A ML model to simulate 3D turbulence in a 2D system applied to wake flows

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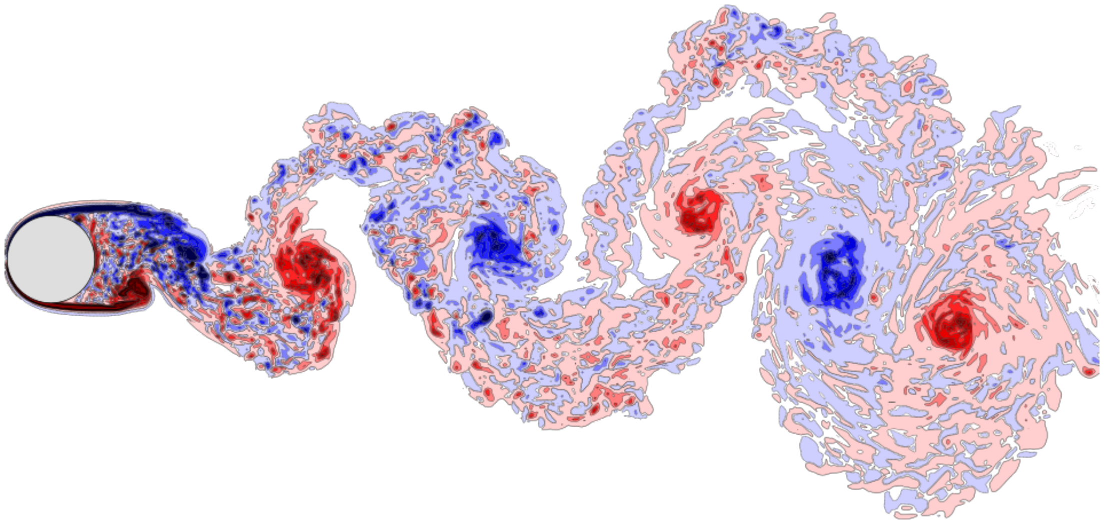
PPPL Computer Science Department's Machine Learning lunch seminar series, 1st September 2021



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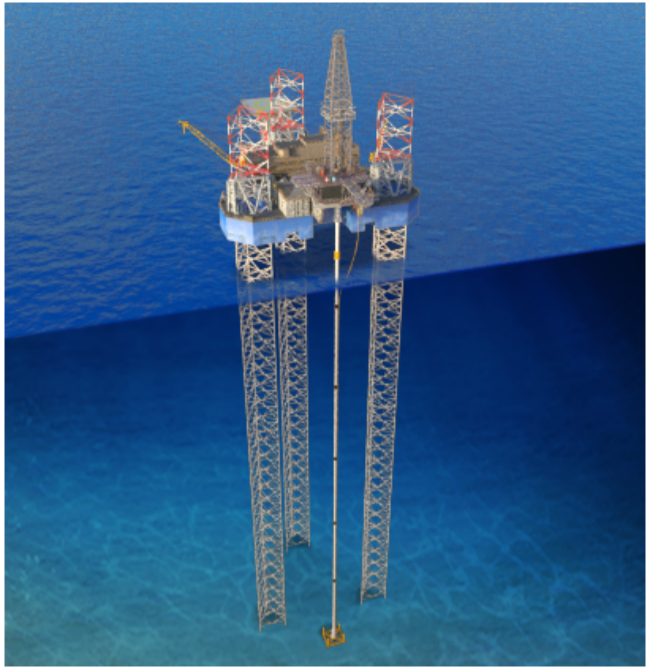
Simulating flow past long flexible structures is computationally expensive



◦ Marine riser

[https://oilstates.com]

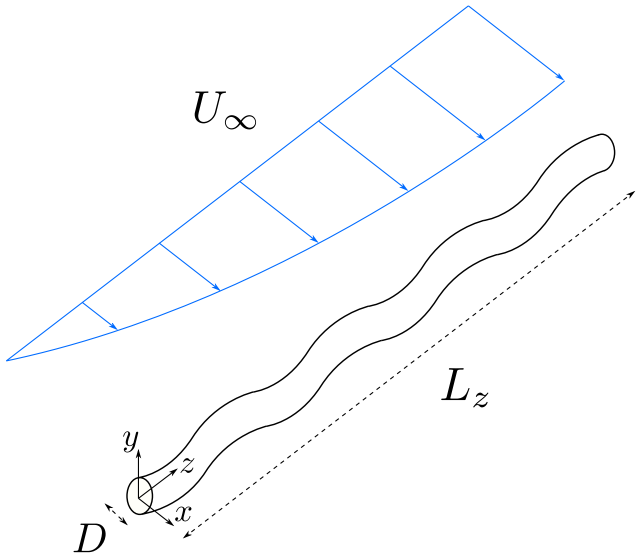
◦ Tall pilar

[http://www.ewea.org]

◦ Aircraft wing

[https://en.wikipedia.org/wiki/Aspect_ratio_(aeronautics)]

It is currently unfeasible to fully resolve all spatial and temporal scales

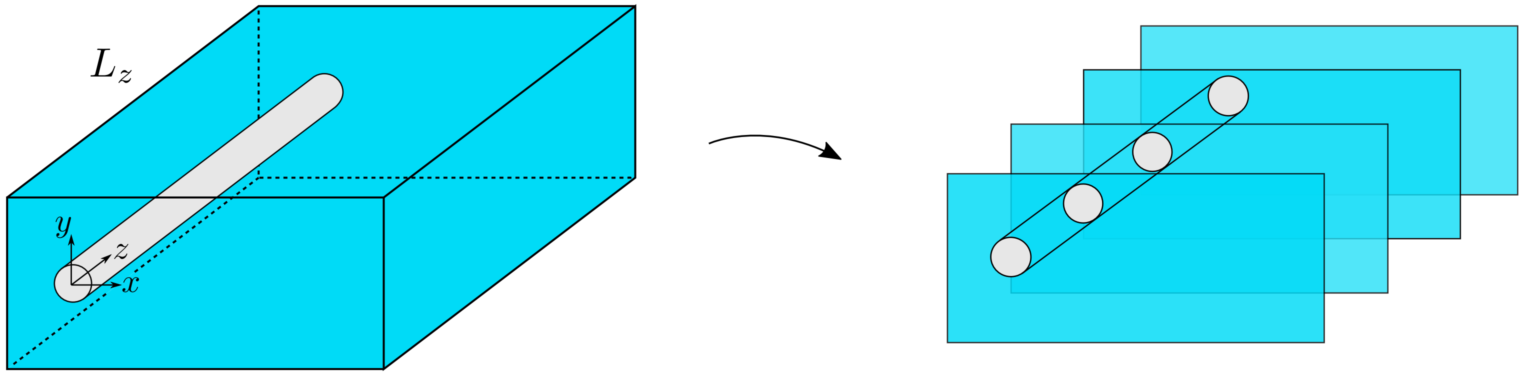


Marine risers:

- $L_z/D > 1000$
- $Re \sim 10^3 - 10^6$
- Vortex-induced vibrations

- **2-D strip theory methods** are used alternatively.

