AMATH 250 — Lecture 16

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Last time

Method of undetermined coefficients for non-homogeneous DEs. We discussed the 12 steps™ to solving it. Today we'll look at more examples of it.

Example 16.0.1. *Find $y_p(x)$ for $y'' - y' - 2y = \sin x$*

Assume $y_p(x) = A\cos(x) + B\sin(x)$.

$$y_p'(x) = -A\sin(x) + B\cos(x)$$
$$y_p''(x) = -A\cos(x) - B\sin(x)$$

Subbing into our DE and solving for A, B, we get

$$y_p(x) = \frac{1}{10}\cos(x) - \frac{3}{10}\sin(x) \qquad \square$$

Example 16.0.2. *Find $y_p(x)$ for $y'' - y' - 2y = 4x + e^x$*

Let $f(x) = f_1(x) + f_2(x)$ where

$$f_1(x) = 4x$$
$$f_2(x) = e^x$$

Due to linearity, we can write $y_p(x) = y_{p1}(x) + y_{p2}(x)$ where

$$y_{pi}'' + y_{pi}' - 2y_{pi} = f_i(x) \qquad i = 1, 2$$

We know from last lecture that $y_{p1}(x) = 1 - 2x$. For $y_{p2}(x) = De^x = y_{p2}'(x) = y_{p2}''(x)$, we solve for $D = -\frac{1}{2}$, and so our solution is

$$y_p(x) = 1 - 2x - \frac{e^x}{2} \qquad \square$$

Example 16.0.3. *Find $y_p(x)$ for $y'' - y' - 2y = 4xe^x$*

Assume $y_p(x) = (Ax + B)e^x$

$$y_p'(x) = (Ax + A + B)e^x$$
$$y_p''(x) = (Ax + 2A + B)e^x$$

Sub into our DE and solve for A and B, and we get $A = -2$ and $B = -1$

$$y_p(x) = (-2x - 1)e^x$$

Example 16.0.4. *Find $y_p(x)$ for $y'' - y' - 2y = 4xe^{-x}$*

We cannot assume $y_p(x) = (Ax + B)e^{-x}$ because this will yield our LHS $= -3A = 4x$, which is wrong because A is a coefficient, not a function! Be^{-x} repeats the term c_2e^{-x} in $y_h(x)$.
We must discuss some exceptions to the method of undetermined coefficients:

If any term in the assumed $y_p(x)$ repeats a term in $y_h(x)$, then multiply your $y_p(x)$ by a factor of x

So we assume $y_p(x) = (Ax^2 + Bx)e^{-x}$

$$y_p'(x) = (-Ax^2 + 2Ax - Bx + B)e^{-x}$$
$$y_p''(x) = (Ax^2 - 4Ax + Bx + 2A - 2B)e^{-x}$$

Solving, we get $A = -\frac{2}{3}$ and $B = -\frac{4}{9}$ and so

$$y_p(x) = -\left(\frac{2}{3}x^2 + \frac{4}{9}x\right)e^{-x} \qquad \square$$

Example 16.0.5. *Height of baseball thrown upward*

$$\frac{d^2h}{dt^2} = -g$$

Our ICs are $h(0) = 0, h'(0) = v_0$. Using the 12 magic™ steps,

1. $h''(t) = 0$
2. $\lambda^2 = 0$
3. $h_1(t) = e^0 = 1,\ h_2 = t$
4. $h_h(t) = c_1 + c_2t$
5. $h_p(t) = t^2A$ (since tA is already in $h_h(t)$)
6. $h_p''(t) = -g = 2A \implies A = -\frac{1}{2}g$
7. $h(t) = c_1 + c_2t - \frac{1}{2}gt^2 \qquad \square$