, 1994	1.0027"	1.0000"	0.9976"	1.0038"
Nov., 13, 1994	1.0002"	1.0002"	0.9943"	1.0001"
Nov., 14, 1994	0.9956"	1.0001"	0.9965"	0.9973"

## 5. PRODUCT DESIGN AND MANUFACTURING REVIEW

Topics:

- 

Objectives:

- 

### 5.1 Introduction

### 5.2 Examples

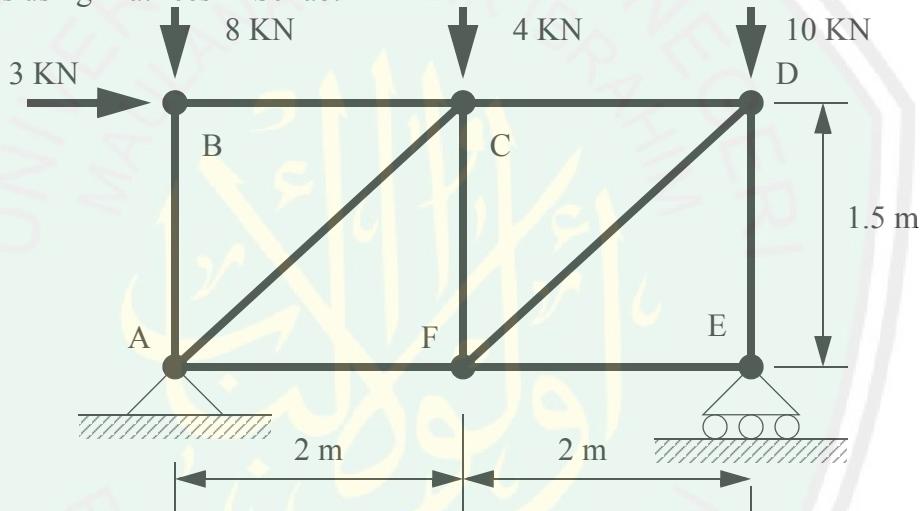
### 5.3 Summary

### 5.4 References/Bibliography

## 5.5 Problems

- F.E. Chapter 10 - Systems of Forces - Problems 1-23
- F.E. Chapter 11 - Trusses - Problems 1-10
- F.E. Chapter 12 - Pulleys, Cables, and Friction - Problems 1-12
- F.E. Chapter 13 - Centroids and Moments of Inertia - Problems 1-18
- F.E. Chapter 18 - Stress and Strain - Problems 1-18
- F.E. Chapter 19 - Thermal, Hoop and Torsional Stress - Problems 1-18
- F.E. Chapter 20 - Beams - Problems 1-15
- F.E. Chapter 21 - Columns - Problems 1-10
- F.E. Chapter 36 - Crystallography and Atomic Bonding - Problems 1-8
- F.E. Chapter 37 - Material Testing - Problems 1-11
- F.E. Chapter 38 - Metallurgy - Problems 1-18

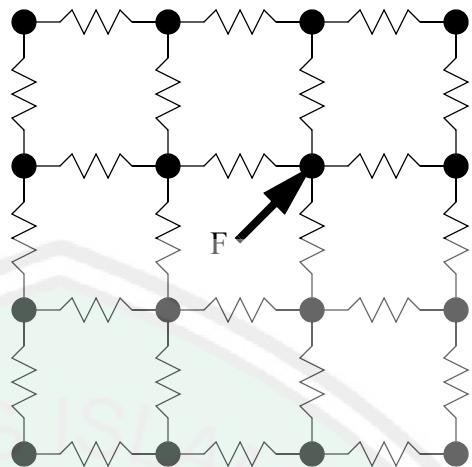
1. Find the forces using Matrices in Scilab.



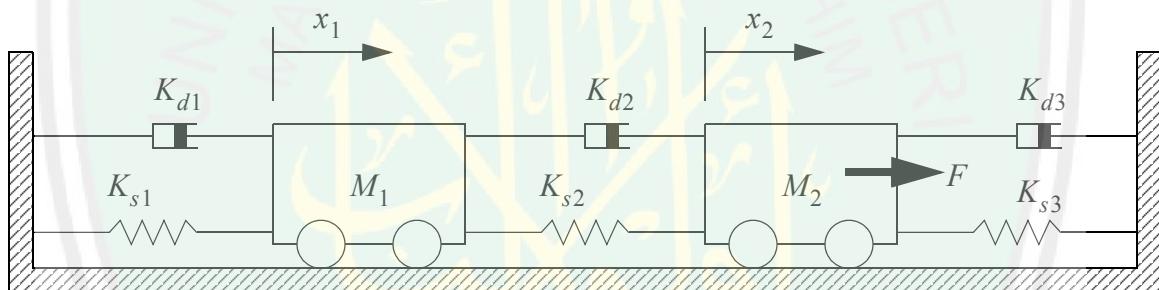
## 5.6 Challenge Problems

1. Write a program to find the deformation of an array of springs (as pictured below) with a force applied to an internal node. Use a matrix method to solve the problem. Assume that the

nodes at the four corners are fixed and unable to move.



2. Write the differential equations for one of the following systems. Simulate the system response using a numerical method, such as Runge-Kutta integration.



## 2. BOOLEAN LOGIC DESIGN

Topics:

- Boolean algebra
- Converting between Boolean algebra and logic gates
- Logic examples

Objectives:

- Be able to simplify designs with Boolean algebra and Karnaugh maps

### 2.1 Introduction

Boolean algebra provides the tools needed to analyze and design logical systems.

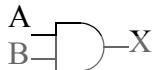
### 2.2 Boolean Algebra

Boolean algebra was developed in the 1800's by James Bool, an Irish mathematician. It was found to be extremely useful for designing digital circuits, and it is still heavily used by electrical engineers and computer scientists. The techniques can model a logical system with a single equation. The equation can then be simplified and/or manipulated into new forms. The same techniques developed for circuit designers adapt very well to circuit and program.

Boolean equations consist of variables and operations and look very similar to normal algebraic equations. The three basic operators are AND, OR and NOT; more complex operators include exclusive or (EOR), not and (NAND), not or (NOR). Small truth tables for these functions are shown in Figure 2.1. Each operator is shown in a simple equation with the variables A and B being used to calculate a value for X. Truth tables are a simple (but bulky) method for showing all of the possible combinations that will turn an output on or off.

Note: By convention a false state is also called off or 0 (zero). A true state is also called on or 1.

AND



$$X = A \cdot B$$

A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

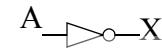
OR



$$X = A + B$$

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

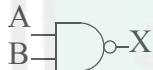
NOT



$$X = \bar{A}$$

A	X
0	1
1	0

NAND



$$X = \overline{A \cdot B}$$

A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

NOR



$$X = \overline{A + B}$$

A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

EOR



$$X = A \oplus B$$

A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

Note: The symbols used in these equations, such as + for OR are not universal standards and some authors will use different notations.

Note: The EOR function is available in gate form, but it is more often converted to its equivalent, as shown below.

$$X = A \oplus B = A \cdot \bar{B} + \bar{A} \cdot B$$

Figure 2.1 Boolean Operations with Truth Tables and Gates

In a Boolean equation the operators will be put in a more complex form as shown in Figure 2.2. The variable for these equations can only have a value of 0 for false, or 1 for

true. The solution of the equation follows rules similar to normal algebra. Parts of the equation inside parenthesis are to be solved first. Operations are to be done in the sequence NOT, AND, OR. In the example the NOT function for C is done first, but the NOT over the first set of parentheses must wait until a single value is available. When there is a choice the AND operations are done before the OR operations. For the given set of variable values the result of the calculation is false.

given

$$X = \overline{(A + B \cdot C)} + A \cdot (B + \bar{C})$$

assuming A=1, B=0, C=1

$$X = \overline{(1 + 0 \cdot 1)} + 1 \cdot (0 + \bar{1})$$

$$X = \overline{(1 + 0)} + 1 \cdot (0 + 0)$$

$$X = \overline{(1)} + 1 \cdot (0)$$

$$X = 0 + 0$$

$$X = 0$$

*Figure 2.2 A Boolean Equation*

The equations can be manipulated using the basic axioms of Boolean shown in Figure 2.3. A few of the axioms (associative, distributive, commutative) behave like normal algebra, but the other axioms have subtle differences that must not be ignored.

Idempotent

$$A + A = A$$

$$A \cdot A = A$$

Associative

$$(A + B) + C = A + (B + C)$$

$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

Commutative

$$A + B = B + A$$

$$A \cdot B = B \cdot A$$

Distributive

$$A + (B \cdot C) = (A + B) \cdot (A + C)$$

$$A \cdot (B + C) = (A \cdot B) + (A \cdot C)$$

Identity

$$A + 0 = A$$

$$A + 1 = 1$$

$$A \cdot 0 = 0$$

$$A \cdot 1 = A$$

Complement

$$A + \bar{A} = 1$$

$$\overline{(A)} = A$$

$$A \cdot \bar{A} = 0$$

$$\bar{1} = 0$$

DeMorgan's

$$\overline{(A + B)} = \bar{A} \cdot \bar{B}$$

$$\overline{(A \cdot B)} = \bar{A} + \bar{B}$$

Duality

interchange AND and OR operators, as well as all Universal, and Null sets. The resulting equation is equivalent to the original.

*Figure 2.3 The Basic Axioms of Boolean Algebra*

An example of equation manipulation is shown in Figure 2.4. The distributive axiom is applied to get equation (1). The idempotent axiom is used to get equation (2). Equation (3) is obtained by using the distributive axiom to move C outside the parentheses, but the identity axiom is used to deal with the lone C. The identity axiom is then used to simplify the contents of the parentheses to get equation (4). Finally the Identity axiom is

used to get the final, simplified equation. Notice that using Boolean algebra has shown that 3 of the variables are entirely unneeded.

$$\begin{aligned}
 A &= \bar{B} \cdot (C \cdot (\bar{D} + E + C) + \bar{F} \cdot C) \\
 A &= \bar{B} \cdot (\bar{D} \cdot C + E \cdot C + C \cdot C + \bar{F} \cdot C) \quad (1) \\
 A &= \bar{B} \cdot (\bar{D} \cdot C + E \cdot C + C + \bar{F} \cdot C) \quad (2) \\
 A &= \bar{B} \cdot C \cdot (\bar{D} + E + 1 + \bar{F}) \quad (3) \\
 A &= \bar{B} \cdot C \cdot (1) \quad (4) \\
 A &= \bar{B} \cdot C \quad (5)
 \end{aligned}$$

Figure 2.4 Simplification of a Boolean Equation

Note: When simplifying Boolean algebra, OR operators have a lower priority, so they should be manipulated first. NOT operators have the highest priority, so they should be simplified last. Consider the example from before.

$$\begin{aligned}
 X &= \overline{(A + B \cdot C)} + A \cdot (B + \bar{C}) \\
 X &= \overline{(A)} + \overline{(B \cdot C)} + A \cdot (B + \bar{C}) \quad \text{The higher priority operators are put in parentheses} \\
 X &= \overline{(A)} \cdot \overline{(B \cdot C)} + A \cdot (B + \bar{C}) \quad \text{DeMorgan's theorem is applied} \\
 X &= \bar{A} \cdot (\bar{B} + \bar{C}) + A \cdot (B + \bar{C}) \quad \text{DeMorgan's theorem is applied again} \\
 X &= \bar{A} \cdot \bar{B} + \bar{A} \cdot \bar{C} + A \cdot B + A \cdot \bar{C} \quad \text{The equation is expanded} \\
 X &= \bar{A} \cdot \bar{B} + (\bar{A} \cdot \bar{C} + A \cdot \bar{C}) + A \cdot B \quad \text{Terms with common terms are collected, here it is only NOT C} \\
 X &= \bar{A} \cdot \bar{B} + \bar{C} \cdot (\bar{A} + A) + A \cdot B \quad \text{The redundant term is eliminated} \\
 X &= \bar{A} \cdot \bar{B} + \bar{C} + A \cdot B \quad \text{A Boolean axiom is applied to simplify the equation further}
 \end{aligned}$$

Note: A simplified expression will generally reduce the number of operations required....

$$A + B\bar{A} = A + B$$

## 2.3 Logic Design

Design ideas can be converted to Boolean equations directly, or with other techniques discussed later. The Boolean equation form can then be simplified or rearranged, and then converted into ladder logic, or a circuit.

If we can describe how a controller should work in words, we can often convert it directly to a Boolean equation, as shown in Figure 2.5. In the example a process description is given first. In actual applications this is obtained by talking to the designer of the mechanical part of the system. In many cases the system does not exist yet, making this a

challenging task. The next step is to determine how the controller should work. In this case it is written out in a sentence first, and then converted to a Boolean expression. The Boolean expression may then be converted to a desired form. The first equation contains an EOR, which is not available in ladder logic, so the next line converts this to an equivalent expression (2) using ANDs, ORs and NOTs. The circuit shown is for the second equation. In the conversion the terms that are ANDed are in series..... The last equation (3) is fully expanded and the circuit for it is shown in Figure 2.6. This illustrates the same logical control function can be achieved with different, yet equivalent, circuits.



**Process Description:**

A heating oven with two bays can heat one ingot in each bay. When the heater is on it provides enough heat for two ingots. But, if only one ingot is present the oven may become too hot, so a fan is used to cool the oven when it passes a set temperature.

**Control Description:**

If the temperature is too high and there is an ingot in only one bay then turn on fan.

**Define Inputs and Outputs:**

B1 = bay 1 ingot present

B2 = bay 2 ingot present

F = fan

T = temperature overheat sensor

**Boolean Equation:**

$$F = T \cdot (B_1 \oplus B_2)$$

$$F = T \cdot (B_1 \cdot \overline{B_2} + \overline{B_1} \cdot B_2) \quad (2)$$

$$F = B_1 \cdot \overline{B_2} \cdot T + \overline{B_1} \cdot B_2 \cdot T \quad (3)$$

**Circuit for Equation (2):**

XXXXXXX

Note: the result for conditional logic is a single step in the ladder

Warning: in spoken and written English OR and EOR are often not clearly defined. Consider the traffic directions "Go to main street then turn left or right." Does this *or* mean that you can drive either way, or that the person isn't sure which way to go? Consider the expression "The cars are red or blue.", Does this mean that the cars can be either red or blue, or all of the cars are red, or all of the cars are blue. A good literal way to describe this condition is "one or the other, but not both".

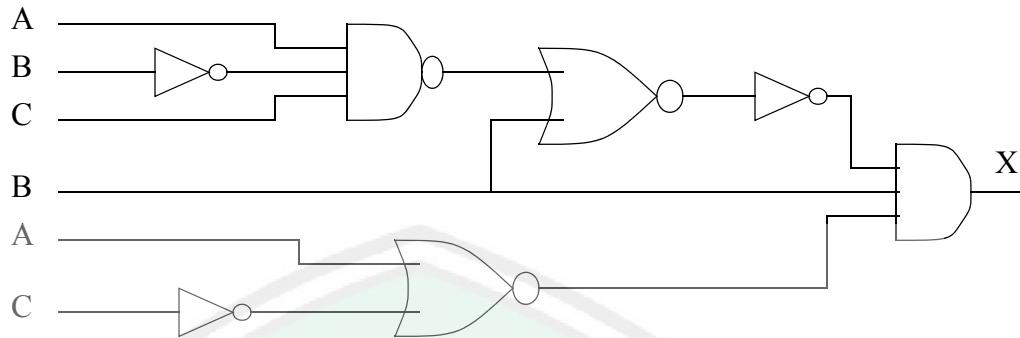
*Figure 2.5 Boolean Algebra Based Design of Ladder Logic*

Circuit for Equation (3):

XXXXXXXXXXXXXXXXXX

Figure 2.6 Alternate Circuit

Boolean algebra is often used in the design of digital circuits. Consider the example in Figure 2.7. In this case we are presented with a circuit that is built with inverters, nand, nor and, and gates. This figure can be converted into a boolean equation by starting at the left hand side and working right. Gates on the left hand side are *solved* first, so they are put inside parentheses to indicate priority. Inverters are represented by putting a NOT operator on a variable in the equation. This circuit can't be directly converted to ladder logic because there are no equivalents to NAND and NOR gates. After the circuit is converted to a Boolean equation it is simplified, and then converted back into a (much simpler) circuit diagram.



The circuit is converted to a Boolean equation and simplified. The most nested terms in the equation are on the left hand side of the diagram.

$$X = \overline{((A \cdot \bar{B} \cdot C) + B)} \cdot B \cdot \overline{(A + \bar{C})}$$

$$X = (\bar{A} + B + \bar{C} + B) \cdot B \cdot (\bar{A} \cdot C)$$

$$X = \bar{A} \cdot B \cdot \bar{A} \cdot C + B \cdot B \cdot \bar{A} \cdot C + \bar{C} \cdot B \cdot \bar{A} \cdot C + B \cdot B \cdot \bar{A} \cdot C$$

$$X = B \cdot \bar{A} \cdot C + B \cdot \bar{A} \cdot C + 0 + B \cdot \bar{A} \cdot C$$

$$X = B \cdot \bar{A} \cdot C$$

This simplified equation is converted back into a circuit.

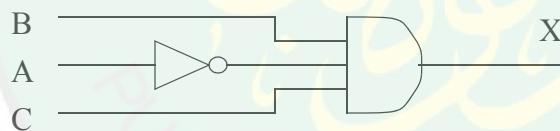


Figure 2.7 Reverse Engineering of a Digital Circuit

To summarize, we will obtain Boolean equations from a verbal description or existing circuit or ladder diagram. The equation can be manipulated using the axioms of Boolean algebra. after simplification the equation can be converted back into a circuit diagram. Circuits can behave the same even though they are in different forms. When simplifying Boolean equations that are to be implemented in circuits there are a few basic rules.

1. Eliminate NOTs that are for more than one variable. This normally includes replacing NAND and NOR functions with simpler ones using DeMorgan's theorem.

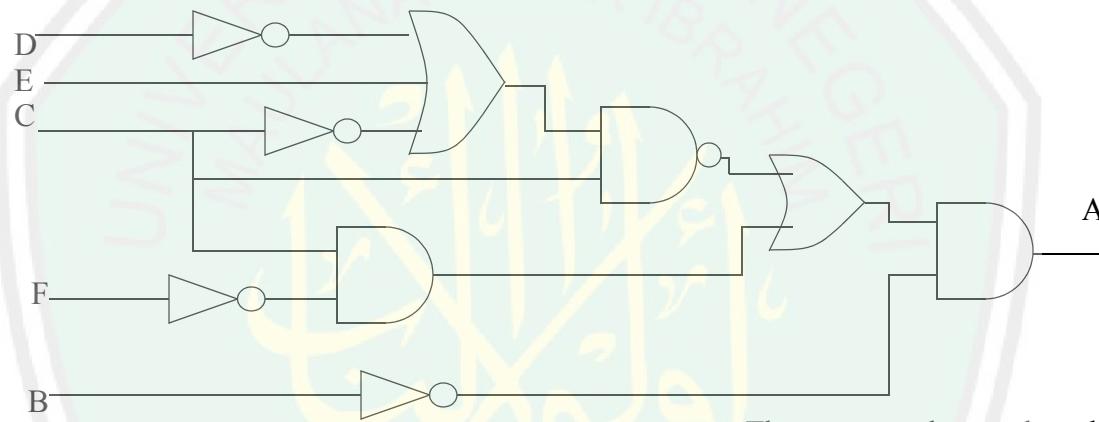
## 2. Eliminate complex functions such as EORs with their equivalent.

These principles are reinforced with another design that begins in Figure 2.8. Assume that the Boolean equation that describes the controller is already known. This equation can be converted into both a circuit diagram and ladder logic. The circuit diagram contains about two dollars worth of integrated circuits. If the design was mass produced the final cost for the entire controller would be under \$50. The prototype of the controller would cost thousands of dollars.

Given the controller equation;

$$A = \bar{B} \cdot (\overline{C \cdot (\bar{D} + E + \bar{C})} + \bar{F} \cdot C)$$

The circuit is given below, and equivalent ladder logic is shown.



The gates can be purchased for about \$0.25 each in bulk.  
Inputs and outputs are typically 5V

Figure 2.8 A Boolean Equation and Derived Circuit

The initial equation is not the simplest. It is possible to simplify the equation to the form seen in Figure 2.8.

$$A = \bar{B} \cdot C \cdot (\bar{D} + E + \bar{F})$$

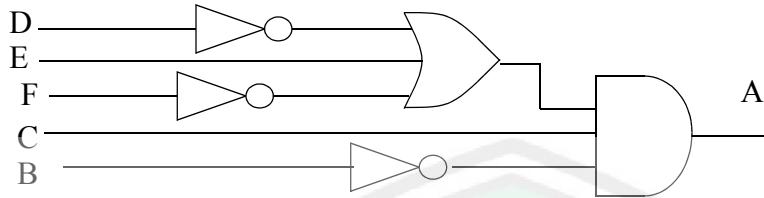


Figure 2.9 The Simplified Form of the Example

The equation can also be manipulated to other forms that are more routine but less efficient as shown in Figure 2.10. The equation shown is in disjunctive normal form - in simpler words this is ANDed terms ORed together. This is also an example of a canonical form - in simpler terms this means a standard form. This form is more important for digital logic. For example, when an equation is simplified, it may not look like the original design intention, and therefore becomes harder to rework without starting from the beginning.

$$A = (\bar{B} \cdot C \cdot \bar{D}) + (\bar{B} \cdot C \cdot E) + (\bar{B} \cdot C \cdot \bar{F})$$

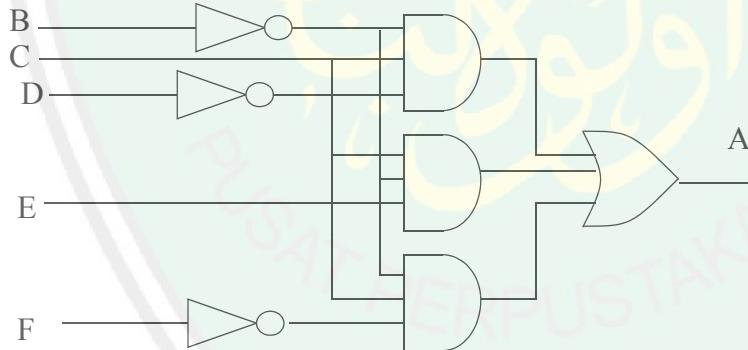


Figure 2.10 A Canonical Logic Form

### 2.3.1 Boolean Algebra Techniques

There are some common Boolean algebra techniques that are used when simplifying equations. Recognizing these forms are important to simplifying Boolean Algebra

with ease. These are itemized, with proofs in Figure 2.11.

$A + C\bar{A} = A + C$	proof:	$A + C\bar{A}$
		$(A + C)(A + \bar{A})$
		$(A + C)(1)$
		$A + C$
$AB + A = A$	proof:	$AB + A$
		$AB + A1$
		$A(B + 1)$
		$A(1)$
		$A$
$\overline{A + B + C} = \bar{A}\bar{B}\bar{C}$	proof:	$\overline{A + B + C}$
		$\overline{(A + B) + C}$
		$\overline{(A + B)}\bar{C}$
		$(\bar{A}\bar{B})\bar{C}$
		$\bar{A}\bar{B}\bar{C}$

Figure 2.11 Common Boolean Algebra Techniques

## 2.4 Common Logic Forms

Knowing a simple set of logic forms will support a designer when categorizing control problems. The following forms are provided to be used directly, or provide ideas when designing.

### 2.4.1 Complex Gate Forms

In total there are 16 different possible types of 2-input logic gates. The simplest are AND and OR, the other gates we will refer to as *complex* to differentiate. The three popular complex gates that have been discussed before are NAND, NOR and EOR. All of these can be reduced to simpler forms with only ANDs and ORs, as shown in Figure 2.12.

NAND

$$X = \overline{A \cdot B}$$

$$X = \overline{A} + \overline{B}$$

NOR

$$X = \overline{A + B}$$

$$X = \overline{A} \cdot \overline{B}$$

EOR

$$X = A \oplus B$$

$$X = A \cdot \overline{B} + \overline{A} \cdot B$$

Figure 2.12 Conversion of Complex Logic Functions

### 2.4.2 Multiplexers

Multiplexers allow multiple devices to be connected to a single device. These are very popular for telephone systems. A telephone *switch* is used to determine which telephone will be connected to a limited number of lines to other telephone switches. This allows telephone calls to be made to somebody far away without a dedicated wire to the other telephone. In older telephone switch boards, operators physically connected wires by plugging them in. In modern computerized telephone switches the same thing is done, but to digital voice signals.

In Figure 2.13 a multiplexer is shown that will take one of four inputs bits D1, D2, D3 or D4 and make it the output X, depending upon the values of the address bits, A1 and A2.

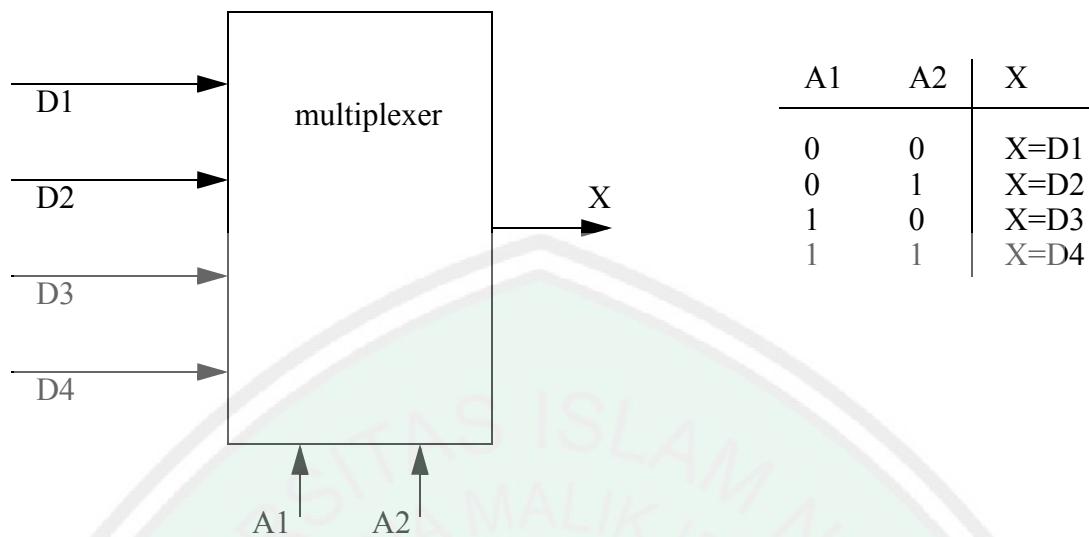


Figure 2.13 A Multiplexer

## 2.5 Simple Design Cases

The following cases are presented to illustrate various combinatorial logic problems, and possible solutions. It is recommended that you try to satisfy the description before looking at the solution.