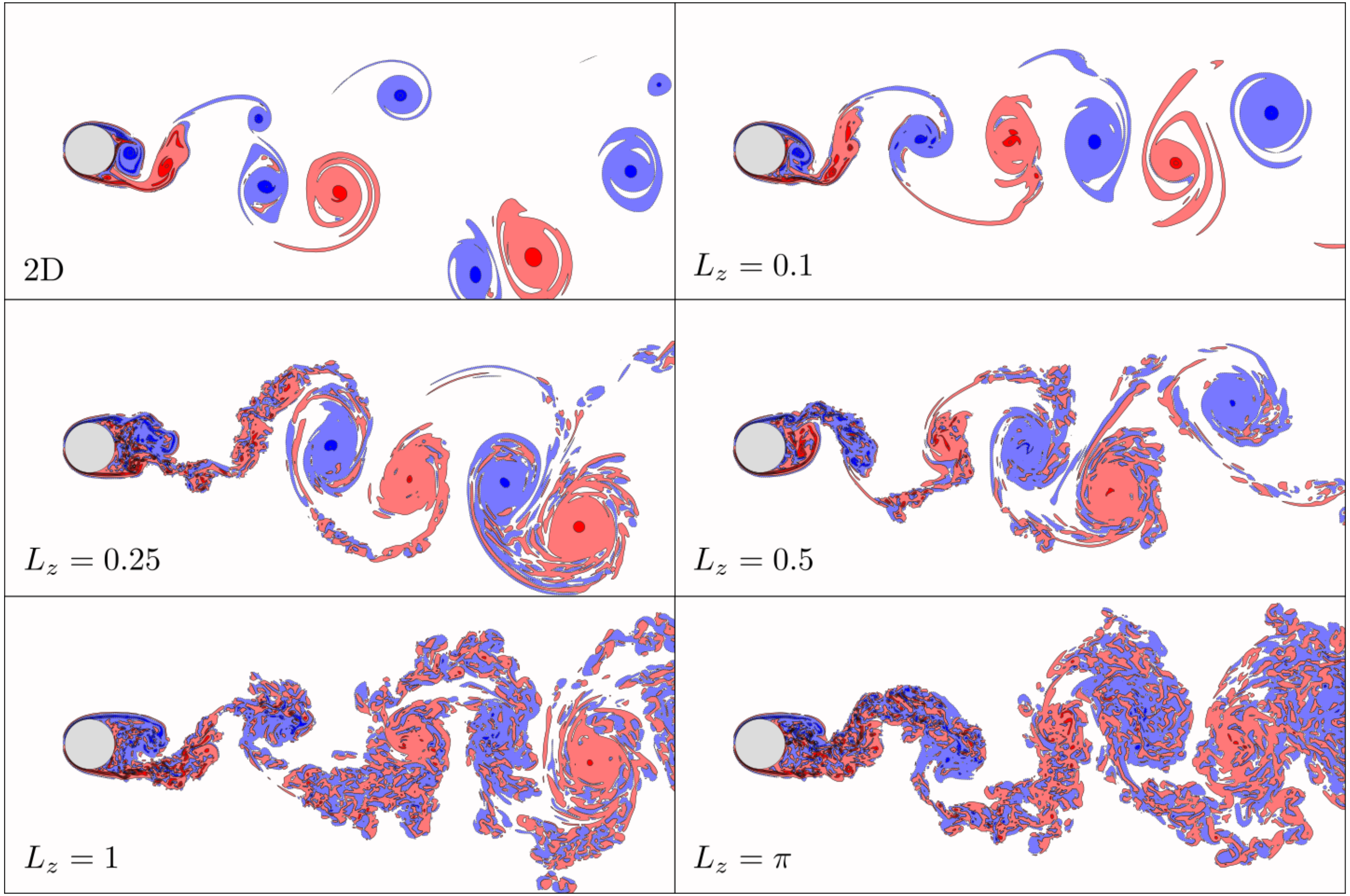
2-D strip-theory methods miss the 3-D turbulence physics



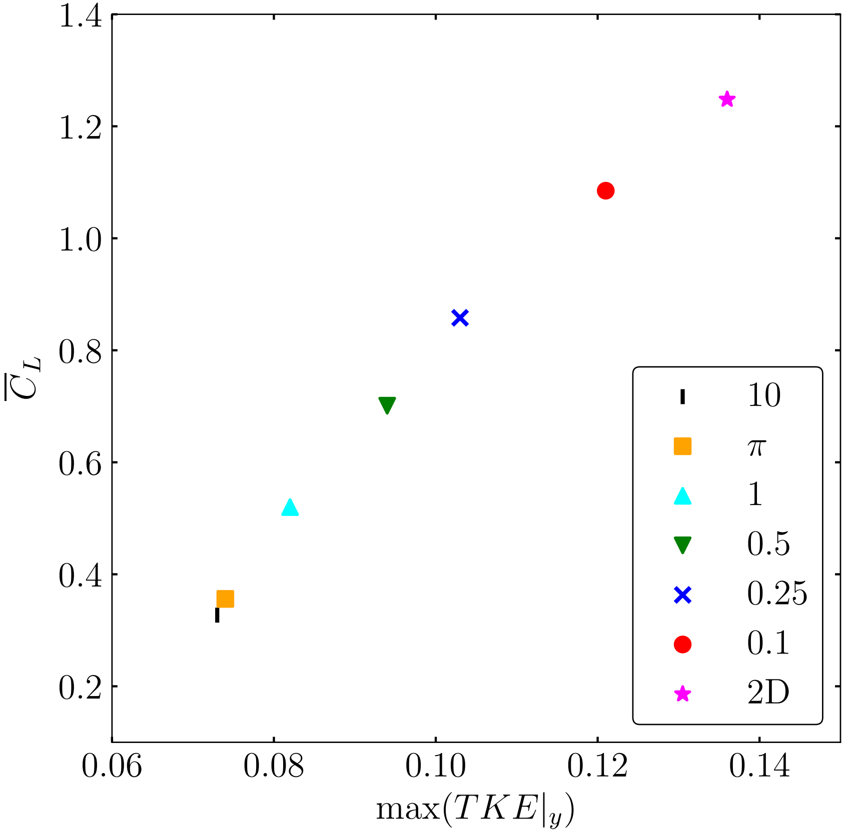
- Different flow physics arising from 3-D vs 2-D turbulence dynamics.
- 2-D simulations over-predict the forces induced to the cylinder.[1] ▶ movie

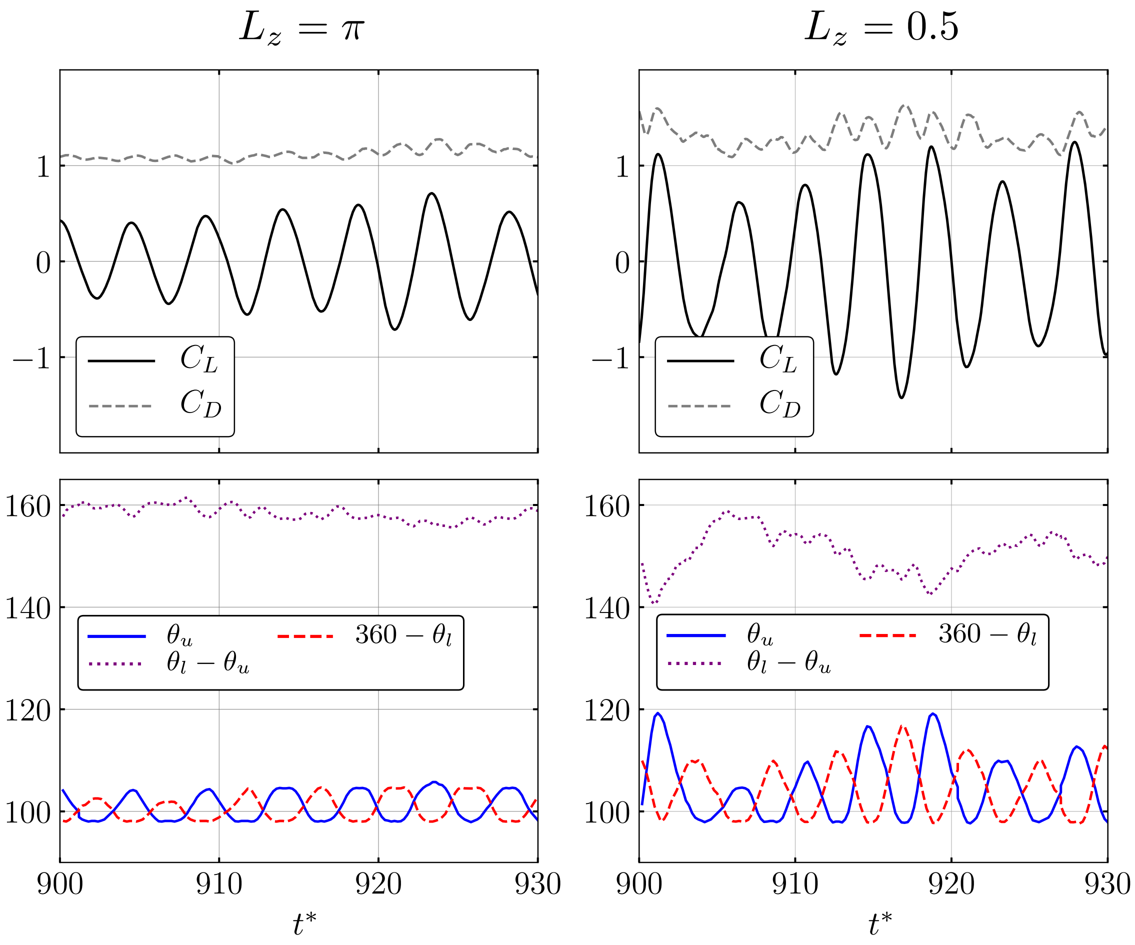
[1] Font, B., Weymouth, G., Nguyen, V., & Tutty, O. (2019). Span effect on the turbulence nature of flow past a circular cylinder. J. Fluid Mech. 878, 306-323.

