MATH 239 — Tutorial 6

Bartosz Antczak *February 15, 2017*

Matchings

Recall that a **matching** is a set of edges that share no end points. A **vertex cover** is a set C of vertices such that every edge has at least one end in the set. For all G, the max matching is less than or equal to the min vertex cover.

Algorithm for max matching in bipartite graphs

1. Begin with any matching M
2. Construct X and Y:
   (a) X_0 is the set of vertices in A that are unsaturated by M
   (b) Z is the set of vertices reachable from X_0 by an alternating path
   (c) $X = A \cap Z$, and $Y = B \cap Z$
3. If there's an unsaturated $v \in Y$, find an augmenting path P ending at v; use it to construct a larger matching M'. Replace M by M' and go to step 2.
4. If every vertex is saturated, then stop. M is a max matching.

Example: Problem Set 8.3 - Q5

On our first iteration, we have:

1. $X_0 = \{1, 2\}$
2. $Z = \{1, 2, 3, 4, 5, a, b, c, d, e\}$ (i.e., every vertex), so $A \cap Z = A = X$ and $B \cap Z = B = Y$ are
3.