### 2.5.1 Basic Logic Functions

Problem: Develop a program that will cause output D to go true when switch A and switch B are closed or when switch C is closed.

Solution:

$$D = (A \cdot B) + C$$

Figure 2.14 Sample Solution for Logic Case Study A

Problem: Develop a program that will cause output D to be on when push button A is on, or either B or C are on.

Solution:

$$D = A + (B \oplus C)$$

Figure 2.15 Sample Solution for Logic Case Study B

## 2.5.2 Car Safety System

Problem: Develop a circuit for a car door/seat belt safety system. When the car door is open, and the seatbelt is not done up, the ignition power must not be applied. If all is safe then the key will start the engine.

Solution:

*Figure 2.16 Solution to Car Safety System Case*

### 2.5.3 Motor Forward/Reverse

Problem: Design a motor controller that has a forward and a reverse button. The motor forward and reverse outputs will only be on when one of the buttons is pushed. When both buttons are pushed the motor will not work.

Solution:

$$F = BF \cdot \overline{BR}$$

where,

$$R = \overline{BF} \cdot BR$$

F = motor forward  
R = motor reverse  
BF = forward button  
BR = reverse button

*Figure 2.17 Motor Forward, Reverse Case Study*

### 2.5.4 A Burglar Alarm

Consider the design of a burglar alarm for a house. When activated an alarm and

lights will be activated to encourage the unwanted guest to leave. This alarm be activated if an unauthorized intruder is detected by window sensor and a motion detector. The window sensor is effectively a loop of wire that is a piece of thin metal foil that encircles the window. If the window is broken, the foil breaks breaking the conductor. This behaves like a normally closed switch. The motion sensor is designed so that when a person is detected the output will go on. As with any alarm an activate/deactivate switch is also needed. The basic operation of the alarm system, and the inputs and outputs of the controller are itemized in Figure 2.18.

The inputs and outputs are chosen to be;

- A = Alarm and lights switch (1 = on)
- W = Window/Door sensor (1 = OK)
- M = Motion Sensor (0 = OK)
- S = Alarm Active switch (1 = on)

The basic operation of the alarm can be described with rules.

1. If alarm is on, check sensors.
2. If window/door sensor is broken (turns off), sound alarm and turn on lights

Note: As the engineer, it is your responsibility to define these items before starting the work. If you do not do this first you are guaranteed to produce a poor design. It is important to develop a good list of inputs and outputs, and give them simple names so that they are easy to refer to. Most companies will use wire numbering schemes on their diagrams.

Figure 2.18 Controller Requirements List for Alarm

The next step is to define the controller equation. In this case the controller has 3 different inputs, and a single output, so a truth table is a reasonable approach to formalizing the system. A Boolean equation can then be written using the truth table in Figure 2.19. Of the eight possible combinations of alarm inputs, only three lead to alarm conditions.

Inputs			Output
S	M	W	A
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

note the binary sequence

alarm off

alarm on/no thief

alarm on/thief detected

Figure 2.19 Truth Table for the Alarm

The Boolean equation in Figure 2.20 is written by examining the truth table in Figure 2.19. There are three possible alarm conditions that can be represented by the conditions of all three inputs. For example take the last line in the truth table where when all three inputs are on the alarm should be one. This leads to the last term in the equation. The other two terms are developed the same way. After the equation has been written, it is simplified.

$$\begin{aligned}
 A &= (S \cdot \bar{M} \cdot \bar{W}) + (S \cdot M \cdot \bar{W}) + (S \cdot M \cdot W) \\
 \therefore A &= S \cdot (\bar{M} \cdot \bar{W} + M \cdot \bar{W} + M \cdot W) \\
 \therefore A &= S \cdot ((\bar{M} \cdot \bar{W} + M \cdot \bar{W}) + (M \cdot \bar{W} + M \cdot W)) \\
 \therefore A &= (S \cdot \bar{W}) + (S \cdot M) = S \cdot (\bar{W} + M)
 \end{aligned}$$

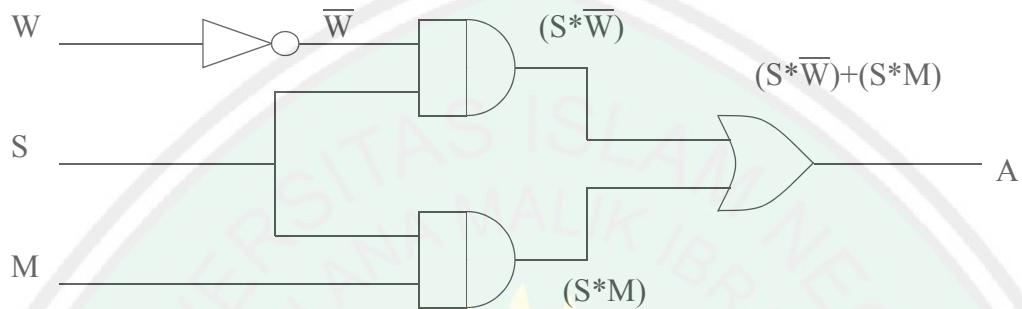


Figure 2.20 A Boolean Equation and Implementation for the Alarm

The equation and circuits shown in Figure can also be further simplified, as shown in Figure 2.21.

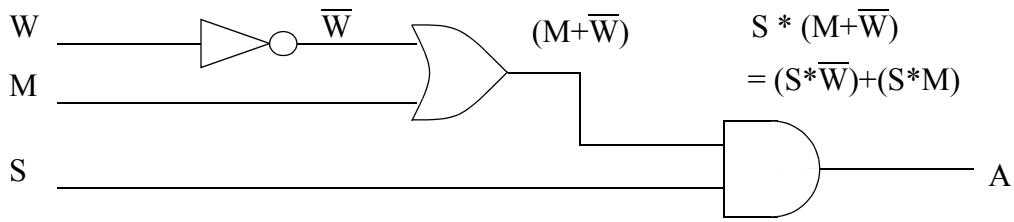


Figure 2.21 The Simplest Circuit and Ladder Diagram

Aside: The alarm could also be implemented in programming languages. The program below is for a Basic Stamp II chip. ([www.parallaxinc.com](http://www.parallaxinc.com))

```
w = 1; s = 2; m = 3; a = 4
input m; input w; input s
output a
loop:
if (in2 = 1) and (in1 = 0 or in3 = 1) then on
low a; goto loop 'alarm off
on:
high a; goto loop 'alarm on
```

Figure 2.22 Alarm Implementation Using A High Level Programming Language

## 2.6 Summary

- Logic can be represented with Boolean equations.
- Boolean equations can be converted to (and from) digital circuits.
- Boolean equations can be simplified.
- Different controllers can behave the same way.
- Common logic forms exist and can be used to understand logic.

- Truth tables can represent all of the possible state of a system.

## 2.7 Problems

1. Draw a circuit that will cause output D to go true when switch A and switch B are closed or when switch C is closed.
2. Draw a circuit that will cause output D to be on when push button A is on, or either B or C are on.
3. Design a circuit for a car that considers the variables below to control the motor M. Also add a second output that uses any outputs not used for motor control.
  - doors opened/closed (D)
  - keys in ignition (K)
  - motor running (M)
  - transmission in park (P)
  - ignition start (I)
4. Make a simple circuit that will turn on the outputs with the binary patterns when the corresponding buttons are pushed. Inputs X, Y, and Z will never be on at the same time.

OUTPUTS									INPUTS
H	G	F	E	D	C	B	A		
1	1	0	1	0	1	0	1	Input X on	
1	0	1	0	0	0	0	1	Input Y on	
1	0	0	1	0	1	1	1	Input Z on	

5. Convert the following Boolean equation to the simplest possible circuit.

$$X = A \cdot (\overline{A} + \overline{A} \cdot B)$$

6. Simplify the following boolean equations.

a)  $A(B + AB)$

b)  $\overline{A(\overline{B} + AB)}$

c)  $\overline{A}(B + AB)$

d)  $\overline{A}(\overline{B} + \overline{AB})$

7. Simplify the following Boolean equations,

- a)  $\overline{(A + B)} \cdot \overline{(A + \bar{B})}$
- b)  $ABCD + \bar{A}BCD + ABC\bar{D} + AB\bar{C}\bar{D}$

8. Simplify the Boolean expression below.

$$((A \cdot \bar{B}) + (\bar{B} + A)) \cdot C + (\bar{B} \cdot C + B \cdot C)$$

9. Given the Boolean expression a) draw a digital circuit and b) simplify the expression.

$$X = A \cdot \bar{B} \cdot C + \overline{(C + B)}$$

10. Simplify the following Boolean equation and write a corresponding circuit.

$$Y = \overline{(AB\bar{C}D + AB\bar{C}\bar{D} + \bar{A}BCD + \bar{A}\bar{B}CD)} + D$$

11. For the following Boolean equation,

$$X = A + B(A + C\bar{B} + D\bar{A}C) + ABCD$$

- a) Write the logic circuit for the unsimplified equation.
- b) Simplify the equation.
- c) Write the circuit for the simplified equation.

12. a) Write a Boolean equation for the following truth table. (Hint: do this by writing an expres-

sion for each line with a true output, and then ORing them together.)

A	B	C	D	Result
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

- b) Write the results in a) in a Boolean equation.  
 c) Simplify the Boolean equation in b)

13. Simplify the following Boolean equation, and create the simplest circuit.

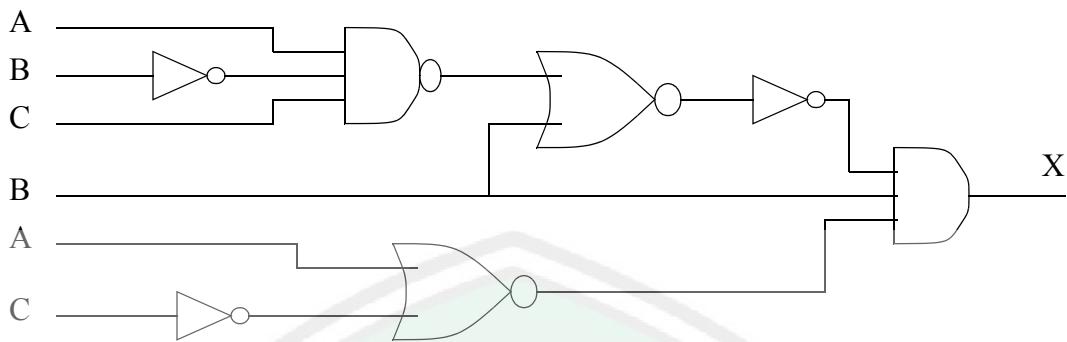
$$Y = \overline{\bar{C}} \left( \overline{\bar{A} + \left( \overline{\bar{A} + (\bar{B}\bar{C}(A + \overline{B}\bar{C}))} \right)} \right)$$

14. Simplify the following boolean equation with Boolean algebra and write the corresponding circuit.

$$X = \overline{(A + B \cdot \bar{A}) + (\bar{C} + D + E\bar{C})}$$

15. a) Develop the Boolean expression for the circuit below.  
 b) Simplify the Boolean expression.

c) Draw a simpler circuit for the equation in b).



16. Given a system that is described with the following equation,

$$X = A + (B \cdot (\bar{A} + C) + C) + A \cdot B \cdot (\bar{D} + \bar{E})$$

- a) Simplify the equation using Boolean Algebra.
- b) Implement the original and then the simplified equation with a digital circuit.

17. Simplify the following and implement the original and simplified equations with gates.

$$A + (\bar{B} + \bar{C} + \bar{D}) \cdot (B + \bar{C}) + A \cdot B \cdot (\bar{C} + \bar{D})$$

18. Simplify the following Boolean equation and implement it in a circuit.

$$X = A + BA + B\bar{C} + \bar{D} + C$$

19. Use Boolean equations to develop simplified circuit for the following truth table where A, B, C and D are inputs, and X and Y are outputs.

A	B	C	D	X	Y
0	0	0	0	0	0
0	0	0	1	1	0
0	0	1	0	0	0
0	0	1	1	1	0
0	1	0	0	0	0
0	1	0	1	0	1
0	1	1	0	0	1
0	1	1	1	0	1
1	0	0	0	0	0
1	0	0	1	1	0
1	0	1	0	0	0
1	0	1	1	1	0
1	1	0	0	0	0
1	1	0	1	1	1
1	1	1	0	0	1
1	1	1	1	1	1

20. Convert the truth table below to a Boolean equation, and then simplify it. The output is X and the inputs are A, B, C and D.

A	B	C	D	X
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

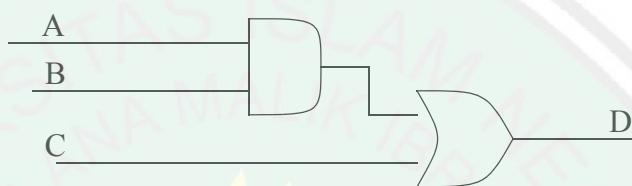
21. Simplify the following Boolean equation. Convert both the unsimplified and simplified equa-

tions to a circuit.

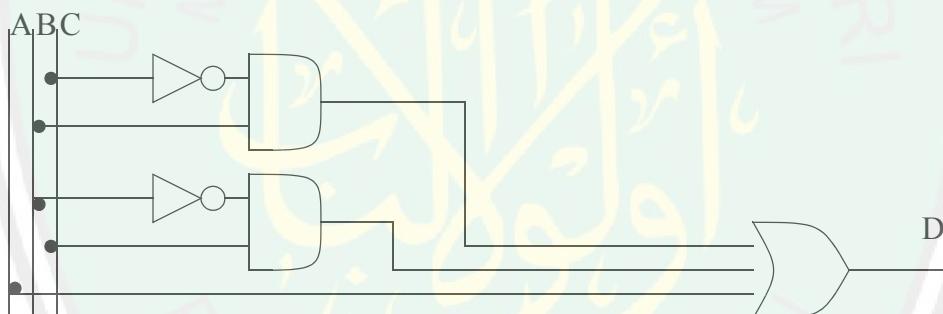
$$X = \overline{(AB\bar{C})}(A + BC)$$

## 2.8 Problem Solutions

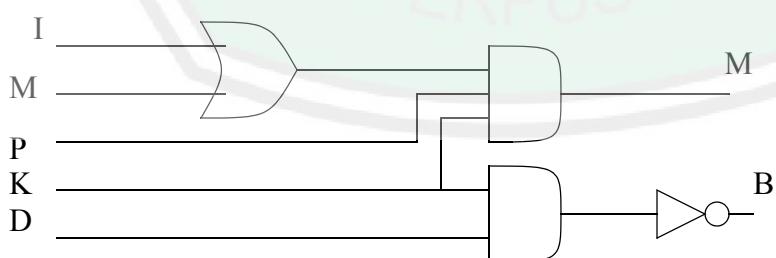
1.



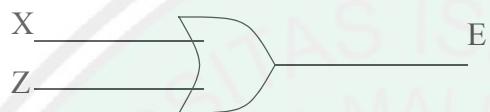
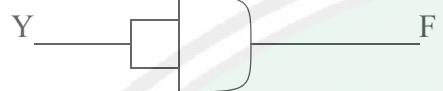
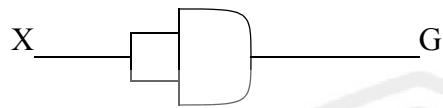
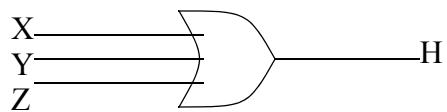
2.



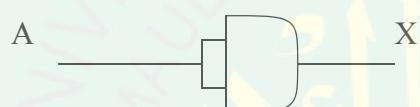
3.



4.



5.



6.

- a)  $AB$
- b)  $\bar{A} + B$
- c)  $\bar{A}\bar{B}$
- d)  $A + B$

7.

a)  $(A + B) \cdot (\bar{A} + \bar{B}) = (\bar{A}\bar{B})(\bar{A}B) = 0$

b)  $ABCD + \bar{A}BCD + ABC\bar{D} + AB\bar{C}\bar{D} = BCD + AB\bar{D} = B(CD + A\bar{D})$

8. C

9.

$$X = \bar{B} \cdot (A \cdot C + \bar{C})$$

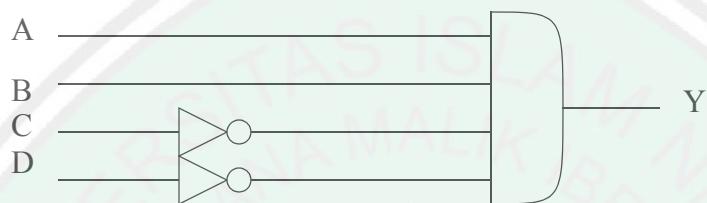
10.

$$Y = \overline{(AB\bar{C}D + AB\bar{C}\bar{D} + \bar{A}BCD + \bar{A}\bar{B}CD)} + D$$

$$Y = (AB\bar{C}D + AB\bar{C}\bar{D} + \bar{A}BCD + \bar{A}\bar{B}CD)\bar{D}$$

$$Y = (0 + AB\bar{C}\bar{D} + 0 + 0)\bar{D}$$

$$Y = AB\bar{C}\bar{D}$$



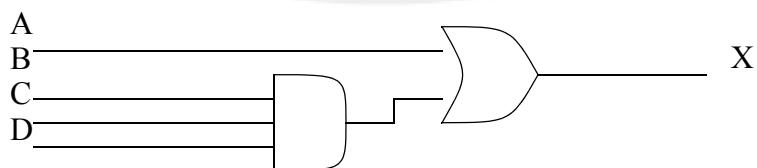
11.

a)

b)

$$X = A + DCB$$

c)



12.

$$\begin{aligned}
 & \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}CD + \bar{A}B\bar{C}D + \bar{A}BCD + A\bar{B}\bar{C}\bar{D} + A\bar{B}CD + ABC\bar{D} + ABCD \\
 & \bar{B}\bar{C}\bar{D} + \bar{A}CD + \bar{B}CD + \bar{A}BD + BCD + ACD + ABC \\
 & \bar{B}\bar{C}\bar{D} + CD(\bar{A} + A) + CD(\bar{B} + B) + \bar{A}BD + ABC \\
 & \bar{B}\bar{C}\bar{D} + D(C + \bar{A}B) + ABC
 \end{aligned}$$

13.

$$Y = \overline{\bar{C}} \left( \overline{\bar{A} + \left( \overline{\bar{A} + (\bar{B}\bar{C}(A + \bar{B}\bar{C}))} \right)} \right)$$

$$Y = \overline{\bar{C}} \left( \overline{\bar{A} + \left( \overline{\bar{A} + (\bar{B}\bar{C}(\bar{A} + \bar{B} + C))} \right)} \right)$$

$$Y = \overline{\bar{C}} \left( \overline{\bar{A} + (\overline{\bar{A} + (\bar{B}\bar{C}\bar{A}\bar{B}\bar{C})})} \right)$$

$$Y = \overline{\bar{C}} \left( \overline{\bar{A} + (\overline{\bar{A} + \bar{0}})} \right)$$

$$Y = \overline{\bar{C}}(\bar{A} + (\bar{A} + 1))$$

$$Y = \overline{\bar{C}}(\bar{A} + (\bar{1}))$$

$$Y = \overline{\bar{C}}(\bar{A} + 0)$$

$$Y = \overline{\bar{C}A}$$

$$Y = C + \bar{A}$$



14.

$$X = \overline{(A + B \cdot \bar{A})} + \overline{(C + D + E\bar{C})}$$

OR

$$X = \overline{(A + B \cdot \bar{A})(C + D + E\bar{C})}$$

$$X = (\bar{A})(\overline{B \cdot \bar{A}})(C + D + E\bar{C})$$

$$X = (\bar{A})(\overline{B \cdot \bar{A}})(C + D + E\bar{C})$$

$$X = \bar{A}\bar{B}(C + D + E\bar{C})$$

$$X = \bar{A}\bar{B}(C + D + E)$$

$$X = \overline{(A + B \cdot \bar{A})} + \overline{(C + D + E\bar{C})}$$

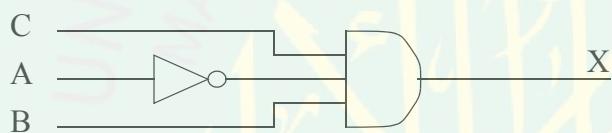
$$X = \overline{A + B \cdot \bar{A} + \bar{C}\bar{D}(\bar{E} + C)}$$

$$X = \overline{A + B + \bar{C}\bar{D}\bar{E}}$$

$$X = \bar{A}\bar{B}(\overline{\bar{C}\bar{D}\bar{E}})$$

$$X = \bar{A}\bar{B}(C + D + E)$$

15.

 $C\bar{A}B$ 

16.

a)

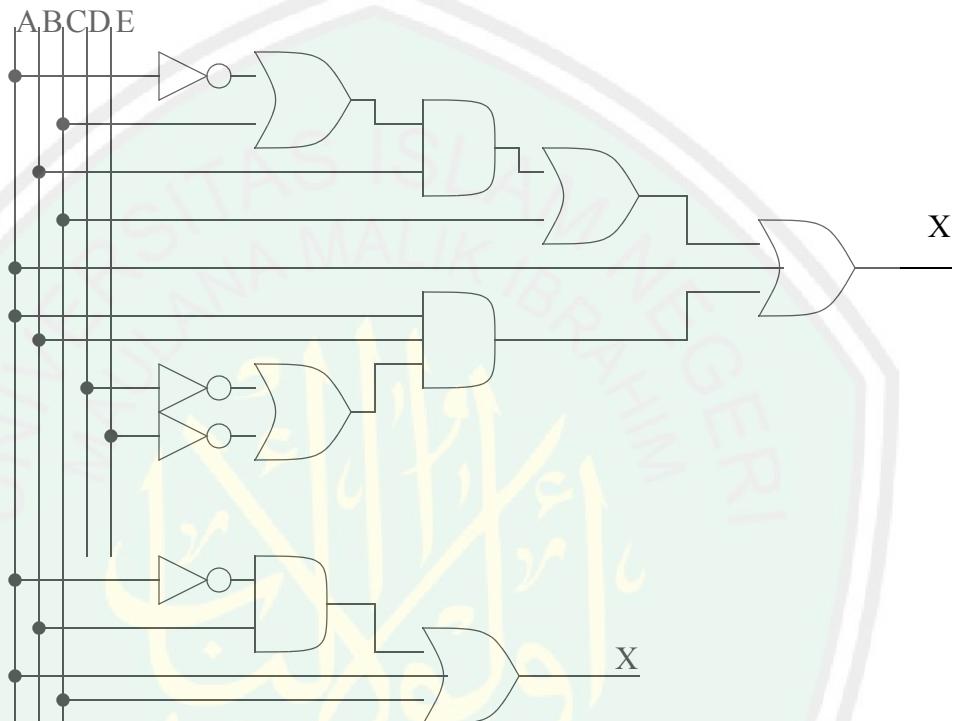
$$X = A + (B \cdot (\bar{A} + C) + C) + A \cdot B \cdot (\bar{D} + \bar{E})$$

$$X = A + (B \cdot \bar{A} + B \cdot C + C) + A \cdot B \cdot \bar{D} + A \cdot B \cdot \bar{E}$$

$$X = A \cdot (1 + B \cdot \bar{D} + B \cdot \bar{E}) + B \cdot \bar{A} + C \cdot (B + 1)$$

$$X = A + B \cdot \bar{A} + C$$

b)



17.

$$A + (\bar{B} + \bar{C} + \bar{D}) \cdot (B + \bar{C}) + A \cdot B \cdot (\bar{C} + \bar{D})$$

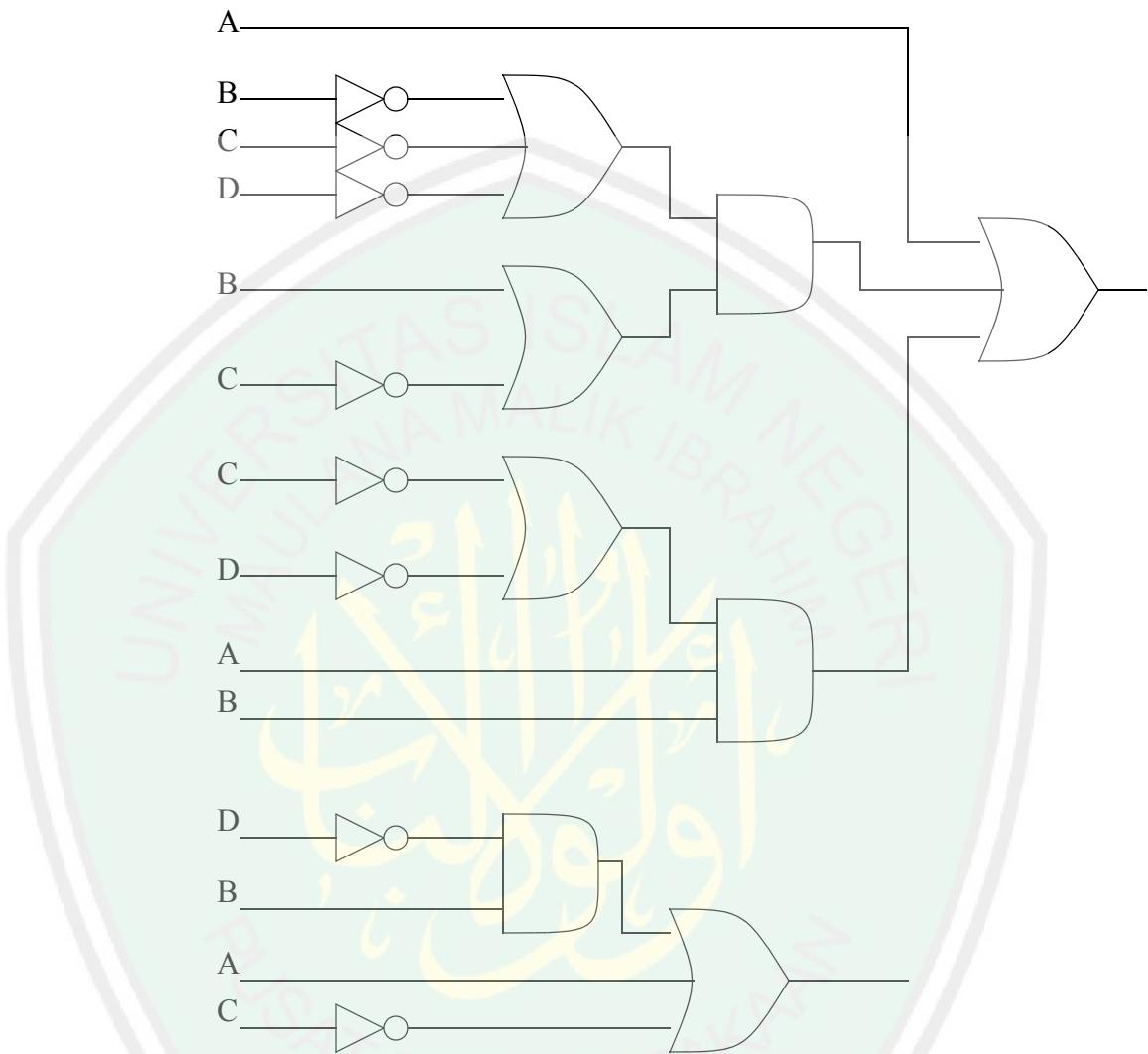
$$A \cdot (1 + B \cdot (\bar{C} + \bar{D})) + (\bar{B} + \bar{C} + \bar{D}) \cdot B + (\bar{B} + \bar{C} + \bar{D}) \cdot \bar{C}$$

$$A + (\bar{C} + \bar{D}) \cdot B + \bar{C}$$

$$A + \bar{C} \cdot B + \bar{D} \cdot B + \bar{C}$$

$$A + \bar{D} \cdot B + \bar{C}$$

$$A + \bar{D} \cdot B + \bar{C}$$



## 2.9 Challenge Problems

1. Write a program that implements the following Boolean expression in Scilab or C. The user should be able input values and the results should be printed.

$$A + (\bar{B} + \bar{C} + \bar{D}) \cdot (B + \bar{C}) + A \cdot B \cdot (\bar{C} + \bar{D})$$



### 3. NUMBERS AND DATA

Topics:

- Number bases; binary, octal, decimal, hexadecimal
- Binary calculations; 2s compliments, addition, subtraction and Boolean operations
- Encoded values; BCD and ASCII
- Error detection; parity, gray code and checksums

Objectives:

- To be familiar with binary, octal and hexadecimal numbering systems.
- To be able to convert between different numbering systems.
- To understand 2s compliment negative numbers.
- To be able to convert ASCII and BCD values.
- To be aware of basic error detection techniques.