Is there a critical span for the onset of 2-D turbulence?


2D
$L_z = 0.1$
$L_z = 0.25$
$L_z = 0.5$
$L_z = 1$
$L_z = \pi$

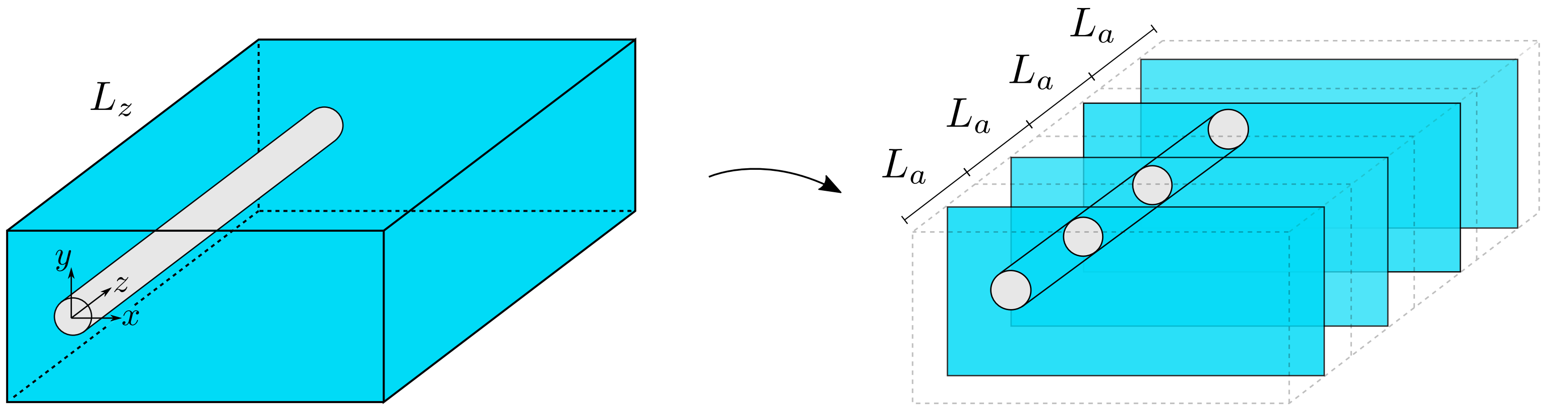
Forces induced to the cylinder are over-predicted when 2-D turbulence dominates

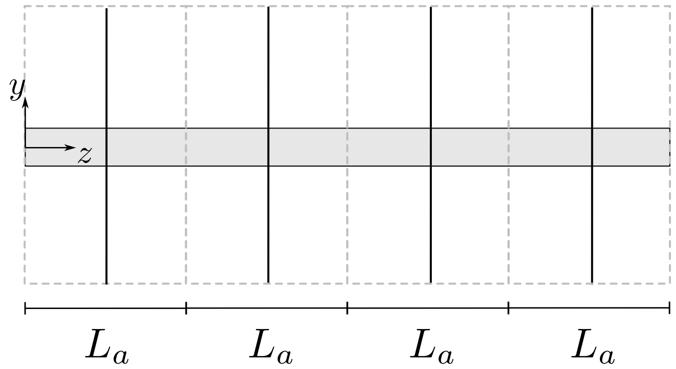




So how can we incorporate the 3-D turbulence effect into 2-D systems?

The spanwise-averaged Navier–Stokes (SANS) equations





- 3-D effects can be can be recovered in 2-D with an additional source term[2]:

$$\langle \text{3-D} \rangle = \text{2-D} + \mathcal{S}^R$$

[2] Font, B., Weymouth, G., Nguyen, V., & Tutty, O. (2020) Deep learning of the spanwise-averaged Navier–Stokes equations. J. Comput. Phys. 434, 110199.

SANS derivation

- Flow decomposition

  $u = \langle u \rangle + u'$

- Spanwise-average operator

  $\langle u \rangle (x, y, t) = \frac{1}{L_a} \int_0^{L_a} u(x, y, z, t)\, \mathrm{d}z$

$$\text{3-D NS:} \quad \partial_t \mathbf{u} + \overbrace{\mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + Re^{-1} \nabla^2 \mathbf{u}}^{\mathcal{S}}$$

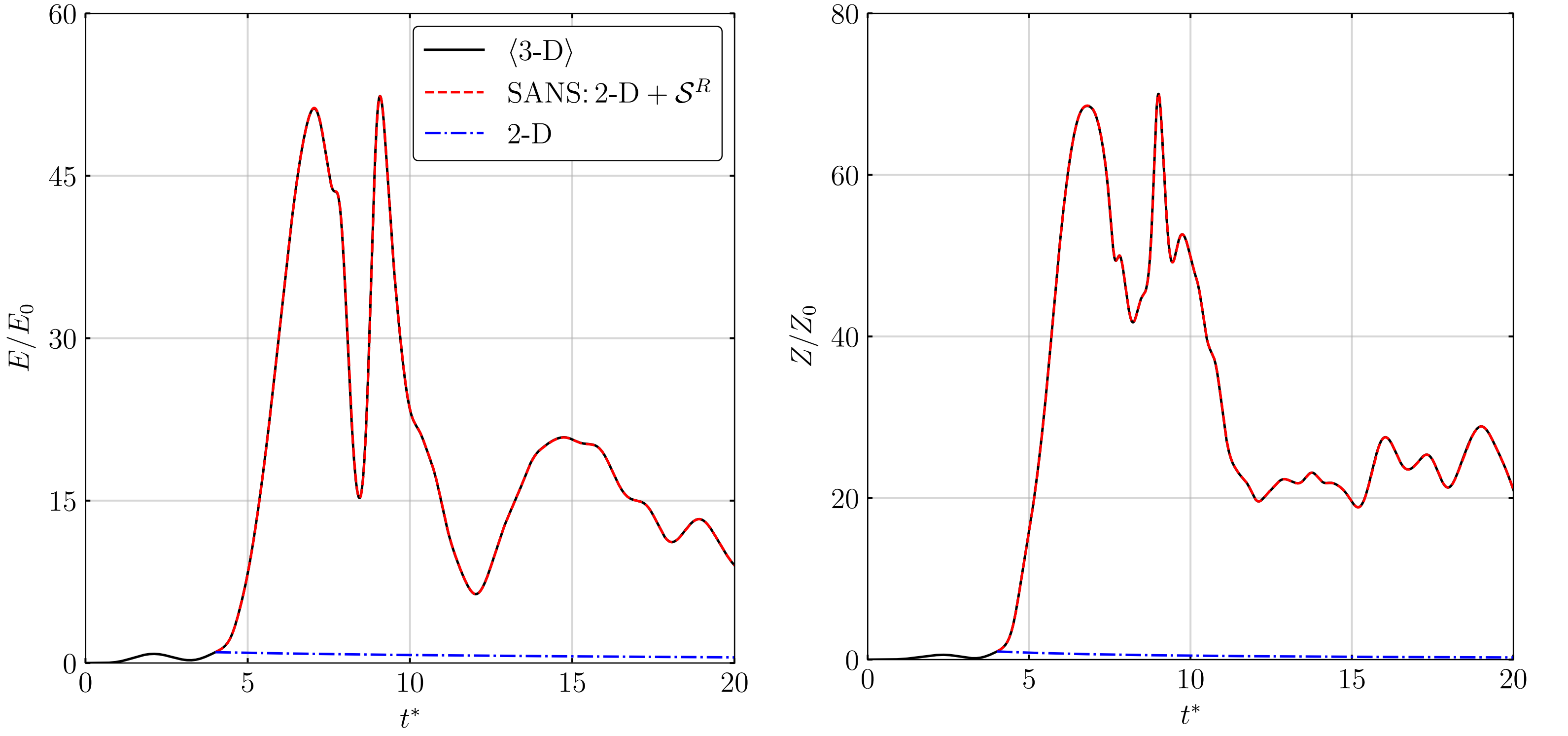
$$\Big\downarrow \ \text{decomposition + spanwise average}$$

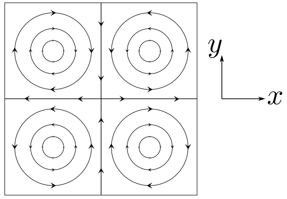
$$\text{2-D SANS:} \quad \partial_t \langle \mathbf{u} \rangle + \overbrace{\langle \mathbf{u} \rangle \cdot \nabla \langle \mathbf{u} \rangle = -\nabla \langle p \rangle + Re^{-1} \nabla^2 \langle \mathbf{u} \rangle}^{\tilde{\mathcal{S}}} - \nabla \cdot \boldsymbol{\tau}_{ij}^R$$

$$\nabla \cdot \boldsymbol{\tau}_{ij}^R = \nabla \cdot \begin{pmatrix} \langle u'u' \rangle & \langle u'v' \rangle \\ \langle u'v' \rangle & \langle v'v' \rangle \end{pmatrix} \qquad \mathcal{S}^R = \tilde{\mathcal{S}} - \langle \mathcal{S} \rangle \equiv -\nabla \cdot \boldsymbol{\tau}_{ij}^R$$

