APMA 4301: Problem Set 4b

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The number of ksp iterations for each solver and solver time per degree of freedom (N^2) for $N = 8, 16, 32, 64$ can be found in the below table. The relative errors for $N = 64$ can be seen in the first graph. The second graph shows the scaling of the truncation error with problem size.

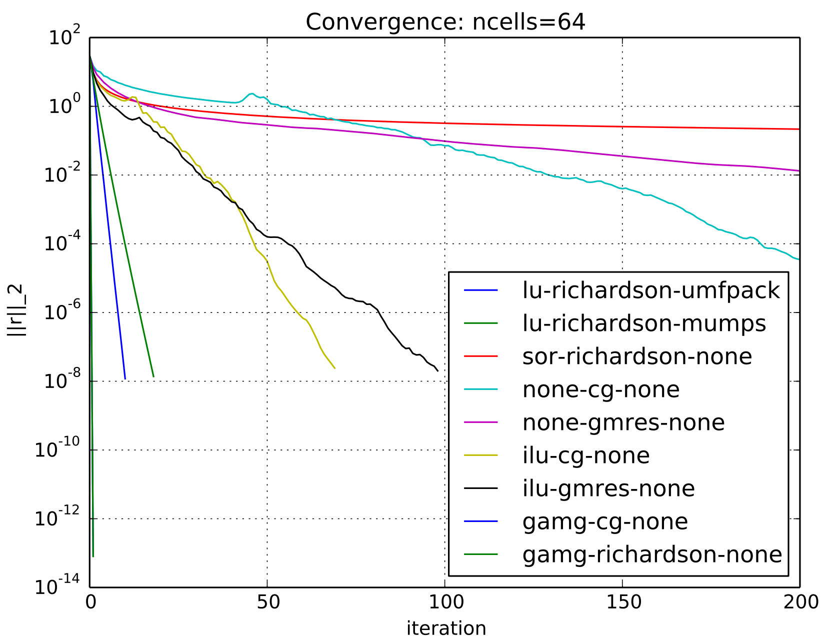
From the first graph, we can see that the LU preconditioned Richardson solvers converge in one step. The two GAMG solvers converge the quickest (in terms of iterations) out of the iterative solvers with CG slightly faster than Richardson. The next quickest are the ILU preconditioned methods, with CG slightly faster than GMRES. CG with no preconditioner is converging to the solution slowly, while unpreconditioned GMRES and SOR preconditioned Richardson both do not appear to be converging (or at least are converging very very slowly).

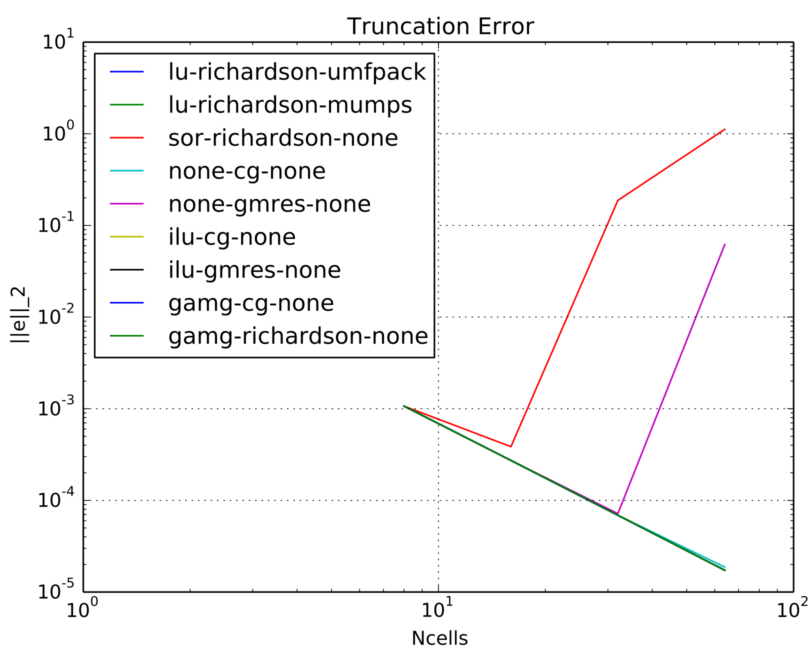
From the second graph, we see that nearly all the solvers converge to the exact solution with very similar errors. The only outliers are SOR preconditioned Richardson, which diverges for $N = 32, 64$, and GMRES with no preconditioner, which diverges only for $N = 64$.