#### 3.1 Introduction

Base 10 (decimal) numbers developed naturally because the original developers (probably) had ten fingers, or 10 digits. Now consider logical systems that only have wires that can be on or off. When counting with a wire the only digits are 0 and 1, giving a base 2 numbering system. Numbering systems for computers are often based on base 2 numbers, but base 4, 8, 16 and 32 are commonly used. A list of numbering systems is give in Figure 3.1. An example of counting in these different numbering systems is shown in Figure 3.2.

Base	Name	Data Unit
2	Binary	Bit
8	Octal	Nibble
10	Decimal	Digit
16	Hexadecimal	Byte

Figure 3.1 Numbering Systems

decimal	binary	octal	hexadecimal
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	a
11	1011	13	b
12	1100	14	c
13	1101	15	d
14	1110	16	e
15	1111	17	f
16	10000	20	10
17	10001	21	11
18	10010	22	12
19	10011	23	13
20	10100	24	14

Note: As with all numbering systems  
most significant digits are at left,  
least significant digits are at right.

Figure 3.2 Numbers in Decimal, Binary, Octal and Hexadecimal

The effect of changing the base of a number does not change the actual value, only how it is written. The basic rules of mathematics still apply, but many beginners will feel disoriented. This chapter will cover basic topics that are needed to use more complex programming instructions later in the book. These will include the basic number systems, conversion between different number bases, and some data oriented topics.

## 3.2 Numerical Values

### 3.2.1 Binary

Binary numbers are the most fundamental numbering system in all computers. A single binary digit (a bit) corresponds to the condition of a single wire. If the voltage on the wire is true the bit value is 1. If the voltage is off the bit value is 0. If two or more wires are used then each new wire adds another significant digit. Each binary number will have an equivalent digital value. Figure 3.3 shows how to convert a binary number to a decimal equivalent. Consider the digits, starting at the right. The least significant digit is 1, and is

in the 0th position. To convert this to a decimal equivalent the number base (2) is raised to the position of the digit, and multiplied by the digit. In this case the least significant digit is a trivial conversion. Consider the most significant digit, with a value of 1 in the 6th position. This is converted by the number base to the exponent 6 and multiplying by the digit value of 1. This method can also be used for converting the other number system to decimal.

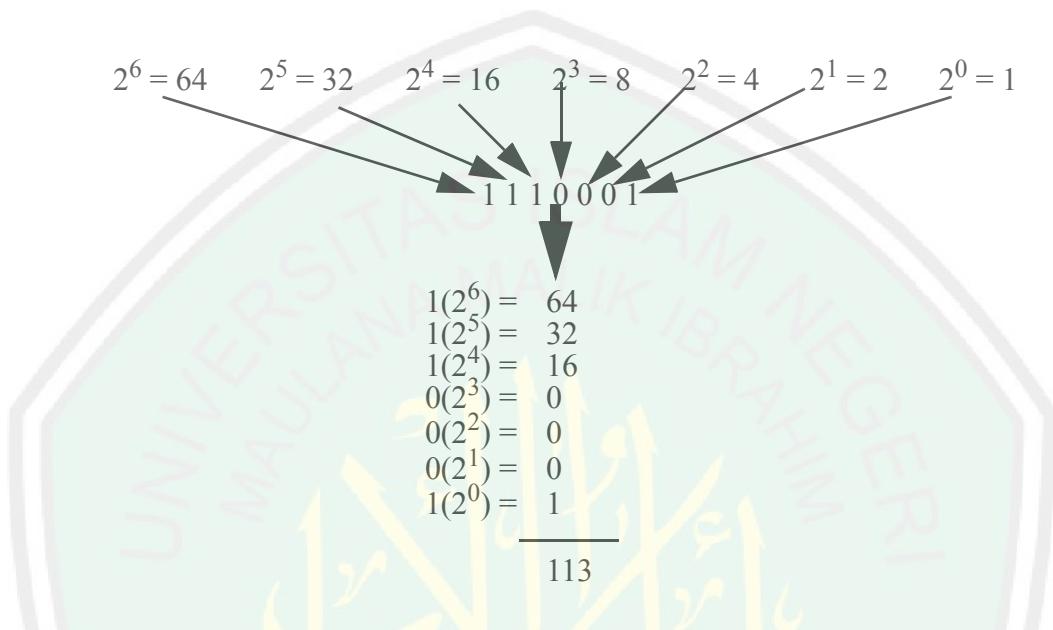
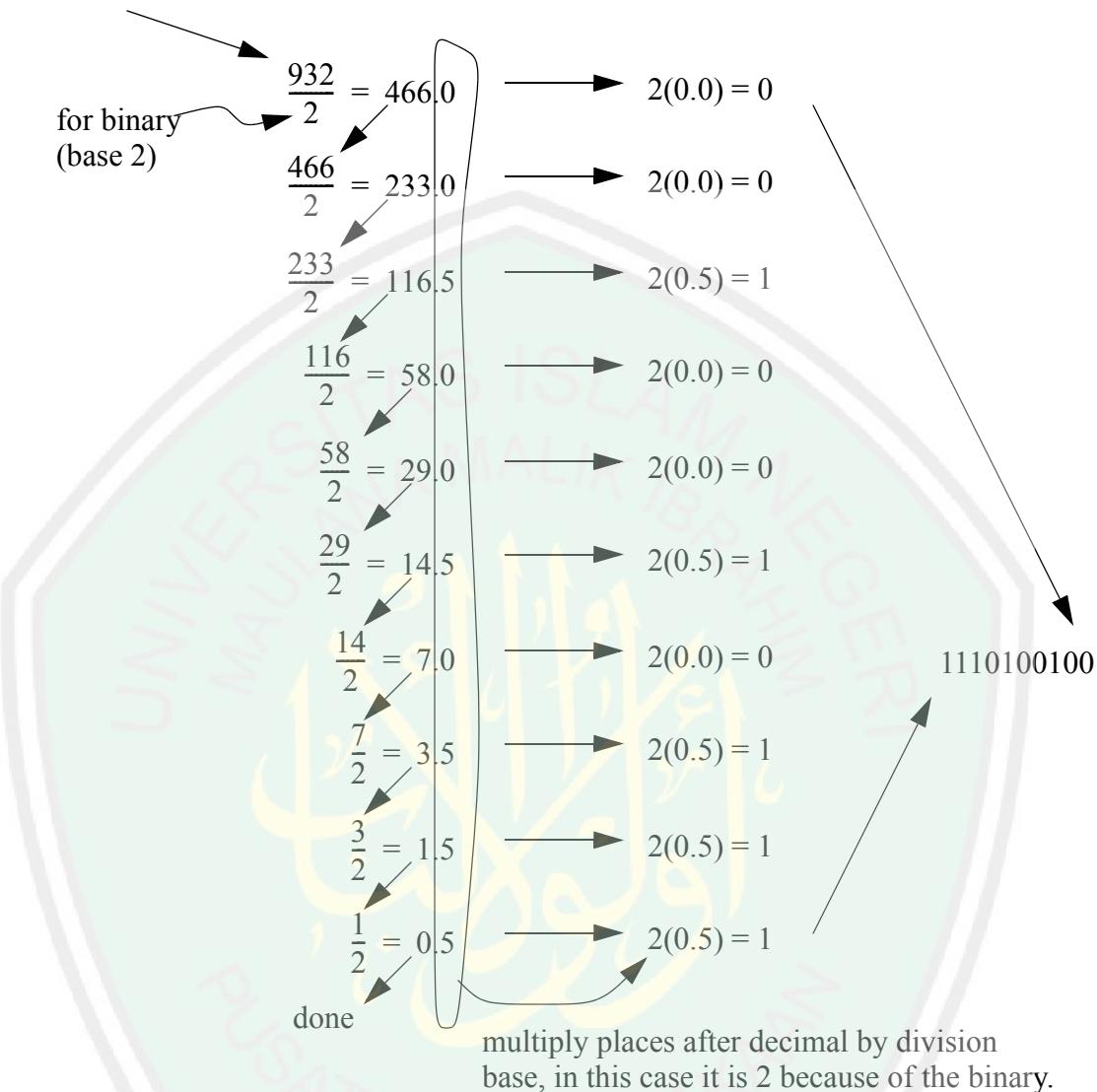


Figure 3.3 Conversion of a Binary Number to a Decimal Number

Decimal numbers can be converted to binary numbers using division, as shown in Figure 3.4. This technique begins by dividing the decimal number by the base of the new number. The fraction after the decimal gives the least significant digit of the new number when it is multiplied by the number base. The whole part of the number is now divided again. This process continues until the whole number is zero. This method will also work for conversion to other number bases.

start with decimal number 932



\* This method works for other number bases also, the divisor and multipliers should be changed to the new number bases.

Figure 3.4 Conversion from Decimal to Binary

Most scientific calculators will convert between number bases. But, it is important to understand the conversions between number bases. And, when used frequently enough the conversions can be done in your head.

Binary numbers come in three basic forms - a bit, a byte and a word. A bit is a single binary digit, a byte is eight binary digits, and a word is 16 digits. Words and bytes are

shown in Figure 3.5. Notice that on both numbers the least significant digit is on the right hand side of the numbers. And, in the word there are two bytes, and the right hand one is the least significant byte.

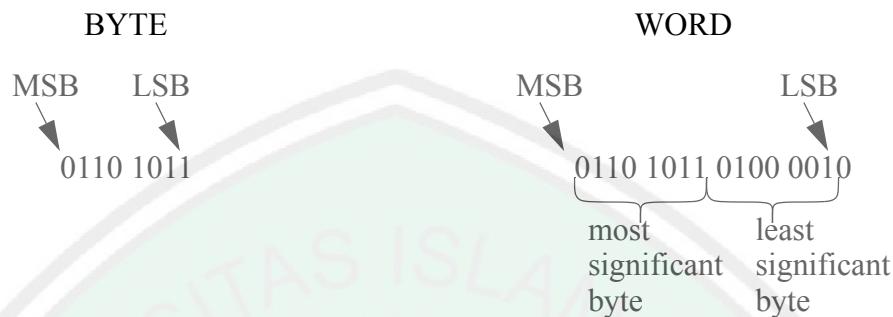


Figure 3.5 Bytes and Words

Binary numbers can also represent fractions, as shown in Figure 3.6. The conversion to and from binary is identical to the previous techniques, except that for values to the right of the decimal the equivalents are fractions.

binary: 101.011

$$\begin{aligned}
 1(2^2) &= 4 & 0(2^1) &= 0 & 1(2^0) &= 1 & 0(2^{-1}) &= 0 & 1(2^{-2}) &= \frac{1}{4} & 1(2^{-3}) &= \frac{1}{8} \\
 &&&&&&&&&&
 \end{aligned}$$

$$= 4 + 0 + 1 + 0 + \frac{1}{4} + \frac{1}{8} = 5.375 \text{ decimal}$$

Figure 3.6 A Binary Decimal Number

### 3.2.1.1 - Boolean Operations

In the next chapter you will learn that entire blocks of inputs and outputs can be used as a single binary number (typically a word). Each bit of the number would correspond to an output or input as shown in Figure 3.7.

There are three motors  $M_1$ ,  $M_2$  and  $M_3$  represented with three bits in a binary number. When any bit is on the corresponding motor is on.

100 = Motor 1 is the only one on

111 = All three motors are on

in total there are  $2^n$  or  $2^3$  possible combinations of motors on.

*Figure 3.7 Motor Outputs Represented with a Binary Number*

We can then manipulate the inputs or outputs using Boolean operations. Boolean algebra has been discussed before for variables with single values, but it is the same for multiple bits. Common operations that use multiple bits in numbers are shown in Figure 3.8. These operations compare only one bit at a time in the number, except the shift instructions that move all the bits one place left or right.

Name	Example	Result
AND	0010 * 1010	0010
OR	0010 + 1010	1010
NOT	0010	1101
EOR	0010 eor 1010	1000
NAND	0010 * 1010	1101
shift left	111000	110001 (other results are possible)
shift right	111000	011100 (other results are possible)
etc.		

*Figure 3.8 Boolean Operations on Binary Numbers*

### 3.2.1.2 - Binary Mathematics

Negative numbers are a particular problem with binary numbers. As a result there are three common numbering systems used as shown in Figure 3.9. Unsigned binary numbers are common, but they can only be used for positive values. Both signed and 2s compliment numbers allow positive and negative values, but the maximum positive values is reduced by half. 2s compliment numbers are very popular because the hardware and software to add and subtract is simpler and faster. All three types of numbers will be found in PLCs.

Type	Description	Range for Byte
unsigned	binary numbers can only have positive values.	0 to 255
signed	the most significant bit (MSB) of the binary number is used to indicate positive/negative.	-127 to 127
2s compliment	negative numbers are represented by complimenting the binary number and then adding 1.	-128 to 127

Figure 3.9 Binary (Integer) Number Types

Examples of signed binary numbers are shown in Figure 3.10. These numbers use the most significant bit to indicate when a number is negative.

decimal	binary byte
2	00000010
1	00000001
0	00000000
-0	10000000
-1	10000001
-2	10000010

Note: there are two zeros

Figure 3.10 Signed Binary Numbers

An example of 2s compliment numbers are shown in Figure 3.11. Basically, if the number is positive, it will be a regular binary number. If the number is to be negative, we start the positive number, compliment it (reverse all the bits), then add 1. Basically when these numbers are negative, then the most significant bit is set. To convert from a negative 2s compliment number, subtract 1, and then invert the number.

decimal	binary byte
2	00000010
1	00000001
0	00000000
-1	11111111
-2	11111110

**METHOD FOR MAKING A NEGATIVE NUMBER**

1. write the binary number for the positive

for -30 we write 30 = 00011110

2. Invert (compliment) the number

00011110 becomes 11100001

3. Add 1

$$11100001 + 00000001 = 11100010$$

Figure 3.11 2s Compliment Numbers

Using 2s compliments for negative numbers eliminates the redundant zeros of signed binaries, and makes the hardware and software easier to implement. As a result most of the integer operations in a PLC will do addition and subtraction using 2s compliment numbers. When adding 2s compliment numbers, we don't need to pay special attention to negative values. And, if we want to subtract one number from another, we apply the twos compliment to the value to be subtracted, and then apply it to the other value.

Figure 3.12 shows the addition of numbers using 2s compliment numbers. The three operations result in zero, positive and negative values. Notice that in all three operation the top number is positive, while the bottom operation is negative (this is easy to see because the MSB of the numbers is set). All three of the additions are using bytes, this is important for considering the results of the calculations. In the left and right hand calculations the additions result in a 9th bit - when dealing with 8 bit numbers we call this bit the carry C. If the calculation started with a positive and negative value, and ended up with a carry bit, there is no problem, and the carry bit should be ignored. If doing the calculation on a calculator you will see the carry bit, but when using a PLC you must look elsewhere to find it.

$$\begin{array}{r}
 00000001 = 1 \\
 + 11111111 = -1 \\
 \hline
 C+00000000 = 0
 \end{array}
 \quad
 \begin{array}{r}
 00000001 = 1 \\
 + 11111110 = -2 \\
 \hline
 11111111 = -1
 \end{array}
 \quad
 \begin{array}{r}
 00000010 = 2 \\
 + 11111111 = -1 \\
 \hline
 C+00000001 = 1
 \end{array}$$

ignore the carry bits

Note: Normally the carry bit is ignored during the operation, but some additional logic is required to make sure that the number has not *overflowed* and moved outside of the range of the numbers. Here the 2s compliment byte can have values from -128 to 127.

Figure 3.12 Adding 2s Compliment Numbers

The integers have limited value ranges, for example a 16 bit word ranges from -32,768 to 32,767. In some cases calculations will give results outside this range, and the Overflow *O* bit will be set. (Note: an overflow condition is a major error, and the PLC will probably halt when this happens.) For an addition operation the Overflow bit will be set when the sign of both numbers is the same, but the sign of the result is opposite. When the signs of the numbers are opposite an overflow cannot occur. This can be seen in Figure 3.13 where the numbers two of the three calculations are outside the range. When this happens the result goes from positive to negative, or the other way.

$$\begin{array}{r}
 01111111 = 127 \\
 + 00000011 = 3 \\
 \hline
 10000010 = -126 \\
 C = 0 \\
 O = 1 \text{ (error)}
 \end{array}
 \quad
 \begin{array}{r}
 10000001 = -127 \\
 + 11111111 = -1 \\
 \hline
 10000000 = -128 \\
 C = 1 \\
 O = 0 \text{ (no error)}
 \end{array}
 \quad
 \begin{array}{r}
 10000001 = -127 \\
 + 11111110 = -2 \\
 \hline
 01111111 = 127 \\
 C = 1 \\
 O = 1 \text{ (error)}
 \end{array}$$

Note: If an overflow bit is set this indicates that a calculation is outside and acceptable range. When this error occurs the PLC will halt. Do not ignore the limitations of the numbers.

Figure 3.13 Carry and Overflow Bits

These bits also apply to multiplication and division operations. In addition the PLC will also have bits to indicate when the result of an operation is zero *Z* and negative *N*.

### 3.2.2 Other Base Number Systems

Other number bases are typically converted to and from binary for storage and mathematical operations. Hexadecimal numbers are popular for representing binary values because they are quite compact compared to binary. (Note: large binary numbers with a long string of 1s and 0s are next to impossible to read.) Octal numbers are also popular for inputs and outputs because they work in counts of eight; inputs and outputs are in counts of eight.

An example of conversion to, and from, hexadecimal is shown in Figure 3.14 and Figure 3.15. Note that both of these conversions are identical to the methods used for binary numbers, and the same techniques extend to octal numbers also.

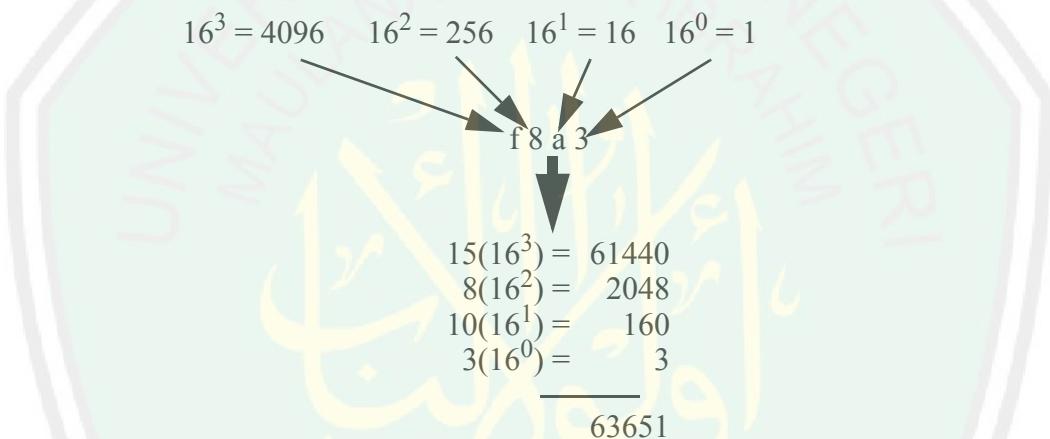
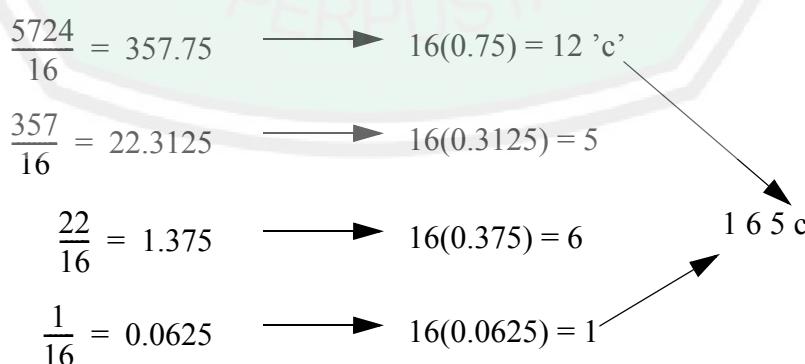


Figure 3.14 Conversion of a Hexadecimal Number to a Decimal Number



*Figure 3.15* Conversion from Decimal to Hexadecimal

### 3.2.3 BCD (Binary Coded Decimal)

Binary Coded Decimal (BCD) numbers use four binary bits (a nibble) for each digit. (Note: this is not a base number system, but it only represents decimal digits.) This means that one byte can hold two digits from 00 to 99, whereas in binary it could hold from 0 to 255. A separate bit must be assigned for negative numbers. This method is very popular when numbers are to be output or input to the computer. An example of a BCD number is shown in Figure 3.16. In the example there are four digits, therefore 16 bits are required. Note that the most significant digit and bits are both on the left hand side. The BCD number is the binary equivalent of each digit.

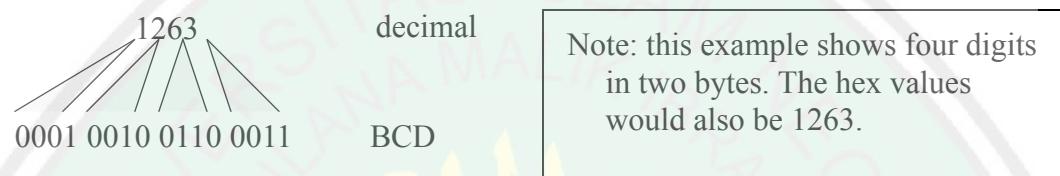


Figure 3.16 A BCD Encoded Number

Most PLCs store BCD numbers in words, allowing values between 0000 and 9999. They also provide functions to convert to and from BCD. It is also possible to calculations with BCD numbers, but this is uncommon, and when necessary most PLCs have functions to do the calculations. But, when doing calculations you should probably avoid BCD and use integer mathematics instead. Try to be aware when your numbers are BCD values and convert them to *integer* or binary value before doing any calculations.

## 3.3 Data Characterization

### 3.3.1 ASCII (American Standard Code for Information Interchange)

When dealing with non-numerical values or data we can use plain text characters and strings. Each character is given a unique identifier and we can use these to store and interpret data. The ASCII (American Standard Code for Information Interchange) is a very common character encryption system is shown in Figure 3.17 and Figure 3.18. The table includes the basic written characters, as well as some special characters, and some control codes. Each one is given a unique number. Consider the letter *A*, it is readily recognized by most computers world-wide when they see the number 65.

decimal	hexadecimal	binary	ASCII	decimal	hexadecimal	binary	ASCII
0	0	00000000	NUL	32	20	00100000	space
1	1	00000001	SOH	33	21	00100001	!
2	2	00000010	STX	34	22	00100010	"
3	3	00000011	ETX	35	23	00100011	#
4	4	00000100	EOT	36	24	00100100	\$
5	5	00000101	ENQ	37	25	00100101	%
6	6	00000110	ACK	38	26	00100110	&
7	7	00000111	BEL	39	27	00100111	'
8	8	00001000	BS	40	28	00101000	(
9	9	00001001	HT	41	29	00101001	)
10	A	00001010	LF	42	2A	00101010	*
11	B	00001011	VT	43	2B	00101011	+
12	C	00001100	FF	44	2C	00101100	,
13	D	00001101	CR	45	2D	00101101	-
14	E	00001110	S0	46	2E	00101110	.
15	F	00001111	S1	47	2F	00101111	/
16	10	00010000	DLE	48	30	00110000	0
17	11	00010001	DC1	49	31	00110001	1
18	12	00010010	DC2	50	32	00110010	2
19	13	00010011	DC3	51	33	00110011	3
20	14	00010100	DC4	52	34	00110100	4
21	15	00010101	NAK	53	35	00110101	5
22	16	00010110	SYN	54	36	00110110	6
23	17	00010111	ETB	55	37	00110111	7
24	18	00011000	CAN	56	38	00111000	8
25	19	00011001	EM	57	39	00111001	9
26	1A	00011010	SUB	58	3A	00111010	:
27	1B	00011011	ESC	59	3B	00111011	;
28	1C	00011100	FS	60	3C	00111100	=
29	1D	00011101	GS	61	3D	00111101	>
30	1E	00011110	RS	62	3E	00111110	^
31	1F	00011111	US	63	3F	00111111	?