- $\nabla \cdot \boldsymbol{\tau}_{ij}^R$ is the spanwise-stress redisual (SSR) tensor.
- $\mathcal{S}^R$ is the perfect residual closure: $\partial_t \langle \mathbf{u} \rangle + \tilde{\mathcal{S}} = \tilde{\mathcal{S}} - \langle \mathcal{S} \rangle$

SANS on the Taylor–Green vortex predicts the unsteady spanwise-averaged flow

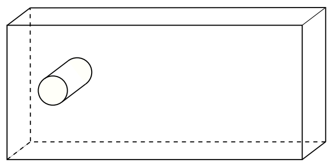


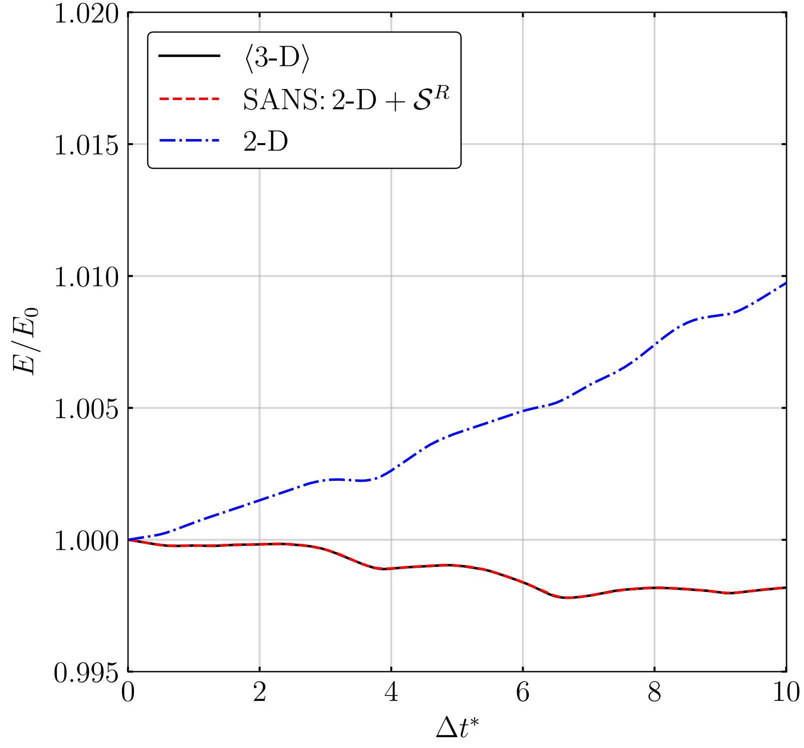


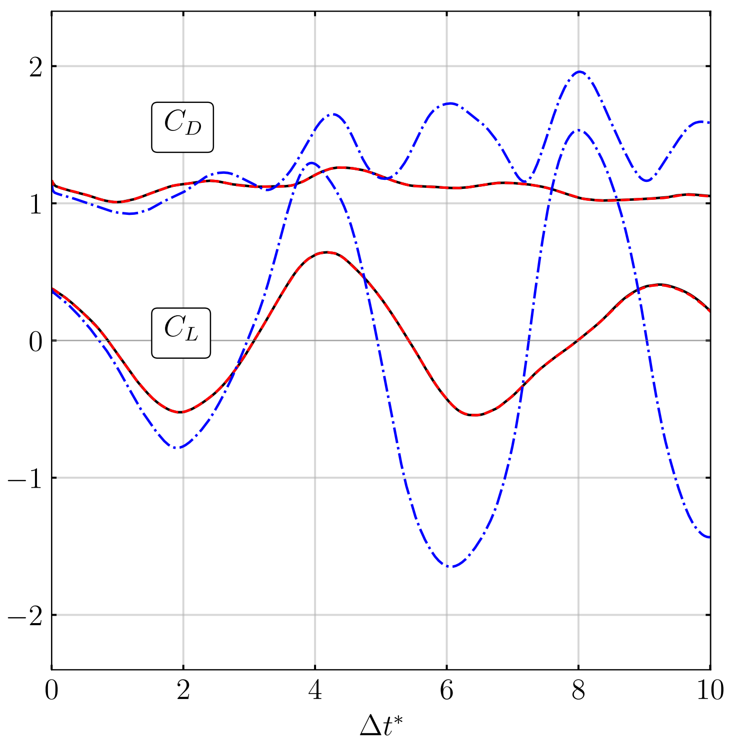
▶ movie

- The perfect closure recovers the spanwise-averaged flow.
- Kinetic energy (E) and enstrophy (Z) (2-D inviscid invariants) are no longer conserved.

SANS on flow past a circular cylinder at Re=10000



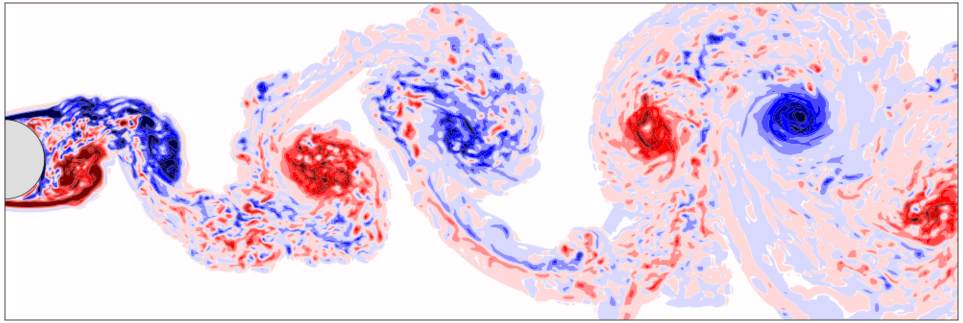




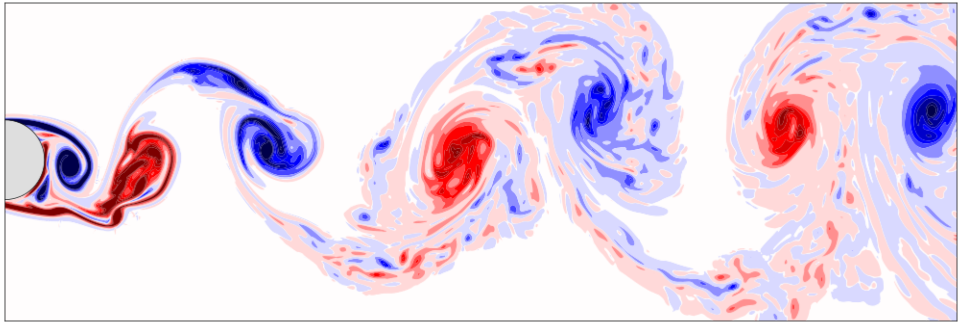
- SANS recovers the spanwise-averaged flow & forces induced to the cylinder as in a 3-D simulation.

