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1. Buktikan bahwa $W = \{[a,b,c] \mid c = 2a\}$ merupakan bagian dari $\mathbb{R}^3$

$\Rightarrow$ misal : $\vec{u}, \vec{v} \in W$

$\vec{u} = [a,b,c] \rightarrow c = 2a$

$\vec{v} = [d,e,f] \rightarrow f = 2d$

$$\vec{u} + \vec{v} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} + \begin{pmatrix} d \\ e \\ f \end{pmatrix} = \begin{pmatrix} a+d \\ b+e \\ c+f \end{pmatrix}$$

$c + f = 2d + 2a$

$c + f = 2(d + a)$

$\therefore \vec{u} + \vec{v} \in W$

misal : k = skalar, $\vec{u} \in W$

$\vec{u} = [a,b,c] \rightarrow c = 2a$

$k\vec{u} = k(c) = k(2a)$

$k(c) = 2ka$

$\therefore k\vec{u} \in W$

Jadi, W ruang bagian dari $\mathbb{R}^3$

2. Buktikan bahwa $W = \{[a,b,c] \mid b = a^2\}$ bukan ruang bagian dari $\mathbb{R}^3$

misal : $\vec{u}, \vec{v} \in W$

$\vec{u}$ $[a,b,c] \rightarrow b = a^2$

$\vec{v}$ $[d,e,f] \rightarrow e = d^2$

$$\vec{u} + \vec{v} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} + \begin{pmatrix} d \\ e \\ f \end{pmatrix} = \begin{pmatrix} a+d \\ b+e \\ c+f \end{pmatrix}$$

$\rightarrow e + b = a^2 + d^2$

$= (a+d)^2$

hasil tidak sama

Jadi W bukan ruang bagian $\mathbb{R}^3$

3. Tentukan apakah himp-2 berikut merupakan bagian dari $\mathbb{R}^2$ / bukan

a. $W = \{[a,b] \mid a - b = 0\}$

misal :

$\vec{u} = [a,b] \rightarrow a - b = 0$

$\vec{v} = [d,e] \rightarrow d - e = 0$

$$\vec{u} + \vec{v} = \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} d \\ e \end{pmatrix} = \begin{pmatrix} a+d \\ b+e \end{pmatrix}$$

$(a+d) - (b+e)$

$a + d - b - e$

$a - b + d - e = 0 + 0 = 0$

misal k skalar, $\vec{u} \in W$

$\vec{u} = [a,b] \rightarrow a - b = 0$

$$k\vec{u} = k\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} ka \\ kb \end{pmatrix}$$

$ka - kb = k(a-b)$

$= k(0)$

$= 0$

$\therefore k\vec{u} \in W$

Jadi W ruang bagian dari $\mathbb{R}^2$

b. $W = \{[a,b] \mid 5ab = 0\}$

misal : $\vec{u}, \vec{v} \in W$

$\vec{u} = [a,b] \rightarrow 5ab = 0$

$\vec{v} = [d,e] \rightarrow 5de = 0$

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$\vec{u} + \vec{v} = \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} d \\ e \end{pmatrix} = \begin{pmatrix} a+d \\ b+e \end{pmatrix}$

$5(a+d)(b+e) = 5(ab + ae + bd + de)$

$= 5(0 + ae + bd + 0)$

$5(a+d)(b+e) \neq 0$

$\therefore \vec{u} + \vec{w} \notin w$

jadi w bukan ruang bagian dari $\mathbb{R}^2$

c. $W = \{[a,b] | a = 4+b\}$

misal

$\vec{u}, \vec{v} \in W$

$\vec{u} = [a,b] \rightarrow a = 4+b$

$\vec{v} = [d,e] \rightarrow d = 4+e$

$\vec{u} + \vec{v} = \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} d \\ e \end{pmatrix} = \begin{pmatrix} a+d \\ b+e \end{pmatrix}$

$a+d = 4+b + 4+e$

$4 + (d+e) \neq 4+b+4+e$

$\therefore$ jadi w bukan ruang bagian dari $\mathbb{R}^2$

d. $W = \{[a,b] | b = 2a+1\}$

misal :