Figure 3.17 ASCII Character Table

decimal	hexadecimal	binary	ASCII	decimal	hexadecimal	binary	ASCII
64	40	01000000	@	96	60	01100000	`
65	41	01000001	A	97	61	01100001	a
66	42	01000010	B	98	62	01100010	b
67	43	01000011	C	99	63	01100011	c
68	44	01000100	D	100	64	01100100	d
69	45	01000101	E	101	65	01100101	e
70	46	01000110	F	102	66	01100110	f
71	47	01000111	G	103	67	01100111	g
72	48	01001000	H	104	68	01101000	h
73	49	01001001	I	105	69	01101001	i
74	4A	01001010	J	106	6A	01101010	j
75	4B	01001011	K	107	6B	01101011	k
76	4C	01001100	L	108	6C	01101100	l
77	4D	01001101	M	109	6D	01101101	m
78	4E	01001110	N	110	6E	01101110	n
79	4F	01001111	O	111	6F	01101111	o
80	50	01010000	P	112	70	01110000	p
81	51	01010001	Q	113	71	01110001	q
82	52	01010010	R	114	72	01110010	r
83	53	01010011	S	115	73	01110011	s
84	54	01010100	T	116	74	01110100	t
85	55	01010101	U	117	75	01110101	u
86	56	01010110	V	118	76	01110110	v
87	57	01010111	W	119	77	01110111	w
88	58	01011000	X	120	78	01111000	x
89	59	01011001	Y	121	79	01111001	y
90	5A	01011010	Z	122	7A	01111010	z
91	5B	01011011	[	123	7B	01111011	{
92	5C	01011100	yen	124	7C	01111100	
93	5D	01011101	]	125	7D	01111101	}
94	5E	01011110	^	126	7E	01111110	r arr.
95	5F	01011111	_	127	7F	01111111	l arr.

Figure 3.18 ASCII Character Table

This table has the codes from 0 to 127, but there are more extensive tables that contain special graphics symbols, international characters, etc. It is best to use the basic codes, as they are supported widely, and should suffice for all controls tasks.

An example of a string of characters encoded in ASCII is shown in Figure 3.19.

e.g. The sequence of numbers below will convert to

A	W	e	e
T	e	s	t

A	65
space	32
W	87
e	101
e	101
space	32
T	84
e	101
s	115
t	116

Figure 3.19 A String of Characters Encoded in ASCII

When the characters are organized into a string to be transmitted and *LF* and/or *CR* code are often put at the end to indicate the end of a line. When stored in a computer an ASCII value of zero is used to end the string.

### 3.3.2 Parity

Errors often occur when data is transmitted or stored. This is very important when transmitting data in noisy factories, over phone lines, etc. Parity bits can be added to data as a simple check of transmitted data for errors. If the data contains error it can be retransmitted, or ignored.

A parity bit is normally a 9th bit added onto an 8 bit byte. When the data is encoded the number of true bits are counted. The parity bit is then set to indicate if there are an even or odd number of true bits. When the byte is decoded the parity bit is checked to make sure it that there are an even or odd number of data bits true. If the parity bit is not satisfied, then the byte is judged to be in error. There are two types of parity, even or odd. These are both based upon an even or odd number of data bits being true. The odd parity bit is true if there are an odd number of bits on in a binary number. On the other hand the Even parity is set if there are an even number of true bits. This is illustrated in Figure 3.20.

	data bits	parity bit
Odd Parity	10101110	1
	10111000	0
Even Parity	00101010	0
	10111101	1

Figure 3.20 Parity Bits on a Byte

Parity bits are normally suitable for single bytes, but are not reliable for data with a number of bits.

Note: Control systems perform important tasks that can be dangerous in certain circumstances. If an error occurs there could be serious consequences. As a result error detection methods are very important for control system. When error detection occurs the system should either be *robust* enough to recover from the error, or the system should *fail-safe*. If you ignore these design concepts you will eventually cause an accident.

### 3.3.3 Checksums

Parity bits are suitable for a few bits of data, but checksums are better for larger data transmissions. These are simply an algebraic sum of all of the data transmitted. Before data is transmitted the numeric values of all of the bytes are added. This sum is then transmitted with the data. At the receiving end the data values are summed again, and the total is compared to the checksum. If they match the data is accepted as good. An example of this method is shown in Figure 3.21.

DATA	
124	
43	
255	
9	
27	
47	

CHECKSUM	
505	

Figure 3.21 A Checksum

Checksums are very common in data transmission, but these are also hidden from the average user. If you plan to transmit data to or from a PLC you will need to consider parity and checksum values to verify the data. Small errors in data can have major consequences in received data. Consider an oven temperature transmitted as a binary integer ( $1023d = 0000\ 0100\ 0000\ 0000b$ ). If a single bit were to be changed, and was not detected the temperature might become ( $0000\ 0110\ 0000\ 0000b = 1535d$ ) This small change would dramatically change the process.

### 3.3.4 Gray Code

Parity bits and checksums are for checking data that may have any value. Gray code is used for checking data that must follow a binary sequence. This is common for devices such as angular encoders. The concept is that as the binary number counts up or down, only one bit changes at a time. Thus making it easier to detect erroneous bit changes. An example of a gray code sequence is shown in Figure 3.22. Notice that only one bit changes from one number to the next. If more than a single bit changes between numbers, then an error can be detected.

ASIDE: When the signal level in a wire rises or drops, it induces a magnetic pulse that excites a signal in other nearby lines. This phenomenon is known as *cross-talk*. This signal is often too small to be noticed, but several simultaneous changes, coupled with background noise could result in erroneous values.

decimal	gray code
0	0000
1	0001
2	0011
3	0010
4	0110
5	0111
6	0101
7	0100
8	1100
9	1101
10	1111
11	1110
12	1010
13	1011
14	1001
15	1000

Figure 3.22 Gray Code for a Nibble

### 3.4 Summary

- Binary, octal, decimal and hexadecimal numbers were all discussed.
- 2s compliments allow negative binary numbers.
- BCD numbers encode digits in nibbles.
- ASCII values are numerical equivalents for common alphanumeric characters.
- Gray code, parity bits and checksums can be used for error detection.

### 3.5 Problems

1. Why are binary, octal and hexadecimal used for computer applications?
2. Is a word is 3 nibbles?
3. What are the specific purpose for Gray code and parity?
4. Convert the following numbers to/from binary

a) from base 10: 54,321

b) from base 2: 110000101101

5. Convert the BCD number below to a decimal number,

0110 0010 0111 1001

6. Convert the following binary number to a BCD number,

0100 1011

7. Convert the following binary number to a Hexadecimal value,

0100 1011

8. Convert the following binary number to a octal,

0100 1011

9. Convert the decimal value below to a binary byte, and then determine the odd parity bit,

97

10. Convert the following from binary to decimal, hexadecimal, BCD and octal.

a) 101101

c) 1000000001

b) 11011011

d) 0010110110101

11. Convert the following from decimal to binary, hexadecimal, BCD and octal.

- |    |    |    |       |
|----|----|----|-------|
| a) | 1  | c) | 20456 |
| b) | 17 | d) | -10   |

12. Convert the following from hexadecimal to binary, decimal, BCD and octal.

- |    |    |    |     |
|----|----|----|-----|
| a) | 1  | c) | ABC |
| b) | 17 | d) | -A  |

13. Convert the following from BCD to binary, decimal, hexadecimal and octal.

- |    |           |    |                     |
|----|-----------|----|---------------------|
| a) | 1001      | c) | 0011 0110 0001      |
| b) | 1001 0011 | d) | 0000 0101 0111 0100 |

14. Convert the following from octal to binary, decimal, hexadecimal and BCD.

- |    |    |    |       |
|----|----|----|-------|
| a) | 7  | c) | 777   |
| b) | 17 | d) | 32634 |

15.

- a) Represent the decimal value thumb wheel input, 3532, as a Binary Coded Decimal (BCD) and a Hexadecimal Value (without using a calculator).
- BCD
  - Hexadecimal
- b) What is the corresponding decimal value of the BCD value, 1001111010011011?

16. Add/subtract/multiply/divide the following numbers.

- |                                      |  |
|--------------------------------------|--|
| a) binary 101101101 + 01010101111011 | i) octal 123 - 777                         |
| b) hexadecimal 101 + ABC             | j) 2s complement bytes 10111011 + 00000011 |
| c) octal 123 + 777                   | k) 2s complement bytes 00111011 + 00000011 |
| d) binary 110110111 - 0101111        | l) binary 101101101 * 10101                |
| e) hexadecimal ABC - 123             | m) octal 123 * 777                         |
| f) octal 777 - 123                   | n) octal 777 / 123                         |
| g) binary 0101111 - 110110111        | o) binary 101101101 / 10101                |
| h) hexadecimal 123-ABC               | p) hexadecimal ABC / 123                   |

17. Do the following operations with 8 bit bytes, and indicate the condition of the overflow and carry bits.

- |                        |                         |
|------------------------|-------------------------|
| a) 10111011 + 00000011 | d) 110110111 - 01011111 |
| b) 00111011 + 00000011 | e) 01101011 + 01111011  |
| c) 11011011 + 11011111 | f) 10110110 - 11101110  |

18. Consider the three BCD numbers listed below.

1001 0110 0101 0001  
0010 0100 0011 1000  
0100 0011 0101 0001

- a) Convert these numbers to their decimal values.
- b) Convert the decimal values to binary.
- c) Calculate a checksum for all three binary numbers.
- d) What would the even parity bits be for the binary words found in b).

19. Is the 2nd bit set in the hexadecimal value F49?

20. Explain where grey code occurs when creating Karnaugh maps.

21. Convert the decimal number 1000 to a binary number, and then to hexadecimal.

### 3.6 Problems Solutions

1. base 2, 4, 8, and 16 numbers translate more naturally to the numbers stored in the computer.
2. no, it is four nibbles
3. Both of these are coding schemes designed to increase immunity to noise. A parity bit can be used to check for a changed bit in a byte. Gray code can be used to check for a value error in a stream of continuous values.
4. a) 1101 0100 0011 0001, b) 3117
5. 6279
6. 0111 0101
7. 4B
8. 113

9. 1100001 odd parity bit = 1

10.

binary	101101	11011011	10000000001	0010110110101
BCD	0100 0101	0010 0001 1001	0001 0000 0010 0101	0001 0100 0110 0001
decimal	45	219	1025	1461
hex	2D	5D	401	5B5
octal	55	333	2001	2665

11.

decimal	1	17	20456	-10
BCD	0001	0001 0111	0010 0000 0100 0101 0110	-0001 0000
binary	1	10001	0100 1111 1110 1000	1111 1111 1111 0110
hex	1	11	4FE8	FFF6
octal	1	21	47750	177766

12.

hex	1	17	ABC	-A
BCD	0001	0010 0011	0010 0111 0100 1000	-0001 0000
binary	1	10111	0000 1010 1011 1100	1111 1111 1111 0110
decimal	1	23	2748	-10
octal	1	27	5274	177766

13.

BCD	1001	1001 0011	0011 0110 0001	0000 0101 0111 0100
binary	1001	101 1101	1 0110 1001	10 0011 1110
decimal	9	93	361	0574
hex	9	5D	169	23E
octal	11	135	551	1076

14.

octal	7	17	777	32634
binary	111	1111	1 1111 1111	0011 0101 1001 1100
decimal	7	15	511	13724
hex	7	F	1FF	359C
BCD	0111	0001 0101	0101 0001 0001	0001 0011 0111 0010 0100

15. a)  $3532 = 0011 0101 0011 0010 = \text{DCC}$ , b0 the number is not a valid BCD

16.

- |                        |                        |
|------------------------|------------------------|
| a) 0001 0110 1110 1000 | i) -654                |
| b) BBD                 | j) 0000 0001 0111 1010 |
| c) 1122                | k) 0000 0000 0011 1110 |
| d) 0000 0001 1000 1000 | l) 0001 1101 1111 0001 |
| e) 999                 | m) 122655              |
| f) 654                 | n) 6                   |
| g) 1111 1110 0111 1000 | o) 0000 0000 0001 0001 |
| h) -999                | p) 9                   |

17.

- |  |   |
|--|---|
| a) $10111011 + 00000011 = 10111110$              | d) $110110111 - 01011111 = 01011000 + \text{C+O}$ |
| b) $00111011 + 00000011 = 00111110$              | e) $01101011 + 01111011 = 11100110$               |
| c) $11011011 + 11011111 = 10111010 + \text{C+O}$ | f) $10110110 - 11101110 = 11001000$               |

18. a) 9651, 2438, 4351, b) 0010 0101 1011 0011, 0000 1001 1000 0110, 0001 0000 1111 1111, c) 16440, d) 1, 0, 0

19. The binary value is 1111 0100 1001, so the second bit is 0

20. when selecting the sequence of bit changes for Karnaugh maps, only one bit is changed at a time. This is the same method used for grey code number sequences. By using the code the bits in the map are naturally grouped.

21.

$$1000_{10} = 1111101000_2 = 3e8_{16}$$

### 3.7 Challenge Problems

1.



## 4. TRANSFORMS

\*\*\*\*\* This contains additions and sections by Dr. Andrew Sterian.

Topics:

- 

Objectives:

- 

### 4.1 Laplace Transforms

- The Laplace transform allows us to reverse time. And, as you recall from before the inverse of time is frequency. Because we are normally concerned with response, the Laplace transform is much more useful in system analysis.
- The basic Laplace transform equations is shown below,

$$F(s) = \int_0^{\infty} f(t)e^{-st} dt$$

where,

$f(t)$  = the function in terms of time t

$F(s)$  = the function in terms of the Laplace s

#### 4.1.1 Laplace Transform Tables

- Basic Laplace Transforms for operational transformations are given below,

TIME DOMAIN	FREQUENCY DOMAIN
$Kf(t)$	$Kf(s)$
$f_1(t) + f_2(t) - f_3(t) + \dots$	$f_1(s) + f_2(s) - f_3(s) + \dots$
$\frac{df(t)}{dt}$	$sf(s) - f(0^-)$
$\frac{d^2f(t)}{dt^2}$	$s^2f(s) - sf(0^-) - \frac{df(0^-)}{dt}$
$\frac{d^n f(t)}{dt^n}$	$s^n f(s) - s^{n-1}f(0^-) - s^{n-2}\frac{df(0^-)}{dt} - \dots - \frac{d^n f(0^-)}{dt^n}$
$\int_0^t f(t)dt$	$\frac{f(s)}{s}$
$f(t-a)u(t-a), a > 0$	$e^{-as}f(s)$
$e^{-at}f(t)$	$f(s-a)$
$f(at), a > 0$	$\frac{1}{a}f\left(\frac{s}{a}\right)$
$tf(t)$	$-\frac{df(s)}{ds}$
$t^n f(t)$	$(-1)^n \frac{d^n f(s)}{ds^n}$
$\frac{f(t)}{t}$	$\int_s^\infty f(u)du$

- A set of useful functional Laplace transforms are given below,

TIME DOMAIN	FREQUENCY DOMAIN
$A$	$\frac{A}{s}$
$t$	$\frac{1}{s^2}$
$e^{-at}$	$\frac{1}{s+a}$
$\sin(\omega t)$	$\frac{\omega}{s^2 + \omega^2}$
$\cos(\omega t)$	$\frac{s}{s^2 + \omega^2}$
$te^{-at}$	$\frac{1}{(s+a)^2}$
$e^{-at} \sin(\omega t)$	$\frac{\omega}{(s+a)^2 + \omega^2}$
$e^{-at} \cos(\omega t)$	$\frac{s+a}{(s+a)^2 + \omega^2}$
$Ae^{-at}$	$\frac{A}{s-a}$
$Ate^{-at}$	$\frac{A}{(s-a)^2}$
$2 A e^{-\alpha t} \cos(\beta t + \theta)$	$\frac{A}{s+\alpha-\beta j} + \frac{A^{\text{complex conjugate}}}{s+\alpha+\beta j}$
$2t A e^{-\alpha t} \cos(\beta t + \theta)$	$\frac{A}{(s+\alpha-\beta j)^2} + \frac{A^{\text{complex conjugate}}}{(s+\alpha+\beta j)^2}$

- Laplace transforms can be used to solve differential equations.

## 4.2 z-Transforms

- For a discrete-time signal  $x[n]$ , the two-sided z-transform is defined by  $X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$ .

The one-sided z-transform is defined by  $X(z) = \sum_{n=0}^{\infty} x[n]z^{-n}$ . In both cases, the z-transform is a polynomial in the complex variable  $z$ .

- The inverse z-transform is obtained by contour integration in the complex plane  $x[n] = \frac{1}{j2\pi} \oint X(z)z^{n-1} dz$ . This is usually avoided by partial fraction inversion techniques, similar to the Laplace transform.
- Along with a z-transform we associate its region of convergence (or ROC). These are the values of  $z$  for which  $X(z)$  is bounded (i.e., of finite magnitude).

- Some common z-transforms are shown below.

**Table 1: Common z-transforms**

Signal $x[n]$	z-Transform $X(z)$	ROC
$\delta[n]$	1	All $z$
$u[n]$	$\frac{1}{1 - z^{-1}}$	$ z  > 1$
$nu[n]$	$\frac{z^{-1}}{(1 - z^{-1})^2}$	$ z  > 1$
$n^2 u[n]$	$\frac{z^{-1}(1 + z^{-1})}{(1 - z^{-1})^3}$	$ z  > 1$
$a^n u[n]$	$\frac{1}{1 - az^{-1}}$	$ z  >  a $
$na^n u[n]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$ z  >  a $
$(-a^n)u[-n-1]$	$\frac{1}{1 - az^{-1}}$	$ z  <  a $
$(-na^n)u[-n-1]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$ z  <  a $
$\cos(\omega_0 n)u[n]$	$\frac{1 - z^{-1} \cos \omega_0}{1 - 2z^{-1} \cos \omega_0 + z^{-2}}$	$ z  > 1$
$\sin(\omega_0 n)u[n]$	$\frac{z^{-1} \sin \omega_0}{1 - 2z^{-1} \cos \omega_0 + z^{-2}}$	$ z  > 1$
$a^n \cos(\omega_0 n)u[n]$	$\frac{1 - az^{-1} \cos \omega_0}{1 - 2az^{-1} \cos \omega_0 + a^2 z^{-2}}$	$ z  >  a $

**Table 1: Common z-transforms**

Signal $x[n]$	z-Transform $X(z)$	ROC
$a^n \sin(\omega_0 n) u[n]$	$\frac{az^{-1} \sin \omega_0}{1 - 2az^{-1} \cos \omega_0 + a^2 z^{-2}}$	$ z  >  a $
$\frac{n!}{k!(n-k)!} u[n]$	$\frac{z^{-k}}{(1-z^{-1})^{k+1}}$	$ z  > 1$

- The z-transform also has various properties that are useful. The table below lists properties for the two-sided z-transform. The one-sided z-transform properties can be derived from the ones below by considering the signal  $x[n]u[n]$  instead of simply  $x[n]$ .

**Table 2: Two-sided z-Transform Properties**

Property	Time Domain	z-Domain	ROC
Notation	$x[n]$ $x_1[n]$ $x_2[n]$	$X(z)$ $X_1(z)$ $X_2(z)$	$r_2 <  z  < r_1$ $ROC_1$ $ROC_2$
Linearity	$\alpha x_1[n] + \beta x_2[n]$	$\alpha X_1(z) + \beta X_2(z)$	At least the intersection of $ROC_1$ and $ROC_2$
Time Shifting	$x[n-k]$	$z^{-k} X(z)$	That of $X(z)$ , except $z = 0$ if $k > 0$ and $z = \infty$ if $k < 0$
z-Domain Scaling	$a^n x[n]$	$X(a^{-1}z)$	$ a r_2 <  z  <  a r_1$
Time Reversal	$x[-n]$	$X(z^{-1})$	$\frac{1}{r_1} <  z  < \frac{1}{r_2}$
z-Domain Differentiation	$nx[n]$	$-z \frac{dX(z)}{dz}$	$r_2 <  z  < r_1$

**Table 2: Two-sided z-Transform Properties**

Property	Time Domain	z-Domain	ROC
Convolution	$x_1[n]*x_2[n]$	$X_1(z)X_2(z)$	At least the intersection of $ROC_1$ and $ROC_2$
Multiplication	$x_1[n]x_2[n]$	$\frac{1}{j2\pi} \oint X_1(v)X_2\left(\frac{z}{v}\right)v^{-1}dv$	At least $r_{1l}r_{2l} <  z  < r_{1u}r_{2u}$
Initial value theorem	$x[n]$ causal	$x[0] = \lim_{z \rightarrow \infty} X(z)$	

### 4.3 Fourier Series

- These series describe functions by their frequency spectrum content. For example a square wave can be approximated with a sum of a series of sine waves with varying magnitudes.
- The basic definition of the Fourier series is given below.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[ a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right) \right]$$

$$a_n = \frac{1}{L} \int_{-L}^L f(x) \cos\left(\frac{n\pi x}{L}\right) dx \quad b_n = \frac{1}{L} \int_{-L}^L f(x) \sin\left(\frac{n\pi x}{L}\right) dx$$

### 4.4 Problems

8a. Find  $y(t)$ .

$$\frac{y(s)}{x(s)} = \frac{s^2 + 4s}{s^2 + 6s + 9} \quad x(t) = 5$$

## 4.5 Challenge Problems



## 5. GEOMETRY

Topics:

- 

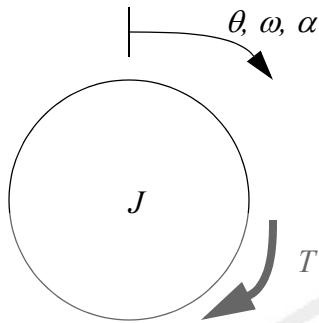
Objectives:

- 

### 5.1 Introduction

#### 5.1.1 Inertia

When unbalanced torques are applied to a mass it will begin to accelerate, in rotation. The sum of applied torques is equal to the inertia forces shown in Figure 1.23.



$$\sum T = J_M \alpha \quad (6)$$

$$J_M = I_{xx} + I_{yy} \quad (7)$$

$$I_{xx} = \int y^2 dM \quad (8)$$

$$I_{yy} = \int x^2 dM \quad (9)$$

*Note: The 'mass' moment of inertia will be used when dealing with acceleration of a mass. Later we will use the 'area' moment of inertia for torsional springs.*

Figure 1.23 Summing moments and angular inertia

The mass moment of inertia determines the resistance to acceleration. This can be calculated using integration, or found in tables. When dealing with rotational acceleration it is important to use the mass moment of inertia, not the area moment of inertia.

The center of rotation for free body rotation will be the centroid. Moment of inertia values are typically calculated about the centroid. If the object is constrained to rotate about some point, other than the centroid, the moment of inertia value must be recalculated. The parallel axis theorem provides the method to shift a moment of inertia from a centroid to an arbitrary center of rotation, as shown in Figure 1.24.

$$J_M = \tilde{J}_M + Mr^2$$

where,

$J_M$  = mass moment about the new point

$\tilde{J}_M$  = mass moment about the center of mass

$M$  = mass of the object

$r$  = distance from the centroid to the new point

Figure 1.24 Parallel axis theorem for shifting a mass moment of inertia

$$J_A = \tilde{J}_A + Ar^2$$

where,

$J_A$  = area moment about the new point

$\tilde{J}_A$  = area moment about the centroid

$A$  = mass of the object

$r$  = distance from the centroid to the new point

Figure 1.25 Parallel axis theorem for shifting a area moment of inertia

*Aside: If forces do not pass through the center of an object, it will rotate. If the object is made of a homogeneous material, the area and volume centroids can be used as the center. If the object is made of different materials then the center of mass should be used for the center. If the gravity varies over the length of the (very long) object then the center of gravity should be used.*

An example of calculating a mass moment of inertia is shown in Figure 1.26. In this problem the density of the material is calculated for use in the integrals. The integrals are then developed using slices for the integration element  $dM$ . The integrals for the moments about the  $x$  and  $y$  axes, are then added to give the polar moment of inertia. This is then shifted from the centroid to the new axis using the parallel axis theorem.

The rectangular shape to the right is constrained to rotate about point A. The total mass of the object is 10kg. The given dimensions are in meters. Find the mass moment of inertia.

First find the density and calculate the moments of inertia about the centroid.

$$\rho = \frac{10\text{Kg}}{2(5\text{m})2(4\text{m})} = 0.125\text{Kgm}^{-2}$$

$$I_{xx} = \int_{-4}^4 y^2 dM = \int_{-4}^4 y^2 \rho 2(5\text{m}) dy = 1.25\text{Kgm}^{-1} \frac{y^3}{3} \Big|_{-4}^4$$

$$\therefore = 1.25\text{Kgm}^{-1} \left( \frac{(4\text{m})^3}{3} - \frac{(-4\text{m})^3}{3} \right) = 53.33\text{Kgm}^2$$

$$I_{yy} = \int_{-5}^5 x^2 dM = \int_{-5}^5 x^2 \rho 2(4\text{m}) dx = 1\text{Kgm}^{-1} \frac{x^3}{3} \Big|_{-5}^5$$

$$\therefore = 1\text{Kgm}^{-1} \left( \frac{(5\text{m})^3}{3} - \frac{(-5\text{m})^3}{3} \right) = 83.33\text{Kgm}^2$$

$$J_M = I_{xx} + I_{yy} = 53.33\text{Kgm}^2 + 83.33\text{Kgm}^2 = 136.67\text{Kgm}^2$$

The centroid can now be shifted to the center of rotation using the parallel axis theorem.

$$J_M = \tilde{J}_M + Mr^2 = 136.67\text{Kgm}^2 + (10\text{Kg})((-2.5\text{m})^2 + (-1\text{m})^2) = 209.2\text{Kgm}^2$$

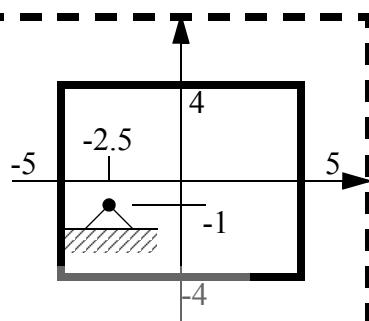
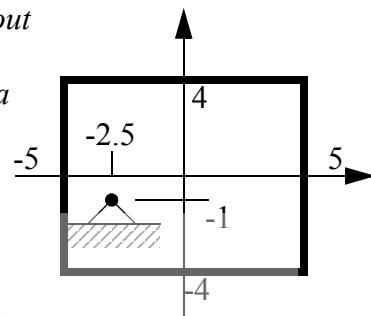


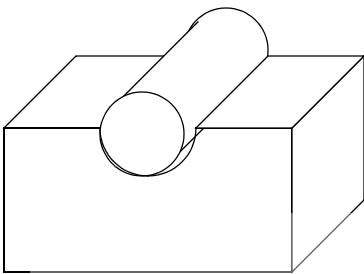
Figure 1.26 Mass moment of inertia example

The rectangular shape to the right is constrained to rotate about point A. The total mass of the object is 10kg. The given dimensions are in meters. Find the mass moment of inertia WITHOUT using the parallel axis theorem.