SANS on flow past a circular cylinder at Re=10000

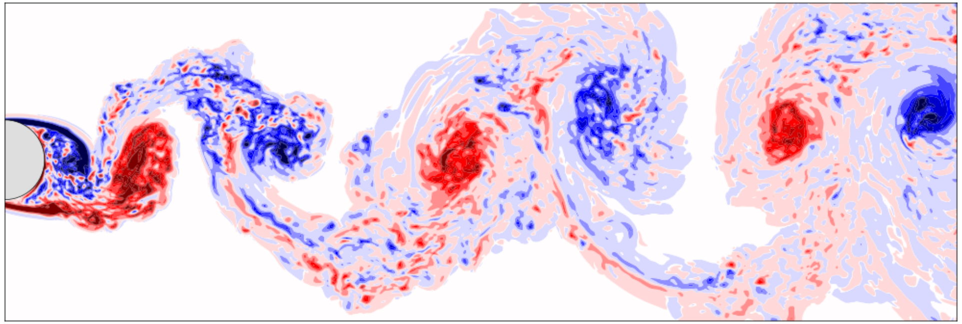
$\langle$3-D$\rangle$ (t_0^*)



2-D $(\Delta t^* = 2)$



SANS : 2-D + $\mathcal{S}^R$ $(\Delta t^* = 2)$



Modelling the SANS closure terms

Classical turbulence models fail to predict the SANS stresses

$$\mathcal{S}^R = \tilde{\mathcal{S}} - \langle \mathcal{S} \rangle \equiv -\nabla \cdot \boldsymbol{\tau}^R_{ij}$$

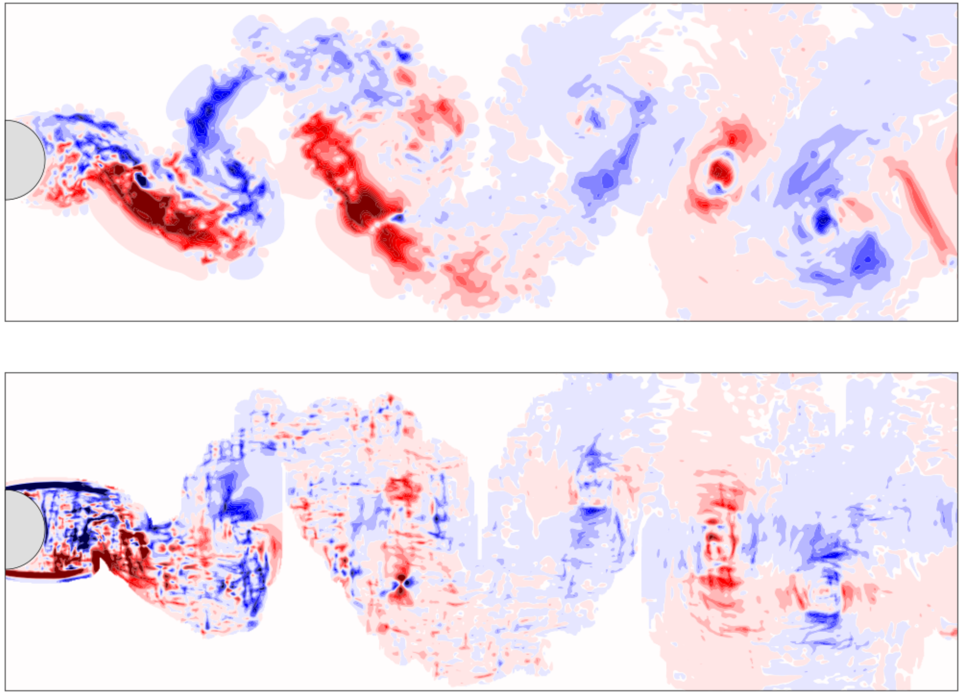
$$\boldsymbol{\tau}^r_{ij} \equiv \langle \boldsymbol{u}' \otimes \boldsymbol{u}' \rangle - \frac{2}{3} k \delta_{ij} = -2\nu_t \langle S_{ij} \rangle$$

$$\nu_t = (C_s \Delta)^2 \sqrt{2 \langle S_{ij} \rangle \langle S_{ij} \rangle}$$

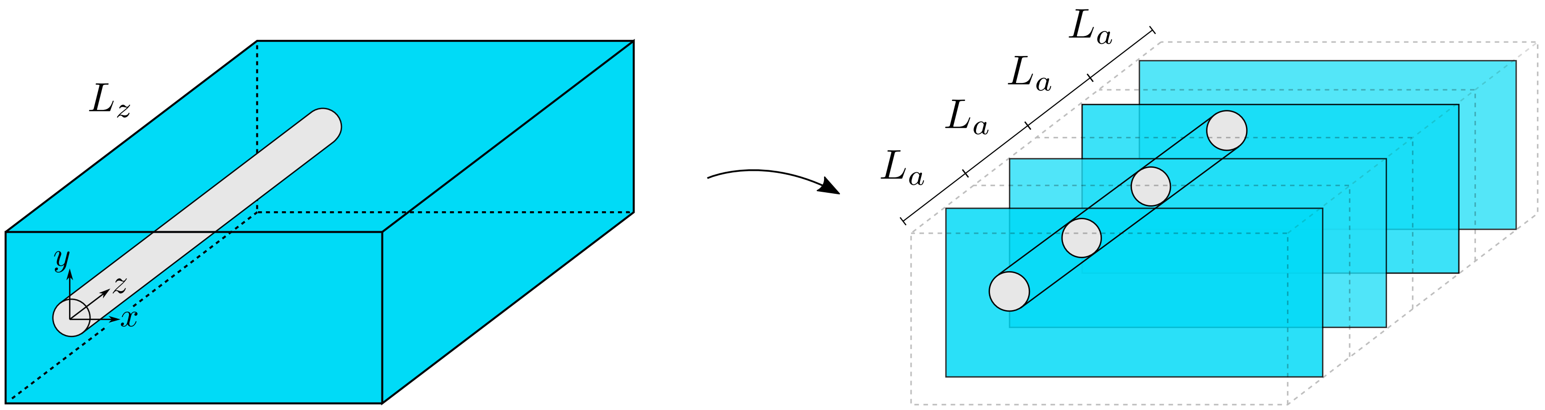
- Eddy-viscosity models (EVM): $\boldsymbol{\tau}^r_{ij} = f(\nu_t, \boldsymbol{U})$

$\boldsymbol{\tau}^r_{12}$

$\boldsymbol{\tau}^{r,\mathrm{EVM}}_{12}$



The SANS-based strip-theory framework



$\{\boldsymbol{U}, \mathcal{S}^R, \boldsymbol{\tau}_{ij}^R\}$ → f_{CNN} → SANS

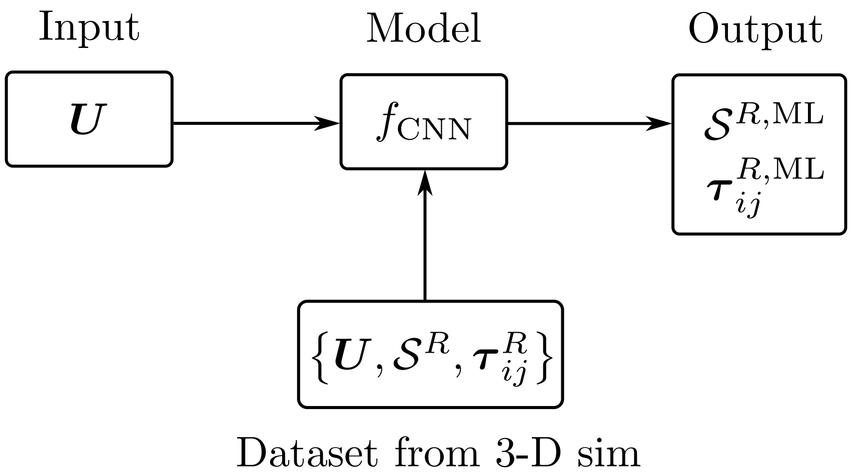
Generate dataset in L_a 3-D simulation

Train CNN

Solve SANS at every strip

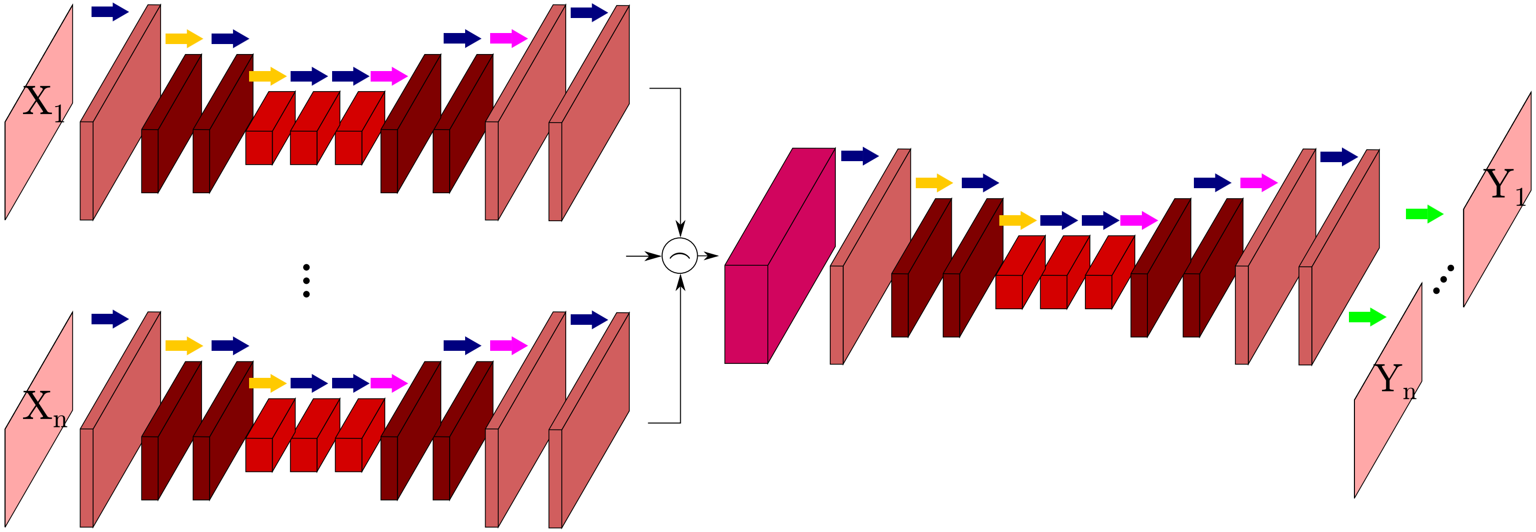
7000 snapshots (1216x540), 85-15 split

Modelling the SANS stresses through a deep convolutional neural network (CNN)



- The CNN can reveal deep correlations for which physics are a-priori unknown.
- CNN translation invariance is important for flow field modelling.