We can see that although GMRES with no preconditioner for $N = 32$ and CG with no preconditioner for $N = 64$ took all 200 iterations and did not have $||r||_2$ <1e-6, they still had truncation errors in line with all the other solvers. Both had relative error of order 1e-5, indicating that we may only need to choose a relative error of 1e-5 instead of 1e-6.

From the table, we see that the LU solvers converge the quickest. Out of the iterative methods, the two GAMG solvers converge the quickest. Since this problem is not very large, the LU sparse direct solvers are the optimal way to solve the problem, with umfpack being the faster of the two. However, as the size of the problem increases, the GAMG preconditioned solvers begin to perform similarly to the LU solvers, indicating that these may be the optimal choice for larger problems.

Solver	nCells	nIts	$\|\|r\|\|_2$	$\|\|e\|\|_2$	KSP time/DoF (s)
lu-richardson-umfpack	8	1	5.884e-15	1.067729e-03	1.037e-05
lu-richardson-umfpack	16	1	1.475e-14	2.729133e-04	2.910e-06
lu-richardson-umfpack	32	1	3.710e-14	6.885276e-05	9.990e-07
lu-richardson-umfpack	64	1	8.125e-14	1.728282e-05	2.458e-07
lu-richardson-mumps	8	1	9.491e-15	1.067729e-03	3.358e-05
lu-richardson-mumps	16	1	2.096e-14	2.729133e-04	1.492e-05
lu-richardson-mumps	32	1	3.694e-14	6.885276e-05	3.105e-05
lu-richardson-mumps	64	1	8.026e-14	1.728282e-05	2.900e-05
sor-richardson-none	8	113	4.999e-09	1.067731e-03	1.468e-03
sor-richardson-none	16	200	1.303e-04	3.854372e-04	6.916e-04
sor-richardson-none	32	200	7.146e-02	1.874269e-01	1.478e-04
sor-richardson-none	64	200	2.162e-01	1.117586e+00	1.055e-04
none-cg-none	8	34	9.482e-09	1.067729e-03	3.078e-04
none-cg-none	16	70	1.331e-08	2.729133e-04	2.669e-04
none-cg-none	32	138	2.076e-08	6.885277e-05	7.127e-05
none-cg-none	64	200	3.478e-05	1.869412e-05	1.344e-04
none-gmres-none	8	35	6.620e-09	1.067729e-03	1.561e-04
none-gmres-none	16	108	1.449e-08	2.729212e-04	4.288e-04
none-gmres-none	32	200	2.417e-06	7.169367e-05	3.126e-04
none-gmres-none	64	200	1.321e-02	6.188559e-02	1.254e-04
ilu-cg-none	8	12	6.463e-09	1.067729e-03	1.684e-04
ilu-cg-none	16	20	9.301e-09	2.729134e-04	5.261e-05
ilu-cg-none	32	35	2.147e-08	6.885273e-05	4.023e-05
ilu-cg-none	64	69	2.410e-08	1.728281e-05	2.271e-05
ilu-gmres-none	8	12	6.800e-09	1.067729e-03	9.381e-05
ilu-gmres-none	16	20	1.120e-08	2.729134e-04	3.668e-05
ilu-gmres-none	32	38	1.063e-08	6.885300e-05	3.229e-05
ilu-gmres-none	64	98	2.011e-08	1.728412e-05	7.492e-05
gamg-cg-none	8	8	1.452e-08	1.067729e-03	1.287e-04
gamg-cg-none	16	9	8.700e-09	2.729134e-04	3.658e-05
gamg-cg-none	32	9	5.198e-08	6.885250e-05	7.754e-06
gamg-cg-none	64	10	1.188e-08	1.728281e-05	8.274e-06
gamg-richardson-none	8	14	1.836e-08	1.067730e-03	1.029e-04
gamg-richardson-none	16	18	5.672e-09	2.729139e-04	3.380e-05
gamg-richardson-none	32	18	1.536e-08	6.885382e-05	3.172e-05
gamg-richardson-none	64	18	1.377e-08	1.728375e-05	6.999e-06


Convergence: ncells=64
||r||_2
iteration
lu-richardson-umfpack
lu-richardson-mumps
sor-richardson-none
none-cg-none
none-gmres-none
ilu-cg-none
ilu-gmres-none
gamg-cg-none
gamg-richardson-none


Truncation Error
||e||_2
Ncells
lu-richardson-umfpack
lu-richardson-mumps
sor-richardson-none
none-cg-none
none-gmres-none
ilu-cg-none
ilu-gmres-none
gamg-cg-none
gamg-richardson-none