# AMATH 250 — Lecture 6

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**Last Time**

Skydiver problem and physical quantities.

## 6.1 More on Physical Quantities

Consistency requirements to be satisfied by all equations having physical content:

1. **Principle of Dimensional Homogeneity:** one may only add, subtract, and equation quantities that have the same dimension
2. Quantities having different dimensions may only be combined by multiplication or division
3. The argument of a function must be dimensionless. For example, consider the value $e^{\frac{\alpha}{m}t}$. The dimension $\frac{\alpha}{m}t$ must have a dimension of [1].
4. The value of each function must also be dimensionless

## 6.2 Newton's Law of Gravitation and the Problem of Escape Velocity

Recall Newton's gravitational equation

$$F = G\frac{Mm}{r^2}$$

where $r = r(t)$ is the distance from the centre of Earth. Also, let $R$ be the radius of the Earth (constant). When $r = R$, then

$$F = -mg$$

and so

$$g = \frac{GM}{R^2}$$

In more general terms, we have

$$\begin{aligned} F &= G\frac{Mm}{r^2} && (6.1) \\ &= m\left(\frac{MG}{R^2}\right)\frac{R^2}{r^2} && (6.2) \\ &= mg\frac{R^2}{r^2} && (6.3) \end{aligned}$$

From here we create our DE for $v(t)$:

$$m\frac{dv}{dt} = mg\frac{R^2}{r^2} \tag{6.4}$$
$$\frac{dv}{dt} = g\frac{R^2}{r^2} \tag{6.5}$$
$$\frac{d^2r}{dt^2} = g\frac{R^2}{r^2} \tag{6.6}$$

Here we have a 2nd-order DE for $r(t)$. We can solve this by making an assumption that $v(t) = v(r(t))$ is a composite function of $t$. We utilize the chain rule:

$$\frac{dv}{dt} = \frac{dv}{dr} \cdot \frac{dr}{dt} = \frac{dv}{dr}v$$

We now sub this value into (6.5):

$$\frac{dv}{dr}v = g\frac{R^2}{r^2} \tag{6.7}$$
$$\int v\ dv = -gR^2 \int \frac{dr}{r^2} \tag{6.8}$$
$$\frac{1}{2}v^2 = g\frac{R^2}{r} + c \tag{6.9}$$