# MATH 239 — Tutorial 10

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## Problem Set 2.8

3. Write an unambiguous expression generating the the following binary strings:

   (a) $\{0,1\}-$*strings with no substrings of 0s of length 3 and no substrings of 1s of length 2*

   **Solution**

   $$S = \{\varepsilon, 1\} \quad (\{0, 00\}\{1\})^* \quad \{\varepsilon, 0, 00\}$$

   (b) $\{0,1\}-$*strings with no 0 blocks of length 3 and no 1 blocks of length 2*

   **Solution**

   $$S = \{\varepsilon, 1, 111, 1111, \cdots\} \quad (\{0, 00, 0000, \cdots\}\{1, 111, 1111\})^* \quad \{\varepsilon, 0, 00, 0000, \cdots\}$$

   (c) *The set of binary string in which 011 doesn't occur*

   **Solution (using more star notation in this one)**

   $$S = \{1\}^* \quad (\{0\}\{0\}^*\{1\})^* \quad \{0\}^*$$

   (d) Skipped this one because Prof. Postle thought it was too hard.

   (e) *The set of binary strings where the 0 blocks have even length and the 1 blocks have odd length*

   **Solution**

   Other than maybe the first and last pieces, if we break up our strings into pieces at the end of each 0 block we get pieces of the form: $\{1, 111, 11111, \cdots\}\{00, 0000, \cdots\}$:

   $$S = \{\varepsilon, 00, 0000, \cdots\} \quad (\{1, 111, 11111, \cdots\}\{00, 0000, \cdots\})^* \quad \{\varepsilon, 1, 111, 11111, \cdots\}$$

   Simplified (by using more star notation), we get:

   $$\{00\}^* \quad (\{1\}\{11\}^*\{00\}^*)^* \quad \{\varepsilon, 1, 111, 11111, \cdots\}$$

   (f) *Set of all binary strings where*

   - *each odd length block of 0s is followed by a non-empty even length block of 1s,* **and**
   - *each even length block of 0s is followed by an odd length block of 1s*

**Solution**

Keep in mind we don't consider blocks of length 0, because they're not blocks. So it's safe to assume that all of these blocks are non-empty.

We'll decompose our strings after blocks of 1s. With perhaps the exception of the first and last pieces, our pieces will look like:

i) $\{0, 000, 00000, \cdots\}\{11, 1111, 111111, \cdots\} = \{0\}\{00\}^*\{11\}\{11\}*$

ii) $\{00, 0000, \cdots\}\{1, 111, 11111, \cdots\} = \{00\}\{00\}^*\{1\}\{11\}^*$

Our expression will look like:

$$S = \{1\}^* \quad (\{0\}\{00\}^*\{11\}\{11\}^* \cup \{1\}\{11\}^*\{00\}\{00\}^*)^*$$

5. (a) Show that the generating series by length (i.e., the weight function is the length of the string) for binary strings in which every block of zeros has length $\geq 2$ and every block of ones has length $\geq 3$ is

$$\frac{(1-x-x^3)(1-x+x^2)}{1-2x+x^2-x^5}$$

**Solution**

Our expression for this set of binary strings is:

$$S = \{\varepsilon, 111, 1111, \cdots\} \quad (\{00\}\{0\}^*\{111\}\{1\}^*)^*\{\varepsilon, 00, 0000, \cdots\}$$

We define the generating series of each part of the expression (begin, mid, end):

$$\Phi_{begin}(x) = 1 + x^3 + x^4 + \cdots = \frac{1}{1-x} - x - x^2$$

$$\Phi_{end}(x) = \left(\frac{1}{1-x}\right) x^2 \left(\frac{1}{1-x}\right) x^3 = y = \frac{1}{1-y}\frac{x^5}{(1-x)^2}$$

$$\Phi_{end}(x) = 1 + x^2 + x^3 + \cdots = \frac{1}{1-x} - x$$