Modelling the SANS stresses through a deep convolutional neural network (CNN)



- Conv2D 5x5 + Batch normalisation + ReLU function
- MaxPooling 2x2
- UpSampling 2x2
- Conv2D 1x1 + Batch normalisation + Linear function

∘ $\mathrm{X}_n = \{\langle p\rangle, \langle u\rangle, \langle v\rangle\}$

∘ $\mathrm{Y}_n = \{\langle u'u'\rangle, \langle u'v'\rangle, \langle v'v'\rangle\}$

$\mathrm{Y}_n = \{\mathcal{S}_x^R, \mathcal{S}_y^R\}$

∘ $\mathrm{Loss} = \sum_{i,j=1}^{n,m} \left(\mathrm{Y}_{i,j} - \mathrm{Y}_{i,j}^{\mathrm{ML}}\right)^2$

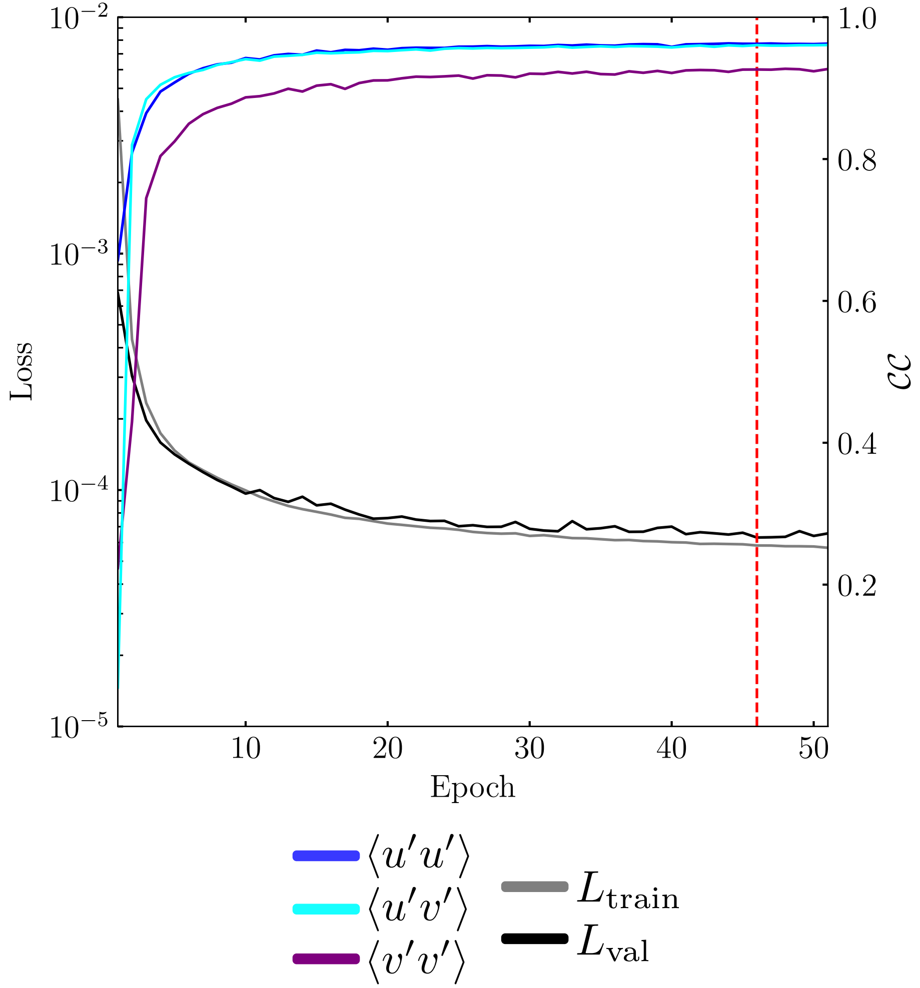
Font, B., Weymouth, G., Nguyen, V., & Tutty, O. (2020). Turbulent wake prediction using deep convolutional neural networks. In 33rd Symposiumon Naval Hydrodynamics. Office of Naval Research, US

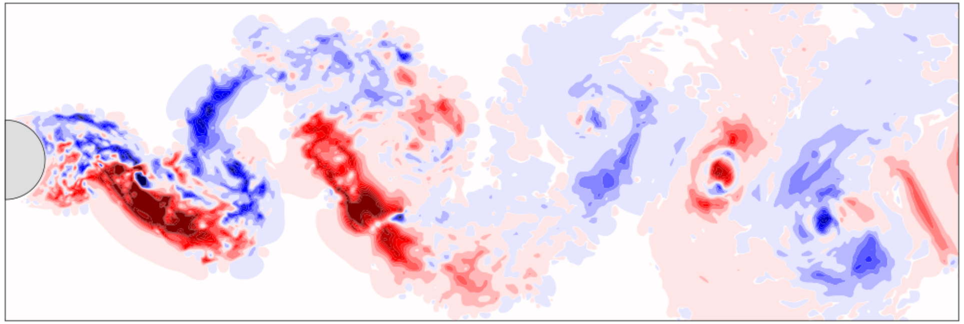
Hyper-parametric study of the CNN

Architecture	Loss	Activation	# Filters	Kernel size	Normalization	Input set
M1	SSE	ReLU	16	3x3	True	$\{U, V, P\}$
M2	SAE	sigmoid	32	5x5	False	$\{\Omega_z, d\}$
M3		tanh	64	7x7		$\{\nabla \boldsymbol{U}, \nabla P\}$