$\vec{u}, \vec{v} \in W$

$\vec{u}\ [a,b] \rightarrow b = 2a+1$

$\vec{v} = [d,e] \rightarrow e = 2d+1$

$\vec{u} + \vec{v} = \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} d \\ e \end{pmatrix} = \begin{pmatrix} a+d \\ b+e \end{pmatrix}$

$b+e = (2a+1) + (2d+1)$

$b+e = 2(a+d) + 2$

$2(a+d) + 1 \neq 2a+d+2$

$\therefore$ jadi w $\neq$ ~~bag~~ ruang bagian dari $\mathbb{R}^2$

4. tentukan apakah himp² berikut merupakan ruang bagian dari $\mathbb{R}^3$

a. $W = \{[a,b,c] | a+b = 2c\}$

misal

$\vec{u}, \vec{v} \in W$

$\vec{u} = [a,b,c] \rightarrow a+b = 2c$

$\vec{v} = [d,e,f] \Rightarrow d+e = 2f$

$\vec{u} + \vec{v} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} + \begin{pmatrix} d \\ e \\ f \end{pmatrix} = \begin{pmatrix} a+d \\ b+e \\ c+f \end{pmatrix}$

$a+b+d+e = 2c+2f$

$2(c+f) = 2c+2f$

$\therefore \vec{u} + \vec{v} \in W$

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Misal $= k$ skalar, $\vec{u} \in w$

$\vec{u} = [a, b, c] \rightarrow a+b = 2c$.

$k\vec{u} = k\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} ka \\ kb \\ kc \end{pmatrix}$

$ka + kb = k(a+b) \Rightarrow k(2c) \Rightarrow 2(kc)$

$\therefore k\vec{u} \in w$.

Jadi w ruang bagian dari $\mathbb{R}^3$

b. $w = \{[a, b, c] \mid a = b = c\}$

Misal

$\vec{u}, \vec{v} \in w$

$\vec{u} = [a, b, c] \rightarrow a = b = c$.

$\vec{v} = [d, e, f] \rightarrow d = e = f$

$\vec{u} + \vec{v} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} + \begin{pmatrix} d \\ e \\ f \end{pmatrix} = \begin{pmatrix} a+d \\ b+e \\ c+f \end{pmatrix}$

$a+d = b+e = c+f$

$\therefore \vec{u} + \vec{v} \in w$

Misal: k skalar, $\vec{u} \in w$

$\vec{u} = [a, b, c] \rightarrow a = b = c$.

$k\vec{u} = k\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} ka \\ kb \\ kc \end{pmatrix}$

$ka = kb = kc$

$\therefore k\vec{u} \in w$

Jadi w ruang bagian dari $\mathbb{R}^3$

c. $w = \{[a, b, c] \mid b = c - a\}$

Misal: $\vec{u}, \vec{v} = w$

$\vec{u} = [a, b, c] \rightarrow b = c-a$

$\vec{v} = [d, e, f] \rightarrow e = f - d$

$\vec{u} + \vec{v} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} + \begin{pmatrix} d \\ e \\ f \end{pmatrix} = \begin{pmatrix} a+d \\ b+e \\ c+f \end{pmatrix}$

$\therefore \vec{u} + \vec{v} \in w$ $\quad b+e = (c+f) - (a+d)$

Misal k skalar $\vec{u} \in w$

$\vec{u} = [a, b, c] \rightarrow b = c - a$

$k\vec{u} = k\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} ka \\ kb \\ kc \end{pmatrix}$

$kc - ka = k(c - a)$

$k(c-a) = kb$.

$\therefore k\vec{u} \in w$

Jadi w ruang bagian dari $\mathbb{R}^3$

d. $w = \{[a, b, c] \mid b^2 = a^2 + c^2\}$

Misal: $\vec{u}, \vec{v} = w$

$\vec{u} = [a, b, c] \Rightarrow b^2 = a^2 + c^2$

$\vec{v} = [d, e, f] \Rightarrow e^2 = d^2 + f^2$

$\vec{u} + \vec{v} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} + \begin{pmatrix} d \\ e \\ f \end{pmatrix} = \begin{pmatrix} a+d \\ b+e \\ c+f \end{pmatrix}$

$b^2 + e^2 = a^2 + d^2 + d^2 + f^2 \neq (a+d)^2 + (d+f)^2$

$\therefore \vec{u} + \vec{v} \notin w$; Jadi w bukan ruang bagian dari $\mathbb{R}^3$