MATH 239 — Tutorial 11

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Problem Set 3.2

1. (a) *Consider* $c_n = 5c_{n-1} - 3c_{n-2} - 9c_{n-3}$, $n \geq 3$ *with initial conditions* $c_0 = 1, c_1 = 1, c_2 = 29$. *Find* c_n *explicitly*

   Solution

   Rewrite it as $c_n - 5c_{n-1} + 3c_{n-2} + 9c_{n-3} = 0$. By theorem 3.2.1, we define $Q(x) = 1 - 5x + 3x^2 + 9x^3$. We define our characteristic polynomial as $C(x) = x^3 - 5x^2 + 3x + 9 = (x+1)(x-3)^2$. Here, we have two roots $-1,\ 3$ with multiplicities $1,\ 2$ respectively. By theorem 3.2.2,

   $$c_n = (-1)^n A + (3)^n (Bn + C)$$

   Using our initial conditions and stuff, we have:

   $$c_n = 2(-1)^n + (3)^n (2n - 1)$$

   (b) *Find* b_n *explicitly, where* $b_n - 5b_{n-1} + 8b_{n-2} - 4b_{n-3} = 0$ *with initial conditions* $b_0 = b_1 = 2,\ b_2 = 0.$. We define out characteristic polynomial as

   $$C(x) = x^3 - 5x^2 + 8x - 4 = (x-2)^2(x-1)$$

   This polynomial has two roots: $1,\ 2$ with respective multiplicities $1,\ 2$. By theorem 3.2.2, our recurrence relation in closed form is written as

   $$b_n = a + (bn + c)2^n$$

   Solving for these unknown constants gives us the closed form

   $$b_n = (-2)2^n + n2^n$$