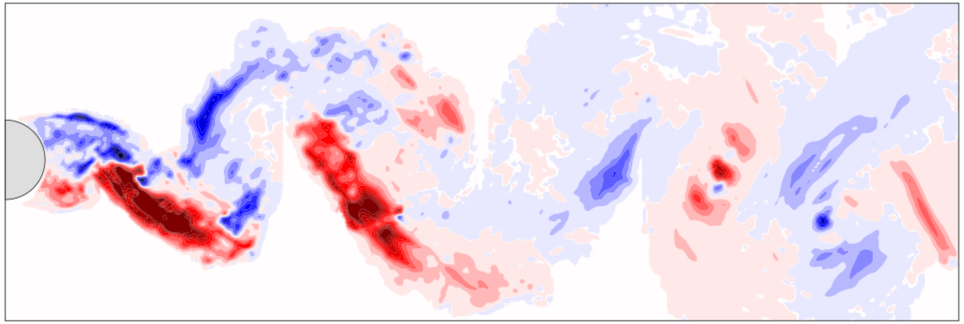
- Base-case parameters
- Best results

A-priori results: CNN can find high correlation coefficients





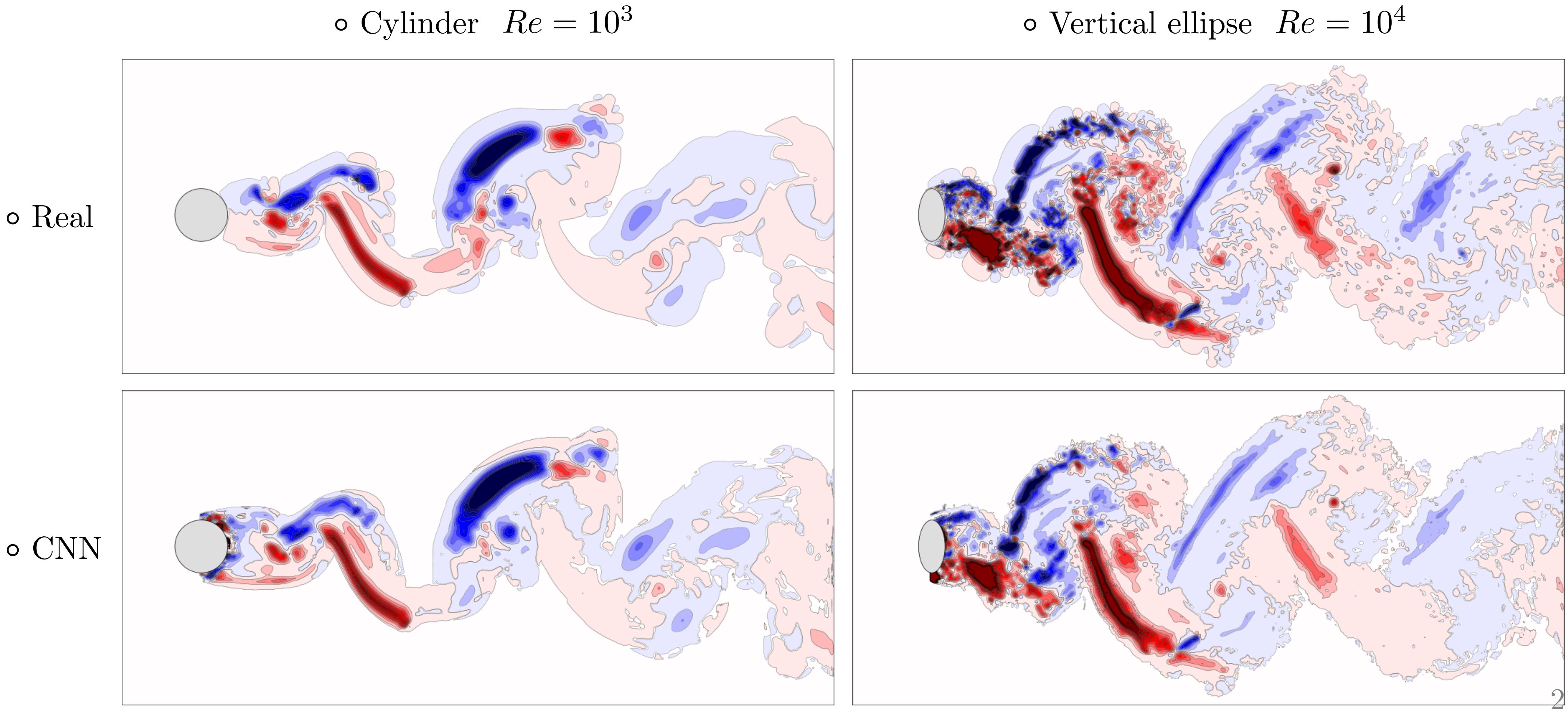
$\boldsymbol{\tau}^{r}_{12}$



$\boldsymbol{\tau}^{r,\mathrm{ML}}_{12}$

▶ movie

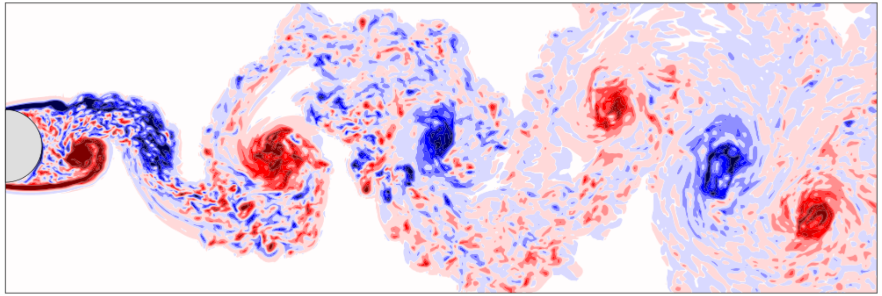
A-priori results: CNN model generalises to lower *Re* and geometries



A-posteriori results: SANS displays more 3-D-like structures at a fraction of its computational cost

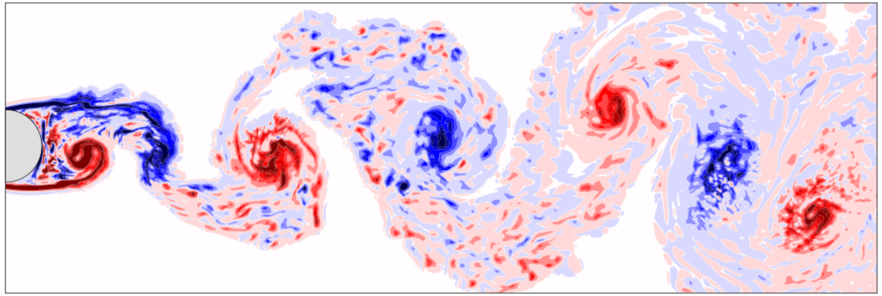
CPU time

$\langle \text{3-D} \rangle$ $(\Delta t^* = 2)$



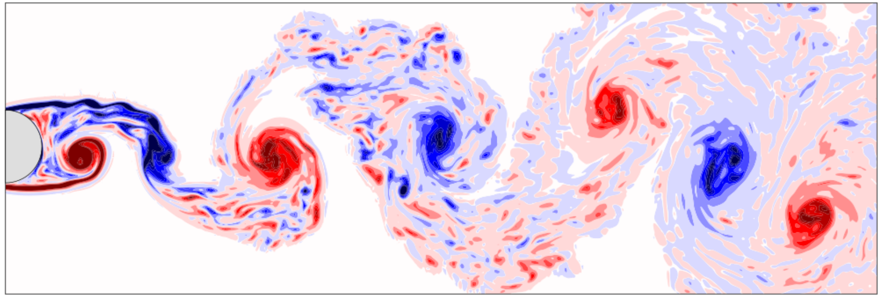
16 h.

SANS : 2-D + $\mathcal{S}^R$ $(\Delta t^* = 2)$



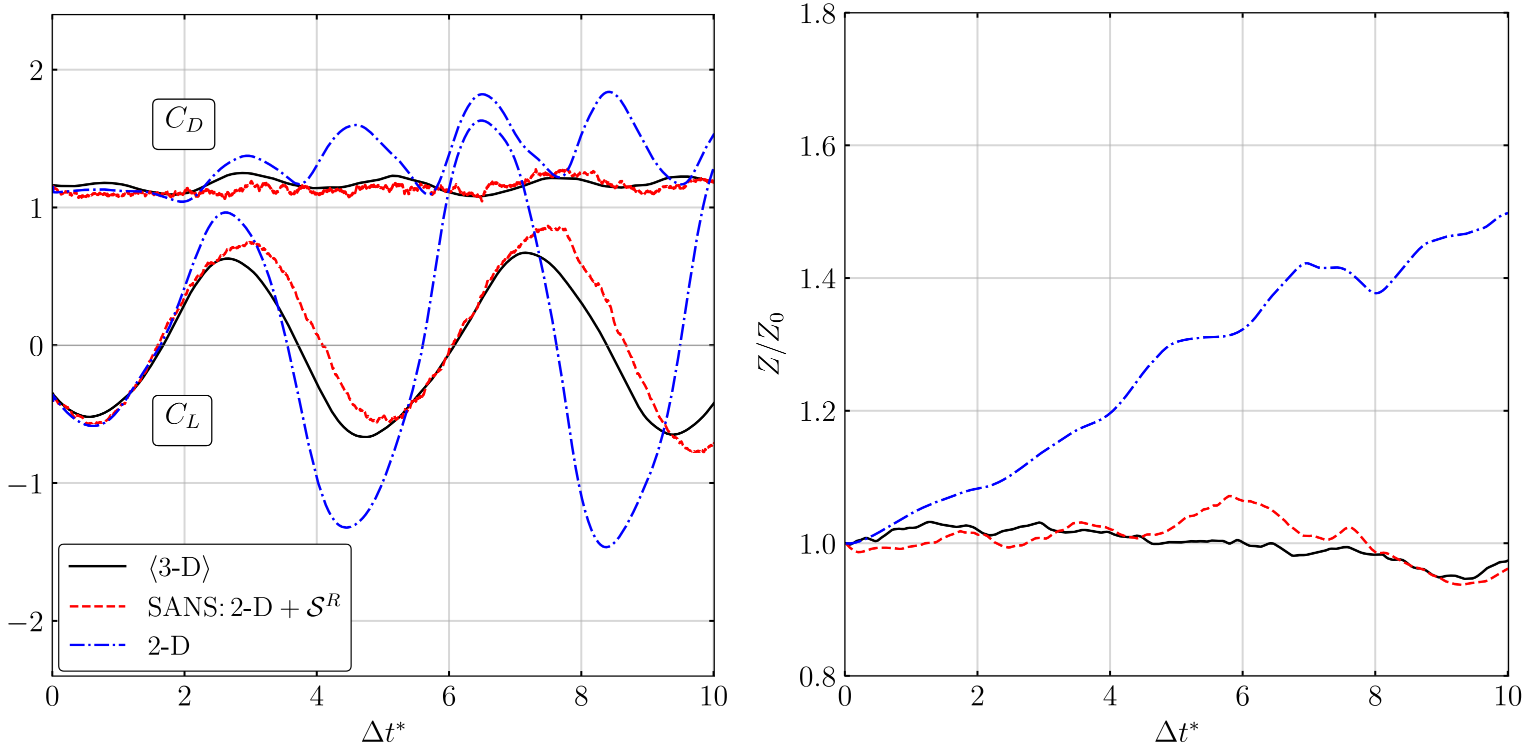
5 min.