<i>ans.</i>	$I_{M_x} = 66.33 \text{Kgm}^2$
	$I_{M_y} = 145.8 \text{Kgm}^2$
	$J_M = 209.2 \text{Kgm}^2$

Figure 1.27 Drill problem: Mass moment of inertia calculation



The 20cm diameter 10 kg cylinder to the left is sitting in a depression that is effectively frictionless. If a torque of 10 Nm is applied for 5 seconds, what will the angular velocity be?

$$\text{ans. } \theta(5s) = 312.5 \text{ rad}$$
$$\omega(5s) = 125 \frac{\text{rad}}{\text{s}}$$

Figure 1.28 Drill problem: Find the velocity of the rotating shaft

- A set of the basic 2D and 3D geometric primitives are given, and the notation used is described below,

$A$  = contained area

$P$  = perimeter distance

$V$  = contained volume

$S$  = surface area

$x, y, z$  = center of mass

$\bar{x}, \bar{y}, \bar{z}$  = centroid

$I_x, I_y, I_z$  = moment of inertia of area (or second moment of inertia)

#### AREA PROPERTIES:

$$I_x = \int_A y^2 dA = \text{the moment of inertia about the y-axis}$$

$$I_y = \int_A x^2 dA = \text{the moment of inertia about the x-axis}$$

$$I_{xy} = \int_A xy dA = \text{the product of inertia}$$

$$J_O = \int_A r^2 dA = \int_A (x^2 + y^2) dA = I_x + I_y = \text{The polar moment of inertia}$$

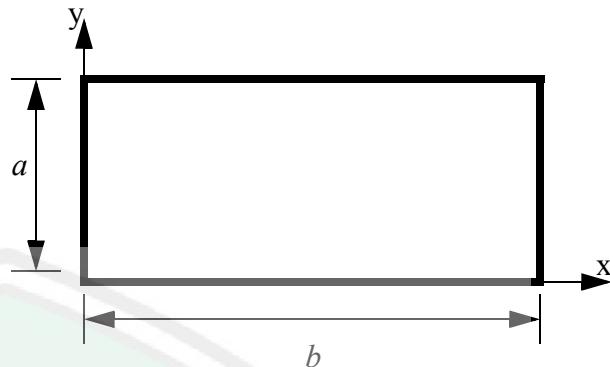
$$\bar{x} = \frac{\int_A x dA}{\int_A dA} = \text{centroid location along the x-axis}$$

$$\bar{y} = \frac{\int_A y dA}{\int_A dA} = \text{centroid location along the y-axis}$$

Rectangle/Square:

$$A = ab$$

$$P = 2a + 2b$$



Centroid:

$$\bar{x} = \frac{b}{2}$$

$$\bar{y} = \frac{a}{2}$$

Moment of Inertia  
(about centroid axes):

$$\bar{I}_x = \frac{ba^3}{12}$$

$$\bar{I}_y = \frac{b^3a}{12}$$

$$\bar{I}_{xy} = 0$$

Moment of Inertia  
(about origin axes):

$$I_x = \frac{ba^3}{3}$$

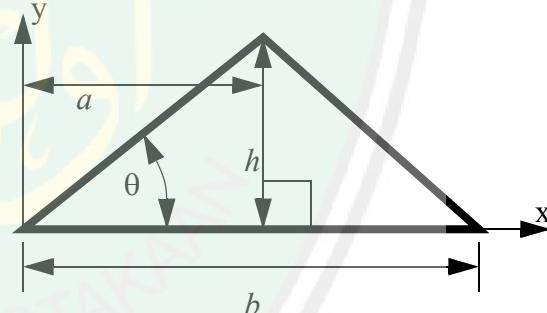
$$I_y = \frac{b^3a}{3}$$

$$I_{xy} = \frac{b^2a^2}{4}$$

Triangle:

$$A = \frac{bh}{2}$$

$$P =$$



Centroid:

$$\bar{x} = \frac{a+b}{3}$$

$$\bar{y} = \frac{h}{3}$$

Moment of Inertia  
(about centroid axes):

$$\bar{I}_x = \frac{bh^3}{36}$$

$$\bar{I}_y = \frac{bh}{36}(a^2 + b^2 - ab)$$

$$\bar{I}_{xy} = \frac{bh^2}{72}(2a - b)$$

Moment of Inertia  
(about origin axes):

$$I_x = \frac{bh^3}{12}$$

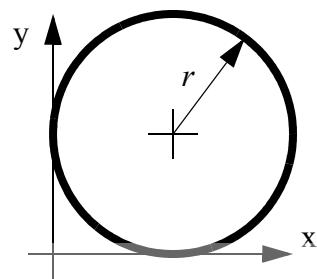
$$I_y = \frac{bh}{12}(a^2 + b^2 - ab)$$

$$I_{xy} = \frac{bh^2}{24}(2a - b)$$

Circle:

$$A = \pi r^2$$

$$P = 2\pi r$$



Centroid:

$$\bar{x} = r$$

$$\bar{y} = r$$

$$\bar{I}_{xy} = 0$$

Moment of Inertia  
(about centroid axes):

$$\bar{I}_x = \frac{\pi r^4}{4}$$

$$\bar{I}_y = \frac{\pi r^4}{4}$$

Moment of Inertia  
(about origin axes):

$$I_x =$$

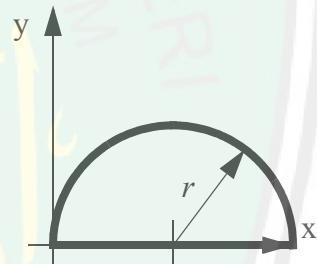
$$I_y =$$

$$I_{xy} =$$

Half Circle:

$$A = \frac{\pi r^2}{2}$$

$$P = \pi r + 2r$$



Centroid:

$$\bar{x} = r$$

$$\bar{y} = \frac{4r}{3\pi}$$

$$I_{xy} = 0$$

Moment of Inertia  
(about centroid axes):

$$\bar{I}_x = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right) r^4$$

$$\bar{I}_y = \frac{\pi r^4}{8}$$

Moment of Inertia  
(about origin axes):

$$I_x = \frac{\pi r^4}{8}$$

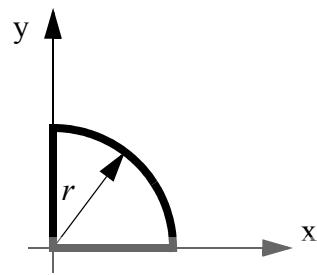
$$I_y = \frac{\pi r^4}{8}$$

$$I_{xy} = 0$$

Quarter Circle:

$$A = \frac{\pi r^2}{4}$$

$$P = \frac{\pi r}{2} + 2r$$



Centroid:

$$\bar{x} = \frac{4r}{3\pi}$$

$$\bar{y} = \frac{4r}{3\pi}$$

Moment of Inertia  
(about centroid axes):

$$\bar{I}_x = 0.05488r^4$$

$$\bar{I}_y = 0.05488r^4$$

$$\bar{I}_{xy} = -0.01647r^4$$

Moment of Inertia  
(about origin axes):

$$I_x = \frac{\pi r^4}{16}$$

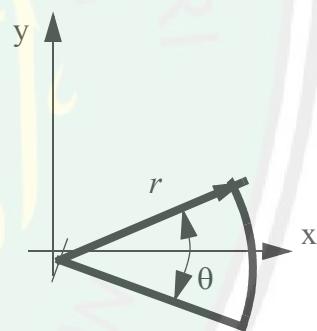
$$I_y = \frac{\pi r^4}{16}$$

$$I_{xy} = \frac{r^4}{8}$$

Circular Arc:

$$A = \frac{\theta r^2}{2}$$

$$P = \theta r + 2r$$



Centroid:

$$\bar{x} = \frac{2r \sin \frac{\theta}{2}}{3\theta}$$

$$\bar{y} = 0$$

Moment of Inertia  
(about centroid axes):

$$\bar{I}_x =$$

$$\bar{I}_y =$$

$$\bar{I}_{xy} =$$

Moment of Inertia  
(about origin axes):

$$I_x = \frac{r^4}{8}(\theta - \sin \theta)$$

$$I_y = \frac{r^4}{8}(\theta + \sin \theta)$$

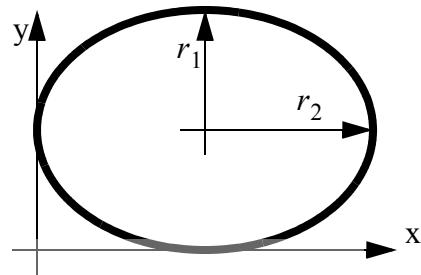
$$I_{xy} = 0$$

Ellipse:

$$A = \pi r_1 r_2$$

$$P = 4r_1 \int_0^{\frac{\pi}{2}} \sqrt{1 - \frac{\sqrt{r_1^2 + r_2^2}}{a} (\sin \theta)^2} d\theta$$

$$P \approx 2\pi \sqrt{\frac{r_1^2 + r_2^2}{2}}$$



Centroid:

$$\bar{x} = r_2$$

$$\bar{y} = r_1$$

Moment of Inertia  
(about centroid axes):

$$\bar{I}_x = \frac{\pi r_1^3 r_2}{4}$$

$$\bar{I}_y = \frac{\pi r_1 r_2^3}{4}$$

$$\bar{I}_{xy} =$$

Moment of Inertia  
(about origin axes):

$$I_x =$$

$$I_y =$$

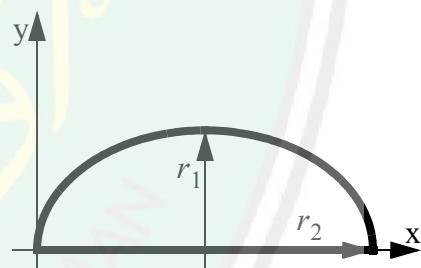
$$I_{xy} =$$

Half Ellipse:

$$A = \frac{\pi r_1 r_2}{2}$$

$$P = 2r_1 \int_0^{\frac{\pi}{2}} \sqrt{1 - \frac{\sqrt{r_1^2 + r_2^2}}{a} (\sin \theta)^2} d\theta + 2r_2$$

$$P \approx \pi \sqrt{\frac{r_1^2 + r_2^2}{2}} + 2r_2$$



Centroid:

$$\bar{x} = r_2$$

$$\bar{y} = \frac{4r_1}{3\pi}$$

Moment of Inertia  
(about centroid axes):

$$\bar{I}_x = 0.05488 r_2 r_1^3$$

$$\bar{I}_y = 0.05488 r_2^3 r_1$$

$$\bar{I}_{xy} = -0.01647 r_1^2 r_2^2$$

Moment of Inertia  
(about origin axes):

$$I_x = \frac{\pi r_2 r_1^3}{16}$$

$$I_y = \frac{\pi r_2^3 r_1}{16}$$

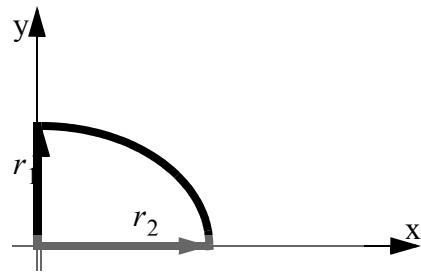
$$I_{xy} = \frac{r_1^2 r_2^2}{8}$$

Quarter Ellipse:

$$A = \frac{\pi r_1 r_2}{4}$$

$$P = r_1 \int_0^{\frac{\pi}{2}} \sqrt{1 - \frac{r_1^2 + r_2^2}{a^2} (\sin \theta)^2} d\theta + 2r_2$$

$$P \approx \frac{\pi}{2} \sqrt{\frac{r_1^2 + r_2^2}{2}} + 2r_2$$



Centroid:

$$\bar{x} = \frac{4r_2}{3\pi}$$

$$\bar{y} = \frac{4r_1}{3\pi}$$

Moment of Inertia  
(about centroid axes):

$$\bar{I}_x =$$

$$\bar{I}_y =$$

$$\bar{I}_{xy} =$$

Moment of Inertia  
(about origin axes):

$$I_x = \pi r_2 r_1^3$$

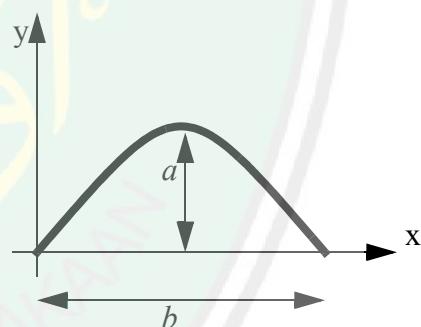
$$I_y = \pi r_2^3 r_1$$

$$I_{xy} = \frac{r_2^2 r_1^2}{8}$$

Parabola:

$$A = \frac{2}{3}ab$$

$$P = \frac{\sqrt{b^2 + 16a^2}}{2} + \frac{b^2}{8a} \ln\left(\frac{4a + \sqrt{b^2 + 16a^2}}{b}\right)$$



Centroid:

$$\bar{x} = \frac{b}{2}$$

$$\bar{y} = \frac{2a}{5}$$

Moment of Inertia  
(about centroid axes):

$$\bar{I}_x =$$

$$\bar{I}_y =$$

$$\bar{I}_{xy} =$$

Moment of Inertia  
(about origin axes):

$$I_x =$$

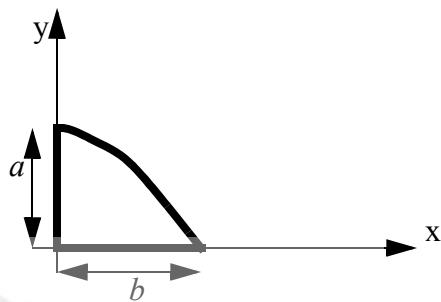
$$I_y =$$

$$I_{xy} =$$

Half Parabola:

$$A = \frac{ab}{3}$$

$$P = \frac{\sqrt{b^2 + 16a^2}}{4} + \frac{b^2}{16a} \ln\left(\frac{4a + \sqrt{b^2 + 16a^2}}{b}\right)$$



Centroid:

$$\bar{x} = \frac{3b}{8}$$

$$\bar{y} = \frac{2a}{5}$$

Moment of Inertia  
(about centroid axes):

$$\bar{I}_x = \frac{8ba^3}{175}$$

$$\bar{I}_y = \frac{19b^3a}{480}$$

$$\bar{I}_{xy} = \frac{b^2a^2}{60}$$

Moment of Inertia  
(about origin axes):

$$I_x = \frac{2ba^3}{7}$$

$$I_y = \frac{2b^3a}{15}$$

$$I_{xy} = \frac{b^2a^2}{6}$$

- A general class of geometries are conics. This form is shown below, and can be used to represent many of the simple shapes represented by a polynomial.

$$Ax^2 + 2Bxy + Cy^2 + 2Dx + 2Ey + F = 0$$

Conditions

$$A = B = C = 0$$

$$B = 0, A = C$$

$$B^2 - AC < 0$$

$$B^2 - AC = 0$$

$$B^2 - AC > 0$$

straight line

circle

ellipse

parabola

hyperbola

**VOLUME PROPERTIES:**

$$I_x = \int_V r_x^2 dV = \text{the moment of inertia about the x-axis}$$

$$I_y = \int_V r_y^2 dV = \text{the moment of inertia about the y-axis}$$

$$I_z = \int_V r_z^2 dV = \text{the moment of inertia about the z-axis}$$

$$\bar{x} = \frac{\int_V x dV}{\int_V dV} = \text{centroid location along the x-axis}$$

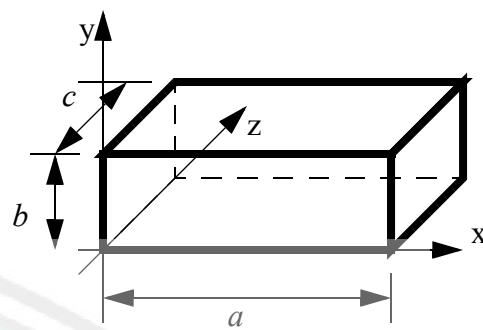
$$\bar{y} = \frac{\int_V y dV}{\int_V dV} = \text{centroid location along the y-axis}$$

$$\bar{z} = \frac{\int_V z dV}{\int_V dV} = \text{centroid location along the z-axis}$$

Parallelepiped (box):

$$V = abc$$

$$S = 2(ab + ac + bc)$$



Centroid:

$$\bar{x} = \frac{a}{2}$$

$$\bar{y} = \frac{b}{2}$$

$$\bar{z} = \frac{c}{2}$$

Moment of Inertia  
(about centroid axes):

$$\bar{I}_x = \frac{M(a^2 + b^2)}{12}$$

$$\bar{I}_y = \frac{M(a^2 + c^2)}{12}$$

$$\bar{I}_z = \frac{M(b^2 + c^2)}{12}$$

Moment of Inertia  
(about origin axes):

$$I_x =$$

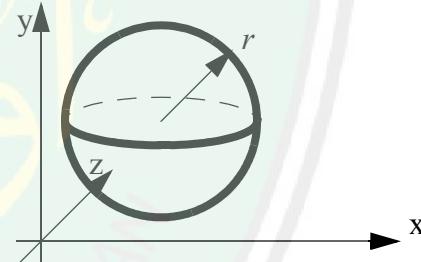
$$I_y =$$

$$I_z =$$

Sphere:

$$V = \frac{4}{3}\pi r^3$$

$$S = 4\pi r^2$$



Centroid:

$$\bar{x} = r$$

$$\bar{y} = r$$

$$\bar{z} = r$$

Moment of Inertia  
(about centroid axes):

$$\bar{I}_x = \frac{2Mr^2}{5}$$

$$\bar{I}_y = \frac{2Mr^2}{5}$$

$$\bar{I}_z = \frac{2Mr^2}{5}$$

Moment of Inertia  
(about origin axes):

$$I_x =$$

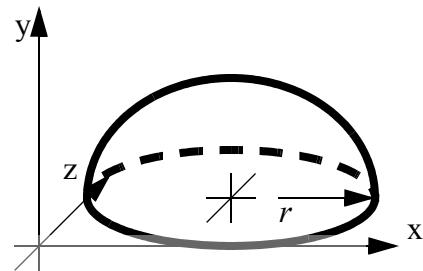
$$I_y =$$

$$I_z =$$

Hemisphere:

$$V = \frac{2}{3}\pi r^3$$

$$S =$$



Centroid:

$$\bar{x} = r$$

$$\bar{y} = \frac{3r}{8}$$

$$\bar{z} = r$$

Moment of Inertia  
(about centroid axes):

$$\bar{I}_x = \frac{83}{320}Mr^2$$

$$\bar{I}_y = \frac{2Mr^2}{5}$$

$$\bar{I}_z = \frac{83}{320}Mr^2$$

Moment of Inertia  
(about origin axes):

$$I_x =$$

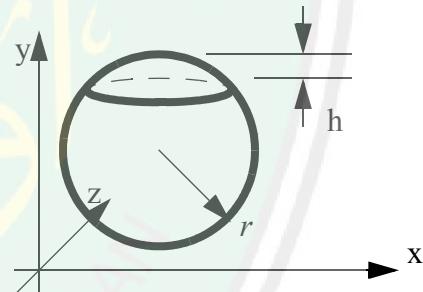
$$I_y =$$

$$I_z =$$

Cap of a Sphere:

$$V = \frac{1}{3}\pi h^2(3r - h)$$

$$S = 2\pi rh$$



Centroid:

$$\bar{x} = r$$

$$\bar{y} =$$

$$\bar{z} = r$$

Moment of Inertia  
(about centroid axes):

$$\bar{I}_x =$$

$$\bar{I}_y =$$

$$\bar{I}_z =$$

Moment of Inertia  
(about origin axes):

$$I_x =$$

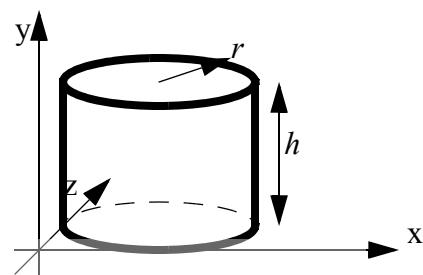
$$I_y =$$

$$I_z =$$

Cylinder:

$$V = h\pi r^2$$

$$S = 2\pi rh + 2\pi r^2$$



Centroid:

$$\bar{x} = r$$

$$\bar{y} = \frac{h}{2}$$

$$\bar{z} = r$$

Moment of Inertia  
(about centroid axis):

$$\bar{I}_x = M\left(\frac{h^2}{12} + \frac{r^2}{4}\right)$$

$$\bar{I}_y = \frac{Mr^2}{2}$$

$$\bar{I}_z = M\left(\frac{h^2}{12} + \frac{r^2}{4}\right)$$

Moment of Inertia  
(about origin axis):

$$I_x = M\left(\frac{h^2}{3} + \frac{r^2}{4}\right)$$

$$I_y =$$

$$I_z = M\left(\frac{h^2}{3} + \frac{r^2}{4}\right)$$

Cone:

$$V = \frac{1}{3}\pi r^2 h$$

$$S = \pi r \sqrt{r^2 + h^2}$$



Centroid:

$$\bar{x} = r$$

$$\bar{y} = \frac{h}{4}$$

$$\bar{z} = r$$

Moment of Inertia  
(about centroid axes):

$$\bar{I}_x = M\left(\frac{3h^3}{80} + \frac{3r^2}{20}\right)$$

$$\bar{I}_y = \frac{3Mr^2}{10}$$

$$\bar{I}_z = M\left(\frac{3h^3}{80} + \frac{3r^2}{20}\right)$$

Moment of Inertia  
(about origin axes):

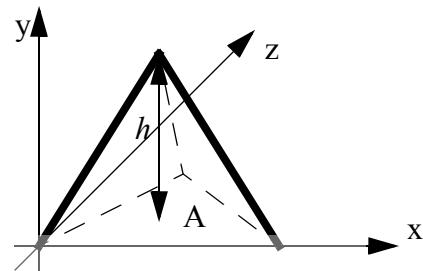
$$I_x =$$

$$I_y =$$

$$I_z =$$

Tetrahedron:

$$V = \frac{1}{3}Ah$$



Centroid:

$$\bar{x} =$$

$$\bar{y} = \frac{h}{4}$$

$$\bar{z} =$$

Moment of Inertia  
(about centroid axes):

$$\bar{I}_x =$$

$$\bar{I}_y =$$

$$\bar{I}_z =$$

Moment of Inertia  
(about origin axes):

$$I_x =$$

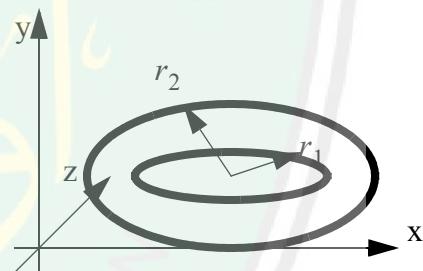
$$I_y =$$

$$I_z =$$

Torus:

$$V = \frac{1}{4}\pi^2(r_1 + r_2)(r_2 - r_1)^2$$

$$S = \pi^2(r_2^2 - r_1^2)$$



Centroid:

$$\bar{x} = r_2$$

$$\bar{y} = \left(\frac{r_2 - r_1}{2}\right)$$

$$\bar{z} = r_2$$

Moment of Inertia  
(about centroid axes):

$$\bar{I}_x =$$

$$\bar{I}_y =$$

$$\bar{I}_z =$$

Moment of Inertia  
(about origin axes):

$$I_x =$$

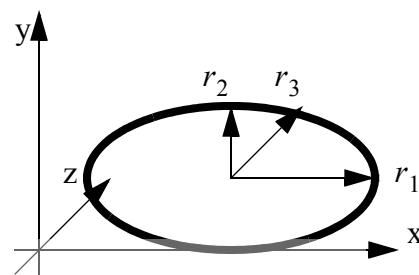
$$I_y =$$

$$I_z =$$

Ellipsoid:

$$V = \frac{4}{3}\pi r_1 r_2 r_3$$

$$S =$$



Centroid:

$$\bar{x} = r_1$$

$$\bar{y} = r_2$$

$$\bar{z} = r_3$$

Moment of Inertia  
(about centroid axes):

$$\bar{I}_x =$$

$$\bar{I}_y =$$

$$\bar{I}_z =$$

Moment of Inertia  
(about origin axes):

$$I_x =$$

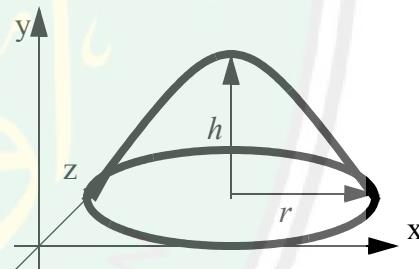
$$I_y =$$

$$I_z =$$

Paraboloid:

$$V = \frac{1}{2}\pi r^2 h$$

$$S =$$



Centroid:

$$\bar{x} = r$$

$$\bar{y} =$$

$$\bar{z} = r$$

Moment of Inertia  
(about centroid axes):

$$\bar{I}_x =$$

$$\bar{I}_y =$$

$$\bar{I}_z =$$

Moment of Inertia  
(about origin axes):

$$I_x =$$

$$I_y =$$

$$I_z =$$

## 5.2 Problems

### 5.2.1 Challenge Problems



## 6. FINANCIAL

Topics:

- 

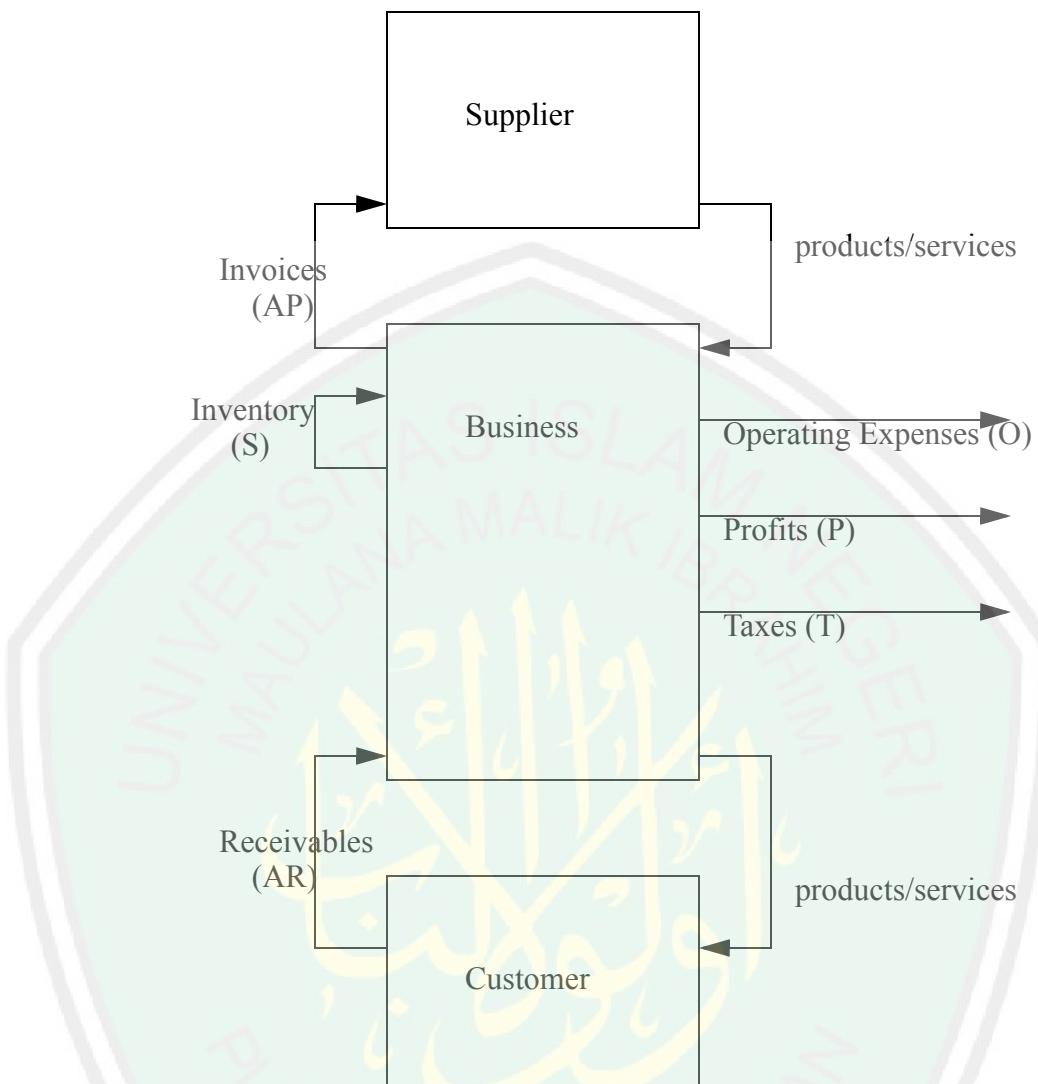
Objectives:

- 

### 6.1 Introduction

- The primary object of a public corporation is to generate the maximum amount of profit possi-

ble. A simple model is shown below.



$$T = 0.5P \quad (\text{assumes 50\% tax on profit})$$

$$\sum = AR - AP - O - P - T = 0$$

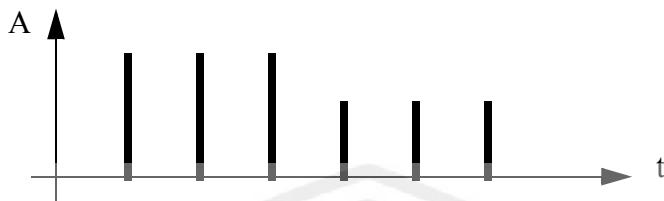
$$\therefore AR - AP - O - P - 0.5P = 0$$

$$\therefore P = \frac{AR - AP - O}{1.5}$$

- Operating expenses include a number of factors such as,
  - Depreciation on capital equipment

- Wages/salaries

- When cash flow occurs over a number of years it is often shown with cash flow diagrams.



- When considering the economic value of a decision, one method is the payback period.

$$N = \frac{C_I}{S_A}$$

where,

$C_I$  = initial investment (\$)

$S_A$  = savings per year (\$/yr)

$N$  = payback period (years)

- Simple estimates for the initial investment and yearly savings are,

$$C_I = C_E - I_S$$

where,

$C_E$  = cost of new equipment

$I_S$  = revenue from sale of old equipment (salvage)

$$S_A = (L_0 H_0 - L_1 H_1) + (M_0 - M_1)$$

where,

$L_0, L_1$  = labor rate before and after

$H_0, H_1$  = labor hours before and after

$M_0, M_1$  = maintenance costs before and after

- There are clearly more factors than can be considered, including,
  - changes in material use
  - opportunity cost
  - setup times
  - change in inventory size
  - material handling change
- The simple models ignore the conversion between present value and future value. (ie, money now is worth more than the same amount of money later)

$$PW = C_0 + \sum [(R_{A_j} - C_{A_j})(P/F, i, j)]$$

$$(P/F, i, j) = \frac{1}{(1+i)^j} \quad (P/A, i, n) = \sum (P/F, i, j) = \frac{(1+i)^n - 1}{i(1+i)^n}$$

where,

$PW$  = present worth of the money (in todays dollars)

$R_{A_j}$  = Annual revenues (income) for year  $j$

$C_{A_j}$  = Annual costs (expenses) for year  $j$

$j$  =  $j$  years in the future

$i$  = interest rate (fractional)

$n$  = number of years for consideration

- The future value of money can be evaluated using,

$$F = P(F/P, i, n) \quad (F/P, i, n) = (1+i)^n$$

where,

$FW$  = future worth of the money

e.g.,

$$i = 5\% \quad P = 1000 \quad n = 3$$

$$F = P(F/P, i, n) = 1000(F/P, 5\%, 3) = 1000(1 + 0.05)^3 =$$

- Quite often a Rate of Return (ROR) will be specified by management. This is used in place of interest rates, and can include a companies value for the money. This will always be higher

than the typical prime interest rate.

- So far we haven't considered the effects of taxes. Basically corporate taxes are applied to profits. Therefore we attempt to distribute expenses evenly across the life of a project (even though the majority of the money has been spent in the first year). This distribution is known as depreciation.

$$A = B - T = B - (\text{tax}_{\text{rate}} C) = B(1 - \text{tax}_{\text{rate}}) + D\text{tax}_{\text{rate}}$$

where,

$A$  = after tax cash (\$/yr)

$B$  = before tax cash (\$/yr)

$D$  = depreciation of equipment (\$/yr)

$\text{tax}_{\text{rate}}$  = the corporate tax rate

- Methods for depreciation are specified in the tax laws. One method is straight line depreciation.

$$D = \frac{C_E - I_S}{n}$$

where,

$D$  = The annual depreciation

- Another methods is the accelerated cost recovery (ACRS) method that is based on US tax law. A similar version is the Modified ACRS (MACRS) system.

$$D_j = C \times f_j$$

where

$D_j$  = The annual depreciation for year j

$f_j$  = the depreciation factor for year j

year	recovery % for a given depreciation period			
	3 yrs	5 yrs	7 yrs	10 yrs
1	33.3	20.0	14.3	10.0
2	44.5	32.0	24.5	18.0
3	14.8	19.2	17.5	14.4
4	7.4	11.5	12.5	11.5
5		11.5	8.8	9.2
6		5.8	8.9	7.4
7			8.9	6.6
8			4.5	6.6
9				6.5
10				6.5
11				3.3

(copied from Lindberg FE Review Manual)

- The 'book value' for an item is calculated as,

$$BV = C - \sum D_j$$

- Rate of Return (ROR) Analysis -

- Present worth analysis shifts all of the future costs and incomes into a present day sum.
- Benefit Cost Analysis for choosing between alternatives, All costs must be converted to the present values.

$$\frac{B_2 - B_1}{C_2 - C_1} > 1 \quad \text{Choice 2 is better than 1}$$

- Break Even Analysis

$$C - PBP \times NAP$$

where,

$PBP$  = Payback period

$NAP$  = Net annual profit

- Return On Investment,

$$ROI = \frac{B - C}{C} 100\%$$

- Consider an assembly line that is currently in use, and the system proposed to replace it. The product line is expected to last 5 years, and then be sold off. The corporate tax rate is 50% and the company policy is to require a 17% rate of return. Should we keep the old line, or install the new one?

**Current Manual Line:**

- used 2000 hrs/yr with 10 workers at \$20/hr each
- maintenance is \$20,000/yr
- the current equipment is worth \$20,000 used

**Proposed Line:**

- the equipment will cost \$100,000 and the expected salvage value at the end of the project is \$10,000
- 2 workers are required for 1000 hours year at \$40/hr each
- yearly maintenance will be \$40,000

**6.2 Problems**

1. If \$100,000 were borrowed for 3 years at a %10 interest rate, how much would be due at the end of the loan. (ans. \$133,100)
2. If \$100,000 were borrowed for 3 years at a %10 interest rate, how much would be due at the end of the loan if \$20,000 were repaid each year. (ans. \$66,900)
3. A machine was purchased for \$100,000 and generates \$20,000 per year income. How many years would be required to break even if the company charged a 10% internal interest rate. (ans. 7.27 yrs)
4. If a machine is purchased for \$100,000 and the company charges %10 for the use of money, what annual return is required for the machine to break even in 3 years. (ans. \$40,211.49)

5. A machine costs \$100,000 and will be sold for salvage value in 3 years for \$30,000. What is the Equivalent Uniform Annual Cost for the machine assuming the company uses an interest rate of 10%? (ans. \$31,148.69)

6. A machine costs \$100,000 and will be sold for salvage value in 3 years for \$30,000. The alternative is to lease a machine for \$40,000 per year. If the company uses an interest rate of 10% which option should be chosen? (ans. purchasing is a better option)

7. An existing manual production line costs \$100,000 to operate per year. A new piece of automated equipment is being considered to replace the manual production line. The new equipment costs \$150,000 and requires \$30,000 per year to operate. The decision to purchase the new machine will be based upon a 3 year period with a 25% interest rate. Compare the present value of the two options. (ans. PV<sub>manual</sub> \$195,200, PV<sub>new</sub> \$208,560)

8. Write a general computer program to solve the following project costing problem. Test the program using the numbers provided. The program should accept the initial cost of equipment (C), an annual maintenance cost (M), an annual income (R), a salvage value (S), and an interest rate (I). The program should then calculate a present worth and the ROI.

Test values:

$$C = 100,000$$

$$M = 20,000$$

$$R = 150,000$$

$$S = 10,000$$

$$I = 10\%$$

$$L = 3 \text{ yrs}$$

9. Write a program that determines the ROI for a project given the project length, initial cost, salvage value, and projected income. To test the program assume that the project lasts for 36 months. The company standard interest rate is 18%. The equipment will cost \$100,000 new and have a slavage value of \$10,000. The annual income will be \$50,000.

### 6.3 Challenge Problems

1. Consider Moore's law that predicts that every 18 months the basic speed and capacity of semiconductors will double. We have a customer that wants.....

## 7. OPTIMIZATION

Topics:

- 

Objectives:

- 

### 7.1 Introduction

- This is normally used when there is no clearly optimal solution to a problem.
- The basic procedure is,
  1. Identify the major variables in a decision
  2. Model the system or decisions to be made with equations
  3. Assign a cost (objective) function for the system
  4. Select an optimization method and search
  5. Analyze the results and search again if necessary

### 7.2 System Modeling and Variable Identification

- Typical decision variables in systems include,
  - mass
  - volume
  - power consumption
  - component cost
  - factor of safety
  - signal-to-noise ratio

- transmission rates
- etc.

- A model of the system should relate the decisions to be made mathematically. These relationships often include performance measures,

- cutting speeds
- cycle times
- power
- capacity
- flow rates
- etc.

### 7.3 Cost Functions and Constraints

- Cost functions quantify things that we want to minimize, or maximize. These should be aligned with system objectives. Typical cost functions include,

- money
- time
- some combination of factors

- Systems also have constraints that limit available solutions

- Expressed as a function of variables that provides a value

- Consider the example of building a fenced pasture. In this case when the area becomes too large, there is a reduced value. We want to maximize the value of V.

$$C_{fence} = 20 \left( \frac{\$}{m} \right) (2w + 2d)$$

where,

$C_{fence}$  = cost to construct the fence (\$)

$w$  = width of the pasture (m)

$d$  = depth of the pasture (m)

$$C_{land} = 0.35 \left( \frac{\$}{m^2} \right)$$

where,

$C_{land}$  = cost for the land

$$R = 0.05 \left( \frac{\$}{m^2 \text{ yr}} \right) wd$$

$$wd < 300 m^2$$

$$R = 0.01 \left( \frac{\$}{m^2 \text{ yr}} \right) (wd - 300 m^2) + 60 \$$$

$$wd \geq 300 m^2$$

where,

$R$  = revenue generated by pasture land

$$V = R - C_{fence} - C_{land}$$

where,

$V$  = total value

Figure 1.29 Example cost function for building a fence around a pasture

- The cost function can be written as..

```

double cost(double w, double d){
    double value;
    double cfence, cland, R;

    Cfence = 40*(w + d);
    Cland = 0.05*w*d;
    if (w*d < 300){
        R = 0.05 * w * d;
    } else {
        R = 0.01 * (w * d - 300) + 60;
    }
    value = R - Cfence - Cland

    return value;
}

```

Figure 1.30 A subroutine for cost function calculation

- Constraints are boundaries that cannot be crossed.
- Example of constraints, the pasture cannot be larger than one 1600m by 1600m because of the constraints of an existing road system.

$$w \leq 1600m$$

$$d \leq 1600m$$

Figure 1.31 Example constraint functions for a pasture

- The cost function can be written as..

```

double cost(double w, double d){
    double value;
    double cfence, cland, R;

    Cfence = 40*(w + d);
    Cland = 0.05*w*d;
    if (w*d < 300){
        R = 0.05 * w * d;
    } else {
        R = 0.01 * (w * d - 300) + 60;
    }
    value = R - Cfence - Cland

    if(w > 1600) value = 1000000;
    if(d > 1600) value = 1000000;

    return value;
}

```

Figure 1.32 A subroutine for cost function calculation

- Slack variables allow constraints to be considered as part of the cost function. Helps with a system with many local minimum.

$$C_{penalty} = \left( \frac{w}{1600m} \right)^4 + \left( \frac{d}{1600m} \right)^4$$

$$V = R - C_{fence} - C_{land} - C_{penalty}$$

Figure 1.33 Example of slack variables for including constraints

- The cost function can be written as..

```

double cost(double w, double d){
    double value;
    double cfence, cland, R;
    double slack;

    Cfence = 40*(w + d);
    Cland = 0.05*w*d;
    if (w*d < 300){
        R = 0.05 * w * d;
    } else {
        R = 0.01 * (w * d - 300) + 60;
    }
    // slack variable
    slack = pow((w/1600), 4) + pow((d/1600), 4);
    value = R - Cfence - Cland - slack;

    return value;
}

```

Figure 1.34 A subroutine for cost function calculation

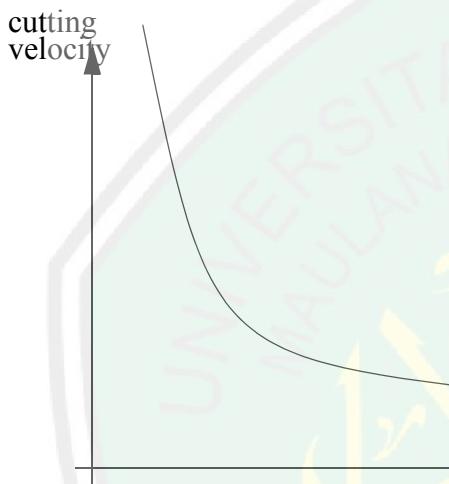
## 7.4 Single Variable Searches

- For simple single value problems use derivatives and find the maxima/minima.

### 7.4.1 Example Problem

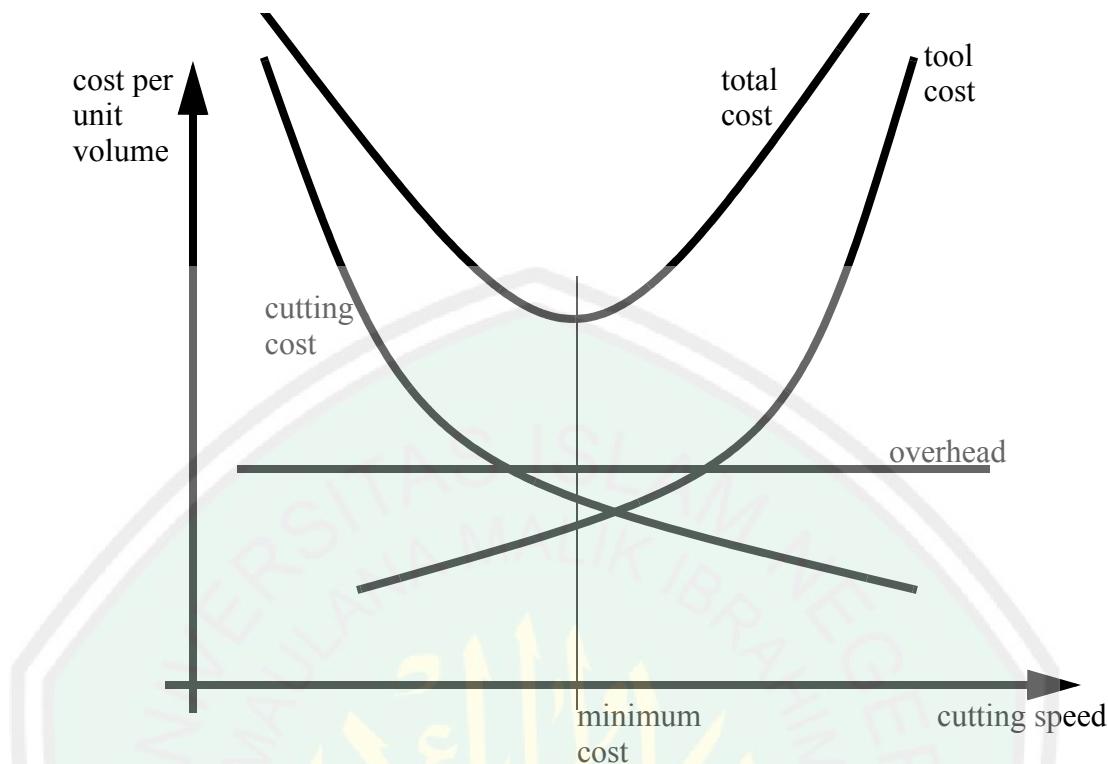
- As with most engineering problems we want to get the highest return, with the minimum investment. In this case we want to minimize costs, while increasing cutting speeds.
- EFFICIENCY will be the key term - it suggests that good quality parts are produced at reasonable cost.
- Cost is primarily affected by,
  - tool life
  - power consumed
- The production throughput is primarily affected by,
  - accuracy including dimensions and surface finish
  - mrr (metal removal rate)

- The factors that can be modified to optimize the process are,
  - cutting velocity (biggest effect)
  - feed and depth
  - work material
  - tool material
  - tool shape
  - cutting fluid
- We previously considered the log-log scale graph of Taylor's tool life equation, but we may also graph it normally to emphasize the effects.



This graph is representative for most reasonable cutting speeds. The velocities at the high and low ranges do not necessarily exhibit the same relationship.

- There are two basic conditions to trade off,
  - Low cost - exemplified by low speeds, low mrr, longer tool life
  - High production rates - exemplified by high speeds, short tool life, high mrr
- \*\*\* There are many factors in addition to these, but these are the most commonly considered



- A simplified treatment of the problem is given below for optimizing cost,

First lets look at costs for a cutting tool over the life of a tool,

$$C_t = c_1 + c_2 + c_3$$

where,

$C_t$  = cost per cutting edge

$c_1$  = the cost to change a tool

$c_2$  = the cost to grind a tool per edge

$c_3$  = the cost of the tool per edge

and,

$$c_1 = t_1 \times R_c$$

$$c_2 = t_2 \times \frac{R_s}{N_1}$$

$$c_3 = \frac{C_T}{N_1 \times (N_2 + 1)}$$

where,

$t_1$  = tool change time

$t_2$  = tool grind time in minutes

$R_c$  = cutting labour + overhead cost

$R_s$  = grinding labor + overhead cost

$C_T$  = cost of the original tool

$N_1$  = the number of cutting edges to grind

$N_2$  = the maximum number of regrinds

and,

$$C_c = R_c \times T$$

where,

$C_c$  = cutting operation cost over life of tool, per edge

$T$  = tool life

Next, let's consider the effects of metal removal rate,

$$Q_T = V \times T \times f \times c \quad (1)$$

where,

$Q_T$  = metal removal rate per edge

$V$  = cutting velocity

$f$  = tool feed rate

$c$  = depth of width of the cut

consider the life of the tool,

$$\begin{aligned} V \times T^n &= C \text{ (Taylors tool life equation)} \\ \therefore V &= \frac{C}{T^n} \end{aligned} \quad (2)$$

Now combine tool life (2) with the mrr (1),

$$Q_T = V \times T \times f \times c = \frac{C}{T^n} \times T \times f \times c = \frac{C \times f \times c}{T^{n-1}}$$

At this point we have determined functions for cost as a function of tool life, as well as the metal removal rates. We can now proceed to find cost per unit of material removed.

$$C_u = \frac{C_c + C_t}{Q_T} = \frac{T^{n-1}}{C \times f \times c} (R_c \times T + C_t)$$

Using some basic calculus, we can find the minimum cost with respect to tool life.

$$\frac{dC_u}{dT} = \left( \frac{1}{C \times f \times c} \right) (R_c \times n \times T^{n-1} + C_t \times (n-1) \times T^{n-2}) = 0$$

$$\therefore R_c \times n \times T = -C_t \times (n-1)$$

$$\boxed{\therefore T = \frac{-C_t \times (n-1)}{R_c \times n} = \frac{C_t}{R_c} \left( \frac{1-n}{n} \right)}$$

- We can also look at optimizing production rates,

There are two major factors here when trying to increase the mrr. We can have a supply of tools by the machine, and as the tools require replacement, the only down-time involved is the replacement of the tool.

This gives us an average rate of production,

$$R_p = \frac{Q_T}{T + t_1}$$

where,

$R_p$  = average rate of production

recall from before that,

$$Q_T = \frac{Cfc}{T^{n-1}}$$

now substituting in gives,

$$R_p = \frac{\left(\frac{Cfc}{T^{n-1}}\right)}{T + t_1} = Cfc(T^n + t_1)^{-1}$$

We can now optimize the production rate,

$$\frac{dR_p}{dT} = Cfc[-(T^n + t_1)^{-2} + (nT^{n-1} + t_1)(T^n + t_1)^{-1}] = 0$$

$$\therefore (T^n + t_1)^{-2} = (nT^{n-1} + t_1)(T^n + t_1)^{-1}$$

$$\therefore 1 = (nT^{n-1} + t_1)(T^n + t_1)$$

$$\therefore 1 = nT^{2n-1} + nt_1T^{n-1} + t_1T^n + t_1^2$$

$$\therefore \log(1) = \log(nT^{2n-1}) + \log(nt_1T^{n-1}) + \log(t_1T^n) + \log(t_1^2)$$

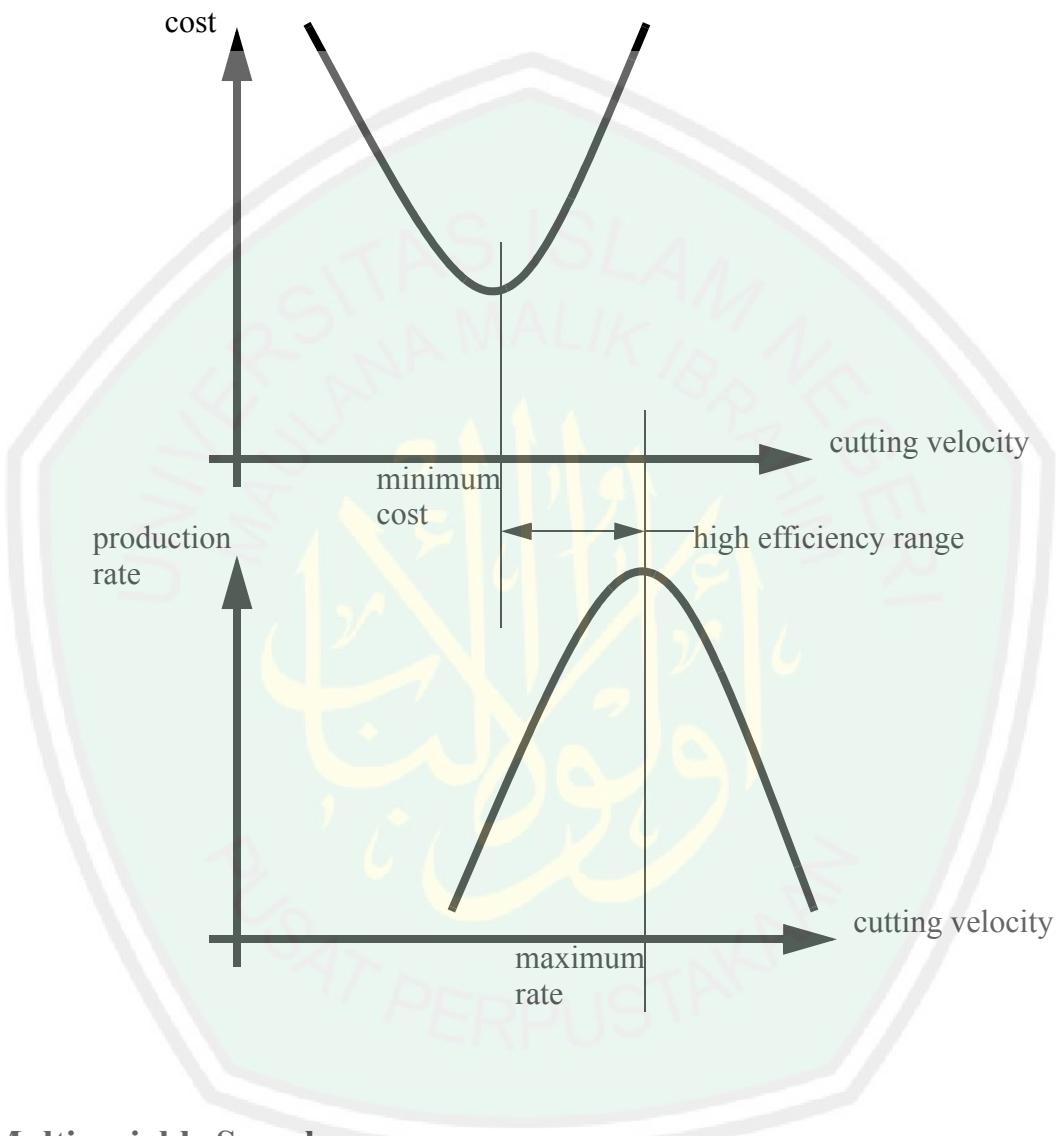
$$\therefore 0 = \log(n) + (2n-1)\log(T) + \log(nt_1) + (n-1)\log(T) + \log(t_1) + n\log(T) + \log(t_1^2)$$

$$\therefore 0 = 2\log(n) + (4n-2)\log(T) + 4\log(t_1)$$

$$\therefore \log(T) = \frac{\log(n) + 2\log(t_1)}{1 - 2n}$$

- We can now put the two optima in perspective,

Since,  $t_1 < C_t/R_c$  then tool life for maximum production is less than economical tool life and as a result, cutting velocity for maximum production is  $>$  velocity for lowest cost



## 7.5 Multivariable Searches

- Local Search Space
- A topographical map shows the relationship between search parameters and cost values.

