AMATH 250 — Tutorial 2

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2.1 Application of DEs

Example 2.1.1. *Consider a pond that initially contains 10 million gallons of fresh water. Water containing an undesirable chemical flows into the pond at the rate of 5 million gal/year, and the mixture in the pond flows out at the same rate. The concentration $\gamma(t)$ of chemical in the incoming water varies periodically with time according to the expression $\gamma(t) = 2 + \sin(2t)\frac{g}{gal}$. Construct a mathematical model of this flow process and determine the amount of chemical in the pond at any time.*

Define $Q(t)$ as the amount of chemical in the tank at time t. By this definition, we see that $\frac{dQ}{dt}$ = rate coming in - rate coming out. This can be modelled as

$$\frac{dQ}{dt} = (5 \times 10^6 \frac{gal}{yr})\gamma(t) - (5 \times 10^6)\left(\frac{Q(t)}{10^7}\right) \tag{2.1}$$

$$= (5 \times 10^6)(2 + \sin(2t)) - \frac{Q(t)}{2} \tag{2.2}$$

Let $q(t) = \frac{Q(t)}{10^6}$

$$\frac{dq}{dt} + \frac{1}{2}q = 10 + 5\sin(2t)$$

From here, simply solve the DE. That's the easy part — constructing the DE is the hard part.