2-D $(\Delta t^* = 2)$



2.5 min.

A-posteriori results: SANS offers 1-10% error vs 3-D simulations at 0.5% of their cost



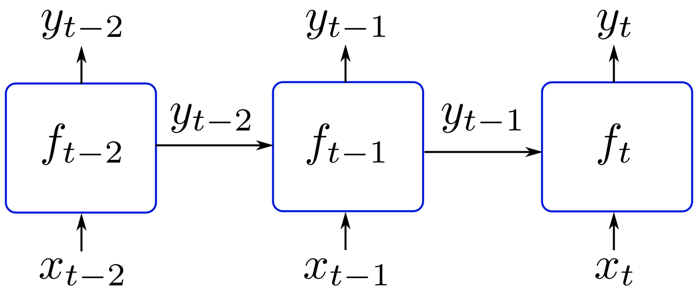
- ∘ Pure 2-D simulations rapidly change the flow dynamics leading to larger forces.

Limitations

- Strip-thoery method: flow angle, number of planes per structural mode.
- ML model: long temporal dynamics.

Future work

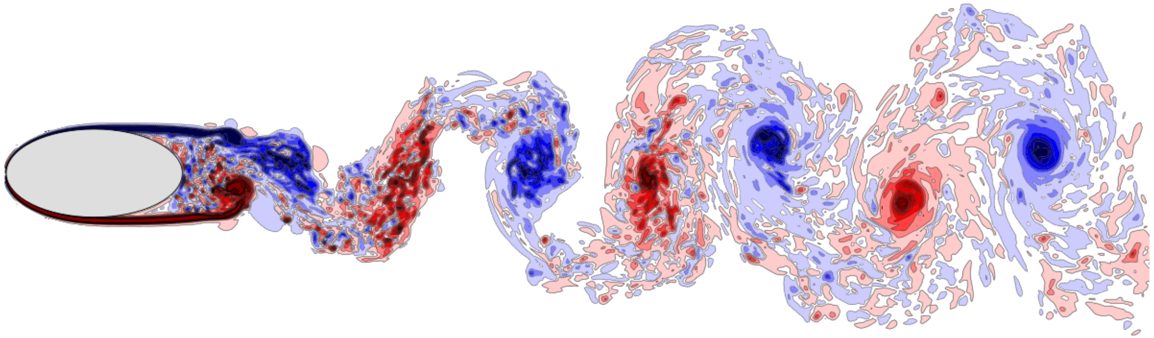
- Recurrent networks: learn temporal dynamics of spanwise stresses (e.g. LSTM).



- Stochastic modelling: target invariant metrics during training.

Remarks

- 2-D simulations over-predict the forces induced to the cylinder, and the minimum span to sustain 3-D turbulence is the Mode B instability wavelength.
- Adding the SANS stresses in a 2-D simulation recovers the spanwise-averaged flow solution.
- Modelling the SANS stresses with a CNN yields 90% correlation w.r.t. target data.
- A-posteriori results show 1-10% error vs 3-D simulations at 0.5% of the their cost.



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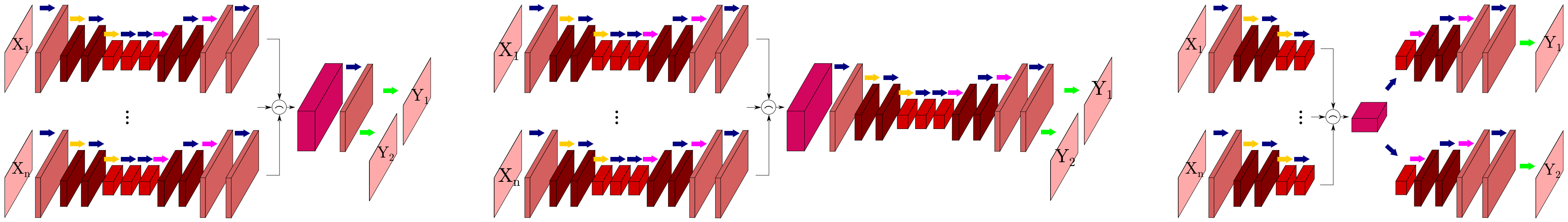
Model architecture and data normalization



M1 M2 M3

- All models provide a correlation coefficient (CC) > 70% between target and predicted fields.
- Models with more trainable parameters yield higher CC: M3 > M2 > M1.
- Normalizing the input data with its standard score helps speeding up convergence.

Activation and loss function

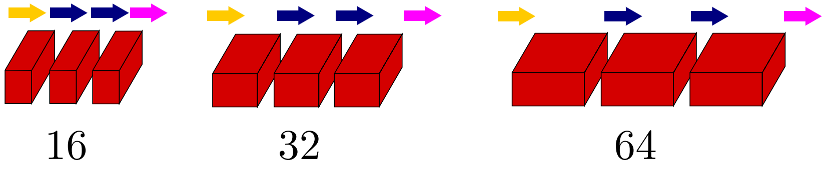
- ReLU outperfoms sigmoid and tanh activation functions providing higher CC.
- Loss functions:

$$\mathrm{SSE} = \sum_{i,j}^{n,m} \left(\mathrm{Y}_{i,j} - \mathrm{Y}_{i,j}^{\mathrm{ML}}\right)^2 \qquad \mathrm{SAE} = \sum_{i,j}^{n,m} \left|\mathrm{Y}_{i,j} - \mathrm{Y}_{i,j}^{\mathrm{ML}}\right|$$

SSE outperforms SAE, possibly linked to SSE being smoother with no singularities (0).

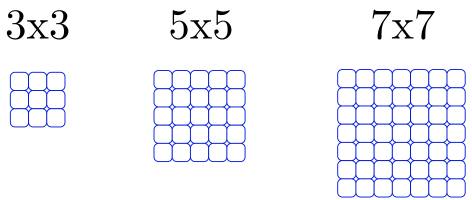
Number of filters and kernel size

- Maximum # filters at the auto-encoder latent space:



  - Faster convergence and higher CC can be achieved by increasing the number of filters.

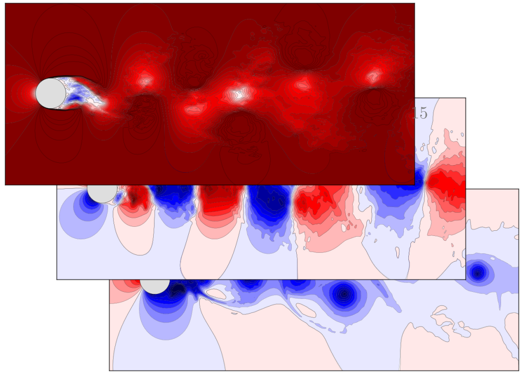
- Kernel size:



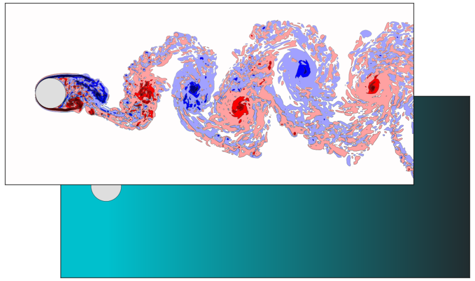
  - The 5x5 kernel yields higher CC to target data, balancing small- and large-scale detection.

Input set

$\{U, V, P\}$



$\{\Omega_z, d\}$



$\{\nabla \boldsymbol{U}, \nabla P\}$ (6 fields)

- The primitive quantities and gradients sets yield the highest CC (86%).
  - The CNN might be learning the spatial derivatives on its own.

- The vorticity + distance function converges to CC=50%.