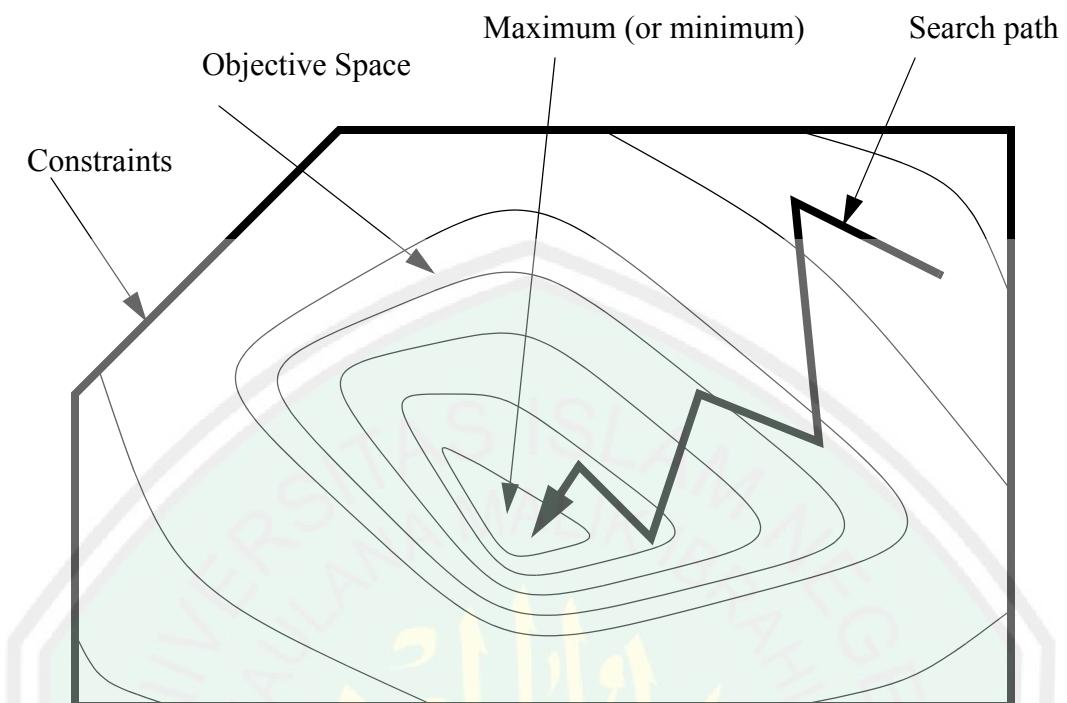


Figure 1.35 Local searches

- Global Search space. In this case the system becomes 'stuck' in a local mimina.

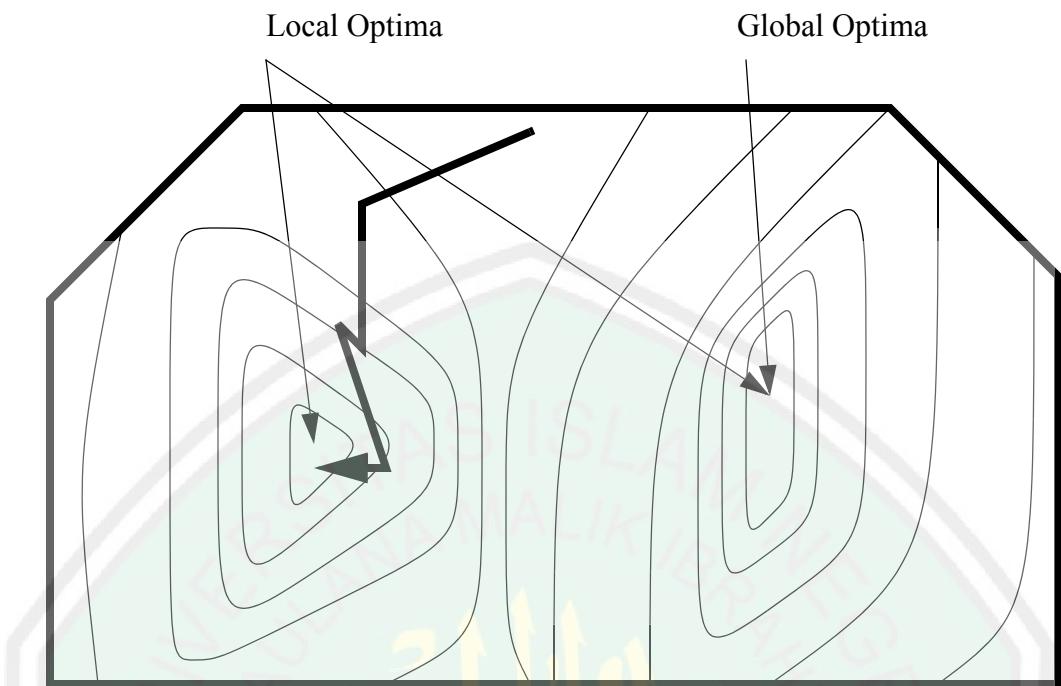


Figure 1.36 Global searches

- Global Search space. In this case the system searches all maxim.

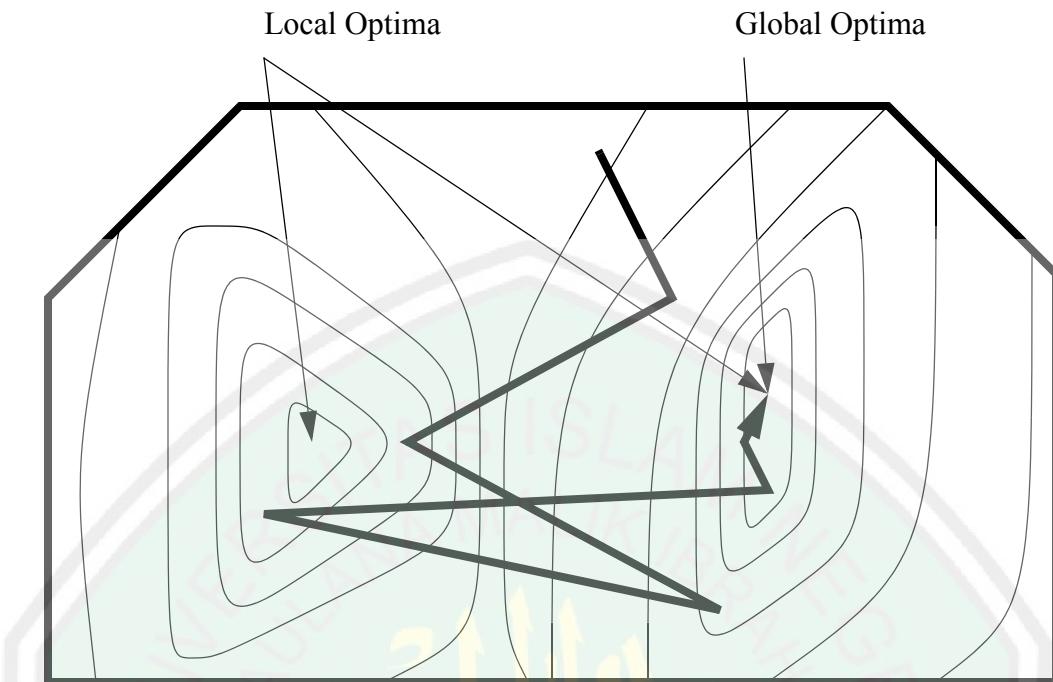


Figure 1.37 A Global Search

### 7.5.1 Algorithms

- The search algorithms change system parameters and try to lower system parameters.
- The main question is how to change the system parameters to minimize the system value.

### 7.5.2 Random Walk

### 7.5.3 Gradient Decent

### 7.5.4 Simplex

### 1.6 Summary

- 

### 1.7 Problems

1. A production facility molds plastic parts. Ideally these parts are made with new plastic pellets. However rejected plastic parts are ground into (regrind) pellets and mixed in with new plastic pellets. New plastic costs \$1.00 per pound. If a part is discarded (not reground) the total cost of the material is lost. To regrind and dry scrap parts there is a cost of \$0.10 per pound. The customer demands that the percentage of regrind cannot exceed 30%. Statistical data was used to find the following relationship between the percentage of regrind and the scrap rate. Write a program to determine the optimum quantity of regrind to minimize the cost per part.

$$S = 2.0 + 3.0 \times 2^{\frac{R+5}{4}}$$

where,

$R$  = percentage of reground material [0, 100%]

$S$  = percentage of parts that are scrap

### 1.8 Challenge Problems

1. Rocket Fuel Burn
2. Power plant fuel mixture

3. Write a program that recommends the optimum cutting speed for a machining process. Model the program on the example in the chapter.



## 2. PROJECT SCHEDULING

Topics:

- 

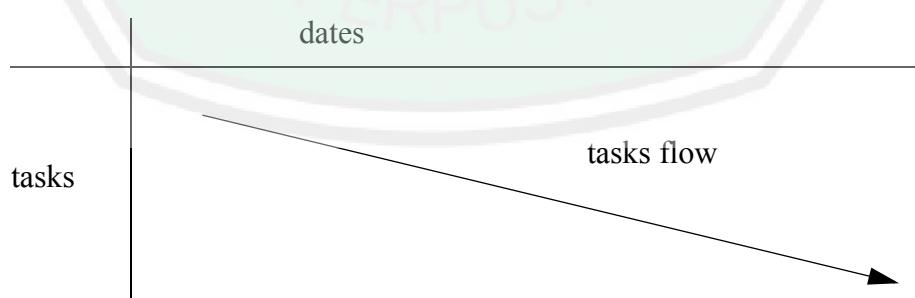
Objectives:

- 

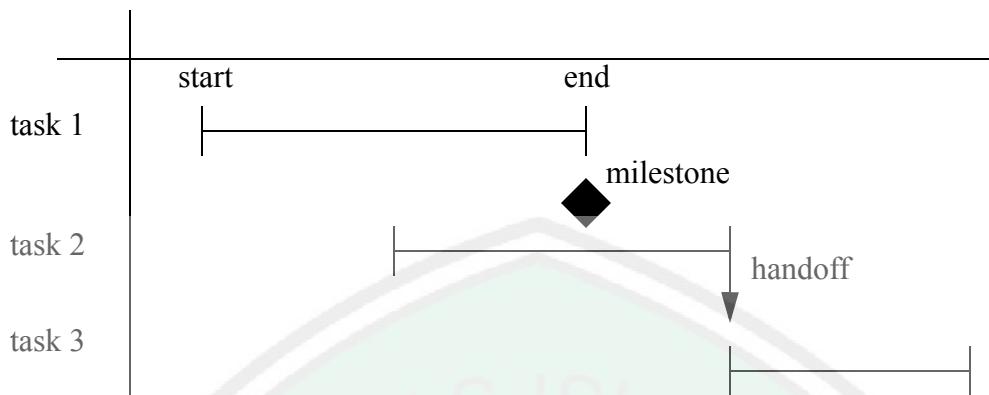
### 2.1 Introduction

### 2.2 Gantt Charts

- General form,



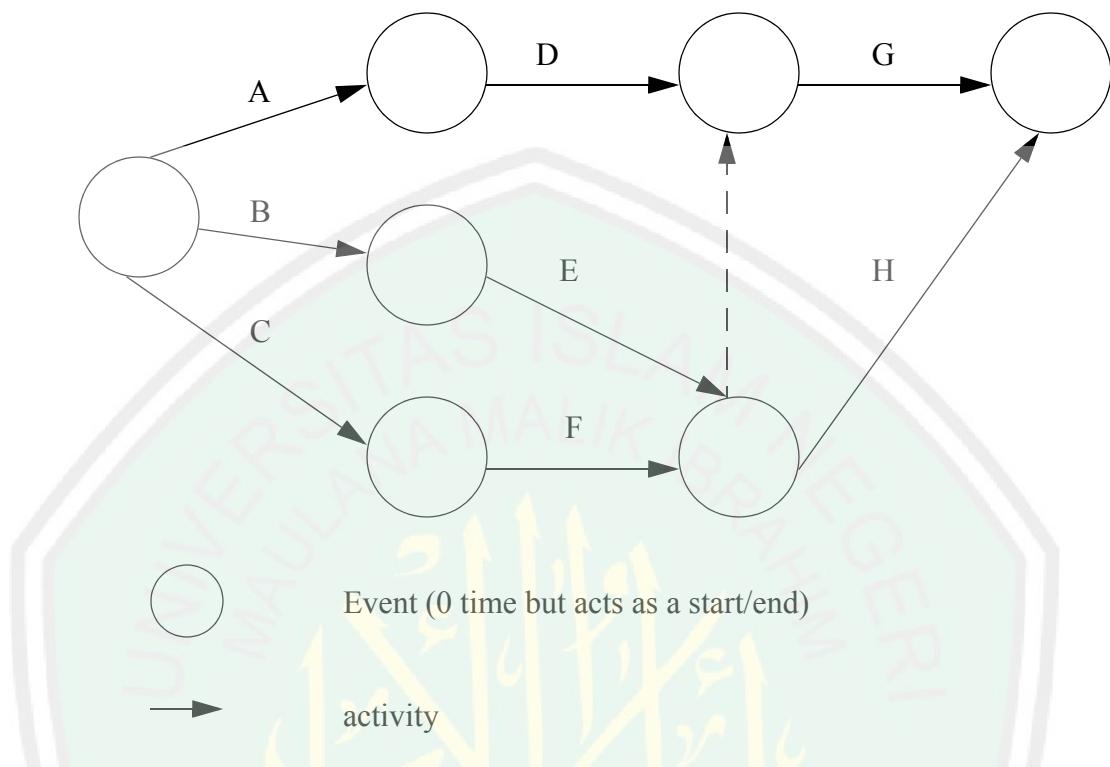
- tasks



- These charts should be updated on a regular basis to track the progress and completion of each task.
- Things to look for when doing Gantt charts include,
  - gaps when no tasks are being done
  - too many concurrent tasks
  - too much/too little detail (charts can be broken into subcharts to isolate detail)
  - associate people and resources to tasks
- Good ideas when constructing Gantt Charts,
  - identify critical paths and move forward to create slack time
  - delay costly components to reduce WIP

### 2.3 Critical Path Method (CPM)

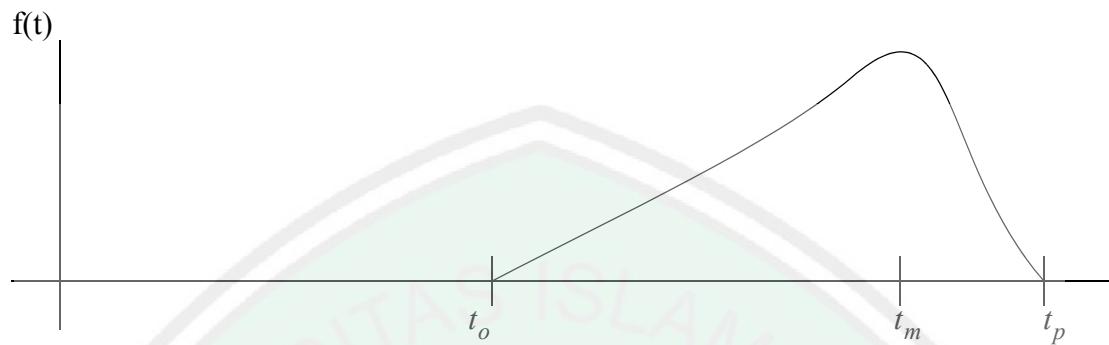
- Tasks (possibly from a Gantt chart) can be put in a network diagram.



- Earliest Start (ES) - the earliest time a task can start.
- Minimum Project Duration - the shortest time to complete the project based upon the longest path.
- Desired Project Duration - the planned time for completing the project.
- Latest Start (LS) - the latest a task can start for a Desired Project Duration.
- Total Float (TF) = LS - ES
- Critical Path - the sequence of tasks that take the longest, and dictate the Minimum Project Duration.

## 2.4 Program Evaluation and Review Technique (PERT)

- In CPM we assume that each activity has a fixed time, in practice the task lengths vary. This variation can be shown with the Beta distribution.



where,

$t_o$  = optimistic time estimate

$t_m$  = most likely time estimate

$t_p$  = pessimistic time estimate

- task times are expressed with three numbers separated by dashes to-tm-tp to represent task times.

- the mean (effective) time can be found with,

$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

- A Standard Deviation for each activity time can be approximated using,

$$\sigma_i = \left| \frac{t_{p_i} - t_{o_i}}{6} \right|$$

- The te values can be used to do a CPM analysis of a network diagram. Once the Critical Path is

identified the overall task time and variance can be calculated using,

$$\sigma_T = \sqrt{\sum \sigma_i^2}$$

$$T_e = \sum t_{e_i}$$

- To find the chance that the process will be done by the time  $T_s$ , the z value can be calculated. The z value can then be used to find the probability of completion using the cumulative normal distribution function.

$$z = \frac{T_s - T_e}{\sigma_T}$$

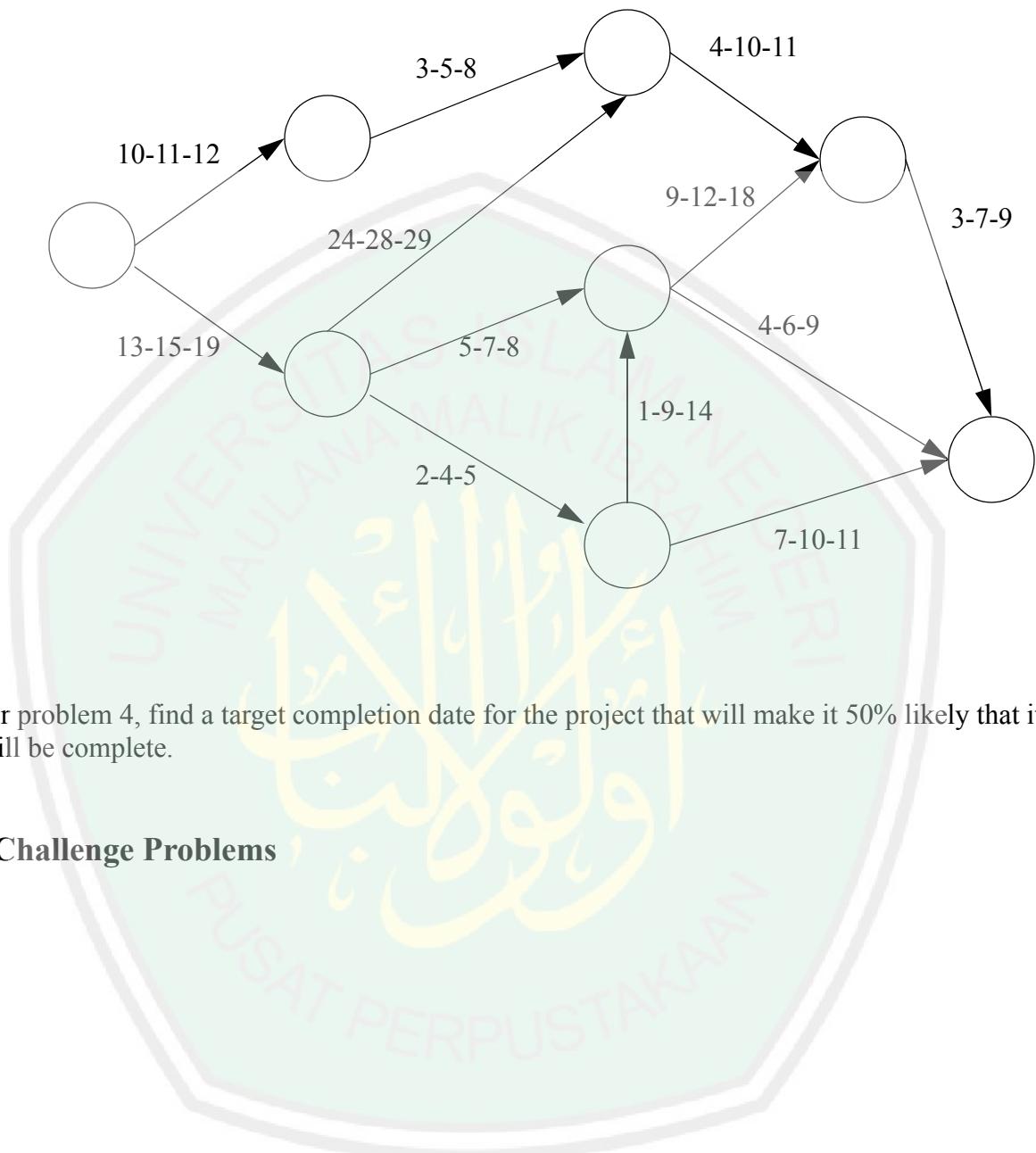
## 2.5 Problems

1. A new building is being constructed and the following tasks are required. The normal workdays are 7am-3pm, Monday to Friday. Overtime is possible, however the costs make it highly undesirable. Write a Gantt chart for completion of the job in 3 months.

Site Preparation - 1 month  
 Foundations - 1 month - after Site Preparation  
 Framing - 2 weeks - after Foundations  
 Plumbing - 5 weeks - after Framing  
 Electrical - 6 weeks - after Framing  
 Inspection - after Plumbing and Electrical  
 Drywall - 2 weeks - after Inspection  
 Painting - 1 week - after Drywall  
 Hardware - 1 week - after Painting  
 Carpet - 3 days - after Painting

2. Develop a project activity network for problem 1.
3. Identify the critical path for problem 2.
4. Consider the PERT network diagram below and find the likelihood that the project will be

complete in 40 days.



5. For problem 4, find a target completion date for the project that will make it 50% likely that it will be complete.

## 2.6 Challenge Problems

### 3. DIRECTED GRAPHS

Topics:

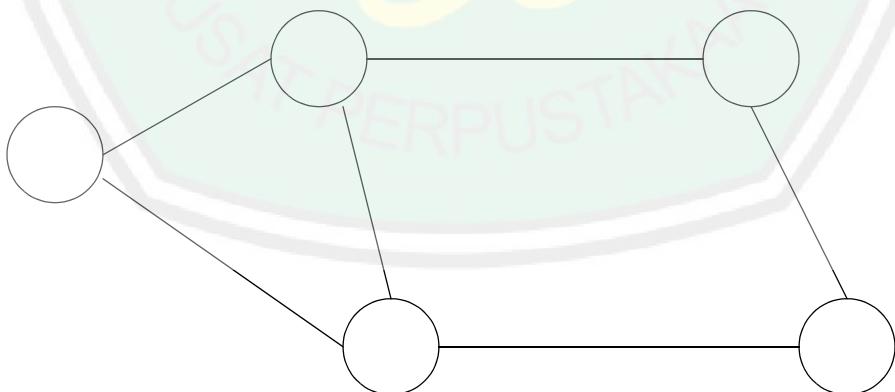
- 

Objectives:

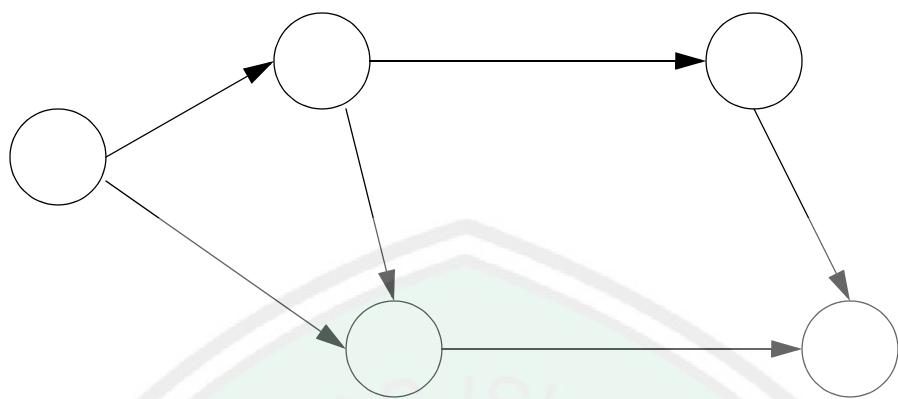
- 

#### 3.1 Introduction

- Graphs are normally data structures that have vertices (circles) and edges (lines) as shown below.



- In a directed graph the edges will have a direction assigned, as shown below.

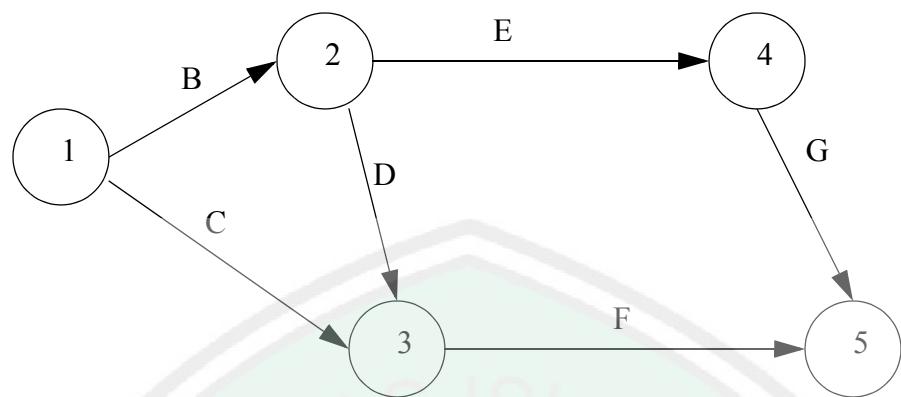


## 3.2 Data Structures

- A simple data structure for representing a graph is shown below.

Vertex list:	Edge list:		
Vertex	Edge	From	To

- Consider the graph below.



Vertex list:

Vertex

1  
2  
3  
4  
5

Edge list:

Edge

B  
C  
D  
E  
F  
G

From vert.

1  
1  
2  
2  
3  
4

To vert.

2  
3  
3  
4  
5  
5

- Representing a graph in Scilab

```

function foo = find_index(_list, name) // a function to find list numbers given names
    foo = -1; // use -1 to indicate no match found yet
    A = size(_list); // find the rows and columns in the list
    cnt = A(1, 1); // get the rows in the list
    for i = 1 : cnt, // loop through the rows
        if name == _list(i) then // look for a name match
            foo = i; // record the matching row number
            break; // no point continuing the for loop
        end,
    end
    if foo == -1 then mprintf("ERROR: list name %s not found\n", name); end
endfunction

vertex_names = ["1" ; "2" ; "3" ; "4" ; "5"]; // define the vertices

function foo = vert_number(name) // a function to find vertex numbers given names
    foo = find_index(vertex_names, name);
endfunction

edge_names = ["B" ; "C" ; "D" ; "E" ; "F" ; "G"]; // define the edges.

function foo = edge_number(name) // a function to find edge numbers given names
    foo = find_index(edge_names, name);
endfunction

edge = [edge_number("B"), vert_number("1"), vert_number("2")];
edge = [edge ; [edge_number("C"), vert_number("1"), vert_number("3")]];
edge = [edge ; [edge_number("D"), vert_number("2"), vert_number("3")]];
edge = [edge ; [edge_number("E"), vert_number("2"), vert_number("4")]];
edge = [edge ; [edge_number("F"), vert_number("3"), vert_number("5")]];
edge = [edge ; [edge_number("G"), vert_number("4"), vert_number("5")]];

```

- Representing a graph in C

```

int find_number(char list[][], char *name){
    int i;
    if(name == NULL) return -1;
    for(i = 0; list[i][0] != NULL; i++){
        if(strcmp(name, list[i]) == 0){
            return i;
        }
    }
    printf("ERROR: search name %s not found \n", name);
    return -1;
}

char vert_names[][] = {{"1"}, {"2"}, {"3"}, {"4"}, {"5"}, NULL};
int vert_number(char *name){ return find_number(vert_names, name);}

char edge_names[][] = {{"B"}, {"C"}, {"D"}, {"E"}, {"F"}, {"G"}, NULL};
int edge_number(char *name){ return find_number(edge_names, name);}

#define MAX_EDGES 10
int edge_cnt = 0;
int edges[MAX_EDGES][3];

void add_edge(char *edge, char *from_vert, char *to_vert){
    edges[edge_cnt][0] = edge_number(edge);
    edges[edge_cnt][1] = vert_number(from_vert);
    edges[edge_cnt][2] = vert_number(to_vert);
    edge_cnt++;
}

int main(){
    add_edge("B", "1", "2");
    add_edge("C", "1", "3");
    add_edge("D", "2", "3");
    add_edge("E", "2", "4");
    add_edge("F", "3", "5");
    add_edge("G", "4", "5");

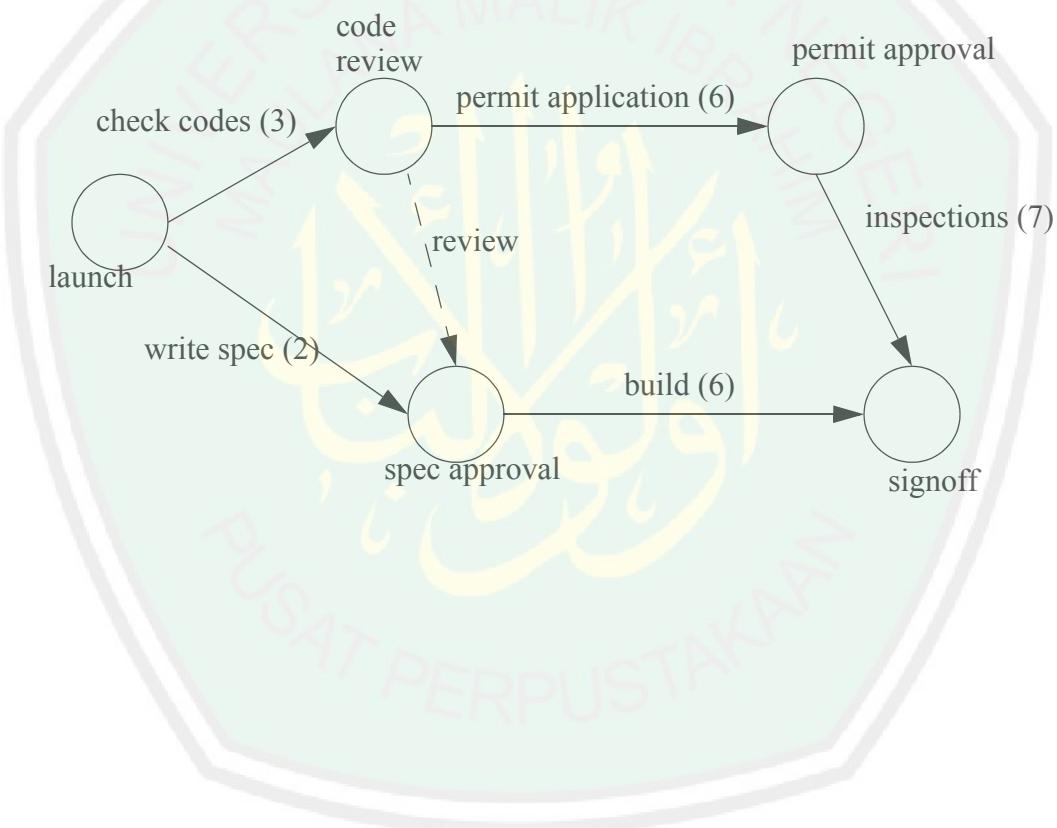
    return 0;
}

```

### 3.3 Applications

#### 3.3.1 Precedence Identification

- To determine a precedence we add a precedence count for each vertex. The basic check is,
  1. Set all the precedence values to 1 (lowest precedence)
  2. Cycle through and check each edge. For each edge ensure that the 'to' vertex has a precedence that is at least one greater than the 'from' vertex.
- Consider a Network Diagram for a CPM problem.



- The algorithm implemented in Scilab (assuming the 'find\_index' function is the same as before)

```

vert_names = ["launch" ; "code review" ; "spec approval" ; "permit approval" ; "signoff"]; // vertices
A = size(vert_names); // find the rows and columns in the list
vert_cnt = A(1, 1); // get the rows in the list

edge_names = ["check codes" ; "write spec" ; "permit application" ; "review" ; "build" ; "inspections"]; / define the edges.
A = size(edge_names); // find the rows and columns in the list
edge_cnt = A(1, 1); // get the rows in the list

//----- Previous Code -----//

function foo = find_index(_list, name) // a function to find list numbers given names
    foo = -1; // use -1 to indicate no match found yet
    A = size(_list); // find the rows and columns in the list
    cnt = A(1, 1); // get the rows in the list
    for i = 1 : cnt, // loop through the rows
        if name == _list(i) then // look for a name match
            foo = i; // record the matching row number
            break; // no point continuing the for loop
        end,
    if foo == -1 then mprintf("ERROR: list name %s not found\n", name); end
endfunction

function foo = vert_number(name) // a function to find vertex numbers given names
    foo = find_index(vert_names, name);
endfunction
function foo = edge_number(name) // a function to find edge numbers given names
    foo = find_index(edge_names, name);
endfunction

//-----

// Note: an extra column is added to include the time for each activity
edge = [edge_number("check codes"), vert_number("launch"), vert_number("code review"), 3];
edge = [edge ; [edge_number("write spec"), vert_number("launch"), vert_number("spec approval"), 2]];
edge = [edge ; [edge_number("permit application"), vert_number("code review"), vert_number("permit approval"), 6]];
edge = [edge ; [edge_number("review"), vert_number("code review"), vert_number("spec approval"), 0]];
edge = [edge ; [edge_number("build"), vert_number("spec approval"), vert_number("signoff"), 6]];
edge = [edge ; [edge_number("inspections"), vert_number("permit approval"), vert_number("signoff"), 7]];

// initialize all precedence values to 1
precedence = [1];
for i = 2 : vert_cnt,
    precedence = [precedence ; 1];
end

// Loop through and update precedences
for i = 1 : edge_cnt - 1,
    for j = 1 : edge_cnt,
        prev_vert = edge(j, 2); // find the previous and current vertices
        next_vert = edge(j, 3);
        prev_precedence = precedence(prev_vert); // find the precedence of previous/next vertices
        next_precedence = precedence(next_vert);
        if (prev_precedence + 1) > next_precedence then // check to see if the precedence needs to be updated
            precedence(next_vert) = prev_precedence + 1;
        end,
    end
end

```

### 3.3.2 Graph Searching

- Identifying the vertex costs.

```

// Assumes the code for vertices has already been defined and run.

// initialize all vertex cost values to 0
cost = [0];
for i = 2 : vert_cnt,
    cost = [cost ; 0];
end

maximum_cost = 0;
last_vert = 0;

// Loop through and update precendences
for i = 1 : edge_cnt - 1,
    for j = 1 : edge_cnt,
        prev_vert = edge(j, 2); // find the previous and current vertices
        next_vert = edge(j, 3);
        edge_cost = edge(j, 4);
        prev_cost = cost(prev_vert); // find the cost of previous/next vertices
        next_cost = cost(next_vert);
        if (prev_cost + edge_cost) > next_cost then // check to see if the cost
needs to be updated
            cost(next_vert) = prev_cost + edge_cost;
        end,
        if cost(next_vert) > maximum_cost then
            maximum_cost = cost(next_vert);
            last_vert = next_vert;
        end
    end,
end

mprintf("The critical path cost is %f \n", maximum_cost);

```

### 3.4 Other Topics

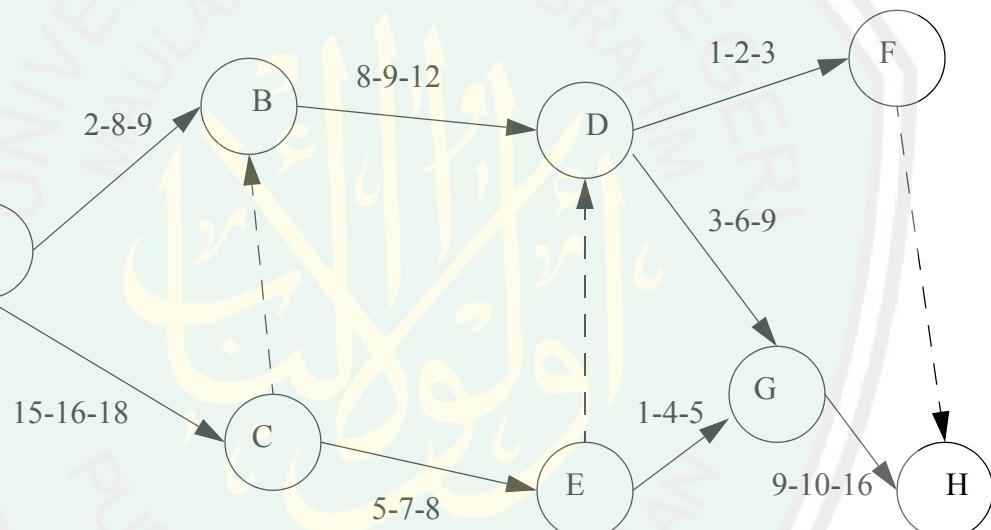
- ADD: resource limited plans

## 1.5 Summary

.

## 1.6 Problems

1. Write a general program that will accept a network diagram entered as vertices and arcs. The program will then do a PERT analysis to determine the  $T_e$  value and the standard deviation. Test the program with the following network diagram.



(ans.  $T_e=42.333$ , S.D.=1.826)

## 1.7 Challenge Problems

## 2. TREE DATA STRUCTURES

Topics:

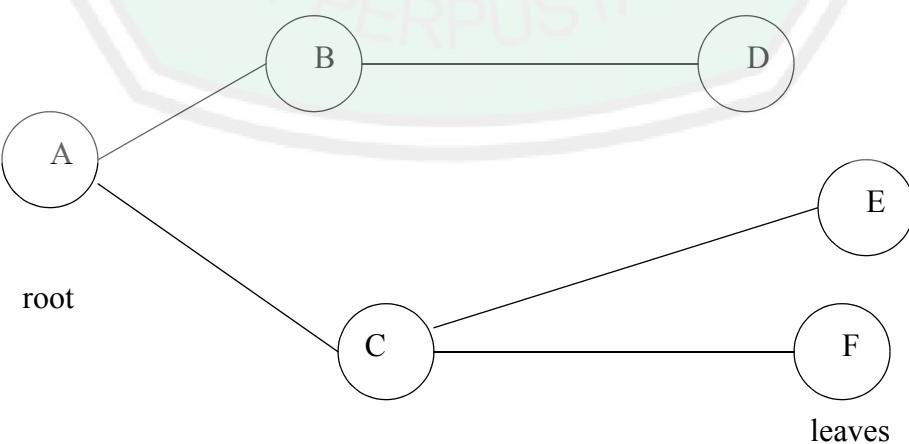
- 

Objectives:

- 

### 2.1 Introduction

- Trees are a special case of a graph data structure. The connections radiate out from a single root without cross connections.
- The tree has nodes (shown with circles) that are connected with branches. Each node will have a parent node (except for the root) and may have multiple child nodes.



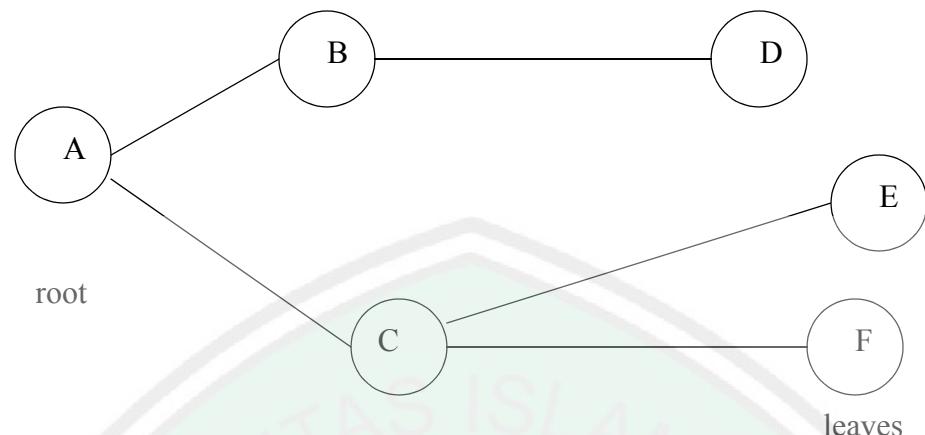
- In an unordered tree the children of a node can be unordered, however in an ordered tree the children take priority, often by listing them in order.
- A Boolean tree applies operators for each of the nodes

## 2.2 Data Structures

- A simple data structure for representing a tree is shown below.

Node list:		Branch list:	
Node		Parent	Child

- Consider the graph below.



Node #	Node Name	Parent	Child
1	A		
2	B	1	2
3	C	1	3
4	D	2	4
5	E	3	5
6	F	3	6

- Representing a tree in Scilab

```

function foo = find_index(_list, name) // a function to find list numbers given names
    foo = -1; // use -1 to indicate no match found yet
    A = size(_list); // find the rows and columns in the list
    cnt = A(1, 1); // get the rows in the list
    for i = 1 : cnt, // loop through the rows
        if name == _list(i) then // look for a name match
            foo = i; // record the matching row number
            break; // no point continuing the for loop
        end
    end
    if foo == -1 then mprintf("ERROR: list name %s not found\n", name); end
endfunction

node_names = ["A" ; "B" ; "C" ; "D" ; "E" ; "F"]; // define the nodes

function foo = node_number(name) // a function to find node numbers given names
    foo = find_index(node_names, name);
endfunction

branch = [node_number("A"), node_number("B")];
branch = [branch ; [node_number("A"), node_number("C")]];
branch = [branch ; [node_number("B"), node_number("D")]];
branch = [branch ; [node_number("C"), node_number("E")]];
branch = [branch ; [node_number("C"), node_number("F")]];

```

- Representing a graph in C

```

int find_number(char list[][], char *name){
    int i;
    if(name == NULL) return -1;
    for(i = 0; list[i][0] != NULL; i++){
        if(strcmp(name, list[i]) == 0){
            return i;
        }
    }
    printf("ERROR: search name %s not found \n", name);
    return -1;
}

char node_names[][] = {{"A"}, {"B"}, {"C"}, {"D"}, {"E"}, {"F"}, NULL};
int node_number(char *name){ return find_number(node_names, name);}

#define MAX_BRANCHES 10
int branch_cnt = 0;
int branch[MAX_BRANCHES][2];

void add_branch(int parent, int child){
    branch[branch_cnt][0] = node_number(parent);
    branch[branch_cnt][1] = node_number(child);
    branch_cnt++;
}

int main(){
    add_branch("A", "B");
    add_branch("A", "C");
    add_branch("B", "D");
    add_branch("C", "E");
    add_branch("C", "F");

    return 0;
}

```

## 2.3 Applications

### 2.3.1 Precedence Identification

- To determine a precedence we add a precedence count for each node. The basic check is,
  1. Set all the precedence values to 1 (lowest precedence)
  2. Cycle through and check each branch. For each branch ensure that the child vertex has a precedence that is one greater than the parent vertex.

- Consider the previous example.

```

function foo = find_index(_list, name) // a function to find list numbers given names
    foo = -1; // use -1 to indicate no match found yet
    A = size(_list); // find the rows and columns in the list
    cnt = A(1, 1); // get the rows in the list
    for i = 1 : cnt, // loop through the rows
        if name == _list(i) then // look for a name match
            foo = i; // record the matching row number
            break; // no point continuing the for loop
        end,
    end
    if foo == -1 then mprintf("ERROR: list name %s not found\n", name); end
endfunction

node_names = ["A" ; "B" ; "C" ; "D" ; "E" ; "F"]; // define the nodes
node_cnt = 6; // get the rows in the list

function foo = node_number(name) // a function to find node numbers given names
    foo = find_index(node_names, name);
endfunction

branch = [node_number("A"), node_number("B")];
branch = [branch ; [node_number("A"), node_number("C")]];
branch = [branch ; [node_number("B"), node_number("D")]];
branch = [branch ; [node_number("C"), node_number("E")]];
branch = [branch ; [node_number("C"), node_number("F")]];
branch_cnt = 5;

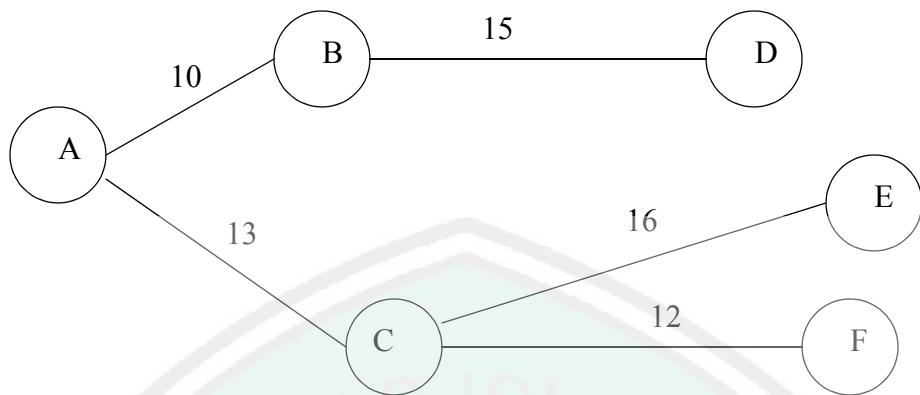
for i=1:node_cnt, // set all precedence values to 1
    precedence(i) = 1;
end

for i=1:node_cnt-1, // loop through and update precedence values
    for j=1:branch_cnt,
        if precedence(branch(j,1)) >= precedence(branch(j,2)) then
            precedence(branch(j,2)) = precedence(branch(j,1)) + 1;
        end
    end
end

```

### 2.3.2 Tree Searching

- Assume that each branch has a cost associated. We can then find the cost of each of the nodes.



- Identifying the vertex costs.

```

function foo = find_index(_list, name) // a function to find list numbers given names
    foo = -1; // use -1 to indicate no match found yet
    A = size(_list); // find the rows and columns in the list
    cnt = A(1, 1); // get the rows in the list
    for i = 1 : cnt, // loop through the rows
        if name == _list(i) then // look for a name match
            foo = i; // record the matching row number
            break; // no point continuing the for loop
        end,
    end
    if foo == -1 then mprintf("ERROR: list name %s not found\n", name); end
endfunction

node_names = ["A" ; "B" ; "C" ; "D" ; "E" ; "F"]; // define the nodes
node_cnt = 6; // get the rows in the list

function foo = node_number(name) // a function to find node numbers given names
    foo = find_index(node_names, name);
endfunction

branch = [node_number("A"), node_number("B"), 10];
branch = [branch ; [node_number("A"), node_number("C")], 13];
branch = [branch ; [node_number("B"), node_number("D")], 15];
branch = [branch ; [node_number("C"), node_number("E")], 16];
branch = [branch ; [node_number("C"), node_number("F")], 12];
branch_cnt = 5;

for i=1:node_cnt, // set all node cost values to 0
    cost(i) = 0;
end

for i=1:node_cnt-1, // loop through and update cost values
    for j=1:branch_cnt,
        cost(branch(j, 2)) = cost(branch(j, 1)) + branch(j, 3);
    end
end

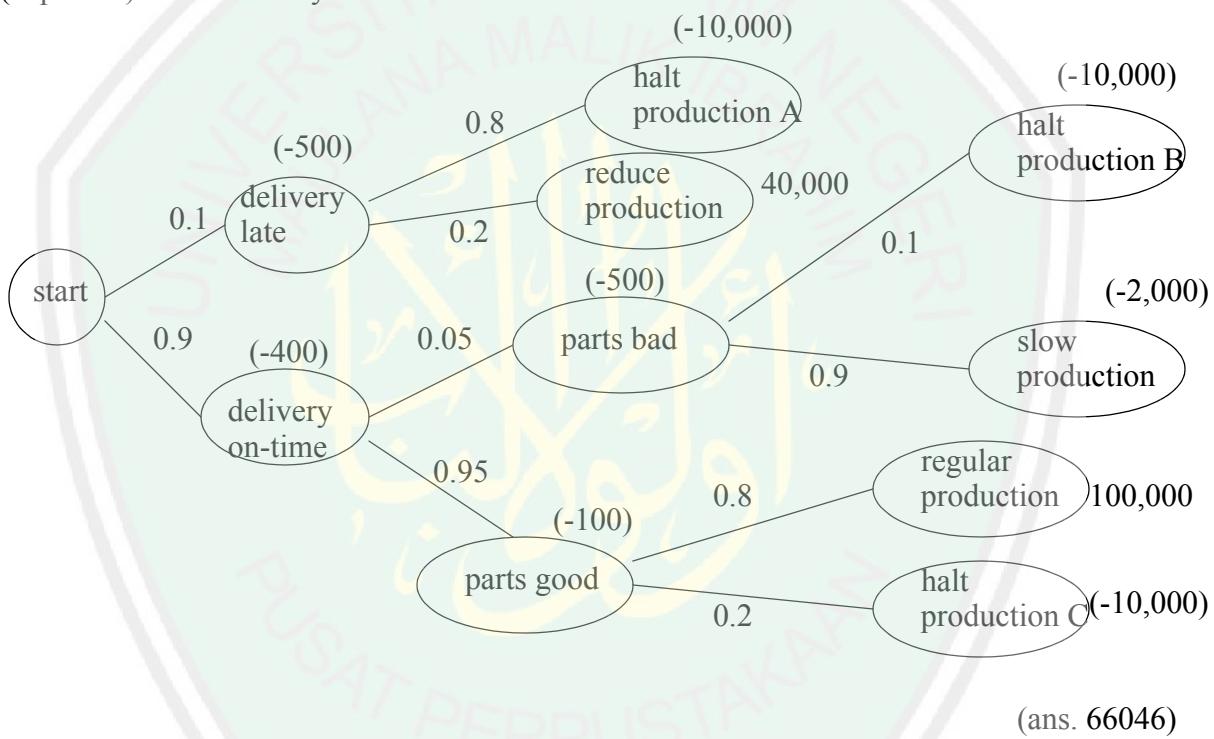
```

## 1.4 Summary

.

## 1.5 Problems

1. 1. Consider the following probability tree for a single day in a production facility. There is a 10% chance that 'delivery late' will occur, and an 90% chance that 'delivery on-time' will occur. The values on the branches indicate the probability of an event occurring. For example, if there is 'delivery on-time' there will be a net cost of \$400 to process the incoming parts. Write a program that will use the probabilities and costs to calculate the effective income (expenses) for the facility.



## 1.6 Challenge Problems